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Abstract

The generalization of quark and lepton generation mixing model of the type suggested by Berezhiani and Chkareuli is considered. The model leads to realistic picture of quark mixing, to not small t-quark mass and admits SU(5) or $SU(5)_{flip} \times U(1)$ grand unification. It allows to calculate elements of the KM matrix of quark mixing and yields, in particular, the ratio $R=V_{ub}/V_{cb}V_{us}$ (which is equal to $|s_\gamma/s_\beta s_\theta|\exp(i\delta)$ in the Maiani representation) in the form $|R|\exp i\delta = \left[\frac{b+1}{M_0} - \exp(-i\vartheta)\right]\frac{m_t}{M_0 b}e^{-i\vartheta}$, where δ is CP violating phase, $M_0 = (m_b/m_s)m_c \simeq 48$ GeV, $b \sim 2$-3 and ϑ can not be calculated.

Recently, Berezhiani and Chkareuli [1,2] have developed different types of models based on violated gauge symmetry $SU(3)_H$ (or $SU(3)_H \times U(1)$) of generations of quarks and leptons. One of these models [2] uses the see-saw mechanism and can be formulated in the framework of the grand unification SU(5) or $SU(5)_{flip} \times U(1)$ scheme. Below, the generalization of it is considered, which results in not small t-quark mass and gives some new information on KM matrix elements (see also [3]).

The spontaneous violation of the "horizontal", or generations symmetry was assumed in [2] to be due, below some scale $Q=M_I$ of order of tens TeV, to appearing of nonzero vacuum expectation values (VEV) of scalars $\chi^{\alpha\beta} = \chi^{\{\alpha,\beta\}} + \chi^{[\alpha\beta]}$, carrying indices of generations $\alpha,\beta=1,2,3$. The symmetric in α,β scalar field $\chi^{\{\alpha\beta\}}$ (the sixtet) has VEV of the form $\langle\chi^{\{\alpha\beta\}}\rangle = \begin{pmatrix} \chi & & \\ & \rho & \\ & & \sigma \end{pmatrix}$ with $\chi \gg |\rho| \gg \sigma$, while the antisymmetric one is the superposition of antitriplets $\chi^{[\alpha\beta]} = \varepsilon^{\alpha\beta\gamma}[a\zeta_\gamma + b\xi_\gamma + c\eta_\gamma]$ with VEV $\langle\chi^{[\alpha\beta]}\rangle = \begin{pmatrix} 0 & c\eta & a\zeta \\ -c\eta & 0 & b\xi \\ -a\zeta, & -b\xi & 0 \end{pmatrix}$, where $\zeta = -\langle\zeta_2\rangle$, $\xi = \langle\xi_1\rangle$, $\eta = \langle\eta_3\rangle$, $\eta \gg \zeta \gg \xi$ (but $\zeta < |\rho|$) and coefficients a,b,c are, in general, different in the Yukawa coupling with different fermions. (Thus $\chi^{\alpha\beta} \to \chi_f^{\alpha\beta}$ will be provided below by the index f=u,d,e, of the corresponding fermions and normalization of VEV's ζ, ξ, η will be used which corresponds to $a_d = b_d = c_d = 1$ for $f=d$)

The violated gauge symmetry of generations means the existance in the model of massive vector (neutral) gauge bosons, with mass of order of M_I, changing the flavour, i.e. the generation indices, of quarks and leptons (see [1,2])

1. The content of fermions and Yukawa interaction Lagrangian.

Besides left-hand fields of usual quarks and leptons $q_L^\alpha = (10^\alpha, \bar{5}^\alpha, e^{c\alpha})_L$ (in SU(5) classification) carrying at the top the generation index $\alpha = 1,2,3$ the model [2] contains more two kinds of heavy quarks and leptons, with masses of order of M_I, in each generation: $Q_{\alpha L} = (X_\alpha, \bar{V}_\alpha, E^c_\alpha)_L$ and $\bar{Q}_{\alpha L} = (\bar{X}_\alpha, V_\alpha, \bar{E}^c_\alpha)_L$ having the generation index α below, i.e. transforming as antitriplets of the $SU(3)_H$ group. Let us remind the content of 10 and $\bar{5}$-plets of the $SU(5)_{flip} \times U(1)$ model [5]:

$$10^\alpha_L = \left(d^c, \begin{pmatrix} d \\ u \end{pmatrix}, \nu^c\right)^\alpha_L, \quad X_{\alpha L} = \left(D^c, \begin{pmatrix} D \\ U \end{pmatrix}, N^c\right)_{\alpha L}, \quad \bar{X}_{\alpha L} = \left(\bar{D}^c, \begin{pmatrix} \bar{D} \\ \bar{U} \end{pmatrix}, \bar{N}^c\right)_{\alpha L},$$

