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Results on R in e^+e^- Annihilation

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Abstract

We discuss the determination of the Standard Model parameters α_s, M_Z, and $\sin^2\theta_W$ from the total hadronic cross section in e^+e^- annihilation. At the highest TRISTAN energies, the tail of the Z^0 resonance is increasing R already by 50%, which allows a direct measurement of M_Z. We find: $M_Z = 89.0(88.5, 88.1) \pm 1.0$ GeV for a top mass of 60 (120,180) GeV and fixed $\sin^2\theta_W$ =0.23. For the strong coupling constant we find: $\alpha_s(34^2\,GeV^2) = 0.143 \pm 0.015$, if $O(\alpha_s^3)$ QCD corrections are taken into account. We study the effect of changing the renormalization scale of α_s and find that the third order QCD corrections, which are larger than the second order ones for the usual scale $Q = \sqrt{s}$, are smaller than the second order contributions at other scales. A Monte Carlo study shows that with 3 energy scan points around the Z^0 resonance peak and a total of about 100 events, one can determine the Z^0 mass and width with an error of about 250 MeV, if one fits the absolute cross section instead of the shape of the resonance.

1 Introduction

Some time ago the CELLO Collaboration has determined α_s [1] from an analysis of all data available from the PEP and PETRA experiments on the total hadronic cross section in e^+e^- annihilation using a complete error correlation matrix. The importance of such a determination of α_s stems from the fact that it is independent of fragmentation models and that the results can be calculated reliably in QCD. These α_s determinations have been updated last year[2]-[6] with new experimental data and new theoretical progress, namely a third order calculation of the QCD corrections by S.G. Gorishny et al.[7]. The third order contribution turns out to be larger than the second order contribution in the commonly used $\overline{MS}$ scheme, if the strong coupling constant is evaluated at the scale $Q = \sqrt{s}$. However, this is not the typical scale for gluon radiation. Therefore we studied the scale dependence of α_s as determined from R and find that at other scales the third order contributions are smaller than the second order ones.

New experimental data has become available from TRISTAN experiments up to center of mass energies of 60.8 GeV. At these energies the tail of the Z^0 resonance is increasing R already by 50%, thus allowing a direct measurement of the Z^0-mass. Such a mass determination has been published by the AMY Collaboration[8] and presented at various conferences[9,10,11]. Quoted Z^0 masses range around 89 GeV (error $\pm$ 1.3 GeV), which is slightly lower than the results from $p\overline{p}$ experiments ($M_Z = 92.0 \pm 1.8$ GeV[12]). Recently D'Agostini, de Boer and Grindhammer[25] repeated the fit to R after applying the radiative corrections in a consistent way.

At present all data has been corrected only for first order QED corrections, i.e. at most one photon radiated in the initial state. If one worries about third order QCD corrections, one should also worry about higher order QED corrections. Recently the higher order QED initial state radiative corrections have been calculated[14], which allows for the first time a comparison between these exact calculations and other approximate methods. The report has been organized as follows: we first summarize the Standard Model (SM) formulae, then discuss the fitting method and results. We finish with a discussion of the higher order radiative corrections, a Monte Carlo study of expected errors on mass and width of data points around the Z^0 resonance, and a summary.

2 Standard Model Formulae

Here we summarize the formulas used in fitting the hadronic cross section. The normalized cross section R is defined as the ratio

$$R \equiv \frac{\sigma[e^+e^- \to \gamma, Z^0 \to hadrons]}{\sigma[e^+e^- \to \gamma \to \mu^+\mu^-]}$$

The $\mu^+\mu^-$ cross section is the lowest order pointlike QED cross section of massless spin $\frac{1}{2}$ particles, and is equal to $4\pi\alpha^2/3s$, where s is the square of the centre of mass energy, while the hadronic cross section has been defined before.

The total hadronic cross section at the Born level is given by the sum of the following contributions:

$$\sigma^{\gamma}_{had} = \frac{4\pi\alpha^2}{3s} r_{QCD} \sum_{q=1}^{5} e_e^2 e_q^2 \quad (1)$$

$$\sigma^{\gamma Z}_{had} = 8\pi\alpha r_{QCD} \frac{K(s - M_Z^2)}{(s - M_Z^2)^2 + s^2\Gamma_{tot}^2/M_Z^2} \sum_{q=1}^{5} e_e e_q v_e v_q \quad (2)$$

$$\sigma^{Z}_{had} = 12\,\pi r_{QCD} \frac{K^2\, s}{(s - M_Z^2)^2 + s^2\Gamma_{tot}^2/M_Z^2}(v_e^2 + a_e^2) \sum_{q=1}^{5} (v_q^2 + a_q^2) \quad (3)$$

The superscripts indicate the contribution from photon exchange, Z^0 exchange and their interference and the sum is taken over five quark flavours, thus assuming the top quark is too heavy; v and a represent the vector and axial vector couplings of the quarks (subscript q) and electrons (subscript e) and Γ_{tot} is the total width of the Z^0. For simplicity we have neglected small mass effects in the formulae above, but they have been taken into account in the analysis, using the formulae in Ref.[1]. The factor r_{QCD} represents the effect from gluon radiation and is given in the $\overline{MS}$ scheme by[7,13]:

$$r_{QCD} = 3\left[1 + \frac{\alpha_s}{\pi} + (1.986 - 0.115n_f)\left(\frac{\alpha_s}{\pi}\right)^2 + (70.985 - 1.2n_f - 0.005n_f^2)\left(\frac{\alpha_s}{\pi}\right)^3\right] \quad (4)$$

The factor 3 on the righthand side accounts for the color of the quarks.

