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The Unitary Gauge Fermion in the Chiral Schwinger Model

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Abstract

The behaviour of the fermion in the chiral Schwinger model is studied by considering two different fermion operators which appear in the original Jackiw-Rajaraman formulation. These are understood as referring to different Hilbert spaces and therefore to different models. It is emphasized that only one of the fermion operators can be looked upon as derived by fixing the unitary gauge in the gauge invariant formulation of the chiral Schwinger model. This fermion shares in several respects the properties of the fermion in the Landau gauge, in particular, one cannot deduce the existence of asymptotically free fermion states.

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1. Introduction

The chiral Schwinger model (CSM), i.e. a chiral fermion coupled to a gauge field in (1+1) dimensions, was proposed by Jackiw and Rajaraman [1] as an example of an anomalous gauge theory which can nevertheless be quantized in a consistent manner. The divergence of the current coupling to the gauge field is not identical to zero, but, after solving the model and inserting the solution for the gauge field into the anomaly, it vanishes. The constraints of this model were shown to be second class [2,3], exhibiting the lack of gauge invariance. We will call this formulation "anomalous" throughout this paper. The CSM was intensively studied during the last years (see e.g. Refs. [2-11,15-24]), and soon it was shown that the proposal of Faddeev and Shatashvili [14] to quantize an anomalous theory by introducing a Wess-Zumino term can be realized here naturally by adjusting the Faddeev-Popov procedure [11-13]. Thereby, one gains a gauge invariant formulation with only first class constraints, which seems to include the original anomalous formulated model as a special gauge, the unitary gauge [7]. In this formulation it was shown by different methods that the CSM has properties similar to the vector Schwinger Model (SM) [18-22], in particular, the chiral charge is shielded just as the vector charge of the SM [18,22,36]. That does not explicitly contradict the earlier result for the CSM in its anomalous formulation, where asymptotic fermions are shown to exist, because the corresponding operator does not carry chiral charge [6]. However, these states are absent in the SM, and the proposed equivalence between the chiral model and the related vector model as well as the inclusion of the anomalous formulation as a special gauge of the gauge invariant formulation becomes dubious when we look at the fermions.

As the specific fermion considered in the anomalous formulation seems to be a composite object and exhibits so-called noncanonical features [21,24], it deserves closer examination whether it is the right object to deduce the existence of asymptotic fermions in the CSM. Within the gauge invariant formulation, the usual way to decide on this point would be to look at a gauge invariant propagator of the fermion. However, there were proposed different ones which led to contrary interpretations [22,23].

Clarifying this situation is important with respect to the aim to quantize a theory consisting of chiral fermions coupled to gauge fields in (1+3) dimensions. One cannot solve these models by bosonization, i.e. without caring for the fermions [1]. The knowledge of the fermions seems to depend on the choice between the two formulations mentioned above, and therefore, a criterion for judging them may be derived from the CSM.

The purpose of the present paper is to show that within a gauge invariant formulated CSM there is no (non-composite) fermion operator whose propagator becomes a free one in the asymptotic region. The unitary gauge fermion of this formulation coincides with one of the two possible fermion operators in the anomalous formulation, the *non*renormalized one in contrast to the renormalized one. In the anomalous formulation, the latter is the sounder object; it leads to the conclusion that there are asymptotic fermion states [6]. However, within the gauge invariant formulation the unitary fermion must not be renormalized, because renormalization may be seen as a change of the model. In fact, this nonrenormalized unitary fermion endowed with a suitable regularization shares the short distance behaviour of the Landau gauge fermion, which has no need for renormalization.

The Lagrangian of the CSM is [1]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ + e\sqrt{\pi}A\!\!\!/(1-\gamma^5)\right)\psi \quad . \tag{1}$$

We disregard the noninteracting right-handed sector in this paper; in the path-integral approach its importance is restricted to technical questions, leaving the right-handed fermion uncoupled after quantization [15,18]. The quantum field theory corresponding to $\mathcal{L}$ is defined by its Wightman functions

$$< \psi_L(x_1)\ldots\psi_L(x_n)\bar{\psi}_L(y_1)\ldots\bar{\psi}_L(y_n)A\!\!\!/(z_1)\ldots A\!\!\!/(z_m) > \quad , \tag{2}$$

and the algebra of the operators ψ_L, $\bar{\psi}_L$ and $A\!\!\!/$ is then given on a Hilbert space $\mathcal{H}$, which in principle is fixed by these functions via the reconstruction theorem [47]. This $\mathcal{H}$ need not be the direct product of the Fock spaces of the basic fields used to solve the CSM [1], which in the anomalous formulation would be a free massive boson field σ and a free massless one h, i.e. $\mathcal{H}_{dir} = \mathcal{H}_\sigma \otimes \mathcal{H}_h$.

Recently, Morchio et al. showed that the SM constructed on a Hilbert space of the kind $\mathcal{H}_{dir}$ contains redundant features, as e.g. the so-called "bleached" (fermion) field [40], which are absent in the theory corresponding to $\mathcal{H}$ [27]. A long time ago, Schroer drew similar conclusions for a model consisting of a massive fermion coupled to a massless boson [28].

It will be demonstrated that the CSM also should be seen with respect to $\mathcal{H}$ rather than $\mathcal{H}_{dir}$, since only then the picture of the interacting $\psi_L - A_\mu$ - system survives quantization, the anomalous formulation is contained as the unitary gauge in the gauge invariant formulation, and the noncanonical features disappear.

