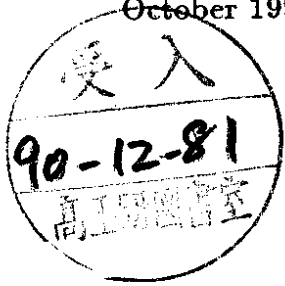


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A note on gluon distribution at small x

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Abstract

We compare the small x behaviour of gluon distribution resulting from the Balitskij-Lipatov equation and from the Monte Carlo approach proposed by Marchesini and Webber. The role of the momentum conservation constraint is discussed.

The forthcoming collider projects (HERA, TEVATRON, SSC) will be able to investigate experimentally the kinematical region of small x ($x < 10^{-2} - 10^{-3}$) which is presently unreachible. Because of the semi-hard nature of the processes in this range of x , the cross sections will be large in contrast to the standard cross sections of hard processes. It is hoped that this region can be treated by standard perturbative QCD techniques and that some new aspects of fundamental theory will be investigated, for example the Regge limit of QCD. On the other hand one can expect new phenomena to appear, like the recombination and shadowing processes of partons within a hadron.

It is of interest to investigate how well the standard methods of perturbative QCD work in the region of small x , say below 10^{-2} [1]. Since this x region will be mainly populated by gluons, we will discuss in the following only various methods used in predicting the behaviour of gluon distribution. We will not touch on the problem of the screening corrections which describe recombination and shadowing phenomena. They are known to depend crucially on the type (flat or singular) of gluon distribution [2]. Once the shape of this distribution is established, the need for those corrections may appear.

Following Kwicinski [1] we investigated the Balitskij-Lipatov equation [3] which is believed to describe properly the region of small x and intermediate Q^2 . In the standard form

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it gives the gluon distribution in the leading logarithmic $1/x$ approximation ($LL(1/x)$) since it sums all leading $\ln(1/x)$ terms. One can also investigate the large Q^2 limit of this equation which leads to the double-leading logarithmic (DLL) approximation. Using this equation it is also possible to obtain the well known leading logarithmic Q^2 ($LL(Q^2)$) approximation for the gluon distribution. This last approach is equivalent to the one based on the Gribov-Lipatov-Altarelli-Parisi equation (GLAP) [4], which is used as the standard description of the parton densities at existing energies. On the other hand by an extension of the original Balitskij-Lipatov equation [5,1] one can go beyond the leading $\ln(1/x)$ approximation. This was done for example in [1] by taking into account some next-to-nonleading $\ln(1/x)$ terms. This approach we call the BLL($1/x$) approximation.

The standard $LL(Q^2)$ approximation neglects various terms proportional to powers of $\ln(1/x)$, namely those which are not accompanied to leading powers of the corresponding $\ln(Q^2)$. These terms may become important in the small x region, especially those which are leading from the point of view of $\ln(1/x)$ contributions. The $LL(1/x)$ approximation of the Balitskij-Lipatov sums all of these terms.

In order to get some insight into physical content of the various approximations we considered them by starting with the same input at some Q_0^2 . Summing large $\ln(Q^2)$ we used also as an independent method the evolution program written by Tung [6] which solves the GLAP equation in the $LL(Q^2)$. It leads to similar results as the $LL(Q^2)$ approximation of the Balitskij-Lipatov equation. Some of our results can be found in [7] and more detailed discussions of all these approximations will be published elsewhere [8].

Here we will limit ourselves to comparison of our results with recent Monte Carlo calculations of Marchesini and Webber [9]. In their approach, which is dedicated to the small x region, only gluons were taken into account. These authors considered the $LL(1/x)$ and DLL approximations in the probabilistic framework formulated by Catani et al. [10]. They calculated gluon distribution in the small x region, down to 10^{-5} , imposing the following constraint on the gluon behaviour

$$\int_0^1 dx x G(x, Q^2) = 0.4 \quad (1)$$

at all Q^2 . The constraint follows from the fact that in the real case besides gluons there are also quarks which carry more than a half of the momentum. The result obtained in [9] is rather surprising. The two approaches, $LL(1/x)$ and DLL, give gluon distributions which are very similar in a whole range of x down to the smallest investigated, i.e. $x \sim 10^{-5}$. This is true both at Q^2 equal to 10^3 and at 10^6 GeV², and for flat and singular input gluon distributions.

