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A Practical Introduction to Electroweak Radiative Corrections

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A Practical Introduction to Electroweak Radiative Corrections *

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Abstract

This is a brief introduction into electroweak radiative corrections within the Standard Model, with the emphasis on performing actual calculations. To this end, a complete set of expressions is given that allows the computation of the ρ parameter, the W mass, and $Z \to f\bar{f}$ decays for massless fermions, where the $\overline{MS}$ scheme has been used. I conclude with an assessment of what we have learned so far from electroweak precision experiments, and a brief outlook.

*Based on a talk given at the Second Workshop on High Energy Physics Phenomenology, Calcutta, January 1991; to appear in the Proceedings.

1) Introduction

This is an introductory review of electroweak radiative corrections, mostly within the Standard Model (SM). A lot has recently been said and written about this topic [1]. I don't suffer from the delusion that this paper can contribute much original material to this field. Instead, I decided to focus on the practical aspects, and tried to write a self-contained note; the goal is that it should contain all information necessary to compute corrections to the ρ-parameter, m_W, and $Z \to f\bar{f}$ decays. The drawback is that most of the more subtle aspects cannot be treated in this limited space; the advantage is that this note will hopefully allow readers to get into the game of computing radiative corrections without having to consult many different papers. The only "new" parts are a first attempt to include QCD corrections to top loops in the $\overline{MS}$ scheme, as well as a fairly subjective discussion of the present status of the field.

Generally speaking, electroweak radiative corrections can be grouped into four classes. Numerically most important are effects due to radiation of soft and/or collinear photons. (Of course, virtual QED corrections have to be added in order to cancel infrared divergencies.) They reduce the cross section at the Z^o peak in e^+e^- annihilation by as much as 30%. The reason is that the emission of photons from the initial state smears out the effective beam energy, so that even if nominally the machine operates at $\sqrt{s} = m_Z$, in many events the actually available energy is somewhat smaller. This is a pure QED effect; moreover, its actual size depends on experimental cuts, e.g. on the angular acceptance for photons. These corrections are therefore routinely 'undone' in the published experimental results, using elaborate Monte Carlo programs. While it is both very important and highly nontrivial [2] to understand these corrections in detail, they will not be treated in this note.

The second class of corrections contains those that can be absorbed into the running of the electromagnetic coupling α. These contributions are due to vacuum polarization graphs with a light fermion f in the loop, and give contributions of order $\frac{\alpha}{\pi}\log\left(\frac{m_Z}{m_f}\right)$; notice that $\log\left(\frac{m_Z}{m_e}\right) = 12.1$, which is a large enhancement. However, only charged particles with mass (much) less than m_Z contribute here; these particles have all been identified a long time ago. Hence these corrections don't teach us anything about "new physics"; nevertheless, because of their size they have to be computed to high precision.

Contributions of massive particles to the gauge boson two-point functions constitute the third class of corrections. It is here that "new physics" can make a sizeable contribution. However, this will happen only if the new particle does not decouple from low-energy physics in the limit where its mass becomes very large, which is true for the top quark, or if there are many only moderately heavy new particles, which might be the case in technicolor models or supersymmetric theories. The SM Higgs boson fulfills the first criterion; nevertheless its contributions are suppressed due to the "screening theorem" [3], which states that to one-loop order and for large Higgs mass m_H, physical amplitudes can only depend logarithmically on m_H, as well as due to the approximate "custodial $SU(2)_V$" symmetry of the SM discussed in sec.3.

The final class of radiative corrections is formed by vertex and box diagrams. With the noteable exception of the $Zb\bar{b}$ vertex [4], which for reasons of space cannot be treated here, they are generally quite small, although not always negligible.

The remainder of this paper is organized as follows. In Sec.2, the renormalization of the electroweak sector of the SM is briefly reviewed. I chose the $\overline{MS}$ definition of the weak mixing angle, since it appears to be somewhat more practical for applications to

Z physics, which is where most experimental progress is expected in the near future. Sec.3 is devoted to the electroweak ρ parameter and the custodial $SU(2)_V$ symmetry. The computation of the physical W mass is discussed in Sec.4, and Sec.5 treats $Z \to f\bar{f}$ decays (widths and asymmetries). Finally, Sec.6 contains an assessment of what, to my opinion, we have learned so far from electroweak precision data, as well as a brief outlook. Explicit expressions for the relevant two-point functions are listed in Appendix A.

2) Renormalization

In this section I discuss the renormalization of the electroweak interactions. I closely follow the classic paper of Sirlin [5].

As well known, in field theory one has to specify one renormalization condition for every free parameter of the fundamental Lagrangian. The electroweak sector of the SM has three such parameters, e.g. the SU(2) and U(1) gauge coupling constants g and g' and the mass m_W of the W boson. For practical purposes it is, however, more convenient to chose the proton charge e, and the masses m_W and m_Z as the three "fundamental" quantities whose counterterms are fixed by renormalization conditions.

Sirlin [5] fixed these counterterms as follows:

$$\delta m_W^2 = -Re\Pi_{WW}^T(m_W^2); \tag{1a}$$

$$\delta m_Z^2 = -Re\Pi_{ZZ}^T(m_Z^2). \tag{1b}$$

Here, Π_{WW}^T is i times the transverse part of the W two-point function, and correspondingly for Π_{ZZ}^T. Note that these functions have been defined such that their imaginary part is positive; e.g., $Im\Pi_{WW}^T(m_W^2) = m_W\Gamma_W$. The Π_{ij}^T thus differ from Sirlin's A_{ij} by an overall sign. The conditions (1) guarantee that m_W and m_Z are indeed the *physical* masses, defined by the condition that the real part of the propagator vanish for $q^2 = m^2$.

The renormalization condition for e reads in Feynman gauge [5]:

$$\frac{2\delta e}{e} = -\lim_{q^2\to 0} \frac{\Pi_{\gamma\gamma}^T(q^2)}{q^2} - \frac{e^2}{4\pi^2}(\log m_W^2 - \Delta), \tag{2}$$

where $\Delta = \frac{2}{4-D} + \gamma_E - \log\pi$ diverges in $D = 4$ dimensions; the r.h.s. of eqs.(1) contain similar divergencies. The condition (2) guarantees that in the limit $q^2 \to 0$, the classical Thomson formula for ep scattering is reproduced, which thus serves as the definition of the electric charge. The first term in eq.(2) is due to vacuum polarization graphs; the second, due to vertex corrections.

In principle, all physical quantities can be expressed in terms of m_W, m_Z and e. However, it is more convenient to introduce the weak mixing angle θ_W as an auxiliary parameter. Several definitions of $sin^2\theta_W$ exist in the literature. In his classic paper [5], Sirlin defined it via the gauge boson masses,

$$sin^2\theta_W = 1 - m_W^2/m_Z^2; \tag{3}$$

this is the "on-shell" definition. With this definition, θ_W is clearly a physical parameter. However, it is subject to sizeable radiative corrections, which depend strongly on the as yet unknown mass m_t of the top quark. In 1982, Sirlin and collaborators [6] therefore

suggested a new definition, where only the *infinite* terms are kept in the counterterm to $sin^2\theta_W$; this resembles the $\overline{MS}$ renormalization. [1] This leads to the definition [7]

$$\hat{s}^2 = s^2\left[1 - \frac{c^2}{s^2}Re\left(\frac{\Pi_{WW}^T(m_W^2)}{m_W^2} - \frac{\Pi_{ZZ}^T(m_Z^2)}{m_Z^2}\right)_{\overline{MS}}\right], \tag{4}$$

where $s^2 = sin^2\theta_W$ is defined in eq.(3), $c^2 = cos^2\theta_W$, and $\hat{s}^2 = sin^2\theta_W(m_Z)_{\overline{MS}}$. The subscript $\overline{MS}$ in eq.(4) means that the divergence Δ has to be replaced by $\log m_Z^2$.

The practical computation of $\hat{s}^2$ from m_Z will be discussed in Sec.4. It should already be clear, however, that this and other calculations of electroweak radiative corrections will necessitate the knowledge of the gauge boson two-point functions at various external momenta q. I therefore close this section by giving explicit expressions for these functions, which are valid in the SM in Feynman gauge. One has [8]

$$\Pi_{\gamma\gamma}^T(q^2) = \frac{e^2}{16\pi^2}q^2\left\{-12B_3(q^2,m_W,m_W) - B_0(q^2,m_W,m_W) - \frac{2}{3} + 8q^2\sum_f Q_f^2 B_3(q^2,m_f,m_f)\right\}; \tag{5a}$$

$$\Pi_{\gamma Z}^T(q^2) = \frac{e^2}{16\pi^2 sc}\left\{(12s^2-10)q^2B_3(q^2,m_W,m_W) - \frac{2}{3}c^2q^2 + \left[q^2\left(s^2-\frac{3}{2}\right) - 2m_W^2\right]B_0(Q^2,m_W,m_W) + 4q^2\sum_f\left[\left(Q_fI_{3_f} - 2Q_f^2s^2\right)B_3(q^2,m_f,m_f)\right]\right\}; \tag{5b}$$

$$\begin{aligned}\Pi_{ZZ}^T(q^2) = \frac{e^2}{16\pi^2s^2c^2}\Bigg\{&\left(-9+20s^2-12s^4\right)q^2B_3(q^2,m_W,m_W) - \frac{2}{3}c^4q^2 + \\ &\left[2\left(2s^2-1\right)m_W^2 - \left(\frac{7}{4}-3s^2+s^4\right)q^2\right]B_0(q^2,m_W,m_W) \\ &+m_Z^2B_0(q^2,m_Z,m_H) - \frac{1}{4}B_5(q^2,m_Z,m_H) \\ &+\sum_f\Big[4q^2\left(I_{3_f}^2 - 2s^2Q_fI_{3_f} + 2s^4Q_f^2\right)B_3(q^2,m_f,m_f) \\ &-2m_f^2I_{3_f}^2B_0(q^2,m_f,m_f)\Big]\Bigg\};\end{aligned} \tag{5c}$$

$$\begin{aligned}\Pi_{WW}^T(q^2) = \frac{e^2}{16\pi^2s^2}\Bigg\{&\left(2s^2-\frac{9}{4}\right)B_5(q^2,m_W,m_Z) - \frac{2}{3}q^2 \\ &+\left(m_Z^2 - 3m_W^2 - 4c^2q^2\right)B_0(q^2,m_W,m_Z) \\ &-2s^2\left[B_5(q^2,m_W,0) + 2q^2B_0(q^2,m_W,0)\right]\end{aligned}$$