$$\bar{5}^\alpha_L = \left(u^c, \begin{pmatrix} \nu \\ e \end{pmatrix}\right)^\alpha_L, \quad \bar{V}_{\alpha L} = \left(U^c, \begin{pmatrix} N \\ E \end{pmatrix}\right)_{\alpha L}, \quad V_{\alpha L} = \left(\bar{U}^c, \begin{pmatrix} \bar{N} \\ \bar{E} \end{pmatrix}\right)_{\alpha L}$$

and represent[1] the Lagrangian of their Yukawa interaction $\mathcal{L}_{Yuk} = \mathcal{L}_{10} + \mathcal{L}_5 + \mathcal{L}_1$ in the following general form:

$$\begin{cases} \mathcal{L}_{10} = f \cdot 10^\alpha X_\alpha H_d + h_d \chi_d^{\alpha\beta} X_\alpha \bar{X}_\beta + \tau I \bar{X}_\alpha 10^\alpha + h.c., \\ \mathcal{L}_5 = f_1 H_u 10^\alpha \bar{V}_\alpha + f_1' H_u X_\alpha \bar{5}^\alpha + h_u \chi_u^{\alpha\beta} V_\alpha \bar{V}_\beta + \tau_1 I \bar{5}^\alpha V_\alpha + h.c., \\ \mathcal{L}_1 = f_0 H_d \bar{5}^\alpha E^c_\alpha + f_0' H_d V_\alpha e^{c\alpha} + h_e \chi_e^{\alpha\beta} E^c_\alpha \bar{E}^c_\beta + \tau_0 I \bar{E}^c_\alpha e^{c\alpha} + h.c., \end{cases} \tag{1}$$

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1) In the presented form model admits $SU(5)_{flip} \times U(1)$ (or $-$ SO(10)) grand unification leading just to the observed value of the Weinberg angle $\sin^2\Theta_W = 0.23$ at any number of the usual or heavy 16-plets. Note also that the condition of cancellation of the triangular anomaly connected with the "horizontal" $SU(3)_H$ group can be satisfied [2] by introduction of 3x16 right-handed heavy neutral fermions - singlets of $SU(5)_{flip}$. They can be [2], for instance, φ^α_{aR} and $(\varphi^{(\alpha\beta)}_{\gamma d})_R$, a=1,2,3 (three triplets and 15-plets) and have Yukawa coupling with neutrino fields which, due to see-saw mechanism [7] will reduce the masses of physical neutrino to very small value.

where the SU(5) indices are omitted: $f, h, \tau, f_1, f_1', h_1$, etc. are Yukawa coupling constants which will be assumed below to be real, $I = I(x)$ is SU(5) scalar which acquires VEV $\langle I \rangle = I_0 \sim M_I$ and $H_u = (S_u, \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix})$, $H_d = (S_d, \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix})$ are 5-plet Higgs fields of standard theory (in which S_u, S_d are triplets of coloured scalars). As was explained above $\chi_f^{\alpha\beta} = (\chi_d^{\alpha\beta}, \chi_u^{\alpha\beta}, \chi_e^{\alpha\beta})$, f=d,u,e, are scalars having non-diagonal VEV's of the form $\zeta_f = a_f \zeta$, $\xi_f = b_f \xi$, $\eta_f = c_f \eta$, with $a_d = b_d = c_d = 1$, $\zeta = \langle \zeta_2 \rangle$, $\xi = \langle \xi_1 \rangle$, $\eta = \langle \eta_3 \rangle$.

Lagrangian (1) is invariant under U(1)-transformations $q^\alpha \to q^\alpha \exp(iY_q\varphi)$, $(Q_\alpha \text{ or } \bar{Q}_\alpha) \to (Q_\alpha \text{ or } \bar{Q}_\alpha)\exp(-iY_q\varphi)$, $\chi^{\alpha\beta} \to \chi^{\alpha\beta}\exp(iY_\chi\varphi)$ and $(H_u, H_d, I) \to (H_u, H_d, I)$ at the following values of hyperchanges: $Y_q = 1/2$, $Y_\chi = 1$.

