DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 89-115
HLRZ 89-45
September 1989

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ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

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Abstract

Investigation of the coupled Higgs and gauge fields on the lattice has elucidated the gauge invariant formulation and several non-perturbative aspects of the Higgs mechanism, in particular its properties for strong gauge coupling and its relationship to confinement. However, until now no indication has been found for the gauge field to inhibit the vanishing of the Φ^4 coupling in the limit of infinite cut-off. The scalar sector dominates the properties of the Higgs mechanism, and the cut-off cannot be removed.

With the gauge sector treated therefore only perturbatively, extensive analytic and numerical calculations in the pure scalar sector of the Standard Model have been performed recently on the lattice. The results indicate that the cut-off parameter in this regularization can be substantially greater than the Higgs boson mass only if this mass is not much bigger than 640 GeV, and the scalar sector is not strongly interacting.

Lattice studies of the Higgs system including fermions have been initiated. Modifications of the phase diagram due to the spontaneous chiral symmetry breaking for fermions with vectorial gauge coupling have been observed. In models with strong Yukawa coupling the fermion masses increase as the expectation value of the Higgs field decreases. Therefore a method of putting chiral fermions on the lattice in a gauge invariant way and removing the unwanted fermion doublers by means of a strong coupling of the Wilson-Yukawa type to the Higgs field is very promising and currently under investigation.

Review talk given at the 8th INFN Eloisatron Project Workshop *Higgs Particle(s) - Physics Issues and Searches in High Energy Collisions* in Erice, July 1989. To be published, ed. A. Ali, Plenum.

Contents

1 Introduction

According to our contemporary knowledge, in the SU(2)⊗U(1) gauge theory of electroweak interactions the masses of gauge bosons and fermions are generated via the Higgs mechanism [1], which relies on the occurence of spontaneous symmetry breaking (SSB) in the 4-dimensional Φ^4 theory. However, the work of Wilson, Aizenman, Fröhlich, Lang [2] and of many others (for a review see [3]) strongly suggests (though a rigorous proof is still lacking) that this theory is non-interacting ("trivial") when a regularization parameter is removed. It seems, therefore, that we have to accept that the electroweak theory is an "effective theory", containing inherently some finite cut-off parameter Λ.

Thus it is quite natural to investigate the electroweak theory on the euclidean space-time lattice with a finite lattice spacing $a = \Lambda^{-1}$. In addition, the lattice regularization has two important technical advantages: it preserves gauge invariance [4] and allows the use of non-perturbative computational methods like the strong coupling expansion and, in particular, the numerical Monte Carlo simulation. These methods, as well as several very useful concepts (phase transition, critical behaviour, correlation, and, in particular, universality) originate in statistical mechanics, which contributes in a profound way to our modern understanding of quantum field theory.

The exciting possibility is that Λ is not far beyond the reach of the next generation accelerators, as we would then have to invent a new theory soon. An indication for such a low value of Λ would be a large mass of the Higgs boson, as the renormalized quartic self-interaction of the scalar field would have to be rather large. Similar reasoning might apply also to a large mass of the top quark or some further heavy fermion. Thus there is a strong motivation to investigate the Higgs systems non-perturbatively with the lattice methods and to determine how large the Higgs boson mass and the quartic coupling could actually be if the cut-off parameter should be substantially larger than the Higgs boson mass.

Another possibility is that non-perturbative effects of the gauge or fermion fields change the non-interacting character of the Φ^4 theory and that the cut-off can be removed. Also this requires a lattice investigation.

This lecture reviews the results and present activities in the lattice studies of the Higgs systems, both without and with fermion fields. It is intended for non-specialists in lattice gauge theories. There exist several reviews which elaborate on various aspects of the subject of this lecture in much more detail [5-13] and I shall refer to them in the corresponding parts of this text.

2 Non-perturbative properties of the lattice Higgs models

The lattice Higgs models (coupled scalar and gauge fields) have been the subject of interest since the beginning of the lattice gauge theory in 1974 [4], and the numerical studies started nearly 10 years ago. I can describe only briefly the most important results. Numerous further details can be found in the review articles [5-7,9,13].

2.1 Formulation of the Higgs systems on the lattice

The lattice Higgs models are defined by means of the path integral on the euclidean hypercubic lattice. Let us first summarize the notations we use for the scalar fields on such a lattice:

$x = (\vec{x}, x_4)$: sites on the euclidean space-time lattice
$V = L^4$: lattice volume or size
a : lattice constant
$\mu = 1, \ldots, 4$: directions on the lattice
$x + \mu$: site which is the nearest neighbour to x in direction μ
φ_x^α : dimensional real scalar field defined at sites
$\alpha = 0, \ldots, 3$: O(4) index (absent for the one-component field)
$\Phi_x^\alpha = a.\varphi_x^\alpha/\sqrt{2\kappa}$: dimensionless real scalar field defined at sites
$\hat{\Phi}_x$: 2×2 matrix composed of the components Φ_x^α
κ and λ : bare hopping parameter and quartic coupling
$(\Phi_{x+\mu} - \Phi_x)/a$: lattice derivative.

It is usual to interpret $\Lambda = 1/a$ or sometimes π/a (the maximal value of one component of the momentum in the Brillouin zone) as a cut-off parameter.

The Lagrangian density of a one-component scalar field on the lattice can have, up to a simple transcription of the derivative, the same form as in the euclidean continuum,

$$\sum_\mu \frac{1}{2}\left(\frac{\varphi_{x+\mu} - \varphi_x}{a}\right)^2 + \frac{1}{2}m_0^2\varphi_x^2 + \frac{1}{4!}g_0\varphi_x^4. \qquad (2.1)$$

But on the lattice it is convenient to reparametrize the scalar field theory by rescaling the field and the quartic coupling and introducing the hopping parameter κ instead of m_0^2:

$$\varphi_x = \sqrt{2\kappa}\,\Phi_x/a, \qquad (2.2)$$

$$g_0 = \frac{6\lambda}{\kappa^2}, \qquad (2.3)$$

$$m_0^2 = \frac{1 - 2\lambda - 8\kappa}{a^2\kappa}. \qquad (2.4)$$

The action is then

$$S_\Phi = -2\kappa \sum_{x\mu} \Phi_x\Phi_{x+\mu} + \lambda \sum_x (\Phi_x^2 - 1)^2 + \sum_x \Phi_x^2. \qquad (2.5)$$

As is usual on the lattice, we mostly set numerically $a = 1$, i.e. the dimensional quantities will be determined in lattice units.

To get some feeling for the meaning of the values of the dimensionless hopping parameter κ ($\kappa \geq 0$) we note that the case $g_0 = 0$, $m_0^2 = 0$ corresponds to $\kappa = 1/8$. The limits $m_0^2 \to +\infty$ and $m_0^2 \to -\infty$ at fixed g_0 correspond to $\kappa \to 0$ and $\kappa \to \infty$, respectively. This can be easily derived from the relations (2.3) and (2.4).

Thus we have obtained a system which is very similar to the statistical mechanics models. An approach to the continuum limit, i.e. the cut-off removal, requires that some correlation length ξ grows much larger than a (scaling region). This is possible only in the vicinity of a critical point corresponding to a phase transition of second or higher order. As $m = 1/\xi$ has the physical interpretation of a mass, it means that some mass in lattice units has to approach zero, $am \to 0$. Dimensionless ratios of physical observables like masses should approach constants during the continuum limit.

The first term in the action (2.5) is the nearest neighbour (nn) coupling between the field variables. Let me remark that one can put the scalar field on a lattice in many different ways, varying e.g. the lattice geometry or including other coupling terms like a next-to-the-nearest-neighbour coupling. According to the universality hypothesis, these differences are unimportant in the scaling regions and the continuum limits should be the same.

It is instructive to consider the model (2.5) in the limit

$$\lambda \longrightarrow \infty, \quad \kappa \text{ fixed.} \tag{2.6}$$

The factor $exp\{-\lambda(\Phi_x^2 - 1)^2\}$ in each integral $\int d\Phi_x$ of the path integral vanishes unless $\Phi_x^2 = 1$. Thus in the sum over the field configurations only those with all

$$\Phi_x = \pm 1 \tag{2.7}$$

do really contribute. Therefore the path integral reduces to the partition function of the Ising model on the $d = 4$ lattice. The parameter κ corresponds to the inverse temperature in this model.

As we know from statistical physics that the Ising model has a second order phase transition, we conlude that the scalar field theory on the $d = 4$ lattice has for $\lambda = \infty$ a critical point

$$\kappa = \kappa_c = 0.0748. \tag{2.8}$$

For $\kappa < \kappa_c$ the system is disordered, whereas for $\kappa > \kappa_c$ the symmetry symmetry is broken spontaneously, $\langle\Phi\rangle \neq 0$.

It has been demonstrated non-perturbatively (cf.[14]) that a similar situation arises also for λ finite. Thus there is a line of critical points $\kappa = \kappa_c(\lambda)$ in the κ, λ plane, and equivalently in the m_0^2, g_0 plane. Therefore the lattice Φ^4 theory in $d = 4$ has two phases: the symmetric phase and the SSB phase. Warning: There is no SSB in the continuum limit of the lattice Φ^4 theory as in this limit the Φ^4 theory is presumably non-interacting in four-dimensional space-time.

A generalization to the 4-component scalar field model is straightforward. The action is in this case

$$S = -2\kappa \sum_{x\mu} \Phi_x^\alpha \Phi_{x+\mu}^\alpha + \lambda \sum_x \left[(\Phi_x^\alpha)^2 - 1\right]^2 + \sum_x (\Phi_x^\alpha)^2, \tag{2.9}$$

where the summation over α is implied. In the $\lambda = \infty$ limit it reduces to the O(4) non-linear σ-model with $\kappa_c = 0.304$.

The gauge fields are introduced following the original proposal of Wilson [4]. The parallel transporters $U_{x,\mu}$, called link variables or simply gauge fields, are defined on the lattice links connecting the neighbour points x and $x + \mu$. Their values are from the fundamental

representation G of a unitary gauge group and their relation to the gauge potentials $A_\mu(x)$ is, for small a,

$$U_{x,\mu} \simeq e^{-igaA_\mu(x)}, \tag{2.10}$$

g being the gauge coupling constant. Note that on the lattice *no gauge fixing is necessary* as the path integral is performed over the compact manifold of the gauge group. The local gauge transformations are, in the typical cases,

$$U_{x,\mu} \to G_x\, U_{x,\mu} G_{x+\mu}^{-1}, \tag{2.11}$$

$$\hat{\Phi}_x \to G_x \hat{\Phi}_x. \tag{2.12}$$

where $\hat{\Phi}_x$ is a suitable combination of the scalar field components transforming according to the fundamental representation G, too. There are numerous lattice Higgs models varying in the choice of the gauge group and also of the representation under which the scalar fields transform [5]. I shall mainly discuss the SU(2) Higgs model arising from the four-component scalar field model (2.9) when the SU(2) gauge field is introduced. The action is

$$\begin{aligned} S_H = & -\frac{\beta}{4} \sum_P Tr(U_P + U_P^\dagger) \\ & -\kappa \sum_x \sum_{\mu=1}^{4} \frac{1}{2} (Tr \hat{\Phi}_x^\dagger U_{x,\mu} \hat{\Phi}_{x+\mu} + c.c.) \\ & +\lambda \sum_x (Tr \hat{\Phi}_x^\dagger \hat{\Phi}_x - 1)^2 + \sum_x Tr \hat{\Phi}_x^\dagger \hat{\Phi}_x \ . \end{aligned} \tag{2.13}$$

Here β is the gauge coupling parameter usually used on the lattice. It is related to the bare continuum coupling constant g,

$$\beta = \frac{4}{g^2}. \tag{2.14}$$

U_P is the product of $U_{x,\mu}$ along a lattice plaquette P and $\hat{\Phi}$ is the 2×2 matrix

$$\hat{\Phi} = \begin{pmatrix} \Phi^0 + i\Phi^1 & i\Phi^3 + \Phi^2 \\ i\Phi^3 - \Phi^2 & \Phi^0 - i\Phi^1 \end{pmatrix} \tag{2.15}$$

which is proportional to a SU(2) matrix. The O(4) symmetry of (2.9) is, due to the isomorphy $O(4) \simeq SU(2) \otimes SU(2)/Z(2)$, extended to the symmetry

$$SU(2)^{(local)} \otimes SU(2)^{(global)} \tag{2.16}$$

of the action (2.13).