[1] It should be noted that the electroweak "$\overline{MS}$" scheme still uses the *physical* gauge boson masses, which differ from the $\overline{MS}$ renormalized masses. This might be somewhat confusing from the purely field theoretical point of view, but practically speaking the introduction of unphysical $\overline{MS}$ masses is an unecessary complication.

$$+m_W^2 B_0(q^2, m_W, m_H) - \frac{1}{4} B_5(q^2, m_W, m_H) + \qquad (5d)$$

$$\left. \sum_{u,d} |V_{ud}|^2 \left[2q^2 B_3(q^2, m_u, m_d) - B_4(q^2, m_u, m_d)\right] \right\}.$$

Here, m_H is the mass of the single physical Higgs boson of the SM. Explicit expressions for the functions B_0, B_3, B_4 and B_5 are listed in Appendix A.

Some comments are in order. First of all, it does not matter (to one-loop order) which definition of s^2 is used in evaluating the r.h.s. of eqs.(5); a constant $s^2 = 0.23$ will do perfectly fine. Furthermore, the fermion loop contributions, which give the largest contributions to physical quantities within the SM, are contained in the last line of each of the eqs.(5). Here, f stands for any SM fermion with charge Q_f and weak isospin $I_{3_f} (= \pm 1/2)$. Similarly u (d) stands for any SM fermion with $I_{3_f} = +1/2$ ($-1/2$), and V_{ud} is the weak mixing matrix (which for leptons is equal to the unit matrix, of course). Note that quarks have to be counted thrice in eqs.(5), due to the color degree of freedom.

3) The ρ-Parameter

The first quantity that I will discuss is the electroweak ρ-parameter, defined [9] via the ratio of neutral and charged current interactions. On tree level, $\rho = m_W^2/m_Z^2 cos^2\theta_W = 1$ in the SM; in fact, this has been assumed in the on-shell definition (3) of $sin^2\theta_W$. On the one-loop level, $\delta\rho = \rho - 1$ gets contributions from vacuum polarization graphs as well as from vertex and box corrections [10]. Obviously these latter contributions depend on which vertex exactly enters the relevant interaction, and hence will be different for different processes. I will briefly return to them at the end of this section. In contrast, the vacuum polarization graphs affect *all* NC processes equally; in this sense they are process-independent. Traditionally, ρ has been measured in neutrino experiments, at squared momentum transfer $q^2 \ll m_W^2$. In this case the two-point function contribution to $\delta\rho$ can be written as

$$\delta\rho = \frac{\Pi_{ZZ}^T(0) + 2s/c\Pi_{\gamma Z}^T(0)}{m_Z^2} - \frac{\Pi_{WW}^T(0)}{m_W^2}. \qquad (6)$$

The term proportional to $\Pi_{\gamma Z}^T(0)$, which only receives contributions from W-loops, has been inserted to render the expression finite.

Essentially $\delta\rho$ measures the *difference* in loop corrections to Z and W two-poiont functions. Clearly such differences cannot exist in the limit of unbroken $SU(2)$ gauge invariance, where the W and Z bosons reside in the same triplet of $SU(2)$. What is more, in 1979 Weinberg and Susskind independently discovered [11] that in the absence of hypercharge interactions the bosonic sector of the SM is symmetric under the "custodial $SU(2)_V$" symmetry even *after* gauge symmetry breaking; a simple proof is given in ref.11. The SM Higgs field transforms like the sum of an isosinglet and an isotriplet under $SU(2)_V$, while the W and Z bosons form an isotriplet.

Note that without hypercharge interactions, there is no $\gamma - Z$ mixing. The proper limit in which $SU(2)_V$ is exact in the bosonic sector of the SM is thus defined by

$$sin^2\theta_W \to 0, \ \frac{\alpha}{sin^2\theta_W} \to const. \ \Rightarrow \ \alpha \to 0, \ m_W \to m_Z. \qquad (7)$$

If one wants to extend this symmetry into the fermionic sector, left- and right-handed fields have both to be placed [12] into $SU(2)_V$ doublets. One can then easily see that unequal Yukawa couplings for up-type and down-type quarks, or charged and neutral leptons, break $SU(2)_V$ explicitly.

Classifying the parts of the Lagrangian $\mathcal{L}$ into terms that respect $SU(2)_V$ and terms that don't is useful since in the limit of exact $SU(2)_V$, $\delta\rho$ of eq.(6) *vanishes*. Notice that apart from the $\Pi_{\gamma Z}^T$-term, which only gets contributions from gauge loops, the r.h.s. of eq.(6) depends on $sin^2\theta_W$ only in the overall pre-factor. (The factor $1/c^2$ in front of Π_{ZZ}^T, eq.(5c), combines with $1/m_Z^2$ to give $1/m_W^2$, leaving $\alpha/(sin^2\theta_W m_W^2)$ as overall factor.) Thus even in the real world of nonzero θ_W and broken $SU(2)_V$, terms in $\mathcal{L}$ that don't break $SU(2)_V$ don't contribute to $\delta\rho$, at least on one-loop level. This is particularly helpful when contributions from new physics, i.e. new terms in $\mathcal{L}$, are analyzed, but also helps to understand the structure of the SM contributions to $\delta\rho$. They are given by [9, 10] (in the relevant case $m_t \gg m_b$)

$$\delta\rho_{SM} = \frac{3\alpha}{16\pi s^2 m_W^2} \left\{ m_t^2 \left[1 - 0.91\alpha_s(m_t^2)\right] + c^2 m_H^2 \left[\frac{\log(c^2/\xi)}{c^2 - \xi} + \frac{1}{c^2} \frac{\log \xi}{1 - \xi} \right] \right\} - 1.5 \cdot 10^{-4}, \qquad (8)$$

where the last term is the contribution from gauge loops in Feynman gauge, and $\xi = m_H^2/m_Z^2$. Notice that the term in square brackets, which is the finite part of the Higgs contribution, vanishes in the limit $sin^2\theta_W \to 0$. Relative to the top-quark contribution, given by the first term, it is thus suppressed by an additional factor of $sin^2\theta_W$, as dictated by $SU(2)_V$ invariance. Furthermore, for $m_H^2 \gg m_Z^2$ the Higgs contribution only grows logarithmically with m_H, as stipulated by the screening theorem [3]. As a result of these two theorems, the Higgs contribution, which is always negative, amounts to at most $-2 \cdot 10^{-3}$, for $m_H \leq 1\,TeV$; for larger values of m_H, tree-level unitarity breaks down [13] in the scattering of longitudinal vector bosons, which via the equivalence theorem [14] is related to Higgs self-interactions. This indicates that perturbation theory in the Higgs sector breaks down for $m_H \geq 1\,TeV$. A recent calculation [15] which uses non-perturbative Higgs propagators indicates that increasing the parameter [‡] m_H beyond 1 TeV does *not* increase the size of the loop corrections from the Higgs sector. In contrast, the contribution of the top quark can be of order 10^{-2}. They are used to place upper bounds on m_t within the SM; this will be discussed in more detail in sec.6. The QCD corrections, which reduce the top contribution to $\delta\rho$ by about 10%, have been taken from ref. [17].

Finally, for a given process $\delta\rho$ also receives contributions from vertex and box diagrams, as mentioned earlier. For the case of neutrino scattering off isoscalar nuclear targets, they only amount [10] to $+6.1 \cdot 10^{-4}$ and are thus negligible. However, in the theoretically cleaner case of neutrino lepton scattering they contribute [10] $+4.9 \cdot 10^{-3}$, which is no longer negligible.

4) Predicting m_W

In the renormalization framework of sec.2, the W mass is considered to be one of the fundamental parameters of the SM. One can therefore not speak of "radiative corrections

[‡]Note that in this case probably no physical particle with mass m_H will show up in the spectrum; rather, one expects some sort of resonance structure [16].

to m_W". Unfortunately, the direct measurements of m_W at $p\overline{p}$ colliders still have an error of about 0.5%. In contrast, the experimental error on the muon decay constant G_μ, which is a derived quantity, is only about 2 parts in 10^5. It is therefore tempting to try and predict m_W from G_μ.

