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E.V.Shuryak

THE ROLE OF INSTANTONS IN QUANTUM CHROMODYNAMICS I. PHYSICAL VACUUM

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THE ROLE OF INSTANTONS IN QUANTUM CHROMODYNAMICS I.

PHYSICAL VACUUM

E.V.Shuryak

Institute of Nuclear Physics

Novosibirsk 630090

Abstract

We argue that in physical QCD vacuum the "instanton liquid" develops with the instanton density $dn/d\rho \simeq n_c\,\delta(\rho-\rho_c)$ where $\rho_c \simeq 1/600$ Mev and $n_c \simeq 8\cdot 10^{-4}$ Gev^4, corresponding to empirical value of the gluon condensate. Such parameters lead to chiral symmetry breaking and give $\langle\bar\Psi\Psi\rangle \simeq -1\cdot 10^{-2}$ Gev^3 and quark effective mass $\simeq 200$ Mev, which is very reasonable. It is shown that factorization works well for quark condensate but is strongly violated for gluon one. No place is seen for first order transition in external field, while the instanton-induced charge renormalization is in agreement with strong coupling expansion. Important to all that is that instantons occupy only about 1/20 part of the space, so they are not deformed too much. This can be traced to the presence of light quarks, so such property is absent in gluodynamics and large N_c case, being therefore very different from real QCD.

1. Introduction

The main problem of strong interaction theory is that of vacuum structure. The studies of last several years have shown that it is very complicated medium filled by strong nonperturbative fluctuations of quark and gluon fields.

Their semiclassical treatment is based on the famous topologically nontrivial solution of Yang-Mills equations found by Belavin, Polyakov, Schwartz and Tyupkin [1], the so called instanton. In another classical paper by t'Hooft [2] the pre-exponent has been calculated, and the instanton physics was open for applications.

Callan (jr.), Dashen and Gross [3] have considered wide range of instanton-induced effects, connected with U(1) problem, chiral symmetry breaking, charge renormalization etc. In their later works they have considered instantons in external field [4] in which they have found first order phase transition, and compared the instanton-induced renormalization with strong coupling expansion [5]. They have systematically used the so called dilute gas approximation. However later studies (see for example [6]) have shown that its validity is more restricted than assumed by CDG, therefore some results become questionable.

The works by Shifman, Vainshtein and Zakharov [7,8] have pointed in the same direction. They have considered the effect of quark and gluon condensates on the instanton density. Their conclusion is that the effects under consideration are very strong and to speak about instantons with radii larger than $\rho_{max} \simeq 1/500$ Mev is meaningless for they are deformed too much.

The main problem of the instanton calculus is the stabilization of rapidly growing density $dn/d\rho$ at some critical value ρ_c and the calculation of the structure of resulting "instanton liquid". This problem is very complicated and remains open so far.

The very important aspect of this problem, which we discuss in the present paper, is the connection between ρ_c and ρ_{max}. If the former is larger, the instantons "are melted" in physical vacuum and do not exist as well defined objects. In the opposite case one still has some liquid, not gas, but at least made of well defined constituents - the instantons. Our knowledge concerning ordinary condenced matter tell us that both possibilities may happen to exist, depending on the dynamical details.

Using some theoretical considerations and the empirical value of the gluon condensate we argue in this paper that ρ_c is much smaller than assumed by previuos works: $\rho_c \simeq 1/600$ Mev $\simeq 1/6\ \Lambda$. It has very important consequences.

First, instantons occupy rather small fraction $f \simeq 1/20$ of the space and therefore are deformed reasonably weakly. Therefore, calculations with instantons are aproximate but still valuable.

Second, ρ_c introduces new important scale in the theory, different from the confinement length $R_{conf} \simeq 1/200$ Mev. As a result, one can speak about two different components of vacuum and hadronic structure. In the first case it is the "instanton liquid" and more long-wave fluctuations, leading to confinement and not discussed in this work. For hadrons these two componets are the constituent quarks and hadrons.

Of course, these two components are not completely independent. In particular note, that R_{conf} is close to instanton separation in the "instanton liquid", and long-wave fluctuations considered now in lattice studies as some closed electric strings can, in principle be also treated as some collective excitations of the "instanton liquid".

Below we discuss chiral symmetry breaking along this line. We remind the reader that for more than one quark flavour it can not happen at one-instanton level and some medium with finite density of instantons and antiinstantons is needed in order to produce it. We show that our parameters are sufficient for this purpose. Moreover, the calculated values for quark condensate and effective quark mass turns out to be in agreement with what is known phenomenologically. The rather nontrivial fact is the so called factorization property, also reproduced with our parameters, which tells us that the quark condensate is very homogeneous in space-time. On the contrary, quantities connected with single instantons like the gluon condensate $(G^a_{\mu\nu})^2$ are very ingomogeneous and for them factorization is strongly violated. This prediction is not tested so far.

In this and the next paper we show that the picture of two components is in agreement with vast phenomenology of vacuum and hadronic structure. More deep question is about the physical reasons of vacuum "diluteness", reflected in smallness of packing fraction f mentioned above. We suggest that the answer is connected with the presence in real QCD of several light quark flavours, which are known to suppress instantons. For example, reducing the mass of the strange quark $m_s \simeq 150$ Mev to zero makes f to become two times smaller. Going in the opposite direction and increasing the quark masses we rather soon found no small parameter f and have to conclude that in pure gluodynamics the physical picture of vacuum and hadrons is completely different. The same conclusion is also valid for large N_c limit [15] . So, the imaginary worlds invented to make the theory simpler are not only far from reality, but also more complicated as far as instanton-induced effects are considered. And we argue below, that such effects in fact dominate in many aspects of strong interactions.