2.2 Phase diagram of the SU(2) Higgs model on the lattice

The first step in the investigation of a lattice model is the understanding of its phase structure. I shall only summarize what we know since several years about the model (2.13), and refer to the review articles [5, 7, 9] for details and earlier references. It is actually sufficient to understand the phase structure for a fixed λ when λ is large ($\lambda > 1$), as shown in Fig. 1. The limit cases are:

(i) $\underline{\beta = \infty}$ (i.e. $g = 0$), where the O(4) Φ^4 model with its symmetric and SSB phases and the second order phase transition at $\kappa = \kappa_c(\infty)$ is found.

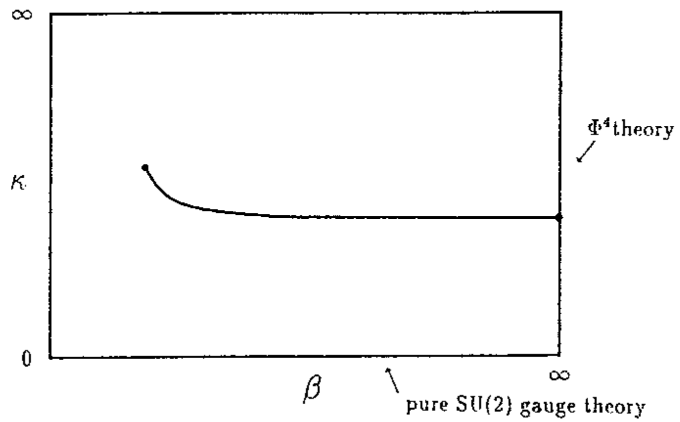


Figure 1: *Schematic phase diagram of the $SU(2)$ Higgs model for fixed large λ. The line of Higgs phase transitions has critical points at both ends, otherwise it is most probably of first order.*

(ii) $\kappa = 0$ (infinitely heavy scalar field), where the model reduces to the pure SU(2) gauge field on the lattice that, as far as we know, has no phase transition and is confining for all β.
(iii) $\beta = 0$ (infinitely strong gauge coupling), where the gauge fields $U_{x,\mu}$ enter the action (2.13) only linearly and can be thus easily integrated out in the path integral. For $\lambda = \infty$ the model is thus easily solved with the result that no phase transition occurs. (This is not so for very small λ, but this fact is of no particular relevance for the continuum physics, so I will not consider that case).
(iv) $\kappa = \infty$, which corresponds to $m_0^2 = -\infty$. The scalar field is deeply in the SSB regime and, as seen in the unitary gauge, the gauge fields are frozen. Thus there is no phase transition there.

The inside of the phase diagram can now be nearly guessed: The SSB phase transition of the pure Φ^4 model at $\beta = \infty$ extends for $\beta < \infty$ as a line of Higgs phase transitions to low β-values, where it ends at certain $\beta > 0$. The region above this line, the ***Higgs region***, is the place where the Higgs mechanism operates. We believe that we understand this region quite well perturbatively with gauge fixing, using e.g. the unitary gauge. It should contain the Higgs boson H and the massive gauge boson triplet W. In particular, $\langle\Phi\rangle \neq 0$.

The region below the line is the *confinement region*, called so because it has a strong resemblance with the lattice QCD. Think about QCD where the SU(3) color gauge group is replaced by SU(2) and the quarks, transforming as the fundamental representation of the gauge group, are replaced by a massive (as κ is small) scalar SU(2) doublet. Up to the spin (and baryons) the analogy is remarkable. As spin should not be relevant for the existence of confinement, this system should be confining. Let us call it "small QCD". From the physical point of view, the most interesting fact, which is established rigorously for local observables [15, 16], is the *analytic connection* between the confinement and Higgs regions.

2.3 Gauge invariant formulation of the Higgs mechanism and its relation to the confinement

If two regions of a phase diagram belong to the same phase, then they must have many similar properties, in particular the spectrum must contain the same states, though the energy differences can be quite different in various regions. Thus we should be able to understand the spectrum in the Higgs region by thinking in the QCD-like concepts. Is this possible?

The "small QCD" tells us that in the confinement region there are "mesons", consisting of scalar "quarks". Confinement implies that there is no free particle ("quark") created from the vacuum by the field Φ. The same must be true also in the Higgs region. Only states which can be produced from the vacuum by gauge invariant operators with compact support like

$$Tr\ \hat{\Phi}_x^\dagger \hat{\Phi}_x, \quad Tr\ \hat{\Phi}_x^\dagger U_{x,\mu} \hat{\Phi}_{x+\mu}, \ \text{etc.,} \tag{2.17}$$

are expected in the asymptotic spectrum. Therefore both the Higgs particle and the intermediate W-boson have to be related to the operators (interpolating fields) like (2.17) [17-20]. This relationship has been discussed rigorously by Fröhlich, Morchio and Strocchi [21]. Note that the operators usually used in the unitary gauge are quite complicated (non-local) if written in a gauge invariant form. For example for the Higgs boson we would have to take the square root of the first expression in (2.17).

There is still another difficulty we have to clarify: The regions are connected analytically, and therefore there exists no local quantity (order parameter) which would vanish in one region and be non-zero in the other. As the model (2.13) is gauge invariant, and on the lattice no gauge fixing is needed, we can calculate $\langle\Phi\rangle$ in the Higgs region in a gauge invariant way and easily convince ourselves that

$$\langle\Phi\rangle \equiv 0 \tag{2.18}$$

identically. This is a consequence of the theorem that there is no spontaneous breakdown of a local gauge symmetry [22]. The usual assumption $\langle\Phi\rangle \neq 0$ is consistent only in some suitable, e.g. unitary gauge.

Thus the possibility of maintaining the gauge invariance forces us to rethink the perturbative picture of the Higgs mechanism, which usually uses the unitary gauge, and sharpen our understanding of the quantum field theory. For example, we are not claiming that the Higgs boson is a composite particle if it is created by a composite operator. A composite operator is a mathematically defined concept, whereas a composite particle in the quantum field theory is an intuitive concept, assuming that we can somehow observe the constituent particles and use them for explaining the properties of the composite particle. Still, the gauge non-invariant Higgs field is not an interpolating field of the Higgs boson (a gauge invariant state). They come together in the unitary gauge only.

Further observation is that we do not have a clear-cut criterion to decide whether the Higgs mechanism operates or not. The non-vanishing gauge invariant quantities like $\langle Tr\hat{\Phi}^\dagger\hat{\Phi}\rangle$, which could substitute for $\langle\Phi\rangle$ in the perturbation theory, do not vanish even below the Higgs phase transition line as they are positive definite. They are only small with respect to their values above this line.

Even more astonishing is the fact that also the W-mass am_W does not vanish below the line, though we do not expect the Higgs mechanism to operate there. Quite to the contrary, it shoots up when κ decreases [23, 24] (Fig. 2). This can be understood within the "small QCD": here W is a kind of "vector meson" which gets heavy as the "quark" mass increases with decresing κ. In the "small QCD" one can also think about "quark"-less states – the gauge invariant glueballs. A glueball with quantum numbers 1^{--} would be a sort of vector boson, too. However, its mass is non-zero, though it remains finite even in the limit $\kappa \to 0$.

One finds that there is no region in the phase diagram where some massless vector boson would exist. The perturbation analysis is, from this point of view, qualitatively wrong, as it ignores non-perturbative effects in the confinement region.

This observation led some time ago Abbott and Farhi [19] to the suggestion that the standard model could be formulated also in the confinement region, without assuming the Higgs mechanism. Indeed, such a "strongly coupled standard model" seems possible, though some problems with the spontaneous chiral symmetry breakdown might arise when fermions are included (see Subsec.4.1). What I want to stress here is that this model is not more confining than the usual Higgs mechanism.

By this statement I mean the concept of confinement in the sense of non-existence of charged states in the asymptotic spectrum. This criterion for confinement in the presence of matter fields has been formulated rigorously by Fredenhagen and Marcu [25, 9], and shown numerically to be valid in both regions of the SU(2) Higgs model [26, 27]. The difference between the confinement in both regions is, roughly speaking, that whereas in the confinemet region an introduced charge is screened by essentially only one heavy scalar "quark" carrying the anticharge as in QCD, in the Higgs region it is screened by the scalar field condensate, as in a plasma. Actually the Higgs mechanism has been suggested long ago by Mack to be a model for confinement in gauge theories [17].

2.4 Calculations of the ratio m_H/m_W in the full Higgs model

The results for m_H and m_W shown in Fig. 2 have been obtained at $\beta = 2.4$ which corresponds to a value of g which is quite far from the weak coupling strength, $g^2 \simeq 0.4$. We have to choose $\beta \simeq 8 - 10$ to approach the physically realistic region, which makes the numerical simulation difficult. Two attempts, by Anna Hasenfratz and Neuhaus [28], and by Langguth and Montvay [29], have been made to calculate the ratio m_H/m_W under these conditions. The idea was to estimate the upper bound for m_H/m_W by choosing $\lambda = \infty$, as here the renormalized quartic coupling is expected to be maximal, and then to determine for this value of λ the possible values of the mass ratio. The result of both works is consistent,

$$m_H/m_W \;\leq\; 9. \tag{2.19}$$

The problem with this result is that its reliability is not sufficiently under control. In particular, it has been pointed out [30] that the results even for smaller values of β can be sizeably influenced by the finite size of the lattice used in the numerical simulation. These finite size effects are caused by the small value of am_W for large β (am_W should vanish at $\beta = \infty$ as W then goes over into the massless Goldstone boson) which means that the correlation length is comparable with the size of the lattice. It is not yet clear how to eliminate such effects. As I will discuss in Sec.3., with the present techniques it is more realistic to study numerically the upper bound on m_H/m_W within the O(4) Φ^4 lattice model.

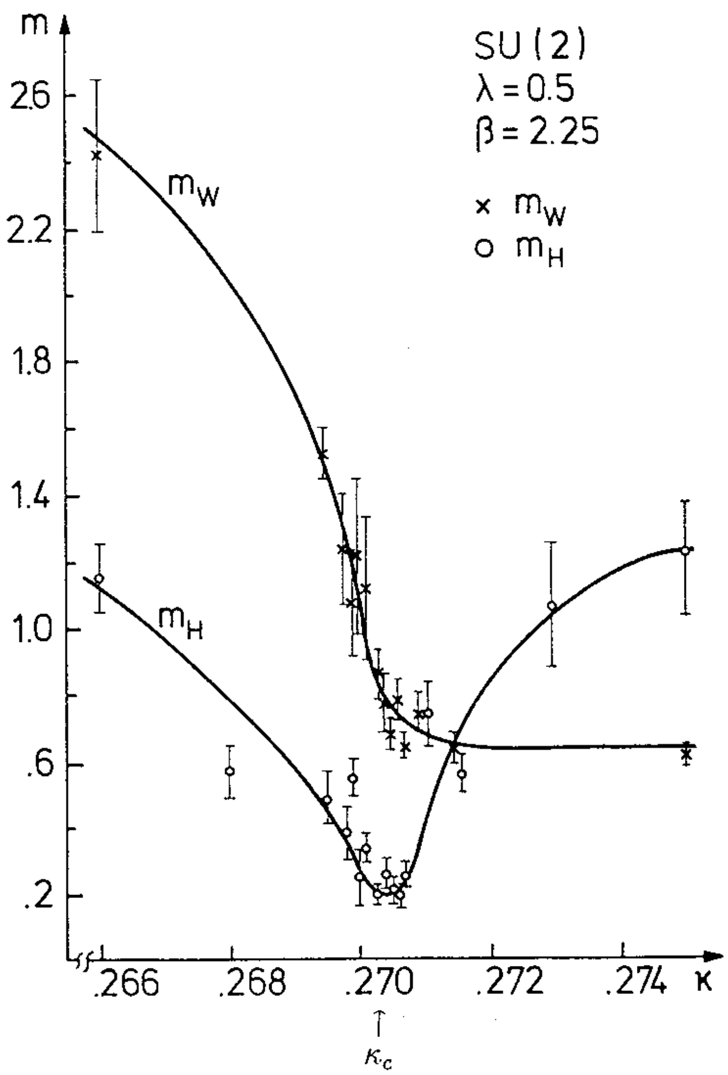


Figure 2: *The κ-dependence of the Higgs boson and W-boson masses in lattice units very close to the Higgs phase transition determined by means of the correlation functions of the operators (2.17). The W-mass is small in the Higgs region immediately above the phase transition, but it rises with increasing κ (not shown in the figure) as expected from perturbative calculations. Below κ_c, in the confinement region, this mass is non-vanishing too, and rises sharply if determined by means of the same correlation function as in the Higgs region. The figure is based on the data obtained in Ref. [24].*

But later one certainly should return to the investigation of the spectrum in the full Higgs model.

