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REGULARIZATION AND FEYNMAN RULES IN NONCOVARIANT GAUGES*

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Abstract

A regularization scheme for defining Feynman rules in the path-integral formalism is proposed. It is applied to pure abelian gauge theory and to obtaining regularized propagators in noncovariant gauges. The propagators are more intricate than those commonly adopted, and the known prescriptions for the noncovariant singularities appear not to be immediate consequences of the scheme. Further studies are required to investigate the renormalization and the practicality of the noncovariant-gauge formalisms.

1 Introduction

Noncovariant gauges have long been recognized for their usefulness in studying certain aspects of gauge field theories. The most commonly-used noncovariant gauges are the Coulomb gauge and algebraic gauges, such as the temporal, the spatial-axial, and the light-cone gauge. Although considerable work has been devoted to providing practical prescriptions for the spurious singularities in the Feynman rules in these gauges for perturbative calculations, there remain great difficulties in proving the correctness of proposed prescriptions. We can only justify a prescription partially by calculating with it, normally only tractable to one-loop level, physical quantities that have been calculated in a covariant gauge. (For a recent review, see ref. [1])

While the Feynman rules in a covariant gauge can be derived either by the canonical formalism or by the path-integral formalism, those in the noncovariant gauges obtained by the path-integral formalism are shown to be inconsistent [2,3]. Thus, here we propose a regularization scheme for defining Feynman rules in the path-integral formalism, with the aim of determining how practical propagators for gauge field in the noncovariant gauges can be derived in the scheme. We should recall that regularization is a procedure consisting mainly in replacing mathematically ill-defined objects with well-defined ones. We then study the latter and prove that some of the properties of interest, if valid for the regularized version, would carry over to their limit. In this context, we should mention two related regularization methods, which were recently considered to provide well-defined Feynman rules in the noncovariant gauges. One method [4] considers gauges that unify covariant gauge with noncovariant gauges by a tunable parameter, and treats the singularities in the noncovariant gauges as regularized by the parameter. The other [5] involves regularizing noncovariant singularities by adding to the required gauges terms that correspond to some boundary conditions. Finally, it should be mentioned that many studies have been made to establish equivalent formulations of Feynman rules in different gauges [3,6,7].

In the next section, we provide a short introduction to the theory of distributions, and define the role of regularizing functions in the theory. For pedagogical reasons, we

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shall consider in section 3 some examples of the application of a regularization scheme. In section 4, the scheme is applied to an abelian gauge theory in the linear covariant gauge and in several noncovariant gauges. Section 5 contains our conclusions.

2 Regularization and the Theory of Distributions

Since the theory of distributions plays a related role in developing a regularization scheme for defining Feynman rules, we give here a short introduction to the theory [8].

We shall refer to $\phi(x) = \phi(x_1, x_2, ..., x_n)$ as a test function if it is infinitely differentiable. For our purpose, we shall consider the Schwartz space S, which is a space of test functions of rapid descent. If $\phi(x) \in S$, then $\phi(x)$ and all its partial derivatives decrease to zero faster than every negative power of $|x| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$.

We define $\tilde{\phi}(x)$ as a generalized function or distribution and represent it by a sequence $\{\phi_\epsilon; \epsilon = 1/j, j = 1, 2, ..\}$. Then $\phi_\epsilon(x) \in S$, and

$$\int_\Omega \tilde{\phi}(x) g(x) dx \equiv \lim_{\epsilon \to 0} \int_\Omega \phi_\epsilon(x) g(x) dx \tag{1}$$

exists for $g(x) \in S$, $x \in \Omega$. For example, the one-dimensional Dirac δ-function $\delta(x)$ is a generalized function and can be defined as

$$\begin{aligned} \delta(x) &\equiv \lim_{\epsilon \to 0} \delta_\epsilon(x) \\ &= \lim_{\epsilon \to 0} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \varphi_\epsilon(k)\, dk, \end{aligned} \tag{2}$$

where $\varphi_\epsilon(k)$ is commonly referred to as a regularizing function. It should be integrable for $\epsilon \neq 0$ and satisfy $\varphi_\epsilon(k, \epsilon = 0) = 1$. If we take $\varphi_\epsilon(k) = e^{-\epsilon^2 k^2}$, then

$$\delta_\epsilon(x) \equiv \frac{1}{\sqrt{4\pi\epsilon^2}} e^{-\frac{x^2}{4\epsilon^2}} \tag{3}$$

and $\delta_\epsilon(x) \in S$, for $\epsilon \neq 0$.