Now we come to more detailed description of the contests of the present work, connected entirely with vacuum structure. The questions conserning hadrons and superdense matter are discussed in its second part as separate publication.

The introductory section 2 collects known formulae about free instantons, while their interaction is discussed in section 3. We give brief discussion of relevent effects such as dipole forces between instanton and aniinstanton [3] , interaction with quark and gluon condensates [7,8] and of less clear core effects. The conclusion is that in general we have something similar to that common for ordinary molecules: weak attraction at large distances and strong repulsion at small ones. Driven by this analogy, some "instanton liquid" is proposed, discussed in section 4. In small scale it has some structure, presumably close to best packing one - 4-dimensional cubic lattice with alterating instantons and antiinstantons. Using as input the empirical value of the gluon condensate [9] we find the following parameters

$$n_c = 8 \cdot 10^{-4}\ \mathrm{Gev}^4 \qquad \rho_c = 1/600\ \mathrm{Mev} \simeq 1/3\ \mathrm{fermi}$$

The next section 5 is devoted to the problem of spontaneous chiral symmetry breaking, in which we found that prameters given above correspond to $\langle\bar{\psi}\psi\rangle \simeq -10^{-2}$ Gev^3 and quark effective mass about 200 Mev, which is very reasonable. More complicated vacuum average values of other operators are briefly discussed in section 6.

Section 7 contains the comparizon with two well known papers by Callan, Dashen and Gross [4,5]. We have found rather small vacuum permeability $\mu \lesssim 1.4$ and therefore no place for first order phase transition in the external field [4]. As for the comparizon with strong coupling expansion, we find that our parameters of instanton density give effect even in better agreement with [13] than original results of the work [5]: instead of crossing of two curves, the strong coupling and instanton-induced ones, the latter turns just in the proper place.

Section 8 is devoted to brief discussion of the imaginary worlds with different quarks and the number of colours. Very strong effect of the strange quark mass, observed in [25], is explained by the instanton suppression by fermion determinant. It is explained that gluodynamics and multicolour QCD seem to be rather different from the real case as far as instanton-induced effects are considered.

The work is summarized and some discussion of open questions is made in section 9.

2. Free instantons

This section is introductory in character and contains known formulae to be used below.

The word instantons is used for particular solution of classical Yang-Mills equations, found by Belavin, Polyakov, Schwartz and Tyupkin [1] . This solution corresponds to Euclidean space-time rather than Minkovsky one, for it describes in semiclassical way the probability of tunneling through some barrier in field configuration space between topologically different classical vacua.

Simple way to find this solution and even extend it to wider class with arbitrary topological number was suggested by t'Hooft [14].Let us start with the following anzatz

$$A^a_\mu = \bar{\eta}^a_{\mu\nu} \partial_\nu \ln \phi \qquad (1)$$

$$\bar{\eta}^a_{\mu\nu} = (-1)^{\delta_{\mu 0} + \delta_{\nu 0}} \eta^a_{\mu\nu} \; : \; \eta^a_{mn} = \epsilon_{amn} \; ; \; \eta^a_{m0} = \delta^a_m$$

Simbols $\bar{\eta}^a_{\mu\nu}$ correspond to SU(2) colour group, a=1,2,3 and its meaning and properties can be found in [2] . Yang-Mills equations are fulfilled if $\Box \phi$ =0 and this equation has the obvious solution

$$\phi = 1 + \sum_{i=1}^{n} \frac{a_i^2}{(x - z_i)^2} \qquad (2)$$

interpreted as n instantons in points z_i . The n=1 case gives the following one-instanton configuration:

$$A^a_\mu(x) = \frac{2}{g} \cdot \frac{\bar{\eta}^a_{\mu\nu}(x-z)_\nu \rho^2}{(x-z)^2[(x-z)^2+\rho^2]} \qquad (3)$$

which is the instanton in the so called singular gauge. The singularity in the instanton center in (3) is pure gauge, but it allows to have strong decrease of A^a_μ at $x\to\infty$. This in turn makes expressions like "scattering on instanton" meaningful.

Solution (2) is the sum of independent terms (3) only for widely separated centers as compared to a_i, but it turns out that the action is exactly n times the action for (3). So, in a sense instantons do not interact.

However, there exists also other class of solutions with $\bar{\eta}^a_{\mu\nu}$ changed to $\eta^a_{\mu\nu}$, the so called antiinstantons. The interaction between instanton and antiinstanton is present and we discuss it in the next section.

The calculation of the preexponent has been made in classical t'Hooft paper [2]. His result is usually formulated [3] as the analog instanton gas with the following density

$$dn(\rho,z) = dz\,\frac{d\rho}{\rho^5}\,d_0(\rho)\,F$$

$$d_0(\rho) = C_{N_c}\left(\frac{8\pi^2}{g^2(\rho)}\right)^{2N_c}\exp\left(-\frac{8\pi^2}{g^2(\rho)}\right) \qquad (4)$$

$$C_{N_c} = \frac{4.6\,\exp(-1.68N_c)}{\pi^2\,(N_c-1)!\,(N_c-2)!}$$

The coupling constant $g(\rho)$ is ρ dependent, and such dependency can be taken [10] with two loop accuracy [16]:

$$\frac{8\pi^2}{g^2(\rho)} = b \ln\left(\frac{1}{\rho\Lambda}\right) + \frac{b'}{b} \ln\ln\left(\frac{1}{\rho\Lambda}\right) + O\left(\frac{1}{\ln(1/\rho\Lambda)}\right) \tag{5}$$

$$b = \frac{11}{3}N_c - \frac{2}{3}N_f \; ; \quad b' = \frac{17}{3}N_c^2 - \frac{13}{6}N_c N_f + \frac{1}{2}\frac{N_f}{N_c}$$

The number of flavours N_f should be taken equal to 3, and in the present work one may completely ignore heavy c,b... quarks.