The connection between these two quantities is given by the famous relation [5]

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{s^2(1-\Delta r)} = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{\hat{s}^2(1-\Delta\hat{r}_W)}, \tag{9}$$

where the second equation is the $\overline{MS}$ equivalent [18] of the perhaps more familiar first equation, which uses the on-shell definition of $sin^2\theta_W$. The quantity G_μ is related to the muon lifetime τ_μ by [5]

$$\frac{1}{\tau_\mu} = \frac{G_\mu{}^2 m_\mu^5}{192\pi^3}\left(1-\frac{8m_e^2}{m_\mu^2}\right)\left[1+\frac{3}{5}\frac{m_\mu^2}{m_W^2}+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]. \tag{10}$$

Note that the "traditional" QED corrections, which are already present in the local $V-A$ theory, have been absorbed in the definition of G_μ. Nevertheless eq.(10) relates G_μ directly to physical, measurable quantities; it is not subject to unknown radiative corrections. Numerically [19], $G_\mu = 1.16637(2)\cdot 10^{-5}\,GeV^{-2}$.

Even if Δr or $\Delta\hat{r}_W$ were known exactly, eq.(9) by itself would not suffice to compute m_W, unless one also knows s^2 or $\hat{s}^2$. In order to fix this parameter, one has to relate it to a third quantity (besides α and G_μ) which is known to high precision. Fortunately, LEP has provided us with such a quantity, i.e. m_Z, which is now known with a relative precision of 2 parts in 10^4: $m_Z = 91.18\pm 0.02\,GeV$. On the other hand, m_Z is also related to G_μ:

$$m_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{s^2c^2(1-\Delta r)} = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{\hat{s}^2\hat{c}^2(1-\Delta\hat{r}_Z)}. \tag{11}$$

Note that Δr in eqs.(9) and (11) is identical since $m_W^2 = c^2 m_Z^2$ by definition, while $\Delta\hat{r}_W$ and $\Delta\hat{r}_Z$ ($=\Delta\hat{r}$ of Sirlin [7]) are different. I will from now on stick to the $\overline{MS}$ scheme, since in this scheme eqs.(9) and (11) automatically re-sum [20] leading higher order corrections. In this scheme one has [18, 7]:

$$\Delta\hat{r}_W = \left[Re\frac{\Pi_{WW}^T(0)-\Pi_{WW}^T(m_W^2)}{m_W^2} - \frac{2\delta e}{e}\right]_{\overline{MS}} + 9.1\cdot 10^{-3}; \tag{12a}$$

$$\Delta\hat{r}_Z = \left[Re\left(\frac{\Pi_{WW}^T(0)}{m_W^2} - \frac{\Pi_{ZZ}^T(m_Z^2)}{m_Z^2}\right) - \frac{2\delta e}{e}\right]_{\overline{MS}} + 9.1\cdot 10^{-3}. \tag{12b}$$

$\delta e/e$ has been defined in eq.(2), and the meaning of the subscript $\overline{MS}$ is as in eq.(4). The constant term in eqs.(12) is due to electroweak vertex and box corrections to muon decay.

The practical procedure for predicting m_W for given m_t and m_H (and a given set of the remaining free parameters in extensions of the SM) is now clear. First one computes the corrections $\Delta\hat{r}_W$, $\Delta\hat{r}_Z$ of eqs.(12) using the expressions (5) for the SM two-point functions (plus additional terms in extensions of the SM). Then one solves eq.(11) for $\hat{s}^2$, which gives

$$\hat{s}^2 = \frac{1}{2}\left\{1-\left[1-\left(\frac{74.571\,GeV}{m_Z}\right)^2\frac{1}{1-\Delta\hat{r}_Z}\right]^{1/2}\right\}. \tag{13}$$

The prediction for m_W can then be computed from eqs.(9), (12a) and (13).

One advantage of the $\overline{MS}$ scheme as compared to the on-shell scheme is that $\Delta\hat{r}_Z$ depends much less [7] on m_t than Δr; both corrections contain terms proportional to m_t^2, but in the on-shell scheme this term has an additional enhancement factor of $c^2/s^2 \simeq 3.3$ which is absent in the $\overline{MS}$ scheme. The correction $\Delta\hat{r}_W$ does not contain [18] any m_t^2 terms; it depends on m_t only logarithmically. Furthermore, its dependence on the Higgs mass is also very mild. As a result, $\hat{s}^2$ is known to much higher precision than s^2, *if* new physics contributions are negligible. I will come back to this point in sec.6.

Notice that in the limit of exact $SU(2)_V$ invariance $\Delta\hat{r}_W = \Delta\hat{r}_Z$, see eqs.(12) and (7). It is easy to see that in that case m_W can be predicted exactly from m_Z, even if the absolute size of $\Delta\hat{r}_W = \Delta\hat{r}_Z$ is unknown; this is inituitively clear, since the W and Z reside in the same triplet of $SU(2)_V$. However, eqs.(9) and (11) explicitly depend on $sin^2\theta_W$ in a nontrivial way. Therefore in the real world where $sin^2\theta_W$ is not zero, new physics contributions will in general change the prediction of m_W even if they do not break $SU(2)_V$ invariance; however, this kind of contribution will be suppressed by an additional factor of s^2.

Before closing this section one technical point has to be clarified: $\delta e/e$ depends on the dynamics of light quarks at zero external momentum, see eq.(2). In the naive quark model, this manifests itself in a logarithmic dependence on the light quark masses, which are not well defined. This difficulty can be circumvented by the following trick [5, 7]: Write

$$\lim_{q^2\to 0}\frac{\Pi_{\gamma\gamma}^T(q^2)_5}{q^2} = \left[\lim_{q^2\to 0}\frac{\Pi_{\gamma\gamma}^T(q^2)_5}{q^2} - \frac{\Pi_{\gamma\gamma}^T(m_Z^2)_5}{m_Z^2}\right] + \frac{\Pi_{\gamma\gamma}^T(m_Z^2)_5}{m_Z^2}, \tag{14}$$

where the subscript 5 refers to the contribution from the 5 light quarks. The expression in square brackets is related to the total cross section for $e^+e^- \to hadrons$, via a dispersion relation; using experimental data it can be computed to [21] 0.0288 ± 0.0009. The second term in eq.(14), as well as the contributions from leptons and the bosonic sector of the SM, can be computed reliably in perturbation theory. Notice that the quantity $\Pi_{WW}^T(0)$ in eqs.(12) does not suffer from this kind of ambiguities, since the contributions from light quarks are proportional to their masses and hence negligible. Finally, it should be mentioned that 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections due to the top quark have been computed recently [17]; present experiments are not sensitive to them, but ultimately it might be necessary to include such terms in the analysis [22]. In the present framwork, this can most easily been done by subtracting the (negative) $\mathcal{O}(\alpha\alpha_s)$ contribution to $\delta\rho$ shown in eq.(8) from $\Delta\hat{r}_Z$, which contains a term $-\delta\rho$; $\Delta\hat{r}_W$, which depends only logarithmically on m_t, need not be modified. In this treatment small terms of $\mathcal{O}(\alpha\alpha_s \log m_t^2/m_W^2)$ are ignored.

5) $Z \to f\overline{f}$ decays

In this section the decay of a Z boson into light SM fermions (other than b-quarks) is discussed. For a full treatment of LEP physics one should really start from the full $e^+e^- \to f\overline{f}$ amplitude, including the γ-exchange diagram and QED radiative corrections; details can, e.g., be found in the recent CERN Yellow Books [1]. However, these QED effects are routinely "undone" by experimenters themselves, who prefer to publish data on the $Z \to f\overline{f}$ partial widths, electroweak asymmetries and/or the effective axial and vector couplings of the fermion f discussed below. To understand these measurements at the one-loop level, one only has to study the decay of on-shell Z bosons.

In the following discussion I will again stick to the $\overline{MS}$ scheme, making use of the results of ref.[23]. There the following expression for the $Z \to f\overline{f}$ partial width can be found:

$$\Gamma(Z \to f\overline{f}) = N_{C_f} \frac{\hat{e}^2}{\hat{s}^2\hat{c}^2} \frac{m_Z}{96\pi} \left[1 - \left(\frac{d}{dq^2} \Pi^T_{ZZ}(q^2) \Big|_{q^2=m_Z^2} \right)_{\overline{MS}} + \frac{\hat{e}^2}{16\pi^2 s^2} \overline{V}_{ff} \right] \sqrt{1 - \frac{4m_f^2}{m_Z^2}}$$
$$\cdot \left\{ 2 \left[\frac{1}{4} + \left(\frac{1}{2} - 4 I_{3_f} Q_f \hat{s}^2 \hat{k}_f \right)^2 \right] - \frac{m_f^2}{m_Z^2} \left[1 + 16 I_{3_f} Q_f s^2 - 16 Q_f^2 s^4 \right] \right\}. \quad (15)$$

The various factors in eq.(15) are defined as follows:

$$N_{C_f} = (3.12 \pm 0.02) \cdot \left(1 + \frac{3\alpha Q_f^2}{4\pi} \right) \quad (16)$$

includes QCD and final-state QED corrections (for leptons, the first bracket has to be replaced by 1, of course);

$$\hat{e}^2 = \frac{e^2}{1 + (\frac{2\delta e}{e})_{\overline{MS}}} \quad (17)$$

is similar to the familiar "running coupling" $\overline{e}^2(m_Z)$, but includes effects from heavy particle loops. $\hat{c}^2$ and $\hat{s}^2$ have already been introduced in the previous section. The derivative of the ZZ two-point function enters due to regularized vacuum polarization diagrams [23], which give an $\frac{1}{q^2 - m_Z^2}$ propagtor; in the given case $q^2 = m_Z^2$ this yields the derivative of $\Pi^T_{ZZ}(q^2)$ at $q^2 = m_Z^2$, as shown. Notice that this derivative does not contain large logarithms or m_t^2 dependent terms. The term

$$\overline{V}_{ff} = -4c^2 \log c^2 + \left[1 - 2s^2 \left(1 - 2 I_{3_f} Q_f \right) \right] f(\frac{1}{c^2}) + 8c^2 g(\frac{1}{c^2})$$
$$- \left(1 - 12 s^2 I_{3_f} Q_f + 12 s^4 Q_f^2 \right) \frac{f(1)}{2c^2} \quad (18)$$

contains the part of the vertex corrections which is proportional to the Z-current. The functions f and g are in general complicated functions [23] of the ratios of q^2 and m_W^2 or m_Z^2, with the property that they vanish as $q^2 \to 0$. Here they are only needed at

$$f(1) = -1.08,\ f(\frac{1}{c^2}) = -1.18,\ g(\frac{1}{c^2}) = -0.22. \quad (19)$$

