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On the problem of scalar mesons in QCD

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Abstract
The effective lagrangian is derived by the conformal bosonization method to describe the low-lying scalar meson nonet in the QCD low-energy region. The mass spectrum of scalar mesons lies in the 1 GeV region. The values of the mass and decay width for isodoublet meson support the existence of the light $\kappa(850)$. The two-photon widths of f_0 and a_0 mesons are in good agreement with the experimental data.

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1. The main features of the quantum chromodynamics (QCD) at low energies are the spontaneous breaking of the chiral symmetry via Namby-Jona-Lasinio mechanism and breaking of the conformal symmetry so that the divergence of the dilatation current

$$\partial_\mu D^\mu = \frac{\beta(g)}{4g}(G_{\mu\nu})^2 + \sum_i (1+\gamma_i) m_i \bar{\psi}\psi,$$

which is valid in all orders of the perturbation theory, does not vanish [1]. The QCD low-energy region is governed by the quark fluctuations leading to a formation of the non-zero values of the quark $<\bar{\psi}\psi>$and gluon $<\frac{\alpha_s}{\pi}G^2_{\mu\nu}>$ condensates. In the case of large numbers of colors N_c one can consider QCD as a effective theory of mesons only [2]. Thus the main goal of the low-energy hadron physics is to develop the effective action for the low-lying mesons and express the masses and coupling constants in terms of the QCD parameters only: quark and gluon condensates, current quark masses and numbers of colors.

The structure of the effective action for the pseudoscalar mesons is well-established in various approaches: the large N_c approximation [3], the chiral quark loop model [4], the Nambu-Jona-Lasinio model [5], the bosonization method [6] and the instanton vacuum model [7].The scalar quarkonium meson lagrangians were considered in papers [8,9,10] on the basis of the QCD scale anomaly. The scalar-dilaton plays an essential role in the formation of the baryon as a soliton [35].

The scalar mesons are some of the interesting and puzzling mesons. From the theoretical point of view the most interesting are the flavor singlet mesons having the same quantum numbers as vacuum. Thus we have a direct test for the QCD to look for the dilatons arising due to the anomalous behaviour of the theory under scale transformations. In the pure gauge Yang-Mills theory the dilaton is a bounded state of the gluons - glueball. The lattice calculations in a pure gauge theory estimate the mass of the glueball in the range of 1.2 - 1.6 GeV [11]. In QCD the dilaton is a mixture of a glueball and isoscalar scalar meson-quarkonium. There is strong believe that the $f_0(1590)$ meson is a real condidate for the glueball state [12].

The low-lying scalar meson nonet (Table 1.) reveals a long standing puzzle for the $\bar{q}q$ assignment in the conventional quark model. The main problems are:

1. The isospin splitting in the nonet is much higher than one can expect from other low-lying multiplets. There is a strong violation of the $SU(6)$ relations for isospin splitting , which predicts for the mass of the isodoublet κ meson : $m_\kappa \approx 800 MeV$ [14]. There is a suggestion [15], that $\kappa(800)$ meson is masked by vector $K^*(895)$ meson having the same isospin $I = \frac{1}{2}$ and the same dominant decay mode into $K\pi$.
2. The conventional schemes predict broad ε meson having the $\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}$ quark content with the mass $m_\varepsilon \approx 600-900 MeV$ and decay width $\Gamma_\varepsilon \geq 300 MeV$ [32](and References therein). The analysis of the $\pi-\pi$ scattering data for the various final states on the base of the unitarity [16] indicates three poles as the isoscalar mesons in 1 GeV region, which are treated as broad $\varepsilon(910)$: $E_\varepsilon = 910 - 350 \cdot i$, $S_2(998)$: $E_{S_2} = 988 MeV$ with $\bar{s}s$ quark content and $S_1(991)$: $E_{S_1} = 991 - 21 \cdot i$ as a glueball state. Such low-lying glueball state suggests a large mixing with quarkonium [17].
3. The $\bar{q}q$ constituent models predict too large two- photon widths $f_0 \to \gamma\gamma$ and $a_0 \to \gamma\gamma$ [18,19,20] as compared with experimental data (Table 2.). The suggestion arised that f_0 and a_0 are not $\bar{q}q$ mesons. It was proposed that the f_0 and a_0 can be either four quark state $\bar{q}^2q^2$ [21] or $\bar{K}K$ molecular state [22], providing the total widths being a magnitude less than in the non-relativistic $\bar{q}q$ models. The hadronic and two-photonic widths of the scalar isovector meson $a_0(980)$ were re-examined [23] in the framework of the QCD duality sum rules for the $\bar{q}q$ and $\bar{q}^2q^2$ assignments, giving the advantage to the $\bar{q}q$ interpretation of a_0 meson. The recent studies in Quark Confined Model [24] and Conformal Bosonization Model [10] show good agreement with experiment whithin $\bar{q}q$ scheme.

