DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 90-107
September 1990

Comparison of Parton Distributions and Structure Functions for the Proton

H. Abramowicz, K. Charchula
Deutsches Elektronen-Synchrotron DESY, Hamburg

M. Krawczyk
Institute of Theoretical Physics, Warsaw University

A. Levy
School of Physics, Tel-Aviv University

ISSN 0418-9833

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY 90-107
September 1990

ISSN 0418-9833

Comparison of parton distributions and structure functions for the proton

H.Abramowicz[1*], K.Charchuła[1†], M.Krawczyk[3], and A.Levy[2]

1. Deutsches Elektronen-Synchrotron, DESY
2. School of Physics, Tel-Aviv University
3. Institute of Theoretical Physics, Warsaw University

September 10, 1990

Abstract

A comparative study of the most popular parton parametrizations is presented. The individual parton distributions as well as the F_2 structure function are discussed with a particular emphasis on the low x region, $10^{-4} < x < 10^{-2}$. The predictions of these parametrizations for the F_2 structure function have a wide spread which persists also in the HERA kinematical region.

*Alexander von Humboldt Fellow, on leave of absence from Institute of Experimental Physics, Warsaw University.

†On leave of absence from Institute of Experimental Physics, Warsaw University.

1 Introduction

Parton distributions play a basic role in the studies of inclusive hadronic processes at energies available at present and envisaged in the future. Their specific features determine the behaviour of hadronic cross sections at high energies. Any reliable estimation of a possible signal of new physics will also depend on these ingredients.

The region of low x, populated by gluons and sea quarks, is expected to give a dominant contribution to the hadronic cross section at energies of the forthcoming experiments. New phenomena arising from interactions between partons belonging to the same hadron may show up in this particular region. Therefore it is of importance to realize what is presently known about parton densities deduced from data by means of perturbative QCD and to what accuracy, especially in the region of small x which seems crucial for future investigations of high energy physics.

A variety of parton parametrizations exists in the literature, obtained under different theoretical assumptions and with different experimental input. We have investigated the most popular parametrizations, including the recent available in the literature, namely :

P1. Duke and Owens (DO) [1]

P2. Eichten, Hinchliffe, Lane and Quigg (EHLQ) [2]

P3. Harriman, Martin, Roberts and Stirling (HMRS) [3]

P4. Diemoz, Ferroni, Longo and Martinelli (DFLM) [4]

P5. Morfin and Tung (MT) [5]

We found important not to limit ourselves to the most recent parametrizations which are expected to be better as compared to the older ones like DO, EHLQ or even DFLM. The reason is that many results are based on those latter ones and they are still widely used.

The aim of this letter is to compare the basic assumptions behind those parametrizations and their predictions for the individual parton densities in the proton as well as for the directly measured physical quantity, the structure function F_2^{em}, in the range $10\ \mathrm{GeV}^2 < Q^2 < 10^5\ \mathrm{GeV}^2$ and $10^{-4} < x < 1$. The predictions for F_2^{em} are also compared to the most recent measurements.

This comparison should constitute a reference for estimating possible effects due to a particular choice of parametrization and a guideline for the precision needed in future experiments.

2 Basic assumptions of the parametrizations

In general terms all the parametrizations can be classified following the basic theoretical assumptions used in the extraction of parton densities at some reference energy scale and in the calculation of their evolution to higher energies. These can be performed either in the leading logarithm(LL) approximation or in the next-to-leading logarithm (NLL) one. The first two parametrizations, **P1**(DO) and **P2**(EHLQ), are of LL type, while **P3**(HMRS) uses the NLL approximations. The two remaining parametrizations, **P4** (DFLM) and **P5** (MT), provide both the LL and the NLL one.

In the approach which is based on the NLL approximation there is freedom in the definition of parton densities, i.e. the relation of parton densities to the measured quantities like structure functions can be chosen. A particular choice determines the so called renormalization scheme. The parametrizations of the NLL type considered by us differ in those definitions: HMRS introduces the $\overline{MS}$ scheme and DFLM uses the DIS scheme. The last parametrization by MT in the NLL version can be obtained in both schemes. A procedure which allows to compare parametrizations obtained in different schemes has been discussed by Tung [7].