Another comment is that the constant in (4) corresponds to Pauli-Villars regularization scheme, and if one like to use another one he should change it to $C'_{N_c} = C_{N_c} \cdot (\Lambda/\Lambda')^{b}$. We also remind relation of Λ parameters: $\Lambda_{PV} \simeq \Lambda_{\overline{MS}} \simeq 2.7\Lambda_{MS} \simeq 0.47\Lambda_{MOM} \simeq 39\Lambda_{LAT}$, where $\Lambda_{\overline{MS}}$, Λ_{MOM} are popular in the theory of hard processes and Λ_{LAT} in lattice studies. Modern data on short distance physics including deep-inelastic scattering and heavy quarkoniums imply that Λ_{PV} is of the order of 100 Mev, but the uncertainty is still rather large, say 50%. Still in this work we use Λ_{PV}=100 Mev in numerical estimates.

The factor F in (4) is not so far explained. It is connected with the so called fermion determinant, which introduces new important physics connected with light quarks and instantons. It has been first discovered by t'Hooft [2] that there exists non-trivial solution of the Euclidean Dirac equation for massless quark in the instanton field, the so called zero mode

$$\psi_0(x) = \frac{\rho}{\pi}\left[(x-z)^2+\rho^2\right]^{-3/2} \frac{(\hat{x}-\hat{z})}{\sqrt{(x-z)^2}} \left(\frac{1-\gamma_5}{2}\right)\chi$$

$$\chi = \text{const}, \quad \chi^+\chi = 1 \tag{6}$$

Because of this the fermionic determinant det($\hat{D}$) is zero, together with the instanton density. The nonzero amplitude is then the production of N_f pairs of quarks, so the instanton can be considered as some vortex with $2N_f$ quark legs. An attempt to close them in loops fails in chiral limit because $\bar{\psi}$ is right-handed and ψ - left-handed. Only the violation of chiral symmetry may help here, either explicit in form of quark mass insertion, or spontaneous in form of quark condensate. The latter possibility we discuss later, and in the former case factor F in (4) is equal to

$$F = \prod_{i=1}^{N_f} f(m_i \rho)$$

$$f(x) = 1.34\, x\,(1 + x^2 \ln x + \ldots) \qquad x \ll 1 \tag{7}$$

$$f(x) = 1 - \frac{2}{75 x^2} + \ldots \qquad x \gg 1$$

where $x = m\rho$. The first term in small x case was found by t'Hooft [2], the second - by Carlitz and Creamer [12], the large x limit is given according to the work by Novikov et al [10], previous calculation by Andrei and Gross [17] was incorrect.

At Fig.1 we have plotted resulting instanton density in the case with absent light u,d,s quarks. In next sections we discuss how the interaction changes this distribution.

3. Instanton interactions

Our starting point is the change in the instanton action due to external weak field $G^a_{\mu\nu}$

$$\Delta S = + \frac{2\pi^2 \rho^2}{g^2} \bar{\eta}^a_{\mu\nu} G^a_{\mu\nu} \tag{8}$$

This formula has been first derived by CDG [3] , and later SVZ [8] have proposed more simple its derivation.

It is evident that only antiselfdual field give the nonzero contribution to (8), so at this level no interaction between instantons is present.

It is then easy to find the interaction of separated instanton-antiinstanton pair with radii $\rho, \bar{\rho}$ [3]:

$$\Delta S = - \frac{32\pi^2}{g^2} \frac{\rho^2 \bar{\rho}^2}{R^6} \bar{\eta}^a_{\mu\nu} \eta^b_{\mu\sigma} T_{ab} R_\nu R_\sigma \tag{9}$$

where R_μ is distance between centers, and T_{ab} is matrix of relative orientation in colour space. Note, that the interaction is of dipole-dipole type.

It is also essential that rather large numerical factor appears in (9). The maximum is reached for some orientation and is $(96\pi^2/g^2)(\rho^2\bar{\rho}^2/R^4)$, while for "random" $T_{ab} = \frac{1}{3}\delta_{ab}$ it is three times smaller. Anyway, instantons interact strongly at rather large distances

$$R \simeq (100\, \rho^2 \bar{\rho}^2)^{1/4} \simeq (3 \div 3.5)\, \rho \tag{10}$$

Another application of the formula (8) has been proposed by SVZ [8] . Let us consider small size instanton in QCD vacuum. The average square of long-range field in it is phenomenologically known from their analysis of charmonium levels (see [9] and references therein):

$$\langle (gG^a_{\mu\nu})^2 \rangle \simeq 0.5 \text{ Gev}^4 \tag{11}$$

This quantity called the gluon condensate will play an important role in what follows.

Expanding the probability change due to (8) one has the following correction to instanton density

$$d(\rho) = d_0(\rho)\left(1 + \frac{\pi^4 \rho^4}{2g^4}\langle (gG^a_{\mu\nu})^2\rangle + \dots\right) \tag{12}$$

Note that it deviates from $d_0(\rho)$ at ρ as small as 1/1.1 Gev. Under certain assuptions (rather questionable, in fact) they have exponentiated correction (12). This give their estimate for upper bound of the instanton radius 1/500 Mev as the point where correction becomes comparable with the initial action.