2. The quark's mass matrices and their diagonalization.

Putting instead of scalars H_d, H_u and $I(x)$ their VEV's $\langle H_u \rangle = (0, \begin{pmatrix} 0 \\ v_u \end{pmatrix})$, $\langle H_d \rangle = (0, \begin{pmatrix} v_d \\ 0 \end{pmatrix})$, $\langle I \rangle = I_0$, where $v_d \sim v_u \ll I_0 \sim M_I$ one can represent L_{Yuk} in the following convenient matrix form

$$\mathcal{L}_{Yuk} = (d^c, D^c, \bar{D}) \begin{pmatrix} 0 & \tau I_0 & f v_d \\ f v_d & h_d \hat{\chi}_d & 0 \\ \tau I_0 & 0 & h_d \hat{\chi}_d^T \end{pmatrix} \begin{pmatrix} d \\ \bar{D}^c \\ D \end{pmatrix} + (u^c, U^c, \bar{U}) \begin{pmatrix} 0 & \tau_1 I_0 & f_1' v_u \\ f_1 v_u & h_u \hat{\chi}_u & 0 \\ \tau I_0 & 0 & h_d \hat{\chi}_d^T \end{pmatrix} \begin{pmatrix} u \\ \bar{U}^c \\ U \end{pmatrix} +$$

$$+ (e^c, E^c, \bar{E}) \begin{pmatrix} 0 & \tau_0 I_0 & f_0' v_d \\ f_0 v_d & h_e \hat{\chi}_e & 0 \\ \tau_1 I_0 & 0 & h_u \hat{\chi}_u^T \end{pmatrix} \begin{pmatrix} e \\ \bar{E}^c \\ E \end{pmatrix} + (\nu^c, N^c, \bar{N}) \begin{pmatrix} 0 & \tau I_0 & f_1 v_u \\ f_1' v_u & h_d \hat{\chi}_d & 0 \\ \tau_1 I_0 & 0 & h_u \hat{\chi}_u^T \end{pmatrix} \begin{pmatrix} \nu \\ \bar{N}^c \\ N \end{pmatrix}, \tag{2}$$

where generation indices α, β =1,2,3 are discarded (they could be restored by substitution $\hat{\chi}_f \to \chi_f^{\alpha\beta}$, f=d,u,e, $I_o \to I_o\delta^\alpha_\beta$, $(v_u, v_d) \to (v_u, v_d)\delta^\alpha_\beta$).

In the case $|\hat{\chi}_f| > I_o, v_u, v_d$ the 3x3 matrices in (2) can be easily diagonalized. This determines massas of heavy quarks D,U and leptons E,N in the form of Berezhiani-Chkareuli-Stech [1] matrices $M_f, \hat{M}_f$:

$$\hat{M}_f = h_f \hat{\chi}_f = h_f \begin{pmatrix} \chi & \eta_f & \zeta_f \\ -\eta_f & \rho & \xi_f \\ -\zeta_f & -\xi_f & \sigma \end{pmatrix}, \quad f = d, e, u \qquad (3)$$

with $h_f = (h_d, h_u, h_e)$, $(\zeta_f, \xi_f, \eta_f) = (a_f\zeta, b_f\xi, c_f\eta)$, and a_f, b_f, c_f – the numerical factors introduced above, where $a_d=b_d=c_d=1$. Note, that non diagonal VEV's ζ, ξ, η can be made here pure imaginary $\zeta = i\zeta_o$, $\xi = i\xi_o$, $\eta = i\eta_o$ by an appropriate choice of quark and lepton field phases and χ – pure real, while $\rho = \rho_1 + i\rho_2$ and $\sigma = \sigma_1 + i\sigma_2$ rests in general complex.

Matrices of light, i.e. physical, quark and lepton Dirac masses are obtained simultaneously in the form of see-saw model matrices [2,7], inverse to (3)

$$\left\{\begin{aligned} \hat{m}_d &= \frac{v_d I_o}{\chi|\rho\sigma|} A_d \, 1/2\left[\Delta_d/\hat{\chi}_d + \Delta_d/\hat{\chi}_d^T\right] = (m_s/\chi|\rho\sigma|)\begin{pmatrix} \sigma\rho+\xi^2 & 0 & \xi\eta \\ 0 & \chi\sigma & 0 \\ \xi\eta & 0 & \chi\rho+\eta^2 \end{pmatrix}, \\ \hat{m}_u &= \frac{v_u I_o}{\chi|\rho\sigma|} A_u \left[(1-\varepsilon_u)\Delta_u/\hat{\chi}_u + \varepsilon_u \Delta_d/\hat{\chi}_d^T\right] = (m_c/\chi|\rho\sigma|)\cdot\begin{pmatrix} \sigma\rho+\bar{\kappa}' b^2\xi^2, & -c\eta\sigma, & \alpha bc\xi\eta \\ c\eta\sigma & \chi\sigma & -b\xi\chi \\ \alpha bc\xi\eta & b\xi\chi & \chi\rho+\kappa c^2\eta^2 \end{pmatrix}, \\ \hat{m}_e &= \frac{v_d I_o}{\chi|\rho\sigma|} A_e \left[(1-\varepsilon_e)\Delta_e/\hat{\chi}_e + \varepsilon_e \Delta_u/\hat{\chi}_u^T\right] = (m_\mu/\chi|\rho\sigma|)\begin{pmatrix} \sigma\rho+\bar{\kappa}' b'^2\xi^2, & -c'\eta\sigma & \alpha' b'c'\xi\eta \\ c'\eta\sigma & \chi\sigma & -b'\xi\chi \\ \alpha' b'c'\xi\eta & b'\xi\chi & \chi\rho+\kappa' c'^2\eta^2 \end{pmatrix}, \end{aligned}\right. \qquad (4)$$