These parametrizations differ also in their approach to the experimental input needed to extract parton densities. The most standard approach is to introduce parton distributions at the level of the global fit to the data. The structure functions are expressed in terms of parton densities parametrized by some free parameters at some reference value Q_0^2. Additional experimental input is needed to constrain the contribution of various flavours: the ratio of the up to down valence quarks, u_v/d_v, that of the strange quarks to the sum of up and down antiquarks, $s/(\bar{u}-\bar{d})$, the charm quark content and so on. The parton densities are then evolved numerically in Q^2 through the Altarelli–Parisi equations in the kinematic regions where the data exist. Finally a global fit is performed to determine the best values for the starting parameters, as well as for the QCD scale parameter Λ. A by-product of these fits, performed on the singlet structure function F_2, is a parametrization of the gluon distribution at the reference scale Q_0^2. The parameters describing the gluon distribution are usually strongly correlated with the Λ value obtained from the fit. This is the approach of the DO, HMRS and MT parametrizations.

The approach of EHLQ and DFLM is to determine the parton densities from structure functions (either parametrized by the experiments (EHLQ) or measured directly in the experiments (DFLM) and parametrized by the authors) at a reference Q_0^2 value. The evolution is then independent of the data, and relies on the Λ value and gluon distribution as determined at Q_0^2 from the appropriate fits to the data.

It is worth to notice that various parametrizations differ not only because they are based on different approaches, but also because they are inferred from different data samples, available at the time of the analysis.

The details of the **P1, P2, P4** parametrizations, the assumptions underlying each of them, the input data used, etc., can be found in an earlier publication [6]. One can also find there a detailed comparison of the parton distributions in the Q^2 range 10-10^4 GeV^2. As for the HRMS and MT parametrizations we refer the interested reader to the original publications [3,5].

3 Assumptions on gluon parametrizations

The main uncertainty in parton parametrizations arises from our rather poor knowledge about the gluon distribution and the fact that deep inelastic scattering measurements do not

constrain significantly this quantity. A precise knowledge of parton densities is crucial for understanding the capacity of future projects to clarify the yet unresolved problems of elementary interactions. In that respect the gluons are especially important since they populate the region of low x, which gives the dominant contribution to interactions at high energies. This need has triggered a renewed interest in developing theories that could cope with this region, which seem to lie on the boundary of perturbative QCD and could shed some light on the transition region.

Different theoretical pictures of the structure of hadrons at large energies lead to different assumptions on the gluon densities at small x, e.g. $1/x^{3/2}$ instead of the conventional $1/x$ behaviour. The small x behaviour of the parton densities is related to the intercept of the appropriate Regge trajectories. The $1/x$ behaviour for gluon densities was originally motivated by the belief that the Pomeron has an intercept of 1. Further studies pointed to the possibility that this intercept could be as big as 1.5, which would correspond to a $1/x^{3/2}$ dependence for the gluon density at small x. Since the existing deep inelastic data do not constrain significantly the gluons at low x, it is very important to have good data for gluon densities in the range of small x in order to test the existing theoretical ideas and moreover to perform a reliable parton parametrization which could be extended into the unmeasured regions.

The situation has somewhat improved since more precise data on direct photon production in pp interactions became available [8]. In fact these data were used in the most recent parametrizations of HMRS. But even those data are not sufficient to fix the shape of the gluon density with adequate accuracy, especially at small x.

Thus at present it is rather a matter of taste which parametrization of the input gluon distributions to choose. Some of the authors choose even to fit different shapes of the input gluon densities, each with its own Λ value. This leads to different sets of parameters within each of the parametrizations. For the sake of completeness, all the input parametrizations of gluons and the corresponding Λ values as used in **P1 – P5** are compiled in table 1. For those parametrizations which adopt a less conventional form for the gluon distribution at the input scale, we have fitted a $x^{-\alpha}$ type of behaviour in the range $10^{-4} < x < 10^{-2}$. The results are also included in the table. Note the 'singular' behaviour of the gluon parametrization for $x \to 0$ in the case of HMRS(E−), DFLM and MT. This choice influences strongly the behaviour of the gluon distributions at higher energy scales and leads to the main differences in the predictions of existing parton parametrizations for other partons. It can be anticipated that the estimates for processes sensitive to small x, especially to the region of $x < 10^{-2}$, can change quite drastically.

