DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 89-111
September 1989

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

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*Talk at the Brighton Conference on Electroweak Radiative Corrections, July 9-14, 1989.
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Abstract

We discuss the determination of the Standard Model parameters α_s, M_Z, and $\sin^2\theta_W$ from the total hadronic cross section in e^+e^- annihilation at centre of mass energies between 7 and 93 GeV. At the highest TRISTAN energies, the tail of the Z^0 resonance does increase R by 50%. Such a sizeable increase allows a direct measurement of M_Z. However, this measurement turns out to be somewhat below the direct value measured at SLC at energies around the Z^0-resonance. This has led to speculation about possible new physics, although statistically the effect is not too significant (less than 2 standard deviations). We study in detail if other effects, like normalization errors or higher order radiative corrections could reduce the difference. The radiative corrections at TRISTAN energies can be calculated more precisely with the new knowledge of the Z^0- mass and the correspondingly improved value for the top mass ($120\pm35\pm20$ GeV/c^2), resulting from the combination with the value of $\sin^2\theta_W$ from low energy measurements.

If third order QCD contributions are taken into account, one finds $\Lambda_{\overline{MS}}=0.26^{+0.16}_{-0.13}$ GeV, which corresponds to $\alpha_s(34\text{ GeV}) = 0.144\pm0.015$ and $\alpha_s(91\text{ GeV}) = 0.122\pm0.011$. From the combined R measurements one finds that near the Z^0-resonance the infinite QCD series $1+\alpha_s/\pi+..$ is 1.046 ± 0.006. We study the effect of changing the renormalization scale of α_s and find that the third order QCD corrections, which are larger than the second order ones for the usual scale $Q=\sqrt{s}$, are smaller than the second order contributions at other scales.

1 Introduction

The most precise measurement of the Z^0 mass (91.11 ± 0.23 GeV/c^2) has been obtained from the recent observation of the Z^0 s-channel resonance in e^+e^- annihilation by the MARK-II Collaboration[1].The data has been obtained at the Stanford Linear Collider (SLC). A comparison with other values is shown in Fig. 1 and Table 1. The M_Z values from SLC and the $p\bar{p}$ collider[2] experiments are in good agreement. However, the M_Z value from R data up to 60.8 GeV is about 2 standard deviations below the one from SLC data, implying a somewhat higher electroweak contribution to the hadronic cross section than expected from the Standard Model. On the other hand, the combined leptonic and hadronic data from TRISTAN yields M_Z values much closer to the SLC value (see Table 1), implying that the leptonic sector has a smaller electroweak contribution than the expectation from the Standard Model, as has been published[3,4] and discussed at various Conferences[5]-[13]. This has led to speculations about a possible Z' heavy boson, which through its interference with the Z^0 could lead to a decrease in the leptonic cross section, but simultaneously an increase in the hadronic cross section[14]. In this paper we try to see if more mundane effects could have contributed to the difference between 'low' (i.e. below $\sqrt{s}$=61 GeV) and 'high' energy data. In particular, we consider

- New constraints on the top mass from the new values of M_Z combined with the low energy measurements from $\sin^2\theta_W$ (θ_W is the electroweak mixing angle or Weinberg angle). Knowledge of M_Z and M_{top} is important for a precise knowledge of the radiative corrections at 'low' energies.
- Possible effects from higher order radiative corrections, which have not been applied neither to the published low energy data nor to the SLC data.

Data	Ref	$M_Z(\mathrm{GeV}/c^2)$
MARK II	[1]	91.11 ± 0.23
UA1	[2]	$93.1 \pm 1.0 \pm 3.1$
UA2	[2]	$91.5 \pm 1.2 \pm 1.7$
CDF	[2]	$90.9 \pm 0.3 \pm 0.2$
R + leptons TRISTAN	[13]	$90.4^{+1.7}_{-1.9}$
R	[*thiswork*]	88.9 ± 1.2

Table 1: **M_Z values from various experiments.**

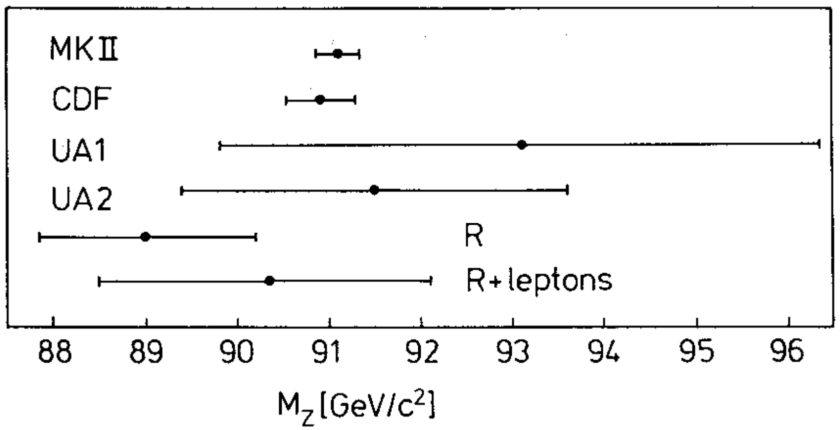


Figure 1: **Comparison of M_Z values.**

- Possible effects from common normalization errors.

The report has been organized as follows: we first summarize the Standard Model (SM) formulae and radiative corrections, then discuss the results on M_Z from 'low'and 'high' energy data. We conclude with the determination of α_s and the scale dependence of α_s.

2 Standard Model Formulae

The normalized cross section R is defined as the ratio

$$R \equiv \frac{\sigma[e^+e^- \to \gamma, Z^0 \to hadrons]}{\sigma[e^+e^- \to \gamma \to \mu^+\mu^-]}$$

The $\mu^+\mu^-$ cross section is the lowest order pointlike QED cross section of massless spin $\frac{1}{2}$ particles, and is equal to $4\pi\alpha^2/3s$, where s is the square of the centre of mass energy.

Fermion	I_3^L	I_3^R	a	v	e_f
neutrino	1/2	0	1	1	0
e, μ, τ lepton	−1/2	0	−1	$-1 + 4\sin^2\theta_W = -0.08$	−1
u, c, t quarks	1/2	0	1	$+1 - \frac{8}{3}\sin^2\theta_W = 0.39$	+2/3
d, s, b quarks	−1/2	0	−1	$-1 + \frac{4}{3}\sin^2\theta_W = -0.69$	−1/3

Table 2: **Summary of the couplings between the Z^0 and various leptons for $\sin^2\theta_W$ = 0.23.**

The total hadronic cross section at the Born level is given by the sum of the following contributions:

$$\sigma^{\gamma}_{had} = \frac{4\pi\alpha^2}{3s} r_{QCD} \sum_{q=1}^{5} e_e^2 e_q^2 \tag{1}$$

$$\sigma^{\gamma Z}_{had} = 8\pi\alpha r_{QCD} \frac{K(s - M_Z^2)}{(s - M_Z^2)^2 + s^2\Gamma_{tot}^2/M_Z^2} \sum_{q=1}^{5} e_e e_q v_e v_q \tag{2}$$

$$\sigma^{Z}_{had} = 12\pi r_{QCD} \frac{K^2 s}{(s - M_Z^2)^2 + s^2\Gamma_{tot}^2/M_Z^2} (v_e^2 + a_e^2) \sum_{q=1}^{5} (v_q^2 + a_q^2) \tag{3}$$

The superscripts indicate the contribution from photon exchange, Z^0 exchange and their interference and the sum is taken over five quark flavours, thus assuming the top quark is too heavy; v and a represent the vector and axial vector couplings of the quarks (subscript q) and electrons (subscript e) and Γ_{tot} is the total width of the Z^0. For simplicity we have neglected small mass effects in the formulae above, but they have been taken into account in the analysis, using the formulae in Ref.[15]. The factor r_{QCD} represents the effect from gluon radiation and is given in the $\overline{MS}$ scheme by[16,17]:

$$r_{QCD} = 3[1 + \frac{\alpha_s}{\pi} + (1.986 - 0.115 n_f)\left(\frac{\alpha_s}{\pi}\right)^2 + \left(70.985 - 1.2 n_f - 0.005 n_f^2 - 1.679\frac{(\sum e_q)^2}{3\sum e_q^2}\right)\left(\frac{\alpha_s}{\pi}\right)^3]$$

The factor 3 on the righthand side accounts for the colour of the quarks. The total Z^0 width is determined by:

$$\Gamma_{tot} = K M_Z N_g \left[a_\nu^2 + v_\nu^2 + a_e^2 + v_e^2\right] + K M_Z r_{QCD} \sum_{q=1}^{5} (v_q^2 + a_q^2) \tag{4}$$

while the partial width into a fermion f is:

$$\Gamma_f = K M_Z r_{QCD} (v_f^2 + a_f^2) \tag{5}$$

N_g is the number of generations and the sum in the total width has been taken over 5 quarks, again assuming the top quark is too heavy to contribute.