Finally, $\hat{k}_f$ is given by

$$\hat{k}_f = 1 + \frac{c}{s} \frac{\left(\Pi^T_{\gamma Z}(m_Z^2) + \Pi^T_{\gamma Z}(0) \right)_{\overline{MS}}}{m_Z^2} - \frac{\hat{e}^2}{16\pi^2 s^2} v_f(m_Z^2), \quad (20)$$

where $v_f(m_Z^2)$ contains the remaining vertex corrections:

$$v_f(m_Z^2) = \frac{1}{2} \left[1 - 2s^2 \left(1 - 2 I_{3_f} Q_f \right) \right] f(\frac{1}{c^2}) + 4c^2 g(\frac{1}{c^2})$$
$$- \left(1 - 12 s^2 I_{3_f} Q_f + 8 s^4 Q_f^2 \right) \frac{f(1)}{4c^2}. \quad (21)$$

In eqs.(15) to (21), I have neglected loop corrections to the terms proportional to the squared mass m_f^2 of the final state fermions, as well as all attempts at re-summing higher-order corrections *except* for the large logarithms contained in $\hat{e}^2$, which are not entirely negligible even at present experimental precision. Finally, I mention again that these expressions are *not* valid for the $b\overline{b}$ final state, where the vertex corrections also contain terms that depend quadratically on the top mass.

Eq.(15) can be cast into a form in which all radiative corrections are contained in properly defined effective axial and vector couplings $\overline{g}_{A_f}$ and $\overline{g}_{V_f}$; in fact, often experimenters directly publish values for these effective couplings, as extracted from measurements of the $Z \to f\overline{f}$ partial widths and forward-backward asymmetries. The $Z \to f\overline{f}$ decay width would then, e.g., be written as (for $m_f = 0$):

$$\Gamma(Z \to f\overline{f}) = N_{C_f} G_\mu \frac{m_Z^3}{6\sqrt{2}\pi} \left(\overline{g}_{A_f}^2 + \overline{g}_{V_f}^2 \right). \quad (22)$$

Comparing this to eq.(15) and using eqs.(11) and (12b) one has:

$$\overline{g}_{A_f}^2 = \frac{1}{4} \left[1 - \Delta\hat{r}_Z + \left(\frac{2\delta e}{e} \right)_{\overline{MS}} \right] [\ldots]; \quad (23a)$$

$$\overline{g}_{V_f}/\overline{g}_{A_f} = 1 - 2Q_f \hat{s}^2 \hat{k}_f / I_{3_f}, \quad (23b)$$

where $[\ldots]$ stands for the first square bracket in eq.(15). Notice that the $Z \to f\overline{f}$ width in eq.(22) has been expressed in terms of G_μ, rather than in terms of α; in this way one absorbs the large logarithms contained in $\delta e/e$, which are therefore not part of the effective axial and vector couplings. These couplings only contain genuine electroweak corrections. In particular, the first square bracket in eq.(23a) contains the contributions of eq.(8) to $\delta\rho$, plus additional, logarithmic terms which are absent at zero external momentum. In contrast, $\hat{k}_f - 1$, defined in eq.(20), contains no m_t^2 terms and is thus quite small, at least in the SM. $\hat{s}^2$ is therefore numerically very close to the "effective" quantity [23] $sin^2\theta_{eff} = \hat{s}^2\hat{k}_e$, as well as to $sin^2\theta^*$ of Lynn et al. [24].

6) Where do we stand?

The main objective of electroweak precision experiments is to "test the Standard Model". In order to do so, one obviously needs precise SM predictions for measureable quantities, which are then compared with actual measurements. The formulae presented in the previous four sections, together with the expressions listed in the Appendix, should allow to do this for all quantities that are presently of interest, with the exception of the $Z \to b\overline{b}$ width.

It should be noted that the dependence on the unknown masses of the top quark and Higgs boson originates solely from gauge boson two-point functions. In Figs. 1 a-c I therefore show predictions for two combinations of two-point functions that are directly related to measureable quantities, as well as for the physical mass of the W boson. The first combination of two-point functions is the parameter $\delta\rho$ defined in eq.(6), which can easily be computed from eq.(8). The second combination shown is $s^2(1+\delta\kappa) = \hat{s}^2(1+\delta\hat{\kappa})$, where $\delta\hat{\kappa}$ is the process-independent part of $\hat{k}_f - 1$:

$$\delta\hat{\kappa} = \frac{c}{s} \frac{\left(\Pi^T_{\gamma Z}(m_Z^2) + \Pi^T_{\gamma Z}(0)\right)_{\overline{MS}}}{m_Z^2}. \tag{24}$$

(Strictly speaking, this quantity also contains vertex and box corrections to muon decay, since $\hat{s}^2$ is computed from m_Z using eq.(13).)

It is important to notice that the leading correction in all three cases is proportional to m_t^2 (apart from the large log $\frac{m_t}{m_Z}$ terms, which are known exactly). Therefore these three quantities are strongly correlated, at least within the SM. In order to emphasize this point figs.1 show the SM predictions as curves in the planes spanned by any two of these three quantities, which result when either m_t is varied between 90 and 210 GeV for constant $m_H = 50$, 250 or 1000 GeV (solid lines), or m_H is varied between 50 and 1000 GeV for constant $m_t = 90$, 110, 130, 150, 170, 190 or 210 GeV (dashed curves). All curves have been obtained using the $\overline{MS}$ scheme. The QCD corrections to $\delta\rho$ have been included both in $\delta\rho$ and in $\Delta\hat{r}_Z$, as discussed in sec.4, and hence also in the predictions for m_W and $\hat{s}^2$; recall also that eq.(11) automatically includes leading higher order corrections. Although QCD corrections to terms that depend only logarithmically on m_t have not been included, the resulting predictions for m_W differ by at most 10 MeV from those of ref.[22], where these corrections have been taken into account in the on-shell scheme.

Notice that $\overline{g}_{V_f}/\overline{g}_{A_f}$ of eq.(23b) can be computed from $\hat{s}^2(1+\delta\hat{\kappa})$ by simply adding the vertex correction of eq.(21). Finally, eq.(23a) can be approximately written as

$$\overline{g}^2_{A_f} \simeq \frac{1}{4}\left(1 + \delta\rho + 2.2\cdot 10^{-3} + \frac{\hat{e}^2}{16\pi^2 s^2}\overline{V}_{ff}\right), \tag{25}$$

where $\overline{V}_{ff}$ is given by eq.(19). The smallness of the (approximately) constant term in eq.(25) results from an "accidental" cancellation between the vertex and box corrections to $\Delta\hat{r}_Z$ and the contributions to eq.(23a) that do not depend quadratically on m_t. Within the SM, eq.(25) is accurate to better than 1 part in 10^3, which will be sufficient for all practical purposes for some time to come.

It should be emphasized that predictions as those shown in figs.1 only became possible after the SLC and LEP provided us with a measurement of m_Z with a precision of 2 parts in 10^4; this error is negligible compared to the effects we're looking for. We therefore now know three basic electroweak quantities (α, G_μ and m_Z) precisely, i.e. with (at present) negligible error; the remaining uncertainty in electroweak calculations is then practically exclusively due to the as yet unknown size of quantum corrections from loops involving top quarks and Higgs bosons. *Any* additional measurement can therefore in principle tell us something about the masses of these particles.

Figs.1 show that if one measures any one quantity precisely, one can predict m_t (with a residual uncertainty of about ± 15 GeV due to the Higgs contribution); one can also predict the other two quantities with a relative accuracy of $1-2\cdot 10^{-3}$. This implies

that a combination of two measurements has a much better chance to rule out the SM than both measurements taken separately; the SM is ruled out if the point made up by these two measurements clearly lies outside the region enclosed by the outermost solid and dashed curves in fig.1, even if each measurement by itself falls within the range that is acceptable for the SM. We also see that one has to measure at least one quantity with a relative error of at most 10^{-3} if one wants to see the effect of the SM Higgs boson; in fact, as long as we don't know m_t from other sources (e.g., top production at colliders), *two* quantities will have to be measured with 10^{-3} precision, a truely formidable task.

The real situation is somewhat different, of course. Many quantities have been measured that are affected by electroweak radiative corrections. I don't want to give a list of the most recent values of the W mass, Z partial widths etc., since all these numbers are expected to change in the near future anyway. It is fair to say, however, that (apart from α, G_μ and m_Z) no single measurement has a precision much better than 1%. This is true even for the measurements of m_W if we allow for the fact that the central values reported by CDF [25] and UA2 [26] differ by more than one standard deviation; simply averaging these numbers might thus be dangerous. In order to get a reasonably accurate prediction of m_t within the SM, one therefore has to combine as many different measurements as possible into a global fit.

