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Angular Correlations of 4 Jets in e^+e^--Annihilation: Asymmetries and the 3-Gluon Vertex

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Abstract

We calculate the total and partial cross sections of the four-parton processes $e^+e^- \to \gamma, Z^0 \to q\bar{q}gg$ and $e^+e^- \to \gamma, Z^0 \to q\bar{q}q\bar{q}$ in lowest order QCD perturbation theory. We study the partial cross sections with regard to the forward backward asymmetry. In addition we compare these cross sections to the cross sections of an abelian vector gluon model.

1 Introduction

Studies of jet production in electron-positron-annihilation at high energies revealed much information about the properties of quarks and gluons and the nature of their interactions as described by quantum chromodynamics (QCD).

In QCD the existence of hadron jets in electron-positron-annihilation results from the primordial production of quarks and gluons and their subsequent fragmentation into hadrons. The annihilation of an electron and a positron into quarks and gluons is calculated in QCD perturbation theory.

In lowest order, $(O(\alpha_S^0))$, only $q\bar{q}$ states can occur, in $O(\alpha_S^1)$ the final state is $q\bar{q}g$ and in $O(\alpha_S^2)$ we have the production of $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ states (see figure 1).

The cross sections for the production of these states have been calculated previously [10] including higher order QCD corrections $(O(\alpha_S^2))$ for no jets(total inclusive), 2, 3, and 4 jets.

In this paper we compute the 4 jet cross section in second order $(O(\alpha_S^2))$ with γZ^0-interference and analyse the angular correlations of the cross section.

There are two points of interest concerning this cross section. First, the γZ^0-interference accounts for a forward backward asymmetry. Second, in $O(\alpha_S^2)$ of QCD the 3-gluon vertex comes in. This 3-gluon vertex implies a self-coupling of the gluon field and is a unique feature of non-abelian field theories like QCD.

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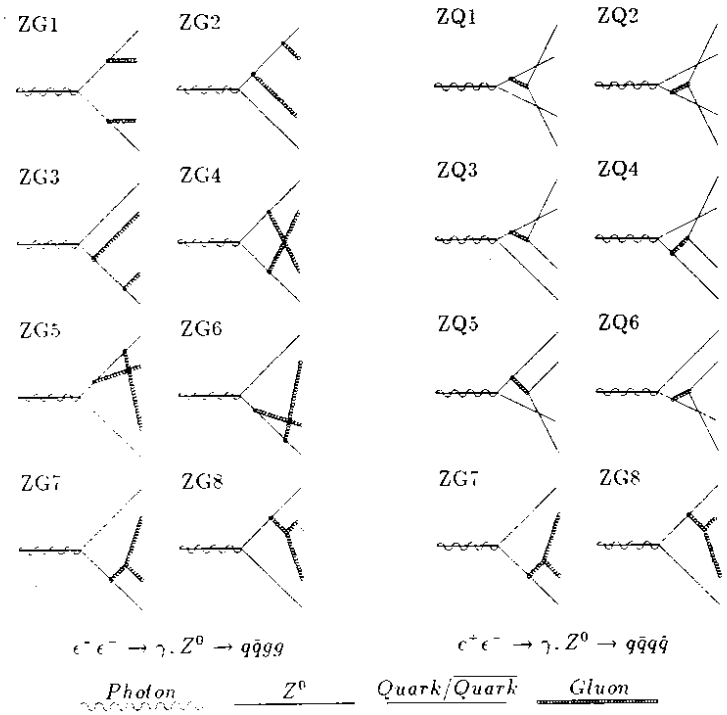


Figure 1: Diagrams of the Reaction $e^+e^- \to \gamma, Z^0 \to 4$ Jets

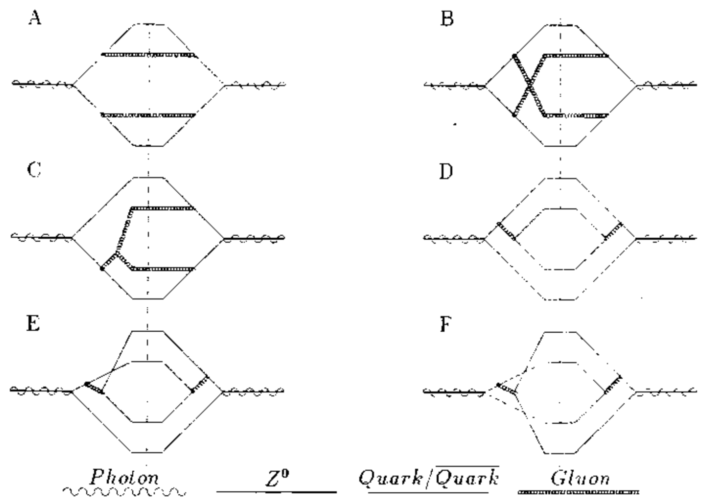


Figure 2: Classification of Diagrams

Both points of interest can be analysed by a separation of the cross section into partial cross sections as described in this paper. A further reason for an analysis of 4 jet events is the fact that LEP (the electron-positron-collider at CERN) is now operating and producing enough 4 jet events for a statistical analysis at a center of mass energy of 91.5 GeV.

The outline of this paper is as follows. In section 2 we describe the calculation of the cross section. In section 3 we separate the cross section into partial cross sections. In section 4 we use this technique to analyse the cross section in general. In section 5 we have a closer look at the forward backward asymmetry. The 3-gluon vertex is analysed in section 6. In section 7 we summarise the results and add some concluding remarks.

2 Calculation of the Cross Section

The cross section of the reaction $e^+e^- \to \gamma, Z^0 \to q\bar{q}q\bar{q}, q\bar{q}gg$ has been calculated first by Clavelli and v. Gehlen[6] in second order QCD with all masses but the Z^0 mass neglected. Since the expressions of the cross section are rather lengthy and have been published, we will repeat the calculations of the cross section in brief and calculate the partial cross sections in detail in the next section.

