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ABSTRACT

We consider fermion-number violating Green's functions involving also gauge and Higgs bosons in the axial (1+1)-dimensional Abelian Higgs model. We show that the Fourier transform of the classical vortex solution has (in the unitary gauge) an *isolated pole* at the position of the Higgs and gauge boson mass. In a certain sense the vortex, an Euclidean *pseudoparticle*, is capable to describe *particles*, i.e. physical gauge and Higgs bosons. From that it follows that, in the leading-order semiclassical approximation around the unit winding number vortex solution, the $S-$matrix elements for the fermion-number violating processes are *local* and of order $v^n e^{-v^2}$, where v is the vacuum expectation value and n is the number of external boson legs. This suggests that, at high energy, fermion-number violating processes with associated production of many gauge and Higgs bosons have large probabilities reaching the unitarity limit, and puts the recent observation of a similar phenomenon in the standard electroweak theory on much firmer ground.

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Already in 1976 it has been shown by 't Hooft [1] that baryon (B) and lepton (L) number are not strictly conserved in the standard electroweak theory. In contrast to the B and L violating processes in grand unified theories the processes in the standard model are of nonperturbative origin which is reflected in the fact that the amplitudes of these processes are proportional to $\exp(-2\pi/\alpha_w)$, where $\alpha_w = g^2/(4\pi)$ is the SU(2) "fine structure" constant. Since this factor is non-analytic at $\alpha_w = 0$ it cannot be obtained by a summation of a finite number of Feynman diagrams.

Not much work has been done in the years following 1976 in connection with B and L violating processes in the standard model. This is basically due to the fact that the numerical value of $\exp(-2\pi/\alpha_w)$ is very small, $\sim 10^{-86}$. Interest in these processes has started essentially only after 1985 when Kuzmin, Rubakov, and Shaposhnikov [2] argued that B and L violating processes occur frequently at high temperatures. In application to the early universe they found that the inclusive rate of B and L violating processes exceeds the Hubble expansion rate of the universe for large temperatures. Semi-quantitative calculations supported these observations and lead to the conclusion that the B and L violating processes were in equilibrium in the cosmic plasma for temperatures larger than about 200 GeV [3 – 5]. This, of course, has tremendous consequences for the evolution of the baryon asymmetry of the universe. So, from a cosmological point of view, B and L violating processes in the standard model cannot be neglected entirely.

Given the fact that at *high temperatures* the inclusive rate of baryon and lepton number violating processes seems to be large, it is natural to ask if a similar behavior is found at *high energies*. First attempts in this direction have been made by Aoyama and Goldberg [6] and by one of the authors [7] and the results are suggestive: it is not entirely excluded that the B and L violating processes in the standard model can be observed in future high-energy collider experiments [6 – 9]. It was found that the relevant B and L violating processes are those with associated production of many, $\mathcal{O}(1/\alpha_w)$, W, Z, and Higgs bosons, which has been conjectured earlier in refs. [10, 11]. The corresponding $S-$matrix elements were found to be *local* in leading-order in the semiclassical expansion, and of order $\sim n! \exp(-2\pi/\alpha_w)$, where n is the number of external bosons [7] (see also ref. [12]). This has the consequence of a strong powerlike increase of the exclusive cross sections with energy leading to a violation of the unitarity bound already at some 10's of TeV [7]. B and L violation would therefore be observable at these energies.

One obvious point of further investigations would be the question of unitarity corrections by multiple scattering. This problem will not be investigated here. Instead we will study another question of consistency which in some way has been left open in ref. [7]. The calculations were based on the expansion of the Euclidean path integral around topologically nontrivial classical gauge and Higgs field configurations which are not solutions of the classical equations of motion, but rather minima of a constrained action (*constrained instanton* [13]). The necessity to introduce a constraint into the path integral stems from the fact that no nontrivial solutions with finite action exist in the electroweak theory. Unfortunately the constrained instanton is not known in detail. The leading asymptotic behavior of the constrained instanton indicates a singularity at the position of the Higgs or W(Z) boson mass shell. The conclusions of ref. [7] are crucially based on the assumption of this singularity being an *isolated pole*. So far this has not been conclusively demonstrated, due to the lack of an explicit construction of the constrained instanton.