The constant K can be either defined as:

$$K_1 = \frac{\sqrt{2} G_F M_Z^2 \kappa_1}{48\pi} \tag{6}$$

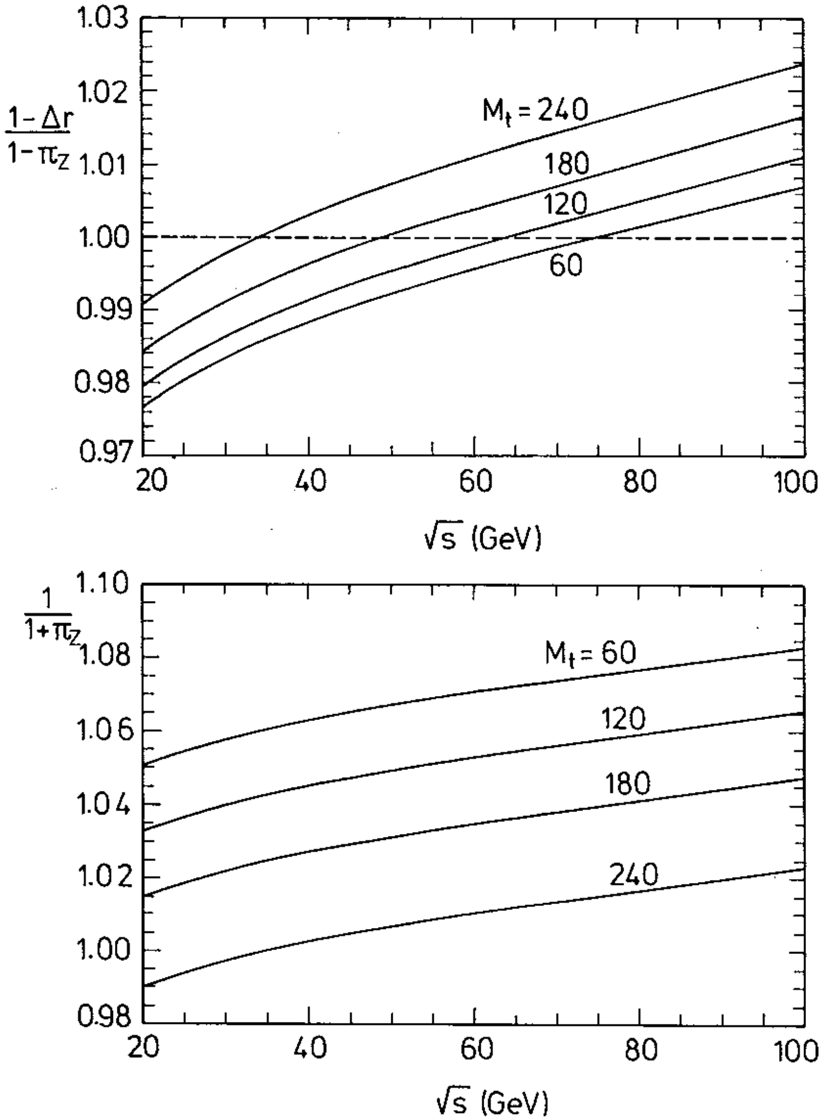


Figure 2: **Electroweak correction factors (κ_1 and κ_2 in text) for the different parametrizations of the Born cross section.**

or

$$K_2 = \frac{\alpha \kappa_2}{48 \sin^2\theta_W \cos^2\theta_W} \quad (7)$$

Here G_F is the Fermi constant, which is well known from muon decay; $\sin^2\theta_W$ defines the electroweak mixing angle, which can be used to define the coupling constants between a pair of fermions and the Z^0 gauge boson (see Table 2):

$$v_f = 2(I_3^L - I_3^R) - 4\,e_f \sin^2\theta_W \quad (8)$$

$$a_f = 2(I_3^L - I_3^R) \quad (9)$$

$I_3^{L(R)}$ is the 3^{th} component of the weak isospin. In the definitions of K we have explicitely included the factor κ which represents the loop corrections to the Z^0 propagator. For example, practically all data from the PEP and PETRA experiments have been corrected with the LUND Monte Carlo program[18], which uses the radiative corrections from Berends et al.[19]. These corrections include the loop corrections for the photon propagator, but not the loop corrections for the Z^0 propagator; the latter can be taken into account by defining[20,21]:

$$\kappa_1 = \frac{1 - \Delta r}{1 + \Pi_Z(s)} \quad (10)$$

or

$$\kappa_2 = \frac{1}{1 + \Pi_Z(s)} \quad (11)$$

where

$$1 - \Delta r = \frac{\alpha(0)}{\alpha(M_W)} + \delta_r(M_t, M_H) \quad (12)$$

and

$$1 + \Pi_Z(s) = \frac{\alpha(0)}{\alpha(M_Z)} + \delta_\Pi(M_t, M_H, s) \quad (13)$$

Here Δr represents the electroweak corrections to the charged gauge boson exchange in muon decay and $\Pi_Z(s)$ represents the electroweak loop corrections to the neutral gauge boson exchange. One sees that the first term in both cases is given by the running of the QED coupling constant coming from the light fermion loops in the photon propagator (hence the indication of the scale in α). For top quark masses below the gauge boson masses this term is dominant in both expressions. E.g. for a top mass of 70 GeV Δr is about 7 % and 6% is coming from the first term alone. However, for a top mass of 230 GeV the latter term $\delta_r(M_{top}, M_H)$ is as large as the first term, but of opposite sign, so the total correction Δr is about zero. $\Pi_Z(s)$ shows a similar behaviour, so that the ratio in κ_1 is much less dependent on the top mass and furthermore close to 1 (see Fig. 2b). This is the advantage of the parametrization with K_1: one can neglect the electroweak corrections to a large extent and the results are insensitive to the unknown top mass. This was the reason why in previous fits to data on R this parametrization has been used, e.g. to determine the strong coupling constant [15]. What was considered further as an advantage compared to the K_2 parametrization was the insensitivity to the Z^0-mass at PETRA energies: the dominant term in both the numerator and denominator in Eq. 3 is proportional to M_Z^6, thus largely canceling the uncertainty in M_Z. However, at TRISTAN-energies one observes the tail of the Z^0-resonance and it becomes possible to make a direct measurement of the Z^0-mass. In this case one obtains much more sensitivity with K_2, since one can measure the pure propagator effect without the compensation from the M_Z^6 factor in the numerator. However, with the K_2 parametrization the electroweak corrections cannot be neglected anymore (κ_2 in this case, see Fig. 2a), since the correction to the total hadronic cross section is of the order of 3% at 60 GeV.

From the definitions of K_1 and K_2 one can deduce the following well known relation between $\sin^2\theta_W$ and M_Z, first derived by Sirlin[22]:

$$\sin^2\theta_W = \frac{1}{2}\left[1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2(1-\Delta r)}}\right] \quad (14)$$

3 Definition of the strong coupling constant

The strong coupling constant α_s is in first order given by

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}. \quad (15)$$

The number of flavours n_f is 5 for the energies considered, and so the Λ value determined is $\Lambda^{(5)}$, while e.g. deep inelastic scattering studies refer to $\Lambda^{(4)}$, the energies there being below the bottom threshold. They are related to each other by $\Lambda^{(5)} \approx 0.7\ \Lambda^{(4)}$[23].

In higher order one not only has to specify the renormalization scheme, but in addition several approximations of α_S have been in use. Within a given scheme (we will use the widely used $\overline{MS}$ scheme) one can define the higher order contributions in terms of the first order coupling constant, i.e. an expansion in $1/ln(Q^2/\Lambda^2)$:

$$\alpha_S(Q^2) = \frac{12\pi}{(33-2n_f)ln(Q^2/\Lambda^2_{\overline{MS}})}\left[1 - 6\frac{153-19n_f}{(33-2n_f)^2}\frac{ln(ln(Q^2/\Lambda^2_{\overline{MS}}))}{ln(Q^2/\Lambda^2_{\overline{MS}})}\right] \quad (16)$$

or one can sum the leading logarithms in this expansion, i.e. terms proportional to $\alpha_S^n\, ln^n(Q^2/\Lambda^2)$, which yields:

$$\alpha_S(Q^2) = \frac{12\pi}{(33-2n_f)ln(Q^2/\Lambda^2_{\overline{MS}}) + 6\frac{153-19n_f}{(33-2n_f)}ln[ln(Q^2/\Lambda^2_{\overline{MS}})]} \quad (17)$$

The two definitions are numerically somewhat different. For example for a given value of α_S=0.15 and n_f=5 the Λ values from the two definitions differ by 14%. Both definitions have been widely used: the first one in the Particle Data Book[24], the second one e.g. in the LUND Monte Carlo[18]. The difference is of some importance in practice. For example, if one would use the definition from the Particle Data Book in the LUND Monte Carlo, one should simultaneously increase the default Λ value from 500 to 570 MeV in order to get the same value of α_S. If one includes the next higher order term, one finds[23]:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 L}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{lnL}{L} + (\frac{\beta_1}{\beta_0^2})^2\frac{1}{L^2}\left\{(lnL-\frac{1}{2})^2 + \frac{\beta_2\beta_0}{\beta_1^2} - \frac{5}{4}\right\}\right] \quad (18)$$

with

$$\begin{aligned} L &= ln(Q^2/\Lambda_{\overline{MS}}{}^2) \\ \beta_0 &= 11 - \tfrac{2}{3}n_f \\ \beta_1 &= 2(51 - \tfrac{19}{3}n_f) \\ \beta_2 &= \tfrac{2857}{2} - \tfrac{5033}{18}n_f + \tfrac{325}{54}n_f^2\,. \end{aligned}$$

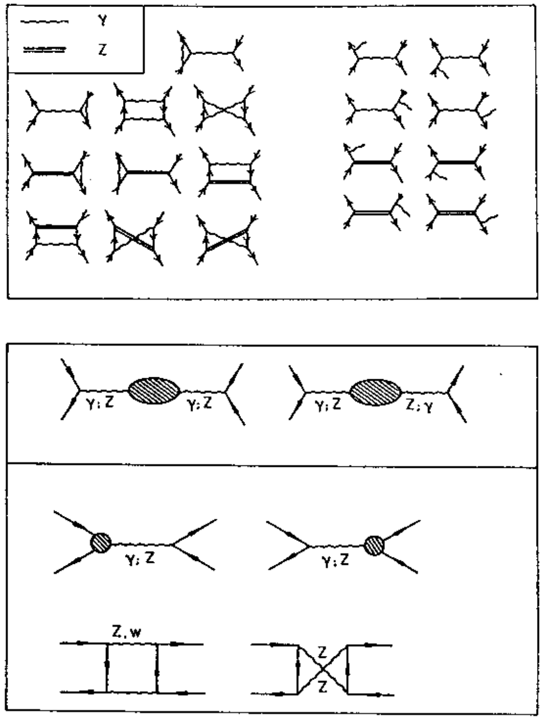


Figure 3: **a) Feynman diagrams for the 'QED corrections' b) Feynman diagrams for the 'electroweak corrections': loop corrections to the propagators and box- and vertex corrections.**

Instead of using a series expansion in α_s, one can solve the renormalization group equation exactly[25]. However, from such a solution one can obtain α_s only numerically for a given $\Lambda_{\overline{MS}}$, which is rather inconvenient and slow in fitting programs. Instead, we have checked that the third order expression Eq. 18 agrees extremely well with the exact solution (better than 0.1% for the energies of interest in this paper). Therefore, we will use this third order expression everywhere, even if we compare with observables calculated only up to second order, since in principle one always should use the correct energy dependence of α_s for all observables, independent of their unknown higher order corrections.