4 Comparison of predictions of various parametrizations

Before any comparison between predictions of the parametrizations **P1 – P5** can be made, some words of caution are needed. When one tries to compare parton densities obtained in different approximations the problem that arises is that the result may depend (as in the

NLL approach) on the renormalization scheme. This dependence is expected to disappear only after parton densities are convoluted with partonic cross sections – scheme dependent as well. Thus a consistent way to compare distributions obtained in two different schemes requires a transformation of the distributions from one scheme to the other, as was discussed in [7,6]. A similar problem appears when one wants to compare parton distributions obtained in the LL and the NLL approximations, since it implies a comparison between objects from two different worlds.

However, for the sake of this study, we will follow the standard procedure of comparing the parton densities directly. We are doing this not only because this was and still is a standard procedure, but also because from this approach one can learn how the type of evolution, the LL or NLL one, influences the parton densities. In that respect both the DFLM and the MT parametrizations are most suitable, since they offer the possibility to compare the LL and the NLL approach based on the same input data sets. This is particularly important at small x [7].

To study the differences between various parametrizations we limit ourselves to comparisons between predictions of the representative sets of parametrizations **P1–P5** for gluons and for the F_2^{em} structure function at $Q^2 = 10, 10^2$ and 10^4 GeV^2.

4.1 Gluons

In Fig. 1 we present the predictions for the gluon distributions. The differences in input shape can be inferred from Fig. 1a, where all the gluon distributions are presented after evolution from the starting Q_0^2 – different for different parametrizations, to $Q^2 = 10$ GeV^2 (starting value for DFLM). At this value of $Q^2 = 10$ GeV^2, one can discern the three distinct groups of input shapes: a relatively flat shape for the standard approach ($xG(x) \sim$ const as $x \to 0$), a slightly steeper behaviour for the MT parametrization ($xG(x) \sim x^{-0.15rel}$ as $x \to 0$), and a much steeper $x^{-1/2}$ distribution introduced by HMRS. It is interesting to note that already at 10 GeV^2 one can observe that the NLL distribution lies below the LL one, as expected [7]. The total spread observed at $x = 10^{-4}$, at this low Q^2 value, is as large as a factor of 10.

All the parametrizations become steeper and steeper as Q^2 rises (see Fig. 1b and 1c for Q^2 of 10^2 and 10^4 GeV^2, respectively), but the overall spread diminishes by a factor of 5, and the $x^{-1/2}$ approach, although still the steepest, is not so much different from the others.

It is important to notice that in the range of HERA energies, because of kinematic limitations, the relevant range of Q^2 for small $x < 10^{-2}$ remains limited to $Q^2 \sim 100$ GeV^2.

4.2 The F_2^{em} structure function

We have also studied the predictions of our chosen set of parametrizations for the measurable quantity F_2^{em}. Special attention is paid to the low x region, populated by sea quarks, where the differences are much greater than for x close to 1, although, as will be shown later, even there differences subsist. The results are presented in Fig. 2. As in the case of the gluons, we

observe a large spread, which, as opposed to gluons, remains almost the same over the whole Q^2 region, and is of the order of 4 to 3 for Q^2 from 10 GeV^2 to 10^4 GeV^2.

This comparison shows that the NLL approach leads to a much faster growth of the structure function at low x than for the LL approach (as has been pointed out by Tung [7]). The bigger the Q^2, the smaller the difference. The HMRS(E-) parametrization based on gluons $xG(x) \sim x^{-1/2}$ is as steep as the NLL MT approach, although smaller in value at $Q^2 = 10$ GeV^2, and overshoots the NLL prediction of MT at $Q^2 = 10$ GeV^2 at very low x, becoming much steeper.

