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Renormalization Scales in Electroweak Physics

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Abstract

Theoretical predictions of photon radiation from quarks depend on the renormalization scale for the electromagnetic coupling constant at the corresponding $q\bar{q}\gamma$ vertex. We argue that in the case of real-photon emission there are no large logarithms to generate the running of that coupling. Consequently, the fine-structure constant as defined through Thomson scattering, $\alpha = 1/137$, provides a natural choice for the description of real bremsstrahlung phenomena.

1 Introduction

The physics of photon radiation from quarks is receiving considerable attention both experimentally and theoretically; for an overview see [1]. In particular, Z bosons are being produced so copiously at LEP 1 that measurements are starting to probe photonic corrections for the hadronic decay modes. The characteristic energy scale of Z-boson physics is clearly $q^2 = M_Z^2$. On the other hand, the four-momentum of a final-state photon satisfies $q^2 = 0$. The question which renormalization scale to use for the coupling of a real photon to a hadronic decay current is, therefore, a nontrivial one and deserves to be considered [2]. In the following we shall ventilate this question and propose an answer to it.

To start with, let us review some general features of electroweak perturbation theory. As a rule, the size of radiative corrections for a given process is determined by the discrepancy between the various mass and energy scales involved in that process. In electroweak physics the central energy scale is $q^2 = M_Z^2$. Sizeable corrections are expected to arise from particles which are either much lighter or much heavier than the Z boson itself. In the Minimal Standard Model the obvious candidates are light fermions, a heavy top quark, and a heavy Higgs boson. Light fermions are known to produce large logarithms of the form $(\alpha/\pi)\ln(M_Z^2/m_f^2)$, either via loop-insertions in internal gauge-boson lines or as collinear "divergencies". The first class of logarithms can in general be resummed to leading order in perturbation theory provided the considered boson mediates the main process. This effect can be taken into account by parametrizing the Born result in terms of running coupling constants. The second class is of kinematical origin. For illustration, consider the inverse propagator of an incoming electron after emission of a bremsstrahlung photon, $N = (p_e - p_\gamma)^2 - m_e^2 = -2p_\gamma^0\left(p_e^0 - |\mathbf{p}_e|\cos\theta\right)$. Integrating over the solid angle yields $\int d\Omega/N \approx -2\pi/\left(p_\gamma^0 p_e^0\right)\ln\left(2p_e^0/m_e\right)$. A heavy top quark does not decouple from electroweak processes but generates power corrections proportional to $(\alpha/\pi)m_t^2/M_Z^2$. The dominant corrections to the ρ parameter [3] and the M_W–M_Z shift are of this kind. Finally, a heavy Higgs boson is a potential source of large corrections. However, the effect is screened in processes which do not involve external Higgs-boson lines, i.e. the leading terms are then of the form $(\alpha/\pi)\ln\left(M_H^2/M_Z^2\right)$ at one loop and $(\alpha/\pi)^2 M_H^2/M_Z^2$ at two loops [4].

Let us return to the light-fermion loops. Their contribution to the dimensionless renormalized photon self energy (in the on-mass-shell scheme) reads

$$\pi(q^2) = \frac{\Pi_{AA}(q^2)}{q^2} - \Pi'_{AA}(0) = \frac{\alpha}{3\pi}\sum_f N_f Q_f^2\left[-\ln\frac{-q^2-i\epsilon}{m_f^2} + \frac{5}{3} + \mathcal{O}\left(\frac{m_f^2}{q^2}\right)\right], \qquad (1)$$

where $\Pi_{AA}(q^2)$ is the unrenormalized photon self energy and $\alpha = 1/137.0359895...$ is defined through Thomson scattering. The set of one-particle reducible diagrams form a geometric series. Performing the Dyson resummation yields

$$\frac{-i}{q^2} + \frac{-i}{q^2}\left(-iq^2\pi(q^2)\right)\frac{-i}{q^2} + \cdots = \frac{-i}{q^2}\frac{1}{1+\pi(q^2)}. \tag{2}$$

This effect can be implemented by using an effective coupling constant, $\alpha(q^2) = \alpha/(1+\pi(q^2))$.

