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Abstract

One of the most fundamental questions in field theory is whether QED has a non-trivial continuum limit. In this talk I shall review recent lattice work which addresses this problem. The key-issue is the determination of the renormalization group flow and the derivation of the Callan-Symanzik β-function. The β-function does not show an ultra-violet stable zero, but is consistent with the prediction of renormalized perturbation theory. This indicates triviality. The question to what extent QED is a valid low-energy theory is raised. Furthermore, the anomalous dimension and the critical exponents of the fermion mass operator are examined.

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1 Introduction

The question whether interacting field theories which are not asymptotically free exist in four dimensions is of great interest for a number of reasons. For the ϕ^4 theory the answer is almost certainly no [1]: it appears that the renormalized coupling goes to zero in the limit of infinite cut-off [2]. It has been speculated that QED suffers the same fate. Indeed, the Callan-Symanzik β-function is positive for weak couplings. If this continues to hold for strong couplings as well, the cut-off can only be taken to infinity if the renormalized charge is zero. Hence, the cut-off free theory is non-interacting. But it is equally well possible that the β-function has a second zero in the strong coupling regime, which would correspond to an ultra-violet stable fixed point and lead to an interacting theory. The only way to find out which of the two possibilities is realized is to solve the theory non-perturbatively.

Progress in this direction has been made by two methods. Analytic studies of a truncated Schwinger-Dyson equation for the fermion propagator indicated that above a critical charge QED undergoes a transition to a phase in which chiral symmetry is broken spontaneously [3]. Though the Schwinger-Dyson equation did not include any vacuum polarization effects, it has been suggested that the critical charge should be regarded as a fixed point of the theory. It has been argued furthermore that the β-function is negative in the broken phase [4]. But this so-called β-function does not coincide with any of the text-book β-functions. Lattice investigations of non-compact QED using staggered fermions have confirmed the existence of a second order chiral phase transition [5,6,7]. The critical charge was found to be surprisingly small: $e_c \approx 2.3$. As a first step in trying to understand the nature of the critical point an effort has been made to compute the critical exponents of the transition [7,8]. They came out to be consistent with the predictions of mean field theory. This led us to suggest that the continuum theory is non-interacting. Recent investigations of a coupled set of truncated Schwinger-Dyson equations which include the effects of fermion loops now also find mean field critical exponents [9,10].

In spite of this progress, the true problem continues to be the calculation of the Callan-Symanzik β-function. An analytic study has shown that the renormalized charge e_R cannot exceed the bare charge e in the lattice regularized theory [11]. This implies that in any continuum limit taken at the above mentioned critical point the fine-structure constant satisfies $\alpha_R \equiv e_R^2/4\pi \leq 0.41$. In particular, if the β-function has a second zero, it must be in this range. In view of the fact that renormalized perturbation theory is applicable in this domain, it thus appears doubtful that the β-function does have an ultra-violet stable zero.

In this talk I shall report on the first full investigation of the renormalization of charge in lattice non-compact QED [12]. I start with a discussion of some technical aspects of the calculation in sec. 2. In sec. 3 I present the results for the renormalized charge and the renormalized fermion mass. These results are used to derive the Callan-Symanzik β-function. It turns out that the renormalized charge vanishes at the critical point. In sec. 4 it is shown that QED is a useful low-energy theory only for very small couplings. In sec. 5 the critical exponents of the chiral phase transition are re-examined. Finally, in sec. 6 I present my conclusions.

2 Preliminaries

As a lattice regularized version of QED we take staggered fermions coupled to a non-compact gauge field A_μ. The action is

$$S = S_G + S_F, \tag{2.1}$$

$$S_G = \frac{\beta}{2}\sum_{x,\mu<\nu} F^2_{\mu\nu}(x),\ F_{\mu\nu}(x) = A_\mu(x) + A_\nu(x+\hat\mu) - A_\mu(x+\hat\nu) - A_\nu(x), \tag{2.2}$$

$$S_F = \sum_x \left\{\frac{1}{2}\sum_\mu (-1)^{x_1+\dots+x_{\mu-1}}[\bar\chi_x e^{iA_\mu(x)}\chi_{x+\hat\mu} - \bar\chi_{x+\hat\mu}e^{-iA_\mu(x)}\chi_x] + m\bar\chi_x\chi_x\right\} \tag{2.3}$$

where $\beta = 1/e^2$. The non-compact formulation of lattice QED is supposed to coincide with the continuum theory in the appropriate limit [13]. In contrast to the compact formulation it is free of topological excitations. As far as the fermions are concerned the lattice is divided into elementary hypercubes. Each hypercube accommodates four Dirac fermions, whose components are linear combinations of the Grassman fields χ_x. The action S_F is invariant under translations of two lattice spacings. In the classical continuum limit it describes four degenerate Dirac fermions minimally coupled to A_μ. For finite lattice spacings the theory has a chiral $U(1)\times U(1)$ symmetry at $m = 0$, while the chiral $SU(4)\times SU(4)$ symmetry is only approximate. In eqs. (2.2) and (2.3) the lattice constant has been set equal to one for convenience, and we will do so in the following. The non-compact character of A_μ does not cause any problems in the numerical simulations as long as we consider only gauge invariant quantities. We have chosen periodic boundary conditions for the gauge fields and periodic (anti-periodic) spatial (temporal) boundary conditions for the Grassmann fields.

In what follows the renormalized charge and the renormalized fermion mass m_R will be of central importance. These quantities have never before been calculated on the lattice. Let me therefore explain how this is done.