With an index R for the kind of reaction ($R = q\bar{q}gg, q\bar{q}q\bar{q}$) the matrix element corresponding to the diagrams on figure 1 is

$$M_R = 16\pi^2\alpha\alpha_S(j^\gamma_\mu \frac{i}{Q^2}(-g^{\mu\nu})J^\gamma_{\nu R} - j^Z_\mu \frac{i}{(4\sin^2 2\vartheta_W)(Q^2 - M_Z^2 + i\Gamma_Z M_Z)}(-g^{\mu\nu} - \frac{q^\mu q^\nu}{M_Z^2})J^Z_{\nu R}) \tag{1}$$

In this equation we have the fine structure constant α and the strong coupling constant $\alpha_S = 0.2$. For the exchange of a photon (Z^0) we get the leptonic current j^γ_μ (j^Z_μ) and the hadronic current

$J^\gamma_{\nu R}$ ($J^Z_{\nu R}$). The Weinberg angle ϑ_W is given by $\sin^2\vartheta_W = 0.23$. The center of mass energy is Q and the 4 vector of the virtual photon/Z^0 is $q^\mu = (Q, 0, 0, 0)$. The mass and width of the Z^0 are M_Z and Γ_Z. Now taking the square of the matrix element, averaging over the spins of the incoming particles and summing over the spins of the outgoing particles we get the equation

$$\frac{1}{4}\sum_{spins_{incoming}}\sum_{spins_{outgoing}}|M_R|^2 = (4\pi\alpha)^2(4\pi\alpha_S)^2\frac{1}{Q^4}\sum_{i,j\in\{\gamma,Z\}}\chi_i\chi_j^* L^{ij}_{\mu\nu}H^{\mu\nu}_{ijR} \tag{2}$$

The leptonic tensor is given by

$$L^{ij}_{\mu\nu} = j^i_\mu j^{*j}_\nu/4 \tag{3}$$

The hadronic tensor is

$$H^{\mu\nu}_{ijR} = J^\mu_{iR}J^{*\nu}_{jR} \tag{4}$$

χ is given by

$$\chi_\gamma = 1, \quad \chi_Z = \frac{1}{4\sin^2 2\vartheta_W}\frac{Q^2}{Q^2 - M_Z^2 + i\Gamma_Z M_Z} \tag{5}$$

To express the leptonic and hadronic tensor we need some further definitions. Let $Z^{xy}_{\mu\nu}$ be the tensor (without coupling constants) of a diagram Z with x coupling ($x \in \{v, a\}$, v=vector, a=axial) in the non-conjugated part and y coupling ($y \in \{v, a\}$) in the conjugated part. For this tensor we can define

$$\begin{aligned} Z^{VV}_{\mu\nu} &= Z^{vv}_{\mu\nu}, & Z^{AA}_{\mu\nu} &= Z^{aa}_{\mu\nu} \\ Z^{VA}_{\mu\nu} &= (Z^{va}_{\mu\nu} - Z^{av}_{\mu\nu})/2, & Z^{I}_{\mu\nu} &= (Z^{va}_{\mu\nu} - Z^{av}_{\mu\nu})/2 \end{aligned} \tag{6}$$

In case of the leptonic tensor we have the relations

$$L^{VV}_{\mu\nu} = L^{AA}_{\mu\nu}, \quad L^I_{\mu\nu} = 0 \tag{7}$$

The 4-vectors of electron (p^-) and positron (p^+) suggest the definition of two new 4-vectors

$$\hat{q} = \frac{p^+ + p^-}{Q}, \quad \hat{r} = \frac{p^+ - p^-}{Q} \tag{8}$$

With these unit vectors we have a symmetric tensor $L^{VV}_{\mu\nu}$ and an antisymmetric tensor $L^{VA}_{\mu\nu}$ with respect to (μ, ν)

$$L^{VV}_{\mu\nu} = -\frac{Q^2}{2}(\hat{r}_\mu\hat{r}_\nu + g_{\mu\nu}), \quad L^{VA}_{\mu\nu} = -\frac{iQ^2}{2}(\epsilon_{\mu\nu\rho\sigma}\hat{q}^\rho\hat{r}^\sigma) \tag{9}$$

Using the different couplings of photon ($q_{e^-} = -1$) and Z^0 ($v_{e^-} = -1 + 4\sin^2\vartheta_W$ and $a_{e^-} = -1$ for the vector and axial vector couplings) we get the following expressions for the leptonic tensor

$$\begin{aligned} L^{\gamma\gamma}_{\mu\nu} &= q_{e^-}q^*_{e^-}L^{VV}_{\mu\nu} \\ L^{\gamma Z}_{\mu\nu} &= q_{e^-}v^*_{e^-}L^{VV}_{\mu\nu} + q_{e^-}a^*_{e^-}L^{VA}_{\mu\nu} \\ L^{Z\gamma}_{\mu\nu} &= v_{e^-}q^*_{e^-}L^{VV}_{\mu\nu} + a_{e^-}q^*_{e^-}L^{VA}_{\mu\nu} \\ L^{ZZ}_{\mu\nu} &= (v_{e^-}v^*_{e^-} + a_{e^-}a^*_{e^-})L^{VV}_{\mu\nu} + (v_{e^-}a^*_{e^-} + a_{e^-}v^*_{e^-})L^{VA}_{\mu\nu} \end{aligned} \tag{10}$$