An equivalent description is based on the renormalization-group equation of QED,

$$\mu^2\frac{\partial}{\partial\mu^2}\alpha(\mu^2) = \frac{\alpha^2(\mu^2)}{3\pi}\sum_{m_f<\mu} N_f Q_f^2. \tag{3}$$

Separating variables and integrating leads to the leading logarithmic approximation

$$\alpha(q^2) = \frac{\alpha\left(m_e^2\right)}{1-\alpha\left(m_e^2\right)/(3\pi)\sum_{m_f^2<q^2} N_f Q_f^2 \ln\left(q^2/m_f^2\right)}. \tag{4}$$

In summary, it suggests itself to absorb *oblique* (=propagator) corrections into Born couplings. In QED this is achieved through the substitution $\alpha \to \alpha(q^2)$. Alternatively, in electroweak physics one can replace $\alpha/s_w^2 \to \sqrt{2}G_F M_W^2/\pi$, where $s_w^2 = 1 - c_w^2 = 1 - M_W^2/M_Z^2$ and G_F denotes the Fermi constant [5]. Note that G_F is not running but fixed at the weak scale, $\mu = \mathcal{O}(M_Z)$.

2 Sirlin's Δr

The on-mass-shell (OMS) scheme is singled out for its transparent input parameters, namely α and the physical particle masses. However, the present experimental error on M_W induces a considerable uncertainty into theoretical predictions. One way out is to replace M_W by G_F, the best-known weak constant of nature, in the set of basic parameters. The technical term *modified* OMS (MOMS) scheme has become customary in this context.

2.1 Definition of Δr

Historically, G_F is defined as the fundamental coupling constant of the phenomenological Fermi Model. An expression of G_F in terms of the OMS parameters can be derived by comparison of the muon lifetime as calculated in the QED-improved Fermi Model with the corresponding analysis on the basis of the Standard Model. The radiatively corrected transition-matrix elements read

$$\mathcal{T}_{FM} = -\frac{G_F}{\sqrt{2}}(1+\delta_{em})\bar{u}_e\gamma^\rho(1-\gamma_5)v_{\nu_e}\,\bar{u}_{\nu_\mu}\gamma_\rho(1-\gamma_5)u_\mu\,, \tag{5}$$

$$\mathcal{T}_{SM} = \frac{e^2}{8s_w^2}\frac{1+\delta_{em}+\delta_{weak}}{0-M_W^2+\hat{\Pi}_{WW}(0)}\bar{u}_e\gamma^\rho(1-\gamma_5)v_{\nu_e}\,\bar{u}_{\nu_\mu}\gamma_\rho(1-\gamma_5)u_\mu\,, \tag{6}$$

respectively, where δ_{em} denotes the traditional photonic corrections,

$$\delta_{weak} = \frac{\alpha}{4\pi s_w^2}\left[\left(\frac{7}{2s_w^2}-2\right)\ln c_w^2 + 6\right] \tag{7}$$

comprises those vertex and box corrections which the Standard Model generates on top of the Fermi Model, and $\hat{\Pi}_{WW}(q^2)$ is the renormalized W-boson self energy. Equating Eqs. 5 and 6 yields

$$G_F = \frac{\pi\alpha}{\sqrt{2}\,s_w^2 M_W^2}\frac{1}{1-\Delta r}, \tag{8}$$

where $\Delta r = \hat{\Pi}_{WW}(0)/M_W^2 + \delta_{weak}$ was originally introduced by Sirlin [6].