The energy dependence (running) of α_s is given by the 3^{rd} order formula[15]:

$$\alpha_s(s) = \frac{4\pi}{\beta_0 \log(\frac{s}{\Lambda^2})}\left\{1 - \frac{\beta_1}{\beta_0^2}\frac{\log[\log(\frac{s}{\Lambda^2})]}{\log(\frac{s}{\Lambda^2})} + (\frac{\beta_1}{\beta_0^2})^2 \frac{1}{\log^2(\frac{s}{\Lambda^2})}\left[(\log[\log(\frac{s}{\Lambda^2})] - \frac{1}{2})^2 + \frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right]\right\} \quad (5)$$

with

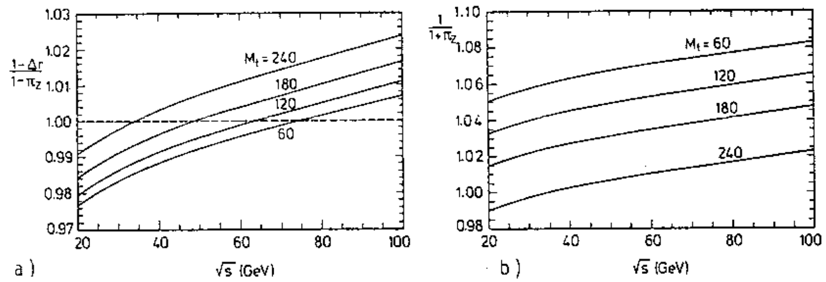

Figure 1: **Electroweak correction factors (κ_1 and κ_2 in text) for the different parametrizations of the Born cross section.**

$$\begin{aligned}\beta_0 &= 11 - \tfrac{2}{3}n_f \\ \beta_1 &= 2(51 - \tfrac{19}{3}n_f) \\ \beta_2 &= \tfrac{2857}{2} - \tfrac{5033}{18}n_f + \tfrac{325}{54}n_f^2\,.\end{aligned}$$

The constant K can be either defined as:

$$K_1 = \frac{\sqrt{2}G_F M_Z^2 \kappa_1}{48\pi} \quad (6)$$

or

$$K_2 = \frac{\alpha\kappa_2}{48\sin^2\theta_W\cos^2\theta_W} \quad (7)$$

Here G_F is the Fermi constant, which is well known from muon decay and $\sin^2\theta_W$ determines the electroweak mixing angle. In the definitions of K we have explicitely included the factor κ which represents the loop corrections to the Z^0 propagator. For example, practically all data from the PEP and PETRA experiments have been corrected with the LUND Monte Carlo program[16], which uses the radiative corrections from Berends et al.[17], thus including the loop corrections for the photon propagator, but not the loop corrections for the Z^0 propagator. In this case the formulae to be fitted to the data should include this κ-factor, which can be written as follows[18,19]:

$$\kappa_1 = \frac{1 - \Delta r}{1 + \Pi_Z(s)} \quad (8)$$

or

$$\kappa_2 = \frac{1}{1 + \Pi_Z(s)} \quad (9)$$

where

$$1 - \Delta r = \frac{\alpha(0)}{\alpha(M_W)} + \delta_r(M_t, M_H) \quad (10)$$

and

$$1 + \Pi_Z(s) = \frac{\alpha(0)}{\alpha(M_Z)} + \delta_\Pi(M_t, M_H, s) \quad (11)$$

Here Δr represents the electroweak corrections to the charged gauge boson exchange in muon decay and $\Pi_Z(s)$ represents the electroweak loop corrections to the neutral gauge boson exchange. One sees that the first term in both cases is given by the running of the QED coupling constant coming from the light fermion loops in the photon propagator (hence the indication of the scale in α). For top quark masses below the gauge boson masses this term is dominant in both expressions. E.g. for a top mass of 70 GeV Δr is about 7 % and 6% is coming from the first term alone. However, for a top mass of 230 GeV the latter term is as large as the first term, but of opposite sign, so the total correction Δr is about zero. $\Pi_Z(s)$ shows a similar behaviour, so that the ratio in κ_1 is much less dependent on the top mass and furthermore close to 1 (see Fig. 1a). This is the advantage of the parametrization with K_1: one can neglect the electroweak corrections to a large extent and the results are insensitive to the unknown top mass. This was the reason why in previous fits to data on R this parametrization has been used, e.g. to determine the strong coupling constant [1]. What was considered further as an advantage compared to the K_2 parametrization was the insensitivity to the Z^0-mass at PETRA energies: the dominant term in both the numerator and denominator in Eq. 3 is proportional to M_Z^6, thus largely canceling the uncertainty in M_Z. However, at TRISTAN-energies one observes the tail of the Z^0-resonance and it becomes possible to make a direct measurement of the Z^0-mass. In this case one obtains much more sensitivity with K_2, since one can measure the pure propagator effect without the compensation from the M_Z^6 factor in the numerator. However, with the K_2 parametrization the electroweak corrections cannot be neglected anymore (κ_2 in this case, see Fig. 1b), since the correction to the total hadronic cross section is of order 3% at 60 GeV, as will be discussed below.

From the definitions of K_1 and K_2 one can deduce the following well known relation between $\sin^2\theta_W$ and M_Z:

$$\sin^2\theta_W = \frac{1}{2}\left[1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2(1-\Delta r)}}\right] \qquad (12)$$

3 Analysis method

Combining the data from different experiments is always a delicate procedure. It requires that:

- all data are have been corrected to the same level and their errors have a similar meaning;
- correlations between the data points within the same experiment and eventually between different experiments, must be considered.