These advantages are gained if one considers the nonrenormalized fermion operator in the anomalous formulation, contrary to the up to now exclusively studied renormalized fermion operator.

This paper is structured as follows. In Sect. 2 we present the gauge invariant formulation of the CSM and write down the fermion propagators in the unitary and the Landau gauge. In Sect. 3 we study the derivation of the unitary gauge fermion operator from the Landau gauge operator by an operator gauge transformation. Section 4 contains a comparision between the CSM and an asymmetric SM, where the role played by the physical massless mode is stressed. In Sect. 5 the Dirac equation and the equal time commutator $[A_0(x), \psi_L(y)]_{ET}$ are considered for the renormalized and the nonrenormalized fermion in the anomalous formulation. A perturbation theory for the fermion propagator is presented in Sect. 6 to give a heuristic argument for the difference between $\mathcal{H}$ and $\mathcal{H}_{dir}$, and Sect. 7 contains a summary of the results and a discussion.

2. Gauge Invariant Formulation of the CSM

In Ref. [11] Harada and Tsutsui derived for the CSM the generating functional

$$Z_{HT} = \int d\psi d\bar{\psi}\mathcal{D}A_\mu dg \; \exp\{i\int d^2x\mathcal{L}[\psi,\bar{\psi},A] + i\alpha_1[A,g^{-1}]\} \tag{3}$$

[1] our conventions are: $\gamma^0 = \sigma^1$, $\gamma^1 = i\sigma^2$, $\gamma^5 = -\gamma^0\gamma^1$, $\epsilon^{01} = -\epsilon_{01} = 1$; $\psi = (\psi_R, \psi_L)^T$

by adapting the Faddeev-Popov procedure to the case of a chiral model. It is gauge invariant, because the noninvariance of the fermionic measure is compensated by the change of the Wess-Zumino term $\alpha_1[A, g^{-1}]$ under gauge transformations. g is an element of the gauge symmetry group. The gauge boson measure reads $\mathcal{D}A_\mu = dA_\mu \Delta_f[A]\delta(f(A))$. Integrating out the fermion, one arrives at

$$Z_{HT} = \int dA_\mu dB d\phi d\theta \; \exp\{i\int d^2x(\mathcal{L}_{eff} + \mathcal{L}_{GF})\}\} \quad , \tag{4}$$

with

$$\begin{aligned}\mathcal{L}_{eff} = \; & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}ae^2A^\mu_\mu \\ & -\frac{1}{2}\phi\Box\phi - e\phi\partial^\mu(g_{\mu\nu} + \epsilon_{\mu\nu})A^\nu \\ & -\frac{1}{2}(a-1)\theta\Box\theta - e\theta\partial^\mu[(a-1)g_{\mu\nu} - \epsilon_{\mu\nu}]A^\nu \quad .\end{aligned} \tag{5}$$

The scalar field ϕ was introduced to get rid of a nonlocal term after integrating out the fermion fields and θ is the Wess-Zumino field. Fixing the unitary gauge, $\mathcal{L}_{GF} = B\theta$, we derive by standard methods (see e.g. Ref. [43]) the photon propagator [1]

$$\mathcal{G}^{\mu\nu} = \frac{i}{k^2 - m^2}\left(-g^{\mu\nu} + \frac{1}{a-1}\frac{1}{k^2}\left[\left(\frac{k^2}{e^2} - 2\right)k^\mu k^\nu - \epsilon^{\mu\alpha}k_\alpha k^\nu - \epsilon^{\nu\alpha}k_\alpha k^\mu\right]\right) \quad , \tag{6}$$

and thereby the fermion propagator [6]

$$S^n_L(x-y) = S^{free}_L(x-y)\; \exp\left[-\frac{4\pi i}{a-1}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m^2 + i0}(e^{-ik(x-y)} - 1)\right] \quad . \tag{7}$$

a is the well-known regularization parameter reflecting an ambiguity linked to the absence of gauge invariance [1]. We use $a > 1$ and $m^2 = e^2a^2/(a-1)$. $S^n_L(x-y)$ is a nonrenormalized propagator, in so far as it contains the ill-defined factor

$$\mathcal{Z} = \exp\left[+\frac{4\pi i}{a-1}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m^2 + i0}\right] = \exp\left[+\frac{4\pi i}{a-1}\Delta_F(0; m^2)\right] \quad . \tag{8}$$

Expression (8) is not even defined as a generalized function, because it gives rise to $\delta(x^2)$-singularity, which is disastrous in (1+1) dimensions. Nevertheless, it will be shown in Sects. 5 and 6 that by different regularization procedures one can deduce some interesting results from the propagator (7) and the fermion operator corresponding to it.

On the other hand, it suggests itself to renormalize the propagator by cancelling $\mathcal{Z}$ [6,8]. This, however, may change the Hilbert space, i.e. the model, in a nontrivial way. We will find several indications that this is exactly what happens. $\mathcal{Z}^{-1}S^n_L$ becomes a free propagator in the asymptotic limit $(-p^2 \to 0)$ and shows anomalous scaling behaviour in the short distance limit $(-p^2 \to \infty)$ [6].