2.2 Properties of Δr

It is useful to identify large contributions of distinct origins. For this end, one writes

$$\Delta r = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{rem}\,, \tag{9}$$

where $\Delta\alpha = -\mathcal{R}e\,\pi\left(M_Z^2\right)$, with $\pi(q^2)$ defined in Eq. 1, embodies the running of α from $q^2 = m_e^2$ to $q^2 = M_Z^2$,

$$\Delta\rho = \frac{3\alpha}{16\pi c_w^2 s_w^2}\frac{m_t^2}{M_Z^2} \tag{10}$$

is the leading correction to the ρ parameter, and

$$\Delta r_{rem} = \frac{11\alpha}{48\pi s_w^2}\ln\frac{M_H^2}{M_Z^2} + \cdots \tag{11}$$

collects the remaining parts, which are free of $\ln(M_Z^2/m_f^2)$ and m_t^2/M_Z^2 terms. For a comprehensive discussion of Δr beyond one loop we refer to Ref. [7] and references therein.

3 Example: $e^+e^- \to ZH(\gamma)$

In the following we shall discuss the difference between large logarithms due to electroweak loops and those due to bremsstrahlung and demonstrate how the first kind of logarithms can be removed. We shall do this on the basis of the so-called bremsstrahlung process, $e^+e^- \to ZH$, which is the dominant Higgs-boson production mechanism at future e^+e^- (super)colliders with centre-of-mass energies $\sqrt{s} \le 500$ GeV. Complete one-loop corrections for this process may be found in Ref. [8].

3.1 Electroweak Loops

We write the perturbation expansion in the form $\mathcal{T} = \mathcal{T}_0 + \mathcal{T}_1 + \mathcal{O}(\alpha^2)$, where

$$\mathcal{T}_0 = \frac{4\pi\alpha M_Z}{c_w s_w}\frac{1}{s-M_Z^2}\varepsilon_\mu^*(p_Z)\bar{v}(p_{e^+})\gamma^\mu(v_e - a_e\gamma_5)u(p_{e^-}) \tag{12}$$

is the Born result in the OMS scheme. Here $v_f = \left(I_f - 2s_w^2 Q_f\right)/(2c_w s_w)$ and $a_f = I_f/(2c_w s_w)$. For the one-loop contribution due to light fermions we find* $\mathcal{T}_1 = \mathcal{T}_0\,\delta$, where

$$\begin{aligned}\delta \;=\; & \Delta\alpha + \left(1 + \frac{c_w s_w}{c_w^2 - s_w^2}\frac{v_e Q_e}{v_e^2 + a_e^2}\right)\Delta r_{rem} \\ & + \frac{\alpha}{3\pi}\sum_f N_f\left[\left(v_f^2 + a_f^2\right)\left(\frac{1}{2} + \frac{s}{s-M_Z^2}\ln\frac{s}{M_Z^2}\right) + v_f Q_f\frac{v_e Q_e}{v_e^2+a_e^2}\ln\frac{s}{M_Z^2}\right].\end{aligned} \tag{13}$$

From Eq. 9 we know that $\Delta\alpha$ is strongly enhanced by large logarithms of the type $\ln(M_Z^2/m_f^2)$. Moreover, the ambiguity associated with the definition of the light-quark masses, m_q, induces an uncertainty proportional to $\delta m_q/m_q$ into the theoretical prediction. This huge and uninteresting effect can be absorbed into the Born coupling by adopting the MOMS scheme. Using Eq. 8, one writes $\mathcal{T}_0 = \bar{\mathcal{T}}_0(1-\Delta r)$, where

$$\bar{\mathcal{T}}_0 = \sqrt{2}G_F M_Z^3\frac{1}{s-M_Z^2}\varepsilon_\mu^*(p_Z)\bar{v}(p_{e^+})\gamma^\mu\left(2I_e - 4s_w^2 Q_e - 2I_e\gamma_5\right)u(p_{e^-}). \tag{14}$$

*Strictly speaking, $\delta = \mathcal{R}e\,\mathcal{T}_0^*\mathcal{T}_1/|\mathcal{T}_0|^2$.