2.5 Search for new critical points caused by the presence of gauge fields

Lattice models differ very much from the field theories in continuum unless all correlation lengths (inverse particle masses) are large with respect to the lattice constant. In the language of statistical mechanics it means having the possibility of tuning the bare parameters to the vicinity of some critical point, i.e. of a phase transition of second (or higher) order. Such critical points have to be found, and it has to be investigated whether the behaviour of the system in their vicinity meets the requirements of the continuum physics. Putting the Higgs system on a lattice we create a complex four-dimensional statistical mechanics model (2.13) with 3 coupling parameters β, κ and λ. With the exception of D. P. Landau and J. L. Xu, who are statistical physicists [26, 27, 31], its investigation has remained in the hands of particle physicists, however.

One line of critical points is well known: it is the phase transition line $\kappa_c(\lambda)$ of the pure Φ^4 model at $\beta = \infty$. But most probably [2] the whole line is in the attraction domain of the Gaussian fixed point at $\lambda = 0$, thus yielding a trivial continuum limit. Let me mention that the inclusion of the gauge fields does not change the character of this perturbative fixed point [32]. We may hope that, due to the presence of gauge fields, at some finite β some another, non-perturbative critical point might exist which could lead to an interacting continuum theory [6, 13]. The only candidate is the manifold of the Higgs phase transition. Unfortunately, it has turned out that this transition is of first order nearly everywhere where it has been investigated more closely [5]. This is consistent with the famous calculation by Coleman and E. Weinberg [33]. Exceptions are the endpoints at small β and a region at large λ and β where the investigation is difficult.

The problem is very technical: how to distinguish a second-order phase transition from a weak first-order one on finite lattices? Statistical physicists have a lot of methods available for that, but they require a variation of the linear lattice size in large intervals, which is not possible in four dimensions.

In a recent investigation [31] a region has been found ($\lambda = 0.5$, $\beta > 2.6$) where no signal of a first order behaviour at the Higgs phase transition has been observed even in large calculations, so that some possibility that a critical point somewhere at $2.6 < \beta < \infty$ exists still remains. In an analogous SU(2) Higgs model, with the scalar field in the adjoint representation, some indication for the existence of such a point has been found by the Monte Carlo renormalization group studies [34]. But we shall not know for long.

The critical point at the lower end of the Higgs phase transition line in Fig. 1 has been recently localized rather precisely [31]. Its properties are not yet known, but it is improbable that it is of use for the electroweak theory as its position at $\beta \simeq 2$ presumably implies a too strong renormalized gauge coupling. It might be of a more general field-theoretical interest, however, though some preliminary investigation by means of the renormalization group method indicates the opposite [35].

It thus seems that the most probable region where the lattice Higgs models can contribute

to the electroweak theory is that of large β. Therefore we now want to consider the Φ^4 theory on the lattice more closely.

3 Striving for precision in the Φ^4 theory on the lattice

The main conclusion from the previous section is that the gauge fields probably play only a passive role in the Higgs mechanism, and so it can be expected that this mechanism can be investigated also quantitatively in the $\beta = \infty$ limit of the Higgs models – in the Φ^4 models. Effects of the gauge fields can then be taken into account perturbatively. As suggested by Dashen and Neuberger [36], the scalar sector should be investigated non-perturbatively, however, in order to determine reliably the upper bound on the Higgs boson mass. The existence of such a bound follows from the triviality of the Φ^4 theory. But triviality à priori does not mean that the interaction is weak, it just cannot be arbitrarily strong. How strong it can be, i.e. how high the Higgs boson mass could be, is still a non-perturbative problem, as one has to investigate the extreme case of the largest possible coupling.

Some non-perturbative renormalization group calculations in continuum have been performed by Hasenfratz and Nager [37], but their reliability is not known. The problem seems to be clearly the case for numerical simulations on the lattice. However, it has been demonstrated by Lüscher and Weisz [38, 39] that in the Φ^4 models on the lattice there exists a non-perturbative *analytic* technique which provides reliable and remarkably accurate results even in this extreme situation. The results are quite competitive with high precision Monte Carlo calculations, so I will discuss both. For more details on this topics I refer to the review articles [10, 12].

Let me recall that in order to investigate the upper bound in the Dashen-Neuberger [36] approximation, we have to calculate the ratio

$$R_\Phi = \frac{m_\sigma}{F}, \tag{3.1}$$

where m_σ is the σ-boson (Higgs boson) mass and

$$F = \frac{\langle\Phi\rangle}{\sqrt{Z}} = \langle\Phi_R\rangle, \tag{3.2}$$

Z being the renormalization constant of the field Φ. As the vector boson mass is [10]

$$m_W = \frac{1}{2}\, g_R F, \tag{3.3}$$

where g_R denotes the renormalized SU(2) gauge coupling constant, $g_R^2 \simeq 0.4$, we have finally

$$\frac{m_H}{m_W} = \frac{2R_\Phi}{g_R}. \tag{3.4}$$

Thus in calculations in the O(4) model we have to determine 3 quantities: $\langle\Phi\rangle, Z$ and m_σ. The renormalized coupling in the broken phase can be most conveniently defined [38] as

$$\lambda_R = \frac{1}{2}\, R_\Phi^2, \tag{3.5}$$

which has the virtue that the relation between m_σ, F and λ_R remains the same as on the tree level.

3.1 Analytic results

In a series of papers, Lüscher and Weisz [38, 39] investigated the Φ^4 models with Z(2) and O(N) symmetries and nn coupling on an infinite hypercubic lattice, eqs. (2.5) and (2.9), both in the symmetric and broken phases. Let me describe briefly the crucial steps of their analysis:

i) The (non-perturbative) κ-expansion of several quantities like the renormalized mass m_R and the renormalized coupling λ_R is performed to the 14-th order and truncation errors estimated. It turns out that even if κ is so close below κ_c that the correlation length $\xi = m_R^{-1}$ is about $\xi \simeq 2a$, the expansion is still controlled and λ_R is already quite small.

ii) This allows to perform the second step perturbatively. They assume that the only fixed point of the Φ^4 theory is a Gaussian fixed point, i.e. that the continuum limit is non-interacting. Using the 3-loops β-function the quantities are continued by means of the renormalization group equations from the $\xi \simeq 2a$ region until $\kappa = \kappa_c$. This is consistent because the renormalization group drives λ_R to still smaller values as κ approaches κ_c. As ξ grows, the lattice effects get unimportant and the quantities show typical scaling behaviour, known from the perturbative RG analysis in the continuum theory, like

$$am_R = C\,(\beta_0\lambda_R)^{-\beta_1/\beta_0^2}\exp(-1/\beta_0\lambda_R). \tag{3.6}$$

Only the coefficient C carries the non-perturbative information.

iii) Similar scaling behaviour is found also for $\kappa > \kappa_c$ close to κ_c. The corresponding coefficient C' is related to C by means of the massless Φ^4 theory valid in the critical regime at $\kappa \simeq \kappa_c$, in which the mass is introduced as a perturbation.

iv) Thus the scaling formulae carry the information further into the broken phase, until they become unreliable when the correlation length becomes again small, $\xi < 2a$. This is sufficient, as the rest of the SSB phase, which is without the scaling properties, is of no interest for the continuum physics anyhow.

The results are impressive tables where one can find the values of m_R, λ_R and the renormalization constant Z for arbitrary values of the parameters λ and κ within the scaling strip along the critical line. And all that with error estimates! Let me show for illustration the lines of constant λ_R, Fig. 3, in both phases. It shows quantitatively how lines with large values of λ_R leave the phase diagram at $\lambda = \infty$ at relatively large distances from κ_c. This is a nice illustration of the triviality of the continuum limit, as for a fixed non-vanishing λ_R the correlation length ξ remains limited and the cut-off cannot be completely removed. The theory can still be used for description of the continuum physics, provided that the cut-off is substantially larger than the physical masses and energies. It is then called "effective theory". Requiring that the cut-off is at least twice as high as m_R ($\xi \geq 2a$), Lüscher and Weisz [39] obtain in the O(4) case the upper bound

$$m_H < 630 \text{ GeV}. \tag{3.7}$$

This result has been obtained for a particular regularization of the Φ^4 theory: with the nn coupling on a hypercubic lattice. The dependence on the regularization scheme is not known. Nevertheless, it is the first non-perturbative solution of the Φ^4 theories claiming reliability.

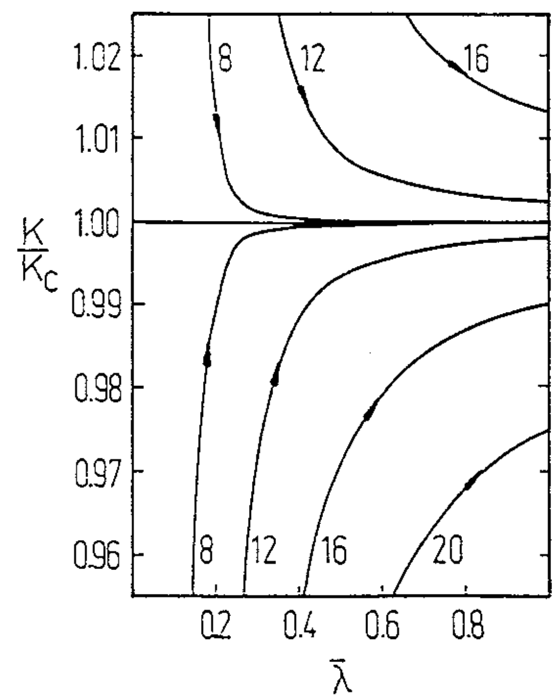


Figure 3: *Lines of constant λ_R in the Φ^4 theory obtained by Lüscher and Weisz* [39]. *(Interval $0 \leq \bar{\lambda} \leq 1$ corresponds to $0 \leq \lambda \leq \infty$.) As for any $\lambda_R > 0$ these lines do not meet the critical line $\kappa = \kappa_c(\lambda)$, it is impossible to construct an interacting continuum field theory.*

3.2 High precision Monte Carlo calculations in some relatively simple cases

With all due respect for the analytic non-perturbative calculations, for $\xi \simeq 2a$ in the SSB phase, i.e. in the region where the upper bound is determined, the numerical simulations should be ultimately more accurate and reliable. But this requires good understanding of all phenomena which might distort the results of Monte Carlo calculations, in particular the control of the finite lattice size effects, as the calculations are performed on finite lattices, usually with periodic boundary conditions. These effects can be close to catastrophic: for example, in finite volumina there cannot be any spontaneous breakdown of symmetry, and without some precaution we shall always find $\langle\Phi\rangle = 0$ in numerical simulations even in the SSB phase.