Correlated errors between measurements can be taken into account by defining the χ^2 via an error correlation matrix[1]:

$$\chi^2 = \Delta^T V^{-1} \Delta \qquad (13)$$

Here Δ is a column vector containing the residuals between the measurements R_i and its estimators R_i^*; V is the NxN error correlation matrix between N measurements. The elements of V can be estimated as follows: Assume the true R values deviate from the measured values by a common normalization factor f, which causes a correlation between the measurements and will make the off-diagonal elements of V nonzero. In this case the best estimator R_i^* from the fit including the correlations will deviate from the best estimator excluding the correlations - called r_i^*- by the same factor f, so $R_i^* = f\, r_i^*$. If the estimator is efficient, r_i^* will just be the averaged R value. The variance of f around 1 is called σ_n^2, where σ_n is the relative normalization error, so $<(f-1)^2>=\sigma_n^2$. Then

$$\begin{aligned}
V_{ii} &= <(R_i - R_i^*)^2> \\
&= <(R_i - r_i^*)^2> + <(r_i^* - f r_i^*)^2> \\
&= \sigma_i^2 + <(1-f)^2> (r_i^*)^2 \\
&= \sigma_i^2 + \sigma_n^2 (r_i^*)^2 \\
V_{ij} &= <(R_i - R_i^*)(R_j - R_j^*)> \\
&= <(R_i - r_i^* + (1-f)r_i^*)(R_j - r_j^* + (1-f)r_j^*)> \\
&= <(1-f)^2> r_i^* r_j^* \\
&= \sigma_n^2 r_i^* r_j^*
\end{aligned}$$

All terms containing $<(R_i - r_i^*)>$ have not been written, since they yield 0 and $\sigma_i^2 = <(R_i - r_i^*)^2)>$ contains the uncorrelated part of the error, which is the sum of both the statistical error - and point to point systematic error squared, but excludes the overall normalization error. Note that σ_n is the error on f, so it is the relative normalization error, which has to be multiplied by r_i^*, while σ_i is the absolute error on R_i.

It should be noted that the procedure of taking correlated errors into account via an error correlation matrix has distinct advantages over a likelihood method, in which the correlations are taken into account by fitting a renormalization constant to the data, as is done e.g. in Ref. [2]: in the first case one has only the physical parameters as free parameters, in the latter case one would have to fit 16 additional parameters in our problem. But what is more important, with a correlation matrix one can define a correlation between every pair of experimental points, thus taking correctly the energy dependence of the correlations into account, which exists for some experiments Furthermore one can study the effects of possible correlations between experiments, e.g. correlations from uncertainties in Monte Carlo programs or radiative corrections. This can be done by setting matrixelements connecting different experiments nonzero. It was found that there is practically no change in the fit results if one includes an overall correlation at the percent level[1]. We have not included the uncertainty from higher order QED radiative corrections in the covariance matrix, since we believe that treating it in a probabilistic way is uncorrect. We prefer to quote the variation of the final results for a given assumption on this correction, which will be discussed in the last chapter.

For some experiments the separation into point-to point and common error was not explicitely given. In these cases it was checked that the numerical values of the fitted parameters were very stable against even large variations of their splittings.

4 Results

The 3 parameters in the total hadronic cross section we would like to determine are: α_s, M_Z, and $\sin^2\theta_W$. At present energies the Z^0 width does not play a role and we fixed its value at 2.5 GeV. We have tried several strategies to determine the other parameters (following Ref. [25], but including newer TRISTAN data[24]):

- The most trivial way is a three parameter fit assuming no connection between the couplings and the mass.
- Make a two parameter fit of M_Z and α_s assuming relation 12 to hold or taking for $\sin^2\theta_W$ the value obtained in neutrino scattering, also taking into account the dependence on the unkown top mass [22,23].
- Determine α_s by using the G_F parametrization, which makes the results insensitive to M_Z and the unkown top mass due to the compensation in κ_1. One could use for $\sin^2\theta_W$ the world average or fit $\sin^2\theta_W$ as a free parameter. In this case $\sin^2\theta_W$ is mainly determined by the vector coupling of the electron, since the sum over all quark couplings is rather insensitive to $\sin^2\theta_W$ (the different electric quark charges cancel largely in the sum: $\sum(v_q^2 + a_q^2) = 10 - 1.33\sin^2\theta_W$). Since the vector coupling of the electron has been tested in various reactions, we will not put the priority on this and use for $\sin^2\theta_W$ the world average.

We first discuss the fit results from the different strategies and then proceed with a discussion of the higher order radiative corrections.

4.1 Determination of M_Z

TRISTAN data up to centre of mass energies of 60.8 GeV have the potential of a Z^0-mass determination better than the one from the $p\bar{p}$-collider. However, the results are rather sensitive to radiative corrections, which -unfortunately - have not been applied yet in a consistent way. For example, data presented at the Munich Conference up to 57 GeV[10] included data corrected with and without electroweak loops and with different Z^0-masses. These differences are rather important as shown in Fig. 2. The curves, representing the ratio of the cross section including radiative corrections and the Born cross section, have been calculated with the programs from Burgers and Hollik[19]. The differences originate from the following physics:

- the lower curve corresponds to initial state and vertex corrections for both γ and Z^0 exchange, but only loop corrections for γ exchange, thus excluding the self energy of the Z^0 propagator. The Z^0-mass was taken to be 90 GeV,