4 Radiative Corrections

4.1 Introduction

Radiative corrections can be subdivided into the following classes:

- 'QED corrections', which consist of those diagrams with an extra photon added to the BORN diagrams either as real bremsstrahlung or as a virtual photon in vertex - or box diagrams (see Fig. 3a).
- 'Electroweak corrections', which collect all other diagrams up to 1 loop: the loop corrections to the propagators, the vertex corrections (excluding the ones with virtual photons), and the box diagrams with two massive gauge bosons (see Fig. 3b).

These two subsets are both gauge invariant[20] and it has sometimes been advertised that experiments should only make the 'QED corrections', which depend on the experimental cuts. Furthermore, the 'electroweak corrections' are unknown as long as the top - and Higgs mass are unknown, but they do not depend on experimental cuts, so in principle one could have 'QED corrected' data, which could be compared with theory for any given top or Higgs mass. This sounds nice. However, in practice the 'QED corrections' depend on the loop corrections, simply through the fact that the cross section after initial state radiation has a reduced centre of mass energy, but this cross section depends on the loop corrections. Therefore, corrected data is only useful, if the original data with applied cuts and correction factors have all been completely specified, including the values used for M_Z, M_{top}, and M_H. Usually the detector acceptances do not change too much, if the top or Higgs mass are changed, so one can than easily recalculate the correction for different values, if the original values have been specified.

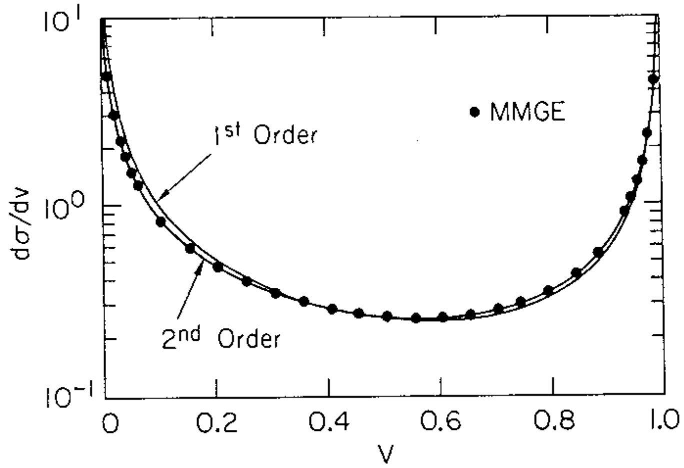
The 'electroweak corrections' are very well approximated by the factors κ in Eq. 6 or 7, since the loop corrections are the dominant ones. Higher order contributions to the loop corrections can be estimated by summing the leading logarithms to a geometric series. The change in cross section by the 'electroweak corrections' depends on energy: at low energy only the loop corrections to the photon (vacuum polarization) are important; they change the cross section typically by 10 %; near the Z^0 resonance peak the effect is small, since on resonance the K^2-factor in both the numerator and denominator cancel each other (see Eqs. 3 and 4). Off-resonance, the loop corrections depend on the choice of K-factor and the top mass as shown in Fig. 2: at $\sqrt{s}$=80 GeV the effect is less than 0.5% with κ_1, but 12% with κ_2 for M_{top}=120 GeV/c^2.

From the 'QED corrections' the initial state radiative corrections are dominant. Final state electromagnetic radiation from the quarks is small, since the Kinoshito-Lee-Nauenberg theorem[26] assures that the procedure of summing over all $q\bar{q}$ final states with an arbitrary number of photons, as is done in the detection of multihadronic events, will cancel all leading logarithms and the remaining radiative correction is of order $O\left(\frac{\alpha}{\pi}\right) \approx 0.2\%$.

For initial state radiation an exact second order calculation has been made by Berends, Burgers and van Neerven[27], which allows a comparison with approximate procedures, based on exponentiation. Such a comparison is important, since it allows to choose an 'optimum' exponentiation procedure, i.e. the one which is closest to the exact second order calculation. Such an 'optimum' procedure can than be applied to e.g. Bhabha scattering, for which no exact second order calculation exists. We describe such an 'optimum' choice in the next section.

4.2 Higher order initial state radiative corrections

In first order the total cross section can be written as:



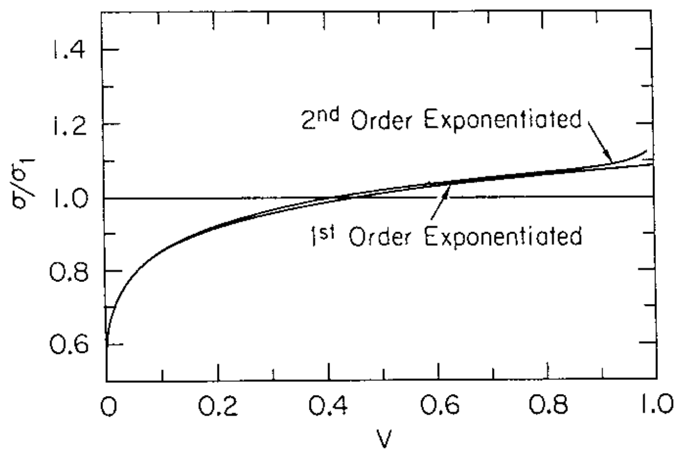


Figure 4: **The first and second order cross sections as function of v = 1 − s′/s, where s and s′ are the invariant masses squared of the initial and final state, respectively. In first order, v is the photon energy normalized to the beam energy. b) The first and second order exponentiated cross sections normalized to the first order non-exponentiated cross section; the latter corresponds to the horizontal line. The first order exponentiated cross section is already close to the second order calculation; exponentiating the second order calculation hardly changes it. One sees, that the higher order calculations are about 30 % below the first order calculation for small v-values, but 10 % above for large v-values.**

$$\sigma_1(s) = \sigma_0(s)\,(1 + \delta_1^v + \beta\, ln\, k_0) + \int_{k_0}^{k_{max}} \beta \left(\frac{1 + (1-k)^2}{2\,k}\right) \sigma_0(s')\, dk \qquad (19)$$

while in second order this is modified to [27]:

$$\begin{aligned} \sigma_2(s) \;=\; & \sigma_0(s)\,(1 + \delta_1^v + \delta_2^v + \beta\, ln\, k_0 + \delta_1\, \beta\, ln\, k_0 + \frac{1}{2}\,\beta^2\, ln^2\, k_0) \\ & + \int_{k_0}^{k_{max}} \sigma_0(s') \Bigg[\beta \left(\frac{1+(1-k)^2}{2\,k}\right)(1 + \delta_1^v + \beta\, ln\, k) \\ & + \left(\frac{\alpha}{\pi}\right)^2 \left\{\left(\frac{1+(1-k)^2}{k}\right) A(k) + (2-k)\, B(k) + (1-k)\, C(k)\right\}\Bigg]\, dk \end{aligned} \qquad (20)$$

Here
$s(s')$ is the centre of mass energy before (after) energy loss from initial state radiation and

$$\beta = 2\alpha/\pi\ (ln\, s/m_e^2 - 1) \qquad (21)$$

$$k = 1 - s'/s \qquad (22)$$

$$\delta_1^v = \frac{\alpha}{\pi}\left[\frac{3}{2}\, ln\,(s/m_e^2) - 2 + \pi^2/3\right] \qquad (23)$$

The expressions for $A(k), B(k), C(k)$ and δ_2^v have been given in Ref. [27].

Note that in first order the energy loss parameter k is just the fractional photon energy E_γ/E_{beam}. In higher orders, when the energy loss is distributed over more photons, these latter have an invariant mass ($M_{\gamma\gamma}$) and the relation between ΣE_γ and k becomes: $k \equiv 1 - s'/s = \Sigma E_\gamma/E_{beam} - M^2_{\gamma\gamma}/s$. Since $M^2_{\gamma\gamma}/s$ is usually small, the difference between k and the total radiated fractional energy $\Sigma E_\gamma/E_{beam}$ becomes small too. This implies that most of the additional photons are either soft or collinear. Therefore, 'simple' Monte Carlo simulations, in which the total radiated energy is given to a single 'effective' photon, do not differ significantly from more elaborate simulations, in which the radiated energy is distributed over multiple photons. In particular, for efficiency determinations of hadronic cross sections they yield similar results, as has been checked explicitely for the generators compared in Ref. [28].