For the Landau gauge, $\mathcal{L}_{GF} = B\partial_\mu A^\mu$, we have

$$\mathcal{G}^{\mu\nu}_{Landau} = \frac{i}{k^2 - m^2}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2}\right) \tag{9}$$

and

$$S_L^{Landau}(x-y) = S_L^{free}(x-y)$$
$$\times \exp\left[-\frac{4\pi i(a-1)}{a^2}\int\frac{d^2k}{(2\pi)^2}\left(\frac{1}{k^2-m^2+i0}-\frac{1}{k^2+i0}\right)\left(e^{-ik(x-y)}-1\right)\right] \quad , \tag{10}$$

respectively [38]. Contrary to expression (7), the "1" in the exponent of (10) plays the role of a normalization constant, which fixes an otherwise necessary infrared cutoff μ to the boson mass m, just as in the SM [40]. Obviously, expression (10) becomes a free propagator in the short distance limit, but not in the asymptotic limit.

Now we report the solution of the CSM in the Landau gauge [17,18]. The effective action (5) describes a model where the gauge field A_μ couples to the conserved current

$$J^\mu \equiv J_L^\mu + J_\theta^\mu = e^2 aA^\mu + e(g^{\mu\nu}-\epsilon^{\mu\nu})\partial_\nu\phi + e[(a-1)g^{\mu\nu}+\epsilon^{\mu\nu}]\partial_\nu\theta \quad , \tag{11}$$

so that no inconsistency arises with respect to the Maxwell equation [18]

$$\partial_\mu F^{\mu\nu} = -J^\nu + \partial^\nu B \quad . \tag{12}$$

The Dirac procedure for quantizing constraint systems can be carried out easily, as only first class constraints appear. The solution for A_μ, ϕ, and θ can be expressed by four basic fields, the massive one F, and the massless fields H, X, and B [17,18]:

$$\begin{aligned} eA_\mu &= +\frac{a-1}{ea^2}\epsilon_{\mu\nu}\partial^\nu F + \frac{1}{a}\epsilon_{\mu\nu}\partial^\nu H + \partial_\mu X + \frac{1}{ea}\partial_\mu B \quad , \\ \phi &= -\frac{a-1}{ea^2}F + \frac{a-1}{a}H - X \quad , \\ \theta &= +\frac{1}{ea^2}F - \frac{1}{a}H - X \quad . \end{aligned} \tag{13}$$

The commutators of F, H, X and B are

$$\begin{aligned} [F(x),F(y)] &= im^2\Delta(x-y;m^2) \quad , \\ [H(x),H(y)] &= iD(x-y) \quad , \\ [X(x),B(y)] &= -ieD(x-y) \quad , \\ [X(x),X(y)] &= i/a\, D(x-y) \quad , \\ [B(x),B(y)] &= 0 \quad , \end{aligned} \tag{14}$$

all other commutators are equal to zero. The left-handed fermion operator can be calculated by demanding that it should be possible to reproduce the anomalous current

$$J_L^\mu = e\sqrt{\pi}\bar\psi(x)\gamma^\mu(\Gamma_V - \gamma^5\Gamma_A)\psi(x) = e^2 aA^\mu(x) + e(g^{\mu\nu}-\epsilon^{\mu\nu})\partial_\nu\phi(x) \tag{15}$$

when suitable phase factors Γ_V and Γ_A are used. This has been done by Miyake and Shizuya [18]; the result is

$$\psi_L(x) =: \exp\{i\sqrt{\pi}\left[2\frac{a-1}{ea^2}F(x) - \left(1-\frac{2}{a}\right)H(x) + \tilde H(x) + 2X(x) + \frac{1}{ea}\left(B(x)+\tilde B(x)\right)\right]\} : u, \tag{16}$$

where $u \equiv [m/(2\pi)]^{1/2}$ and the dual fields are defined by $\partial_\mu f = \epsilon_{\mu\nu}\partial^\nu \tilde f$. The normalization was chosen to match with the one used for the propagator (10), i.e. m is again identified with a necessary infrared cutoff. ψ_L has a vanishing equal time commutator with A_μ and the usual anticommutator $\{\psi_L(x),\psi_L^\dagger(0)\}_{ET} = \delta(x_1)$ [18].

3. Operator Gauge Transformation

Now we try to obtain the unitary fermion by an operator gauge transformation from the well-defined fermion ψ_L (16). This formal manipulation should be carried out with respect to the Hilbert space on which the operators act [30]. The transformation we are interested in is

$$(\Phi_1,\psi_L\Phi_2) \longrightarrow (\Phi_1, e^{i\sqrt{\pi}\lambda}\psi_L\Phi_2) \equiv (\Phi_1,\zeta_L\Phi_2) \quad , \tag{17}$$

where $\Phi_{1,2}$ are state vectors of the physical Hilbert space $\mathcal{H}_{phys}$. We demand that ζ_L is a physical operator, i.e. it should have a vanishing commutator with the operator which defines $\mathcal{H}_{phys}$ by the subsidiary condition

$$\Lambda^{\mu\,(+)}(x)\Phi = 0 \quad . \tag{18}$$

Λ^μ is derived by inserting the solution (13) for A_μ and ϕ in the Maxwell equation following from (1):

$$\partial_\mu F^{\mu\nu} = -J_L^\nu \quad . \tag{19}$$

We arrive at

$$\Lambda^\mu(x) = -\partial^\mu B(x) + e(a-1)\partial^\mu\theta(x) + e\epsilon^{\mu\nu}\partial_\nu\theta(x) \quad . \tag{20}$$