where[2)]

$$\Delta_f = \det\hat{\chi}_f = \chi\rho\sigma z_f, \quad z_f = 1 + \eta_f^2/\chi\rho + \xi_f^2/\chi\sigma \simeq 1 + \eta_f^2/\chi\rho$$

(as $|\xi_f| \ll |\eta_f|$, see below) $f = d, u, e$ and matrices $\Delta_f/\hat{\chi}_f$, $\Delta_f/\hat{\chi}_f^T$ are written in Eq.(4) in an explicit form (at $\zeta = \langle\zeta_3\rangle = 0$, as this VEV is inessential at $\zeta < |\rho|$ for the following), while $A_d = 2f\tau/h_d z_d$, $A_u = f_1 \tau_1/h_u z_u + f_1'\tau/h_d z_d$, $A_e = f_o\tau_o/h_e z_e + f_o'\tau_1/h_u z_u$, with $z_d = 1-x$, $z_u = 1-b_u^2 x$, $z_e = 1-b_e^2 x$ and $x = (i\eta)^2/\chi\rho$ is a single complex parameter ($|x| \sim 1$, see below). The running s,c-quarks and muon masses in Eq.(4) are

$$m_s = v_d I_o A_d/|\rho|, \quad m_c = v_u I_o A_u/|\rho|, \quad m_\mu = v_d I_o A_e/|\rho|$$

and parameters

$$\varepsilon_u = f_1'\tau/h_d z_d A_u, \quad \varepsilon_e = f_o'\tau_1/h_u z_u A_e \qquad (5)$$

determine values of different factors in Eq.(4), in the matrix $\hat{m}_u$:

$$\left\{\begin{aligned} b &= (1-\varepsilon_u)b_u - \varepsilon_u, & \alpha bc &= (1-\varepsilon_u)b_u c_u + \varepsilon_u, \\ c &= (1-\varepsilon_u)c_u - \varepsilon_u, & \kappa c^2 &= (1-\varepsilon_u)c_u^2 + \varepsilon_u, \end{aligned}\right. \qquad (6)$$

and of similar factors c', b', α', k' in $\hat{m}_e$ (for instance, $b' = (1-\varepsilon_e)b - \varepsilon_e$, $k'c'^2 = (1-\varepsilon_e)c_e^2 + \varepsilon_e$ etc.)

In the particular case $f_o' = f_1' = 0$ in Eqs.(1) i.e. at $\varepsilon_u = 1$, $\varepsilon_e = 0$, Eqs.(2)-(4) reproduce the Berezhiani-Chkareuli model[2] of the matrices inverse to their (and Stech) matrices (3), with $\hat{m}_d \sim (\hat{\chi}_d^{-1} + \hat{\chi}_d^{T-1})$, $\hat{m}_u \sim 1/\hat{\chi}_d^T$, $\hat{m}_e \sim 1/\hat{\chi}_e$, which leads to inadmissibly

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2) Neutrino masses are determined by the matrix m from (2), however due to Yukawa interaction with "horizontal" heavy fermions (see footnote 1), the physical neutrino obtains very small masses [2].

small t-quark mass $m_t \simeq m_c (m_s/m_d) \simeq 48$ GeV (see below). Taking $f_0' \ll f_0$, i.e. $|\varepsilon_e| \ll 1$, or $\varepsilon_e = 0$ we consider below the opposite case $f_1' \ll f_1$, when $|\varepsilon_u|$ is small: $|\varepsilon_u| \simeq |(f_1'/f_1)(\tau h_u z_u / \gamma_1 h_d z_d)| < 1$ and m_t is not restricted.