The cross sections in Eqs. 19 and 20 have been split into 2 parts. The first part is the part with only soft photons, so $\sigma(s) = \sigma(s')$; the second part includes hard photon radiation, in which case $\sigma(s') \neq \sigma(s)$, so one has to convolute the energy loss spectrum with $\sigma(s')$, as is done by the expesssions below the integrals. The separation between the soft and hard parts of the spectrum is defined by k_0. For $k_0 \to 0$ the soft part goes to $-\infty$, while the hard part goes to $+\infty$. As long as k_0 is small, the sum of the two parts is independent of the choice of k_0.

It can be shown [29] that the leading terms of the real photon emission always lead to terms $\frac{1}{n!}\,\beta^n\, ln^n\, k$, so summing the leading logs to all orders implies " exponentiating" the cross section, which yields:

$$\sigma_{soft}^{exp}(s) = \sigma_0(s)\,(1 + \delta_1^v + \delta_2^v + ...)\, k^\beta \qquad (24)$$

since

$$k^\beta = e^{\beta\, ln\, k} = 1 + \beta\, ln\, k + \frac{1}{2!}\,\beta^2\, ln^2\, k + ... \qquad (25)$$

This expression clearly reproduces the soft parts of the first and second order cross sections (Eqs. 19 and 20), if the higher order terms are dropped in the expansion.

For the virtual corrections no such simple formulae exist. Therefore they have to be calculated, a difficult task already in second order. Fortunately, numerically the second order vertex correction is already small, so one may hope that the higher orders are small too. For example at $\sqrt{s}$= 29 GeV , δ_1^v = 0.08 and δ_2^v = - 0.005.

If one neglects the small terms proportional to $A(k), B(k)$ and $C(k)$ in Eq. 20, one sees that the hard part in second order contains the factor $(1 + \delta_1^v + \beta\, ln\, k)$ which can be replaced again by the exponentiated version $(1 + \delta_1^v + \ ...)\, k^\beta$ (see Eq. 25), so one finds:

$$\sigma_1^{exp}(s) = \int_0^{k_{max}} \beta\left(\frac{1}{k} - 1 + \frac{k}{2}\right)\,(1 + \delta_1^v)\, k^\beta \sigma_0\,(s')\, dk \qquad (26)$$

Note that the factor k^β regularizes the infinities, if $k \to 0$, so the integral for $k = 0$ is well behaved and one does not have to split the cross section in a hard- and soft part anymore, but integrates from 0 to the kinematical limit. One can regulate the divergencies in the second order cross section too by exponentiating the soft part:

$$\sigma_2^{exp}(s) = \int_0^{k_{max}} \sigma_0(s')\left[\beta\left(\frac{1}{k}\right)\,(1 + \delta_1^v + \delta_2^v)\, k^\beta\right]\, dk + \sigma_{2H} \qquad (27)$$

where σ_{2H} contains the finite part of the cross section (everything except the $1/k$ pole).

Note that in the first order exponentiated cross section (Eq. 26) we have exponentiated the finite part ($-1 + k/2$ term) too, while in second order the finite part is treated exactly. Exponentiating the finite part of the first order cross section is usually not done [30,31], but its justification stems from the fact that in second order this finite part is multiplied by $1 + \delta_1^v + \beta lnk$ (see Eq. 20), and secondly, that σ_1^{exp} is numerically now very close to σ_2^{exp}, as can be seen from Table 3. Thus a very simple procedure for exponentiating a first order Monte Carlo is: weight the hard part of the cross section with the factor $(1 + \delta_1^v)k^\beta$. This works perfect in the case of μ-pairs and is probably the best guess in case of Bhabha scattering, for which no exact second order calculation exists. However, in that case the β factor has to be modified to include final state radiation and the interference, as will be discussed hereafter.

The effect of the higher order contributions on the energy loss is shown in Fig. 4a (taken from Ref. [28]). As expected, one observes a shift of events towards higher energy losses (a decrease at the left hand side and increase at the right hand side). On a logarithmic plot the effect seems small, but if one plots the ratio between the higher order and first order curves, the difference is -30% at small v and 10% at large v (see Fig. 4b). These rather drastic effects on the radiated energy change the cross section considerably near a resonance and exponentiating the second order cross section still changes the cross section a few % on the high side of the Z^0 resonance (see Table 3). However, at all energies the first order exponentiated cross section is very close to the second order exponentiated one, thus exponentiating the first order cross section in the way described before is enough.

4.3 Higher order initial state radiative corrections for Bhabha scattering

Unfortunately there exists no higher order calculation for Bhabha scattering, i.e. at most one photon is allowed from initial or final state radiation. Therefore, the effect of higher order contributions can only be estimated by exponentiation. Since exponentiation procedures are not unique, the exponentiation procedure which 'works best' for μ-pairs has been choosen, i.e. the procedure which gives results closest to the exact second order calculation. As mentioned in the description of the exponentiation of the μ-pair generator, this corresponds to weighting each radiative event (both with soft AND hard photons, not only the soft part) with a factor $(1-\delta_1^s)k^{-\beta}$ and exponentiating the soft part as described before. For Bhabha scattering final state radiation is important, in which case β should be defined is $\beta_i + \beta_f + 2\beta_{int}$, where $\beta_i = \beta_f$ equals β as defined by Eq. 21, and [32]

$$\beta_{int} = \frac{4\alpha}{\pi}\, ln\, tan\frac{\theta}{2}, \qquad (28)$$

where θ is the polar scattering angle.

In the neighbourhood of a resonance the exponentiation procedure becomes somewhat more complicated [32], but this does not play a major role in the luminosity measurements, since even at SLC and LEP the resonance contribution is very small in the angular range considered for luminosity measurements.

4.4 Effect of higher order radiative corrections on R.

R is calculated from the number of multihadron events - N_{MH} - and the number of Bhabha events - N_{BB} - in the following way:

$$R = \frac{\sigma_{BB}}{\sigma_{\mu\mu}}\frac{N_{MH}}{N_{BB}}\frac{\epsilon_{BB}(1+\delta)_{BB}(1+\delta_{VP})_{BB}}{\epsilon_{MH}(1+\delta)_{MH}(1+\delta_{VP})_{MH}}$$

Here ϵ is the detection efficiency, δ_{VP} is the vacuum polarization correction, δ is the correction for initial and final state radiation and vertex graphs, and σ_{BB} and $\sigma_{\mu\mu}$ are the Born cross sections for Bhabha scattering and μ-pair production.

We have factorized the effects of loop corrections and other radiative corrections; this yields a higher order contribution $\delta\delta_{VP}$, which should be neglected in first order, but can be large for multihadrons, since typically $\delta = 0.2$ and $\delta_{VP} = 0.1$. Such a contribution, representing the radiative corrections to graphs including a fermion pair in the propagator, yields the main difference between the first and second order corrections, shown by the upper two curves in Fig. 5. Experimental cuts require typically a visible energy of $0.4\sqrt{s}$, which corresponds to $k_{max} \approx 0.6$. For such a cutoff the difference between first- and second order becomes much smaller as shown by the two lowest curves in Fig. 5. The reason is simple: initial state radiative corrections are rather small in that case and therefore the product $\delta\delta_{VP}$ is small too.

In order to estimate more precisely the effect of higher order radiative corrections, one should consider possible changes in ϵ, since the energy loss distribution is changed considerably by the higher order corrections, as has been shown in Fig. 4b.

We have calculated the change in efficiency for a wide range of experimental cuts and center of mass energies using the LUND fragmentation program [18] after implementing the

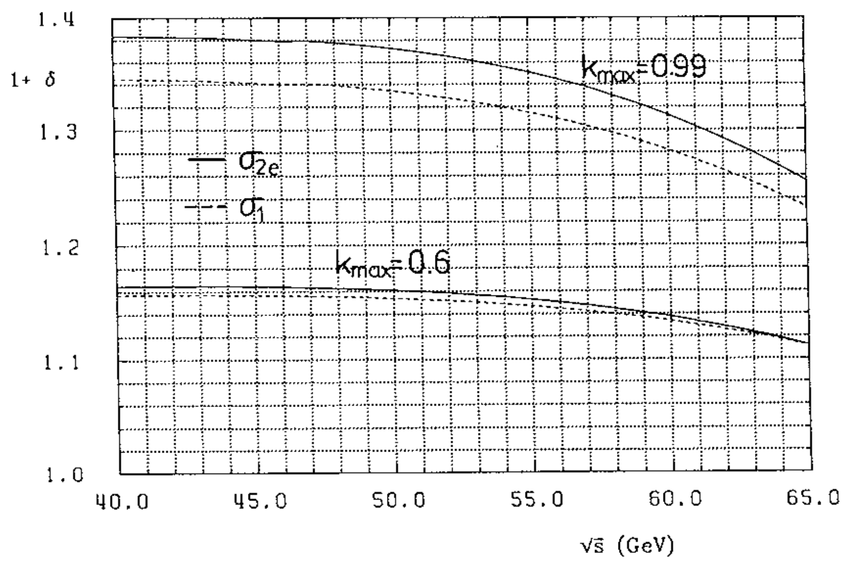


Figure 5: **Radiative correction factors calculated in first and exponentiated second order for k_{max} = 0.99 and 0.6. The M_Z mass is 90 GeV/$c^2$and the top mass 60** GeV/c^2.

higher order radiative corrections in that program[33]. The result is that indeed the product of efficiency and $1+\delta$ hardly changes by the higher orders, as expected from the small differences in the lower curves of Fig. 5.