In this paper we want to discuss fermion-number violating processes with associated production of many gauge and Higgs bosons in the simpler context of an axial (1+1)-dimensional Abelian Higgs model. In particular we want to study the question of the singularity structure of the corresponding Green's functions obtained in the semiclassical approximation of the Euclidean path integral around topologically nontrivial configurations. In contrast to the electroweak theory this model has instanton *solutions* of the classical Euclidean equations of motion, the Nielsen-Olesen vortices [14]. Moreover, if a certain relation between the gauge boson mass and the Higgs mass is fulfilled, one knows even an exact vortex solution [15]. This enables us to investigate leading-order $S-$matrix elements for fermion-number violating processes in much more detail than in the (3+1)-dimensional standard electroweak theory.

Consider the axial Abelian Higgs model in two-dimensional Euclidean space, described by the Lagrangian

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\phi|^2 + \lambda\left(|\phi|^2 - \frac{1}{2}v^2\right)^2 + \mathcal{L}_f, \qquad (1)$$

$$\mathcal{L}_f = i\sum_{i=1}^{n_f}\overline{\psi}_L^{(i)}\hat{D}_L\psi_L^{(i)} + i\sum_{j=1}^{n_f}\overline{\psi}_R^{(j)}\hat{D}_R\psi_R^{(j)}. \qquad (2)$$

A_μ is an Abelian gauge field, ϕ is a complex scalar field, and $\psi_{L(R)}^{(i)}$ are $2n_f$ independent fermion fields of definite handedness, $\gamma_5\psi_{L(R)}^{(i)} = \pm\psi_{L(R)}^{(i)}$. We have introduced as many right-handed as left-handed fermions in order to have no anomaly in the gauge current. The covariant derivative of the scalar field is defined by $D_\mu = \partial_\mu - igA_\mu$, whereas the covariant derivatives of the fermions are given by $\hat{D}_{L(R)} = \gamma_\mu(\partial_\mu \mp igA_\mu)$. γ_μ, $\mu = 1, 2$, are Euclidean Hermitian Dirac matrices, defined by $\gamma_1 = \sigma_1$, $\gamma_2 = \sigma_2$, and $\gamma_5 = -i\gamma_1\gamma_2 = \sigma_3$. We will assume that $v \gg 1$, such that perturbatively a weak coupling analysis is viable.

The model (1) has much in common with the standard electroweak theory: it also has a nontrivial vacuum structure and an anomaly in the gauge invariant fermionic current

$$J_\mu^F = \sum_{i=1}^{n_f}\overline{\psi}_L^{(i)}\gamma_\mu\psi_L^{(i)} + \sum_{j=1}^{n_f}\overline{\psi}_R^{(j)}\gamma_\mu\psi_R^{(j)}, \qquad (3)$$

namely

$$\partial_\mu J_F^\mu = 2n_f\left(\frac{g}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu}\right). \qquad (4)$$

It does have instanton configurations, which moreover are *classical solutions*, with fermion zero modes of definite chirality. These configurations are the so-called Nielsen-Olesen vortices [14]. The particle spectrum of the model includes vector and Higgs bosons with masses $m_w = gv$ and $m_h = \sqrt{2\lambda}v$, respectively, and massless fermions. We note in passing that this model has also been investigated with regards to the fermion-number non-conservation at high temperatures [16].

We are interested in the following Green's function,

$$G(x_1, ..., x_{2n_f}; y_1, ..., y_{n_w}; z_1, ..., z_{n_h}) =$$

$$= \int [dA_\mu][d\phi][d\psi][d\overline{\psi}]e^{-S[A,\phi,\overline{\psi},\psi]}\prod_{i=1}^{n_f}\psi_L^{(i)}(x_i)\prod_{j=1}^{n_f}\psi_R^{(j)}(x_{n_f+j})\prod_{k=1}^{n_w}A_{\mu_k}(y_k)\prod_{l=1}^{n_h}\eta(z_l), \qquad (5)$$

which involves each left-(right-)handed fermion flavor exactly once and in addition n_w gauge bosons and n_h physical Higgs bosons. A nonzero value of this Green's function implies, according to the LSZ reduction formula [17], the existence of the process, for example,

$$\overline{f}_L^{(1)} + \overline{f}_L^{(2)} \to \sum_{i=3}^{n_f} f_L^{(i)} + \sum_{j=1}^{n_f} f_R^{(j)} + n_w\, W + n_h\, H, \qquad (6)$$

where $f^{(i)}_{L(R)}$ are the fermions with definite chirality, W is the gauge boson, and H is the Higgs boson, corresponding to the fields in eq. (5). This process changes the fermion number of each flavor by one unit. From the anomaly (4), which formally integrated gives

$$\triangle F = 2n_f \left(\frac{g}{4\pi} \int d^2x \ \epsilon_{\mu\nu} F_{\mu\nu} \right) \equiv 2n_f \ q, \tag{7}$$

we expect that a nonzero value is induced by gauge fields with Euclidean topological number $q = 1$. This is related to the existence of zero modes of the Dirac operator in $q = 1$ background fields.