The hadronic tensor is calculated by taking the square of the sum of the diagrams on figure 1. The squared diagrams can be classified according to figure 2. These classes depend on the number of colors ($N_C = 3$), the Casimir operator ($C_F = \frac{4}{3}$) and the number of flavours ($n_f = 5, T_R = \frac{n_f}{2}$) in the following way

$$\begin{array}{ll} q\bar{q}gg \left\{ \begin{array}{lll} \text{Class A} & \propto & C_F^2 \\ \text{Class B} & \propto & C_F(C_F - \frac{1}{2}N_C) \\ \text{Class C} & \propto & C_F N_C \end{array} \right. \\ q\bar{q}q\bar{q} \left\{ \begin{array}{lll} \text{Class D} & \propto & T_R C_F \\ \text{Class E} & \propto & C_F(C_F - \frac{1}{2}N_C) \\ \text{Class F} & \propto & \frac{1}{2}C_F \end{array} \right. \end{array} \tag{11}$$

Class C consists of all terms related to the 3-gluon vertex. These terms are a unique feature of QCD as compared to abelian field theories. In class F the γZ^0-interference gives birth to terms proportional to $\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$. This feature is special to reactions with γ, Z^0-interference and at least 4 fermions in the final state. The full hadronic tensor is the sum of the tensor of each class

$$H^{ik}_{\mu\nu} = A^{ik}_{\mu\nu} + B^{ik}_{\mu\nu} + C^{ik}_{\mu\nu} + D^{ik}_{\mu\nu} + E^{ik}_{\mu\nu} + F^{ik}_{\mu\nu}\,. \tag{12}$$

Using the tensors $A^{VV}_{\mu\nu}$ to $F^{AA}_{\mu\nu}$ (these tensors are given explicitly in [6] and as a REDUCE-program in [11]) and the relations

$$\begin{array}{ll} A^{VV}_{\mu\nu} = A^{AA}_{\mu\nu}, & A^{I}_{\mu\nu} = 0 \\ B^{VV}_{\mu\nu} = B^{AA}_{\mu\nu}, & B^{I}_{\mu\nu} = 0 \\ C^{VV}_{\mu\nu} = C^{AA}_{\mu\nu}, & C^{I}_{\mu\nu} = 0 \\ D^{VV}_{\mu\nu} = D^{AA}_{\mu\nu}, & D^{I}_{\mu\nu} = 0 \\ E^{VV}_{\mu\nu} = E^{AA}_{\mu\nu}, & E^{I}_{\mu\nu} = 0 \\ F^{VV}_{\mu\nu} \neq F^{AA}_{\mu\nu}, & F^{I}_{\mu\nu} \neq 0 \end{array} \tag{13}$$

and the couplings

$$\begin{aligned} q_d = q_s = q_b &= -\tfrac{1}{3} \\ q_u = q_c = q_t &= -\tfrac{2}{3} \\ v_d = v_s = v_b &= -1 + \tfrac{4}{3}\sin^2\vartheta_w \\ v_u = v_c = v_t &= +1 - \tfrac{8}{3}\sin^2\vartheta_w \\ a_d = a_s = a_b &= -1 \\ a_u = a_c = a_t &= +1 \end{aligned} \tag{14}$$

we can write the hadronic tensors with explicit couplings. The hadronic tensors of the classes A, B, C, and E have the same structure (A given as an example)

$$\begin{aligned} A^{\gamma\gamma}_{\mu\nu} &= (\textstyle\sum_f q_f q_f^*)A^{VV}_{\mu\nu} \\ A^{\gamma Z}_{\mu\nu} &= (\textstyle\sum_f q_f v_f^*)A^{VV}_{\mu\nu} - (\sum_f q_f a_f^*)A^{VA}_{\mu\nu} \\ A^{Z\gamma}_{\mu\nu} &= (\textstyle\sum_f v_f q_f^*)A^{VV}_{\mu\nu} - (\sum_f a_f q_f^*)A^{VA}_{\mu\nu} \\ A^{ZZ}_{\mu\nu} &= (\textstyle\sum_f v_f v_f^* - a_f a_f^*)A^{VV}_{\mu\nu} - (\sum_f v_f a_f^* + a_f v_f^*)A^{VA}_{\mu\nu}\,. \end{aligned} \tag{15}$$

Because of the inner fermion loop the tensor of class D has a factor n_f

$$\begin{aligned} D^{\gamma\gamma}_{\mu\nu} &= n_f(\textstyle\sum_f q_f q_f^*)D^{VV}_{\mu\nu} \\ D^{\gamma Z}_{\mu\nu} &= n_f(\textstyle\sum_f q_f v_f^*)D^{VV}_{\mu\nu} - n_f(\sum_f q_f a_f^*)D^{VA}_{\mu\nu} \\ D^{Z\gamma}_{\mu\nu} &= n_f(\textstyle\sum_f v_f q_f^*)D^{VV}_{\mu\nu} - n_f(\sum_f a_f q_f^*)D^{VA}_{\mu\nu} \\ D^{ZZ}_{\mu\nu} &= n_f(\textstyle\sum_f v_f v_f^* - a_f a_f^*)D^{VV}_{\mu\nu} - n_f(\sum_f v_f a_f^* + a_f v_f^*)D^{VA}_{\mu\nu}\,. \end{aligned} \tag{16}$$