In this section we suppose that $m^2 \ll 1$, i.e. $\Delta(x;m^2) \simeq D(x)$. $F_{\mu\nu}$ is an example of a physical operator, as it commutes with Λ^μ. To fulfill our demand for a physical unitary fermion operator ζ_L, we transform with $\lambda = 2[e(a-1)]^{-1}\partial_\mu\Box^{-1}\Lambda^\mu$. Then we have

$$(\Phi_1, e^{i\sqrt{\pi}\lambda}\psi_L\Phi_2) \equiv (\Phi_1,\zeta_L\Phi_2) = (\Phi_1,\psi_L\Phi_2) \quad , \tag{21}$$

where

$$\zeta_L(x) \equiv \rho_L^n(x) = \hat{\mathcal{Z}}^{\frac{1}{2}} : \exp\{i\sqrt{\pi}\left[\frac{2}{ea}P(x) - \left(h(x)-\tilde h(x)\right)\right]\} : u \quad , \tag{22}$$

with

$$P \equiv F - \frac{1}{a-1}B, \quad h \equiv H + \frac{1}{a}B \quad . \tag{23}$$

One can easily check that

$$[\rho_L^n(x), \Lambda^{\mu\,(+)}(y)] = 0 \quad . \tag{24}$$

P and h are physical fields and the operator ρ_L^n corresponds to the nonrenormalized fermion propagator S_L^n, (7). $\hat{\mathcal{Z}}$ is equivalent to $\mathcal{Z}$, (8), apart from a change of the analytic structure

$$\hat{\mathcal{Z}} = \exp\left[+\frac{4\pi i}{a-1}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2-m^2+i0k_0}\right] = \exp\left[+\frac{4\pi i}{a-1}\Delta^{(+)}(0;m^2)\right] \quad . \tag{25}$$

The fermion operator ρ_L that reproduces the renormalized propagator $\mathcal{Z}^{-1}S_L^n$ is formally reached from ψ_L by using the transformation prescription $\rho_L =: exp\{i\sqrt{\pi}\bar\lambda\}\psi_L$: with $\bar\lambda = 2\theta$ [24]. However, even if one abstracts from the role played by the B-field this transformation does not lead to a physical operator on the physical Hilbert space defined by means of

$$\bar\Lambda^\mu(x) = e(a-1)\partial^\mu\theta(x) + e\epsilon^{\mu\nu}\partial_\nu\theta(x) \quad , \tag{26}$$

since $[\rho_L(x), \bar\Lambda^{\mu\,(+)}(y)]$ does not vanish.

Things look different if the relation between the transformation function and the subsidiary condition is disregarded. Then one can take the conventional condition [17,18]

$$B^{(+)}(x)\Phi = 0 \qquad (27)$$

to identify the physical Hilbert space and transform the Landau gauge fermion with λ leading to ρ_L, which is physical on $\mathcal{H}_{phys} = \mathcal{H}_{dir} = \mathcal{H}_F \otimes \mathcal{H}_H \otimes \mathcal{H}_B$. However, the equality in (21) does not hold for $\zeta_L = \rho_L$, so that the Landau gauge fermion has vacuum expectation values on its physical Hilbert space that are different from those of the fermion operator ρ_L, which should not be called "unitary gauge" fermion.

The vacuum structure in the SM was studied by Raina and Wanders [42], following an idea of Rothe and Swieca [41] (see also Ref. [27]). In our formulation A^μ plays the role of the longitudinal current, whose positive frequency part defines the physical Hilbert space. There should be an operator generating "large" gauge transformations in the Landau gauge just as the one constructed in Refs. [41,42], and therefore it is reasonable to believe that the degeneration of the vacuum of the physical Hilbert space $\mathcal{H}_{phys} = \mathcal{H} \neq \mathcal{H}_{dir}$ defined by (18,20) appears here as in the SM.

The second possible transformation mentioned above on a physical Hilbert space $\mathcal{H}_{dir}$ different from our $\mathcal{H}$ may well lead to a non-degenerate vacuum, cluster property, and asymptotic fermion states [6,9]. But, if we look at the breakdown of relation (21), one has to say that the transformation does not just result in a different gauge; on the contrary, it changes the physical content of the theory.

4. The Relation of the CSM to an Asymmetric SM

In Ref. [4] Halliday et al. compared two methods to avoid anomalies, the introduction of a Wess-Zumino term and the usual one, i.e. the addition of new fermions. This relation has been further studied recently [20,21]. The Wess-Zumino term is fermionized and gives rise to one left-handed and one right-handed fermion.

From this point of view, the gauge invariant formulated CSM looks like an asymmetric SM (ASM) [33], with one right-handed and two left-handed fermions. In fact, it is well-known that this ASM has the same physical spectrum as the CSM, i.e. one massive and one massless boson. In the present section we want to stress that the massless boson plays a different role in the CSM and the ASM, respectively, when the anomalous formulation of the CSM is chosen.