Another important work of SVZ [7] deals with the effect of quark condensate

$$\langle \bar{\psi}\psi \rangle \simeq -10^{-2} \text{ Gev}^3 \tag{13}$$

on small size instantons. Related questions will be discussed in section 5 , and all we want to say now is that the quark masses in (7) should be substituted by effective ones

$$M_{eff}(\rho) = m - \frac{2\pi^2}{3}\langle\bar{\Psi}\Psi\rangle\rho^2 \qquad (14)$$

Both effects considered lead to strong growth of the instanton density with ρ . The corresponding curve is shown at Fig.1 . Unfortunately, this approach is valid only for small enough instantons.

It is physically clear that such growth should be stabilized by some effect of repulsion. The general reason for its existence is also known since the first CDG work [3] : close instanton-antiinstanton pair means weak fields and their account leads to double counting. The more detailed discussion of such effects can be found in works [18] . Still the explicit introduction of collective coordinate with the meaning of instanton-antiinstanton separation (the dipole moment of the topological charge ?) and quantum calculation of corresponding effective potential is missing.

In works [6, 19] some model-dependent hard core was introduced and its effect on instanton-induced effects was discussed. For example, the choice of Ilgenfritz and Mueller-Preussker [6] is the following

$$U(R) = \begin{cases} 0 \\ \infty \end{cases} \quad R^4 \gtrless a\rho^2\bar{\rho}^2 \qquad (15)$$

It has the virtue that the free parameter in it a is dimensionless, so at least no new scale in introduced by hand. Its value they have tried to fix from large N_c limit (to be discussed in 8-th section) and Gell-Mann-Low function

(to be discussed in section 7). Both considerations point toward rather large a , of the order of 200. It means that core is essential at distances

$$R \lesssim [\, a \rho^2 \bar{\rho}^2 \,]^{1/4} \simeq 3.7 \rho \qquad (16)$$

Note that it is comparable with (10), which is probably not ocasional. It is important, that some numerical parameter $\rho / R \ll 1$ comes into play here, to be utilized below.

The introduction of core has answered some questions posed by SVZ [9] . They have pointed out that artificial cut-off over instanton size violates some general relations, in particular that given by trace anomaly for energy-momentum tensor

$$T_{\mu\mu} = -\frac{b}{32\pi^2} (g G^a_{\mu\nu})^2 \qquad (17)$$

Calculations with the cut-off give identical results for both parts of this equation up to factor (b/4). Since this factor is proportional to N_c, it becomes significant in discussion of large N_c limit.

However it turns out that consistent calculations with core [6,19] automatically give correct relation (17). It means that it was just artifact of cut-off, not the intrinsic difficulty of instanton calculus.

In summary, instanton interaction resembles those for ordinary molecules : they are weak and attractive at large distances and strongly repulsive at small ones.

4. Instanton liquid

The analogy between the instanton interactions and that of ordinary molecules suggests the following picture: at high enough density one has closely packed liquid rather than free gas. Unfortunately it is very difficult to find from first principles the properties even of ordinary liquids, to say nothing about the instanton one. The calculations with some convenient core shape discussed above are the example of such approach.

In this work we have chosen another tactics: we discuss first empirical facts about QCD vacuum and try to find some small parameter, giving some hope for success of future analitical calculations.

Our first object is the gluon condensate (11). Its physical nature was not important in the discussion of preceeding section. Now we make strong assumption that it is caused by our instanton liquid.

Since each instanton contributes to it fixed amount $32\pi^2$ one can find directly the total instanton density in such liquid

$$n_c^{inst} = n_c^{antiinst} = \frac{1}{64\pi^2}\langle (gG^a_{\mu\nu})^2\rangle \simeq 8\cdot 10^{-4} \ \mathrm{Gev}^4 \qquad (18)$$

More sceptical reader may say that at least it can not be larger.

From this number SVZ in their old work [20] have estimated the typical instanton radius to be 1/170 Mev. However, they have used free gas density, which in fact is strongly modified by interaction. Our point here is that in fact it is much smaller and only due to this one can speak about instantons as meaningful objects.

Even in such case and even if instanton interactions are known, the calculation of the properties of instanton liquid is rather difficult problem. The account for global instanton stimulation by average field, as it was done in (12), is not possible for instantons, which themselves dominate in such field. The instanton probability depends on the positions and properties of neighbouring antiinstantons, and the difficulty lyes in multiplicity of configurations. For orientation, one can take one of them, which seems reasonable.

Consider for example the most simple one, being close to most dense packing: cubic 4-dimentional lattice with alterating instantons and antiinstantons (the NaCl structure). Of course we do not propose instanton cristall with long range order, but think that at small scale such model is not too far from reality. We also freeze the instanton radius to be inside the small interval $\rho_c \pm \Delta\rho$ with $\Delta\rho \ll \rho_c$.

All which is left is the integration over relative orientation in colour space. Performing this for dipole-dipole interaction we find the correction to instanton density

$$n_c = 2\Delta\rho \left.\frac{dn_0}{d\rho}\right|_{\rho_c} \left\langle \exp\left[\frac{16\pi^2}{g^2} \bar{\eta}^{a}_{\mu\nu} \eta^{b}_{\mu\sigma} \left(\frac{R_\nu R_\sigma}{R^2}\right) \rho_c^4 n_c T_{ab}\right]\right\rangle \quad (19)$$

Note that it is a kind of selfconsistency equation because the density enters also the exponent. Solving this equation one finds $n_c(\rho_c)$ for our model, and assuming density (18) one obtains then the critical radius ρ_c (for $\Delta\rho = .1\,\rho_c$):

$$\rho_c \simeq 1.6 \text{ Gev}^{-1} \simeq 1/3 \text{ fermi} \quad (20)$$

The resulting instanton distribution is shown at Fig.1.