Starting with the same flat input as Marchesini and Webber:

$$xG(x, Q_0^2) = 2.4(1-x)^5, \quad (2)$$

where Q_0^2 is equal to 5 GeV², we calculated gluon distribution from the Balitskij-Lipatov equation in various approximations. In order to be able to compare our results with those from Marchesini and Webber paper we switched off the quarks at scale Q_0^2 .

We start with the $LL(1/x)$ and DLL approaches, where quarks are not generated during the evolution process from the gluons. Since quarks are switched off at the input scale they

are totally absent at any Q^2 . We have found that gluon distributions obtained according to the $LL(1/x)$ and DLL approximations, which we applied without constraint (1), start to differ at very small x , and that the $LL(1/x)$ results are always bigger than DLL ones. Nevertheless, these differences between $LL(1/x)$ and DLL are not very big, ~ 50 per cent at smallest x . The gluon distributions obtained by us differ from those obtained by Marchesini and Webber being higher at small x (fig.1). Also in our case the predictions of both approximations show stronger dependence of Q^2 . Imposing however the same constraint as Marchesini-Webber did, we obtained results which are very close to their results from the Monte Carlo program (fig.2). The only difference is that our $LL(1/x)$ predictions lie somewhat above DLA results and in [9] the relation is opposite. But this effect is small and we will disregard this small deviation in further considerations.

This comparison shows that the two methods, one based on the analytic solution of the Balitskij-Lipatov equation and the other employing the Monte Carlo approach, lead to similar results. However, this may also suggest that the constraint of the type (1) may influence the predictions for the gluon distribution very much. Indeed, in the case where our distributions were free from the demand that gluons should always carry 0.4 of the proton momentum, they carried much more, as can be seen in table 1.

Continuing our discussion on the content of different approximations we now study the remaining two options of the Balitskij-Lipatov equation: $LL(Q^2)$ and $BLL(1/x)$. Both of them include terms which are beyond leading $\ln(1/x)$. In this case it is a matter of consistency to take into account besides singular, $\sim 1/(N-1)$, also less singular terms in the anomalous dimension for the gluon, i.e. in the γ_N^{gg} , and at the same time the dominating terms in the anomalous dimensions involving quarks, γ_N^{qq} and γ_N^{gq} (see also [1]). This corresponds to the generation of quarks during the evolution of the gluon density, which then via the gluon bremsstrahlung influences the final gluon distribution. This effect is much smaller than this due to the constant term in the γ_N^{gg} but can not be neglected.

Inclusion of more terms, i.e. going beyond leading $\ln(1/x)$ in the Balitskij-Lipatov equation, seems to tame rapid growth of the gluon distribution at small x seen in the $LL(1/x)$ or DLL predictions (see fig.3). Comparing now the $BLL(1/x)$ with the $LL(Q^2)$ results we see (fig.3) that these two predictions are very similar to each other but differ very much from the results obtained using $LL(1/x)$ or DLL approximation. This means that neither $LL(1/x)$ nor DLL approach is very realistic in the kinematical region which we consider (see also [1]) and that the $BLL(1/x)$ approximation taking into account the next-to-leading corrections in $\ln(1/x)$ reconstructs partly the leading logarithms of Q^2 . Now if one looks into the momenta carried by gluons according to the $BLL(1/x)$ or $LL(Q^2)$ approximations (see table 1) one finds that they are close to 0.4, independently on the Q^2 . Therefore it is not surprising that as a result non-constrained predictions for the gluon distribution from $BLL(1/x)$ and $LL(Q^2)$ and the constrained ones of $LL(1/x)$ and DLL, coincide in the $xG(x, Q^2)$ vs x plot (fig.4). Note that the Marchesini-Webber curves will go together with the constrained $LL(1/x)$ or DLL predictions.

One can ask why so different approaches are giving similar predictions for the gluon distributions at small x . In particular the comparison of our non-constrained $LL(1/x)$ and $BLL(1/x)$ results shows big difference in the behaviour of gluon density, whereas the constrained $LL(1/x)$ and $BLL(1/x)$ are very similar. It seems that the implementation of the

constraint on the momentum (eq.(1)) is equivalent to the introduction of some important $\ln(Q^2)$ terms which are crucial in $LL(Q^2)$ or $BLL(1/x)$ approach. And in fact this is the case. Let us write the integral (1) in a general form

$$\int dx x^{N-1} G(x, Q^2) \sim (Q^2/Q_0^2)^{\gamma_N}, \quad (3)$$

where γ_N is an N th anomalous dimension for the gluon distribution. The momentum conservation integral (1) corresponds to $N=2$. Similarity between the $LL(1/x)$ and the DLL approximation in the range of x considered by us suggests that the anomalous dimension which governs the small x behaviour is in both approaches given by