Several such fits have been performed in recent years. Ref.[27] was the first paper in which a precise measurement of m_Z was combined with older data from lower energies used in previous analyses [28], which are statistically dominated by νN DIS experiments. In ref.[27] the best value of m_t was 132 GeV, with an error of about $\pm 35\,GeV$. The most recent fit to all available data that I know of has been performed by Langacker [29]; he found

$$m_t = 139^{+33}_{-39} \pm 16\,GeV, \tag{26}$$

where the second uncertainty is from the Higgs mass. The weak mixing angle is determined to

$$sin^2\theta_W = 0.2272 \pm 0.004; \tag{27a}$$

$$sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2328 \pm 0.0010, \tag{27b}$$

which clearly indicates that θ_W defined in the $\overline{MS}$ scheme is less affected by unknown radiative corrections than the corresponding quantity in the on-shell scheme.[§]

Notice that the error on m_t has not been reduced by the inclusion of LEP data which were not available when ref.[27] was written. This is partly due to the fact that νN data favor a smaller value of m_t than do LEP data or the central value of m_W. This might well be purely accidental; the "discrepancy" between these two data sets is only about one standard deviation. On the other hand, it has recently been found [30] that such a pattern could emerge if supersymmetric contributions to electroweak radiative corrections

[§]This does not imply that the $\overline{MS}$ scheme is "better" than the on-shell or any other scheme. As already stated in sec.2, in principle one could formulate the SM without any θ_W, and express its predictions directly as relations between measureable quantities. The weak mixing angle simply allows a convenient way to compare results of different experiments with each other; of course, the question whether these experiments are compatible with each other and with the SM does not depend on the scheme one uses to analyze their data. For this kind of comparison a weak m_t dependence of θ_W seems advantegeous. It should be noted, however, that heavy particles do not decouple from $\hat{s}^2$even if they do decouple from physical processes at low energies. The minimal SM does not have such particles, but in supersymmetric theories half of the spectrum falls in this category.

are non-negligible, but one nevertheless tries to fit all data within the SM. The point is that superpartners, being heavy, tend to contribute less at lower energies, where νN experiments are peformed, as they do at higher energies relevant for LEP; the effects of SUSY loops is the same as that of increasing m_t. In any case, whether this "discrepancy" is accidental or not, it broadens the χ^2 distribution and hence increases the error on m_t. As a result, the 90% c.l. upper bound [29] of 192 GeV on m_t is almost identical to the bound derived [28] almost five years ago! One therefore has to conclude that all the data amassed during these five years did not help us to pin down one of the most important free parameters of the SM.

Another aspect of "testing the SM" is to derive bounds on new physics. Clearly the Tevatron and LEP have provided us with many new bounds on the masses of hypothetical particles as well as on *tree-level* couplings of well-known ones. However, as far as constraining new contributions to radiative corrections are concerned, the recent data have not yielded any interesting new bounds. This is partly due to the fact that in many "reasonable" models of new physics, the new contributions enter mostly through $\delta\rho$ - which is also true for the top-contribution in the SM, as we have seen; this implies that the bound on new positive contributions to $\delta\rho$ is strongly linked to the top-bound in the SM, which has not improved in recent years, as discussed in the previous paragraph. There are also a few cases (models with two Higgs doublets [31] or heavy Majorana neutrinos [32]) where new *negative* contributions to $\delta\rho$ are possible; in these models it is possible to relax the bound on m_t. Here LEP has yielded some new information, at least in principle [4]: If $\delta\rho$ were the only available quantity, one could cancel arbitrarily large top contributions against negative new contributions. However, the top quark affects the $Z \to b\bar{b}$ width, and hence also the total and hadronic widths of the Z, also via vertex diagrams that are *independent* of $\delta\rho$. This can be used to derive [33] an absolute upper bound on m_t of about 300 GeV, independent of additional negative contributions to $\delta\rho$. This also implies that new contributions to $\delta\rho$ cannot be more negative [34] than about -0.02, *unless* the $Zb\bar{b}$ vertex is also changed. While this is a new bound, it should be admitted that one has to work quite hard to get such large corrections in the first place. (This bound also applies for contributions to $\delta\rho$ from vacuum expectation values from Higgses in triplets or higher representations of SU(2), which could in principle be sizeable; however, the motivation for such models is very weak.)

Much has been made of the recent observation [35] that there can also be sizeable new physics contributions to

$$S = \frac{\left(\Pi^T_{WW}(0) - \Pi^T_{WW}(m_W^2)\right)_{\overline{MS}}}{m_W^2} \simeq \frac{\left(\Pi^T_{ZZ}(0) - \Pi^T_{ZZ}(m_Z^2)\right)_{\overline{MS}}}{m_Z^2}; \qquad (28)$$

this quantity is small in the SM (which is why $\bar{g}_{A_l}$ is essentially determined by $\delta\rho$), but, according to the authors of ref.[35], could be quite large in technicolor models. Since LEP data as well as recent data on atomic parity violation disfavor [35, 36] large S, one might be lead to the conclusion that these data disfavor or even rule out technicolor. This seems impressive. However, one has to keep in mind that simple technicolor is not a realistic model since it cannot produce fermion masses. This problem is solved in so-called Extended Technicolor (ETC) models [37], but one then automatically generates flavor changing neutral currents (FCNC). This used to be considered the death of the technicolor idea, until it was suggested [38] to increase the ETC scale, and thus suppress FCNC's, by changing the behaviour of the technicolor β-function ("walking technicolor"). It should also be mentioned that simple technicolor models, which are essentially scaled-

up versions of QCD, used to predict charged technipions with mass below 30 GeV; for the calculation of ref.[35], their masses were increased "by hand" to 50 GeV. The upshot of these two observations seems to be, at least to me, that "realistic" technicolor models are quite different from QCD (where the coupling doesn't "walk" but "runs"), contrary to the assumption of ref.[35]. In my opinion electroweak data only disfavor technicolor models that were ruled out a long time ago, but say very little about at least potentially realistic ETC models, which seem to require some genuinely new nonperturbative dynamics.

Nevertheless there is one area where LEP results have lead to great progress: We now know the electroweak $\overline{MS}$ gauge coupling to high accuracy, see eq.(27b); the crucial new input here is again the mass of the Z boson, which together with the bound $m_t \leq 200\,GeV$ suffices [7] to pin down $\hat{s}^2$ with an error of ± 0.002. Also, the "running" of the electromagnetic coupling from 1/137 at $q^2 = 0$ to 1/128 at $q^2 = m_Z^2$ is now an experimentally established fact. Using these inputs, the authors of ref. [39] found that in the minimal SM, the SU(3), SU(2) and U(1) gauge couplings do *not* meet at any scale. Again, this rules out a model (minimal SU(5)) that was already ruled out by bounds on proton decay, unless computations of hadronic matrix elements were off by more than an order of magnitude, which seems very implausible. To me at least the second result of ref. [39] is therefore more interesting: The minimal supersymmetric extension of the SM leads to perfect unification of the gauge couplings at an acceptable value of $M_X \simeq 10^{16}\,GeV$. Of course, one can construct non-supersymmetric models with intermediate scales and larger GUT groups, e.g. SO(10), that will still give acceptable unification. The beauty of this result is that the supersymmetric model has essentially *no* free parameter, if one believes the argument [40] that sparticle masses should not be much above 1 TeV.

The next few years should bring us close to the goal of measuring some quantities with relative precision of 10^{-3}. This seems most feasible for the mass of the W boson, where a precision of about 80 to 100 MeV should be reached by experiments at LEP200, in perhaps 5 to 6 years from now. Measurements by the CDF and D0 collaborations at the Tevatron will almost certainly shrink the error on m_W considerably from its present value before the first W pair is produced at LEP200; here the ultimate precision might well be determined by systematic uncertainties, like lack of knowledge of structure functions or of higher order QCD corrections.

A new major neutrino experiment has recently been proposed [41] for the Fermilab neutrino beam. One of its goals is to reduce the present errors on $sin^2\theta_W$ and $\delta\rho$ from neutrino experiments by a factor of three. This necessitates the accurate treatment of charm production, and various ingenious methods for this have been devised by the authors of ref.[41]. While the aimed-for precision is still not quite good enough to establish quantum corrections from the SM Higgs boson, one should keep in mind that the present bound on m_t is still "essentially" determined by the old neutrino experiments; and at least as far as the ρ-parameter is concerned, LEP will probably have a hard time to compete with the precision of $2.3 \cdot 10^{-3}$ that this experiment wants to achieve, as will be argued below. The drawback is that results from this experiment can only be expected in the second half of this decade. This is a pity, since conceptually $\delta\rho$ offers the simplest avenue for testing radiative corrections.