In order to give some impression about numbers involved let us note, that for each neighbour pair $\langle \exp(-\Delta S) \rangle \simeq$ $\simeq 2.4$ or not so large effect. However, there are 8 neighbours and in total this is an increase by the factor 800. Still one should remember that the absolute probability of barrier penetration $\exp(-S_0) \simeq 10^{-8}$ and in fact it is not modified too much.

Of course, the model is very naive. In addition, the total normalization is uncertain due to large power of Λ involved. However, the dependence on ρ_c in (19) is so strong that this parameter is defined with good accuracy, presumably not worse than 20%. The absolute density of instanton liquid is difficalt to calculate, but we can rely here on (18).

In what follows we use the simplified instanton density in the form

$$\frac{dn}{d\rho} \simeq n_c\, \delta(\rho-\rho_c) \tag{21}$$

where parameters n_c, ρ_c are given in (18,20). This provides the possibility to discuss multiple instanton-induced effects on much more simple and quantitative level than it was done before.

It is also interesting, that more heavy calculations with the core taken in form (15) with parameter $a \simeq 200$ fitted to β function give rather similar distribution.

The very important feature of this result is the following. The so called packing fraction f defined in [3] as

$$f = 2\int\left(\frac{\pi^2}{2}\rho^4\right)dn \simeq \pi^2 n_c \rho_c^4 \sim 1/20 \tag{22}$$

(2 for instantons plus antiinstantons and $\pi^2\rho^4/2$ for 4-volume) is rather small: only about 1/20 part of the volume is occupied by instantons. We have still liquid, not gas, but made of well defined constituents, which are not deformed too much.

The question is natural here about the physical origin of this parameter. First, it is due to the fact that (9,15) contain large numerical factors of geometrical origin. The power of π from angular integration cansels in (22) and the candidate can be the number of space-time dimensions d=4. It is interesting in this connection, that in 2-dimensional models considered so far instantons are "melted".

But there is other effect, to be discussed in next section and connected with light quarks. They are effective in instanton suppression and N_f=3 is also important for this parameter.

5. Chiral symmetry breaking and the quark condensate

It is well known now that due to smallness of light quark masses (for u,d,s quarks approximately 4,7, 150 Mev) the strong interactions posess nearly exact chiral symmetry. In physical vacuum it is spontaneously broken, and as the result the Goldstone mode - the pion - is massless, not the nucleon. The explicit manifestation of the breaking is the nonzero quark condensate, the vacuum average of $\bar{\psi}\psi$, for it directly connects right and left handed quarks.

It has been discovered by t'Hooft [2] that instantons do important things with light quarks, namely produce N_f pairs of them. In Euclidean formulation this is seen as zero modes (6), respectively the propagator of quark can be written as

$$S(x,y) = \frac{\bar{\psi}_0(x)\,\psi_0(y)}{-m} + S'(x,y) \qquad (23)$$

where S' is nonsingular at $m \to 0$. The instanton density given in section 2 is proportional to m , so in the case of one flavour quark immediately obtains the effective mass, violating the chiral symmetry [3] .

Things are not so simple at N_f larger than 1. The instanton generated vertex with $2N_f$ legs has the following structure in flavour indices

$$\bar{\psi}^{i_1}\psi_{k_1}\ldots\bar{\psi}^{i_{N_f}}\psi_{k_{N_f}}\cdot\left(\epsilon^{i_1\ldots i_{N_f}}\epsilon_{k_1\ldots k_{N_f}}\right) \qquad (24)$$

Note that upper indices correspond only to right-handed spinors, and lower ones correspond to left-handed ones. So, the rotation of only right-handed quarks affects only one $\epsilon^{i_1\ldots i_N}$ and the determinant appears. This is nontrivial for U(1) but not $SU(N_f)$ part of the rotation. Therefore, only this subgroup of chiral symmetry is violated directly. So, we have qualitative explanation for Weinberg U(1) problem [2], but not to breakdown of chiral symmetry. This problem is open at one-instanton level.

It has been suggested in [3] that such phenomenon can be explained if instantons and antiinstantons are close enough. The idea is that in this case attraction in scalar quark-antiquark channel may become so strong that chirally symmetric vacuum becomes unstable and the quark condensate appears. As always for spontaneous symmetry breaking, the question is quantitative and can be answered only by explicit calculation. So, it is interesting to check whether our parameters determined above can do the job.

We do not give here details of the relevent formulae for they are present in well known work [3]. We only remind the logics of their derivation. First, iteration of Bethe-Salpeter kernel $\Sigma(p)$, as always give

$$\Sigma + \Sigma^2 + \ldots = \Sigma/(1-\Sigma) \tag{25}$$

and if for some momentum p of the quark-antiquark pair $\Sigma(p) > 1$ than the instability (bound state) take place. The diagrams for Σ are constracted from t'Hooft vertexes with $2N_f$ legs. Two of them is the quark-antiquark pair under consideration, all the rest should either be absorbed by the antiinstanton or closed in loops. As mentioned above, in latter case the result is proportional to quark mass. Obviously, this calculation is made in chirally symmetric vacuum without the condensate and quark effective mass: we are going to prove its instability. So, one should use in the instanton density $n_c / \prod_{uds} (M_{eff} \rho_c)$ instead of n_c , and write down all fermionic factors explicitely. Interesting, that it turns out that the largest contribution is obtained if the strange quark is closed in loops while one of remaining light quarks is transfered to antiinstanton. The resulting expression is

$$\sum(p) \simeq \frac{8 m_s \rho_c}{\prod_{uds}(M_{eff}\rho_c)}(\pi^2 n_c \rho_c^4) \exp(-2p\rho_c) \simeq 1{,}8 \exp(-2p\rho_c) \quad (26)$$

where exponent is some approximation for $p\rho_c \lesssim 2$ of some complicated integral (see [3] for details). What is important, it is really larger than unity for small p and the instability is present.