We first write (5) in the form

$$G(x_1, ..., x_{2n_f}; y_1, ..., y_{n_w}; z_1, ..., z_{n_h}) =$$

$$\int [dA_\mu][d\phi] e^{-S[A,\phi]} \ \langle \prod_{i=1}^{n_f} \psi_L^{(i)}(x_i) \prod_{j=1}^{n_f} \psi_R^{(j)}(x_{n_f+j}) \ \rangle_{A_\mu} \ \prod_{k=1}^{n_w} A_{\mu_k}(y_k) \prod_{l=1}^{n_h} \eta(z_l), \tag{8}$$

where the fermionic Green's function in the background of A_μ is given by

$$\langle \prod_{i=1}^{n_f} \psi_L^{(i)}(x_i) \prod_{j=1}^{n_f} \psi_R^{(j)}(x_{n_f+j}) \rangle_{A_\mu} = \int [d\psi][d\overline{\psi}] \prod_{i=1}^{n_f} \psi_L^{(i)}(x_i) \prod_{j=1}^{n_f} \psi_R^{(j)}(x_{n_f+j}) \exp\left[i \int d^2x \ \mathcal{L}_f \right]. \tag{9}$$

The Green's function (9) vanishes in the topological trivial sector, $q = 0$. A nonzero value arises due to Atyah-Singer fermion zero modes [18] in the topologically nontrivial gauge field configurations with winding number $q = 1$. To make the calculations rigorous one has to compactify the system, for example to make a stereographic projection onto S^2. We skip the details and only quote the results [19 – 21]. The gauge field can be written in general as

$$A_\mu = \epsilon_{\mu\nu} \partial_\nu \alpha + \partial_\mu \beta. \tag{10}$$

In the Landau gauge, $\partial_\mu A_\mu = 0$, which we will use in the following, we have $A_\mu = \epsilon_{\mu\nu} \partial_\nu \alpha$. For a $q = 1$ configuration α diverges logarithmically for $r \to \infty$, $\alpha \to -\frac{1}{g} \log \mu r$, where r is the euclidean radius and μ is an arbitrary infrared mass scale introduced to define the logarithm. It is convenient to think of it as the inverse vortex core size (see below). In the $q = 1$ background field, A^+_μ, there exist zero modes of definite chirality, $\hat{D}_{L(R)} \chi_{L(R)} = 0$,

$$\chi_{L(R)}(x) = \sqrt{\frac{\mu}{2\pi}} \ \exp\left[g\alpha(x) \right] \xi_{L(R)}, \tag{11}$$

where $\xi_L = \binom{1}{0}$ and $\xi_R = \binom{0}{1}$. The zero modes behave as $1/r$ as $r \to \infty$, and only appear in the Dirac operator acting on $\psi_{L(R)}$ as opposed to acting on $\overline{\psi}_{L(R)}$. This is a general feature of axial interactions [22, 23].

The standard procedure separating out the zero modes and performing the functional integral over the fermion fields yields [19 – 23]

$$\langle \prod_{i=1}^{n_f} \psi_L^{(i)}(x_i) \prod_{j=1}^{n_f} \psi_R^{(j)}(x_{n_f+j}) \rangle_{A_\mu} = [\mathrm{Det}' \hat{D}(A^+)]^{2n_f} \ \prod_{i=1}^{n_f} \chi_L(x_i; [A^+]) \prod_{j=1}^{n_f} \chi_R(x_{n_f+j}; [A^+]), \tag{12}$$

where

$$\log[\mathrm{Det}' \hat{D}(A^+)] = \frac{g^2}{4\pi} \int d^2x \ \alpha \partial^2 \alpha. \tag{13}$$

Inserting (12) into (8) gives our desired Green's function as

$$G(x_1, ..., x_{2n_f}; y_1, ..., y_{n_w}; z_1, ..., z_{n_h}) = \int_{q=1} [dA_\mu][d\phi] e^{-S[A,\phi]} [\mathrm{Det}' \hat{D}(A)]^{2n_f} \times$$

$$\prod_{i=1}^{n_f} \chi_L(x_i; [A]) \prod_{m=1}^{n_f} \chi_R(x_{n_f+m}; [A]) \prod_{j=1}^{n_w} A_{\mu_j}(y_j) \prod_{l=1}^{n_h} \eta(z_l), \tag{14}$$

where now the path integral is only over gauge fields with $q = 1$.