Many finite size effects can be brought under control by analytic means, and can be even very useful. As an example, think about the mass $m(L)$ of a particle in a finite volume of linear size L with periodic boundary conditions. One vacuum polarization contribution to $m(L)$, which causes its L-dependence, consists in emitting and reabsorbing another particle which makes a trip "around the world". Such a contribution can be estimated e.g. by

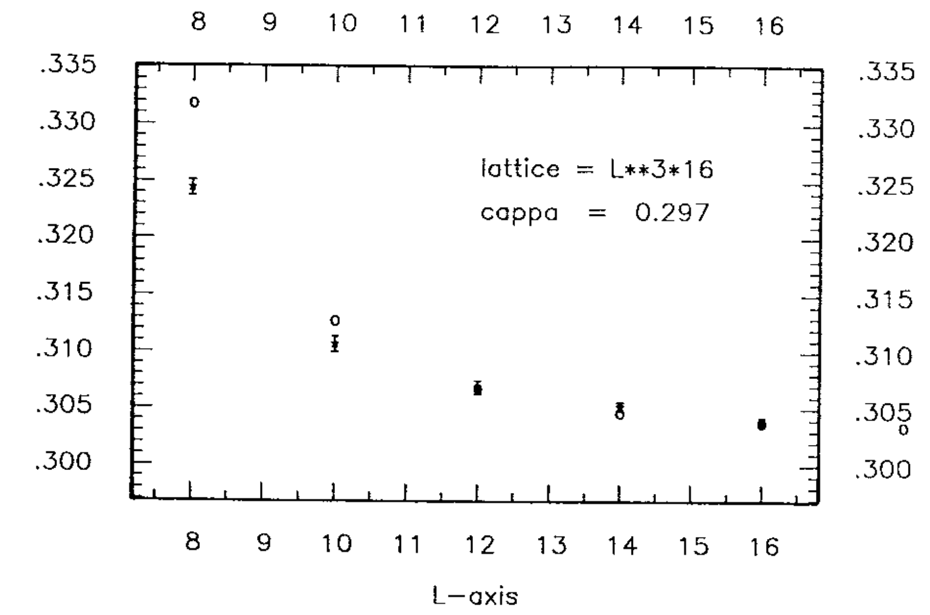


Figure 4: *The particle mass obtained in the O(4) Φ^4 theory in the symmetric phase on lattices of different sizes L (asterisks). The circles indicate the results of perturbative calculations of the mass in finite volumes. The agreement for larger L allows us to interpolate the results to infinite lattices by means of the perturbative formula. The figure is from Ref.* [43].

means of the perturbative quantum field theory in finite volumes [40]. If $m(L)$ is determined numerically in some range of L it provides an invaluable information about the scattering amplitude between the particles [40]. But most important, using the known L-dependence, one can then determine the actual mass m_R in the infinite volume without having to perform calculations on so huge lattices that the finite size effects are completely negligible.

The importance of these techniques is so high that I want at least to mention the exercises made with them in relatively simple cases of the one-component Φ^4 model and of the O(4) Φ^4 model in the symmetric phase, before turning to the real thing – to the O(4) Φ^4 theory in the SSB phase. Montvay and others [41, 42] performed a high statistics investigation of the one-component model (2.5) in the Ising limit ($\lambda = \infty$) in the symmetric phase for various L and found that using the formulae of Refs. [40] one can reliably extract am_R, λ_R and some other quantities on lattices of sizes a few times the correlation length only. This kind of analysis has been recently extended to the O(4) model in the symmetric phase (Fig. 4) [43].

The broken phase of the one-component model presents a further challenge. As a symmetry cannot be broken spontaneously for finite L, the magnetization of the configurations flips ("tunnels") during the Monte Carlo simulation between the two ground states with positive and negative magnetizations. This introduces a new kind of finite size effects which cannot be taken into account by perturbative methods, as those work always in the vicinity of one

ground state only. Kuti and Shen [44] attacked the problem by performing simulations with a constraint which fixes the magnetization of all configurations to some predetermined value during the calculation. This is a method suitable for determining the effective potential [45], from which then m_R and λ_R can be derived. Another approach introduces an external field to stabilize the system [46]. In an elaborate investigation in the Ising limit [47, 48] an opposite attitude has been adopted and the flip rate has been wastly enhanced by means of a wonderful non-local cluster algorithm invented by Swendsen and Wang [49, 42], which can flip whole clusters of spins simultaneously. This allows one to investigate the flip phenomenon quantitatively, and, using its relation to the quantum mechanical tunnel effect, to get it under analytic control. Then, again, one can determine m_R and λ_R in the infinite volume limit [48].

The merit of these investigations is twofold: First, they represent a substantial methodological progress in numerical simulations of quantum field theories. Second, when the observables obtained in Monte Carlo calculations are correctly extrapolated to the infinite volume limit, they agree to an impressive degree with the analytic results of Lüscher and Weisz for the Φ^4 models. This is a demonstration of the reliability of both methods.

3.3 $\langle\Phi\rangle$ and Z in the broken phase of the O(4) model

A naive attempt to determine $\langle\Phi\rangle$ in the broken phase of the O(4) Φ^4 model by calculating $\langle\Phi^a\rangle$ as an average over configurations produced in a Monte Carlo run ends up soon even on quite large lattices with a value consistent with zero. The direction of the magnetization of individual configurations,

$$M^a = \frac{1}{V}\sum_x \Phi^a_x. \tag{3.8}$$

drifts quickly in the O(4) space due to the presence of 3 light Goldstone bosons.

The most obvious remedy is to consider the positive definite quantity $|M^a|$, and to assume that

$$\langle\Phi\rangle \;\simeq\; \langle|M^a|\rangle. \tag{3.9}$$

This has been done in some simulations [50-52]. Alternatively, one can use also the effective potential method [51, 53, 54]. Unfortunately, it is not à priori clear how reliable these approaches are. One can imagine that on larger lattices domains of different magnetization could form which would distort the relation (3.9).

Neuberger [55, 56] was the first who noticed that for the extraction of F (eq. (3.2)) from the data obtained in the O(4) Φ^4 model on finite lattices one can exploit in finite volumes the non-linear σ-model usually applied for description of the low energy properties of Goldstone bosons ("pions"). Actully, several very useful analytic relations due to Gasser and Leutwyler [57, 58] and based on the low energy effective pion Lagrangian have been available in the context of the study of finite size effects in the lattices QCD, and more are to come [59-61]. It just took some time before it has been realized [52,62,10] that they apply to the O(4) Φ^4 model in the broken phase as well. I shall describe only the idea and the most important formula. More details can be found in Refs. [58, 10] and in a work in preparation [59].

The idea behind the method of Gasser and Leutwyler is that the finite size effects in systems with spontaneous breakdown of a continuous symmetry are mainly due to the Goldstone bosons of a very small mass m_G. This assumes that some other possibly present particles – in our case the σ-particle of mass m_σ – are much heavier than m_G, and that the lattice is sufficiently large so that σ itself causes no finite size effects. This amounts to the requirements

$$m_\sigma >> m_G\ ,\ \ 1/m_\sigma << aL. \tag{3.10}$$

Then the system is well approximated by the low energy effective Lagrangian for the Goldstone bosons, in our case the non-linear O(4) σ-model. The model is stabilized against the drift of the system through the set of degenerate ground states by the introduction of a small constant external source j breaking the O(4) symmetry explicitely (inserting the term $j\sum_x \Phi_x^0$ into the action (2.9)). It gives the Goldstone bosons in the infinite volume a non-vanishing mass $m_G \propto j$. The two free parameters of the model are F^2, entering as the coupling constant, and $\langle\Phi\rangle$, which determines the effective strength of the interactions with the external source j. The Langrangian density is

$$\mathcal{L}(x) = \frac{1}{2}F^2\sum_{a=0}^{3}(\partial_\mu U^a(x))^2 \ - \ j\langle\Phi\rangle U^0(x), \qquad \sum_{a=0}^{3} U^a(x)U^a(x) = 1. \tag{3.11}$$

This model is then used in the continuum euclidean space-time of finite volume L^4. In the analytic calculations the quantities $1/L^4$ and j are treated as small expansion parameters of the same order, and suitable power expansions in these quantities are performed. In this way we get useful formulae which describe the properties of the model for $L < \infty$ and $j > 0$ in terms of F and $\langle\Phi\rangle$, the parameters of the infinite volume model with $j = 0$.

An example of such formulae is, in the lowest order of the power expansion, the relation

$$jL^4\,\langle\Phi_x^0\rangle_{L,j} = u^2\eta(u). \tag{3.12}$$

where

$$u \ = \ \langle\Phi\rangle\, jL^4 \tag{3.13}$$

and

$$\eta(u) = \frac{1}{u}\frac{I_2(u)}{I_1(u)}. \tag{3.14}$$

Here I_1 and I_2 are Bessel functions of imaginary argument and $\langle\Phi_x^0\rangle_{L,j}$ is the expectation value of the scalar field for finite L and non-vanishing j. Notice that these formulae include the standard expression for the magnetization in the SSB case, namely

$$\langle\Phi\rangle = \lim_{j\to 0}\lim_{L\to\infty}\langle\Phi_x^0\rangle_{L,j}. \tag{3.15}$$

The immediate conclusion is that if such formulae are to be used, then one should simulate on the lattice the Φ^4 model with an external source (which does not need to be given any physical interpretation). The numerical determination of $\langle\Phi_x^0\rangle_{L,j}$ is straightforward and doing it only for one pair of values of the parameters L and non-vanishing j is, in principle, sufficient to determine $\langle\Phi\rangle$ in the infinite volume and $j = 0$! Let me add that numerical simulations for $j > 0$ are much easier than those for $j = 0$ as the ground state drift is suppressed.

Can such a miraculous method be correct? The obvious test is to fix the bare parameters κ and λ and to perform calculations in the O(4) Φ^4 theory with the external source j for

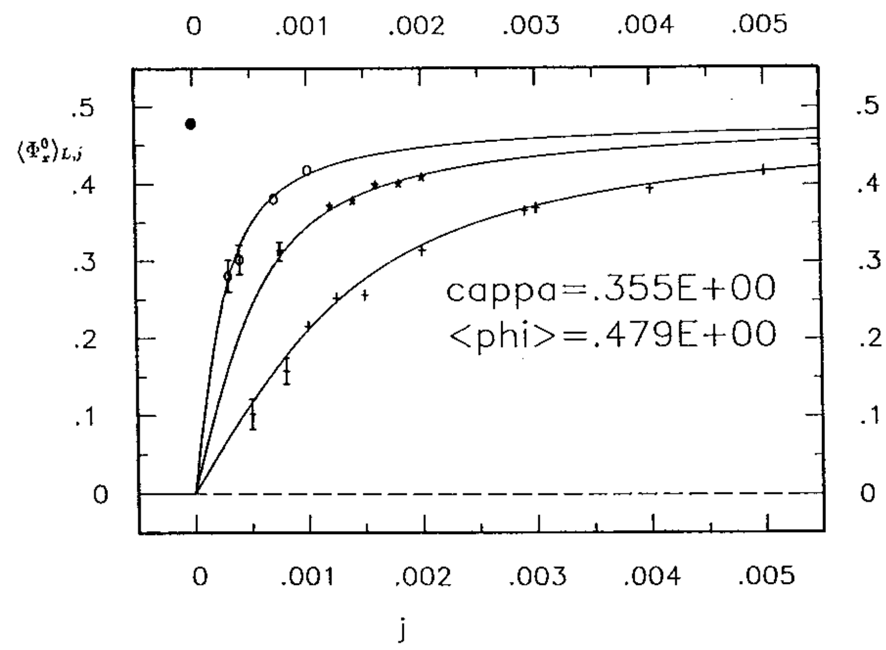


Figure 5: *All data for $\langle\Phi_x^0\rangle_{L,j}$ obtained for various lattice sizes L and external sources j agree with values obtained from $\langle\Phi\rangle$ (the dot in the upper left corner) by means of the Gasser-Leutwyler formula* (3.12) *(including some higher order corrections). The figure is from Ref.* [62].

different values of j and L comparing the results for $\langle\Phi\rangle$. This has been done in a recent study [62, 63] and a typical result is presented in Fig. 5. The figure shows very different values of $\langle\Phi_x^0\rangle_{L,j}$ obtained in simulations with various L and j for one κ-value (we have taken $\lambda = \infty$). The curves correspond to the expression for $\langle\Phi\rangle_{L,j}$ obtained from (3.12) with only one common value of $\langle\Phi\rangle$. As the agreement is excellent, we are confident that $\langle\Phi\rangle$ has been determined reliably. Its value is indicated by the bold dot in the figure. Please, notice how different from $\langle\Phi\rangle_{L,j}$ it is.