In this paper we present the extention of the joint chiral and conformal bosonization method [9,10] to the case of three flavor scale transformations.

2. We consider a quark field interacting with a gluon field G_μ and the external colour singlet vector V_μ, axial A_μ and scalar S fields. The Lagrangian of this system is

$$L_\psi = \bar{\psi}\left(i\gamma^\mu\left[\partial_\mu + V_\mu + \gamma_5 A_\mu + G_\mu\right] - S\right)\psi \equiv \bar{\psi}\hat{\mathbf{D}}\psi \qquad (1)$$

where the external fields $V_\mu = V^a_\mu T^a, \ldots$ take the value in the flavor SU(3) algebra. The gauge fields $G_\mu = G^i_\mu t_i$ take value in the colour group. T_a and t_i are antihermitian generators. The external scalar field $S(x) = m_q + \hat{S}(x)$, includes the quark mass matrix $m_q = diag(m_u\ , m_d\ , m_s\ )$, where $m_u\ , m_d$

and m_s are the current quark masses of the *up* , *down* and *strange* quarks correspondingly.

At the quantum level the generating functional for vacuum Green functions is given by

$$Z(V,A,S) = \int [DG]\, exp(iW_{YM}) \cdot Z_\psi\,(V,A,S,G) \qquad (2)$$

$$Z_\psi\,(V,A,S,G) = \int \left[D\bar{\psi}\right][D\psi] \exp\left(i \int d^4x L_\psi(x)\right). \qquad (3)$$

Here W_{YM} is the Yang-Mills action $W_{YM} = \frac{1}{2g^2}\int d^4x\; tr_c G^2_{\mu\nu}$ and $[DG]$ is a measure with gauge-fixing terms and Faddeev-Popov ghosts. An integration over quarks in (3) is restricted to a low-energy **L** . By its definition [6] , the low-energy region is dominated by non-perturbative quark fluctuations violating the chiral symmetry and leading to the formation of the quark condensate. One can define **L** in Euclidean space in gauge invariant manner by two spectral parameters Λ and M in terms of the eigenvalues K of the Dirac operator $\hat{\mathbf{D}}$ as follows

$$-\Lambda + M \leq \mathbf{K} \leq \Lambda + M, \quad \Lambda \geq M \geq 0 \qquad (4)$$

Both parameters Λ and M are flavour singlets and invariant under local chiral transformations. The chiral fluctuations $\psi(x) \to exp\{i\gamma_5\pi\}\psi(x)$ do not change the quark condensate, but if we introduce scale fluctuations of the quark field $\psi \to exp\{-\sigma\}\psi$, where the hermitean scalar field $\sigma(x) = \sqrt{2/3}\sigma_0(x) \cdot I + \sigma_a(x) \cdot \lambda_a$ is realized in the singlet and the adjoint representations of the flavour group, the these fluctuations change the value of the quark condensate. In order to have stable parameters the flavour singlet scale fluctuations of the quark condensate should be suppressed. The flavour "radial" fluctuations tend to be the origin of the scalar mesons nonet and provide nonequal to each other $<\bar{u}u>$, $<\bar{d}d>$ and $\bar{s}s$ condensates. We can express the mean value of the quark condensate $<\bar{\psi}\psi>_{av} = (<\bar{u}u> + <\bar{d}d> + <\bar{s}s>)/3$ in terms of the parameters Λ and M (for the massless quarks):

$$C_q \equiv <\bar{\psi}\psi>_{av} = -\frac{N_c}{2\pi^2}(\Lambda^2 M - M^3/3) \qquad (5)$$

The stability condition for the low-energy region and its relation to the gluon condensate will be discussed in later after the derivation of the effective potential for the dilaton field.