We may also use the regularizing function $\rho_\alpha(k) = e^{-\alpha|k|}$ and get

$$\delta_\alpha(x) \equiv \frac{1}{\pi} \frac{\alpha}{x^2 + \alpha^2}, \ \alpha > 0. \tag{4}$$

The two representations (3) and (4) are *equivalent*, since for $g(x) \in S$,

$$\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \delta_\epsilon(x) g(x) dx = \lim_{\alpha \to 0} \int_{-\infty}^{\infty} \delta_\alpha(x) g(x) dx = g(0). \tag{5}$$

We note that the function $\rho_\alpha(k)$ is not differentiable at $k = 0$, but the represented function in x-space $\delta_\alpha(x)$ is. Thus, unlike test functions, regularizing functions are not necessarily everywhere differentiable, but should be integrable, both locally and globally. Nevertheless, it is possible to define a regularizing function $\varphi_\epsilon(k)$ such that $\varphi_\epsilon(k) \in S(k)$.

3 Examples

To appreciate the importance of regularization in defining many physical problems, we first consider the simple integral $I = \int_\Omega \frac{1}{r}\exp(i\vec{k}\cdot\vec{r})d\vec{r}$, where Ω is the three-dimensional Euclidean space and $r = |\vec{r}|$. This integral is undefined but bounded, having an arbitrary value in [0,2]. To give it a definite value, we can define it in a regularized sense as

$$I \equiv \lim_{\alpha\to 0}\int_\Omega \frac{1}{r}\exp(i\vec{k}\cdot\vec{r})f_\alpha(r)d\vec{r},\ \alpha > 0, \tag{6}$$

where we have chosen a spherically symmetric $f_\alpha(r)$ as is the potential $1/r$. If we choose $f_\alpha(r) = f(\alpha r) \in \mathcal{S}$, then $I \to 4\pi/\vec{k}^2$ as $\alpha/|\vec{k}| \to 0$. (Note that dimensional regularization can be used here and gives the identical result.)

As a second example we consider the derivation of anomalous Ward identities in gauge theories under local chiral transformation in the path-integral approach, as studied notably by Fujikawa [9]. These identities are consequences of noninvariance of fermion measure with respect to local chiral transformation, and a regularization scheme is required to derive them. Fujikawa has used an exponential regularizing function, and argued that we should choose a regularizing function $f_\alpha(x) = f(y=\alpha x)$, such that $f(0) = 1, f(\infty) = 0$, and all its derivatives evaluated at $x = \infty$ vanish. Recently, Marozov [10] gave the following sufficient condition on the n-th ($n \geq 1$) derivative of $f(y)$:

$$f^{(n)}(y) \leq (constant)/(1+y)^{n+\eta}, \eta > 0, \tag{7}$$

in the limit $y \to \infty$. (For a detailed study of regularization schemes, see ref. [11].)

Finally, we apply the regularization scheme to the field-theoretic φ^4 model. Quantization of this model in the path-integral formalism amounts to considering the generating functional

$$\mathcal{Z}[J] = \int \mathcal{D}[\varphi]\ \exp\left(i\int(\mathcal{L}(x) + J(x)\varphi(x))dx\right), \tag{8}$$

$$\mathcal{L}(x) = \frac{1}{2}\left((\partial_\mu\varphi(x))^2 - m^2\varphi^2(x)\right) - \frac{\lambda}{4!}\varphi^4(x)\ ;\ m\,,\,\lambda > 0.$$