Expressions analogous to eq. (3.12) exist also for the propagators, and can be used for the determination of Z. Also these formulae have been tested successfully [62, 63]. So we conclude that the Gasser-Leutwyler formulae are applicable to the lattice O(4) Φ^4 theory in the SSB phase. The results for $\langle\Phi\rangle$ are also consistent with those obtained by means of $|M^a|$, eq. (3.8), which increases the confidence in earlier results obtained by methods based on the relation (3.9) [50-52]. It should be pointed out, however, that the κ-range already investigated is still quite distant from the critical point $\kappa_c = 0.304$. We have kept $am_\sigma > 0.8$ in order to satisfy safely the constraints (3.10). Further studies closer to κ_c are necessary.

3.4 Higgs boson mass

The two groups which have recently determined the upper bound on the Higgs boson mass in large scale numerical simulations of the O(4) Φ^4 theory on the lattice use different techniques for determining the σ-particle mass m_σ (which is then identified with the Higgs boson mass m_H). One way is to look at the decrease of a two-point function in the configuration space with the distance τ and fit it to the form $A + Be^{-m_\sigma \tau}$ corrected for the lattice periodicity [50, 52, 63]. Another method is to determine the Fourier transformation of the two-point function and compare it with the free particle propagator on the lattice [51, 53, 54]. The second group determines m_σ also by means of the effective potential. The results are consistent on the precision level which presumably suffices for the determination of the upper bound. They also agree with the results of Lüscher and Weisz for the O(4) model [39]. But some details remain to be clarified.

One of them is the dependence of m_σ on the lattice size L. The currently available analytic results within the Gasser-Leutwyler method ignore the σ-particle completely. Kuti, Lin and Shen [64] investigated the L-dependence of m_σ by means of the renormalized perturbation theory for the effective potential obtaining

$$m_\sigma(L) \simeq m_\sigma(\infty) + \frac{A}{L^2}. \qquad (3.16)$$

This formula gives sizeable (about 5 per cent) correction to the values of $m_\sigma(L)$ obtained on lattices of various sizes up to 16^4 [63]. But the formula has not yet been sufficiently verified by means of a systematic investigation of the L-dependence of $m_\sigma(L)$ in analogy to the simpler cases of the symmetric phase [43] or of the Z(2) model [41, 47, 48]. So we have an uncertainty of possibly a few percent in m_σ.

This is related to a methodological problem: how to determine $m_\sigma(L)$ on relatively small lattices with a high accuracy needed for a study of the validity e.g. of the formula (3.16) and for a reliable extrapolation of $m_\sigma(L)$ to $L = \infty$. Due to the presence of the light Goldstone bosons the spectrum of the model in finite volumes is very complex. As has been demonstrated within the framework of the non-linear σ-model, the system has various excited states with energies which can be close to $m_\sigma(L)$ [65]. (The spectrum is actually described by the spectrum of a quantum-mechanical particle moving on the manifold of the SU(2) group and is quite different from a multi-Goldstone spectrum of the infinite volume theory even if the Goldstone particles are assumed to have a finite mass.) Any correlation function suitable for a determination of $m_\sigma(L)$ will get contributions also from some of these states. We do not yet know how to separate these contributions which might distort the results for $m_\sigma(L)$.

The third problem concerns the instability of the Higgs particle. On the lattice the σ-particle does not have any decay channel open until L is so large that the energy of two Goldstone bosons with the lowest non-zero momentum $2\pi/aL$ is approximately equal to m_σ,

$$am_\sigma \simeq 2 \cdot \frac{2\pi}{L}. \qquad (3.17)$$

For $am_\sigma \simeq 0.5$ this amounts to $L \simeq 25$, much too large sizes with respect to what has been possible until now. So we cannot yet investigate the effects of instability of the Higgs boson on the lattice, though the theoretical analytic framework for such studies is ready [66, 67]. Fortunately, it seems that for the Higgs boson masses allowed by the upper bound of not

much more than 600 GeV, the decay width will be no more than about 20 per cent of the mass [38, 10] and its effects thus should not be very important.

3.5 Upper bound on the Higgs mass and tests of the Standard Model

The results on the upper bound obtained by Monte Carlo calculations of the group around Kuti [51, 53, 54], by our collaboration [50, 52, 63] and by analytic methods [39] agree very well if L-dependence of m_σ of the form (3.16) in the numerical calculations on finite lattices is assumed. For definiteness I quote the result of Kuti et al.:

$$m_H \leq (640 \pm 40) GeV, \qquad (3.18)$$

if the ratio of the cut-off to the Higgs mass, Λ/m_H, should be greater than 2. The results of all 3 groups have been summarized by Kuti at the Munich conference [12] (note that Kuti formulated them in terms of the cut-off parameter π/a). The most important aspect of this value for the upper bound is that the scalar sector is actually never strongly interacting, even if one starts with an infinite bare coupling λ. The maximal value of λ_R is about 2/3 of the tree level unitary bound only [39].

I stress that, strictly speaking, this result applies only to the O(4) Φ^4 model regularized on the hypercubic lattice with nn coupling, eq. (2.9). However, some calculations on lattices with other geometries or couplings give similar, though not equal results [68]. Of course, it would be very helpful to have non-perturbative results also for some other regularizations than on the lattice to compare with. Unfortunately, they are not available except the calculation by P. Hasenfratz and Nager [37] whose reliability is not known but whose results are also not far from (3.18). One should also take into consideration that the lattice regularization is fully acceptable from the physical point of view. The deviations from the continuous rotational invariance are not a serious drawback as they can be well controlled and suppressed by a suitable choice of the lattice action [69]. From all that one has the impression that the quantitative results obtained until now by means of the lattice regularization are physically relevant and that the triviality of the Φ^4 theory has really the phenomenological consequence of impossibility of a Higgs boson mass much larger than (3.18) and of the strongly interacting scalar sector in the Standard Model. However, some caution [70] is certainly still in place.

It should be clear that the bound obtained in a given regularization is still slightly dependent on the degree of scaling violations one is ready to allow. Should the ratio of the cut-off to the Higgs mass, Λ/m_H, be greater than 2, or, say, 5? The latter requirement can lower the bound by 100 GeV [53]. It is actually more sensible to look at the "envelope", showing what would be the maximal value of R_4 at a chosen value of the cut-off $\Lambda/m_\sigma = 1/am_\sigma$. In Fig. 6 the analytic results for the envelope [39] (diamonds), the finite-volume corrected (eq. (3.16)) results of Kuti et al. [51, 53, 54] (curve 2) and the similarly corrected results of our group [50, 52, 63] (curve 1) are shown. The shapes of the curves are determined by the assumed scaling laws

$$\begin{aligned} \langle \Phi_R \rangle &\propto \sqrt{t}\ (\ln t)^{1/4} \\ m_\sigma &\propto \sqrt{t}\ (\ln t)^{-1/4} \\ t &= \frac{\kappa - \kappa_c}{\kappa_c}. \end{aligned} \qquad (3.19)$$

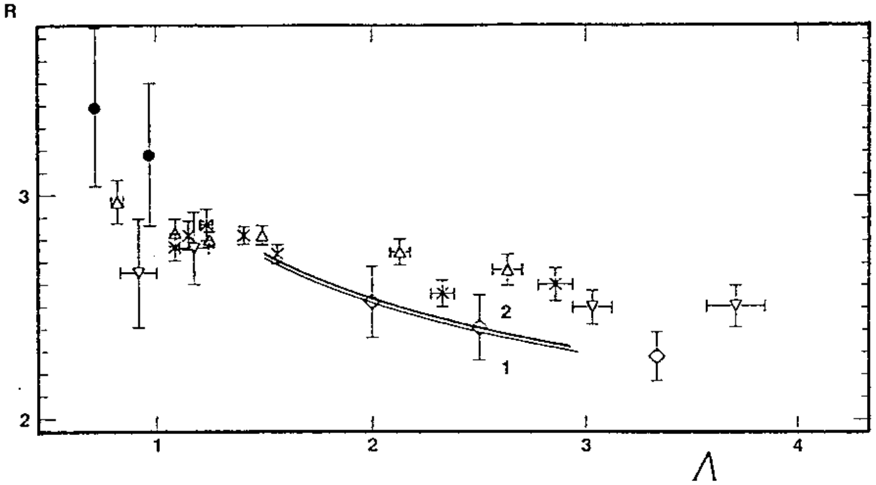


Figure 6: *The envelope of the possible values of the ratio R_Φ (eq. (3.1)) for different values of the cut-off parameter Λ as obtained by Lüscher and Weisz (diamonds), our group (curve 1) and Kuti et al. (curve 2). Curves are drawn according to the scaling laws (3.19) after interpolation to the infinite lattice. Some raw data on finite lattices are shown too. The figure is from Ref.* [63].

Raw data of our group on $12^4 - 16^4$ lattices, without a finite size correction for m_σ, are shown in this figure, too. We can see that the shift of R_Φ due to finite size effects might be about 5 per cent, but we do not know yet for sure.

We have achieved remarkable reliability and accuracy, which can still be improved in the future. What should be our goals? The question of a "precise" value of the upper bound is a question with intrinsic unprecision and contains several uncertainties [70, 71]. The actual question, which has a more precise meaning, is: How accurately can we control the electroweak theory in the case when the renormalized quartic coupling is nearly as large as allowed by the bound? In this case the cut-off is low and it might be possible to detect experimentally some, at first only minor departures from the present day effective theory. Thus the emphasis is on precision of the numerical determination of various quantities in some range of the cut-off values.

In particular, we should study the possibilities of detecting scaling violations, like the dependence on the regularization scheme. We should investigate the differences between the positions and shapes of the envelopes for various lattice formulations [68]. These envelopes have to coincide for large cut-off Λ/m_σ as they all have to obey the same scaling laws (3.19) for small t. But they will differ at lower values of Λ/m_σ [10] indicating that the Φ^4 theory is too much regularization dependent there to be useful as an effective theory for physical phenomena. Therefore I find it more appropriate to quote the upper bounds like (3.18) with error bars (instead of saying that $m_H < 680$ GeV), in order to be able to compare the results for various regularizations.

I conclude that in the future we should try to determine the onset of the common scaling behaviour (3.19) for various regularizations which might require numerical calculations at relatively large $\Lambda/m_\sigma = \xi$ and thus a very good control of the finite size effects for large correlation lengths. Much remains to be done in this respect.

4 Introducing fermions into the Higgs models

Until now we have completely ignored fermions in the electroweak theory. This might be correct quantitatively if no heavy fermions exist, but it is not satisfactory for general theoretical reasons. And if a heavy fermion with strong Yukawa coupling exists, then we must include fermions into the non-perturbative lattice study of the Higgs system to maintain the reliability of our calculations.

Of course, the trouble is that it is so difficult to include fermions on the lattice. Fermion doubling, wrong chirality of some doublers, and difficult computational methods are the obstacles. But things got moving and I shall describe the (mostly preliminary) progress in this field. Useful review articles on this topics are Refs. [7, 8, 11].

Let me just mention that there are presently two methods of lattice formulation of fermionic fields. One is the "staggered fermion" or Kogut-Susskind method, where different Dirac components of a fermion field live at different neighbouring lattice sites. This formulation preserves a continuous remnant of the chiral symmetry of the continuum theory and reduces the number of fermions from 16 to 4. For its description I have to refer to other works, e.g. [72]. The other method is due to Wilson [4, 73], and I will describe it later in some detail, as this method is more suitable for the lattice study of the electroweak interactions. In numerical simulations the fermionic degrees of freedom are often treated in the "quenched" approximation which neglects effects of the virtual fermion loops, i.e. the dynamics of fermions.