The renormalized charge and the bare charge are related by $e_R^2 = Z_3 e^2$, where Z_3 is the charge renormalization constant. It is easy to show that

$$Z_3 = \lim_{k\to 0} D(k), \tag{2.4}$$

$$D(k) = \frac{\beta}{3}\sum_{x,\mu<\nu} e^{ikx}\langle F_{\mu\nu}(x)F_{\mu\nu}(0)\rangle. \tag{2.5}$$

Equation (2.5) is, however, not particularly well suited for numerical applications because of strong fluctuations of A_μ. Instead we have used the expression

$$1 - D(k^{(\mu)}) = \sum_x e^{ik^{(\mu)}x}\langle A_\mu(x)j_\mu(0)\rangle, \tag{2.6}$$

which follows from the photon field equation, where j_μ is the electromagnetic current and $k^{(\mu)}$ is any momentum with $k^{(\mu)}_\mu = 0$. The advantage of eq. (2.6) over eq. (2.5) is that it contains only one power of A_μ.

The renormalized fermion mass is determined from the fermion propagator. Since this is not gauge invariant, we have to fix the gauge. It is customary to choose the Landau gauge, which can be implemented exactly for non-compact gauge fields. This does, however, not eliminate all gauge degrees of freedom in the fermion propagator: we may add a multiple of $2\pi/L_\mu$, where L_μ is the extent of the lattice in μ-direction, to $A_\mu(x)$ for all x without changing the action. To do so we shall restrict the average of the gauge fields to $-\pi/L_\mu < \bar A_\mu < \pi/L_\mu$, $\bar A_\mu = \sum_x A_\mu(x)/V$ for all μ, where V is the lattice volume. We compute the propagator

$$G(t) = \sum_{\vec x}\langle\chi(2x+\omega)\bar\chi(0)\rangle, \tag{2.7}$$

where x labels the hypercube, $\omega = (\vec 0, \omega_4)$ with $\omega_4 = 0, 1$ and $t = 2x_4 + \omega_4$. In trying to fit $G(t)$ with the lattice free fermion propagator we realized that $\langle\bar A_\mu\rangle \neq 0$. This is due to our periodic spatial boundary conditions for the Grassmann fields. For example, for a one-dimensional lattice of length $L = 4$ we find the fermion determinant in the presence of a constant background field to be

$$m^4 + m^2 + \frac{1}{8} \mp \frac{1}{8}\cos 4A,\ -\frac{\pi}{4} < A \leq \frac{\pi}{4}. \tag{2.8}$$

The minus (plus) sign stands for periodic (anti-periodic) boundary conditions. Thus for periodic boundary conditions the determinant assumes its maximal value for

$$A = \frac{\pi}{4}, \tag{2.9}$$

while $A = 0$ for anti-periodic boundary conditions. In four dimensions there will be loops that couple spatial and temporal fields, and the determinant may have several local minima, and tunneling between them may occur. In view of this we have fitted $G(t)$ with the lattice fermion propagator in the presence of a background field B_μ:

$$G(2x_4) = \frac{m_R}{L_4}\sum_{p_4} e^{2ip_4x_4}[\sin^2(p_4+B_4) + \sum_{i=1}^{3}\sin^2 B_i + m_R^2]^{-1}, \tag{2.10}$$

$$G(2x_4+1) = -\frac{1}{2m_R}[e^{iB_4}G(2x_4+2) - e^{-iB_4}G(2x_4)], \tag{2.11}$$

$$p_4 = \frac{2\pi n}{L_4},\ n = \pm\frac{1}{2}, \pm\frac{3}{2}, ..., \pm\frac{1}{2}(L_4-1), \tag{2.12}$$

where B_4 and $\sum_i \sin^2 B_i$ are treated as free parameters. It turned out that B_4 and $\sum_i \sin^2 B_i$ agree well with $\langle \bar{A}_4\rangle$ and $\sum_i \sin^2\langle \bar{A}_i\rangle$, respectively, as it should.

In computing Z_3 from eqs. (2.4) and (2.6), we have explored several ways of extrapolating $D(k)$ to $k = 0$. We shall demonstrate elsewhere that the results do not depend on the specific form of the interpolating function. The procedure we find most satisfactory and which we will use in this paper is based on renormalized perturbation theory. Renormalized perturbation theory predicts to one-loop order

$$\frac{1}{e^2 D(k)} = \frac{1}{e_R^2} + \Pi(0,\infty) - \Pi(k, L_\mu), \tag{2.13}$$

where Π derives from the (lattice) vacuum polarization and e_R^2 refers to the infinite lattice. Since we know m_R, the only free parameter in eq. (2.13) is e_R^2. It turns out that the k^2-dependence of $D(k)$ can be very well fitted by eq. (2.13). Moreover, we obtain the same values within errors for e_R^2 on lattices of different sizes. This means that the finite size effects are also accounted for by the perturbative formula.

We have performed calculations on 8^4, $8^3\cdot 16$ and 12^4 lattices at $\beta = 0.16$, 0.17, 0.18, 0.19, 0.20, 0.21, 0.22 and $m = 0.02$, 0.04, 0.09, 0.16. At $\beta = 0.16$ and 0.22 our runs on the 12^4 lattice have not been completed yet. Therefore we will use the numbers obtained on the 8^4 lattice in these cases. For generating the gauge field configurations we have used the (exact) hybrid Monte Carlo algorithm [14]. Details of the performance of the algorithm for QED can be found in ref. [7]. On the 12^4 and $8^3\cdot 16$ lattices we have accumulated $O(300)$ gauge field configurations, each separated by 5-10 trajectories, for each value of β and m, while on the 8^4 lattices our sample consists of $O(100)$ configurations.