Fig. 3. shows the dependence of the predicted F_2^{em} on Q^2 for the x values 10^{-1}, 10^{-2}, 10^{-3}, and 10^{-4}. The spread of the predictions grows as x becomes smaller, and for a given x value it is constant with Q^2. As noted above, one can see that for small x values the LL and the NLL predictions get closer as Q^2 increases.

5 Comparison of F_2^{em} with data

In the comparison of F_2^{em} determined experimentally and the predictions, we have used only the newest lepton–hadron data [9] of SLAC [10], BCDMS [11] and EMC [12]. There seems to be a general consensus now as to how the data should be presented [9]. It has been shown that if the BCDMS data are renormalized by .975 (allowed by systematic uncertainties on the overall normalization) and the EMC data by 1.075 (same remark as for BCDMS), then there is agreement between those three latest data sets on F_2^{em}. This is how they have been treated in our comparison (see Fig. 4). The range of x relevant for the comparison is from 0.07 up to 0.75. The SLAC data, although not used for any of the parametrizations, extended down to $Q^2 = 0.5$ GeV^2, have been included in Fig. 4 for completeness.

The parametrizations that we chose to study can be classified in two groups. The MT and HMRS parametrizations differ from the others by that they have used among other data those new F_2^{em} measurements, although the treatment of the overall normalization is not the same. All the older parametrizations were obtained with data on structure functions which, compared to the newest results both from lepton–hadron and ν–hadron, suffer from an apparent lower normalization. It is thus not surprising that in the comparison presented in Fig. 4 they tend to lie below the data, especially for $x < 0.4$, as opposed to the new parametrizations which reproduce those data pretty well. Unfortunately the parametrizations cannot be corrected easily to account for the normalization problem. The spread between the predictions remains even at high Q^2, it is almost constant for $x < 0.4$, except for the lowest value of x, where they seem to converge at least in the Q^2 range presented, while at large x, where the quality of the data does not strongly constrain the fits, the spread becomes larger with Q^2. It seems that, due to the improvement in the quality of the data, the latest parametrizations account better for the experimental results on F_2^{em}, and thus the calculations based on the older parametrizations should be taken with caution. However, it should be kept in mind that a good description of F_2^{em} in the range available, does not guarantee a unique description of parton densities.

6 Conclusions

We have performed a comparative study of the most popular parton parametrizations for the proton. We have found that at the x range accessible today in deep inelastic scattering, $0.07 < x < 0.75$, the spread in the predictions for the F_2^{em} structure function of the proton, which is of the order of $\sim 20 - 40\%$, is almost independent of x and Q^2 in the range of $10 < Q^2 < 10^5$ GeV^2.

In the small x region, $10^{-2} < x < 10^{-4}$, there is a very large variation in the predictions for the gluon and the F_2^{em} structure function distributions in the Q^2 range of 10 to 10^4 GeV^2. The fact that the spread of F_2^{em} at lower values is less than for the gluons, reflects the fact that at those Q^2, F_2^{em} gets still substantial contributions from valence quarks, while at higher Q^2 the sea dominates, and the spread of F_2^{em} is directly related to the spread in gluons. Measurements in the HERA kinematic region could be a tool to distinguish between some of the different parametrizations.

Acknowledgements

We would like to thank E.Levin from Leningrad Nuclear Physics Institute for many useful discussions and suggestions. We are indebted to G.Martinelli, J.Stirling and Wu-Ki Tung for making their programs available to us. Two of us (KCh and MK) acknowledge the support of the Polish Government Research Programs CPBP 01.03 and CPBP 01.06 as well as the DESY Directorate for supporting their visit at DESY. One of us (HA) would like to thank the Alexander von Humboldt Stiftung for supporting her stay at DESY. AL would like to thank the Minerva Foundation for the financial support which made his stay at DESY possible.

References

[1] D.Duke, J.Owens: Phys. Rev. D30 (1984) 49.

[2] E.Eichten, I.Hinchliffe, K.Lane, Ch.Quigg: Rev. Mod. Phys. 56 (1984) 579; revised ibid. 58 (1986) 1047.

[3] P.Harriman, A.Martin, R.Roberts, J.Stirling: RAL-90-007, RAL-90-018.