We extend the joint chiral and conformal bosonization method [4] to the case of the flavour scale transformation. The non-invariant fluctuations of the quark fields $\psi \to \Phi\psi$, where $\Phi = exp\frac{1}{2}(\sigma + \gamma_5\pi)$, $\sigma = \sqrt{2/3} \cdot I \cdot \sigma_0 + \sigma_a \cdot \lambda_a$, $\pi = \pi_a\lambda_a$, are treated as local transformations preserving the hermiticy of the Dirac operator in Euclidean space:

$$\hat{\mathrm{D}}_\Phi = \Phi\hat{D}\Phi \tag{6}$$

The invariant against this transformation functional $Z_{inv}^{-1}(\hat{\mathrm{D}})$ is given by

$$Z_{inv}^{-1}(\hat{D}) = \int D\Phi Z_\psi^{-1}(\Phi\hat{D}\Phi) \tag{7}$$

Then the effective action for the chiral $U = exp(\gamma_5\pi)$ and scalar $\sigma(x)$ fields can be defined in the Euclidean space

$$\int D\Phi exp[-W_{eff}(U,\sigma)] = \frac{Z_\psi(\hat{D})}{Z_{inv}(\hat{D})} \tag{8}$$

$$W_{eff}(U,\sigma) = -\int_0^1 ds \int d^4x\, tr[(\sigma + \gamma_5\pi) < x \mid P(\Lambda^2 - (\hat{D}_{\sigma s,\pi s} - M)^2 \mid x >] \tag{9}$$

where P is the projection operator onto the subspace of the eigenvalues K. We calculate the diagonal part of the projector using the finite mode regularisation [29] of the functional integral. We restrict ourselves in this paper to the determination of the masses of scalar particles and their two-photon decay widths. The relevant expression for the effective Lagrangian for the dilaton field σ_0 and the octet of scalar mesons is given by

$$L_{eff}(\sigma) = \frac{F_0^2}{4}tr(\partial_\mu\sigma e^{-\sigma}\partial^\mu\sigma e^{-\sigma}) + \frac{N_c}{4\pi^2}tr[\frac{1}{10}(\partial_\mu\sigma)^2 S^2 + \frac{3}{20}(\partial_\mu\sigma S)^2] +$$

$$+\frac{N_c}{160\pi^2}tr(\partial_\mu^2\sigma\partial_\nu^2\sigma) - V(\sigma) \tag{10}$$

where the constant $F_0^2 = N_c(\Lambda^2 - M^2)/4\pi^2$ is defined by the numbers of colours N_c, spectral parameters Λ, M and coincides with the pion decay

constant F_π. The tachyonic term can be dropped at low energies, but at the scalar meson mass shell it contributes considerably to the meson masses and field normalizations. The effective potential tends to be

$$V(\sigma) = \mathrm{tr_f}[\frac{\mathrm{C_g}}{48}\mathrm{e}^{-4\sigma} - \mathrm{C_q m_q e^{-3\sigma}} + \frac{\mathrm{F_0^2}}{2}(2\mathrm{e}^{-2\sigma}\mathrm{s}^2 + \mathrm{e}^{-\sigma}\mathrm{s}\mathrm{e}^{-\sigma}\mathrm{s}) - \frac{\mathrm{N_c M}}{2\pi^2}\mathrm{e}^{-\sigma}\mathrm{s}^3] -$$

$$\frac{N_f}{24\pi^2}\sqrt{\frac{2}{3}}\sigma_0 tr_c G_{\mu\nu}^2 \tag{11}$$

where N_f is a number of flavors. The quantity C_g is a function of the spectral parameters and is equal to the gluon condensate for the massless quarks

$$C_g = \frac{3N_c}{2\pi^2}(6\Lambda^2 M^2 - \Lambda^4 - M^4)$$

The last term in (11) defines the mixing between the dilaton-quarkonium and the colourless configuration $tr_c G_{\mu\nu}^2$ of the gluon field (glueball).