For Bhabha scattering the complete second order calculation has not been done. We have estimated the higher order radiative corrections by applying the exponentiation procedure described before to the standard Bhabha generator from Berends and Kleiss[34]. Different experiments usually make quite different cuts on acolinearity and visible energy to select the Bhabha sample. It turns out that the higher order corrections are somewhat sensitive to the various cuts, since one integrates over quite different regions of phase space and different parts are affected differently as shown by the curves in Fig. 4. It was found that the product $\epsilon_{BB}(1+\delta)_{BB}$ drops between 0 and 1%, if one considers the various cuts. The experiment dependence makes it difficult to correct the R-values. Instead we use these values to make an estimate. If we vary all experimental points between the limits, the refitted value of α_s drops between 0 and 11%. Estimating the error to be half this range or less, we see that the uncertainty from higher order radiative corrections are smaller than the quoted experimental error of 11 %.

The effect of higher order corrections on M_Z is appreciably smaller, since the energy dependence is rather smooth, so the shape of R versus energy is not changed significantly.

5 Analysis method

Combining the data from different experiments is always a delicate procedure. It requires that:

- all data are have been corrected to the same level and their errors have a similar meaning;
- correlations between the data points within the same experiment and eventually between different experiments, must be considered.

Correlated errors between measurements can be taken into account by defining the χ^2 via an error correlation matrix[15]:

$$\chi^2 = \Delta^T V^{-1} \Delta \qquad (29)$$

Here Δ is a column vector containing the residuals between the measurements R_i and its estimators R_i^*; V is the NxN error correlation matrix between N measurements. The elements of V can be estimated as follows: Assume the true R values deviate from the measured values by a common normalization factor f, which causes a correlation between the measurements and will make the off-diagonal elements of V nonzero. In this case the best estimator R_i^* from the fit including the correlations will deviate from the best estimator excluding the correlations - called r_i^*- by the same factor f, so $R_i^* = f\ r_i^*$. If the estimator is efficient, r_i^* will just be the averaged R value. The variance of f around 1 is called σ_n^2, where σ_n is the relative normalization error, so $< (f-1)^2 >= \sigma_n^2$. Then

$$\begin{aligned}
V_{ii} &= < (R_i - R_i^*)^2 > \\
&= < (R_i - r_i^* + r_i^* - R_i^*)^2 > \\
&= < (R_i - r_i^*)^2 > + < (r_i^* - R_i^*)^2 > \\
&= < (R_i - r_i^*)^2 > + < (r_i^* - f r_i^*)^2 > \\
&= \sigma_i^2 + < (1-f)^2 > (r_i^*)^2 \\
&= \sigma_i^2 + \sigma_n^2 (r_i^*)^2 \\
V_{ij} &= < (R_i - R_i^*)(R_j - R_j^*) > \\
&= < (R_i - r_i^* + r_i^* - R_i^*)(R_j - r_j^* + r_j^* - R_j^*) > \\
&= < (R_i - r_i^* + (1-f)r_i^*)(R_j - r_j^* + (1-f)r_j^*) > \\
&= < (1-f)^2 > r_i^*\ r_j^* \\
&= \sigma_n^2\ r_i^*\ r_j^*
\end{aligned}$$

All terms containing $< (R_i - r_i^*) >$ have not been written, since they do not contribute and $\sigma_i^2 =< (R_i - r_i^*)^2) >$ contains the uncorrelated part of the error, which is the sum of both the statistical error - and point to point systematic error squared, but excludes the overall normalization error. Note that σ_n is the error on f, so it is the relative normalization error, which has to be multiplied by r_i^*, while σ_i is the absolute error on R_i.

It should be noted that the procedure of taking correlated errors into account via an error correlation matrix has distinct advantages over a likelihood method, in which the correlations are taken into account by fitting a renormalization constant to the data, as is done e.g. in

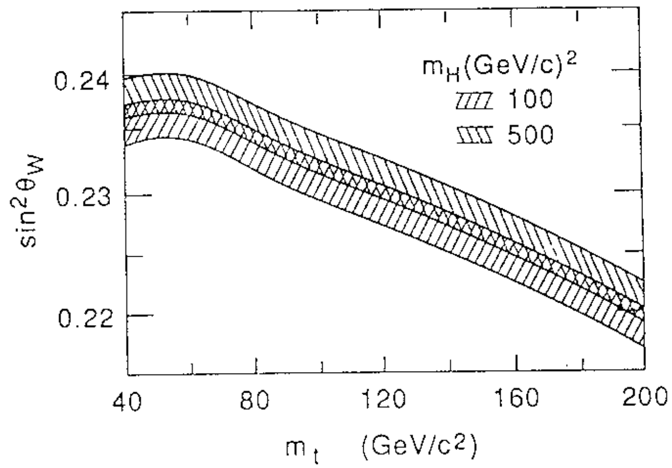


Figure 6: **Display of Sirlin relation between $\sin^2\theta_W$ and M_{top}**

Ref. [35]: in the first case one has only the physical parameters as free parameters, in the latter case one would have to fit 16 additional parameters in our problem. But what is more important, with a correlation matrix one can define a correlation between every pair of experimental points, thus taking correctly the energy dependence of the correlations into account, which exists for some experiments Furthermore one can study the effects of possible correlations between experiments, e.g. correlations from uncertainties in Monte Carlo programs or radiative corrections. This can be done by setting matrixelements connecting different experiments nonzero. It was found that there is practically no change in the fit results if one includes an overall correlation at the percent level[15]. We have not included the uncertainty from higher order QED radiative corrections in the covariance matrix, since we believe that treating it in a probabilistic way is uncorrect. We prefer to quote the variation of the final results for a given assumption on this correction.

For some experiments the separation into point-to-point and common error has not been given explicitely. In these cases it was checked that the numerical values of the fitted parameters were very stable against even large variations of these splittings.

6 Estimating the top mass in the minimal SM

The Sirlin relation (Eq. 14) gives a connection between M_Z, $\sin^2\theta_W$ and Δr. The latter depends on the top and Higgs mass, so if M_Z and $\sin^2\theta_W$ are known, one gets a handle on the unknown values of M_{top} and M_H , of course under the assumptions of the minimal SM. The relation between $\sin^2\theta_W$ and M_{top} is shown in Fig. 6 for the best value of M_Z from MARK-II (Fig. from Ref. [1]). The measured values of $\sin^2\theta_W$ usually depend on M_{top} too via the radiative corrections[36].

However, the most precise value of $\sin^2\theta_W$ stems from the ratio of neutral and charged cur-

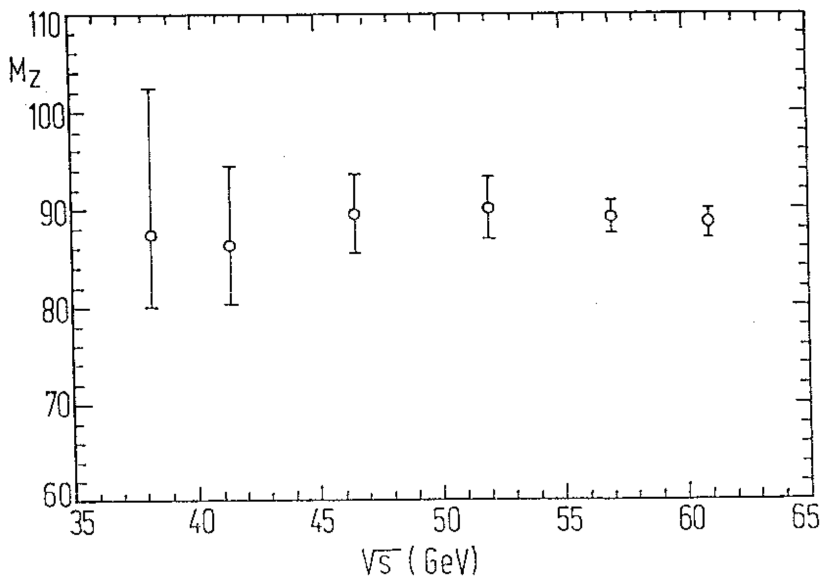


Figure 7: **Fitted M_Z value as function of the maximum fitted centre of mass energy.**

rent cross sections in deep inelastic neutrino quark scattering. This ratio has been measured to a 1% accuracy and the extracted value of $\sin^2\theta_W$ from such a ratio is insensitive to M_{top}[36,37,38]. For large top masses and assuming $\rho = 1$ one finds: $\sin^2\theta_W = 0.2293 \mp 0.0030 \pm 0.0028$[38]. The corresponding values of M_{top} are shown in Table 4 for various Higgs masses. The values range between 100 and 140 GeV, so taking this range into account in the errors, one estimates:

$$\mathbf{M_{top} = 120 \pm 35 \pm 20}\ \mathrm{GeV}/c^2.$$

The latter error covers the present uncertainty in M_Z as well.

7 Determination of M_Z from PEP/PETRA/TRISTAN data

At the highest TRISTAN energies Z^0 exchange does increase R already by 50 %, thus allowing a direct measurement of M_Z. The sensitivity to this direct propagator effect is demonstrated in Fig. 7 (from Ref.[4]): it shows the error on the fitted M_Z as function of the maximum available centre of mass energy for fixed values of the couplings, i.e. $\sin^2\theta_W$, so the only sensitivity is through the propagator.