In contrast to the massless Schwinger model the remaining functional integral over the gauge and Higgs fields can only been done semiclassically. In general one should look for the saddle point of the whole integrand in (14) [8]. We will not do this in the present paper, which deals mainly with the singularity structure of the Green's function, which can only be investigated if the saddle point configurations are known in sufficient detail. We therefore look for a saddle point of the Euclidean bosonic action, i.e. a classical Euclidean solution, with $q = 1$. The existence of such a solution has been demonstrated by Nielsen and Olesen [14]. In the regular gauge it is of the form

$$\phi^{+\mathrm{cl}}(x) = e^{i\varphi(x)} \frac{v}{\sqrt{2}} f(r), \tag{15}$$

$$A_\mu^{+\mathrm{cl}}(x) = \frac{1}{g} \frac{\epsilon_{\mu\nu} x_\nu}{r^2} A^+(r). \tag{16}$$

For the anti-instanton, with $q = -1$, we have $A^- = -A^+$, $\phi^- = (\phi^+)^*$. Here r and φ are polar coordinates. The appropriate boundary conditions for unit-winding-number finite-action vortex solutions are $f(r) \propto r$, $A^+(r) \propto r^2$, for $r \to 0$, and $\lim_{r\to\infty} f(r) = 1$, $\lim_{r\to\infty} A^+(r) = -1$. The corresponding expressions in the (singular) unitary gauge, which are obtained after a gauge transformation $\phi \to e^{-i\varphi}\phi$, $A_\mu \to A_\mu - \frac{1}{g}\partial_\mu\varphi$, $\psi_{L(R)} \to e^{\mp i\varphi}\psi_{L(R)}$, are

$$A_\mu^{+\mathrm{cl}\ \prime}(x) = \frac{1}{g}\frac{\epsilon_{\mu\nu}x_\nu}{r^2}\Big[A^+(r)+1\Big], \tag{17}$$

$$\phi^{+\mathrm{cl}\ \prime}(x) = \frac{v}{\sqrt{2}}f(r), \tag{18}$$

$$\chi_{L(R)}^{\mathrm{cl}\ \prime}(x) = \sqrt{\frac{m_w}{2\pi}}\ \exp\big[g\alpha^+(r)\big]\frac{\gamma_\mu x_\mu}{r}\xi_{R(L)}. \tag{19}$$

$\chi_{L(R)}^{\mathrm{cl}\ \prime}$ are the fermion zero modes in the vortex background (in the unitary gauge). Here α^+ is related to A^+ by $\partial_r\alpha^+ = A^+/(gr)$. We have taken the integration constant to be such that $\alpha^+(r) \to -\frac{1}{g}\log m_w r$ for $r \to \infty$. From (18) we obtain for the shifted classical Higgs field, corresponding to physical excitations,

$$\eta^{+\mathrm{cl}\ \prime}(x) = \frac{v}{\sqrt{2}}[f(r)-1]. \tag{20}$$

The classical action of the vortex can be written in the form

$$S_{\mathrm{cl}} = v^2 Q\Big(\frac{\lambda}{g^2}\Big), \tag{21}$$

where Q is slowly varying. Moreover, de Vega and Schaposnik [15] constructed an exact solution for the special case $m_w = m_h$. In this particular case, i.e. $g^2 = 2\lambda$, we have $Q(\frac{1}{2}) = \pi$.