[4] M.Diemoz, F.Ferroni, E.Longo, G.Martinelli: Z. Phys. C39 (1988) 21.

[5] J.Morfin, Wu-Ki Tung: Fermilab-Pub-90/74.

[6] K.Charchula, M.Krawczyk, H.Abramowicz, A.Levy: DESY-90-019.

[7] Wu-Ki Tung: Nucl. Phys. B315 (1989) 378; Wu-Ki Tung et al.: ANL-HEP-CP-89-01.

[8] WA70 Coll., M.Bonesini et al.: Z. Phys. C38 (1988) 371.

[9] J.Feltesse: DPhPE-89-20, to appear in Proceedings of the XIV Int.Symposium on Lepton and Photon Interactions, Stanford, August 6-12, 1989.

[10] SLAC Coll., L.W. Whitlow et al.: SLAC Pub-5100 (1989); L.W.Whitlow: SLAC-Report-357 (1990).

[11] BCDMS Coll., A.Benvenuti et al.: Phys. Lett. 223B (1989) 485.

[12] EMC Coll., J.Aubert et al.: Nucl. Phys. B259 (1985) 189.

Table 1: Compilation of some of the existing parametrizations for the gluon input distribution.

Parametrization		$xG(x,Q_0^2)$	Type	$\Lambda(GeV)$	$Q_0^2(GeV^2)$
P1	DO(1)	$1.56(1+9x)(1-x)^6$	LL	0.2	4
	DO(2)	$0.879(1+9x)(1-x)^4$	LL	0.4	4
P2	EHLQ(1)	$(2.62+9.17x)(1-x)^{5.9}$	LL	0.2	5
	EHLQ(2)	$(1.75+15.57x)(1-x)^{6.03}$	LL	0.29	5
P3	HMRS(B)	$2.855(1-x)^{5.1}$	NLL	0.19	4
	HMRS(E)	$2.622(1-x)^{4.4}$	NLL	0.1	4
	HMRS(E+)	$4.585x^{1/2}(1+17x)(1-x)^8$	NLL	0.1	4
	HMRS(E−)	$0.357x^{-1/2}(1+11x)(1-x)^5$	NLL	0.1	4
P4	DFLM(1)[1]	$\sim x^{-0.13}$ for $10^{-4} < x < 10^{-2}$	LL	0.2	10
	DFLM(2)	same as LL	NLL	0.3	10
P5	MT(SL)	$e^{1.52}x^{-0.25}(1-x)^{7.01}\ln^{-0.79}(1+\frac{1}{x})$ $[\sim x^{-0.17}$ for $10^{-4} < x < 10^{-2}]$	LL	0.144	4
	MT(S)	$e^{1.88}x^{-0.33}(1-x)^{7.52}\ln^{-1.34}(1+\frac{1}{x})$ $[\sim x^{-0.15}$ for $10^{-4} < x < 10^{-2}]$	NLL	0.212	4

1 The original result is in the form of the gluon Mellin transform. This is only a good description of the input gluon distribution at small x.