The results on M_Z are rather sensitive to radiative corrections, which depend on M_Z and M_{top}, as shown in Fig. 8. The curves represent the ratio of the cross section including radiative corrections and the Born cross section, defined by Eqs. 1-3 with $K = K_2$ and $\kappa_2 = 1$. The

$\sqrt{s}$	σ_B (nb)	$\frac{\sigma_1}{\sigma_B}$	$\frac{\sigma_1^{exp}}{\sigma_B}$	$\frac{\sigma_2}{\sigma_B}$	$\frac{\sigma_2^{exp}}{\sigma_B}$	$\frac{\sigma_1^{exp}}{\sigma_2}$	$\frac{\sigma_2^{exp}}{\sigma_2}$
20	0.2172	1.341	1.367	1.369	1.367	0.998	0.999
30	0.0966	1.360	1.390	1.393	1.390	0.998	0.998
40	0.0546	1.372	1.406	1.408	1.406	0.998	0.998
50	0.0355	1.378	1.413	1.416	1.413	0.998	0.998
60	0.0258	1.370	1.405	1.408	1.405	0.998	0.998
70	0.0222	1.319	1.351	1.352	1.350	0.999	0.998
80	0.0326	1.135	1.152	1.152	1.150	1.000	0.998
87	0.1598	0.882	0.897	0.898	0.894	0.999	0.995
88	0.2636	0.836	0.856	0.857	0.852	0.998	0.994
89	0.5074	0.778	0.808	0.811	0.805	0.996	0.992
90	1.1593	0.701	0.751	0.755	0.748	0.995	0.990
91	2.0002	0.694	0.743	0.742	0.740	1.001	0.997
92	1.1416	0.980	0.943	0.930	0.940	1.014	1.010
93	0.5126	1.311	1.215	1.195	1.211	1.017	1.013
94	0.2732	1.597	1.472	1.451	1.468	1.015	1.012
95	0.1688	1.841	1.706	1.687	1.703	1.011	1.010
96	0.1155	2.050	1.915	1.899	1.913	1.008	1.007
97	0.0849	2.228	2.099	2.088	2.098	1.005	1.005

Table 3: **The total μ-pair cross sections after initial state radiation and electroweak radiative corrections, expressed in units of the Born cross section σ_B, defined as the cross section without any loop- or vertex corrections. The maximum energy loss is limited to 0.99E$_{beam}$. Various levels of initial state radiation are compared (see text). Note the close agreement between σ_1^{exp} and σ_2^{exp}, indicating that simply exponentiating the first order cross section is enough. Note too the difference between first and second order, which is appreciable at all energies, not only near the resonance. The input parameters correspond to $M_Z = 91$ GeV/c^2, $M_H = M_t = 100$ GeV/c^2, and α_s=0.12.**

M_H (GeV/c^2)	M_{top} (GeV/c^2)
10	100 ± 35
100	115 ± 35
500	134 ± 33
1000	143 ± 32

Table 4: **M_{top} values for various M_H values from the Sirlin relation with M_Z=91.11±0.23 GeV/$c^2$and $\sin^2\theta_W$ =0.2293±0.003±0.0028. The errors on M_{top} stem from the uncertainty in $\sin^2\theta_W$. The uncertainty from M_Z introduces an additional error of 15 GeV/c^2.**

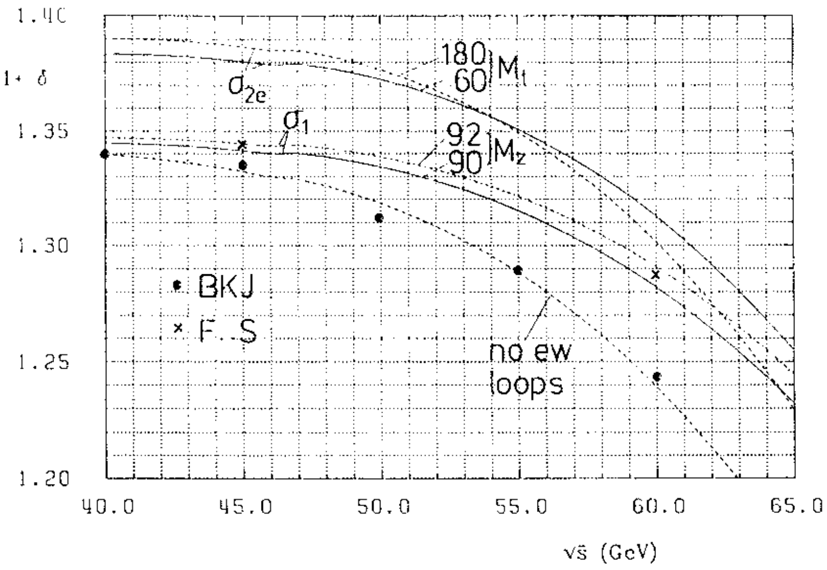


Figure 8: **Radiative correction factors calculated in first and second order for k_{max} = 0.99. The M_Z mass is 90 GeV/$c^2$and the top mass 60 GeV/c^2, unless indicated differently. The lowest curve excludes the Z^0 selfenergy and is comparable with the BKJ radiative corrections as implemented in the Lund Monte Carlo (black dots). The first order full electroweak corrections from Fujimoto et al. (FS) (crosses) are in good agreement with the results from Burgers and Hollik (curves).**

second order exponentiated curve has been calculated with the program ZHADRO from Burgers and Hollik [21] and the other ones with suitably modified versions. The differences originate from the following physics:

- the lower curve corresponds to initial state and vertex corrections for both γ and Z^0 exchange, but only loop corrections for γ exchange, thus excluding the self energy of the Z^0 propagator. The Z^0-mass was taken to be 90 GeV, the Higgs mass 100 GeV and the top mass 60 GeV. Second order QCD corrections have been included using $\alpha_s = 0.12$). Only first order QED graphs have been taken into account ($O(\alpha^3)$). The maximum photon energy allowed corresponds to $k_{max} = E_\gamma/E_{beam} = 0.99$, except for the b-quark, where k_{max} corresponds to the kinematical limit assuming $m_b = 4.7$ GeV. For the other light quarks u, d, s, and c we assumed masses of 0.04, 0.065, 0.3, and 1.5 GeV, respectively. These quark masses have been used to calculate the vacuum polarization. Since the vertex corrections for the Z^0-propagator are small in the on-shell renormalization scheme of Böhm et al. [39], this curve is very close to the results from the well known Berends, Kleis, Jadach (BKJ) results [19] as implemented in the Lund Monte Carlo. These BKJ results, indicated in the figure as solid dots, ignore the Z^0 self

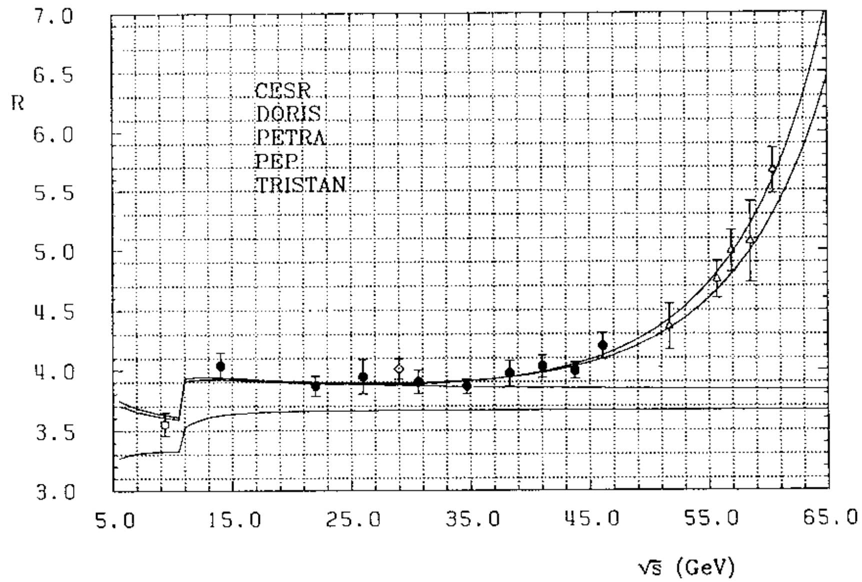


Figure 9: **Experimental data (fully corrected for electroweak contributions) and the Standard Model expectations for M_Z = 88.9 (highest curve) and M_Z = 91.11 GeV/c^2, both using $\sin^2\theta_W$ = 0.2293 and Γ_{tot}=2.5 GeV/c^2. The two lowest curves indicate the contributions from the parton model and the QCD contribution, respectively.**

energy too.

- The two middle curves for two Z^0 masses (90 and 92 GeV/c^2) include in addition the loop corrections from the Z^0-propagator. The sensitivity comes mainly from the graphs with initial state radiation, which depend on the cross section at the energy after radiation and are therefore sensitive to the shape of the cross section, i.e. to the Z^0 mass. The results agree with the calculations from Fujimoto et al.[40], which have been indicated as crosses in the figure. These crosses were calculated for M_Z=91.9 and M_{top}=45 GeV/$c^2$and include final state radiation. It should be noted that in these programs final state radiative corrections are large and cannot be neglected, as is the case for the calculations of Hollik et al.[20]. The reason is the different wave function renormalization, which shifts part of the electroweak loop corrections into the vertex corrections; however, the final corrections in both calculations are very close.

As long as M_Z is poorly known, the radiative corrections cannot be calculated accurately. One only can use the available data to estimate M_Z, apply the corresponding radiative corrections, and iterate this procedure untill a stable value of M_Z has been found. Since the radiative corrections depend on M_{top} too, this procedure has to be repeated for every top mass. The M_Z values found for M_{top} between 60 and 180 GeV/c^2 range between 88.1 and 89.3 GeV/c^2[4,12]. Since the error on each value of M_Z is at least 1 GeV/c^2, there is no strong discrepancy (less than 3 s.d.) with the M_Z value of 91.11 GeV/c^2 from SLC. Therefore, we will use this latter value to calculate the radiative corrections more precisely and use for M_{top} the value given in the previous section ($M_{top} = 120 \pm 35 \pm 20$ GeV/c^2).