The tensor of class F exhibits two special features. First, there is a term F^I proportional to $\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$. Second, the couplings contribute as a product of sums and not as a sum of products as in all the other tensors. The tensor of class F is given by

$$\begin{aligned} F^{\gamma\gamma}_{\mu\nu} &= (\textstyle\sum_f q_f)(\sum_{f'} q_{f'}^*)F^{VV}_{\mu\nu} \\ F^{\gamma Z}_{\mu\nu} &= (\textstyle\sum_f q_f)(\sum_{f'} v_{f'}^*)F^{VV}_{\mu\nu} + (\sum_f q_f)(\sum_{f'} a_{f'}^*)(F^{VA}_{\mu\nu} + F^{I}_{\mu\nu}) \\ F^{Z\gamma}_{\mu\nu} &= (\textstyle\sum_f v_f)(\sum_{f'} q_{f'}^*)F^{VV}_{\mu\nu} + (\sum_f a_f)(\sum_{f'} q_{f'}^*)(F^{VA}_{\mu\nu} - F^{I}_{\mu\nu}) \\ F^{ZZ}_{\mu\nu} &= (\textstyle\sum_f v_f)(\sum_{f'} v_{f'}^*)F^{VV}_{\mu\nu} + (\sum_f v_f)(\sum_{f'} a_{f'}^*)(F^{VA}_{\mu\nu} + F^{I}_{\mu\nu}) \\ &+ (\textstyle\sum_f a_f)(\sum_{f'} v_{f'}^*)(F^{VA}_{\mu\nu} - F^{I}_{\mu\nu}) + (\sum_f a_f)(\sum_{f'} a_{f'}^*)F^{AA}_{\mu\nu}\,. \end{aligned} \tag{17}$$

With these tensors and a statistical factor ($N_{q\bar{q}gg} = 2, N_{q\bar{q}q\bar{q}} = 4$) we get the differential cross section

$$d\sigma = (4\pi\alpha)^2(4\pi\alpha_S)^2\frac{1}{Q^4}\sum_{R\in\{q\bar{q}gg,q\bar{q}q\bar{q}\}}\ \sum_{i,j\in\{\gamma,Z\}}\chi_i\chi_j^* L^{ij}_{\mu\nu}\frac{H^{\mu\nu}_{ijR}}{N_R}d\Omega^{(4)} \tag{18}$$

The phase space factor is given by

$$d\Omega^{(4)} = (2\pi)^4\delta^{(4)}\left[p^+ + p^- - \sum_{i=1}^{4}p_i\right]\prod_{i=1}^{4}\frac{d^3p_i}{2E_i(2\pi)^3}\,. \tag{19}$$

Using the equations

$$\begin{array}{ll} L^{VV}_{\mu\nu}Z^{\mu\nu}_{VA} = 0, & L^{VA}_{\mu\nu}Z^{\mu\nu}_{VV} = 0 \\ L^{VA}_{\mu\nu}F^{\mu\nu}_{AA} = 0, & L^{VA}_{\mu\nu}F^{\mu\nu}_{I} = 0 \end{array} \tag{20}$$

we get

$$d\sigma = \frac{(4\pi\alpha)^2(4\pi\alpha_S)^2}{Q^4}(L^{VV}_{\mu\nu}H^{\mu\nu}_{VV} + L^{VA}_{\mu\nu}H^{\mu\nu}_{VA})d\Omega^{(4)} \tag{21}$$

with the new tensors

$$\begin{aligned} H^{\mu\nu}_{VV} &= \tfrac{g_1}{N_{q\bar{q}gg}}A^{\mu\nu}_{VV} + \tfrac{g_1}{N_{q\bar{q}gg}}B^{\mu\nu}_{VV} + \tfrac{g_1}{N_{q\bar{q}gg}}C^{\mu\nu}_{VV} + \tfrac{n_f g_1}{N_{q\bar{q}q\bar{q}}}D^{\mu\nu}_{VV} + \tfrac{g_1}{N_{q\bar{q}q\bar{q}}}E^{\mu\nu}_{VV} + \tfrac{g_3}{N_{q\bar{q}q\bar{q}}}F^{\mu\nu}_{VV} \\ &+ \tfrac{g_4}{N_{q\bar{q}q\bar{q}}}F^{\mu\nu}_{AA} + \tfrac{g_6}{N_{q\bar{q}q\bar{q}}}F^{\mu\nu}_{I} \\ H^{\mu\nu}_{VA} &= \tfrac{g_2}{N_{q\bar{q}gg}}A^{\mu\nu}_{VA} + \tfrac{g_2}{N_{q\bar{q}gg}}B^{\mu\nu}_{VA} + \tfrac{g_2}{N_{q\bar{q}gg}}C^{\mu\nu}_{VA} + \tfrac{n_f g_2}{N_{q\bar{q}q\bar{q}}}D^{\mu\nu}_{VA} + \tfrac{g_2}{N_{q\bar{q}q\bar{q}}}E^{\mu\nu}_{VA} + \tfrac{g_5}{N_{q\bar{q}q\bar{q}}}F^{\mu\nu}_{VA}\,. \end{aligned} \tag{22}$$

The new functions g_i are given in the appendix. Almost all dependencies on Q, M_Z and Γ_Z are included in these functions. The tensors A_{VV} to F^{AA} are just proportional to Q^2. Therefore and to make it comparable[1] to [10] it makes sense to normalise the differential cross section (and all cross sections derived from it) according to

$$d\tilde{\sigma} = \frac{d\sigma}{\sigma(e^+e^- \to \gamma, Z^0 \to q\bar{q})} \tag{23}$$

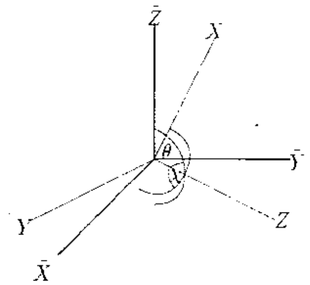
and use the second order QED relations

$$\sigma(e^+e^- \to \gamma, Z^0 \to q\bar{q}) = 3g_1\sigma(e^+e^- \to \gamma \to \mu^+\mu^-) \tag{24}$$

and

$$\sigma(e^+e^- \to \gamma \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3Q^2} = \frac{87nbarn}{Q^2[GeV^{-2}]}\,. \tag{25}$$

[1] The results are comparable because of $\frac{\sigma(e^+e^- \to \gamma \to 4\text{ Jets})}{\sigma(e^+e^- \to \gamma \to q\bar{q})} \approx \frac{\sigma(e^+e^- \to \gamma, Z^0 \to 4\text{ Jets})}{\sigma(e^+e^- \to \gamma, Z^0 \to q\bar{q})}$.