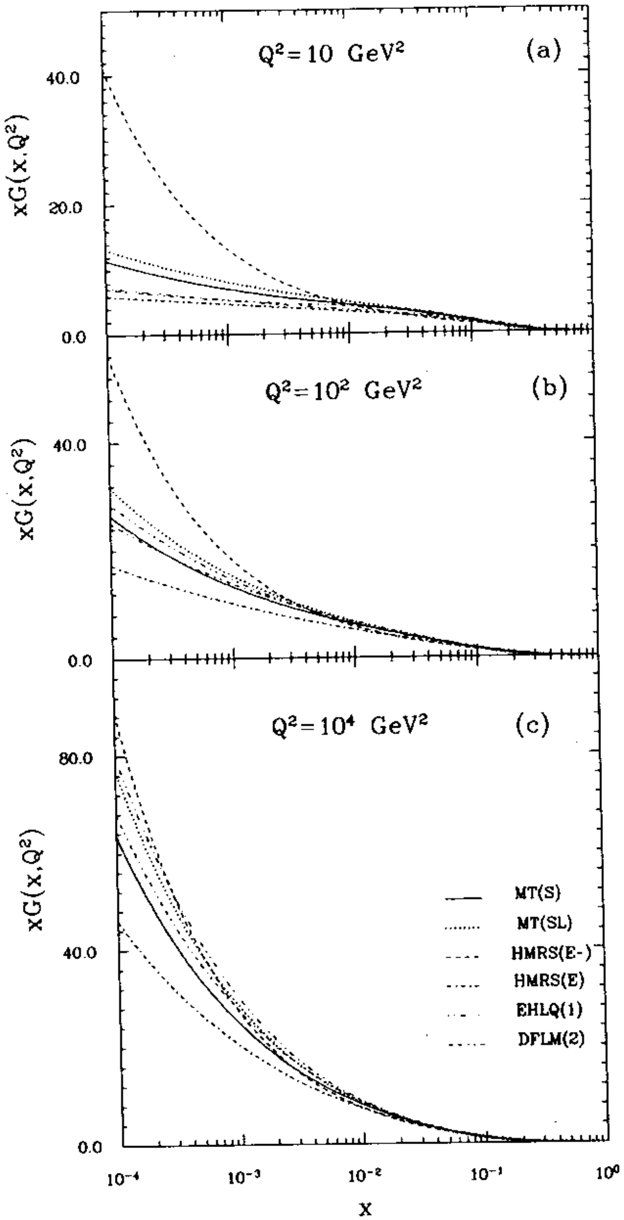

Figure 1: Comparison of the gluon distribution obtained in various parametrizations for (a) $Q^2 = 10$ GeV^2, (b) $Q^2 = 10^2$ GeV^2, and (c) $Q^2 = 10^4$ GeV^2.

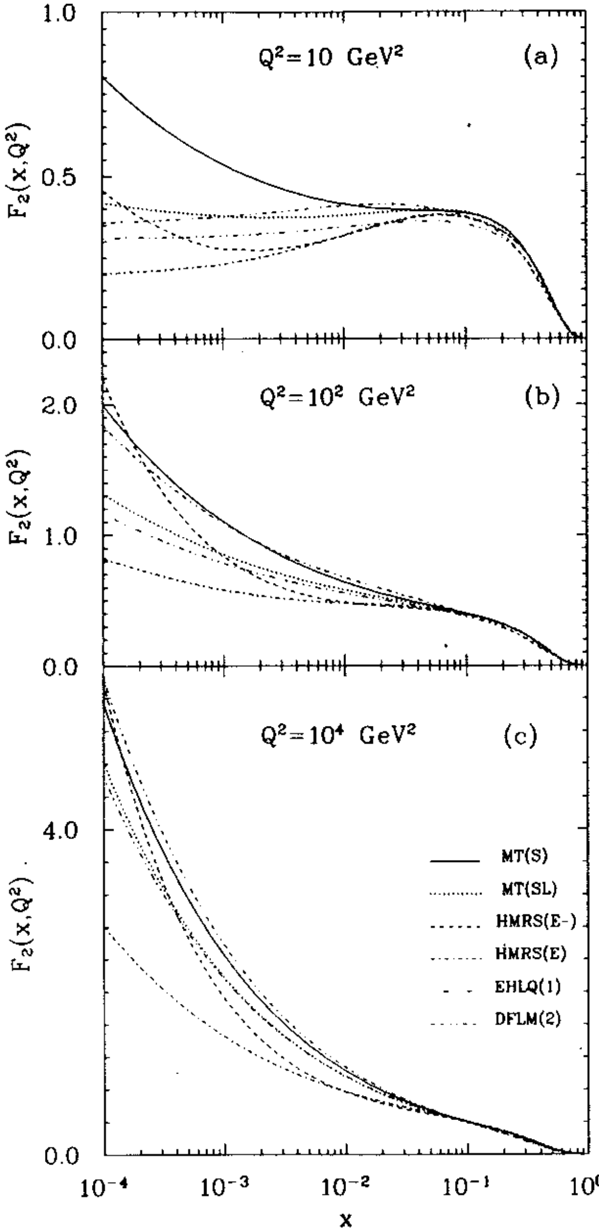

Figure 2: Comparison of the F_2^{em} structure function obtained in various parametrizations for (a) $Q^2 = 10$ GeV^2, (b) $Q^2 = 10^2$ GeV^2, and (c) $Q^2 = 10^4$ GeV^2.