This conclusion was also made in [3] but in fact the differences are significant. The packing fraction (22) present in (26) was not so small in [3] and with their parameters the instability appeared at p as large as 9.2Λ . There is empirical argument against it, to be discussed in next section: quark condensate is very homogeneous in space-time. So, our instability only at small p is wellcomed.

The second problem is to calculate the numerical calue of the condensate. Now we return to physical asymmetric vacuum and write the selfconsistency condition for $\langle\bar\Psi\Psi\rangle$ as

$$\langle\bar\Psi\Psi\rangle = -2\int\frac{dn}{M_{eff}(\rho)} \simeq -\frac{2n_c}{M_{eff}(\rho_c)} \quad (27)$$

It was considered in [3,12] and follows directly from the propagator (23) in the instanton field.

With our simplified instanton density (21) this integral equation become a pair of algebraic ones and from (27) and (14) both the condensate and effective mass can be found:

$$\langle\bar\Psi\Psi\rangle \simeq -\frac{(3n_c)^{1/2}}{\pi\rho_c} \simeq -1\cdot 10^{-2}\ \mathrm{Gev}^3$$

$$M_{eff}(\rho_c) \simeq 2\pi\rho_c\left(\frac{n_c}{3}\right)^{1/2} \simeq 200\ \mathrm{Mev} \tag{28}$$

The classical result of PCAC is (see e.g.[9])

$$\langle\bar\Psi\Psi\rangle \simeq -\frac{f_\pi^2 m_\pi^2}{2(m_u+m_d)} \simeq -1.6\cdot 10^{-2}\ \mathrm{Gev}^3 \tag{29}$$

Another estimate is provided by heavy-to-light quark matching proposed in [9]

$$\langle\bar\Psi\Psi\rangle \simeq -\frac{1}{48\pi^2 m}\langle(gG^a_{\mu\nu})^2\rangle + O(m^{-3}) \tag{30}$$

where m is some large mass. If this formula is extrapolated to some intermediate mass, say 200 Mev, one has $\langle\bar\Psi\Psi\rangle \simeq$ $\simeq -0.6\ 10^{-2}\ \mathrm{Gev}^3$.

In the sum rules for ρ, ω, φ mesons [9] and baryons [21] some four-fermion operators enter, which were reduced to $\langle\bar\Psi\Psi\rangle^2$ by factorization hypotheses (see below) and results again agrees with $\langle\bar\Psi\Psi\rangle$ of the order of 10^{-2} Gev^3. The author recent work [22] deals with hadrons, containing one heavy quark, like B meson, Σ_B, Λ_B baryons. In this case $\langle\bar\Psi\Psi\rangle$ enters directly, and again the results with such $\langle\bar\Psi\Psi\rangle$ are reasonable.

The physics connected with quark effective mass we are going to discuss in the second part of the work, but only note here that the result (28) is agreable as well.

*) Here small "current" masses enter because explicit symmetry violation leads to axial current nonconservation. The spontaneous violation is accounted by the existence of pion.

Let us also note here, that quark effective mass is small in the scale ρ_c, characteristic to instantons. This is some reflection of the fact, that instantons alone are not able to produce it and they are produced rather at scale equal to instanton-antiinstanton separation $R \simeq 3\rho$. The smallness of $\langle\bar{\Psi}\Psi\rangle$ in turn leads to rather dilute instanton liquid. We see once more the selfconsistency of the picture proposed.

Closing this section we may say, that both quark condensate and quark effective mass, calculated in our parametrization of the instanton liquid, are found to be in reasonable agreement with empirical values.

6. Vacuum average values of other operators

Inspired by the results reported above we proceed to calculations of vacuum averages of more complicated operators in our picture, and their comparison with empirical facts. The latter come mostly from SVZ sum rules analysed in [9,21,22]

There are also three theoretical proposals made in [9] for estimation of vacuum averages : instanton gas, factorization with vacuum intermediate state, heavy-to-light quark matching. Now we are going to discuss their consistency and connections with our results.

As for instanton gas estimates, the whole our work can be considered as the development of this idea on more detailed and quantitative level. As the example of the results, appearing on this basis, let us give the expression given in [9]

$$\frac{\langle g^3 f^{abc} G^a_{\mu\nu} G^b_{\nu\sigma} G^c_{\sigma\mu} \rangle}{\langle (g G^a_{\mu\nu})^2 \rangle} \simeq \frac{12}{5\rho_c^2} \simeq 0.9 \ \mathrm{GeV}^2 \qquad (31)$$

However, numerically result is rather large as compared to [9] for we use much larger scale for $1/\rho_c$.