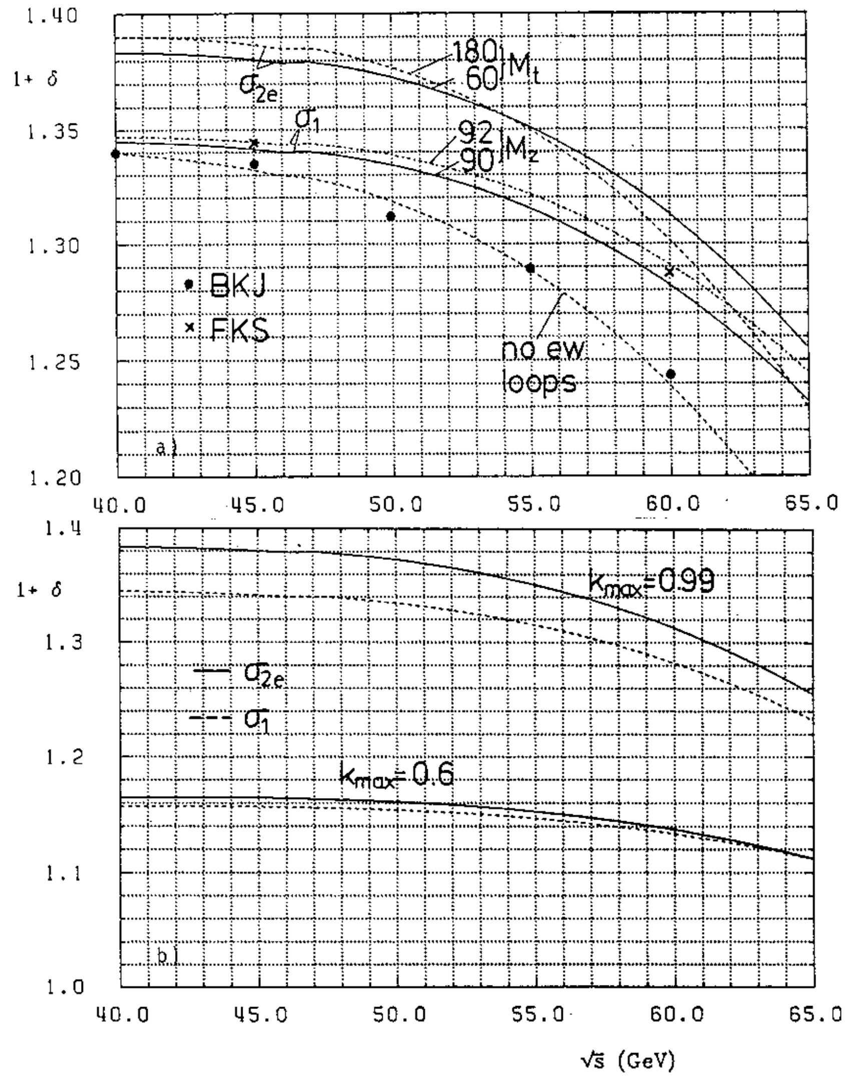

Figure 2: **a) Radiative correction factors calculated in first and second order for $k_{max} = 0.99$. The M_Z mass is 90 GeV and the top mass 60 GeV, unless indicated differently. The lowest curve excludes the Z^0 selfenergy and is comparable with the BKJ radiative corrections [17] as implemented in the Lund Monte Carlo (black dots). The first order full electroweak corrections from Fujimoto et al.[21] (FKS) (crosses) are in good agreement with the results from Burgers and Hollik (curves). b) as in a), but now for $k_{max} = 0.6$.**

$M_t(GeV)$	$\sin^2\theta_W$ *fixed*	$\sin^2\theta_W = f(M_Z)$	$\sin^2\theta_W$ *free*
60	$M_Z = 89.0 \pm 1.0\ GeV$ $\alpha_s = 0.138 \pm 0.017$ $\sin^2\theta_W = 0.231$	$M_Z = 89.3 \pm 1.3\ GeV$ $\alpha_s = 0.134 \pm 0.018$ $\sin^2\theta_W = 0.250 \pm 0.014$	$M_Z = 89.4 \pm 1.3\ GeV$ $\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_W = 0.220^{+0.025}_{-0.020}$
120	$M_Z = 88.5 \pm 1.0\ GeV$ $\alpha_s = 0.138 \pm 0.017$ $\sin^2\theta_W = 0.230$	$M_Z = 88.7 \pm 1.3\ GeV$ $\alpha_s = 0.135 \pm 0.018$ $\sin^2\theta_W = 0.248 \pm 0.014$	$M_Z = 88.9 \pm 1.3\ GeV$ $\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_W = 0.221^{+0.024}_{-0.020}$
180	$M_Z = 88.1 \pm 1.0\ GeV$ $\alpha_s = 0.139 \pm 0.017$ $\sin^2\theta_W = 0.230$	$M_Z = 88.2 \pm 1.2\ GeV$ $\alpha_s = 0.135 \pm 0.018$ $\sin^2\theta_W = 0.246 \pm 0.014$	$M_Z = 88.4 \pm 1.2\ GeV$ $\alpha_s = 0.142 \pm 0.018$ $\sin^2\theta_W = 0.221^{+0.025}_{-0.020}$

Table 1: M_Z and α_s values from various fits.

the Higgs mass 100 GeV and the top mass 60 GeV. Second order QCD corrections have been included using $\alpha_s = 0.12$). Only first order QED graphs have been taken into account ($O(\alpha^3)$). The maximum photon energy allowed corresponds to $k_{max} = E_\gamma/E_{beam} = 0.99$, except for the b-quark, where k_{max} corresponds to the kinematical limit assuming $m_b = 4.7$ GeV. For the other light quarks u, d, s, and c we assumed masses of 0.04, 0.065, 0.3, and 1.5 GeV, respectively. These quark masses have been used to calculate the vacuum polarization. Since the vertex corrections for the Z^0-propagator are small in the on-shell renormalization scheme of Böhm et al.[20], this curve is very close to the results from the well known Berends, Kleis, Jadach (BKJ) results[17] as implemented in the Lund Monte Carlo. These BKJ results, indicated in the figure as solid dots, ignore the Z^0 self energy too.