X, Y, Z	:	Hadronic coordinate system
$\tilde{X}, \tilde{Y}, \tilde{Z}$	:	Leptonic coordinate system
θ	:	Angle between Z- and $\tilde{Z}$-axis
χ	:	Angle between X-Z- and $\tilde{Z}$-Z-plane

Figure 3: Coordinate Systems and Angles

To obtain the full cross sections from the differential cross sections we integrate over the phase space using Monte Carlo methods. However the matrix element becomes singular for infrared gluons ($p_3 = 0$) or collinear quarks ($p_1 p_2 = 0$). To get rid of these singularities we restrict our integration by the condition

$$\frac{2p_i p_j}{Q^2} > y_{\text{Monte Carlo}} \tag{26}$$

This is called an invariant mass cut or y-cut. Since this cut is used for all phase space integrations we take $\int$ as the symbol for phase space integration with y-cut and write

$$\sigma = \int d\sigma \tag{27}$$

Now having the cross section we separate it into partial cross sections in the next section.

3 Separation into Partial Cross Sections

In this section we describe the separation of the cross section into partial cross sections as introduced by Avram and Schiller[3] (see also [2]). In the next three sections we apply this technique.

In order to separate the cross section into partial cross sections, we introduce two coordinate systems and the angles that characterise the transformation from one to the other (see figure 3). The coordinate system $\tilde{K}$ with the axes $\tilde{X}$, $\tilde{Y}$, $\tilde{Z}$ is used to describe the leptonic tensor. Here $\tilde{Z}$ is the direction of the positron. The other coordinate system K with the axes X, Y, Z is used for the hadronic tensor. The choice of the axes depends on the purpose of analysis and will be described in detail in section 4 to 6. Since the hadronic tensor obeys current conservation $q^\mu H_{\mu\nu} = 0$ only the space components of the leptonic and hadronic tensor are relevant. The transformation of the space components of the leptonic tensor L_{ij} from $\tilde{K}$ to K

with a transformation matrix R is

$$L_{ij} = R_{mi} R_{nj} \tilde{L}_{mn} \tag{28}$$

The transformation matrix is given by

$$R_{\tilde{K}\to K} = \begin{pmatrix} \cos\phi\cos\theta\cos\chi - \sin\phi\sin\chi & -\cos\phi\cos\theta\sin\chi - \sin\phi\cos\chi & \cos\phi\sin\theta \\ \sin\phi\cos\theta\cos\chi - \cos\phi\sin\chi & -\sin\phi\cos\theta\sin\chi - \cos\phi\cos\chi & \sin\phi\sin\theta \\ -\sin\theta\cos\chi & \sin\theta\sin\chi & \cos\theta \end{pmatrix} \tag{29}$$

All in all we get the following expressions for the leptonic tensor in both coordinate systems

$$\begin{aligned}
\tilde{L}^{VV} &= -\frac{Q^2}{2}\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
L^{VV} &= -\frac{Q^2}{2}\begin{pmatrix} -\frac{1-\cos^2\theta}{2} - \frac{\sin^2\theta\cos 2\chi}{2} & -\frac{\sin^2\theta\sin 2\chi}{2} & -\frac{\sin 2\theta\cos\chi}{2} \\ -\frac{\sin^2\theta\sin 2\chi}{2} & -\frac{1+\cos^2\theta}{2} - \frac{\sin^2\theta\cos 2\chi}{2} & \frac{\sin 2\theta\sin\chi}{2} \\ -\frac{\sin 2\theta\cos\chi}{2} & \frac{\sin 2\theta\sin\chi}{2} & -\sin^2\theta \end{pmatrix} \\
\tilde{L}^{VA} &= -\frac{iQ^2}{2}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
L^{VA} &= -\frac{iQ^2}{2}\begin{pmatrix} 0 & -\cos\theta & \sin\theta\sin\chi \\ \cos\theta & 0 & \sin\theta\cos\chi \\ -\sin\theta\sin\chi & -\sin\theta\cos\chi & 0 \end{pmatrix}
\end{aligned} \tag{30}$$

By collecting the appropriate angular terms we get the following separation into partial differential cross sections

$$\begin{aligned}
\frac{d^2\sigma}{d\cos\theta d\chi} &= \frac{3}{16\pi}(1-\cos^2\theta)d\sigma_U - \frac{3}{8\pi}\sin^2\theta d\sigma_L \\
&- \frac{3}{8\pi}\sin^2\theta\cos 2\chi d\sigma_{TR} - \frac{3}{8\pi}\sin^2\theta\sin 2\chi d\sigma_{TI} \\
&- \frac{3}{4\sqrt{2}\pi}\sin 2\theta\cos\chi d\sigma_{IR} - \frac{3}{4\sqrt{2}\pi}\sin 2\theta\sin\chi d\sigma_{II} \\
&- \frac{3}{8\pi}\cos\theta d\sigma_A - \frac{3}{2\sqrt{2}\pi}\sin\theta\cos\chi d\sigma_{PR} + \frac{3}{2\sqrt{2}\pi}\sin\theta\sin\chi d\sigma_{PI}
\end{aligned} \tag{31}$$