At present the errors on many quantities measured at LEP are dominated by systematic effects. This is most obvious for the mass of the Z boson, but here the error is already negligible anyway, at least for the time being. (As other errors continue to shrink, it would also be nice to reduce the error on m_Z, which seems possible.) More serious at present is the uncertainty of about 1% of the luminosity measurements, which feeds through to all decay widths. Part of the problem was lack of precision of the calculation for Bhabha-scattering, which is used as a reference process against which all other processes are measured. Fortunately, progress is being made in this field; experts are now confident that the theoretical uncertainty can be reduced to below 10^{-3}, should this become necessary. This leaves us with systematic experimental uncertainties, which at present are typically of order 0.5 to 1%. These are related to the lack of knowledge of the exact response of the small angle counters used for the luminosity measurements. Almost certainly these errors will also shrink with time, but it seems quite unlikely to me that they can be reduced to below 10^{-3}, which, as discussed above, is necessary to see the Higgs contribution. It should be noted that the luxury of having four full-fledged LEP experiments is quite helpful here, since it can reduce the total systematic error by a factor of two. Notice that the error on the luminosity measurement enters the precision with which $\delta\rho$ can be measured at LEP. This is because $\delta\rho$ enters cross sections at LEP only through the pre-factor $(1+\delta\rho+\cdots)^2 \simeq (1+2\delta\rho+\cdots)$, see eqs.(23) and (25), up to the contributions from photon exchange diagrams, which are small on or near the Z pole. ¶

Systematic errors can be greatly reduced by using ratios of widths, or asymmetries. The former would help to isolate the top loop contribution to the $Z \to b\bar{b}$ width, since in such ratios the $\delta\rho$-dependent prefactor of eq.(25) cancels. However, measuring this width even with a precision of 1% is far from easy. The problem is that, at least until now, b-quarks have to be identified via their leptonic decays, the branching ratio of which is known [19] only to a precision of 5%. This problem can be overcome in two ways: Either one measures this branching ratio directly at LEP, ‖ or one uses a different method to tag b-quarks. The first method would require a large sample of $b\bar{b}$ events where both b-quarks decay leptonically; one would thus need at least 5 million Z's if the $Br(b \to \mu)$ is to be measured to 1% accuracy. The microvertex detectors which are now installed at all LEP experiments will certainly help to find new ways to tag b-quarks, but it remains to be seen how well they will perform; in particular, charm events might prove a formidable background if one aims for 1% precision. It is noteworthy that at present the absolute error on the total hadronic width of the Z boson is about two times smaller than that of the $Z \to b\bar{b}$ width.

In any case, the only chance to achieve a precision of 10^{-3} at LEP seems to be to study asymmetries. Unfortunately, the experimentally easiest one, the forward-backward asymmetry of charged leptons, is quite small, since it is quadratic in $\overline{g}_{V_e}$; what is worse, its dependence on $\overline{g}_{V_e}$ is also quite small, since one probes a curve close to its minimum, where the slope is small. Again, b-quarks are theoretically promising [1], and again there is a practical problem: $B-\overline{B}$ mixing also contributes to this asymmetry; it will be quite difficult to separate these two contributions. The left-right asymmetry of charged leptons is linear in $\overline{g}_{V_e}$ and easy to measure, *if* one can collect at least 10,000 Z-events with longitudinally polarized beams. The prospects for this are uncertain. Finally, the τ polarization asymmetry [1] is also linear in $\overline{g}_{V_e}$, and its measurement does not rely on

¶The Bhabha process $e^+e^- \to e^+e^-$ is an exception. For very small scattering angles it is dominated by photon exchange in the t-channel, which is not affected by the type of corrections we're interested in; this is why this process is used as a standard, of course. At larger angles, one enters a region where both s-channel Z-exchange and t-channel photon exchange are important; in principle this allows one to determine the absolute strength of the Z-exchange contribution, i.e. $\delta\rho$, either via asymmetries or by comparing this process to other, pure Z-exchange reactions. Possible obstacles are the difficulty to compute QED corrections to this channel to high precision, and the need for excellent angular resolution for the electrons.

‖Measuring it at a B factory will not do, since here the mix of b-flavored hadrons will be quite different than at LEP; this could easily produce an error of 1%.

polarized beams. Recently the ALEPH collaboration announced [42] a first measurement of this quantity; the error on $\hat{s}^2$ from this measurment is still six times the world average of eq.(27b), but it should be noted that it measures $\hat{s}^2$ "directly" rather than via m_Z. However, at least 10^7 Z events are needed in order to reach the desired precision of 10^{-3} from this measurement alone.

So my conclusion is that there is a reasonable chance to determine both m_t and m_H - the latter with a large error - if we can get about 10^5 polarized Z's, at least 10^7 unpolarized ones and a few thousand W pairs. Of course, it is entirely possible that the top quark and/or the Higgs boson will be found in the next few years. Once m_t is known, playing with radiative corrections will become a completely different ball game: One will then be able to focus on looking for contributions from truely unknown particles, like the Higgs boson(s), rather than "merely" pin down the parameters of the Standard Model.

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Appendix A

In this appendix I give explicit analytic expressions for the B functions that appear in the vector boson two-point functions in eqs(5). Following Passarino and Veltman [8], I define the following form factors with dimensional regularization:

$$A(m^2) = \int \frac{d^Dk}{i\pi^2} \frac{1}{k^2 - m^2 + i\varepsilon}, \tag{A.1}$$

$$\{B_0, B_\mu, B_{\mu\nu}\}(q^2, m_1^2, m_2^2) = \int \frac{d^Dk}{i\pi^2} \frac{\{1, k_\mu, k_\mu k_\nu\}}{(k^2 - m_1^2 + i\varepsilon)[(k-q)^2 - m_2^2 + i\varepsilon]}, \tag{A.2}$$

$$B_\mu = q_\mu B_1, \tag{A.3}$$

$$B_{\mu\nu} = q_\mu q_\nu B_2 + g_{\mu\nu} B_{22}. \tag{A.4}$$

where the Bjorken-Drell metric has been used. (Note that the definitions (A.2) - (A.4) of the B functions differ from those of ref.[8] by signs.) The invariant functions B_0, B_1, B_2 are logarithmically divergent, A and B_{22} quadratically divergent. A and B_{22} must eventually cancel leaving only logarithmic divergences. The Feynman parametrization of the functions B_0, B_1, B_2 is

$$B_n(q^2, m_1^2, m_2^2) = \frac{\Delta}{n+1} - \int_0^1 dx x^n \ln[(1-x)m_1^2 + x m_2^2 - x(1-x)q^2 - i\varepsilon], \tag{A.5}$$

for $n = 0, 1, 2$ and

$$\Delta = \frac{2}{4-D} + \gamma_E - \ln\pi, \tag{A.6}$$

where γ_E is the Euler constant. In the $\overline{\mathrm{MS}}$ (modified minimal subtraction) renomalization scheme, the singular piece Δ in these functions is simply replaced by a logarithm of the unit of mass μ:

$$\Delta \xrightarrow{\overline{\mathrm{MS}}} \ln \mu^2. \tag{A.7}$$

It is convenient to define three more B functions

$$B_3(q^2,m_1^2,m_2^2) = B_1(q^2,m_1^2,m_2^2) - B_2(q^2,m_1^2,m_2^2), \tag{A.8}$$
$$B_4(q^2,m_1^2,m_2^2) = m_1^2 B_1(q^2,m_2^2,m_1^2) + m_2^2 B_1(q^2,m_1^2,m_2^2), \tag{A.9}$$
$$\begin{aligned} B_5(q^2,m_1^2,m_2^2) &= 4B_{22}(q^2,m_1^2,m_2^2) - A(m_1^2) - A(m_2^2) \\ &= \{(m_1^2+m_2^2-q^2)B_0 + 4q^2B_3 - 2B_4\}(q^2,m_1^2,m_2^2). \end{aligned} \tag{A.10}$$

All the two point functions of the minimal standard model and most of its extensions can be expressed compactly in terms of the four B functions, B_0, B_3, B_4 and B_5 which all satisfy the symmetry relation

$$B_n(q_1^2,m_2^2,m_1^2) = B_n(q^2,m_1^2,m_2^2) \;\; for \;\; n=0,3,4,5. \tag{A.11}$$

The finite part of these B functions can be defined as follows:

$$B_0(q^2,m_1^2,m_2^2) = \Delta - F_0(q^2,m_1^2,m_2^2), \tag{A.12}$$
$$B_3(q^2,m_1^2,m_2^2) = \frac{\Delta}{6} - F_3(q^2,m_1^2,m_2^2), \tag{A.13}$$
$$B_4(q^2,m_1^2,m_2^2) = \frac{m_1^2+m_2^2}{2}\Delta - F_4(q^2,m_1^2,m_2^2), \tag{A.14}$$
$$B_5(q^2,m_1^2,m_2^2) = -\frac{q^2}{3}\Delta - F_5(q^2,m_1^2,m_2^2), \tag{A.15}$$

where the F functions have Feynman parametrizations

$$F_0(q^2,m_1^2,m_2^2) = \int_0^1 dx \;\ln[(1-x)m_1^2 + xm_2^2 - x(1-x)q^2 - i\varepsilon], \tag{A.16}$$
$$F_3(q^2,m_1^2,m_2^2) = \int_0^1 dx\; x(1-x)\ln[(1-x)m_1^2 + xm_2^2 - x(1-x)q^2 - i\varepsilon], \tag{A.17}$$
$$F_4(q^2,m_1^2,m_2^2) = \int_0^1 dx[(1-x)m_1^2 + xm_2^2]\ln[(1-x)m_1^2 + xm_2^2 - x(1-x)q^2 - i\varepsilon], \tag{A.18}$$

and F_5 is expressed by the others as

$$F_5(q^2,m_1^2,m_2^2) = \{(m_1^2+m_2^2-q^2)F_0 + 4q^2F_3 - 2F_4\}(q^2,m_1^2,m_2^2). \tag{A.19}$$