This functional is ill defined in two instances. Firstly, we encounter divergences in perturbative calculations that need be defined by a regularization scheme, such as dimensional regularization, and a renormalization procedure. Secondly, for perturbative calculations, we need a set of Feynman rules that are yet to be specified by some boundary conditions. To specify them, we can define the functional (8) as

$$\mathcal{Z}[J] = \lim_{\epsilon\to 0}\int \mathcal{D}[\varphi]F_\epsilon[\varphi]\exp\left(i\int(\mathcal{L}(x) + J(x)\varphi(x))dx\right), \epsilon > 0, \tag{9}$$

where $F_\epsilon[\varphi]$ is the non-derivative, integrable functional satisfying $F_\epsilon[\varphi, \epsilon=0] = 1$:

$$F_\epsilon[\varphi] = \exp\left(-\frac{\epsilon}{2}\int\varphi^2(x)dx\right). \tag{10}$$

The functional measure is then the weighted measure $\mathcal{D}[\varphi]F_\epsilon[\varphi]$ with weight $F_\epsilon[\varphi]$. The Feynman rules are now well defined, and the propagator for the scalar field reads

$$G(p,m) = \frac{i}{p^2 - m^2 + i\epsilon}\ , \epsilon > 0. \tag{11}$$

Alternatively, we can define the Feynman rules in Euclidean space by considering (8) as a Wiener integral.

4 Application to Abelian Gauge Theory

Here we shall apply the regularization scheme to the pure abelian theory†

$$\mathcal{L}(x) = -\frac{1}{4}(F_{\mu\nu})^2\ , \tag{12}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength with A_μ being the gauge field. In the path-integral approach, we first consider the naïve generating functional

$$\mathcal{Z}[J_\mu] = \lim_{\epsilon\to 0}\int \mathcal{D}[A_\mu]F_\epsilon[A_\mu]\ \exp\left(i\int(\mathcal{L}(x) + J_\mu(x)A_\mu(x))dx\right), \epsilon > 0, \tag{13}$$

where $F_\epsilon[A_\mu]$ is some properly chosen regularizing functional. But, because of local gauge invariance, we need to fix a gauge. It is crucial that the introduction of a regularization scheme does not correspond to fixing a gauge. To show this, we consider

$$F_\epsilon[A_\mu] = \exp\left[\frac{\epsilon}{2}\int(A_\mu\delta_{\mu\nu}A_\nu)\,dx\right], \epsilon > 0, \tag{14}$$

which behaves as a damping factor for space-like A_μ. This functional is not integrable but can be made so by Wick rotation. With (14), we get the singular propagator

$$G_{\mu\nu}(p) = G(p)\left(-\delta_{\mu\nu} - \frac{p_\mu p_\nu}{i\epsilon}\right)\ ,\ G(p) \equiv \frac{i}{p^2 + i\epsilon}. \tag{15}$$

In a general linear gauge, $\mathcal{F}_\mu A_\mu = \phi(x)$, where $\mathcal{F}_\mu$ is a linear operator and $\phi(x)$ is an arbitrary function, the generating functional can be cast in the form [1]

$$\mathcal{Z}[J_\mu] = \lim_{\epsilon\to 0}\int \mathcal{D}[A_\mu]F_\epsilon[A_\mu]\ \exp\Bigg(i\int(\mathcal{L}(x) - \frac{1}{2\alpha}(\mathcal{F}_\mu A_\mu)^2 + J_\mu(x)A_\mu(x))dx\Bigg), \alpha \neq 0. \tag{16}$$

Using (16), we can now proceed to discuss the theory in the following gauges.

4.1 The general linear covariant gauge

† We employ the metric $\delta_{\mu\nu}$ = diag(1,-1,-1,-1), and summation convention for repeated indices. Greek letters μ, ν are Lorentz indices.