$$\gamma_N = \gamma_N^{gg} \sim \alpha_s/(N-1). \quad (4)$$

This is true exactly only for the DLL case whereas for the $LL(1/x)$ approximation, in principle, the whole series in $\alpha_s/(N-1)$ should appear. Now going to the $BLL(1/x)$ approximation we took into account also other terms in the anomalous dimension. This leads to the value of the anomalous dimension for $N=2$ practically being equal to zero and in consequence the fraction of the momentum carried by gluons is constant. On the other hand the constraint (1) imposed on the predictions of the original $LL(1/x)$ approximation, in which the fraction of momentum carried by gluons is much higher than 0.4 (see table 1) introduces effectively the Q^2 dependent contributions.

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Figure captions

- fig. 1 Gluon distributions obtained in the LL(1/x) (dashed) and DLL (dotted) approximations of the Balitskij-Lipatov equation compared with the corresponding results obtained by Marchesini and Webber[9] (solid and dotted-dashed lines, respectively) at $Q^2 = 10^3$ and 10^5 GeV². Also the input distribution (eq.2) at $Q_0^2=5\text{GeV}^2$ is shown.
- fig. 2 Gluon distributions obtained from the Balitskij-Lipatov equation in the LL(1/x) and DLL approximations at $Q^2 = 10^3$ and 10^5 GeV² in two cases : without constraint from eq.2 (as in fig.1) —LL(1/x) (dashed), DLL (dotted) and with constraint (eq.2) —LL(1/x) (solid), DLL (dotted-dashed). Input as in fig.1.
- fig. 3 Comparison of LL(1/x) (dashed), DLL(dotted), BLL(1/x) (solid) and LL(Q²) (dotted-dashed) predictions of the Balitskij-Lipatov equation for gluon distribution at $Q^2 = 10^3$ and 10^5 GeV². The common input as in fig.1.
- fig. 4 Comparison of the constrained LL(1/x) (dashed) and DLL(dotted) predictions of the Balitskij-Lipatov equation for gluon distribution with BLL(1/x) (solid) and LL(Q²) (dotted-dashed) results at $Q^2 = 10^3$ and 10^5 GeV². The common input as in fig.1.

Table 1: Momentum fraction carried by gluons in various approximations.

Type of approximation	$\int dx x G(x, Q^2)$		
	$Q^2 = 5 \text{ GeV}^2$	$Q^2 = 10^3 \text{ GeV}^2$	$Q^2 = 10^5 \text{ GeV}^2$
LL(1/x)	0.40	1.17	1.95
DLL	0.40	1.14	1.91
BLL(1/x)	0.40	0.40	0.44
LL(Q ²)	0.40	0.38	0.39