4.1 Chiral symmetry breaking in the Higgs models with fermions

It is a well established result of numerical investigation of lattice QCD that at low temperatures the chiral symmetry of QCD is spontaneously broken and the fermion condensate is non-vanishing, $\langle\bar\Psi\Psi\rangle \neq 0$, in the limit of massless quarks. Thus, when the Higgs models are enriched by fermions with vector-like coupling to the gauge field,

$$\frac{1}{2}\sum_{x\mu}\eta_{x,\mu}(\bar\chi_x U_{x,\mu}\chi_{x+\mu} - \bar\chi_{x+\mu}U^\dagger_{x,\mu}\chi_x), \qquad (4.1)$$

χ_x being the "staggered" fermions and $\eta_{x,\mu}$ certain phase factors [72], we expect also in these systems that the chiral symmetry is spontaneously broken as long as the effects of the scalar field are small, i.e. for small κ. Then the question arises, what is the influence of the scalar field on $\langle\bar\Psi\Psi\rangle$ for large values of κ and what is the interplay between the spontaneous chiral symmetry breaking and the Higgs mechanism related to the SSB in the Φ^4 theory.

In the case of the SU(2) Higgs model this question is of direct relevance to the electroweak theory as the SU(2) sector of this theory with chiral fermions can be rewritten in a vector-like

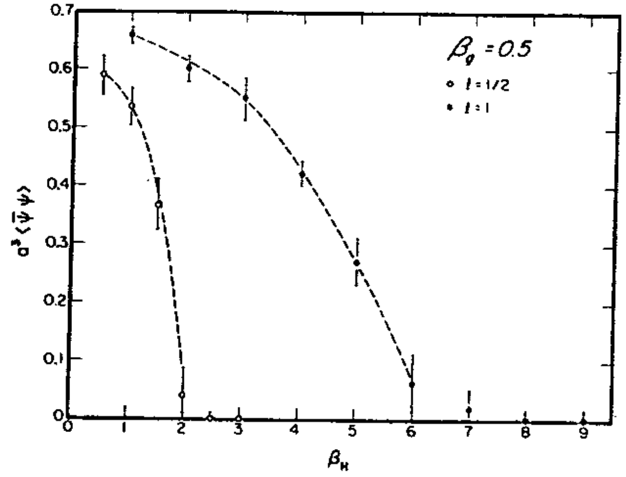


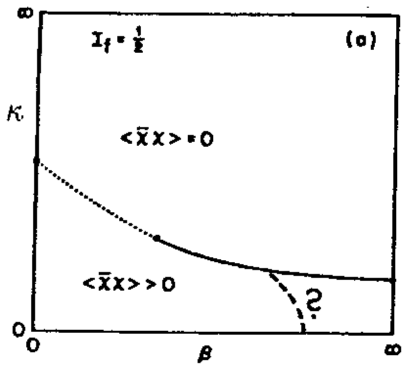
Figure 7: *Chiral condensate $\langle\bar\Psi\Psi\rangle$ as a function of $\beta_H = 2\kappa$ in the $SU(2)$ Higgs model at $\beta = 0.5$ in the quenched approximation. The figure is from Ref.* [75].

form. The argument based on the reality of the group SU(2) is due to Georgi (cited in [20]) and explained in detail in Ref. [74].

The pioneering work has been done by Lee and Shigemitsu [75] who found numerically for staggered fermions in the quenched approximation that for small β the chiral condensate $\langle\bar\Psi\Psi\rangle$ decreases with increasing κ and vanishes at some finite value of κ Fig. 7. This is not in contradiction with the theorem on the analytic connection between the confinement and Higgs regions (Subsec. 2.2) because $\langle\bar\Psi\Psi\rangle$ is, expressed in terms of the scalar and gauge fields when the fermionic integration in the path integral has been carried out, a highly non-local quantity. For fermions with weak isospin $I = 1/2$ this κ value is lower than for those with $I = 1$. Lee and Shrock [76] have shown later analytically, using at $\beta = 0$ the mean field method for the order parameter $\langle\bar\Psi\Psi\rangle$, that this is a genuine phase transition and the single phase of the SU(2) Higgs model thus splits into 2 phases, with $\langle\bar\Psi\Psi\rangle > 0$ for low κ and $\langle\bar\Psi\Psi\rangle = 0$ for large κ. This result, and a systematic analytic and numerical investigation of the chiral phase transition line for $\beta \geq 0$ in the $SU(2)$ model by several authors [76-81] lead to the following picture (see Fig.8):

As κ increases and the system approaches the Higgs region, the configurations of the gauge field are increasingly restricted by the interaction with the scalar field in such a way that the effective fermion-fermion interaction gets weaker. This causes the restoration of the chiral symmetry, which takes place at higher κ if the fermion field has larger effective gauge coupling for higher I.

The very interesting fact demonstrated by De and Shigemitsu [78] is that the chiral phase transition for $I = 1/2$ seems to coincide with the Higgs phase transition at $\beta = 2.3$ (the Higgs phase transition line extends from $\beta = \infty$ until $\beta \simeq 2$). The chiral transition



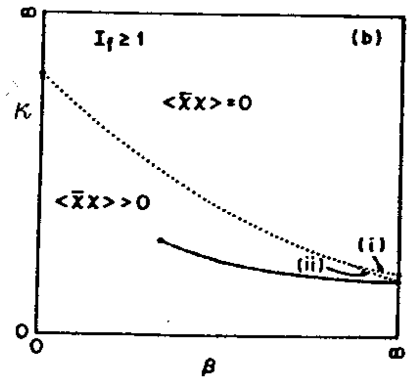


Figure 8: *Schematic phase diagrams for the $SU(2)$ Higgs model with (a) $I = 1/2$ and (b) $I = 1$ fermions, as they are expected on the basis of calculations performed for $\beta \leq 2.3$. For larger β, i.e. in the physically relevant region, the positions of the chiral phase transition (doted and dashed lines) is not really known, however. The figure is adapted from Ref.* [82].

line at low β also looks like an analytic continuation of the Higgs transition line to lower β until $\beta = 0$ (Fig. 8). Thus for $I = 1/2$ the Higgs region is apparently free of the chiral condensate and fermion masses can arise only through the Yukawa coupling – as expected in the continuum electroweak theory. On the contrary, the Abbott-Farhi [19, 20] scenario (the "strongly coupled standard model") does not seem to be realized in this theory as fermions would acquire large mass of the order of the weak interaction scale, $O(10^2\text{GeV})$, below the Higgs transition due to the chiral symmetry breaking. It should be stressed, however, that the phase diagram at large β is not yet accessible to numerical simulations, and that there is no real evidence that the chiral phase transition continues along the Higgs phase transition for large β [81]. The phase diagram could also be as indicated in Fig. 8 by the dashed line [11]. In this case the Abbott-Farhi scenario would not be excluded.

Similar argument can be also used to speculate about absence of the $I = 1$ fermions in the physical spectrum [82]. If the phase diagram for $I = 1$ looks really to be as shown in Fig. 8, and for $\beta = 2.3$ this is the case [78], then an approach to the continuum limit in the vicinity of the Higgs phase transition would have to be performed with $I = 1$ fermions in the broken chiral symmetry phase with the result that such fermions would end up with too large mass to have already been observed.

The effect of the Yukawa coupling y on the chiral phase transition in the SU(2) Higgs model is to extend the region without the chiral symmetry breaking. According to some strong β coupling calculations [74], for sufficiently high y the chiral symmetry breaking completely vanishes.

The U(1) Higgs model with vector-like coupling of fermions to the gauge field is perhaps less interesting from the point of view of the electroweak theory, but it is an excellent laboratory for the study of the above effects as this model on the lattice also has the Higgs-confinement phase with SSB of chiral symmetry in the confinement region. Much understanding of the chiral symmetry breaking in the lattice Higgs models has been achieved also within this model [83, 84]. Good reviews of the whole topic are in Refs. [7, 11].

4.2 Yukawa coupling in simple scalar field theories with fermions

Non-perturbative investigations of the fermion mass generation through the Yukawa coupling to scalar fields, and of some other effects of this coupling, has become recently one of the focuses of research in the lattice quantum field theories. The most important long term reasons for this interest are:

(i) Investigation of the question whether there is an upper bound on the Yukawa generated fermion masses [85], in analogy to the upper bound on the Higgs boson mass, and possibly its determination.

(ii) Study of the influence of the strong Yukawa coupling on the scalar sector, in particular on the upper bound on the Higgs boson mass [11, 54] and on the envelope.

(iii) Search for new critical points suitable for the construction of a – possibly non-trivial – continuum limit of some lattice formulation of electroweak theory [13].

Most of the work until now has been done with toy models with Z(2) and U(1) symmetry only, disregarding the problems with fermion doubling and other phenomenological shortcomings of these models and leaving out gauge fields. The methodological contributions of these studies are invaluable, however, and some qualitative aspects of the results might well be of some relevance to physics, too.

As an example, let us take the Z(2) lattice Yukawa model with one naive lattice fermion field [86], which in the continuum limit describes 16 degenerate fermions:

$$\begin{aligned} S &= -2\kappa \sum_{x\mu} \Phi_x \Phi_{x+\mu} + \lambda \sum_x (\Phi_x^2 - 1)^2 + \sum_x \Phi_x^2 \\ &+ \frac{1}{2} \sum_{x\mu} \bar{\Psi}_x \gamma_\mu (\Psi_{n+\mu} - \Psi_{n-\mu}) + y \sum_x \Phi_x \bar{\Psi}_x \Psi_x . \end{aligned} \tag{4.2}$$

The model is symmetric with respect to the global "chiral" $Z(2)$ transformation

$$\Phi_x \to -\Phi_x, \quad \Psi_x \to i\gamma_5 \Psi_x, \quad \bar{\Psi}_x \to \bar{\Psi}_x i\gamma_5, \tag{4.3}$$

which is a generalization of the Z(2) symmetry of the one-component scalar field model. Alternatively, one can use staggered fermions [87-90] instead of the naive ones, reducing the number of degenerate fermions to four. One cannot add an explicite fermion mass term without violating the Z(2) symmetry.

Let us consider the fermion mass m_F. For small y we expect in the SSB phase

$$am_F \simeq y \,\langle\Phi\rangle \tag{4.4}$$

and

$$am_F \to 0 \quad \text{as} \quad \kappa \searrow \kappa_c. \tag{4.5}$$

For large y the situation is different. Rescaling the fermion field, $\sqrt{y} \cdot \Psi \to \Psi$, the fermionic kinetic term in eq. (4.2) gets the factor $1/y$. Performing then the strong coupling expansion

in the powers of $1/y$, which is a standard tool in the lattice theories, one finds that m_F *increases* as $\kappa \searrow \kappa_c$ [86]. Also a mean field calculation (within a more complex model discussed in Subsec. 4.4) at strong y gives [91]

$$m_F \propto \frac{1}{a^2 \langle\Phi\rangle}. \tag{4.6}$$

For some recent analytic arguments see also Ref. [92].

Thus there are two qualitatively different regions of Yukawa coupling. Firstly, the weak y (perturbative) region, where the relation (4.4) holds. Secondly, the strong y region, where m_F increases as κ approaches the critical point in the SSB phase. In the limit of the infinite cut-off the fermions get presumably infinitely heavy there. Numerical calculations should clarify what separates these regions – some cross-over or even a new phase transition with some interesting critical points?