The result from a 3 parameter fit to the data on R between 7 and 60.8 GeV is:

$$
\begin{aligned}
M_Z &= 89.3 \pm 1.4 \text{ GeV}/c^2 \\
\alpha_s(34\ GeV) &= 0.142 \pm 0.018 \\
\sin^2\theta_W &= 0.217^{+0.024}_{-0.019}
\end{aligned}
$$

The value of $\sin^2\theta_W$ agrees with the more precise value from deep inelastic neutrino scattering[38]: $\sin^2\theta_W = 0.2293 \pm 0.003 \pm 0.0028$. If we use this value for $\sin^2\theta_W$, we find:

$$
\begin{aligned}
M_Z &= 88.9 \pm 1.2 \text{ GeV}/c^2 \\
\alpha_s(34\ GeV) &= 0.137 \pm 0.017
\end{aligned}
$$

The uncertainty from the top mass entering into the radiative corrections is less than 0.1GeV/c^2, if we use $M_{top} = 120 \pm 35 \pm 20$ GeV/c^2 (see previous Section).

Here we included the data as presented at the Topical Conference at KEK[41]. These data, which are of a preliminary nature and the results should be considered accordingly, included full electroweak corrections using M_{top}=45 GeV/$c^2$and M_Z=91.8 GeV/c^2. However, the results are in excellent agreement with older data, as presented at the Munich Conference[5] and after taking the different radiative correction factors into account [4].

The correlation coefficients between the 3 parameters are (independent of the top mass): $\rho(\sin^2\theta_W, M_Z) = -0.57$, $\rho(\Lambda^{(5)}_{\overline{MS}}, \sin^2\theta_W) = -0.43$ and $\rho(\Lambda^{(5)}_{\overline{MS}}, M_Z) = 0.41$. The χ^2/DF is 70/101; this excellent χ^2 comes mainly from the fact that the common normalization errors might have been overestimated. If we calculate the χ^2 only from the diagonal elements of the matrix V^{-1}, thus ignoring the correlations, but including the complete errors, we find χ^2_D/DF is 94/101.

Fig. 9 shows the fit results together with the expectations from the SM with M_Z=88.9 and 91.11 GeV/c^2, Γ_{tot}=2.5 GeV/c^2, and $\sin^2\theta_W$ =0.2293. For clarity we have averaged the data points within certain energy bins in the following way: we have fitted a constant value to the data points within a certain energy bin using the complete error correlation matrix. This procedure is equivalent to calculating a weighted average by taking correctly into account independent and correlated errors. So the error bars represent the total errors including the correlation and the data have not been renormalized.

One clearly sees that the TRISTAN data is above the fit expected for M_Z=91.11 GeV/c^2. The discrepancy is about 2 standard deviations. There are several possibilities to explain these differences:

- The difference is just a statistical fluctuation: 2 s.d. effects occur in 5% of all comparisons of independent measurements.

- The TRISTAN data is systematically too high for some unknown reason. In the fit we have assumed a 5% normalization error for each experiment and this error is included in the error on M_Z. We have checked that all experiments are consistent with the common fit, so it is not a single experiment, which causes M_Z to come out low. It is hard to explain the difference by a common normalization problem for all TRISTAN

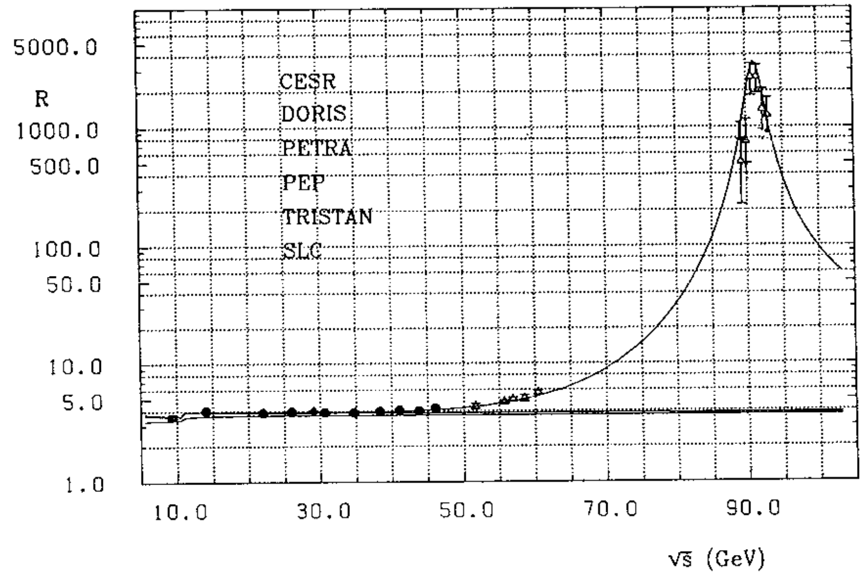


Figure 10: **R values between 7 and 93 GeV (fully corrected) and Standard Model prediction for $\sin^2\theta_W$ =0.2293, M_Z=91.11, and Γ_{tot}=2.5 GeV/c^2.**

experiments for the simple reason that M_Z is determined mainly by the increase in the cross section at the highest energies, so it is the shape, which is important, not the overall normalization. If one lowers all TRISTAN data simultaneously by as much as 6%, one gets M_Z=91 GeV/c^2. However, the χ^2 gets considerably worse in that case, since the lowest TRISTAN data start to disagree with the highest PETRA data.

- The most interesting explanation would be the contribution of new physics. One possible suggestion is the existence of a new heavy neutral gauge boson (Z')[14], which would be able to explain simultaneously a too high hadronic cross section and a too low leptonic cross section; The μ-pair cross section from the combined TRISTAN data is indeed on the low side[13]. If one uses both hadronic and leptonic data from TRISTAN in the fit for M_Z, the value becomes 90.4 GeV[13].

8 Comparison between PEP/PETRA/TRISTAN and SLC data on R

Fig. 10 shows all data on R for centre of mass energies between 7 and 93 GeV. In order to compare the 'low' energy data with the published SLC data, we subtracted the small leptonic 'background' contribution in the latter and applied the second order exponentiated radiative corrections using the program ZHADRO from G. Burgers and W. Hollik[21]. We used

Data	Energy range	$O(\alpha_s^2)$	$O(\alpha_s^3)$
PEP, PETRA	$14 - 47\ GeV$	$\alpha_s = 0.167 \pm 0.025$	$\alpha_s = 0.151 \pm 0.020$
		$\Lambda_{\overline{MS}}^{(5)} = 580^{+470}_{-330}\ MeV$	$\Lambda_{\overline{MS}}^{(5)} = 330^{+240}_{-180}\ MeV$
PEP, PETRA,	$14 - 61\ GeV$	$\alpha_s = 0.172 \pm 0.024$	$\alpha_s = 0.155 \pm 0.018$
TRISTAN		$\Lambda_{\overline{MS}}^{(5)} = 650^{+450}_{-340}\ MeV$	$\Lambda_{\overline{MS}}^{(5)} = 370^{+230}_{-190}\ MeV$
CESR, DORIS,			
PEP, PETRA,	$7 - 61\ GeV$	$\alpha_s = 0.159 \pm 0.019$	$\alpha_s = 0.144 \pm 0.015$
TRISTAN		$\Lambda_{\overline{MS}}^{(5)} = 460^{+290}_{-230}\ MeV$	$\Lambda_{\overline{MS}}^{(5)} = 260^{+160}_{-130}\ MeV$

Table 5: $\alpha_s(34\ GeV)$ and $\Lambda_{\overline{MS}}^{(5)}$ fitted with the G_F parametrization.

M_{top}=115 GeV/c^{2}and M_H=100 GeV/c^2 and assumed that the detection efficiency, calculated for first order radiative corrections, does not change in second order, which is certainly a good approximation with the present statistical errors.

The curve in Fig. 10 corresponds to $M_Z = 91.11$ GeV/c^2 and $\Gamma_{tot} = 2.5$ GeV/c^2. Here we have used the SM width (Γ_{tot}=2.5 GeV/c^2), which is close to the width calculated from the fitted number of neutrino species: $\Gamma_{tot} = 2.63 \pm 0.23$ GeV/c^2 corresponding to $N_\nu = 3.8 \pm 1.4$[1]. The directly fitted width ($1.61^{+0.60}_{-0.43}$ GeV/c^2[1]) is slightly lower, because of the different assumptions involved: in the first case one assumed all couplings known (Table 2), while in the fit of Γ_{tot} one only assumed the lepton couplings to be known. In the latter case the couplings to hadrons are determined by: $\Gamma_f = \Gamma_h + f(\Gamma_\mu + \Gamma_\tau) = \Gamma_{tot} - (1-f)(\Gamma_\mu + \Gamma_\tau) - \Gamma_e - N_\nu\Gamma_\nu$ with f=0.556. Γ_{tot}=1.61 leads to $\Gamma_h \approx 0.8$ GeV/c^2, which has to be compared with 1.6 GeV expected for 5 quarks and 3 colours. However, neither value of Γ_{tot} (1.6 or 2.6) deviates by more than 1.5 s.d. from the expected SM value of 2.5 GeV/c^2, so there is no disagreement. One just has to wait for more statistics, especially to get better limits on the number of neutrino species.

9 Determination of α_s

For the determination of α_s it is convenient to use the G_F parametrization (i.e. combining Eqs. 1-3 and 6,10), which makes the results insensitive to M_Z and the unkown top mass owing to the compensation in κ_1. In Table 5a we give the fit results of α_s with the G_F parametrization. We fixed $\sin^2\theta_W$ to the world average, since all fits for $\sin^2\theta_W$ are consistent with that value (0.2293[38]). We used M_Z=91.11 GeV/c^2 and M_{top}=100 GeV/c^2, but as mentioned before, the results are insensitive to the latter values. The values of α_s obtained from the different energy regimes are consistent. From the fitted scale parameter $\Lambda_{\overline{MS}} = 0.260^{+0.160}_{-0.130}$ GeV one finds in third order:

$$\alpha_s(34\ GeV) = 0.144 \pm 0.015$$

$$\alpha_s(91\ GeV) \quad 0.122 \pm 0.011$$

Here we have taken the third order QCD corrections into account, which lower the α_s values about 10% (see Table 5) with respect to the second order one.