For the second proposal mentioned above, the factorization, the situation is not so simple. Let us illustrate it by two relations for operators $(gG^a_{\mu\nu})^4$, $(\bar\Psi\Psi)^2$ being the complete squares. For them the factorization with vacuum intermediate state reads as follows:

$$\langle (gG^a_{\mu\nu})^4\rangle \simeq \langle (gG^a_{\mu\nu})^2\rangle^2$$

$$\langle (\bar\Psi\Psi)^2\rangle \simeq \langle \bar\Psi\Psi\rangle^2 \qquad (32)$$

where we have neglected numerically small factorized contribution of crossing channels (other pairing).

In general, our picture of the instanton liquid does not imply relations of such kind. Moreover, smallness of packing fraction (22) means rather inhomogeneous vacuum, in which simple relations like (32) should be strongly violated. The explicite calculation for instantons give all averages to be proportional to n_c , and not to $(n_c)^2$ as suggested by (32). It is rather simple to calculate $\langle (gG^a_{\mu\nu})^4\rangle$ in our picture:

$$\langle (gG^a_{\mu\nu})^4\rangle / \langle (gG^a_{\mu\nu})^2\rangle = \frac{192}{7\rho_c^4} \simeq 3.6\ \mathrm{GeV}^4 \qquad (33)$$

which is indeed essentially larger than $\langle (gG^a_{\mu\nu})^2\rangle$, entering relation (32). So we conclude that for gluonic operators the factorization hypothesis of SVZ [9] is in contradiction with our results. In principle, effect of higher order gluonic operators on charmonium levels can be studied and therefore this conclusion can be tested, but it is not done so far.

However, the analogous calculation for $\langle (\bar\Psi\Psi)^2\rangle$ gives :

$$\frac{\langle(\bar{\psi}\psi)^2\rangle}{\langle\bar{\psi}\psi\rangle} \simeq -\frac{1}{8\pi^2 \rho_c^3 (NS)} \simeq -4.3\cdot 10^{-2}\ \mathrm{Gev}^3 \qquad (34)$$

which is indeed very close to what is predicted by the factorization relation (32). Strong difference in numerical coefficients are caused by the fact, that fermion zero mode, unlike $(gG^a_{\mu\nu})^2$, are normalized to unity.

The conclusion, that for quark condensate factorization works well is also in agreement with our discussion in the preceeding section, where we have shown that such condensate, in contrast to gluon one, is formed at distances of the order of instanton separation and is rather homogeneous.

We have also noted above, that phenomenologically factorization works well for ρ, ω, φ [9] and N, Δ [21,22] where $(\bar{\psi}\psi)^2$ is present, and also for B - type mesons with heavy quark [22], where its first power appears. Moreover, in the discussion by SVZ [9] of vector mesons mixing it is suggested that such factorization is valid with accuracy of the order of 1/20. Our result (34), of course, supports it only on order-of-magnitude level. So, what can be the origin of so high accuracy ?

Strictly speaking, one should calculate both parts of (32) with similar accuracy in order to answer this question, with more detailed knowledge of the instanton configurations. However, one may at least try some model, for example the cubic instanton-antiinstanton lattice discussed in section 4. The interference of zero modes of instantons (right-handed) and the antiinstantons (left-handed) produce the condensate. What is observed is that it is very homogeneous in space between instantons, on the level of 15-20%, but is changed of the order unity inside instantons which, however, occupy only 1/20 part of the volume (22). Introducing some fluctuations of instanton size and positions, one obviously finds smaller inhomogenuities. Although we have not done it explicitely, we suggest that high accuracy of factorization for quark condensate can be explained in our, rather dilute instanton liquid.

The last point of this section is the so called heavy-to-light quark matching [9] , used for order-of-magnitude estimates of some vacuum averages. We have already given such result for the quark condensate (30), which for reasonable intermediate matching mass $m_q \simeq 200$ Mev gives values 2-3 times smaller than empirical and our values of $\langle\bar{\Psi}\Psi\rangle$. Another important result obtained in the same way in [23], it deals with the operator $ig\bar{\Psi}\sigma_{\mu\nu}G^a_{\mu\nu}t^a\Psi$ important for applications to baryons [21] as well as for hadrons with heavy quark [22] . The result of [23] is

$$\langle ig\bar{\Psi}\sigma_{\mu\nu}G^a_{\mu\nu}t^a\Psi\rangle \simeq -\frac{1}{12\pi^2 m_q}\langle g^3 f^{abc}G^a_{\alpha\beta}G^b_{\beta\gamma}G^c_{\gamma\alpha}\rangle \simeq \tag{35}$$

$$\simeq -\frac{64}{5}\frac{n_c}{m_q\rho_c^2} \simeq -1.8\cdot 10^{-2}\ \mathrm{Gev}^5$$

where we have substituted m_q=200 Mev and our value (31). At the same time direct estimation in instanton calculus gives $\langle\bar{\Psi}\Psi\rangle(8/\rho_c^2) \simeq -3\cdot 10^{-2}$ Gev^5, so the agreement is reasonable.

Summarizing this section we may conclude, that factorization is valid for quark condensate but not for gluon one in our picture. The first statement is very nontrivial and its agreement with data is encouraging. The second prediction is new, but so far not tested experimentally.

6. Phase transition in external field and charge renormalization

In this section we discuss the connection between the proposed picture of QCD vacuum and two well known works by Callan (Jr) Dashen and Gross concerning phase transition in external field [4] and comparison of the instanton-induced charge renormalization with strong coupling expansion [5]. The basis of both works is the discussion of vacuum permeability μ in the instanton analog gas, which is treated by the Onzager method for dipole interactions.