Here the tilde labels MOMS expressions. Consequently,

$$\begin{aligned}\mathcal{T} &= \mathcal{T}_0(1+\delta)+\mathcal{O}(\alpha^2)\\ &= \tilde{\mathcal{T}}_0(1+\tilde{\delta})+\mathcal{O}(\alpha^2),\end{aligned} \tag{15}$$

where $\tilde{\delta} = \delta - \Delta r$ is manifestly free of mass singularities for $m_f \to 0$ and does not fake huge light-fermion effects any more.

3.2 Bremsstrahlung

Since the final state of $e^+e^- \to ZH$ is neutral, QED corrections to first order affect the incoming particles only. Initial-state corrections for the cross section of e^+e^- annihilation have been known for a long time [9]. They comprise virtual-photon exchange and (soft and hard) real bremsstrahlung, $\delta_{\rm QED} = \delta_{\rm virt} + \delta_{\rm soft} + \delta_{\rm hard}$. The individual contributions read

$$\delta_{\rm virt} = \frac{\alpha}{\pi}Q_e^2\left[-(L_e-1)\ln\frac{m_e^2}{m_\gamma^2} - \frac{L_e^2}{2} + \frac{3}{2}L_e + 4\zeta(2) - 2\right], \tag{16}$$

$$\delta_{\rm soft} = \frac{\alpha}{\pi}Q_e^2\left[(L_e-1)\ln\frac{4\Delta^2}{m_\gamma^2} - \frac{L_e^2}{2} + L_e - 2\zeta(2)\right], \tag{17}$$

$$\begin{aligned}\delta_{\rm hard} &= \frac{\alpha}{\pi}Q_e^2(L_e-1)\int_{s_{\rm min}/s}^{1-2\Delta/\sqrt{s}} dx\frac{1+x^2}{1-x}\sigma_0(xs)/\sigma_0(s)\\ &= \frac{\alpha}{\pi}Q_e^2(L_e-1)\left(\ln\frac{s}{4\Delta^2}+F\right),\end{aligned} \tag{18}$$

where $L_f = \ln(s/m_f^2)$, m_γ is an infrared regulator, Δ discriminates between soft and hard photons, $\sigma_0(s)$ is the Born cross section, and F is some function with values of $\mathcal{O}(1)$. The sum,

$$\delta_{\rm QED} = \frac{\alpha}{\pi}Q_e^2\left[(L_e-1)F + \frac{3}{2}L_e + 2\zeta(2) - 2\right], \tag{19}$$

is finite and linear in L_e. (The latter is in compliance with Sudakov's theorem [10], which states that QED corrections in $\mathcal{O}(\alpha^n)$ assume the form of polynomials of degree n in L_f if $|s| \gg m_f^2$.) Note that this logarithm reflects a collinear divergency. It is of kinematical origin and has nothing to do with the running of α.

Let us take a look at initial-state corrections to second order in QED. A possible candidate for a diagram which generates a running α is the one which is obtained by inserting a light-fermion loop into the virtual-photon line connecting the incoming legs. Its contribution to the annihilation cross section reads [11]

$$\delta_{\rm virt} = \left(\frac{\alpha}{\pi}\right)^2 N_f Q_e^2 Q_f^2\left[-\frac{L_f^3}{18} + \frac{19}{36}L_f^2 + \left(\frac{2}{3}\zeta(2) - \frac{265}{108}\right)L_f - \frac{2}{3}\zeta(3) - \frac{19}{9}\zeta(2) + \frac{3355}{648}\right]. \tag{20}$$

Note that the L_f^3 term cancels when Eq. 20 is combined with the adjoint $f\bar{f}$ bremsstrahlung contribution so that Sudakov's theorem is indeed satisfied. Obviously, these logarithms cannot be taken into account by a simple rescaling of the coupling in the corresponding seed diagram. Here the photon virtuality is a loop variable, not a constant fixed by external kinematics as in Sec. 3.1. On the other hand, a fermion-loop insertion into a real-photon line is exactly compensated by its counterterm.