3 Renormalization of charge

The evolution of the renormalized charge towards the critical point is described by the renormalization group equation

$$-\Lambda\frac{\partial e_R^2}{\partial\Lambda}\bigg|_{e\ \text{fixed}} = \beta(e_R^2). \tag{3.1}$$

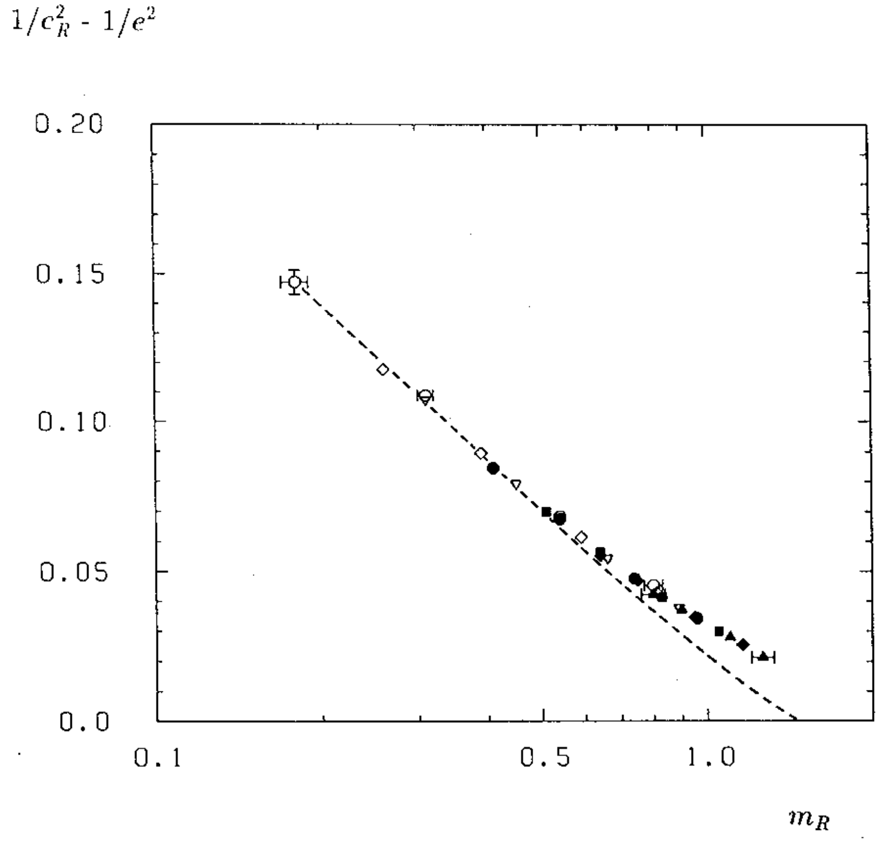

Fig. 1: The relationship between the renormalized charge, the bare charge and the renormalized fermion mass. The symbols refer to the different values of β: $\beta = 0.16$ (▲), 0.17 (◆), 0.18 (■), 0.19 (●), 0.20 (▽), 0.21 (◇), 0.22 (○). The open symbols are for β values above β_c, while the solid symbols are for β values below β_c. The dashed line is the prediction of the one-loop lattice β-function shifted to fit the data point at the smallest value of m_R, which corresponds to $m = 0.02$.

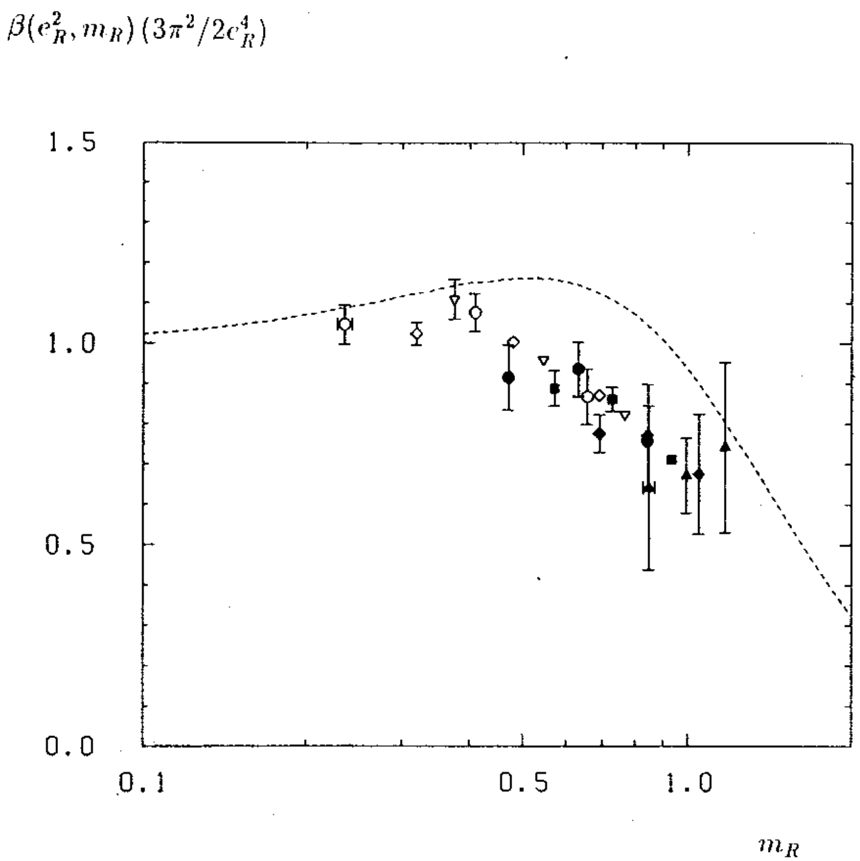

Fig. 2: The β-function. We show $\beta(e_R^2, m_R)\,(3\pi^2/2e_R^4)$ found by differentiating the data in fig. 1. The symbols are the same as in fig. 1. This is compared with the one-loop lattice β-function indicated by the dashed line.