As one cannot deduce the effective Lagrangian for the glueball fields from the low-energy QCD we prefer to integrate out the gluon fields with some plausible assumptions. The full generating functional

$$Z_L = \int DG\mathrm{e}^{i\mathrm{W_{YM}}} \int D\Phi \mathrm{e}^{i\mathrm{W_{eff}}(\sigma,\mathrm{G})}$$

includes the integration over the gluon fields with the weight $exp[-iW_{YM}]$. We combine the last term in (11) with the effective Yang-Mills Lagrangian and find that a variation of the averaged value of the dilaton field $<\sigma_0>= \sigma_c$ changes the effective coupling constant

$$(\frac{1}{2g^2} + \frac{N_f\sigma_c}{24\pi^2})tr_c G_{\mu\nu}^2 \equiv \frac{1}{2g_{eff}^2}tr_c G_{\mu\nu}^2 \tag{13}$$

At large positive values of σ_c the effective coupling constant g_{eff} decreases, going over to the regime of the asymptotic freedom. This obstacle enables us to calculate the effective potential not only in the region of low energies but also in the perturbative region. Due to the asymptotic freedom, one can solve the renormalization group equation for the vacuum energy in one-loop approximation for the Gell-Mann-Low β function [30] $\beta(g) \simeq -bg^3$, $b =$

$-(33-2N_f)/48\pi^2$ using the correlation between the vacuum energy and the trace of the energy-momentum tensor

$$E_{vac} = \frac{1}{4} < T^{\mu}_{\mu} >; \quad T^{\mu}_{\mu} = \frac{\beta(g)}{2g}(G^a_{\mu\nu})^2 \qquad (14)$$

and find the principal contribution to the effective potential in the asymptotic freedom region (large σ_c) [31]:

$$V_G(\sigma_0) = - < \frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2 > \frac{11N_c - 2N_f}{32 \cdot N_c} e^{\varepsilon\sigma_0}, \quad \varepsilon = N_f/6\pi^2 b \qquad (15)$$

which replaces the last term in Eq.(11). In order to have stable low-energy spectral parameters Λ and M or equivalently F_π and C_q against the flavor-singlet scale fluctuations of the mean value of the quark condensate the effective potential $V(\sigma)$ has to have minimum at $\sigma = 0$, which provides us with the requirement of the positive definiteness of the gluon condensate and the expression for the $< \frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2 >$ in terms of the vacuum expectation value $< \sigma_8 >$:

$$< \frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2 > \equiv \bar{C}_g = \frac{C_g}{t(\varphi)}; t(\varphi) \equiv exp(-\frac{4\varphi}{\sqrt{3}}) \qquad (17)$$

for the massless *up* and *down* quarks.

3. The non-zero vacuum expectaqtion value of the σ_8 field : $\varphi \equiv < \sigma_8 >$ and is determined by the equation:

$$\frac{1}{6}\bar{C}_g(t^3(\varphi) - 1) - 6C_q m_s t^{3/2}(\varphi) + 6F_0^2 m_s^2 t(\varphi) - \frac{N_c M}{\pi^2} m_s^3 t^{1/2} = 0 \qquad (18)$$

We shift the σ_8 field $\hat{\sigma}_8 = \sigma_8 - \varphi$ in order to acquire zero-value expectation $< \hat{\sigma}_8 > = 0$ and then drop hat. In the wide range of values $C_q = -(160 - 200MeV)^3, C_g = (330 - 400MeV)^4$ and $m_s = 50 - 140MeV$ the solution of the Eq.18 can be represented within the 10% accuracy in the the form:

$$\varphi \equiv < \sigma_8 > = \frac{3\sqrt{3}C_q m_s}{C_g - 18C_q m_s} \qquad (19)$$