This gauge is specified by $\partial_\mu A_\mu(x) = \phi(x)$. The quantized theory is defined by the functional (16). Using the regularizing functional (14), we get the propagator

$$G_{\mu\nu}(p) = G(p)d_{\mu\nu} \equiv G(p)\left[-\delta_{\mu\nu} + \frac{(1-\alpha)p_\mu p_\nu}{p^2 + i\epsilon\alpha}\right], \tag{17}$$

where $G(p)$ is given in (15). An immediate consequence of our regularization scheme is the factorization of the physical and the unphysical poles. If we demand that Wick rotation be possible also with the unphysical poles defined by $p^2 + i\epsilon\alpha = 0$, imposing the familiar boundary conditions on the propagator, we must have $\alpha > 0$. Therefore, we should consider, even in the Landau gauge ($\alpha = 0$), the limits $\epsilon \to 0$ and $\epsilon' \equiv \epsilon\alpha \to 0$ only after momentum integrations.

A slightly more complicated regularizing functional that depends on additional parameters may be considered. For example,

$$F_\epsilon[A_\mu] = \exp\left[\frac{\epsilon}{2}\int \left(A_\mu\left[\delta_{\mu\nu} + \beta n_\mu n_\nu\right]A_\nu\right)dx\right], \epsilon > 0, \tag{18}$$

with $n_\mu = (1,0,0,0)$ and $\beta \geq 0$ may be used. We note that while (14) has a light-like boundary $A_\mu\delta_{\mu\nu}A_\nu = 0$, (18) has a modified boundary $A_\mu[\delta_{\mu\nu}+\beta n_\mu n_\nu]A_\nu = 0$. Clearly, a space-like vector, which is regularized by (14), is not necessarily regularized by (18).

Returning to the functional (18), we get the polarization sum

$$\begin{aligned} d_{\mu\nu} = &-\delta_{\mu\nu} + D\left\{(1-\alpha)(p^2 + i\epsilon + i\beta\epsilon)p_\mu p_\nu \right. \\ &\left. - i\beta\epsilon p\cdot n(1-\alpha)\left(p_\mu n_\nu + p_\nu n_\mu\right) + i\beta\epsilon(p^2 + i\alpha\epsilon)n_\mu n_\nu\right\}. \end{aligned} \tag{19}$$

There are four poles defined by the quartic equation

$$D^{-1} \equiv (p^2 + i\alpha\epsilon)(p^2 + i\epsilon + i\beta\epsilon) - i\beta\epsilon(1-\alpha)(p\cdot n)^2 = 0. \tag{20}$$

Thus, the functional (18) offers no practical advantage. We see that, unlike in the scalar field theory, we have a freedom to define the Feynman rules for a vector theory. In particular, the use of a different $F_\epsilon[A_\mu]$ leads to a different representation of a gauge theory, and could lead to calculational difficulties and complicated renormalization.

4.2 The Coulomb gauge

The inhomogeneous Coulomb gauge is specified by $\vec\nabla\cdot\vec A = (\partial_\mu - n\cdot\partial n_\mu)A_\mu = \phi(x)$, where $n_\mu = (1,0,0,0)$. We consider the generating functional (16) with (14) and obtain $G_{\mu\nu}(p) = G(p)c_{\mu\nu}$, where $G(p)$ is defined in (15), and $c_{\mu\nu}$ is given by

$$\begin{aligned} c_{\mu\nu} = &-\delta_{\mu\nu} + D\left\{\left[(\alpha-1)(p^2 + i\epsilon) + (p\cdot n)^2\right]p_\mu p_\nu \right. \\ &\left. - p\cdot n\left[(p\cdot n)^2 - p^2 - i\epsilon\right]\left(p_\mu n_\nu + p_\nu n_\mu\right) - i\epsilon(p\cdot n)^2 n_\mu n_\nu\right\}. \end{aligned} \tag{21}$$

The spurious poles are defined by

$$D^{-1} \equiv \left[(p\cdot n)^2 - p^2 - i\epsilon\alpha\right](p^2 + i\epsilon) - (p\cdot n)^2\left[(p\cdot n)^2 - p^2\right] = 0 \tag{22}$$

or, explicitly, read

$$p_0^\pm = \pm(1+i)\sqrt{\frac{1}{2\epsilon\alpha}(\vec p^2 - i\epsilon)(\vec p^2 - i\epsilon\alpha)}, \epsilon > 0. \tag{23}$$

To allow for Wick rotation, the parameter α must be negative and should not be set to zero prematurely. As always, in the path-integral formalism, one should consider the homogeneous Coulomb gauge $\vec\nabla\cdot\vec A = 0$ only in the limit $\alpha \to 0$.