where Λ is the cut-off and $\beta(e_R^2)$ is the Callan-Symanzik β-function. The lattice equivalent of this equation is

$$m_R \frac{\partial e_R^2}{\partial m_R}\bigg|_{e\ \mathrm{fixed}} = \beta(e_R^2, m_R). \tag{3.2}$$

At the critical point $m_R \to 0$ in lattice units. Similarly, one can define a bare β-function by

$$-m_R \frac{\partial e^2}{\partial m_R}\bigg|_{e_R\ \mathrm{fixed}} = \beta_0(e^2, m_R), \tag{3.3}$$

which describes how the bare charge runs as the cut-off goes to infinity. In lowest order perturbation theory both β-functions are equal. If $\beta_0(e^2, 0)$ has a second zero it will be at $e^2 = e_c^2$, while the zero e_R^{*2} of $\beta(e_R^2, 0)$, if any, must satisfy $e_R^{*2} \leq e_c^2$ due to the fact that [11] $Z_3 \leq 1$.

The results of the lattice calculation concerning the Callan-Symanzik β-function are summarized in the first three figures. In fig. 1 I have plotted our data on the renormalized charge and fermion mass in the form $1/e_R^2 - 1/e^2$ versus m_R. The open symbols refer to $\beta > \beta_c$ ($\beta_c = 1/e_c^2$), while the solid symbols refer to $\beta < \beta_c$. For each symbol the data point with the smallest value of m_R corresponds to $m = 0.02$. We find that the data lie on an almost universal curve. The slope of the curve is $-\beta(e_R^2, m_R)/e_R^4$. The most important point to notice is that this slope is negative over the entire range investigated, which results in

$$\beta(e_R^2, m_R) > 0. \tag{3.4}$$

The β-function found by differentiating our data is shown in fig. 2. This is compared with the one-loop β-function on the infinite lattice.[1] At $m_R = 0$ the latter is

$$\beta(e_R^2, 0) = \frac{2e_R^4}{3\pi^2}, \tag{3.5}$$

in agreement with the continuum β-function for four degenerate Dirac fermions (flavors). At small m_R we find that the observed β-function is the same as the one-loop β-function, whereas at larger m_R the observed β-function is slightly smaller than the one-loop result. I believe that this difference is due to higher order effects.

Let us go back to fig. 1 now, where I have compared the data with the integrated one-loop β-function normalized to coincide with the data point at the smallest value of m_R. For $m_R \lesssim 0.5$ we find good agreement between the data and the one-loop

[1] Note that our data for e_R has been corrected for finite size effects and thus refer to the infinite lattice.

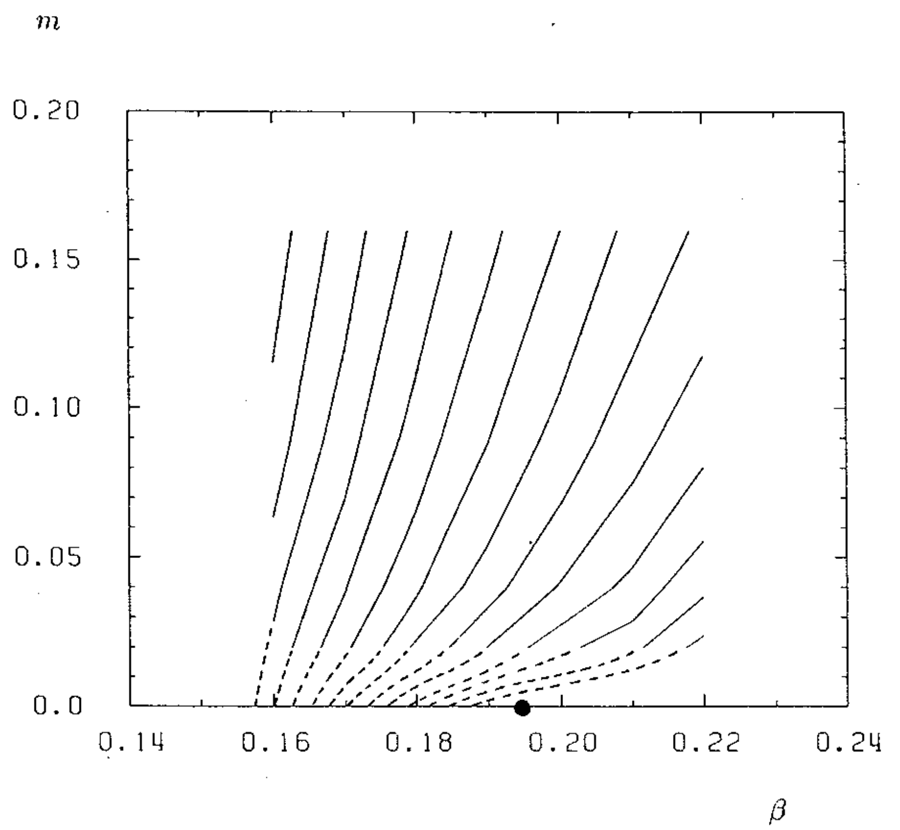

Fig. 3: The renormalization group flow in the region of parameters covered by our calculations. The solid lines are lines of constant renormalized charge, where e_R^2 ranges from $e_R^2 = 2.8$ (lower right-hand corner) to $e_R^2 = 5.4$ (upper left-hand corner) in steps of 0.2. The uncertainty in the position of the flow lines is about 5 per cent of the spacing between lines. The dashed lines arise from integrating the renormalization group equation down to $m = 0$. The solid circle indicates the position of the critical point.

result, which indicates that we can use the one-loop β-function to extrapolate our results to $m_R = 0$. The fact that the data points fall on a universal curve tells us furthermore that the Callan-Symanzik β-function and the bare β-function are approximately equal, i.e.

$$\beta(e_R^2, m_R)/e_R^4 \approx \beta_0(e^2, m_R)/e^4, \tag{3.6}$$

as one would expect from lowest order perturbation theory.