- The two middle curves for two Z^0 masses (90 and 92 GeV) include in addition the loop corrections from the Z^0-propagator. The sensitivity comes mainly from the graphs with initial state radiation, which depend on the cross section at the energy after radiation and are therefore sensitive to the shape of the cross section, i.e. to the Z^0 mass. The results agree with the calculations from Fujimoto et al.[21], which have been indicated as crosses in the figure. These crosses were calculated for M_Z=91.9 and m_t=45 GeV and include final state radiation. It should be noted that in these programs final state radiative corrections are large and cannot be neglected, as is the case for the calculations of Hollik et al.[18]. The reason is the different wave function renormalization, which shifts part of the electroweak loop corrections into the vertex corrections;

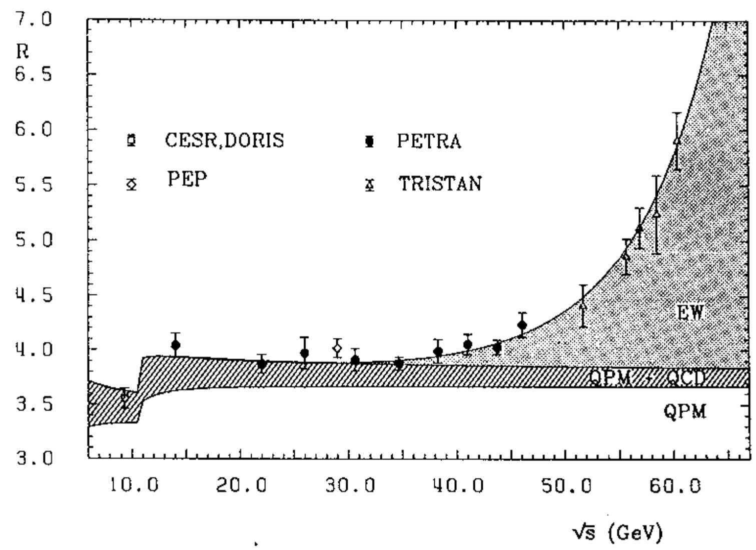

Figure 3: **Experimental data and result of a 3 parameter fit for $M_t = 60$ GeV yielding $M_Z = 89.4 \pm 1.3$, $\alpha_s = 0.142 \pm 0.018$, and $\sin^2\theta_W = 0.220^{+0.025}_{-0.020}$ (from Ref. [25]).**

however, the final corrections in both calculations are very close.

- The upper two curves to second order exponentiated cross sections for two different top masses. One observes that the higher order corrections are important, if one integrates over all possible photon momenta. However, experimental cuts require typically a visible energy of $0.4\sqrt{s}$, which corresponds to $k_{max} \approx 0.6$. For such a cut off the difference between first- and second order becomes much smaller as shown in Fig. 2b.

The fit results with both M_Z and α_s as free parameters are presented in Table 1. Here we included the data as presented at the Topical Conference at KEK[24]. These data, which are of a preliminary nature and the results in Table 1 should be considered accordingly, included full electroweak corrections. However, the results are in excellent agreement with older data, as presented at the Munich Conference[10] and after taking the different radiative correction factors into account [25].

The radiative corrections were handled in the following way: the data is corrected only for QED radiative corrections, i.e. we have undone the electroweak corrections to the TRISTAN data (the difference between the lowest and middle curves in Fig. 2a). This has two advantages:

- The TRISTAN data is now corrected to the same level as the PEP and PETRA data, so they can be treated equally in the fits.
- The data corrected in such a way do not depend anymore on the top mass, so we can perform a fit with a function including the top mass dependence (through κ_2 in Eq. 7).

As shown in Fig. 2, the radiative correction factors depend on the Z^0 mass and top mass. Therefore, we have iterated the fit each time by applying the correct correction factor for the best fitted Z^0-mass (and using the top mass given in the table). In addition to the changes in radiative corrections as function of M_Z, we have considered possible changes in the detection efficiency. Using the Lund Monte Carlo, we find that varying the Z^0 mass from 92 to 89 GeV increases the detection efficiency about 0.3 % at $\sqrt{s}$=60 GeV for typical experimental cuts. Without iterations we would find $M_Z \approx 90$ GeV for M_t=60 GeV, which is about 1 GeV higher than the comparable values quoted before[11].

It should be noted that the main sensitivity to M_Z comes from the propagator effect in the TRISTAN energy range, while at PEP/PETRA energies the sensitivity came only through the couplings and the use of relation 12.

The top mass dependence found for M_Z comes from the radiative corrections to the Z^0 exchange present in the κ_2 parametrization. This cannot be avoided by the use of the κ_1 parametrization, since in this case the sensitivity to the Z^0 mass is reduced (the fit would give an error on M_Z of around 3 GeV). Table 1 shows the result from the three parameter fit too. The correlation coefficients between the 3 parameters are (independent of the top mass): $\rho(\sin^2\theta_W, M_Z) = -0.64$, $\rho(\Lambda^{(5)}_{\overline{MS}}, \sin^2\theta_W) = -0.42$ and $\rho(\Lambda^{(5)}_{\overline{MS}}, M_Z) = 0.44$. The χ^2/DF is 70/101; this excellent χ^2 comes mainly from the fact that the common normalization errors might have been overestimated. If we calculate the χ^2 only from the diagonal elements of the matrix V^{-1}, thus ignoring the correlations, but including the complete errors, we find χ^2_D/DF is 95/101. The fit result is shown in Fig. 3. For clarity we have averaged the data points within certain energy bins in the following way: we have fitted a constant value to the data points within a certain energy bin using the complete error correlation matrix (we have checked that this procedure exactly corresponds to make a weighted average, taking correctly into account independent and correlated errors). So the error bars represent the total errors including the correlation and the data have not been renormalized.

4.2 Determination of α_s

In Table 2 we give the fit results with the G_F parametrization. The values of α_s obtained from the different energy regimes are consistent. Comparing the $O(\alpha_s{}^2)$ and $O(\alpha_s{}^3)$ fits in Table 2, one observes a systematic reduction of the third order α_s values by 11 − 12% for all energy regimes in agreement with the estimates given originally in Ref. [7]. This reduction varies between 6.6% and 16.5% in Ref. [2] for different energy regimes, which is strange, although the variations are within errors.