We may also use a different regularizing functional, such as (18). But, we may then have to work with a complicated propagator.

4.3 The temporal gauge

This gauge, also called the Hamiltonian gauge, is characterized by $A_0 = 0$ [12], or by $n\cdot A = 0$, where $n_\mu = (1,0,0,0)$. The inhomogeneous temporal gauge is defined by $n\cdot A = \phi(x)$.

We define the theory in this gauge as a regularized path integral given by (16) with $F_\epsilon[A_\mu]$ in (14). (Other forms of $F_\epsilon[A_\mu]$ may also be considered.) The propagator in this case reads $G_{\mu\nu}(p) = G(p)t_{\mu\nu}$, where $G(p)$ is given in (15) and

$$t_{\mu\nu} = -\delta_{\mu\nu} - D\left\{\left[\alpha(p^2 + i\epsilon) + 1\right]p_\mu p_\nu - p\cdot n\left(p_\mu n_\nu + p_\nu n_\mu\right) - i\epsilon n_\mu n_\nu\right\}, \tag{24}$$

with $D^{-1} = (p\cdot n)^2 + i\epsilon\alpha(p^2 + i\epsilon) + i\epsilon$. For $\alpha \approx 0$, the poles are

$$p_0^\pm \approx \pm(1-i)\sqrt{\frac{\epsilon}{2}(1 - \alpha\vec p^2)}, \tag{25}$$

which are seen to permit Wick rotation.

Setting $\alpha = 0$ (the pure temporal gauge), we get the polarization sum

$$t^0_{\mu\nu} = -\delta_{\mu\nu} - D_0\left[p_\mu p_\nu - p\cdot n\left(p_\mu n_\nu + p_\nu n_\mu\right) - i\epsilon n_\mu n_\nu\right], \tag{26}$$

where $D_0^{-1} = p_0^2 + i\epsilon$. More simply, the propagator reads

$$\begin{aligned} G_{00}(p) = G_{0j}(p) &= G_{j0}(p) = 0, \\ G_{ij}(p) = G(p)&\left[\delta_{ij} - \frac{p_i p_j}{p_0^2 + i\epsilon}\right], i,j = 1,2,3. \end{aligned} \tag{27}$$

This propagator can also be found in several publications [7,13]. It can be derived by considering the three-component lagrangian obtained by setting $A_0 = 0$ in (12). Then we introduce the regularizing functional (14) with $A_0 = 0$.

4.4 The spatial-axial gauge

Like the temporal gauge, this gauge can be defined as $n\cdot A = 0$, where $n_\mu = (0,0,0,1)$. The inhomogeneous gauge is specified by $n\cdot A(x) = \phi(x)$.

Using $F_\epsilon[A_\mu]$ in (14) and the inhomogeneous gauge, we get the propagator $G_{\mu\nu}(p) = G(p)a_{\mu\nu}$, with $G(p)$ defined in (15) and

$$a_{\mu\nu} = -\delta_{\mu\nu} - D\left\{\left[\alpha(p^2 + i\epsilon) - 1\right]p_\mu p_\nu - p\cdot n\left(p_\mu n_\nu + p_\nu n_\mu\right) - i\epsilon n_\mu n_\nu\right\}, \tag{28}$$

where $D^{-1} = (p{\cdot}n)^2 + i\epsilon\alpha(p^2 + i\epsilon) - i\epsilon$. The propagator is more complicated than those commonly used. In particular, it contains additional tensorial terms and its singularities are not defined by any known prescription, such as the principal-value prescription.