In fig. 3 I show the lines of constant renormalized charge. Since we know e_R only on the grid of points in the (β, m)-plane quoted in sec. 2, we have to interpolate between them. For the interpolation in β and m we have used the formulae $1/e_R^2 = a + b\,\beta$ and $1/e_R^2 = c + d \ln m$, respectively, which is suggested by fig. 1. The uncertainty is about five per cent of the spacing between the trajectories. As m, and so m_R, is decreased the curves tend to smaller values of β on both sides of the critical point. In the symmetric phase, $\beta \geq \beta_c$, m_R vanishes as $m \to 0$, and so we obtain from a positive β-function that $e_R = 0$ in this limit. In particular, $e_R = 0$ at the critical point. In the broken phase, $\beta < \beta_c$, m_R stays finite as $m \to 0$, and so e_R is finite. Therefore all trajectories will end at $m = 0$ in the broken phase. The consistency of our data with the one-loop result at small m_R and the observed universality (fig. 1) for all values of m_R allows us to complete this figure by integrating the renormalization group equation down to $m = 0$. The result is shown by the dotted lines. In the limit $e_R = 0$ the trajectories will approach the line $m = 0, \beta \geq \beta_c$.

For any finite e_R we find a lower limit on m_R:

$$m_R \geq (26 \pm 3) e^{-3\pi^2/2e_R^2}, \tag{3.7}$$

which implies an upper bound on the cut-off. Note that the Landau pole is at

$$m_R = e^{-3\pi^2/2e_R^2}, \tag{3.8}$$

(in first order perturbation theory and for four flavors). Hence, it will never be reached. A consequence of this result is that massless QED does not exist: in the limit of zero (renormalized) fermion mass the electric charge gets totally screened. Coleman and Weinberg [15] did not think of this possibility.

4 On the validity of QED as a cut-off theory

Though being a trivial theory, QED can be a valid description of charged particles and their interactions up to some finite energy scale. We know that this is the case

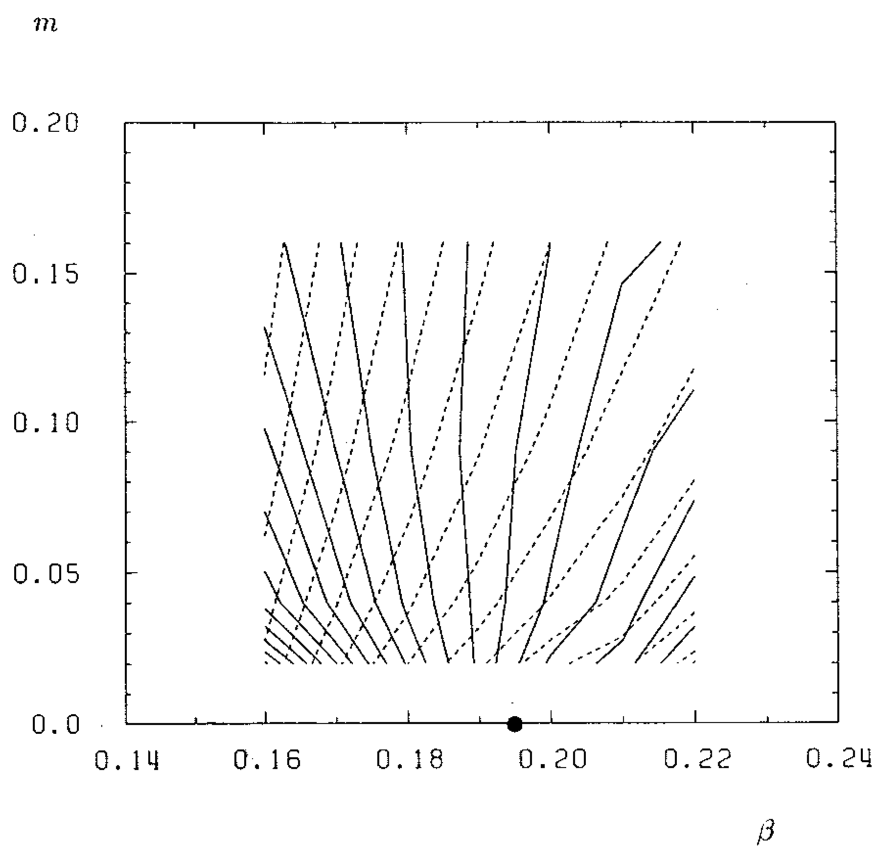

Fig. 4: The renormalization group flow in the region of parameters covered by our calculations. The solid lines are lines of constant m_R/m_{PS} ranging from 0.4 (lower right-hand corner) to 2.1 (lower left-hand corner) in steps of 0.1. The dashed lines are lines of constant renormalized charge as in fig. 3. The uncertainty in the position of the mass-ratio lines is about 15 per cent of the spacing between the lines. The solid circle indicates the position of the critical point.

for $\alpha_R = 1/137$ up to presently available energies. But the situation may be less favorable if the number of charged particles is increased or if α_R is taken to be larger.