where the partial differential cross sections are given by

$$\begin{aligned}
d\sigma_U &= \frac{Q^2}{3}L^U_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}(\epsilon^{*+}_\mu\epsilon^+_\nu - \epsilon^{*-}_\mu\epsilon^-_\nu)H^{\mu\nu}_{VV}d\Omega^{(4)} \\
d\sigma_L &= \frac{Q^2}{3}L^L_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}(\epsilon^{*0}_\mu\epsilon^0_\nu)H^{\mu\nu}_{VV}d\Omega^{(4)} \\
d\sigma_{TR} &= \frac{Q^2}{6}L^{TR}_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}\Re[(\epsilon^{*+}_\mu\epsilon^-_\nu)H^{\mu\nu}_{VV}d\Omega^{(4)}] \\
d\sigma_{TI} &= \frac{Q^2}{6}L^{TI}_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}\Im[(\epsilon^{*+}_\mu\epsilon^-_\nu)H^{\mu\nu}_{VV}d\Omega^{(4)}] \\
d\sigma_{IR} &= \frac{Q^2}{6\sqrt{2}}L^{IR}_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}\Re[(\frac{-\epsilon^{*+}_\mu\epsilon^0_\nu+\epsilon^{*0}_\mu\epsilon^-_\nu}{2})H^{\mu\nu}_{VV}d\Omega^{(4)}] \\
d\sigma_{II} &= \frac{Q^2}{6\sqrt{2}}L^{II}_{\mu\nu}H^{\mu\nu}_{VV}d\Omega^{(4)} = \frac{Q^2}{3}\Im[(\frac{-\epsilon^{*+}_\mu\epsilon^0_\nu+\epsilon^{*0}_\mu\epsilon^-_\nu}{2})H^{\mu\nu}_{VV}d\Omega^{(4)}] \\
d\sigma_A &= \frac{Q^2}{3}L^A_{\mu\nu}H^{\mu\nu}_{VA}d\Omega^{(4)} = \frac{Q^2}{3}(-\epsilon^{*+}_\mu\epsilon^+_\nu + \epsilon^{*-}_\mu\epsilon^-_\nu)H^{\mu\nu}_{VA}d\Omega^{(4)} \\
d\sigma_{PR} &= \frac{Q^2}{6\sqrt{2}}L^{PR}_{\mu\nu}H^{\mu\nu}_{VA}d\Omega^{(4)} = \frac{Q^2}{3}\Re[(\frac{\epsilon^{*+}_\mu\epsilon^0_\nu+\epsilon^{*0}_\mu\epsilon^-_\nu}{2})H^{\mu\nu}_{VA}d\Omega^{(4)}] \\
d\sigma_{PI} &= \frac{Q^2}{6\sqrt{2}}L^{PI}_{\mu\nu}H^{\mu\nu}_{VA}d\Omega^{(4)} = \frac{Q^2}{3}\Im[(\frac{\epsilon^{*+}_\mu\epsilon^0_\nu+\epsilon^{*0}_\mu\epsilon^-_\nu}{2})H^{\mu\nu}_{VA}d\Omega^{(4)}]
\end{aligned} \tag{32}$$

The polarisation vectors in this definition are

$$\epsilon^0_\nu = (0,0,0,1) \quad \epsilon^-_\nu = (0, \frac{-1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0) \quad \epsilon^-_\mu = (0, \frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}, 0) \tag{33}$$

		$Q = 40.0$ GeV	$Q = 60.0$ GeV	$Q = 91.5$ GeV
$\tilde{\sigma}$	$y = 0.01$	0.341	0.341	0.341
	$y = 0.02$	0.119	0.119	0.119
	$y = 0.05$	0.0136	0.0135	0.0135
$\tilde{\sigma}_U$	$y = 0.01$	0.322	0.322	0.322
	$y = 0.02$	0.110	0.110	0.110
	$y = 0.05$	0.0118	0.0117	0.0117
$\tilde{\sigma}_L$	$y = 0.01$	0.0189	0.0189	0.0189
	$y = 0.02$	0.00898	0.00898	0.00898
	$y = 0.05$	0.00176	0.00175	0.00175
$\tilde{\sigma}_{TR}$	$y = 0.01$	0.00693	0.00693	0.00693
	$y = 0.02$	0.00276	0.00276	0.00276
	$y = 0.05$	0.000307	0.000306	0.000306
$\tilde{\sigma}_{IR}$	$y = 0.01$	0.00891	0.00891	0.00891
	$y = 0.02$	0.00329	0.00329	0.00329
	$y = 0.05$	0.000379	0.000378	0.000378

Table 1: Cross sections for $e^+e^- \to 4$ Jets in the energy ordered coordinate system

The tensors in the definition are given by

$$L^U_{mn} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad L^L_{mn} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad L^{TR}_{mn} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$L^{TI}_{mn} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad L^{IR}_{mn} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad L^{II}_{mn} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \tag{34}$$
$$L^A_{mn} = \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad L^{PR}_{mn} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad L^{PI}_{mn} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

Integrating these partial differential cross sections over the configuration space yields partial cross sections (σ_U given as an example)

$$\sigma_U = \int d\sigma_U \tag{35}$$

For σ we have the relation

$$\sigma = \sigma_U + \sigma_L \tag{36}$$

Having now all the prerequisites we will start with the analysis of the cross section in general.

4 General Cross Sections

Some well known jet parameters in e^+e^--annihilation are the momentum and the direction of the most energetic jets. Using these parameters of the two most energetic jets ($E_1 = |\vec{p}_1|$

		$Q = 40.0$ GeV	$Q = 60.0$ GeV	$Q = 91.5$ GeV
$\tilde{\sigma}$	$y = 0.01$	0.325	0.325	0.325
	$y = 0.02$	0.112	0.112	0.112
	$y = 0.05$	0.0126	0.0126	0.0126
$\tilde{\sigma}_U$	$y = 0.01$	0.295	0.295	0.295
	$y = 0.02$	0.0981	0.0982	0.0982
	$y = 0.05$	0.0104	0.0104	0.0104
$\tilde{\sigma}_L$	$y = 0.01$	0.0300	0.0301	0.0301
	$y = 0.02$	0.0132	0.0133	0.0133
	$y = 0.05$	0.00221	0.00221	0.00221
$\tilde{\sigma}_{TR}$	$y = 0.01$	0.0128	0.0129	0.0129
	$y = 0.02$	0.00522	0.00522	0.00522
	$y = 0.05$	0.000705	0.000705	0.000705
$\tilde{\sigma}_{IR}$	$y = 0.01$	0.0172	0.0172	0.0172
	$y = 0.02$	0.00622	0.00622	0.00622
	$y = 0.05$	0.000716	0.000715	0.000715
$\tilde{\sigma}_A$	$y = 0.01$	0.0843	0.197	-0.0373
	$y = 0.02$	0.0275	0.0642	-0.0121
	$y = 0.05$	0.00279	0.00651	-0.00123
$\tilde{\sigma}_{PR}$	$y = 0.01$	0.00910	0.0213	-0.00404
	$y = 0.02$	0.00360	0.00841	-0.00159
	$y = 0.05$	0.000499	0.00116	-0.000219