The present situation of the numerical study of this and some related questions is as follows. The weak coupling region of the Z(2) model with staggerd fermions has been investigated quite early by Shigemitsu [87] in the quenched approximation. She found that am_F and also the renormalized Yukawa coupling,

$$y_R = \frac{am_F}{\langle\Phi\rangle_R}, \tag{4.7}$$

decrease for $\kappa \searrow \kappa_c$. The situation resembles that in the pure Φ^4 theory ($m_F \leftrightarrow m_\sigma, y_R \leftrightarrow \lambda_R$). Later Anna Hasenfratz and Neuhaus [86] found a distinctly different behavior for large y, illustrated in Fig. 9. Here it is demonstrated that for $y > 2$ the fermion mass am_F increases as $\langle\Phi\rangle$ decreases. This calculation also has been performed in quenched approximation, but that approximation should be reasonable good both for small y (i.e. the weakly coupled fermions) and large y (fermions so heavy that their loops are strongly suppressed). Recently similar phenomenon has been observed also in the SU(2) model [93, 94]. Thus the existence of two different regions seems to be established.

The question what separates these regions must be answered in calculations with dynamical fermions. Kuti and his co-workers observed in such calculations with staggered fermions [90], when increasing y, a smooth behaviour of several observables and no signal for a phase transition even for quite large y, the results staying consistent with the perturbation theory.

However, we know that the phase diagram of the Z(2) model with dynamical fermions is quite different from that in the quenched case [88,89,92,95,96] and, in addition, depends on the way how the Yukawa coupling is transcribed on the lattice. The difference between the phase diagram in the quenched and dynamical cases is indicated schematically in Fig. 10. The phase transition line separating the broken and symmetric phases is found in the dynamical fermion case at κ values much lower than κ_c of the pure Φ^4 model, as the Yukawa coupling tends to order the scalar field and thus enlarges the SSB phase. Similar change of the phase diagram has been found also in the Yukawa model with U(1) symmetry and dynamical fermions [97]. These facts suggest that the separation between the weak and strong y regions can lie at very different places for quenched and dynamical fermions. Further study of this question is thus required. First calculation of the masses in the dynamical fermion case for an U(1) model by Thornton indicate the precence of the strong coupling region indeed [98].

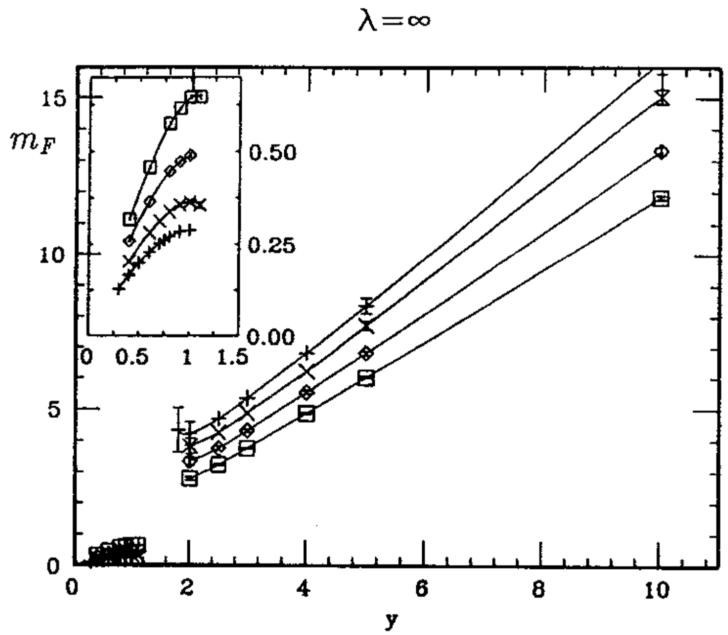


Figure 9: *Fermion mass generated by the Yukawa coupling in the SSB phase of the model* (4.2) *for 4 κ-values as a function of y in the quenched approximation. For y > 2 the fermion mass increases as κ decreases from larger values (squares) to smaller ones (crosses). The figure is from Ref.* [86].

4.3 Spectrum of chiral fermions on the lattice

Now we come to the most difficult problem of the formulation of weak interactions on the lattice – to the problem of the fermion spectrum. With the lattice regularization comes the problem of fermion doubling. Under quite general assumptions Nielsen and Ninomiya [99] have shown that lattice transcriptions of a continuum left-handed Weyl field contain an equal number of left- and right-handed fermions. Karsten and Smit [100] pointed out that this is related to the manifest gauge invariance of the lattice regularization even in the case of anomalous chiral gauge theories, as the doubling results in cancellation of anomalies. These facts do not exclude, however, that the unwanted additional fermions – "doublers" – get very heavy or do not interact with that part of the spectrum which would be phenomenologically acceptable. So there is some hope left, and several interesting suggestions how to circumvent the problem have been made.

I start by recalling how the fermion doubling problem is solved by the Wilson method [4, 73] in the case of vector-like gauge theories like QCD. For clarity I shall reconstitute in this subsection the dimensions of the fields and write explicitely the lattice constant a. The

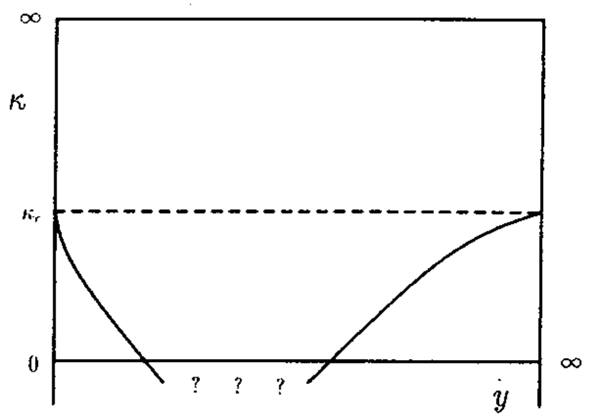


Figure 10: *Schematic phase diagram of the Φ^4 theory with a Yukawa coupling y to fermions in the quenched approximation (dashed line) and for dynamical fermions (full line).*

action S^Ψ of a free Dirac field Ψ is written on the lattice in the form

$$S^\Psi = S^\Psi_n + S^\Psi_W, \tag{4.8}$$

where the first term is called the "naive" action

$$S^\Psi_n a^{-4} = \frac{1}{2a} \sum_{x\mu} \bar\Psi_x \gamma_\mu (\Psi_{x+\mu} - \Psi_{x-\mu}) + \sum_x m_0 \bar\Psi_x \Psi_x, \tag{4.9}$$

and the second one the Wilson term with the dimensionless Wilson parameter r,

$$S^\Psi_W a^{-4} = \frac{r}{a} \sum_{x\mu} (\bar\Psi_x \Psi_x - \frac{1}{2}\bar\Psi_{x+\mu}\Psi_x - \frac{1}{2}\bar\Psi_x \Psi_{x+\mu}). \tag{4.10}$$

The inverse propagator in the momentum space is then

$$S_F^{-1}(k) = \sum_\mu i\frac{1}{a}\gamma_\mu \sin ak_\mu + m_0 + \frac{r}{a}\sum_\mu (1-\cos ak_\mu). \tag{4.11}$$

The sixteen fermions are seen when the momentum components k_μ are either near zero or close to their maximal value within the Brillouin zone,

$$k_\mu = p_\mu + P^D_\mu \ , \quad D = 0, \dots 15, \tag{4.12}$$

$$P^D_\mu = (0,0,0,0)\ , (\frac{\pi}{a},0,0,0), \dots, (\frac{\pi}{a},\frac{\pi}{a},\frac{\pi}{a},\frac{\pi}{a}). \tag{4.13}$$

For each D one recovers for small ap_μ one continuum Dirac field propagator from S_F. If the Wilson term (4.10) is absent ($r = 0$ in (4.11)), all fermions have the same mass $m_F = m_0$. For $r > 0$ the doublers have higher mass m_D,

$$m_F = m_0; \quad m_D = m_0 + 2\frac{r}{a}n_D \ , \quad n_D = 1, \dots, 4 \ \text{ for } D \geq 1. \tag{4.14}$$

This is fine, as in the continuum limit, i.e. for $a \to 0$, the doublers get an infinite mass. (Numerically it is of course quite difficult to achieve sufficiently small a).

In the case of chiral gauge theories the doublers do not only duplicate the spectrum, but in addition some of them transform in a "mirror" way with respect to the chiral transformations (i.e. the transformation properties of the left- and right-handed components are interchanged) so that we must remove them at least from the low mass spectrum. The problem is that the Wilson term (4.10) cannot be used for the removal of the doublers as it is not manifestly invariant under the chiral gauge transformation for the same reason for which also the mass term in (4.9) is non-invariant, namely it contains products of the type $\bar{\Psi}_L \Psi_R$ and $\bar{\Psi}_R \Psi_L$ (the subscripts L and R denote the left- and right-handed fields, respectively).

One way how to circumvent the problem has been proposed by Montvay [101]. He pointed out that, if the doublers with mirror properties have to arise in the spectrum [102, 13], it is better to include them from the very beginning as fundamental fields in order to better control the possibilities of their removal. Thus he deliberately doubles the fermion spectrum by introducing the "mirror" Dirac fermion χ such that χ_R transforms with respect to the chiral transformation in the same way as Ψ_L, and analogously χ_L transforms as Ψ_R. Then one can write invariant terms $\bar{\Psi}_L \chi_R$ and $\bar{\Psi}_R \chi_L$, both local and of the Wilson type (4.10), which removes the doublers of both the usual fermions and the mirrors. Choosing suitably some of the parameters, we can hope to make the mirror fermions sufficiently heavy so that they are consistent with the present day phenomenological knowledge. One should also keep in mind the possibility that heavy mirror fermions do exist in the nature. Montvay discussed some aspects of his approach in his Erice lecture last year [13].

Another idea is to compensate for the gauge non-invariance of the Wilson term by introducing some counter-terms whose coefficients are chosen in such a way that the Ward identities are satisfied in the perturbation expansion up to positive orders of the lattice constant a [103].

It seems to me that the most natural idea is to modify the Wilson term in the case of a chiral gauge theory on the lattice in the same way as the fermion mass term in the continuum theory had to be modified: insert a suitably transforming scalar field φ so that for a Dirac field Ψ the products of the type $\bar{\Psi}_L \varphi \Psi_R$ etc. are manifestly gauge invariant. Then the Wilson term gets the form of a new, Wilson-Yukawa (or "point-split Yukawa") coupling term. Aoki and others [104] suggested that φ can be an auxiliary field without a kinetic part. But the interaction with fermions presumably generates the kinetic term so that one can as well include it from the very beginning.

The most natural possibility is to use the Higgs field itself. This has been suggested in 1980 by Smit [105, 8] and later by Swift [106]. In the simple model (4.2) with the global Z(2) symmetry such a Wilson-Yukawa term can be written (using now, according to (2.2), the dimensional scalar field $\varphi = \sqrt{2\kappa}\Phi/a$) in the form

$$w \sum_{x\mu} (\bar{\Psi}_x \varphi_x \Psi_x - \frac{1}{2}\bar{\Psi}_{x+\mu} \varphi_x \Psi_x - \frac{1}{2}\bar{\Psi}_x \varphi_x \Psi_{x+\mu}). \qquad (4.15)$$

Here w is a new Wilson-Yukawa coupling constant. Local gauge invariance of this term can easily be achieved by inserting suitable gauge field variables on the links in (4.15). Perturbatively, we expect from eq. (4.14) that in the SSB phase

$$m_F = y\langle\varphi\rangle$$

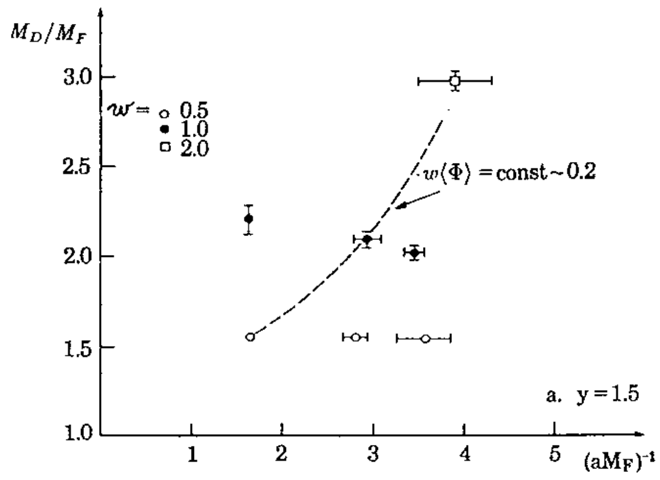


Figure 11: *Ratio of the doubler to fermion masses in the quenched approximation in the one-component Φ^4 theory with the Wilson-Yukawa coupling term* (4.15). *Different symbols denote different values of w. The dashed line indicates the possibility to increase the ratio by increasing w and keeping $w\langle\Phi\rangle$ constant. The figure is from Ref.* [107].

$$m_D = y\langle\varphi\rangle + 2w\langle\varphi\rangle n_D , \quad D = 1, \ldots, 15. \qquad (4.16)$$

Notice that for dimensional reasons the factor $1/a$ present in eqs. (4.10) and (4.14) is absent in (4.15) and (4.16). As we have to perform the continuum limit in such a way that the expectation value of the scalar field with physical dimension, $\langle\varphi\rangle \simeq 250$ GeV, remains constant, the perturbative formula (4.16) suggests that the doublers can be made heavy only by increasing w. But it could also be, and recent results indicate that this is the case [94], that for large w the relationship between m_D and $\langle\varphi\rangle$ is very different from (4.16), rather analogous to (4.6), and values of m_D of the order of the cut-off Λ can be even achieved for large but fixed w. This is similar to the non-perturbative behaviour of the fermion mass m_F for strong Yukawa coupling described in Subsec. 4.2. In any case this means that the question of giving the doublers a high mass is a non-perturbative dynamical problem in the coupling w. Fortunately, the lattice regularization is suitable for dealing with such problems, so this is not prohibitive. Thus, at least in principle, there *is* a possibility to use the lattice regularization in the chiral gauge models [105, 8, 106], it has just been overlooked because of its relative complexity.