3. Particles' mass s and parameters of matrices χ_f, t=d,u,e.

At the hierarhie $\chi > \eta > |\rho| >> \xi \gtrsim |\sigma|$ and with ,b , ,b 1 the matrices (4) can be easily diagonalized

$$(\hat{m}_f)^{diag} = \hat{U}_f^+ \hat{m}_f \hat{V}_f$$

where V_f are matrices of rotation of left fermions, U_f - of right ones and for f=d,u

$$V_d = \begin{pmatrix} 1 & 0 & (\xi\eta/\lambda)^* \\ 0 & 1 & 0 \\ -\xi\eta/\lambda & 0 & 1 \end{pmatrix}, \qquad V_u = \begin{pmatrix} 1, & -\eta_1/\chi & (\alpha\xi_1\eta_1/\lambda_1)^* \\ \eta_1/\chi & 1 & -(\xi_1\chi/\lambda_1)^* \\ (1-\alpha)\xi_1\eta_1/\lambda_1 & \xi_1\chi/\lambda_1 & 1 \end{pmatrix} \quad (7)$$

with $\lambda = \chi\rho(1-x)$, $\xi_1 = b\xi$, $\eta_1 = c\eta$, $\lambda_1 = \chi\rho(1-kc^2x)$, and with similar form of the matrix V_e (defined in terms of $\xi_1' = b'\xi$, $\eta_1' = c'\eta$ and $\lambda' = \chi\rho(1-k'c'^2x)$), while the explicit form of matries U_f is inessential for the following. The elements of $(\hat{m}_f)^{diag}$ practically coincide (up to the small, inessential terms) with the diagonal terms of matrices given in Eq.(4) except for the first small element in $(m_u)^{diag}$ and $(m_e)^{diag}$, which are equal to $m_u = m_s/\chi\sigma|\sigma\rho + c^2\eta^2\sigma/\chi| = \frac{m_s|\rho|}{\chi}|1-c^2x|$ and $m_e = m_\mu|\rho/\chi|\cdot|1-c'^2x|$, correspondingly.

Therefore, the running massas of quarks and electrons are

$$\begin{cases} m_d = m_s|\rho/\chi|, & m_u = m_c|\rho/\chi||1-c^2x|, \quad m_e = m_\mu|\rho/\chi||1-c'^2x|, \\ m_b = m_s|\rho/\sigma||1-x|, & m_t = m_c|\rho/\sigma||1-\kappa c^2x|, \quad m_\tau = m_\mu|\rho/\sigma||1-k'c'^2x| \end{cases} \quad (8)$$

where $|\rho/\chi| = m_d/m_s \simeq 1/20$ (this ratio is determined according to PCAC) and

$$|1-c^2x| = (m_u/m_c)|\chi/\rho| = 1/15.6, \quad |1-c'^2x| \simeq 20\, m_e/m_\mu \simeq 1/10.3$$

as $m_c/m_u \simeq 313$, $m_\mu/m_e \simeq 207$. At $\varepsilon_e \simeq 0$ when $b'=b_e$, $C'=C_e$, $k'=1$ Eqs.(8) for m_e and m_τ give $|\rho/\sigma| \simeq (m_\tau/m_e)(m_d/m_s) \simeq 174$ (as $m_\tau/m_e \simeq 3490$) and from equations for m_b and m_t one finds: $|1-x| = (m_b/m_s)/|\rho/\sigma| = (m_b/m_d)(m_e/m_\tau) \simeq 1/4.3$ and

$$|\tau| = \left|\frac{1-\kappa c^2 x}{1-x}\right| = m_t/[m_c(m_b/m_s)] \simeq m_t^0/M_0 \quad (9)$$

where $m_t^0 = [m_t(q^2)]_{q^2=m_t^2}$ is the physical t-quark mass (one can show [8] that $m_t/m_c \simeq m_t^0/1.17$ GeV) and $M_0 = (m_b/m_s)1.17$ GeV $\simeq 48$ GeV is that value of m_t^0 which wou'l correspond to $\varepsilon_u = 1$ in Eqs.(6),(9).