Table 2: Cross sections for $e^-e^- \to q\bar{q}gg$ in the $q\bar{q}$ coordinate system

$E_2 = |\vec{p}_2|$) we can define the axes of the energy ordered coordinate system in the following way

$$\begin{aligned} Z &= \frac{\vec{p}_1}{|\vec{p}_1|} \\ Y &= \frac{\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1 \times \vec{p}_2|} \\ X &= Z \times Y \end{aligned} \tag{37}$$

Using this coordinate system we get the non-zero partial cross sections of table 1. In the given normalisation the cross sections show almost no dependence on energy. The cross section $\tilde{\sigma}$ reproduces some previous results [10] without γZ^0 interference. As we see $\tilde{\sigma}_U$ contributes most to $\tilde{\sigma}$. The contribution of $\tilde{\sigma}_L$ depends on the y-cut and ranges from 5 percent for $y = 0.01$ to 13 percent for $y = 0.05$. The other contributions are small. Since there is no easy interpretation of this choice of coordinate system we go on and analyse the forward backward asymmetry and the 3-gluon vertex in the next two sections. In addition to that we might analyse the terms proportional to $\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$ in class F. But the contributions of these terms are so small, that they are swamped[2] by the noise of the Monte Carlo integration even in the case of coordinate systems specific to these terms.

5 Forward Backward Asymmetry

[2] Using the methods of [6] the best signal noise ratio obtained was about 10^{-3}.

		QCD	QAD
$\tilde\sigma$	$y = 0.01$	0.325	0.291
	$y = 0.02$	0.112	0.0987
	$y = 0.05$	0.0127	0.0112
$\tilde\sigma_U$	$y = 0.01$	0.309	0.282
	$y = 0.02$	0.104	0.0941
	$y = 0.05$	0.0112	0.0103
$\tilde\sigma_L$	$y = 0.01$	0.0163	0.00947
	$y = 0.02$	0.00772	0.00455
	$y = 0.05$	0.00145	0.000861
$\tilde\sigma_L/\tilde\sigma$	$y = 0.01$	0.050	0.033
	$y = 0.02$	0.069	0.046
	$y = 0.05$	0.114	0.077

Table 3: Cross sections for $e^+e^- \to q\bar{q}gg$ in the momentum difference coordinate system

In this section we analyse the forward backward asymmetry. This analysis implies an experimental method to distinguish the jets according to their primordial particles (quark, antiquark or gluon). Given this method we can sort out all the events of the reaction $e^+e^- \to \gamma, Z^0 \to q\bar{q}gg$ and measure the 4 vectors of the quark (p_q) and antiquark jet ($p_{\bar{q}}$). For the events of this reaction we can define the $q\bar{q}$ coordinate system by

$$\begin{aligned} Z &= \frac{\vec{p}_q}{|\vec{p}_q|} \\ Y &= \frac{\vec{p}_q \times \vec{p}_{\bar{q}}}{|\vec{p}_q \times \vec{p}_{\bar{q}}|} \\ X &= Z \times Y \end{aligned} \tag{38}$$

Using this coordinate system we obtain the non-zero partial cross sections given in table 2. As can be seen from table 2 and 1 the contribution of the reaction $e^+e^- \to q\bar{q}gg$ to $e^+e^- \to$ 4 Jets is about 95 percent of $\tilde\sigma$. In table 2, $\tilde\sigma_L$ corresponds to γ or Z^0 longitudinally polarised in the q directions. The $\tilde\sigma_L$ contribution to $\tilde\sigma$ ranges from 9 to 17 percent for y-cut of 0.01 to 0.05. The ratio $\tilde\sigma_{TR}/\tilde\sigma_L$ is cut dependent and ranges from 0.32 ($y = 0.05$) to 0.43 ($y = 0.01$). For the reaction $e^+e^- \to \gamma, Z^0 \to q\bar{q}g$ the ratio has been calculated [9] in first order ($O(\alpha_S)$) as $\tilde\sigma_{TR}/\tilde\sigma_L = 0.5$. $\tilde\sigma_A$ corresponds to the ordinary forward backward asymmetry with terms proportional to $\cos\theta$. The values of $\tilde\sigma_A$ are cut-dependent and have the magnitude of 20 to 25 percent of $\tilde\sigma$. All the other partial cross sections are small. Let us now analyse the 3 gluon vertex in the next section.