Data	Energy range	$O(\alpha_s^2)$	$O(\alpha_s^3)$
PEP, PETRA	$14-47\ GeV$	$\alpha_s = 0.168 \pm 0.025$ $\Lambda^{(5)}_{\overline{MS}} = 590^{+470}_{-340}\ MeV$	$\alpha_s = 0.151 \pm 0.020$ $\Lambda^{(5)}_{\overline{MS}} = 320^{+240}_{-180}\ MeV$
PEP, PETRA, TRISTAN	$14-57\ GeV$	$\alpha_s = 0.170 \pm 0.025$ $\Lambda^{(5)}_{\overline{MS}} = 620^{+460}_{-340}\ MeV$	$\alpha_s = 0.152 \pm 0.019$ $\Lambda^{(5)}_{\overline{MS}} = 330^{+230}_{-180}\ MeV$
CESR, DORIS, PEP, PETRA, TRISTAN	$7-57\ GeV$	$\alpha_s = 0.158 \pm 0.020$ $\Lambda^{(5)}_{\overline{MS}} = 440^{+300}_{-230}\ MeV$	$\alpha_s = 0.143 \pm 0.015$ $\Lambda^{(5)}_{\overline{MS}} = 240^{+150}_{-120}\ MeV$

Table 2: α_s and $\Lambda^{(5)}_{\overline{MS}}$ fitted with the G_F parametrization for for $\sin^2\theta_W = 0.231$ (M_t =60 GeV) and M_Z=89.3 GeV (from Ref. [25]).

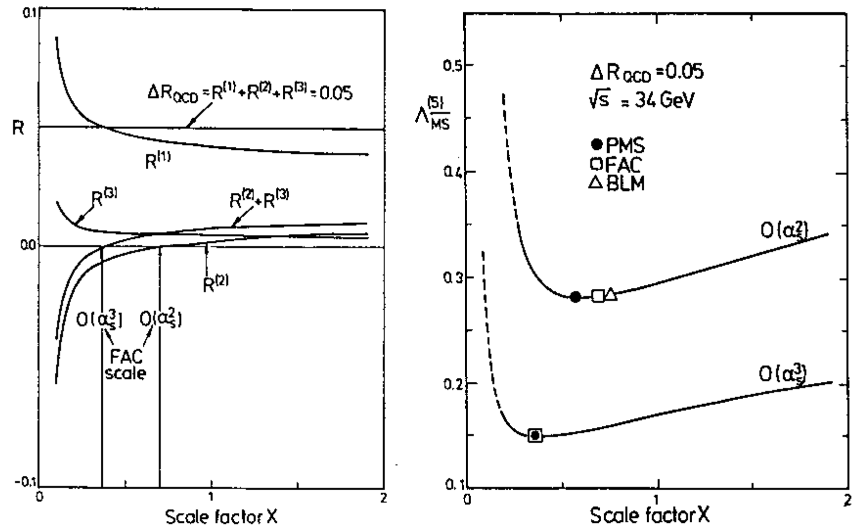

Figure 4: **Contributions to R (a) and Λ determinations in second - and third order (b) as a function of the renormalization scale factor x.**

In the definition of α_s we have made the usual choice $Q^2 = \sqrt{s}$, since this is the only large scale in the process. However, one may argue that gluon radiation occurs at a much smaller scale and there is no reason to use such a large scale. A large amount of literature exists with arguments for the choice of scale[26,27]. The main arguments are based on the size of the higher order corrections and/or the sensitivity to the renormalization scheme. Since the third order QCD contributions are large for R, we have studied these contributions as function of scale in contrast to previous results for a few specific scales[28]. This can be done easily as follows. Suppose a variable is given in a certain renormalization scheme and for a given Q^2 scale to be:

$$R = r_1\alpha_s + r_2\alpha_s^2 + r_3\alpha_s^3 + O(\alpha_s^4) \tag{14}$$

If we choose the coupling at a different scale, we get:

$$R' = r_1'\alpha_s' + r_2'\alpha_s'^2 + r_3\alpha_s'^3 + O(\alpha_s'^4) \tag{15}$$

If one neglects the terms of order $O(\alpha_s^4)$, then

$$R' - R = dR = r_1 d\alpha_s + \alpha_s dr_1 + \alpha_s^2 dr_2 + 2\alpha_s r_2 d\alpha_s + \alpha_s^3 dr_3 = 0 \tag{16}$$

This can only be zero, if coefficient for each power of α_s equals zero, which yields 3 equations for the 3 unknowns r_1', r_2', r_3'. After calculating $d\alpha_s$ from the renormalization group equation:

$$\mu\frac{\partial\alpha_s(\mu)}{\partial\mu} = -\beta_0\alpha_s^2(\mu) - \beta_1\alpha_s^3(\mu) + O(\alpha_s^4), \tag{17}$$

we find for the coefficients at a different scale $Q' = xQ$:

$$\begin{aligned} r_1' &= r_1 \\ r_2' &= r_2 + r_1\beta_0 ln\ x \\ r_3' &= r_3 + r_1\beta_1 ln\ x + 2r_2\beta_0 ln\ x \end{aligned} \tag{18}$$

The β-factors are renormalization scheme independent and given by:

$$\begin{aligned} \beta_0 &= \frac{1}{6\pi}[33 - 2n_f] \\ \beta_1 &= \frac{1}{12\pi^2}[153 - 19n_f] \end{aligned}$$

The various QCD contributions to R as function of the renormalization factor x are shown in Fig. 4a, assuming the total contribution to be constant ($R_{QCD} = 0.05$). One observes that at $x = 1$ the third order contribution is indeed larger than the second order contribution, but at small and large x the absolute value of the second order contribution is larger. However, at all scales the first order contribution is dominant. We have indicated the scales at which the second order or the second plus third order contributions become zero. These are the so-called FAC scales (Fastest Apparent Convergence)[29]. After recalculating the coefficients at a new scale, one can redetermine the corresponding α_s from the measured R-value and recalculate the corresponding Λ value. The result is shown in Fig. 4b. The minimum in this

curve is the PMS scale corresponding to the point of minimal sensitivity, a concept introduced by Stevenson[27]. One observes that at all scales the Λ values in third order are roughly a factor two below the Λ values in second order, indicating that for this reaction there is nothing like an optimum scale, where the higher orders are not important.