In leading-order of the semiclassical expansion around the unit-winding-number vortex we obtain from (14)

$$G(x_1,...,x_{2n_f};y_1,...,y_{n_w};z_1,...,z_{n_h}) = \int d^2R\ \gamma(m_w,m_h,g)e^{-S_{\mathrm{cl}}}[\mathrm{Det}'\hat{D}(A^{+\ \mathrm{cl}})]^{2n_f}\times$$
$$\times\prod_{i=1}^{n_f}\chi_L^{\mathrm{cl}\ \prime}(x_i-R)\prod_{m=1}^{n_f}\chi_R^{\mathrm{cl}\ \prime}(x_{n_f+m}-R)\prod_{j=1}^{n_w}A_{\mu_j}^{+\mathrm{cl}\ \prime}(y_j-R)\prod_{l=1}^{n_h}\eta^{+\mathrm{cl}\ \prime}(z_l-R). \tag{22}$$

Here $\gamma(m_w,m_h,g)$ results from doing the gauge boson and scalar field fluctuations around the vortex. It has dimension of a mass squared, $\gamma \sim m_w^2$ [24 – 27]. R_μ denotes the position of the vortex. Since we are interested in an $S-$matrix element for the process (6) we have

used for the explicit classical configurations in (22) the expressions in the unitary gauge. Note that in expanding around a single $q=1$ instanton rather than around superpositions of n_+ instantons and n_- anti-instantons with $n_+ - n_- = q = 1$ we have implicitly assumed that the instanton gas is dilute. This is true if $v \gg 1$, because the average distance between instantons is given by $\gamma^{-1/2}e^{S_{\mathrm{cl}}} \sim m_w^{-1}e^{v^2}$, which is large against the natural scale m_w^{-1}. In this case (22) really represents the leading-order result in the dilute gas approximation.

Fourier transforming the Green's function neatly undoes the integral over the instanton location,

$$\tilde{G}(p_1,...,p_{n_f};q_1,...,q_{n_w};k_1,...,k_{n_h}) = (2\pi)^2\delta^{(2)}\Big(\sum_{i=1}^{2n_f}p_i + \sum_{j=1}^{n_w}q_j + \sum_{l=1}^{n_h}k_l\Big)\times$$
$$\times\gamma e^{-S_{\mathrm{cl}}}\ [\mathrm{Det}'\hat{D}(A^{+\ \mathrm{cl}})]^{2n_f}\prod_{i=1}^{n_f}\tilde{\chi}_L^{\mathrm{cl}\ \prime}(p_i)\prod_{m=1}^{n_f}\tilde{\chi}_R^{\mathrm{cl}\ \prime}(p_{n_f+m})\prod_{j=1}^{n_w}\tilde{A}_{\mu_j}^{+\mathrm{cl}\ \prime}(q_j)\prod_{l=1}^{n_h}\tilde{\eta}^{+\mathrm{cl}\ \prime}(k_l), \tag{23}$$

where the Fourier transforms of the classical fields (17), (19), and (20) are given explicitly by

$$\tilde{A}_\mu^{+\mathrm{cl}\ \prime}(q) = \frac{2\pi i}{g}\frac{\epsilon_{\mu\nu}q_\nu}{|\,q\,|}\int_0^\infty dr\ J_1(|\,q\,|\,r)\big[A^+(r)+1\big], \tag{24}$$

$$\tilde{\chi}_{L(R)}^{\mathrm{cl}\ \prime}\ (p) = 2\pi i\sqrt{\frac{m_w}{2\pi}}\ \frac{\gamma_\mu p_\mu}{|\,p\,|}\int_0^\infty dr\ r\ J_1(|\,p\,|\,r)\exp\big[g\alpha^+(r)\big]\xi_{R(L)}. \tag{25}$$

$$\tilde{\eta}^{+\mathrm{cl}\ \prime}(k) = 2\pi\frac{v}{\sqrt{2}}\int_0^\infty dr\ r\ J_0(|\,k\,|\,r)\big[f(r)-1\big]. \tag{26}$$

We are now ready to discuss the singularity structure of the Green's function (23), when analytically continued to Minkowski space-time. In order that our expression for the leading-order contribution of the Green's function, eq. (23), really makes a contribution to the $S-$matrix element for the process (6) it should have *isolated poles* at the positions of the gauge and Higgs boson masses, $q_j^2 = -m_w^2$ and $k_l^2 = -m_h^2$, respectively. For the massless fermions the situation is more complicated because branch cuts may also start at $p_i^2 = 0$.