The ASM is given by the Lagrangian

$$\mathcal{L}^{ASM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi_{L1}\left(i\partial\!\!\!/ + e_{L1}\sqrt{\pi}A\!\!\!/\right)\psi_{L1} + \bar\psi_{L2}\left(i\partial\!\!\!/ + e_{L2}\sqrt{\pi}A\!\!\!/\right)\psi_{L2} + \bar\psi_R\left(i\partial\!\!\!/ + e_R\sqrt{\pi}A\!\!\!/\right)\psi_R, \qquad (28)$$

where $e_R^2 = e_{L1}^2 + e_{L2}^2$ must be fulfilled to guarantee that no anomaly emerges [33]. With the definitions $e_R = ea/\sqrt{a-1}, e_{L1} = 2e$, and $e_{L2} = e(2-a)/\sqrt{a-1}$ we could connect this ASM to the CSM ($m^2 = e_R^2$) [20]; however, we will see that the relation can be maintained only when we make an unconventional choice for the fermions in the ASM. The fermions are integrated out and nonlocal terms are traded for three scalar fields, which are not independent degrees of freedom due to the relation between the coupling constants [33]. Then one may

recast the bosonic Lagrangian equivalent to (28) into a form where it becomes obvious that only one of the two necessary scalar fields couples to the gauge field.

$$\mathcal{L}^{ASM}_{boson} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} -\frac{1}{2}\varphi\Box\varphi - \frac{1}{2}\omega\Box\omega + 2e_R\omega\epsilon_{\mu\nu}\partial^\mu A^\nu \quad . \qquad (29)$$

The field ω represents the massive degree of freedom. The massless one decouples and has no influence on the dynamics of the ASM, which is equal to the one of the SM [34].

What does that mean to the left-handed unitary fermion propagator? Of course, the answer depends on the fermion we consider. Let us take the charges given above to relate the ASM to the CSM. Although in the ASM there is no need for a Wess-Zumino term, one may mimick the procedure for the CSM, i.e. a regularization parameter a and a Wess-Zumino term with the coefficient $(a-1)$ is introduced, so that for $a = 1$ an effective Lagrangian without gauge fixing would result. We choose $a = 2$, i.e. $m^2 = 4e^2$. The photon propagator is given by

$$G^{\mu\nu}_{ASM} = \frac{i}{k^2 - m^2}\left(-g^{\mu\nu} + \frac{1}{2e^2}\left(-k^2 + 2e^2\right)\frac{k^\mu k^\nu}{k^2}\right) \quad , \qquad (30)$$

and the fermion propagators read

$$S^{ASM,n}_{Li,R}(x-y) = S^{free}_{L,R}(x-y) \times \exp\left[-\frac{\pi i}{2}\left(\frac{e_{Li,R}}{e}\right)^2 \int \frac{d^2k}{(2\pi)^2}\left(\frac{1}{k^2 - m^2 + i0} - \frac{1}{k^2 + i0}\right)\left(e^{-ik(x-y)} - 1\right)\right] . \qquad (31)$$

For ψ_{L1} and ψ_R one has a propagator looking like the one of the SM in Landau gauge, and for ψ_{L2} a free propagator. Wagner stressed in Ref. [21] that the propagator $\mathcal{Z}^{-1}S_L^n$ (7) has to be looked at as the result of a composite object within the gauge invariant sector. Indeed, for $a = 2$ one may write $\exp\{i\sqrt{\pi}2\theta\}\psi_L = K\bar\chi^R_{WZ}\chi^L_{WZ}\psi_L$, with a renormalization constant K [21]. One can infer from (31) that this object really reproduces the left-handed propagator of the CSM; however, this is only true for the renormalized propagator. Therefore, we have no reason to consider ρ_L^n (22) as a composite object. The same is valid for ρ_L within the anomalous formulation [6].

The CSM is therefore different from other generalizations of the SM, where gauge invariance can be guaranteed by a suitable choice of the coupling constants. In contradistinction to the latter models, the massless mode in the CSM may have nontrivial physical effects, but only in the anomalous formulation. It gives e.g. rise to asymptotic fermions due to the renormalized operator ρ_L [6]. The existence of this anomalous formulation makes up the difference to all other Schwinger models. Only if we confine ourselves to the gauge invariant formulation of the CSM we can say that there is no generalized SM (CSM, ASM, multifermion SM, ...) with non-composite fermion operators giving rise to asymptotic fermions.

5. Dirac Equation

Now the Dirac equation which describes the interaction between the fermion and the gauge boson is studied. We know that the Landau gauge fermion ψ_L satisfies the Dirac equation [18]

$$i\partial\!\!\!/\psi_L(x) + 2e\sqrt{\pi} : (A\!\!\!/\psi_L)(x) := 0 \quad , \tag{32}$$

where the normal ordering can be replaced by [31]

$$: (A\!\!\!/\psi_L)(x) := \frac{1}{2}\gamma^\mu \lim_{\epsilon\to 0}[A_\mu(x+\epsilon)\psi_L(x) + \psi_L(x)A_\mu(x-\epsilon)] \quad . \tag{33}$$

How about the fermion in the anomalous formulation? The renormalized fermion is rewritten as (see (22))

$$\rho_L(x) \equiv \hat{Z}^{-1}\rho_L^n(x) =: \exp[i\sqrt{\pi}\frac{2}{ea}P(x)] : \rho_L^{free}(x) \quad . \tag{34}$$

In what follows, V_μ is the photon field which is derived from (13) by using the transformation $V_\mu = A_\mu + [e^2(a-1)]^{-1}\,\partial_\mu\partial_\nu\Box^{-1}\Lambda^\nu$. The first term of (32) leads to

$$i\partial\!\!\!/\rho_L(x) = -\frac{2\sqrt{\pi}}{ea} : \partial\!\!\!/P(x)\exp[i\sqrt{\pi}\frac{2}{ea}P(x)] : \rho_L^{free}(x) \quad . \tag{35}$$

For the second we use

$$V\!\!\!/(1-\gamma^5) = \frac{1}{e^2a}\partial\!\!\!/P(1-\gamma^5) \quad , \tag{36}$$

i.e.