In order that QED can be regarded as a useful theory, the low-energy physics must not depend on the cut-off. This requires the existence of lines of constant physics. To find out whether this is the case, one needs to compare the flow of different dimensionless quantities. We have computed the ratio m_R/m_{PS}, where m_{PS} is the pseudoscalar Goldstone boson mass, on our grid of points and interpolated the result to find the lines of constant mass ratios. In fig. 4 these lines are compared with the lines of constant e_R. The two flows are obviously completely different in the parameter range studied. The trajectories of constant mass ratios flow into the critical point in contrast to the lines of constant e_R. In perturbation theory $m_{PS} = m_R f(\alpha_R)$, and the two sets of trajectories would agree. The inconsistency is most striking for $\beta < \beta_c$, where the e_R trajectories move in the direction of lower β, while the mass-ratio trajectories move in the direction of larger β, but it is also clearly present for $\beta > \beta_c$. The correlation length does not have to be very large before the difference between the flows becomes apparent. For example at $e_R^2 = 3.6$ ($\alpha_R = 0.29$) the difference becomes marked when $m_R \lesssim 0.5$ (i.e. at a cut-off, which is only two times as large as the fermion mass). When e_R^2 is smaller, we can reach smaller values of m_R (larger correlation lengths) before this effect is seen. Thus, there are no lines of constant physics in the critical region except possibly for very small values of e_R^2. This contradicts renormalizability: a change in cut-off cannot be compensated for by a change in the bare parameters. It may be possible to restore renormalizability by adding another interaction to the action. If the extended action is renormalizable, it will have true trajectories of constant physics in the larger space of bare parameters. Various authors have suggested adding a four-fermi interaction [16].

5 Anomalous dimensions and critical exponents

Having shown that the photon decouples at the critical point, one may ask whether the fermion and the bound states still interact at this point. One can imagine, for example, that the theory generates a four-fermi interaction. The anomalous dimension of the fermion mass operator and the critical exponents of the transition, which I will discuss in this section, provide some information on the nature of the effective continuum theory.

In the lattice notation the anomalous dimension of the fermion mass reads

$$\gamma_m = \frac{m_R}{m} \frac{\partial m}{\partial m_R}\bigg|_{e\ \text{fixed}} - 1. \tag{5.1}$$

Similarly, the anomalous dimension of $\bar{\chi}\chi$ is given by

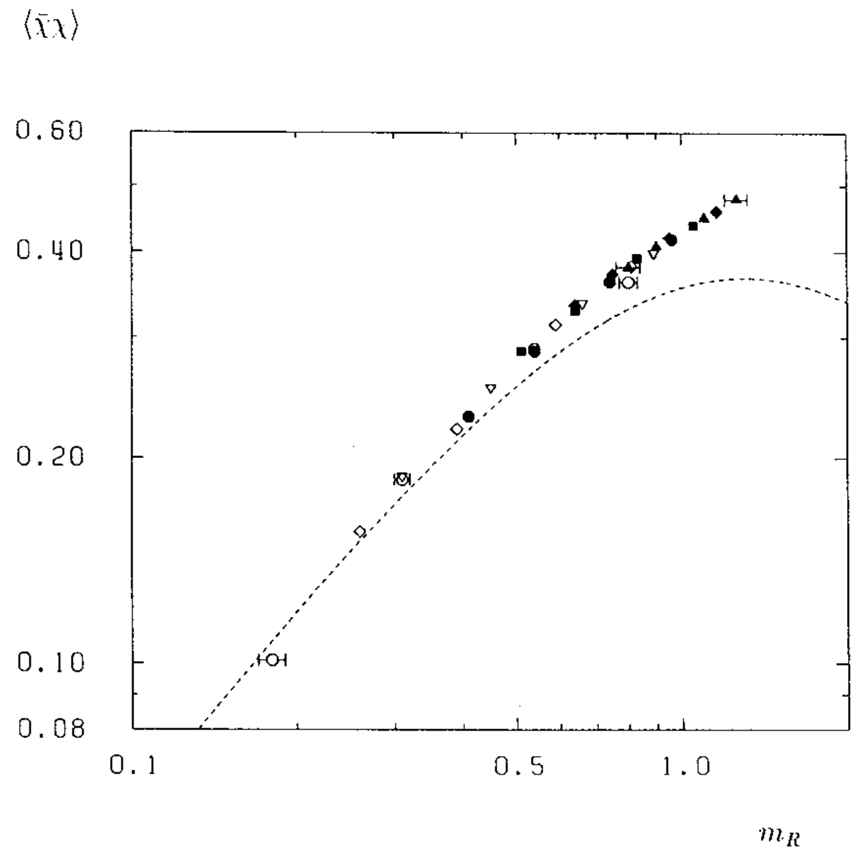

Fig. 5: The chiral condensate against the renormalized fermion mass. The symbols are the same as in fig. 1. The open symbols are in the chirally symmetric phase, while the solid symbols are in the broken phase. The dashed line is the one-loop lattice result, which is included for comparison.

$$\gamma_{\bar\chi\chi} = \frac{m_R}{\langle\bar\chi\chi\rangle}\frac{\partial\langle\bar\chi\chi\rangle}{\partial m_R}\bigg|_{e\ \mathrm{fixed}} - 3. \tag{5.2}$$

At the critical coupling, $\beta = \beta_c$, we expect $\gamma_m + \gamma_{\bar\chi\chi} = 0$. In fig. 5 I have plotted $\langle\bar\chi\chi\rangle$ as a function of m_R. This is compared with the one-loop free field theory result on the infinite lattice, which for small m_R is

$$\langle\bar\chi\chi\rangle = 0.62 m_R + O(m_R^2). \tag{5.3}$$

We find good agreement between the data and this curve for $m_R \lesssim 0.5$ and for all values of β. This indicates that the scaling dimension of $\bar\chi\chi$ is one and that $\gamma_{\bar\chi\chi} = -2$. The anomalous dimension γ_m is expected to go to infinity as $m \to 0$ in the broken phase, because $m_R > 0$ there. At $\beta = \beta_c$ we expect $\gamma_m = 2$, while $\gamma_m = 0$ for $\beta > \beta_c$ and $m = 0$ due to the fact that $\alpha_R = 0$. I have computed γ_m for two values of m by differentiating the data. The result is shown in fig. 6. We find the trend of the data to be in accordance with our expectations.