We have to study eqs. (24) - (26) near the mass shells, i.e. for $q^2 + m_w^2 \to 0$, $p^2 \to 0$, and $k^2 + m_h^2 \to 0$, respectively. For this end we make a variable substitution and write

$$\tilde{A}_\mu^{+\mathrm{cl}\ \prime}(q) = \frac{2\pi i}{g}\frac{\epsilon_{\mu\nu}q_\nu}{|\,q\,|\sqrt{q^2+m_w^2}}\int_0^\infty dx\ J_1\Big(\frac{|\,q\,|\,x}{\sqrt{q^2+m_w^2}}\Big)\Big[A^+\Big(\frac{x}{\sqrt{q^2+m_w^2}}\Big)+1\Big], \tag{27}$$

$$\tilde{\chi}^{\mathrm{cl}\,\prime}_{L(R)}\,(p) = 2\pi i\sqrt{\frac{m_w}{2\pi}}\,\frac{\gamma_\mu p_\mu}{|\,p\,|^3}\int_0^\infty dx\; x\; J_1(x)\exp\Big[g\alpha^+\Big(\frac{x}{|\,p\,|}\Big)\Big]\xi_{R(L)}, \tag{28}$$

$$\tilde{\eta}^{+\mathrm{cl}\,\prime}(k) = 2\pi\frac{v}{\sqrt{2}}\frac{1}{k^2+m_h^2}\int_0^\infty dx\; x\; J_0\Big(\frac{|\,k\,|\,x}{\sqrt{k^2+m_h^2}}\Big)\Big[f\Big(\frac{x}{\sqrt{k^2+m_h^2}}\Big)-1\Big]. \tag{29}$$

Near the mass shell we can replace A^+, α^+, and f in the integrals by their asymptotic forms for large r. What one normally can show without explicitly knowing the solution is only exponential decay, $A^+ + 1 \sim e^{-m_w r}$, $f-1 \sim e^{-m_h r}$. This alone is not sufficient for showing the correct singularity structure. For this end one needs an exact solution which also gives the next-to-leading terms in the asymptotic expansion. Here we make explicit use of the exact solution for $m \equiv m_w = m_h$, found by de Vega and Schaposnik [15]. In this particular case we have, for $mr \to \infty$,

$$A^+(r) + 1 = Z_1 mr K_1(mr) + \mathcal{O}(Z_1^2 e^{-2mr}), \tag{30}$$

$$f(r) - 1 = -Z_1 K_0(mr) + \mathcal{O}(Z_1^2 e^{-2mr}), \tag{31}$$

where $Z_1 = 1.7079\ldots$. Working only with the leading terms we get for the Fourier transforms near the mass shell

$$\tilde{A}^{+\mathrm{cl}\,\prime}_\mu(q) = \frac{2\pi i}{g}Z_1\frac{\epsilon_{\mu\nu}q_\nu}{q^2+m^2}\ ,\ \text{for } q^2+m^2 \to 0, \tag{32}$$

$$\tilde{\chi}^{\mathrm{cl}\,\prime}_{L(R)}\,(p) = \sqrt{\frac{2\pi}{m}}\,i\frac{\gamma_\mu p_\mu}{p^2}\xi_{R(L)}\ ,\ \text{for } p^2 \to 0, \tag{33}$$

$$\tilde{\eta}^{+\mathrm{cl}\,\prime}(k) = -\frac{v}{\sqrt{2}}\,2\pi Z_1\frac{1}{k^2+m^2}\ ,\ \text{for } k^2+m^2 \to 0. \tag{34}$$

Since the next-to-leading terms in eqs. (30) and (31) are decaying with e^{-2mr} they can produce branch cuts only for $q^2(k^2) \geq 4m^2$. This shows that the Fourier transforms of the pseudoparticle in the unitary gauge have *isolated poles* at the positions of the gauge and Higgs boson mass shells. Therefore (23) really makes a contribution to the $S-$matrix element for the process (6) with physical gauge and Higgs bosons in the final state. In this sense we can say that the *pseudoparticle* not only describes tunneling between topologically inequivalent vacua but also the *particles*, i.e. physical gauge and Higgs bosons, in a process like (6).