$$\begin{aligned} 2e\sqrt{\pi} : (V\!\!\!/\rho_L)(x) := \; & \frac{\sqrt{\pi}}{ea}\gamma^\mu \lim_{\epsilon\to 0}\Big(\partial_\mu P(x+\epsilon) : \exp[i\sqrt{\pi}\frac{2}{ea}P(x)] : \\ & + : \exp[i\sqrt{\pi}\frac{2}{ea}P(x)] : \partial_\mu P(x-\epsilon)\Big)\,\rho_L^{free}(x) \quad . \end{aligned} \tag{37}$$

This is recast with the help of the relations

$$\begin{aligned} &A : e^B :=: (A + [A^{(+)}, B^{(-)}])e^B : \quad , \\ &[A, e^B] = [A, B]e^B \quad . \end{aligned}$$

We come out with

$$\begin{aligned} 2e\sqrt{\pi} : (V\!\!\!/\rho_L)(x) &:= -i\partial\!\!\!/\rho_L(x) + \frac{2\pi i}{e^2a^2}\lim_{\epsilon\to 0} G(x;\epsilon)\rho_L(x) \quad , \\ G(x;\epsilon) &= [\partial\!\!\!/P^{(+)}(x+\epsilon), P^{(-)}(x)] - [\partial\!\!\!/P^{(-)}(x-\epsilon), P^{(+)}(x)] \quad . \end{aligned} \tag{38}$$

The $G(x;\epsilon)$-term prevents that ρ_L fulfills the Dirac equation (32). However, introducing the renormalization constant $\hat{Z}^{1/2}$ in a smeared form [30]

$$\rho_L^n(x) = \lim_{\epsilon\to 0}\rho_L^n(x;\epsilon) \equiv \lim_{\epsilon\to 0}\exp\{-\frac{2\pi}{e^2a^2}[P^{(+)}(x+\epsilon), P^{(-)}(x)]\}\rho_L(x) \quad , \tag{39}$$

one finds that $i\partial\!\!\!/\rho_L^n$ produces an extra term:

$$\begin{aligned} i\partial\!\!\!/\rho_L^n(x) &= i\hat{Z}^{\frac{1}{2}}\partial\!\!\!/\rho_L(x) - \frac{2\pi i}{e^2a^2}\lim_{\epsilon\to 0}\partial\!\!\!/[P^{(+)}(x+\epsilon), P^{(-)}(x)]\rho_L^n(x) \\ &= i\hat{Z}^{\frac{1}{2}}\partial\!\!\!/\rho_L(x) - \frac{2\pi i}{e^2a^2}\lim_{\epsilon\to 0} G(x;\epsilon)\rho_L^n(x) = -2e\sqrt{\pi} : (V\!\!\!/\rho_L^n)(x) : \quad . \end{aligned} \tag{40}$$

Therefore, the unitary fermion ρ_L^n satisfies the Dirac equation (32).

Another possibility of checking on the persistance of the fermion gauge boson picture after quantization is to look at their kinematic independence, i.e.

$$[\eta_L(x), V\!\!\!/(y)]_{ET} = 0 \quad . \tag{41}$$

One can infer from (34) and (36) that this relation is not fulfilled for $\eta_L \equiv \rho_L$ due to the V_0-component [24]. However, using ρ_L^n in its smeared form (39) one has

$$[\rho_L^n(x), V_0(y)]_{ET} = \lim_{\epsilon_1\to 0}\left(\frac{2i\sqrt{\pi}}{a-1}\right)\delta(x_1 - y_1)[\rho_L^n(x;\epsilon)]_{\epsilon_0=0} = 0 \quad , \tag{42}$$

because while $\lim_{\epsilon\to 0}\exp[+\frac{2\pi i}{a-1}\Delta^{(+)}(\epsilon;m^2)]$ is ill-defined we have

$$\begin{aligned} &\lim_{\epsilon_1\to 0}\exp[+\frac{2\pi i}{a-1}\Delta^{(+)}(\epsilon;m^2)]_{\epsilon_0=0} \\ &= \lim_{\epsilon_1\to 0}\exp\left[\left(+\frac{2\pi}{a-1}\right)\left(+\frac{1}{4\pi}\right)\left(\ln(m^2|\epsilon^2|) - \frac{i}{4}\mathrm{sign}(\epsilon_0)\Theta(\epsilon^2)\right)\right]_{\epsilon_0=0} = 0 \quad . \end{aligned} \tag{43}$$

It should also be mentioned that contrary to ρ_L the nonrenormalized operator ρ_L^n , (39), has the same anticommutator as the Landau gauge fermion : $\{\rho_L^n(x), \rho_L^{n+}(0)\}_{ET} = \delta(x_1)$.

Here it has to be emphasized that the noncanonical features [24] of the renormalized "unitary gauge" version are nothing unusual. In fact, the same features appear in (1+3) dimensional QED when the Coulomb gauge is chosen [45].[‡]

In the next section we once again demonstrate that it is the nonrenormalized fermion ρ_L^n which is adequate to the picture of a fermion interacting with a gauge boson.