The non-zero square mass matrix elements one can obtain from Eqs.(11)and (16)

$$F_{00}^2 m_{00}^2 = \frac{1}{2}(1 + \frac{t^3(\varphi)}{3} - \frac{\varepsilon}{2})\bar{C}_g - 4C_q m_s t^{3/2}(\varphi) + 2m_s^2 F_0^2 t(\varphi)$$

$$F_{a_0}^2 m_{a_0}^2 = \frac{2}{3}\bar{C}_g$$

$$F_{88}^2 m_{88}^2 = \frac{1}{3}(1 + t^3(\varphi))\bar{C}_g - 8C_q m_s t^{3/2}(\varphi) + 4F_0^2 m_s^2 t(\varphi) \qquad (20)$$

$$F_\kappa^2 m_\kappa^2 = \frac{2}{3}\bar{C}_g - 9C_q m_s t^{3/2}(\varphi)$$

$$F_{00}F_{88}m_{08}^2 = \sqrt{2}[\frac{\bar{C}_g}{6} - \frac{\bar{C}_g}{6}t^3(\varphi) + 4C_q m_s t^{3/2}(\varphi) - 2F_0^2 m_s^2 t(\varphi)]$$

where the field normalizations F_i have the values:

$$F_{00}^2 = F_0^2(\frac{2t^{1/2}(\varphi)}{3} + \frac{1}{3t(\varphi)}) + \frac{N_c}{12\pi^2}m_s^2$$

$$F_{a_0}^2 = F_0^2 t^{1/2}(\varphi)$$

$$F_8^2 = F_0^2(\frac{t^{1/2}}{3} + \frac{2}{3t(\varphi)}) + \frac{N_c}{6\pi^2}M_s^2 \qquad (21)$$

$$F_\kappa^2 = \frac{F_0^2}{2}(t^{1/2}(\varphi) + \frac{1}{t(\varphi)}) + \frac{7N_c}{40\pi^2}m_s^2$$

The nondiagonal element m_{08}^2 reproduces the $\sigma_0\sigma_8$ mixing . The $\sigma_0\sigma_8$ mixing angle α is defined by the equation

$$tg2\alpha = \frac{2 \cdot m_{08}^2}{m_{00}^2 - m_{88}^2} \qquad (22)$$

For the values of the gluon condensate $< \frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2 > = (385\ MeV)^4$, quark condensate $C_q = -(185MeV)^3$ and quark mass $m_s = 105MeV$ we obtain the value of the octet-singlet mixing angle $\alpha \approx -33°$. Thus one of the isoscalar mesons has dominant $\bar{s}s$ quark content, while the other has SU(2)-like quark structure.

Due to the tachyon term $\sim tr_f(\partial_\mu^2\sigma\partial_\nu^2\sigma)$ present in (10) the masses and field normalizations acquire the expression [9]

$$\hat{m}_\lambda^2 = \frac{20\pi^2 F_0^2}{3}\left[\sqrt{1+\frac{3m_\lambda^2}{10\pi^2 F_0^2}} - 1\right] \quad (23)$$

$$\hat{F}_\lambda^2 = \frac{3m_\lambda^2}{20\pi^2} + F_\lambda^2; \quad (24)$$

In Table 3 we present the calculation of the masses of the scalar mesons with and without the contribution from the tachyon term. The results obtained for the meson spectra show the strong dependence on the contribution from the tachyon term. The field normalizations $\bar{F}_\alpha$ strongly depend on the contribution of the tachyon term, so that on the mass shell

$$\bar{F}_\sigma \approx (1.5 - 2.1)F_\pi$$

It is hard to make predictions for the masses of the σ_0 and σ_8 mesons due to the strong dependance on the dilaton (quarkonium)-glueball mixing. At some choice of the parameter ε these masses can be degenerated. But we can estimate the region of the varying of the isoscalar masses due to the change of the quark $<\bar{\psi}\psi> = -(150-210MeV)^3$, gluon $<\alpha/\pi(G^a_{\mu\nu})^2> = (340-410MeV)^4$ condensates and current strange mass quark $m_s = (50-145)MeV$.

But , one can make predictions for the masses of the isotriplet and isodoublet of scalar mesons (see Table 3), which do not mix with the glueball. We identify the $\sigma(I=1)$ state with the $a_0(980)$ meson [13]. For the above mentioned range of QCD parameters one can state that

$$\frac{m_\sigma(I=\frac{1}{2})}{m_\sigma(I=1)} < 1$$

One can calculate [10] the decay width

$$\Gamma(\sigma(I=\frac{1}{2}) \to K\pi) \approx 30 - 60\ MeV$$

At this point we support the suggestion of Bramon and Scadron [15], that such $\sigma(I=1/2)$ scalar meson could be almost masked by $K^*(895)$ vector

meson:

1. it has the same isospin I=1/2;
2. it has nearly the same mass and width ;
3. it decays into the same dominant $K\pi$ mode;
4. it is supported by the SU(6) relations for isospin splitting [14].