Note, that parameters c,b are almost real, this is seen from Eqs.(6) written in the form: $c = c_u - (c_u+1)\varepsilon_u$, $b = b_u - (b_u+1)\varepsilon_u$, and similar for α and k:

$$\alpha - 1 = (1+1/b)(1+1/c)\varepsilon_u', \quad k-1 = (1+1/c)^2\varepsilon_u', \quad (10)$$

where $\varepsilon_u' = \varepsilon_u/(1-\varepsilon_u) \simeq \varepsilon_u$ is really a small complex parameter (estimation [3] of it gives $|\varepsilon_u| = |\tau|/20 \sim 1/10$) and $c_u \sim b_u \sim 1$. From the small value of $|1-c^2x| \simeq 1/16$ found above it follows that $|c^2x| \simeq 1 \pm 1/16$, i.e. that $Re(x) \sim 1$ and $Im(x) \lesssim 1/16$. Similarly, the equality $|1-x| \simeq 1/4.3$ yields two solutions: $|x| \simeq 1-1/4.3 \simeq 0.77$, $c = 1.11$, or $|x| \simeq 1+1/4.3 = 1.23$, $c = 0.90$. Since $|\chi/\rho| \simeq 20$ and $|x| = |\eta^2/\chi\rho| \simeq 1$, then $\chi/|\eta| = \sqrt{|\chi/\rho x|} \simeq \sqrt{20} \simeq 4.6$ and, in the normalization in which $|\rho| \simeq 1$, we obtain $\chi \simeq 20$ and $\eta \simeq 4.6$, while $|\sigma| \simeq |\rho|/174 = 1/174$ and, as is shown below, $|\xi| \sim |\sigma| \sim 10^{-2}$.

4. Quark mixing angles and CP-violation phase.

As is known, quark mixing Kobayashi-Maskawa matrix (KM) is: $V_{KM} = V_u^{*} \cdot V_d$, i.e. from Eq. (7):

$$V_{KM} = \begin{array}{c c} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{pmatrix} 1 & s_\theta & s_\theta s_\gamma R^{*} \\ -s_\theta & 1 & s_\gamma \\ s_\theta s_\gamma (1-R) & -s_\gamma & 1 \end{pmatrix} \end{array} = \begin{pmatrix} 1 & (\eta_1/\chi)^{*} & \left[\xi\eta/\lambda + (1-\alpha)\xi_1\eta_1/\lambda_1\right]^{*} \\ -\eta_1/\chi & 1 & (\xi_1\chi/\lambda_1)^{*} \\ \alpha\xi_1\eta_1/\lambda_1 - \eta/\lambda & -\xi_1\chi/\lambda_1 & 1 \end{pmatrix}, \quad (11)$$

where s_θ, s_γ - are quark mixing angles in the Maiani [9] representation in which $R = |R| \exp(i\delta)$ is a complex number, $R = V_{ub}/V_{cb}V_{us}$, and δ is the CP - violating, Maiani representation phase. The right-hand part of Eq. (10) contains the product of unitary matrices (7) and only leading terms in s_θ, s_γ were retained in all elements of matrices. Comparing both parts in Eq. (11) one obtains putting $\eta_1 = c\eta$, $\xi_1 = b\xi$, $\lambda_1 = \chi\rho(1-kc^2x)$, $\lambda = \chi\rho(1-x)$:

$$\begin{cases} V_u = s_\theta = |c\eta/\chi| = \sqrt{c^3x/(\sigma/\rho)} \simeq \sqrt{1/20} = 0.22, \\ V_{cb} = -V_{ts} = s_\gamma = |b\xi\chi/\lambda_1| = |b\xi/\rho| / |1-kc^2x|, \end{cases}$$

and $V_{ub}^{*} = s_\theta s_\gamma R$, $V_{td} = s_\theta s_\gamma (1-R)$, where

$$R = |R| e^{i\delta} = 1 - \alpha + \lambda_1/bc\lambda = 1 - \alpha + \tau/bc, \quad \tau = \frac{1-kc^2x}{1-x}. \quad (12)$$

According to experiental data [10] $V_{cb} \simeq 1/22$; this gives $|b\xi/\rho| = |V_{cb}| \frac{m_t^o}{M_o} |1-x| = (m_t^o/M_o)(22 \cdot 4.3)^{-1} \simeq m_t^o/95M_o$, or $|b\xi/\rho| \simeq 1/40$ if $|\tau| = m_t^o/M_o \simeq 2$-$2.5$. Using the most natural value of $b \simeq 3-4$ one then

obtains $|\xi/\rho| \simeq 10^{-2}$, what is close to $|\sigma/\rho| \simeq 1/174$. Value of the parameter b larger than unity is need to form on Eq. (11) small $|R| = |V_{ub}/V_{cs} V_{us}|$, satisfying restriction $|R| < 0.6$ of ARGUS [10], or $|R| < 0.8$ of CLEO [11].