6. Perturbation Theory

In this section we want to present an approximate perturbative derivation of the unitary fermion propagator (7). This seems to be impossible, as the ϵ^{-2}-pole of the exact photon propagator (6) indicates that it has no free analogue (with $m^2 = 0$), which should be used for the quantum corrections of the free fermion propagator. However, we will now show that even for the perturbative derivation of the Landau gauge fermion propagator (10) one has to use the *exact* photon propagator (9), instead of the free one. S_L^{Landau} agrees with the fermion propagator of the SM in the Landau gauge when a is set equal to 2 [18-22,39]. Expanding (10) with respect to m^2 and Fourier transforming by convolutions we arrive at

$$\begin{aligned} \tilde{S}_L^{Landau}(p) = i\frac{p\!\!\!/}{p^2}\; & \left[1 - \frac{a-1}{a^2}\frac{m^2}{p^2} + \left(\frac{a-1}{a^2}\right)^2\frac{m^4}{p^4} + 2\left(\frac{a-1}{a^2}\right)^2\frac{m^4}{p^4}\ln\left(\frac{m^2}{-p^2}\right)\right. \\ & \left. + O\left(\frac{m^6}{p^6}\ln\left(\frac{m^2}{-p^2}\right)\right)\right] \quad , \end{aligned} \tag{44}$$

where standard formulas of the dimensional regularization method are used to calculate the convolutions. On the other hand, taking the exact photon propagator (9) and the vertex $ie\sqrt{\pi}$ it is possible, though tedious, to derive (44) by summing the contributions of the Feynman

[‡] I would like to thank D.Buchholz for bringing this fact to my attention.

diagrams of Fig.1, which has been done with the help of the algebraic programming system *Reduce* [48].

This shows that the perturbation theory in the Landau gauge CSM (or SM) has to be carried out with the exact photon propagator from the start, and not just to regularize diagram d., Fig.1, as proposed for the light cone gauge SM in Ref. [29]. Diagram b. would give no contribution to (44) when the free photon propagator is used and so the term of order m^2 could not be reproduced.

As the perturbation theoretical calculation of the fermion propagator has to be done with the exact photon propagator even in the Landau gauge, where a free photon propagator exists, it poses no problem that there is no free one in the unitary gauge, respectively the anomalous formulation. Expansion of S_L^n, (7), followed by a Fourier transformation gives the same result as summing the contributions of the diagrams a.-e., Fig. 1, with the high energy version of (6),

$$\hat{G}^{\mu\nu} = \frac{i}{k^2 - m^2}\frac{1}{a-1}\frac{k^\mu k^\nu}{e^2} \quad . \tag{45}$$

Using dimensional regularization in d dimensions with $\epsilon \equiv d - 2$ and γ being the Euler constant, we arrive at

$$\tilde{S}_L^n(p) = i\frac{\not p}{p^2}\Bigg[1 - \frac{1}{a-1}\left(\frac{2}{\epsilon} + \gamma + \ln\left(\frac{-p^2}{\mu^2}\right)\right) + \frac{1}{(a-1)^2}\left(\left(\ln\left(\frac{-p^2}{\mu^2}\right)\right)^2 + \gamma^2 + 2\frac{\gamma}{\epsilon} + 2\left(\gamma + \frac{1}{\epsilon}\right)\ln\left(\frac{-p^2}{\mu^2}\right) + \frac{2}{\epsilon^2} - \frac{1}{2}\frac{\pi^2}{6}\right) + \ldots\Bigg] \, , \tag{46}$$

where $\mu \equiv e/\hat{e}$ with the dimensionless coupling constant $\hat{e}$ was introduced. This result is divergent as $\mathcal{Z}$, (8), is involved. Of course, it is easy to renormalize the exact propagator (7), and the perturbation theory can be modified accordingly. This, however, has to be interpreted as a shift from $\mathcal{H}$ to $\mathcal{H}_{dir}$, i.e. as replacing the old model by a new one.

To clarify this point let us look at the fermion propagator $\mathcal{Z}^{-1}S_L^n$ (see expression (7)). Expanding the exponent in momentum space we have

$$\mathcal{Z}^{-1}\tilde{S}_L^n(p) = i\frac{\not p}{p^2} * \left[\delta(p) - \frac{4\pi i}{a-1}\frac{1}{p^2 - m^2 + i0} - \frac{16\pi^2}{2(a-1)^2}\frac{1}{p^2 - m^2 + i0} * \frac{1}{p^2 - m^2 + i0} + \ldots\right] \, . \tag{47}$$

When the convolution between the boson propagators is evaluated, one finds that the expression in square brackets has the same analytical structure as the Green's function of an interacting meson system [46]: a pole at $p^2 = m^2$ and a cut on the positive real axis starting at $p^2 = 4m^2$. Therefore, one can look at the renormalized propagator of the fermion in the anomalous formulation of the CSM as referring to the Hilbert space $\mathcal{H}_{dir}$, i.e. a space with asymptotically free massive and massless bosonic states, where the latter may be considered as fermions in this picture [6,32].

On the other hand, the picture of an interacting $\psi_L - A_\mu$ - system presupposed by perturbation theory seems to refer to the Hilbert space $\mathcal{H}$ and is realized by the nonrenormalized fermion propagator S_L^n, (7).

7. Summary and Discussion

In summarizing the results presented in this paper one can say that there are two approaches towards the CSM in the anomalous formulation, which imply a different behaviour of the fermion.