It is therefore sufficient to give closed analytic expressions for the three F functions, F_0, F_3, and F_4. One has

$$F_0(q^2,m_1^2,m_2^2) = \ln(m_1m_2) - \delta\ln\frac{m_2}{m_1} - 2 + \beta L, \tag{A.20}$$

$$F_3(q^2,m_1^2,m_2^2) = \frac{1}{6}\ln(m_1m_2) - \frac{3\sigma-2\delta^2}{6}\delta\ln\frac{m_2}{m_1} - \frac{5}{18} - \frac{\sigma-\delta^2}{3} + \frac{1+\sigma-2\delta^2}{6}\beta L, \tag{A.21}$$

$$\begin{aligned} F_4(q^2,m_1^2,m_2^2) &= \frac{m_1^2+m_2^2}{2}[\ln(m_1m_2) - 2 + \beta L] \\ &+ \frac{m_1^2-m_2^2}{2}[(\delta^2-2\sigma)\ln\frac{m_2}{m_1} + \delta(1-\beta L)], \end{aligned} \tag{A.22}$$

where

$$\delta = \frac{m_1^2-m_2^2}{q^2}, \tag{A.23}$$
$$\sigma = \frac{m_1^2+m_2^2}{q^2}, \tag{A.24}$$
$$\beta = |1-2\sigma+\delta^2|^{\frac{1}{2}}, \tag{A.25}$$

and the function L is defined as

$$L(q^2,m_1^2,m_2^2) = \begin{cases} \frac{1}{2}\ln\frac{1+\beta-\sigma}{1-\beta-\sigma} - i\pi & \text{for } q^2 > (m_1+m_2)^2, \\ \frac{1}{2}\ln\frac{1+\beta-\sigma}{1-\beta-\sigma} & \text{for } q^2 < (m_1-m^2)^2, \\ \tan^{-1}\frac{1-\delta}{\beta} + \tan^{-1}\frac{1+\delta}{\beta} & \text{for } (m_1-m_2)^2 < q^2 < (m_1+m_2)^2. \end{cases} \tag{A.26}$$

It is useful to find several limiting cases of the above three functions. When one of the masses m_1^2 or m_2^2 is zero, one has

$$F_0(q^2,m^2,0) = \ln(m^2) - 2 + (1-r)[\ln|1-\frac{1}{r}| - i\pi\theta(q^2-m^2)], \tag{A.27}$$

$$F_3(q^2,m^2,0) = \frac{1}{6}\ln(m^2) - \frac{5}{18} - \frac{r-r^2}{3} + \frac{1-3r^2+2r^3}{6}[\ln|1-\frac{1}{r}| - i\pi\theta(q^2-m^2)], \tag{A.28}$$

$$F_4(q^2,m^2,0) = \frac{m^2}{2}\{\ln(m^2) - 2 + r + (1-r)^2[\ln|1-\frac{1}{r}| - i\pi\theta(q^2-m^2)]\}, \tag{A.29}$$

with

$$r = \frac{m^2}{q^2}. \tag{A.30}$$

When the two masses are equal, $m_1 = m_2$, the generic expressions (A.20)-(A22) allow smooth limits as $\delta \to 0$ at $q^2 \neq 0$. When both masses are zero, one finds

$$F_0(q^2,0,0) = -2 + \ln|q^2| - i\pi\theta(q^2), \tag{A.31}$$
$$F_3(q^2,0,0) = -\frac{5}{18} + \frac{1}{6}[\ln|q^2| - i\pi\theta(q^2)], \tag{A.32}$$
$$F_4(q^2,0,0) = 0. \tag{A.33}$$

Also important are the values at $q^2 = 0$:

$$F_0(0,m_1^2,m_2^2) = \ln(m_1m_2) - \frac{m_1^2+m_2^2}{m_1^2-m_2^2}\ln\frac{m_2}{m_1} - 1, \tag{A.34}$$

$$F_3(0,m_1^2,m_2^2) = \frac{1}{6}\ln(m_1m_2) - \frac{(m_1^2+m_2^2)(m_1^4+m_2^4-4m_1^2m_2^2)}{6(m_1^2-m_2^2)^3}\ln\frac{m_2}{m_1} + \frac{m_1^2m_2^2}{3(m_1^2-m_2^2)^2} - \frac{5}{36}, \quad (A.35)$$

$$F_4(0,m_1^2,m_2^2) = \frac{m_1^2+m_2^2}{2}\{\ln(m_1m_2) - \frac{m_1^4+m_2^4}{m_1^4-m_2^4}\ln\frac{m_2}{m_1} - \frac{1}{2}\}. \quad (A.36)$$

For specific mass values, this becomes

$$F_0(0,m^2,m^2) = \ln(m^2), \quad (A.37)$$

$$F_3(0,m^2,m^2) = \frac{1}{6}\ln(m^2), \quad (A.38)$$

$$F_4(0,m^2,m^2) = m^2\ln(m^2), \quad (A.39)$$

and

$$F_0(0,m^2,0) = \ln(m^2) - 1, \quad (A.40)$$

$$F_3(0,m^2,0) = \frac{1}{6}\ln(m^2) - \frac{5}{36}, \quad (A.41)$$

$$F_4(0,m^2,0) = \frac{m^2}{2}\{\ln(m^2) - \frac{1}{2}\}. \quad (A.42)$$

Notice that the expressions (A.27) - (A.42) for special cases in practice have to be used for finite ranges of the arguments. E.g., eq.(A.35) has to be expanded up to terms of fourth order to arrive at eq.(A.38); this indicates that large cancellations occur, which can give numerically wrong answers if (A.35) is used for cases where m_1 and m_2 are almost, but not exactly, equal. Similarly, in order to compute the derivative of B_5 at $q^2 = 0$, which enters $\delta e/e$, one should compute F_5 analytically, rather than numerically, from eq.(A.19), for very small q^2 and small *or* equal masses.

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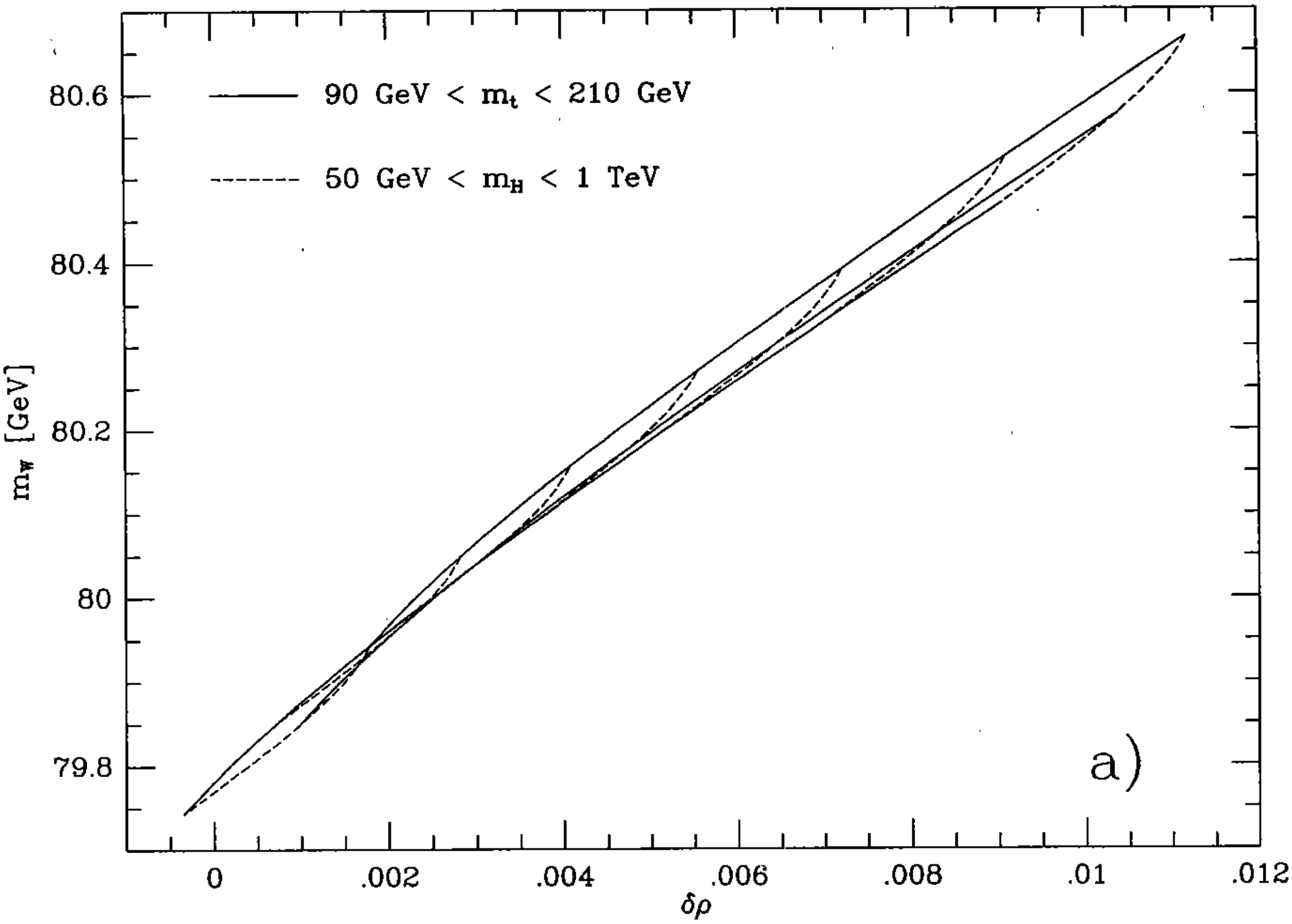
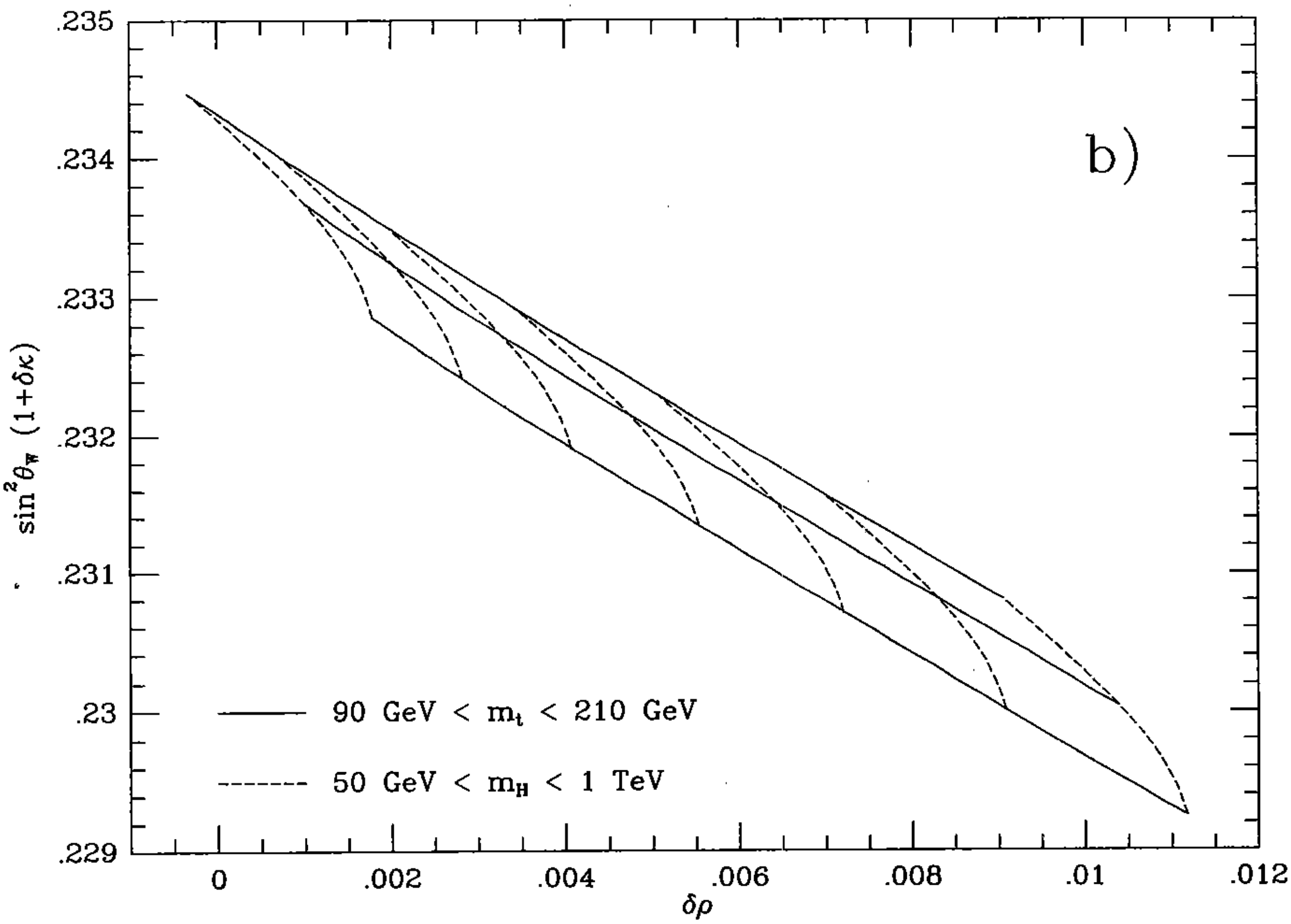
Figure Caption