We use the expression for the quark condensate

$$<\bar{\psi}\psi> = i\frac{\delta Z^L_\psi}{\delta S}|_{S=m_q} \quad (25)23)$$

to obtain values of the condensates for the u, d and s quarks

$$<\bar{u}u> = C_q\cdot(1-3<\sigma_3>)(1-\sqrt{3}<\sigma_8>)$$
$$<\bar{d}d> = C_q\cdot(1+3<\sigma_3>)(1-\sqrt{3}<\sigma_8>) \quad (26)$$
$$<\bar{s}s> = C_q\cdot(1+2\sqrt{3}<\sigma_8>),$$

where we switch on the masses of *up* and *down* quarks, so, that the vacuum expectation value of the σ_3 field is proportional to the quark mass difference $\Delta m = (m_u - m_d)/2$ [10] (see next section) . It is convenient to introduce the parameters β, and ρ, which characterize the difference between the $<\bar{u}u>$, $<\bar{d}d>$ and $<\bar{s}s>$condensates

$$\beta = 1 - \frac{<\bar{d}d>}{<\bar{u}u>}$$

$$\rho = 1 - \frac{<\bar{s}s>}{<\bar{d}d>}$$

The dependance on the value of $<\sigma_8>$ cancels in the definition of β and it recovers the SU(2) value : $\beta \approx 2\cdot 10^{-2}$ [10].

The expression

$$L_l(\sigma, V) = \frac{N_c M}{4\pi^2} tr\partial_\mu\sigma D^\mu S \quad (27)26)$$

where we introduce the covariant derivative $D_\mu = \partial + [V_\mu, *]$, defines the leptonic decay constants of the $a_0(980)$and κ. Using the definition of the leptonic decay constant f^l_a via the divergence of the vector current as $<0 \mid \partial_\mu J^\mu \mid a_0> \equiv \sqrt{2}f^l_a m^2_a$ and analogous formula for the f^l_κ we obtain the values $f^l_a = 1.54MeV$ and $f^l_\kappa = 33MeV$, consistent with the QCD spectral sum rules estimate $1.3-1.6 MeV$ and $34 MeV$ correspondingly [34], though the latter value of f^l_κ was obtained in Ref.34 for $m_\kappa = 1.35 GeV$ and $m_s = 250 MeV$.

4. **SU(2) case.** As one can see from Eq.(11) , the third component of the scalar meson triplet acquires the non-zero expectation value due to the quark mass difference Δm:

$$< \sigma_3 > \equiv \varphi = \frac{6 C_q \Delta m}{(-\frac{2}{3} C_g + 18 C_q \hat{m})} + O((\Delta m)^2) \quad (28)$$

We shift the σ_3 field $\hat{\sigma}_3 = \sigma_3 - \varphi$ in order to acquire zero-value expectation $< \hat{\sigma}_3 >= 0$ and then drop hat. We obtain from Eqs.(11) and (15) the following non-zero square mass matrix elements $m^2_{ij} = \delta^2 V / \delta\sigma_i \delta\sigma_j$ (neglecting for a while the tachyon term)

$$f_0^2 m_{00}^2 = \frac{2}{3} C_g - \frac{\varepsilon}{6} < \frac{\alpha_s}{\pi} (G^a_{\mu\nu})^2 > -18 C_q \hat{m} + 54 C_q \Delta m \cdot \varphi$$

$$f_+^2 m_{++}^2 = \frac{2}{3} C_g - 18 C_q \hat{m} + 18 C_q \Delta m \cdot \varphi$$

$$f_-^2 m_{--}^2 = f_+^2 m_{++}^2 \quad (29)$$

$$f_3^2 m_{33}^2 = \frac{2}{3} C_g - 18 C_q \hat{m} + 54 C_q \Delta m \cdot \varphi$$

$$f_0 f_3 m_{03}^2 = -\frac{2}{3} C_g \cdot \varphi$$

where the field normalizations f_i have the values:

$$f_+^2 = f_-^2 = F_0^2 + \frac{N_c (\hat{m}^2 - \Delta m^2)}{4\pi^2}$$

$$f_0^2 = f_3^2 = F_0^2 + \frac{N_c (\hat{m}^2 + \Delta m^2)}{4\pi^2} \quad (30)$$

the nondiagonal element m_{03}^2 reproduces the $\sigma_0 \sigma_3$ mixing which is proportional to the quark mass difference Δm , being small in the case of the $SU(2)$ group. The $\sigma_0 \sigma_3$ mixing angle α is defined by the equation

$$tg 2\alpha = -\frac{C_g \cdot \varphi}{\frac{\varepsilon}{8} < \frac{\alpha_s}{\pi} (G^a_{\mu\nu})^2 >} \quad (31)$$

so that the width of the G-parity violating process $\sigma_3 \to \pi\pi$ is suppressed by the factor $sin^2\alpha \sim (\Delta m)^2$. For the values of the gluon condensate $< \frac{\alpha_s}{\pi} (G^a_{\mu\nu})^2 >= (385\ MeV)^4$, quark condensate $C_q = -(185 MeV)^3$ and quark masses $m_u = 5 MeV$, $m_d = 10 MeV$ we obtain the value $sin(\alpha) = 0.04$. In proceeding we shall neglect the mixing of σ_0 and σ_3 fields. Due to the tachyon term $\sim tr_f(\partial^2_\mu \sigma \partial^2_\nu \sigma)$ present in (10) the masses and field normalizations acquire the expression (23, 24) correspondingly.

In Table 4 we present the calculation of the masses of the scalar mesons with and without the contribution from the tachyon term. The results obtained for the meson spectra show the strong dependence on the contribution from the tachyon term and exhibits the degeneracy of the masses of the triplet $a_0(980)$

$$\hat{m}_\pm^2 - \hat{m}_3^2 = 36 \frac{C_q}{\hat{F}_0^2} \Delta m \cdot \varphi \sim (\Delta m)^2 \quad (32)$$

The mean value of the mass for the triplet $a_0(980)$ and the degeneracy of the mass spectra are in good agreement with the experimental data [10]. The mass of the flavor-singlet meson is less $\approx 50 MeV$ than the mean value mass of the triplet and the mass difference

$$\frac{\hat{m}_+^2 + \hat{m}_-^2 + \hat{m}_3^2}{3} - \hat{m}_0^2 = \frac{\varepsilon}{6 \hat{F}_0^2} < \frac{\alpha_s}{\pi} (G^a_{\mu\nu})^2 > + O((\Delta m)^2) \quad (33)$$

is essentially governed by the values of the gluon condensate and parameter ε, which was determined in one-loop approximation for the $\beta-$ function. The field normalizations $\bar{F}_\alpha$ strongly depend on the contribution of the tachyon term, so that on the mass shell $\bar{F}_\alpha \approx 2 F_0$.

If we identify the $G(1590)$ state with the 0^{++} glueball field [20] we can calculate the mixing angle between the glueball and dilaton-quarkonium states $\Theta_{0g} \approx 15^\circ - 20^\circ$ to be of the same magnitude as the QCD sum rule estimate [33].

5. **The two-photon decay of the scalar meson** is defined by the gauge-invariant expression

$$L_{\gamma\gamma}(\sigma, B) = \frac{N_c}{24 \pi^2 \hat{F}_0} tr \sigma F^2_{\mu\nu}(\hat{B}) \quad (34)$$

where $F_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu$ is a stress tensor of the electromagnetic field $\hat{B}_\mu = e(B^3_\mu \cdot \lambda_3 + B^8_\mu \cdot \lambda_8/\sqrt{3})$. In table 2. we present the calculation of the $\gamma\gamma$ widths of the low-lying scalar mesons in comparison with the experimental results and theoretical expectations. The calculation of the width $\Gamma(f_0 \to \gamma\gamma)$ on the base of the Eq.(34) gives $0.6KeV$. If we take into account the additional factors in $L_{\gamma\gamma}$ arising from the glueball-quarkonium we can obtain the less value $0.34KeV$.