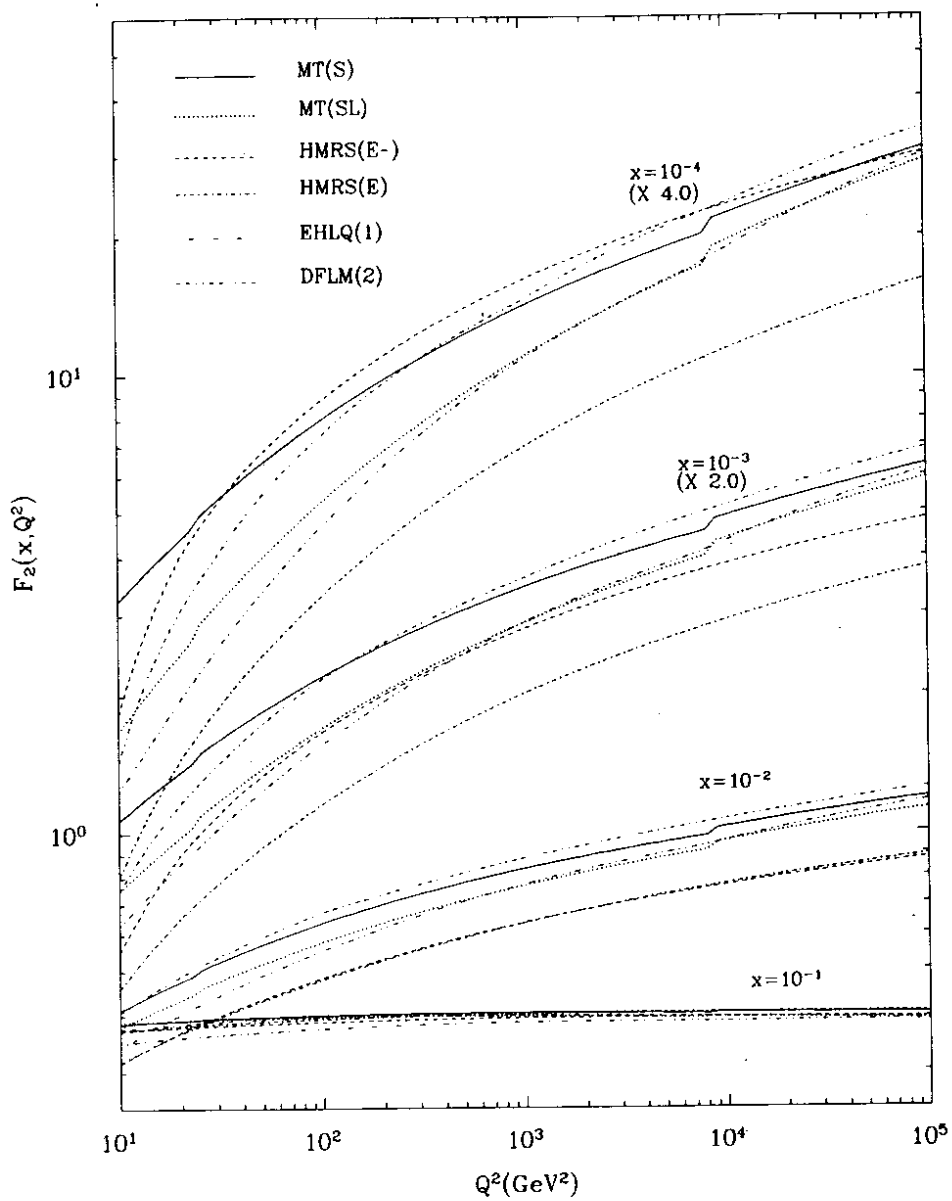

Figure 3: Comparison of the F_2^{em} structure function obtained in various parametrizations for $x = 10^{-4}$, $x = 10^{-3}$, $x = 10^{-2}$, and $x = 10^{-1}$.