The first one, which implicitly refers to the Hilbert space $\mathcal{H}_{dir}$, leads to the conclusion that there are asymptotically free fermion states in the interacting sector [6]. The corresponding fermion operator $\rho_L \equiv \hat{\mathcal{Z}}^{-1/2}\rho_L^n$ cannot be seen as a special gauge version of the fermion belonging to the gauge invariant formulation in the sense of Refs. [11-13]. If this formulation is considered in its own right, it is surely reasonable. In particular, one does not have to deal with the renormalization constant on the right-hand side of relation (7). It suffers, however, from the above mentioned peculiarities in its short distance behaviour (see also Refs. [24,37]); the picture of independent quantum fields ψ_L and A_μ breaks down and therefore it is not a reasonable frame for comparison with the non-anomalous SM or for using a perturbation theory. These statements for the anomalous formulation are also valid for the gauge invariant formulation chosen in Ref. [23]. It differs from the gauge invariant formulation referred to throughout the present paper mainly by the choice of the physical relevant Green's functions. Although ρ_L is the correct choice for the fermion operator in the anomalous formulation, it has to be considered as a composite operator in the gauge invariant formulation [21,24]. The lacking relation to the SM becomes even more explicit in this approach by the impossibility of deriving Green's functions in ordinary vector theories along these lines.[§]

The second approach is linked to the original gauge invariant formulation of the CSM, where the usual Faddeev-Popov procedure for vector theories is contained as a limiting case. As demonstrated in Sect. 3, the anomalous formulation of the CSM [1] can be looked at as the $\theta = 0$ - gauge of the former not only on the bosonized level [7], but also when the fermion is considered. The unitary fermion was shown to be connected by an operator gauge transformation to the Laudau gauge fermion if $m^2 \ll 1$. Note that this unitary fermion operator is the *non*renormalized ρ_L^n, (22,39), which cannot be seen as a composite object (Sect. 4). We found that in several aspects it behaves just like the Landau gauge fermion if the ill-defined factor $\hat{\mathcal{Z}}^{1/2}$ is suitably regularized. The second approach towards the CSM seems not to correspond to a formulation on $\mathcal{H}_{dir}$, i.e. the Wightman functions (2) define their own specific Hilbert space $\mathcal{H}$ (see also [27,28]).

The unitary gauge fermion operator has no anomalous dimension in this formulation; it is introduced by renormalization [6], which obviously changes the short distance behaviour of the fermion propagator (7). At first sight it seems unlikely that an ultraviolet renormalization procedure can influence the existence of asymptotic fermion states. But, as we saw, it modifies the model referring to the generating functional (3). Stated differently, it signals that we are working with a composite object [21,24] of unclear physical relevance if we renormalize the fermion without changing the physical Hilbert space to $\mathcal{H}_{dir}$. Not a single effect of the left-handed fermions can be found if the correct gauge invariant formulation is chosen, i.e. the physical Hilbert space $\mathcal{H}$ and the physical fermion operator ρ_L^n.

From this point of view motivated by the gauge invariant formulation the anomalous formulation is just an unsuitable choice of gauge [44], in so far as one has to work with the regularized, nonrenormalized fermion operator ρ_L^n on $\mathcal{H}$. On may leave the gauge invariant approach by renormalizing in a correct way, i.e. choosing ρ_L on $\mathcal{H}_{dir}$, but one should not

[§]This was recognized by N.K. Falck.

renormalize the fermion without changing the physical Hilbert space to $\mathcal{H}_{dir}$.

Usually the unitary gauge is chosen to reveal the particle content of a model. In the CSM, however, other gauges lead to the same conclusion about the physical spectrum. It is not problematic to prove the existence of a massless degree of freedom, but to decide whether it gives rise to an unscreened fermion [32]. This is exactly what happens in the formulation with the fermion ρ_L based on $\mathcal{H}_{dir}$ or $\mathcal{H}$, in contradistinction to the correct gauge invariant formulation with ρ_L^n referring to $\mathcal{H}$. ρ_L gives rise to asymptotic fermions and restores cluster property [9], but in the gauge invariant formulation on $\mathcal{H}$ these properties may be due to the fact that it is a composite object. This in turn corresponds to the relation between the CSM and an asymmetric SM in that case. A special broken symmetry in this ASM can restore cluster property in spite of the vacuum degeneration [35]. Here one may think of the $U(1)_{WZ}$-symmetry in Refs. [20,24]. Once again it should be emphazised that ρ_L is a perfectly well-defined, non-composite fermion operator for the anomalous formulation on $\mathcal{H}_{dir}$ [6].

As the gauge invariant formulation of the CSM leads automatically to a model with canonical features where a perturbation theory can be used, this formulation should be preferred if one tries to quantize models with chiral fermions coupled to gauge fields. This is particularly important with respect to more than (1+1) dimensions, when no exact solution exists. In the gauge invariant formulation there is no difference between the behaviour of non-composite left-handed fermion operators in the vector SM and the CSM.

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Figure Caption

Fig. 1 : The lowest order Feynman diagrams for the calculation of the fermion propagator with the exact photon propagator

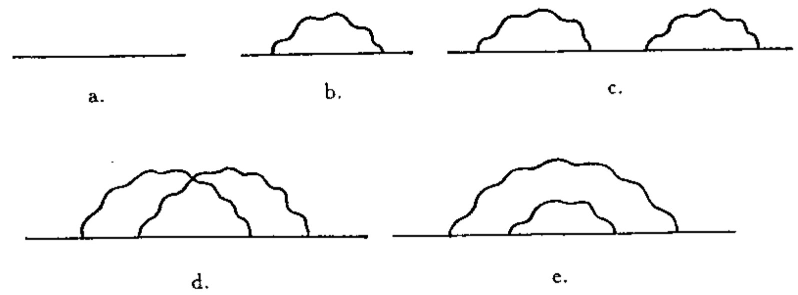

Figure 1