Let us consider the real case of the degenerated masses of the f_0 and a_0 mesons and neglect mixing of f_0 with the glueball, then we obtain for the ratio of the two-photon decay widths

$$\frac{\Gamma(f_0 \to \gamma\gamma)}{\Gamma(a_0 \to \gamma\gamma)} = 2, \tag{35}$$

in contrast with the constituent quark model prediction 25/9 for this ratio [18,19,20] The deviation from this ratio , being proportional to the $\cos^2 \Theta_{0g}$ can provide with the information about the quarkonium-glueball mixing.

Resume.

1. Spectrum of scalar mesons in the Conformal Bosonization Model lies within the 1 GeV region.
2. There is a strong dependance of the values of masses and normalization constants (and thus other physical quantaties) on the contribution of the tachyon term.
3. The isovector $\sigma(I = 1)$ meson can be identified with the $a_0(980)$ meson.
4. There is a broad , dominantly SU(2) , isoscalar meson with $m_\epsilon \approx 750 - 900MeV$ and $\Gamma_\epsilon \approx 200 - 250MeV$ [10] , which one can identify with the $S_1(910)$ state of Ref.16.
5.The estimate of the mass and decay width of the $\sigma(I = 1/2)$ supports the suggestion of the [15] about the existence of the light $\kappa(850)$ meson, being masked by the vector $K^*_0(895)$ meson.
6. The two-photon decay widths of f_0 and a_0 mesons are in good agreement with the experimental data. The ratio $\Gamma(f_0 \to \gamma\gamma)/\Gamma(a_0 \to \gamma\gamma)$ can give information about the value of the quarkonium-glueball mixing angle Θ_{0g}.

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Table 1. The spectrum of scalar mesons from PDG view [13] and some candidates for low-lying scalar mesons.

Isospin	Particle data group [13]	Comment
$I=0$	$f_0(975)$	$\varepsilon(910)$, $\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}$
	$f_0(1400)$	$S_2(998)$, $\bar{s}s$ [16]
	$f_0(1590)$	$S_1(991)$, (gg)
$I=1$	$a_0(980)$	-
$I=\frac{1}{2}$	$K_0^*(1430)$	$\kappa(850)$ [15]

Table 2. Theoretical and experimental results for $\Gamma(f_0\to\gamma\gamma)$ and $\Gamma(a_0\to\gamma\gamma)$

Content	Theoretical model	$\Gamma(f_0\to\gamma\gamma), KeV$	$\Gamma(a_0\to\gamma\gamma), KeV$
$\bar{q}q$	Berger-Feld [18]	$\frac{25}{9}\Gamma(a_0\to\gamma\gamma)$	$2.5\div 3.8$
	Budnev-Kaloshin [19]	$\frac{25}{9}\Gamma(a_0\to\gamma\gamma)$	4.8
	Barnes [20]	3.0	1.1
	Volkov, Osipov [26]	1.7×10^{-3}	1.3
	Efimov, Ivanov [24]	0.37	0.41
	Dominguez,Paver [23]	–	~ 0.6
	This model [10]	$0.34-0.6$	0.23
$\bar{q}^2q^2$	Achasov et al.[21]	0.27	0.27
	Narison [25]	-	$(2-5)\times10^{-4}$
$K\bar{K}$	Barnes [22]	0.6	0.6
Exp	[27]		0.19 ± 0.07
	MARK II [28]	$0.24\pm0.06\pm0.15$	
	Crystal Ball[28]	$0.31\pm0.14\pm0.09$	
	JADE [28]	≤ 0.6	$0.29\pm0.05\pm0.14$

Table.3. SU(3) mass spectrum of scalar mesons . Parameter ε is defined by the Eq.15. The values of current quark masses:$m_u=m_d=0$; $m_s=105\ MeV$

Isospin	masses with tachyon contribution, MeV	masses without tachyon contribution, MeV
$I=0$	810	1005
	905	1180
$I=1$	982	1360
$I=1/2$	850	1080

Table.4. SU(2) mass spectrum of scalar mesons . The values of the current quark masses:$m_u=8.7MeV$, $m_d=15.4MeV4$

Particle	masses with tachyon contribution, MeV	masses without tachyon contribution, MeV
$\sigma(I=0)$	927	1228
$\sigma(I=1)$ charged	982	1335
$\sigma(I=1)$ neutral	981	1334