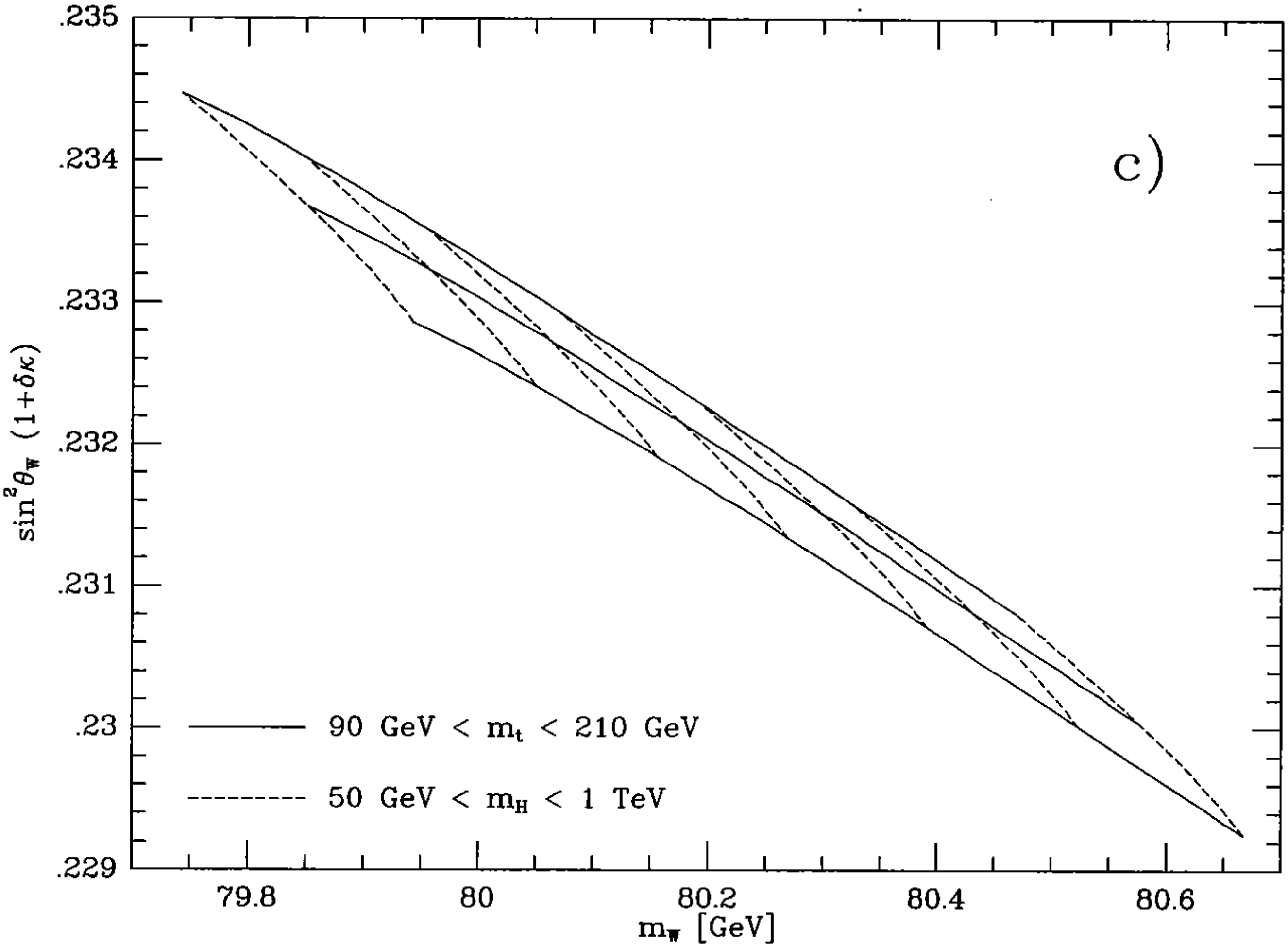
The figures show predictions of the SM in the planes spanned by $(\delta\rho, m_W)$ (a), $(\delta\rho, \hat{s}^2(1+\delta\hat{\kappa}))$ (b) and $(m_W, \hat{s}^2(1+\delta\hat{\kappa}))$ (c). The solid lines are curves of constant $m_H = 50$, 250 or 1000 GeV, where m_t is varied between 90 and 210 GeV; increasing values of m_H correspond to decreasing values of $\delta\rho$ and m_W, but increasing values of $\hat{s}^2(1+\delta\hat{\kappa})$. The dashed lines emerge when m_H is varied between 50 and 1000 GeV, for fixed m_t of 90, 110, 130, 150, 170, 190 or 210 GeV; increasing values of m_t correspond to increasing values of $\delta\rho$ and m_W, but decreasing values of $\hat{s}^2(1+\delta\hat{\kappa})$. The lower values of m_t and m_H are the experimental lower bounds that can be derived from the fact that these particles have not yet been detected at the Tevatron and LEP, respectively. The largest values of these masses used in the figure roughly correspond to theoretical upper bounds, as discussed in sec.3 for the Higgs boson and in sec.6 for the top quark. If a combination of measured values clearly lies outside of the region enclosed by the outermost solid and dashed lines the Standard Model would be ruled out, even if every single measurement can still be accommodated within the SM.



90 GeV < m_t < 210 GeV
50 GeV < m_H < 1 TeV
m_W [GeV]
80.6
80.4
80.2
80
79.8
a)
0
.002
.004
.006
.008
.01
.012
$\delta\rho$



.235
.234
.233
.232
.231
.23
.229
$\sin^2\theta_W\ (1+\delta\kappa)$
b)
90 GeV < m_t < 210 GeV
50 GeV < m_H < 1 TeV
0
.002
.004
.006
.008
.01
.012
$\delta\rho$


c)
.235
.234
.233
.232
.231
.23
.229
$\sin^2\theta_W\ (1+\delta\kappa)$
79.8
80
80.2
80.4
80.6
m_W [GeV]
90 GeV < m_t < 210 GeV
50 GeV < m_H < 1 TeV