This clearly indicates that all the heated discussions about choosing a certain scale mean nothing more than betting on the future: you only can say something seriously about higher order contributions by calculating them, not by fiddling with renormalization scales or schemes.

It is interesting to determine the QCD contribution independently from the definition of α_s. If we assume a linear energy dependence within our energy range, we find this contribution to be $f_{QCD}(\sqrt{s} = 34GeV) = 1.057 \pm 0.008$. An extrapolation of this value to the LEP/SLC energy range yields $f_{QCD}(\sqrt{s} = 91GeV) = 1.046 \pm 0.005$, which is an experimental number for the infinite series $(1 + \alpha_s/\pi + ...)$.

5 Higher order QED radiative corrections.

R is calculated from the number of multihadron events - N_{MH} - and the number of Bhabha events - N_{BB} - in the following way:

$$R = \frac{\sigma_{BB}}{\sigma_{\mu\mu}}\frac{N_{MH}}{N_{BB}}\frac{\epsilon_{BB}(1+\delta)_{BB}(1+\delta_{VP})_{BB}}{\epsilon_{MH}(1+\delta)_{MH}(1+\delta_{VP})_{MH}}$$

Here ϵ is the detection efficiency, δ_{VP} is the vacuum polarization correction, δ is the correction for initial and final state radiation and vertex graphs, and σ_{BB} and $\sigma_{\mu\mu}$ are the Born cross sections for Bhabha scattering and μ-pair production.

We have factorized the effects of loop corrections and other radiative corrections; this yields a higher order contribution $\delta\delta_{VP}$, which should be neglected in first order, but can be large for multihadrons, since typically $\delta = 0.2$ and δ_{VP}=0.1. Such a contribution, representing the radiative corrections to graphs including a fermion pair in the propagator, yields the main difference between the first and second order curves in Fig. 2a. For $k_{max} = 0.6$ δ is small, so one expects the higher order contributions to be smaller too, as proven in Fig. 2b.

In order to estimate more precisely the effect of higher order radiative corrections, one should consider possible changes in ϵ, since the energy loss distribution is changed considerably by the higher order corrections, as shown in Fig. 5.

We have calculated the change in efficiency for a wide range of experimental cuts and center of mass energies using the LUND fragmentation program[16] after implementing the higher order radiative corrections[30]. The result is that indeed the product of efficiency and $1 + \delta$ hardly changes by the higher orders, as expected from the small differences in the lower curves of Fig. 2b.

For Bhabha scattering the complete second order calculation has not been done. We have estimated the higher order radiative corrections by applying the exponentiation procedure of Ref[14] to the Bhabha generator[31]; this exponentiation procedure agrees very well with the exact second order calculation in case of μ-pair

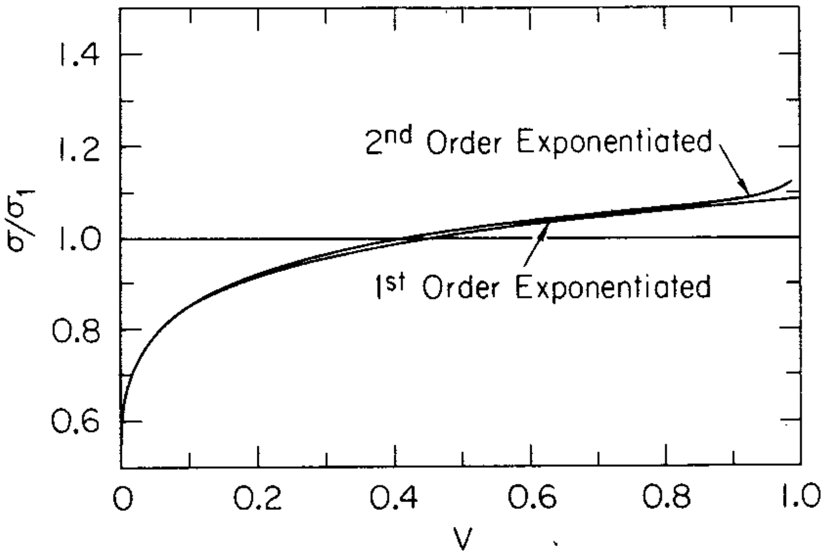

Figure 5: **The first and second order exponentiated cross section normalized to the first order non-exponentiated cross section as function of v = $1 - s'/s$, where s and s' are the invariant masses squared of the initial and final state, respectively. In first order, v is the photon energy normalized to the beam energy. One sees, that the higher order calculations are about 30 % below the first order calculation for small v-values, but 10 % above for large v-values. Exponentiating the first order calculation (horizontal line) gives already results close to the second order calculation; exponentiating the second order calculation hardly changes it.**

production[32]. Different experiments usually make quite different cuts on acolinearity and visible energy to select the Bhabha sample. It turns out that the higher order corrections are somewhat sensitive to the various cuts, since one integrates over quite different regions of phase space and different parts are affected differently as shown by the curves in Fig. 5. It was found that the product $\epsilon_{BB}(1+\delta)_{BB}$ drops between 0 and 1%, if one considers the various cuts. The experiment dependence makes it difficult to correct the R-values. Instead we use these values to make an estimate. If we vary all experimental points between the limits, the refitted value of α_s drops between 0 and 11%. Estimating the error to be half this range or less, we see that the uncertainty from higher order radiative corrections are smaller than the quoted experimental error of 11 %.

The effect of higher order corrections on M_Z is appreciably smaller, since the energy dependence is rather smooth, so the shape of R versus energy is not changed significantly.