Miransky has predicted [3] $\langle\bar\chi\chi\rangle \propto m_R^2$ near the phase transition based on the planar ladder model. This leads to $\gamma_m = 1$ – a result, which has received a lot of attention because it would provide the wanted suppression of flavor changing neutral currents in technicolor models. In the full theory this result is definitely not correct as we have seen.

In order to describe the critical behavior associated with the breaking of chiral symmetry, it is conventional to introduce various critical exponents, which characterize the transition. In mean field theory the equation of state is

$$2\kappa\sigma + 4\zeta\sigma^3 - m = 0, \tag{5.4}$$

where $\sigma = \langle\bar\chi\chi\rangle$ and κ, ζ are analytic functions of β: $\kappa = \kappa_1(\beta-\beta_c)+\kappa_2(\beta-\beta_c)^2+\cdots$, $\zeta = \zeta_0 + \zeta_1(\beta-\beta_c) + \zeta_2(\beta-\beta_c)^2 + \cdots$. Equation (5.4) may be generalized to give arbitrary critical exponents $\dot\beta$ and δ:

$$(\delta - \frac{1}{\dot\beta} + 1)\kappa\sigma^{\delta-\frac{1}{\dot\beta}} + (\delta+1)\zeta\sigma^\delta - m = 0. \tag{5.5}$$

(In order to avoid confusion of the coupling β and the critical exponent named by the same letter I have called the latter $\dot\beta$).

At $\beta = \beta_c$

$$\langle\bar\chi\chi\rangle \propto m^{\frac{1}{\delta}}. \tag{5.6}$$

From eqs. (5.1) and (5.2) we deduce

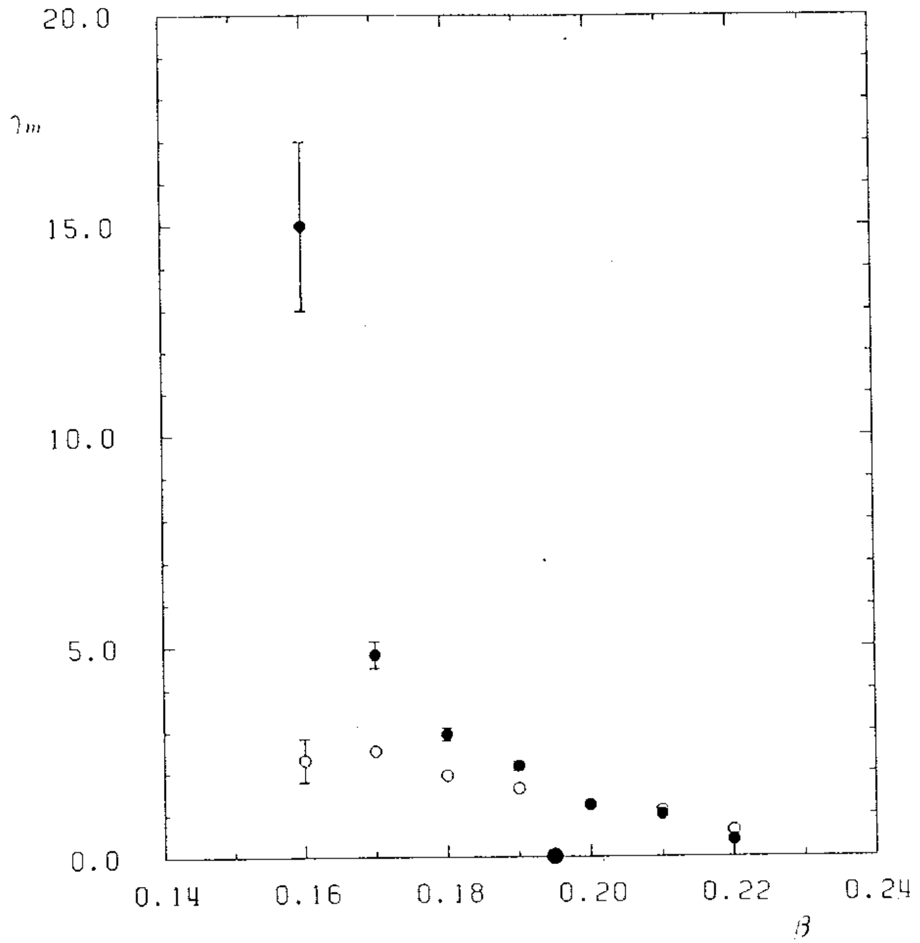

Fig. 6: The γ-function. We show γ_m found by differentiating our data as a function of β for $m = 0.02$ (●) and $m = 0.09$ (○). The solid circle indicates the position of the critical point.

$$\frac{m}{\langle\bar{\chi}\chi\rangle}\frac{\partial\langle\bar{\chi}\chi\rangle}{\partial m}\bigg|_{\beta\ \mathrm{fixed}} = \frac{3+\gamma_{\chi\chi}}{1+\gamma_m}. \tag{5.7}$$

Thus, for $\gamma_m = -\gamma_{\chi\chi} = 2$ we obtain $\delta = 3$, i.e. the mean field theory value, and vice versa.