It is interesting to determine the QCD contribution independently from the definition of α_s. If one assumes a linear energy dependence within the energy range, one finds this contribution to be[4]: $f_{QCD}(\sqrt{s} = 34\ GeV) = 1.057 \pm 0.008$. An extrapolation of this value to the LEP/SLC energy range yields[4] $f_{QCD}(\sqrt{s} = 91\ GeV) = 1.046 \pm 0.006$, which is an experimental number for the infinite series $(1 + \alpha_s/\pi + ...)$.

10 Scale dependence of α_s

In the definition of α_s we have made the usual choice $Q^2 = \sqrt{s}$, since this is the only large scale in the process. However, one may argue that gluon radiation occurs at a much smaller scale and there is no reason to use such a large scale. A large amount of literature exists with arguments for the choice of scale[25,42]. The main arguments are based on the size of the higher order corrections and/or the sensitivity to the renormalization scheme. It is interesting to study the scale dependence for R, since this is the first quantity for which both the second and third order contributions have been calculated, so one can study the scale dependence in second order and check if there are scales for which the third order corrections are small. We have studied these contributions as a function of the scale in contrast to previous results for a few specific scales[43]. This can be done easily as follows. Suppose a variable is given in a certain renormalization scheme and for a given Q^2 scale to be:

$$R = r_1\alpha_s + r_2\alpha_s^2 + r_3\alpha_s^3 + O(\alpha_s^4) \tag{30}$$

If we choose the coupling at a different scale, we get:

$$R' = r_1'\alpha_s' + r_2'\alpha_s'^2 + r_3'\alpha_s'^3 + O(\alpha_s'^4) \tag{31}$$

If one neglects the terms of order $O(\alpha_s^4)$, then

$$R' - R = dR = r_1 d\alpha_s + \alpha_s dr_1 + \alpha_s^2 dr_2 + 2\alpha_s r_2 d\alpha_s + \alpha_s^3 dr_3 = 0 \tag{32}$$

This can only be zero, if the coefficient for each power of α_s equals zero, which yields 3 equations for the 3 unknowns r_1', r_2', r_3'. After calculating $d\alpha_s$ from the renormalization group equation:

$$\mu\frac{\partial\alpha_s(\mu)}{\partial\mu} = -\beta_0\alpha_s^2(\mu) - \beta_1\alpha_s^3(\mu) - \beta_2\alpha_s^4(\mu) + O(\alpha_s^5), \tag{33}$$

we find for the coefficients at a different scale $Q' = xQ$:

$$\begin{aligned} r_1' &= r_1 \\ r_2' &= r_2 + r_1\beta_0 \ln x \\ r_3' &= r_3 + r_1\beta_1 \ln x + 2r_2\beta_0 \ln x \end{aligned} \tag{34}$$

The β-factors are given by[44]:

$$\beta_0 = \frac{1}{6\pi}(33 - 2n_f)$$

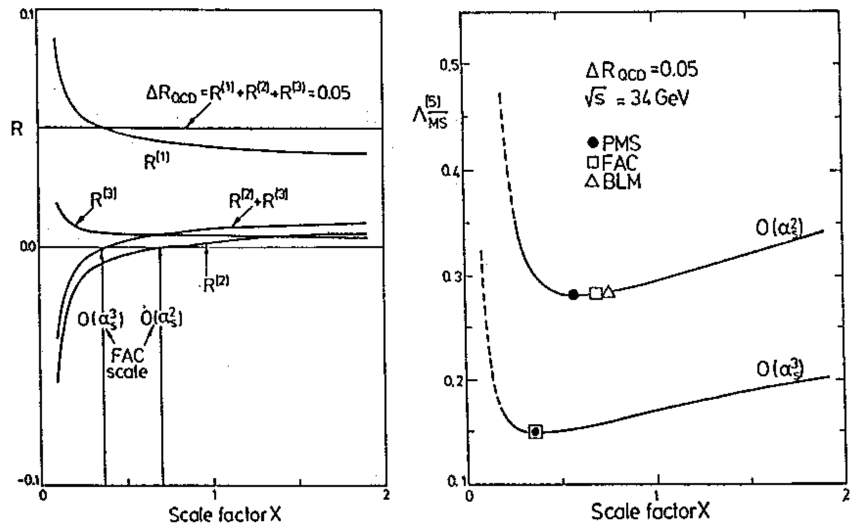


Figure 11: **QCD contributions to R in first, second, and third order ($R^{(1)}$, $R^{(2)}$, $R^{(3)}$, respectively) (a) and Λ determinations in second - and third order as a function of the renormalization scale factor x for R_{QCD}=0.05 (b).**

$$\beta_1 = \frac{1}{12\pi^2}[153 - 19n_f]$$
$$\beta_2 = \frac{1}{64\pi^3}\left[2857 - 5033n_f/9 + 325n_f^2/27\right]$$

The first two β-factors are independent of the renormalization scale. The scale dependence of the last one can be calculated by means of Stevenson's invariants[25]. The QCD contributions to R in first, second, and third order have been displayed in Fig. 11a as function of the renormalization factor x, assuming the total contribution to be constant ($\Delta R_{QCD} = 0.05$). One observes that at $x = 1$ the third order contribution is indeed larger than the second order contribution, but at small and large x the absolute value of the second order contribution is larger. However, at all scales the first order contribution is dominant. We have indicated the scales at which the second order or the second plus third order contributions become zero. These are the so-called FAC scales (Fastest Apparent Convergence)[45]. After recalculating the coefficients at a new scale, one can redetermine the corresponding α_s from the measured R-value and recalculate the corresponding Λ value. The result is shown in Fig. 11b. The minimum in this curve is the PMS scale corresponding to the point of minimal sensitivity, a concept introduced by Stevenson[25]. We have indicated too the scale advocated by Brodsky, Lepage, and Mackenzie (BLM)[46]. One observes that at all scales the Λ values in third order are roughly a factor two below the Λ values in second order, indicating that for this reaction there is nothing like an optimum scale, where the higher orders are not important.

This clearly indicates that all the heated discussions about choosing a certain scale mean

nothing more than betting on the future: you only can say something seriously about higher order contributions by calculating them, not by fiddling with renormalization scales or schemes. Nevertheless, studying the scale dependence in a given order is still very useful, since a strong scale dependence indicates large higher order contributions. On the other hand, a small scale dependence does not guarantee that the higher orders are negligible, as can be seen from the above example of R.

11 Summary

We have determined the Standard Model parameters α_s, M_Z, and $\sin^2\theta_W$ from a 3 parameter fit to the data on R between 7 and 60.8 GeV. The result is:

$$\begin{aligned} M_Z &= 89.3 \pm 1.4\ \mathrm{GeV}/c^2 \\ \alpha_s(34\ GeV) &= 0.142 \pm 0.018 \\ \sin^2\theta_W &= 0.217^{+0.024}_{-0.019} \end{aligned}$$

The value of $\sin^2\theta_W$ is in good agreement with the more precise value from deep inelastic neutrino scattering[38]: $\sin^2\theta_W = 0.2293 \pm 0.003 \pm 0.0028$. If we use this value for $\sin^2\theta_W$, we find:

$$\begin{aligned} M_Z &= 88.9 \pm 1.2\ \mathrm{GeV}/c^2 \\ \alpha_s(34\ GeV) &= 0.137 \pm 0.017 \end{aligned}$$

The error from the uncertainty in the radiative corrections is only 0.1 GeV/c^2, if we calculate these corrections for a fixed M_Z (91.1 GeV/c^2) and use

$$M_{top} = 120 \pm 35 \pm 20\ \mathrm{GeV}/c^2,$$

as obtained from a fit to M_Z and $\sin^2\theta_W$ from deep inelastic scattering using the Sirlin relation Eq. 14.

The SM parametrization of the cross section with the Fermi constant is insensitive to M_Z and M_{top}. This can be exploited to determine the strong coupling constant almost independently of M_Z and M_{top}. From the fitted scale parameter $\Lambda_{\overline{MS}} = 0.260^{+0.160}_{-0.130}$ GeV one finds:

$$\begin{aligned} \alpha_s(34\ GeV) &= 0.144 \pm 0.015 \\ \alpha_s(91\ GeV) &= 0.122 \pm 0.011 \end{aligned}$$

Here we have taken the third order QCD corrections into account, which lower the α_s values about 10% (see Table 5) with respect to the second order one.

The QCD series $1 + \alpha_s/\pi + ...$ has been determined from a direct fit to the data in a model independent way. Extrapolating to the Z^0 region, one finds this factor to be 1.046 ± 0.006[4]. We study the effect of changing the renormalization scale of α_s and find that the third order QCD corrections, which are larger than the second order ones for the usual scale $Q = \sqrt{s}$, are smaller than the second order contributions at other scales.

The M_Z value from 'low'' energy data is about 2 standard deviations below the one from 'on-resonance' data, if one only takes hadronic data. On the other hand, if one includes both hadronic and leptonic data from TRISTAN in the fit for M_Z, the value becomes 90.4 GeV/c^2[13]. This indicates that the hadronic data is below the expectation of the SM, while the leptonic data is below, which is the possible signature for a second neutral heavy gauge boson[14]. A second boson will have little effect on the resonance, but could show up below and should show up in $p\bar{p}$ collisions. Clearly, the whole effect is statistically not very compelling, but it will be interesting to watch future data from TRISTAN at somewhat higher energies.

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