For the discussion of final-state bremsstrahlung, we can appeal to the Kinoshita-Lee-Nauenberg theorem [12], which states that large logarithms due to outgoing particles cancel in the total cross

section, i.e. when all mass-degenerate configurations are summed up. A well-known manifestation of this cancellation mechanism is the $\mathcal{O}(\alpha)$ QED correction for an $f\bar{f}$ final state, $\delta_{\rm QED} = 3\alpha Q_f^2/(4\pi)$. As the running of couplings should not be affected by phase-space integration, it is apparent that logarithms which may still be present in the differential cross section cannot be responsible for the running of some α.

We conclude that photon bremsstrahlung requires the Thomson α, not some running α, regardless in which part of the process the emission takes place. We have got the impression that this goes without saying in the relevant literature, too. Finally, we present a hand-waving argument in favour of this concept. The renormalization scale, μ, of the running α should be Lorentz invariant. The photon energy, q^0, is not a scalar but the invariant mass squared, q^2, is. This motivates the choice $\mu^2 = q^2$ and, in particular, $\mu = 0$ for real photons.

4 Conclusions

In electroweak perturbation theory large logarithms associated with light fermions can be due to loop insertions in internal boson lines or photon bremsstrahlung. The first class of logarithms can (and should) be eliminated by adopting the modified on-mass-shell (MOMS) scheme, which is a "natural" framework for electroweak physics. In this scheme the Born approximations for decay rates, cross sections, and other observables are parametrized in terms of the Fermi constant. The second class of logarithms are of kinematical origin and cannot be absorbed into running couplings. In the high-energy limit the perturbative expansion for initial-state radiation to $\mathcal{O}(\alpha^n)$ is a polynomial of degree n in the characteristic Sudakov logarithm, while the final-state correction to an inclusive process is free of large logarithms. In conclusion, there is no theoretical justification for assuming some coupling different from the Thomson α for real-photon emission.

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Photon Radiation in the Dipole Model and in the Ariadne Program

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Abstract

The treatment of final state photon radiation within the Dipole Cascade Model is presented together with the implementation in the Ariadne program. Some comparisons with conventional Parton Shower models are given.

Introduction

The Colour Dipole Model (CDM) [1] as implemented in the Ariadne program [2] has had considerable success in describing data from both e^+e^- [3] and lepto-production [4] experiments.

The CDM differs from conventional QCD cascade models in that it in a natural way correctly treats most QCD coherence effects by describing the bremsstrahlung in terms of radiation from colour dipoles between partons, instead of treating partons as independent emitters.

The CDM is based on the fact that a gluon emitted from a $q\bar{q}$ pair in an e^+e^- collision can be treated as radiation from the colour dipole between the q and $\bar{q}$, and that, to a good approximation, the emission of a second softer gluon can be treated as radiation from two independent dipoles, one between the q and g and one between the g and $\bar{q}$. In the CDM this is generalized so that the emission of a third, still softer gluon, is given by three independent dipoles etc.

A $q\bar{q}$ pair is, besides a colour dipole, of course also an Electro-Magnetic (EM) dipole, and the CDM can in a natural way be extended to also describe bremsstrahlung of photons from quarks.

The interesting point is then that there are now two competing processes; radiating a gluon from the colour-dipole or a photon from the EM dipole. This competition is governed by the Sudakov form-factor and it will be shown below that the photon radiation is very sensitive to the choice evolution (or ordering) variable used in this form-factor.