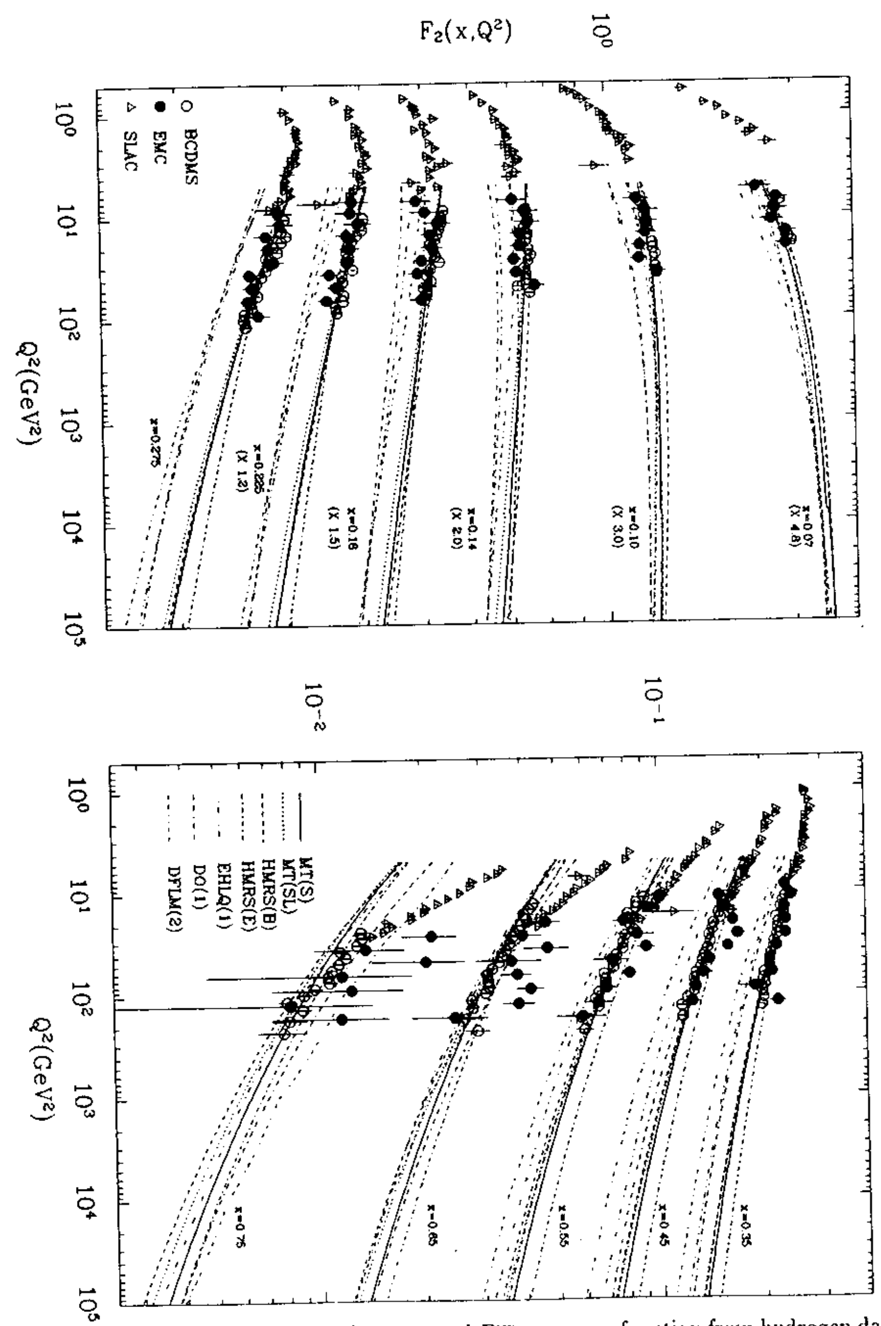

Figure 4: Comparison between the measured F_2^{em} structure function from hydrogen data (see text) and the predictions of various parametrizations for several values of x.