If we set $\alpha = 0$ in the polarization sum (28), we get

$$a^0_{\mu\nu} = -\delta_{\mu\nu} + D_0\left[p_\mu p_\nu + p{\cdot}n\,(p_\mu n_\nu + p_\nu n_\mu) + i\epsilon n_\mu n_\nu\right], \tag{29}$$

where $D_0^{-1} = p_3^2 - i\epsilon$.

Other regularizing functionals that could give practical prescriptions remain to be found.

4.5 The light-cone gauge

This gauge is defined as $A_- \equiv (A_0 - A_3) = 0$ or $n\cdot A = 0$, where $n_\mu = (1,0,0,1)$ is a light-like vector. We define the inhomogeneous gauge by $n\cdot A = \phi(x)$.

Adopting the regularizing functional (14), we get the corresponding propagator $G_{\mu\nu}(p) = G(p)d_{\mu\nu}$, with $G(p)$ given in (15) and

$$d_{\mu\nu} = -\delta_{\mu\nu} - D\left[\alpha(p^2 + i\epsilon)p_\mu p_\nu - p{\cdot}n\,(p_\mu n_\nu + p_\nu n_\mu) - i\epsilon n_\mu n_\nu\right], \tag{30}$$

where $D^{-1} = (p{\cdot}n)^2 + i\epsilon\alpha(p^2 + i\epsilon)$. For $\alpha \neq 0$, the naïve poles given by $(p{\cdot}n)^2 = 0$ are regularized, but not by the Leibbrandt-Mandelstam prescription [14].

However, setting naïvely $\alpha = 0$ gives

$$d^0_{\mu\nu} = -\delta_{\mu\nu} + \frac{1}{(p{\cdot}n)^2}\left[p{\cdot}n\,(p_\mu n_\nu + p_\nu n_\mu) + i\epsilon n_\mu n_\nu\right], \tag{31}$$

and the singularities $(p{\cdot}n)^2 = 0$ are not regularized at all. In fact, the functional

$$F_\epsilon[A_\mu] = \exp\left[\frac{\epsilon}{2}\int\left(A_\mu\left[\delta_{\mu\nu} + \beta(n_\mu n^*_\nu + n^*_\mu n_\nu)\right]A_\nu\right)dx\right], \beta \neq 0, \tag{32}$$

with $n^*_\mu = (1,0,0,-1)$, also leads to (31) if we set $\alpha = 0$. With the functional

$$F_\epsilon[A_\mu] = \exp\left[\frac{\epsilon}{2}\int\left(A_\mu\left[\delta_{\mu\nu} + \beta n^*_\mu n^*_\nu\right]A_\nu\right)dx\right], \beta \neq 0, \tag{33}$$

we get, for $\alpha = 0$, a complicated propagator with spurious poles specified by

$$-4i\beta\epsilon[p{\cdot}n p{\cdot}n^* + i\epsilon] + (p{\cdot}n)^2(p^2 + i\epsilon) = 0. \tag{34}$$

5 Conclusions

By introducing a simple regularization scheme to define Feynman rules in abelian gauge theory quantized in the noncovariant gauges, we found that the propagators are generally more intricate than those commonly used. The propagators contain additional tensorial terms, because these gauges depend on constant vectors. Our preliminary study based on this scheme indicates that there exists a freedom to define Feynman rules for gauge theory. Specifically, the use of a different $F_\epsilon[A_\mu]$ leads to a different propagator. Further studies are, however, required to determine whether the additional tensorial

terms found in this scheme play any significant role in perturbative calculations, and whether the propagators are consistent with physics. Also, since the renormalization of these noncovariant-gauge formalisms remains open, it is necessary to perform some calculations.

Eventually we should like to apply the regularization scheme to non-abelian gauge theory, where we also need to regularize the propagator for the Faddeev-Popov ghosts.

Finally, it should be of interest to see how this scheme can be related to the canonical formulation, and how the previously proposed prescriptions can be incorporated in this scheme.

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