Photons in Ariadne

The implementation of photon emissions in Ariadne is a fairly straight forward procedure, however not completely trivial. The differential cross section for radiating a photon from a $q\bar{q}$ dipole is given by

$$\frac{d\sigma_\gamma}{dx_1 dx_3} = \frac{\alpha_{EM}}{2\pi} e_q^2 \frac{x_1^2 + x_3^2}{(1-x_1)(1-x_3)} \qquad (1)$$

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where $x_i = 2E_i/\sqrt{S_{dip}}$ are the energy fractions after the emission with the two quarks denoted 1 and 3. This cross section has the same form as the one for gluon radiation

$$\frac{d\sigma_g}{dx_1 dx_3} = \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_3^2}{(1-x_1)(1-x_3)} \qquad (2)$$

substituting α_s with α_{EM} and the colour factor 3/4 with the charge of of the quarks squared.

The ordering variable in Ariadne is taken to be the invariant $p_\perp^2$ of an emission defined as

$$p_\perp^2 = S_{dip}(1-x_1)(1-x_3) \qquad (3)$$

Using this in the Sudakov form-factor, the probability for emitting a photon from a $q\bar{q}$ pair can be written as

$$\frac{dP_\gamma(p_\perp^2, y)}{dp_\perp^2 dy} = \frac{\sigma_\gamma(p_\perp^2, y)}{dp_\perp^2 dy} e^{-\int_{p_\perp^2}^{p_{\perp max}^2} dk_\perp^2 (\mathcal{I}_\gamma(k_\perp^2) + \mathcal{I}_g(k_\perp^2))} \qquad (4)$$

where

$$\mathcal{I}_i(k_\perp^2) = \int_{y_{min}(k_\perp^2)}^{y_{max}(k_\perp^2)} dy \frac{d\sigma_i(k_\perp^2, y)}{dk_\perp^2 dy} \qquad (5)$$

and y is the invariant rapidity of an emission defined as

$$y = \frac{1}{2} \ln \frac{1-x_1}{1-x_3} \qquad (6)$$

In words, equation 4 says that the probability of emitting a photon at a phase-space point $(p_\perp^2, y)$ is equal to the naive cross section for photon emission at this point times the probability of not having any emissions of photons or gluons at a higher $p_\perp^2$.

In the variables $p_\perp^2$ and y, the cross sections in equations 1 and 2 can be approximately written as

$$d\sigma_\gamma = \alpha_{EM} d\ln(p_\perp^2) dy \qquad (7)$$

$$d\sigma_\gamma = \alpha_s d\ln(p_\perp^2) dy \qquad (8)$$

and the available phase-space can be approximately described as the triangular region in figure 1a. and the exponent in the Sudakov form-factor corresponds to the integral of the cross sections in the shaded region. Note that while the cross section for photon emission is basically constant over the whole phase-space, the cross section for gluon emission increases for small $p_\perp^2$ due to the running of α_s.

If at first a gluon is emitted from the $q\bar{q}$ dipole, which is very likely due to the relative smallness of α_{EM}, there will be many competing processes in the next stage; emitting a photon from the EM dipole between the q and $\bar{q}$, emitting a gluon from each of the two colour dipoles qg and gq, and splitting the gluon into a new $Q\bar{Q}$ pair.

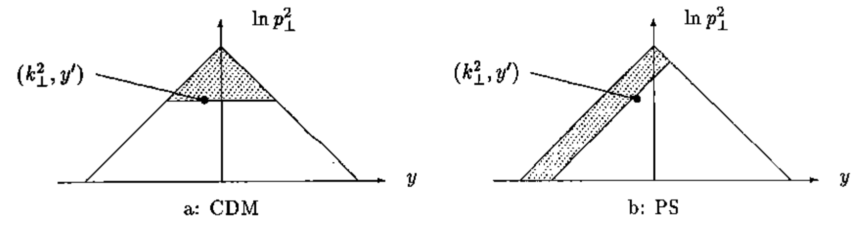

Figure 1: The phase space boundaries for dipole emission. The shaded area corresponds to the integration region in the Sudakov form-factor for the CDM (a) and PS (b) approach.