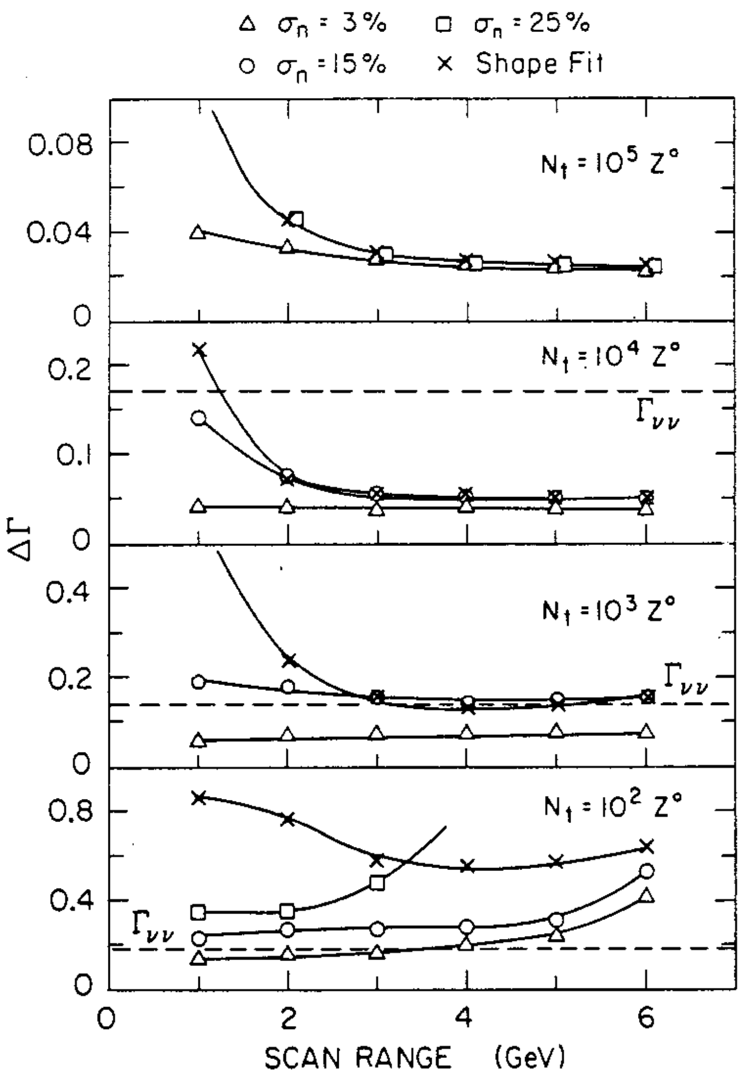

Figure 6: **The expected error on the width as function of scanning range assuming 3 scan points and event samples as indicated by N_t (the number of events which would have been obtained if all the luminosity would have been taken at the peak of the resonance). The different assumptions on the normalization errors have been indicated by the different symbols (crosses imply no absolute normalization measured, so only shape fit). Note that these errors include the uncertainty from the Z^0 mass, which has been fitted simultaneously. The error on M_Z is usually somewhat less than the error on the width. From Ref. [32].**

6 Future M_Z and Γ_Z determinations

Applying the previous analysis method to three hypothetical scan points around the Z^0 resonance would quickly improve the precision of the mass measurement as well as provide a measurement of the width. The expected error on the width as function of scanning range (maximum difference in cm energy between the 3 points) is shown in Fig. 6 for event samples varying between 10^2 and 10^5 Z^0's. The horizontal dashed line indicated the contribution to the width of one new massless neutrino generation. One observes that with 100 Z^0's one can measure the width already with a precision of about 250 MeV, if the absolute normalization error is below 15 %. Similar errors on M_Z can be obtained[32]. For 10^5 Z^0's the precision is limited to about 25 MeV by the systematic poin-to-point uncertainty, which we assumed (optimistically) to be 1% (including the uncertainty from the error on the relative beam energy and reproducibility). Note that if one only fits the shape (crosses in Fig. 6), thus ignoring the knowledge from the absolute normalization (but simultaneously avoiding correlations from the normalization errors), the results are always worse than fits including the normalization. The effect is especially important with low statistics, as expected, since in that case the shape is not well defined, but the width is mainly determined by the events near the peak: the peak cross section is proportional to one over the width squared, so a 10 % error on the peak cross section gives a 5% error on the width (contrary to an eye-fit: the peak cross section determines the width, while events on either side of the peak determine the mass!). For high statistics both fits become equivalent, since then the shape is well determined and the normalization error allows a simultaneous shift of all points without changing the shape. More details can be found in Ref. [32].

7 Summary

We have determined the Standard Model parameters α_s,M_Z, and $\sin^2\theta_W$ from a 3 parameter fit to the data on R between 7 and 60.8 GeV. The result is:

$$\begin{aligned} M_Z &= 89.4 \pm 1.3 \text{ GeV} \\ \alpha_s &= 0.142 \pm 0.018 \\ \sin^2\theta_W &= 0.220^{+0.025}_{-0.020} \end{aligned}$$

Here we assumed a top mass of 60 GeV and a Higgs mass of 100 GeV. For a top mass of 120(180) GeV M_Z is lowered to 88.9(88.4) GeV. There is little dependence on the Higgs mass. A Monte Carlo study shows that with 3 energy scan points around the Z^0 resonance peak and a total of about 100 events, one can determine the Z^0 mass and width already with an error of about 250 MeV, if one fits the absolute cross section instead of the shape of the resonance (dealing with correlated normalization errors in the way described above). The value of $\sin^2\theta_W$ is in good agreement with recent values from deep inelastic neutrino scattering[22]: $\sin^2\theta_W = 0.231 \pm 0.005$. If we use this value for $\sin^2\theta_W$, we find:

$$\alpha_s = 0.143 \pm 0.015.$$

Note that the small error comes partly from the fact that we fit over a large energy range, in which case a reduction of all R values by 1% translates into a reduction in $\alpha_s$of 11% only. Here we have taken the third order QCD corrections into account, which lower the α_s values about 10% (see Table 2) with respect to the second order one. The QCD series $1 + \alpha_s/\pi + \ldots$ has been determined from a direct fit to the data in a model independent way. Extrapolating to the Z^0 region, we find this factor to be 1.046 ± 0.005.

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