Using Eq. (10), for (1-k), one has

$$\begin{cases} \tau = \frac{1-c^2x}{1-x} + \frac{c^2(1-k)x}{1-x} \simeq \mathcal{N} \cdot (c+1)(-\varepsilon_u'), \\ \mathcal{N} = \frac{x(c+1)}{1-x} \simeq \mathcal{N}_0 \exp -i(\pi + \end{cases} \quad (13)$$

where term $\frac{1-c^2x}{1-x}$ was disregarded (as $\left|\frac{1-c^2x}{1-x}\right| \simeq 4{,}3/15{,}6 \simeq 1/3{,}6$ and $|\tau| \simeq m_t/48\,GeV \sim 2-3$), $\mathcal{N}_0 = \left|\frac{x(c+1)}{1-x}\right| \simeq 4{,}3|x(c+1)| \simeq 10$ and the phase ϑ of $1-x = |1-x| \exp[i(\pi+\vartheta)]$ could be not small even at small phase of $x = x\exp(i\lambda_0)$, $\lambda_0 \ll 1$, determined by VEV on Higgs sector. Eq. (13) shows that $|\varepsilon_u'| = (|\tau|/(c+1))/\mathcal{N}_0 \sim 1/\mathcal{N}_0 \sim 1/10$ is really small and gives after substitution in Eq. (12):

$$R = \left[\frac{b+1}{\mathcal{N}_0} - \exp(-i\vartheta)\right] \left|\frac{\tau}{bc}\right| \exp(-i\vartheta), \quad (14)$$

where Eq. (10) for $(1-\alpha)$ was used and $\mathcal{N}_0(c+1)(-\varepsilon_u')$ was written in the form $|\tau| \exp(-i\vartheta)$. The last follows from determination (5) of ε_u, where only the first term is essential in A_u at $|\varepsilon_u| \simeq |\varepsilon_u'| \ll 1$ i.e. from: $\varepsilon_u' \simeq \varepsilon_u = \varepsilon_0 \frac{1-c_u^2x}{1-x}$, with $\varepsilon_0 = f_1' \tau h_u / f_1 \tau_1 h_d$ a real constant. Here $1-c_u^2x$ is a complex number with very small phase (or order of $\lambda_0 \ll 1$) and, thus, $-\vartheta$ is the phase of $\mathcal{N}_0(c+1)(-\varepsilon_u')$. The value of $c \simeq 1/\sqrt{|x|} \simeq 1$ is determined by ϑ, as $|1-x| = 1/4{,}3$.
At given $b \sim 3$-4, $R = |R| e^{i\delta}$ depends on the phase ϑ and on t-quark mass $|\tau| = m_t/48$ GeV. Both of them, ϑ and $|\tau|$, could be easily determined [3] from Eq. (14) and the experimental data on $\varepsilon(K\bar{K})$ and on $x_{B_d}(B\bar{B})$. Expressing them in terms of $|\tau|$ and δ, we have [3]

$$\begin{cases} \varepsilon = (s_\theta^2 s_\gamma^2/\sqrt{2}) |R| \sin\delta \cdot \Phi(|\tau|, |R|\cos\delta) = (2.3 \pm 0.2) \cdot 10^{-3} \\ x_{B_d} = (G_F^2/6\pi^2)(M_B f_B^2 m_t^2/\Gamma_B)\, 0.85\, I(z)\, |V_{td}|^2 = \\ \quad = (|\tau|^2/12.3)\, |1-R|^2 I(z) = 0.73 \pm 0.25 \end{cases} \quad (15)$$

with [12] $\Phi = 1.2 \ln|\tau| + 2.45 + 3.0\,|\tau|^2\, I\left(m_t^2/M_W^2\right)\left(1 - |R|\cos\delta\right)$
and with standard form [13] of the function

$$I(z) = 1 - (3/4)\,z\,(1+z)/(1-z)^2 - (3z^2/2)\ln z/(1-z)^3, \quad z = m_t^2/M_W^2,$$

$I(0) = 1$, $I(1) = 0.75$, $I(2) = 0.71$, $I(3) = 0.69$.
Fig. 1 shows values of $0.48\,|\tau| = m_t/100$ GeV, of $|R| = |\tau/\ell|\,\sin\vartheta/\sin\delta$, of $|1-R|$, of δ and of $\sin\vartheta$ obtained from Eqs. (14), (15) as a function of the single parameter $\ell = N_0\left[\sin(\delta+\vartheta)/\sin\delta\right] - 1$. The details are explained in Ref. [3].
For most natural values of $\ell \simeq 3$-4 (which are larger than unity, but smaller than 10) Fig. 1 leads to

$$m_t = (125 \pm 15)\ \text{GeV}, \quad \delta = 180^\circ - (15 \pm 5)^\circ, \quad |R| = 0.50 \pm 0.05$$