6 3-Gluon Vertex

As previously explained the 3 gluon vertex is a special feature of QCD. So it is of some interest to analyse the effect of this vertex on the cross section. In order to do this we need another theory to compare the results. This theory is Quantum Abelian Dynamics (QAD), a theory built like QED. Accordingly the gluons do not carry colour and do not interact in a 3- or 4-gluon vertex. But every quark flavour consists of a triplet of different colour states. In zeroth order (i.e. $e^+e^- \to q\bar{q}$) there is no difference between the theories. In first order we get a relation[7] for the coupling constant of both theories

$$\alpha_{QAD} = \frac{4}{3}\alpha_S \; . \tag{39}$$

Using this relation we get the QAD results of the reaction $e^+e^- \to \gamma, Z^0 \to q\bar{q}gg$ in terms of α_S by setting $N_C = 0$ in the relations 11. Let us now assume that we can sort out the reaction $e^+e^- \to q\bar{q}gg$ and detect momentum and direction of quark (p_q) and antiquark jet ($p_{\bar{q}}$). Then the axes of the momentum difference coordinate system are given by

$$\begin{aligned} Z &= \frac{\vec{p}_q - \vec{p}_{\bar{q}}}{|\vec{p}_q - \vec{p}_{\bar{q}}|} \\ Y &= \frac{(\vec{p}_q - \vec{p}_{\bar{q}}) \times (\vec{p}_q + \vec{p}_{\bar{q}})}{|(\vec{p}_q - \vec{p}_{\bar{q}}) \times (\vec{p}_q + \vec{p}_{\bar{q}})|} \\ X &= Z \times Y \end{aligned} \tag{40}$$

Using this coordinate system we can calculate the partial cross sections for both theories. The relevant results are given in table 3. As we can see there is little difference between the full cross sections. But there is some difference for the cross section $\tilde\sigma_L$. Since the coupling constant α_S is not a fixed, well known parameter we use a normalisation not depending on α_S and calculate $\tilde\sigma_L/\tilde\sigma$. Here we see a difference between QCD and QAD of about 30 percent. Similar results, a significant difference between QCD and QAD, have been published by some other authors (see the review in [13]). But their methods to detect this difference are not totally satisfying. The method of Bengtsson and Zerwas [4,5] is based on two pairs of jets with opposite direction and the angle between these pairs of jets. This method makes only use of a small set of all available events. The method of Ali et al.[1,2,8] suffers from the same flaw.

7 Summary and Concluding Remarks

We calculated the cross section of the reaction $e^+e^- \to \gamma, Z^0 \to$ 4 Jets in second order QCD ($O(\alpha_S^2)$). The cross section was then separated into partial cross sections. These partial cross sections were used to analyse the forward backward asymmetry. As a result we got in the $q\bar{q}$ coordinate system the relevant asymmetry $\tilde\sigma_A/\tilde\sigma \approx 0.25$. Furthermore we compared QCD to the test-theory QAD and calculated in the momentum difference coordinate system a difference of the longitudinal cross sections in both theories of about 30 percent.

Some further work on these topics may include a full analysis of the LEP experimental data and/or next order ($O(\alpha_S^3)$= 1 loop) calculations of the 4 jet cross section.

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A Representation of the g_i Functions

The function g_i are given by

$$
\begin{aligned}
g_1 &= \lambda_\gamma\chi_\gamma^*(q_{e^-}q_{e^-}^*)(\textstyle\sum_f q_f q_f^*) \\
&+ \lambda_\gamma\chi_Z^*(q_{e^-}v_{e^-}^*)(\textstyle\sum_f q_f v_f^*) + \chi_Z\chi_\gamma^*(v_{e^-}q_{e^-}^*)(\textstyle\sum_f v_f q_f^*) \\
&- \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)(\textstyle\sum_f v_f v_f^* + a_f a_f^*) \\
g_2 &= \lambda_\gamma\chi_Z^*(q_{e^-}a_{e^-}^*)(\textstyle\sum_f q_f a_f^*) + \chi_Z\chi_\gamma^*(a_{e^-}q_{e^-}^*)(\textstyle\sum_f a_f q_f^*) \\
&+ \lambda_Z\chi_Z^*(v_{e^-}a_{e^-}^* + a_{e^-}v_{e^-}^*)(\textstyle\sum_f v_f a_f^* + a_f v_f^*) \\
g_3 &= \lambda_\gamma\chi_\gamma^*(q_{e^-}q_{e^-}^*)(\textstyle\sum_f q_f)(\textstyle\sum_{f'} q_{f'}^*) \\
&+ \lambda_\gamma\chi_Z^*(q_{e^-}v_{e^-}^*)(\textstyle\sum_f q_f)(\textstyle\sum_{f'} v_{f'}^*) + \chi_Z\chi_\gamma^*(v_{e^-}q_{e^-}^*)(\textstyle\sum_f v_f)(\textstyle\sum_{f'} q_{f'}^*) \\
&+ \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)(\textstyle\sum_f v_f)(\textstyle\sum_{f'} v_{e^-}^*) \\
g_4 &= \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)(\textstyle\sum_f a_f)(\textstyle\sum_{f'} a_{f'}^*) \\
g_5 &= \lambda_\gamma\chi_Z^*(q_{e^-}a_{e^-}^*)(\textstyle\sum_f q_f)(\textstyle\sum_{f'} a_{f'}^*) + \chi_Z\chi_\gamma^*(a_{e^-}q_{e^-}^*)(\textstyle\sum_f a_f)(\textstyle\sum_{f'} q_{f'}^*) \\
&+ \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)[(\textstyle\sum_f v_f)(\textstyle\sum_{f'} a_{f'}^*) + (\textstyle\sum_f a_f)(\textstyle\sum_{f'} v_{f'}^*)] \\
g_6 &= \lambda_\gamma\chi_Z^*(q_{e^-}v_{e^-}^*)(\textstyle\sum_f q_f)(\textstyle\sum_{f'} a_{f'}^*) - \chi_Z\chi_\gamma^*(v_{e^-}q_{e^-}^*)(\textstyle\sum_f a_f)(\textstyle\sum_{f'} q_{f'}^*) \\
&+ \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)(\textstyle\sum_f v_f)(\textstyle\sum_{f'} a_{f'}^*) - \lambda_Z\chi_Z^*(v_{e^-}v_{e^-}^* + a_{e^-}a_{e^-}^*)(\textstyle\sum_f a_f)(\textstyle\sum_{f'} v_{f'}^*)
\end{aligned}
\tag{41}
$$