"Amputating" the Fourier transforms, i.e. multiplying them with the inverse free propagators, gives

$$\tilde{A}^{+\mathrm{cl}\,\prime}_{\mu\ \mathrm{amp}}(q) = \frac{2\pi i}{g}Z_1\epsilon_{\mu\nu}q_\nu\ ,\ \text{for } q^2+m^2 \to 0, \tag{35}$$

$$\tilde{\chi}^{\mathrm{cl}\,\prime}_{L(R)\ \mathrm{amp}}\,(p) = \sqrt{\frac{2\pi}{m}}\,\xi_{R(L)}\ ,\ \text{for } p^2 \to 0, \tag{36}$$

$$\tilde{\eta}^{+\mathrm{cl}\,\prime}_{\mathrm{amp}}(k) = -\frac{v}{\sqrt{2}}\,2\pi Z_1\ ,\ \text{for } k^2+m^2 \to 0. \tag{37}$$

From that we obtain for the on-shell amputated Green's function

$$\tilde{\Gamma}(p_1,\ldots,p_{2n_f};q_1,\ldots,q_{n_w};k_1,\ldots,k_{n_h})|_{\mathrm{on-shell}} =$$

$$(2\pi)^2\delta^{(2)}\,\Big(\sum_{i=1}^{2n_f}p_i + \sum_{j=1}^{n_w}q_j + \sum_{l=1}^{n_h}k_l\Big)\ \gamma e^{-S_{\mathrm{cl}}}[\mathrm{Det}'\hat{D}(A^{+\,\mathrm{cl}})]^{2n_f}\times \tag{38}$$

$$\Big(\frac{2\pi}{m}\Big)^{n_f}\ \xi_R\ldots\xi_R\ \xi_L\ldots\xi_L\ \Big(\frac{2\pi i Z_1}{g}\Big)^{n_w}\big[\epsilon_{\mu_1\nu}q_\nu\ldots\epsilon_{\mu_{n_w}\lambda}q_\lambda\big]_{|q_j^2=-m^2}\Big(\frac{-2\pi v Z_1}{\sqrt{2}}\Big)^{n_h}.$$

According to the LSZ reduction formula [17] this has to be multiplied with the appropriate wave functions in order to get the $S-$matrix element for the process (6). From eq. (38) it follows immediately that the leading-order $S-$matrix element is purely *local*, i.e. it does not depend on momentum transfers, and of order $v^n e^{-v^2}$, where $n = n_w + n_h$. The locality leads to the important observation that the probabilities of the anomalous fermion-number violating processes rise with energy like phase space. The prefactor before the WKB tunneling amplitude, which stems from the "normalization" of the pseudoparticle, overcomes the exponential suppression for large n. Both features lead to the fact that at high energies the anomalous processes become strong.

Strictly speaking the results become unreliable when $n = n_w + n_h$ becomes too large. This is because we have looked for a saddle point of the generating functional of the Green's functions without external sources rather than for the saddle point of the whole integral in eq. (14). This becomes invalid when the polynomial in the fields becomes of high order. Also the corrections due to the exchange of, say, virtual Higgs bosons between the external lines become large for large n, because they have as their expansion parameter n/v^2. These problems have been investigated by McLerran, Vainshtein, and Voloshin [8] by taking into account the distortion of the shape of the instanton by the external quanta. They claim that the effect persists and becomes even stronger at high numbers of external boson legs. The singularity structure using the distorted instanton deserves a further study.

Further investigations are also necessary with regards to (i) higher order corrections in the single instanton sector, i.e. higher orders in the semiclassical expansion around the instanton, and (ii) higher order corrections in the dilute instanton-anti-instanton gas expansion, $\sim \exp[-(n_+ + n_-)v^2]$, where $n_\pm$ are the numbers of widely-separated (anti-)instantons, such that $n_+ - n_- = q = 1$. Both types of corrections destroy the strict locality of the vertex. The former lead to a nonlocality by tying together the external legs with propagators in the background of the instanton, whereas the latter give rise to a nonlocality via multiple scattering off from different (anti-)instantons, which are connected by the fermion zero modes. These multiple scatterings are suspected to save unitarity at high energies, when the leading-order result exceeds the unitarity bound.

In conclusion, we have seen that, in the (1+1)-dimensional Abelian Higgs model, the anomalous fermion-number violating processes become strong at high energies. The relevant processes are those with associated production of many gauge and Higgs bosons. From the singularity structure of the Fourier transform of the pseudoparticle in the unitary gauge it follows that it explicitly contributes to the $S-$matrix element for associated production of many gauge and Higgs bosons. The reasons for the strong effect are the *locality* of the vertex and the "normalization" of the instanton, the gauge (Higgs) configuration being proportional to $g^{-1}(v)$. The constrained instanton [10] in the electroweak theory or distortions of it [8] deserve a much more detailed study. On the other hand, the explicit calculation in the (1+1)-dimensional Abelian Higgs model puts similar observations [7] in the (3+1)-dimensional standard electroweak theory on much firmer ground.

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