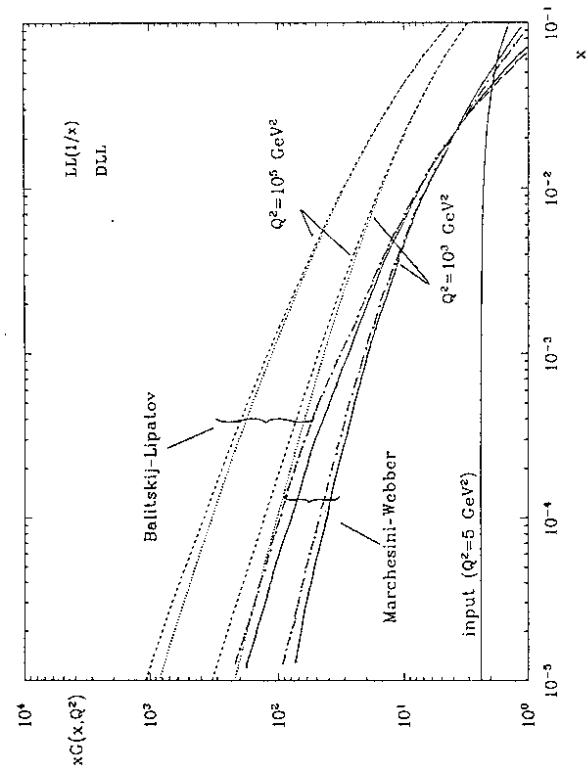


Fig.1

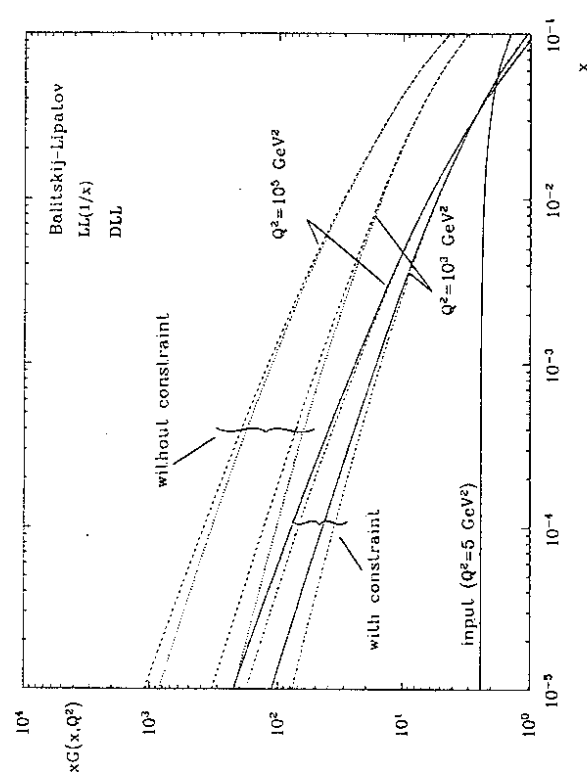


Fig.2

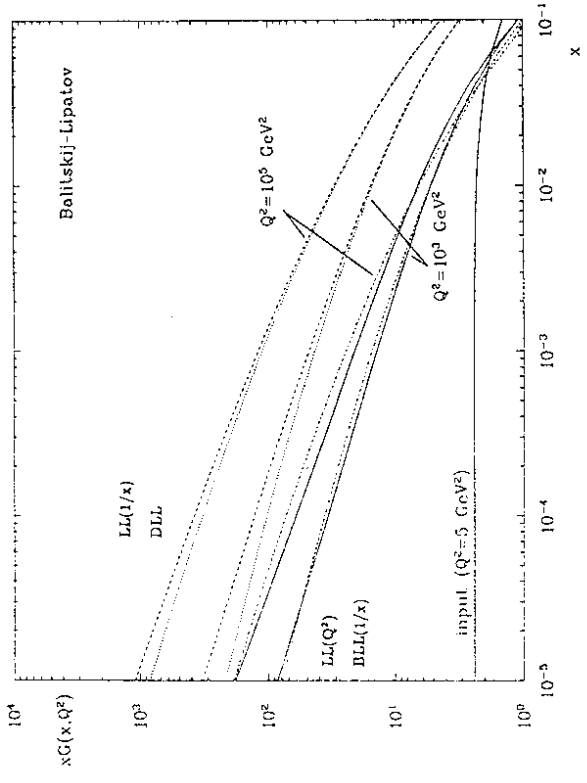


Fig.3

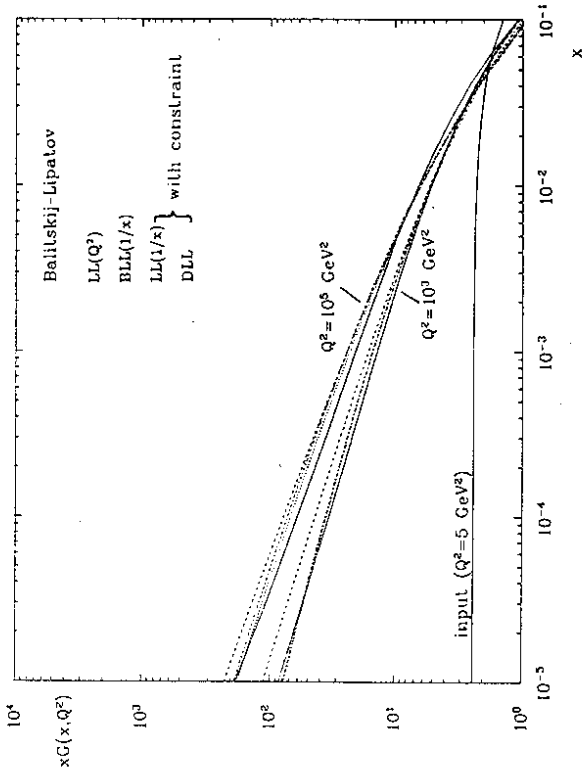


Fig.4