and $|1-R| = |V_{td}/V_{us}V_{cb}| \simeq 1.45 \pm 0.05$. The corresponding values of the ratio

$$\varepsilon'/\varepsilon = \left[13.7 \cdot \Phi\left(|\tau|,\ |R|\cos\delta\right)\right]^{-1} \qquad (16)$$

is shown in Fig. 2. For $\ell \simeq 3$-4 they are close to the old CERN result $a_0 = (\varepsilon'/\varepsilon)^0_{exp} = (3.3 \pm 1.1)10^{-3}$ and do not reproduce the negative values of $\varepsilon'/\varepsilon = (-0.5 \pm 5)\,10^{-3}$ obtained recently.

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Figure Captions

Fig. 1 Dependence on the parameter ℓ of m_t, of CP-phase δ, of $|R| = |V_{ub}/V_{cb}\,V_{us}|$ and of $|1-R| = |V_{td}/V_{cb}\,V_{us}|$, which follows [3] from Eqs. (14) and (15).

Fig. 2 Dependence of $(\varepsilon'/\varepsilon)/a_0$, where $a_0 = 3.3\,10^{-3}$ is the old CERN value of ε'/ε, on the parameter ℓ according to Eq. (16).

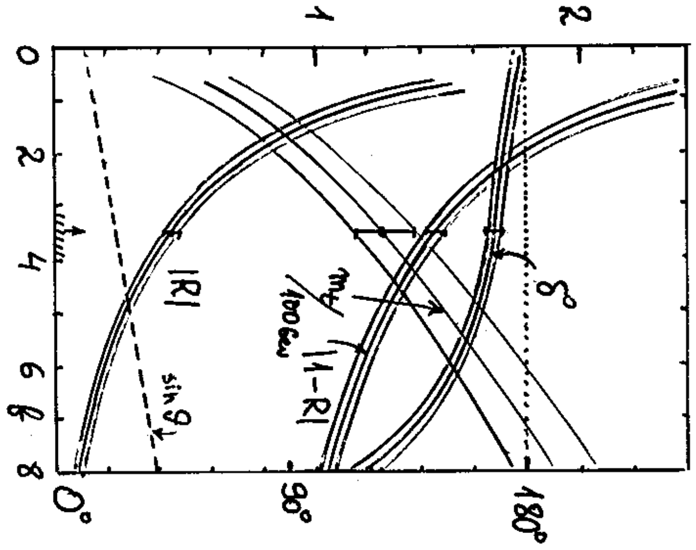

Fig. 1

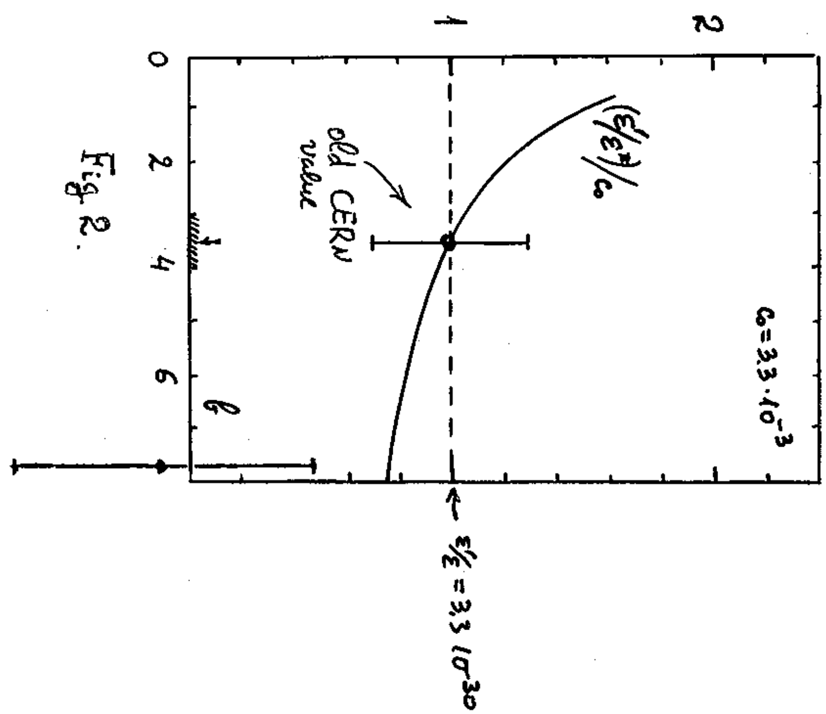

Fig 2.

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