In Ariadne this is handled by first generating a $p_\perp^2$ and a y for each possible emission i according to

$$\frac{dP_i(p_\perp^2, y)}{dp_\perp^2 dy} = \frac{\sigma_\gamma(p_\perp^2, y)}{dp_\perp^2 dy} e^{-\int_{p_\perp^2}^{p_{\perp max}^2} dk_\perp^2 \mathcal{I}_i(k_\perp^2)} \tag{9}$$

and then performing the emission which gave the highest $p_\perp^2$.

As long as only gluons and photons are emitted, the CDM can treat EM bremsstrahlung in a well defined way using $p_\perp^2$ ordering. If however an additional $q\bar{q}$ pair is created in the cascade, the picture becomes very complicated. This would mean that further colour dipole emission would have to compete with EM quadrupole emission for which it is not possible to define a $p_\perp^2$ in the same way. Instead it may be argued that when an additional $q\bar{q}$ pair is created, the original EM dipole is screened and in Ariadne by default, the EM bremsstrahlung is simply switched off. Optionally it is possible to allow the original EM dipole to continue radiating. In any case, the photons that are emitted at this late stage in the cascade are usually drowned in the background of photons from hadronic decays in the jets.

It should be noted that the Ariadne program only describes final state photon radiation in e^+e^- collisions. Initial state photon radiation must be handled by the program performing the hard interaction, and no interferences between initial and final state is possible.

Comparisons with the Parton Shower Approach

Although the CDM is conceptually very different from the conventional parton shower (PS) approach, it can be shown that the two are basically equivalent in the limit of small $p_\perp$[1]. It is instructive to compare the CDM of Ariadne with the PS implemented in JETSET [5], looking only at the first emission of a gluon or a photon, as these two programs here use exactly the same naive cross section (equations 1 and 2).

In the JETSET PS the generation is performed in the variables Q^2 and z, related to x_1 and x_3 through

$$Q^2 = S_{dip}(1 - x_3) \tag{10}$$

$$z = \frac{x_1}{2 - x_3} \tag{11}$$

if the emission is from quark 1. The argument in α_s is for JETSET PS $Q^2 z(1-z)$ which is approximately the same as the $p_\perp^2$ used in Ariadne. The differences between the programs is instead found in the choice of ordering variable.

JETSET PS uses Q^2 as defined in equation 10 which gives the probability to emit a photon

$$\frac{dP_\gamma(Q^2, z)}{dQ^2 dz} = \frac{d\sigma_\gamma(Q^2, z)}{dQ^2 dz} e^{-\int_{Q^2}^{Q_{max}^2} dq^2 (\mathcal{I}_\gamma(q^2) + \mathcal{I}_g(q^2))} \tag{12}$$

where

$$\mathcal{I}_i(q^2) = \int_{z_{min}(q^2)}^{z_{max}(q^2)} dz \frac{d\sigma_i(q^2, z)}{dq^2 dz} \tag{13}$$

In this case the integration region in the Sudakov form-factor corresponds to the shaded region in figure 1b. Comparing with the situation for Ariadne in figure 1a, the first thing to note is that in Ariadne the emission of a photon only has to "compete" with gluon emission at higher $p_\perp^2$, while a corresponding photon emission in JETSET has to compete with gluon emissions with lower $p_\perp^2$ for which the cross section is increased due to the running of α_s. Hence the number of photons emitted in the first emission is larger in Ariadne than in JETSET.

Another consequence of the difference in ordering is that, looking at emissions at constant $p_\perp^2$, the ones generated by JETSET are shifted towards higher values of Q^2 as compared to the ones generated with Ariadne. In the same way, the $p_\perp^2$ spectrum at constant Q^2 is harder in Ariadne than in JETSET.

Conclusions

The analysis of final state radiation of photons in hadronic e^+e^- annihilation events can be used as a probe of the QCD cascade process. It is clear that the yield of photons depend on the choice of evolution variable in the cascade and significant differences have been found comparing the Ariadne and JETSET Monte-Carlo programs [6]. The statistics accumulated so far at LEP is however not yet sufficient to decide which of the two approaches is more correct.

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