For an independent and unbiased derivation of the critical exponents I have fitted eq. (5.5) to our data of $\langle\bar{\chi}\chi\rangle$. The result of a linear fit (in the notation of ref. [7]) is shown in the following table:

fit	mean field theory
β_c=0.194(2)	
δ=3.14(18)	3
$\dot{\beta}$=0.49(2)	0.5

The data and the fitted curves are compared in fig. 7. The agreement with mean field theory is very good. This together with the result of the last paragraph suggests that the continuum theory is non-interacting.

An interesting consequence is that the four-fermi interaction

$$\frac{G}{2}[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2] \tag{5.8}$$

will most likely acquire a scaling dimension of two at the critical point. That means it becomes super-renormalizable. Is it possible now by adding a four-fermi interaction to the QED Lagrangian to find a non-trivial continuum limit? In view of our result it seems likely now that $\gamma_m = 2$ on the whole critical line in the (β, G)-plane, indicating that the theory is trivial everywhere on this line.

One could also include the interactions

$$g\bar{\psi}\gamma_\mu\partial_\nu F_{\mu\nu}\psi \tag{5.9}$$

and

$$\frac{H}{2}[(\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi)^2 + (\bar{\psi}i\gamma_5\sigma_{\mu\nu}F_{\mu\nu}\psi)^2]. \tag{5.10}$$

They are expected to have scaling dimensions three and four, respectively, at the critical point because the photon decouples.

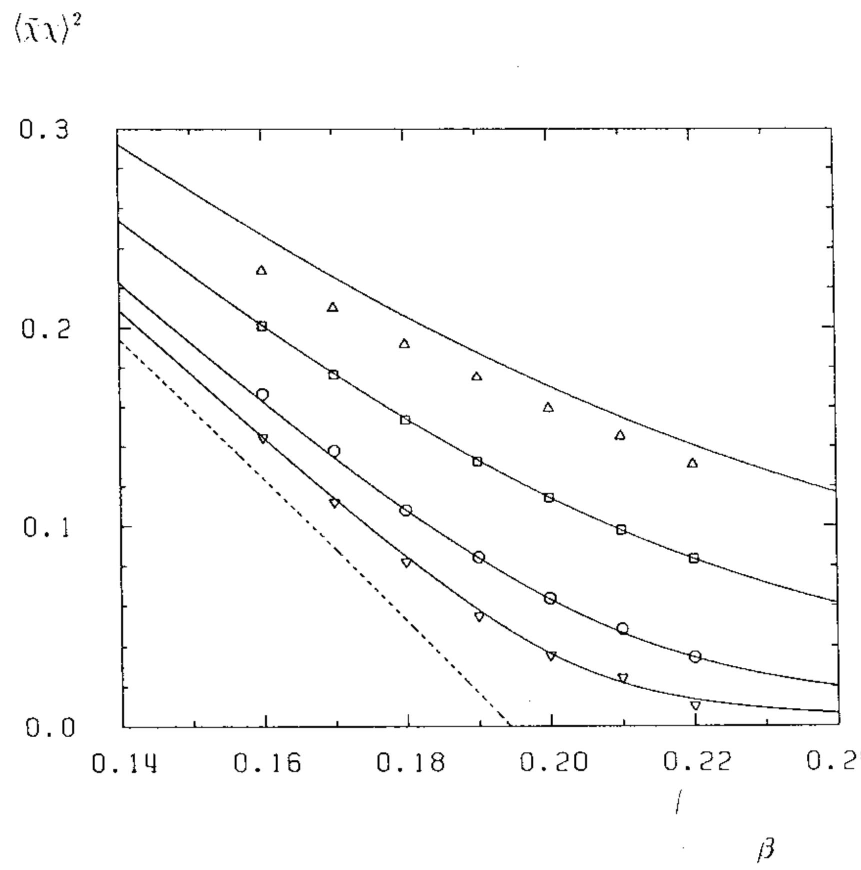

Fig. 7: The chiral condensate against β. We compare the data with a mean field theory fit. The symbols refer to the different masses: $m = 0.02$ (∇), $m = 0.04$ ($\bigcirc$), $m = 0.09$ ($\Box$) and $m = 0.16$ ($\triangle$). The fit did not include the data values at $m = 0.16$. The error bars are smaller than the symbols. The dashed curve is the extrapolation to $m = 0$.

6 Conclusions

The principle conclusions are as follows:

(i) The key-point of the calculation was that we were able to simulate the theory at small values of ϵ_R and m_R, where we find agreement between the measured β-function and that predicted by renormalized perturbation theory. This allowed us to extrapolate to the continuum limit. We find that $\epsilon_R = 0$ at the critical point. A finite charge implies an upper bound on the cut-off.

(ii) We have found no lines of constant physics in the parameter space investigated, i.e. $\alpha_R \geq 0.22$ and $m_R \geq 0.2$. This is a surprise. It means that QED is a valid low-energy theory only for very small values of α_R.

(iii) We have determined the anomalous dimension of the fermion mass operator and the critical exponents of the chiral phase transition. We find good agreement with mean field theory. This indicates that the bound states and the fermion do not interact in the continuum limit.

(iv) A puzzle still is what mechanism causes the spontaneous breakdown of chiral symmetry. Usually this is attributed to the effect of instantons or monopoles which are clearly absent here. Most likely this is due to the strong vector forces which, on the other hand, are absent in QCD. Our data suggests that the main effect of chiral symmetry breaking is simply to give the fermion a mass, which acts in the same way as the mass induced by the bare mass in the symmetric phase.

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