Shigemitsu [107] pioneered the numerical (quenched) investigation of this mechanism in the Z(2) model and found that to some extent it is possible to increase the ratio m_D/m_F simultaneously with decreasing am_F, the fermion mass in lattice units, i.e. when the continuum limit is approached (Fig. 11). Some similar results have been obtained recently also in the 2-component model when the dynamics of the fermions have been taken into account [98]. A study of the physically more realistic chiral SU(2)⊗SU(2) model will be discussed in the next subsection.

4.4 Smit-Swift formulation of the standard model on the lattice

The parameters of the model are: the gauge field coupling parameter β, the hopping parameter κ, the quartic coupling λ, the usual local Yukawa coupling y and the Wilson-Yukawa coupling w. The model has the symmetry

$$SU(2)_L^{(local)} \otimes SU(2)_R^{(global)} \tag{4.17}$$

with respect to the chiral transformations

$$\begin{aligned} \Psi &\rightarrow (V_L P_L + V_R P_R)\Psi, \quad \overline{\Psi} \rightarrow \overline{\Psi}(V_L^\dagger P_R + V_R^\dagger P_L) \\ \hat{\Phi} &\rightarrow V_L \hat{\Phi} V_R^\dagger, \\ V_L &\in SU(2)_L^{(local)}, \quad V_R \in SU(2)_R^{(global)} \end{aligned} \tag{4.18}$$

with $\hat{\Phi}$ defined in eq. (2.15). The action is

$$S = S_H + S_F + S_Y + S_W. \tag{4.19}$$

The meaning of the individual terms in S is the following: S_H is the action of the SU(2) Higgs model, eq. (2.13). S_F describes the "naive" Dirac fermions

$$S_F = \frac{1}{2}\sum_x \sum_{\mu=1}^{4} \{\overline{\Psi}_x \gamma_\mu P_L (U_{x,\mu}\Psi_{x+\mu} - U_{x-\mu,\mu}^\dagger \Psi_{x-\mu}) + \overline{\Psi}_x \gamma_\mu P_R (\Psi_{x+\mu} - \Psi_{x-\mu})\}. \tag{4.20}$$

S_Y is the Yukawa coupling term

$$S_Y = y \sum_x \overline{\Psi}_x (\hat{\Phi}_x P_R + \hat{\Phi}_x^\dagger P_L)\Psi_x, \tag{4.21}$$

and S_W is the Wilson-Yukawa coupling term

$$\begin{aligned} S_W = \; & w \sum_x \sum_{\mu=1}^{4} \{\overline{\Psi}_x (\hat{\Phi}_x P_R + \hat{\Phi}_x^\dagger P_L)\Psi_x \\ & -\frac{1}{2}\overline{\Psi}_x (\hat{\Phi}_x P_R + \hat{\Phi}_{x+\mu}^\dagger P_L)\Psi_{x+\mu} - \frac{1}{2}\overline{\Psi}_{x+\mu}(\hat{\Phi}_{x+\mu} P_R + \hat{\Phi}_x^\dagger P_L)\Psi_x\}. \end{aligned} \tag{4.22}$$

As usual,

$$P_L = \frac{1}{2}(1-\gamma_5), \quad P_R = \frac{1}{2}(1+\gamma_5). \tag{4.23}$$

This lattice model, and its extension to the SU(2)⊗U(1) case, has been suggested by Smit [105, 8] and Swift [106]. Recently, it has been shown that the right-handed massless neutrino present in this model decouples from the physical spectrum [108].

For $w = 0$ this model describes 16 SU(2)-doublets of fermions of the same mass. Some of the 15 doublers have the "mirror" coupling to the gauge field. The hope is that due to the presence of the Wilson-Yukawa coupling w the 15 unwanted doublers get sufficiently massive to be in agreement with the phenomenology. This is an intrinsically non-perturbative problem as the coupling w has to be made large.

Until now the model has been investigated numerically in the limit $\beta = \infty$, i.e. without the gauge fields in the quenched simulation [93, 94]. (For other special cases see Subsec. 3.1.) I just want to mention two results: It has been demonstrated that in the quenched

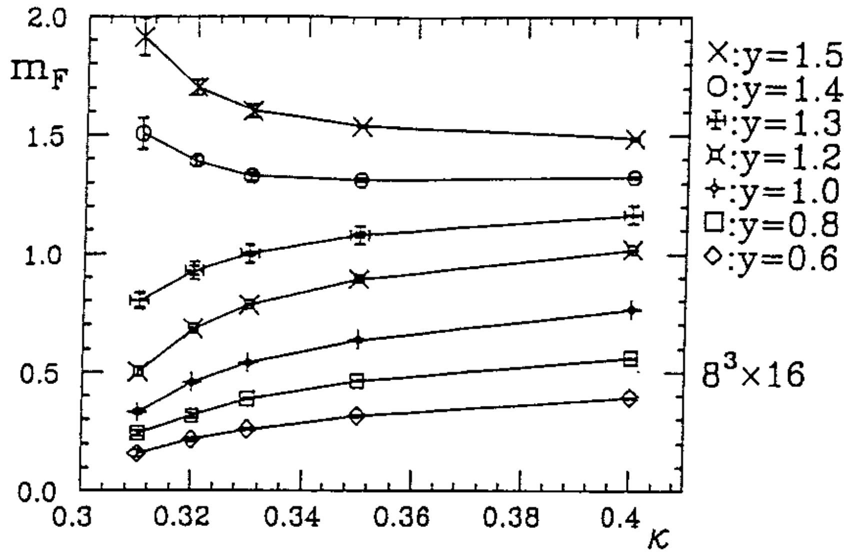


Figure 12: *Fermion mass in the quenched approximation for various fixed values of y as a function of κ in the SSB phase of the $SU(2)$ Smit-Swift model* (4.19) *without the gauge fields and with $w = 0$. For $y \geq 1.4$ the fermion mass increases as κ approaches the critical point at $\kappa_c \simeq 0.304$. The figure is from Ref.* [93].

approximation the model in the SSB phase has both the weak and strong y coupling regions. In the strong coupling region the fermion mass (and also the doubler masses) increase with decreasing distance from the critical line (Fig. 12). First results [94] for various values of w indicate that the doubler masses can be eleminated from the physical part of the spectrum. They can be namely made as large as the cut-off ($am_D \approx 1$) by a proper choice of w even for fixed w while am_F can be made small by a suitable choise of y. These results are very encouraging. (Fig. 13).

5 Outlook

Further work investigating the Smit-Swift model and other lattice models for weak interactions is in progress and the results are promising. Furthermore, a fast "hybrid Monte Carlo" algorithm for fermions has been recently introduced [109]. So I think that in a few years we shall have the non-perturbative properties of the Standard Model on the lattice numerically under control also in the case that some heavy fermions exist. We shall know the phase diagram and whether there exist some new critical points where one might obtain a – possibly non-trivial – continuum limit. An upper bound on the fermion mass – if it exists – will be determined. But the precision of the calculations will remain probably limited for a long time.

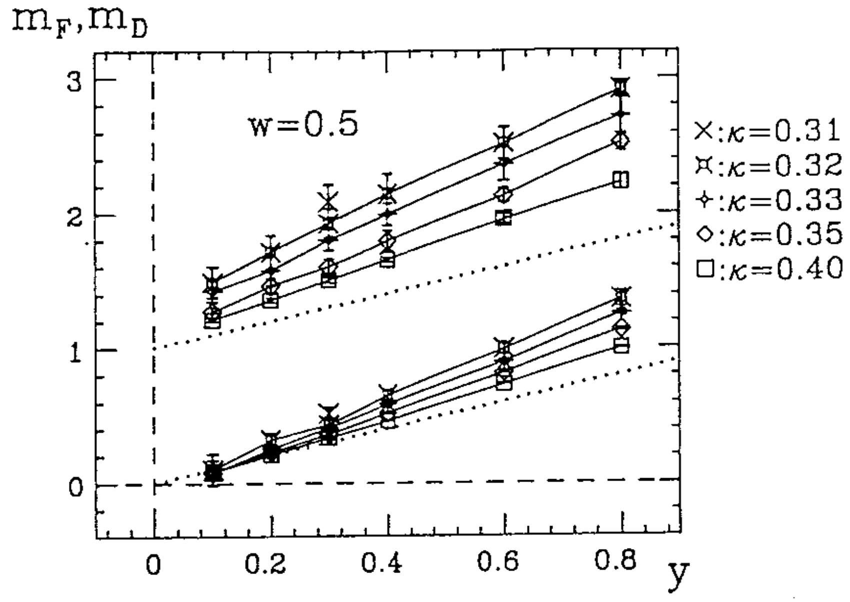


Figure 13: *Fermion and doubler mass as a function of y for various κ in the same model as in Fig. 12 but for $w = 0.5$. It demonstrates that in the quenched approximation it is possible to vary κ and y in such a way that the fermion mass (lower set of lines) approaches zero in lattice units as $\kappa \to \kappa_c$ but the doubler mass (upper set of lines) stays larger than the cut-off ($am_D > 1$). Thus the doublers do not appear in the physical spectrum. The figure is from Ref.* [94].

Concerning the scalar field calculations, we shall be able to achieve a high precision indeed. My optimism is due to the recent development of a new cluster algorithms for the O(4) Φ^4 model [110], based on an idea of Wolff [111], allowing its investigation on substantially larger lattices and with much increased statistical effectiveness. Thus fine tests of the validity of the Standard Model will be possible in the near future in the approximation treating the gauge fields and fermions only perturbatively

Concerning the lattice Higgs models, we do not have substantially improved algorithms for the gauge field yet, so that these models will have to wait for some time for a renewed interest.

Acknowledgement. I thank A. Ali and A. Zichichi for the opportunity to deliver these lectures and for the hospitality in Erice. I am grateful to A.K. De, C.B. Lang and J. Smit for many helpful suggestions. I have also benefited from discussions with A. Ali, M.A.B. Bég, W. Bock, K. Jansen, H.A. Kastrup, H. Leutwyler, I. Montvay and T. Neuhaus. I further thank Justina Smižanská-Jersák for typing these lecture notes and T. Trappenberg for some suggestions and help with the manuscript. A support of the Deutsches Bundesministerium für Forschung und Technologie is acknowledged.

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