Let us start with the comparison of the underlying instanton density. At Fig.1 we schematically show by dotted lines this quantity in three cases according to CDG work. They are: (a) the so called dilute phase, presumably existing inside the "bag"; (b) the instability point, where derivative $dE/d\mathcal{D}$ changes sign; (c) the end of the CDG curve, presumably corresponding to "meron ionization". It is seen, that the curve (b) is rather close to our instanton density, which is however corresponding to zero field case, or physical vacuum. We therefore conclude that further growth in instanton density from (b) to (c) is questionable, as well as phase transition of the first order claimed by CDG. The explicit account for core in [6] has lead to the same conclusion.

Another way to see this is to calculate the permeability

$$\mu(\rho) = 1 + 4\pi^2 \int_0^{\rho} d\rho \, \frac{dn}{d\rho} \left(\frac{\rho^4 \pi^2}{g^2(\rho)} \right) \qquad (36)$$

With our parameters it is very close to unity, $\mu \lesssim 1.4$, so there is no need to use approximations of Onzager type, important only for strongly paramagnetic media $\mu \gg 1$. Then there is no place for phase transition to be developed.

Still the instanton-induced permeability is important effect, and it can be considered as a type of charge renormalization [9]:

$$g^2(r) = g^2_{pert}(r)\,\mu(r) \qquad (37)$$

The standard way to look at it is provided by Gell-Mann - Low function defined as follows

$$-\frac{\beta(g)}{g} = \frac{d \ln g}{d \ln r} \qquad (38)$$

At Fig.2 we have plotted this function versus charge in lattice regularization. Dashed curves marked AF and SC correspond respectevely to asymptotic freedom [16] and strong coupling [13]. The curve marked CDG is what is found in [5], while the solid line corresponds to our results with instanton density as shown at Fig.1.

The important difference between two last curves is that the first one crosses the SC curve, while the second one turns in the proper place. Also important, that according to CDG the crossing point lyes completely inside the region of validity of the calculation. This was so suspicious, that, for example, in the work [6] the core parameters were found in order to prevent such crossing.

Let us note in this connection, than in their later work Ilgenfritz, Kazakov and Mueller-Preussker [24] claim that the instanton-induced renormalization behaves in fact as shown at Fig.2 by the dotted curve without crossing with SC one, and even with some similarity with it.

However, unlike the logarithmic effects, power corrections are not the universal charge renormalization and depend on the particular physical quantity one is interested in. What is calculated in [24] is essentially the three-gluon vortex renormalization constant

$$Z_1(q) = 1 + \frac{8\pi^2}{3q^4}\int\left[\frac{8\pi^2}{g^2(\rho)}\right]^2\left[1-\frac{q^2\rho^2}{2}K_2(q\rho)\right]^3 dn \quad (39)$$

Note that it is of second order in large parameter $(8\pi^2/g^2)$, that is why it starts at much smaller $g \sim 0.9$ or extremely small scale $q \sim 5\cdot 10^4 \Lambda_{LAT} \sim 100\,\mathrm{GeV}$! However, it remains still unclear in what physical process can this effect be seen.

There are also many other definitions of the charge, say classical "force between static charges", but one should compare the one used in lattice theory: the coefficient of $(G^a_{\mu\nu})^2$ term in effective action. The CDG permeability indeed corresponds to it.

Closing this section we conclude, that CDG phase transition is very questionable from our point of view, while the comparison with strong coupling regime becomes even better.

8. Changing quark masses and N_c

In order to simplify real QCD theorists have invented some imaginary worlds with different quarks and the number of colours N_c. In this section we briefly discuss some of them, in particular: (i) the effect of the strange quark mass; (ii) gluodynamics without light quarks; (iii) multicolour QCD.

Transition to the case in which strange quark mass $m_s \simeq 150$ Mev goes to zero was rather thoroughly discussed by Novikov et al [25] . They have calculated derivatives of the gluon condensate over quark mass at its zero value

$$\left.\frac{d\langle (gG^a_{\mu\nu})^2\rangle}{dm}\right|_{m=0} = -\frac{96\pi^2}{b}\langle\bar\psi\psi\rangle$$

$$\left.\frac{d^2\langle (gG^a_{\mu\nu})^2\rangle}{d^2m}\right|_{m=0} = \frac{36}{b}\left(\frac{m_\pi^2}{m_u+m_d}\right)^2 \ln\left(\frac{\Lambda^2}{m_\pi^2}\right) \tag{40}$$

Here Λ is the normalization point which appears because $m_\pi^2 \to 0$ at $m_q \to 0$ limit.

Both derivatives are positive, so the condensate increases with quark mass. Substituting here m_s one finds

$$\langle (gG^a_{\mu\nu})^2\rangle_{m_s=0} \simeq \frac{1}{2}\langle (gG^a_{\mu\nu})^2\rangle_{real} \tag{41}$$

Expansion in $1/m_q$ is also possible, and it shows that with further growth of m_s to infinity the change in gluon condensate is smaller. Therefore, the authors of [25] have concluded that in gluodynamics the condensate is 2-3 times larger than in real world. In this connection it is interesting that recent Monte-Carlo simulations for gluodynamics with $N_c=3$ (see references and discussion in [26]) have given very large value

$$\langle (gG^a_{\mu\nu})^2\rangle_{gluodynamics} = (4 \pm 0.2)\ Gev^4 \tag{42}$$

but the theoretical uncertaities are much larger than given statistical error.

Now, what is the physical reason for so strong effect of the quark mass? Our answer is based on the instanton nature of the condensate, and the instanton density is effectively suppressed by the quark determinant. For the effect of the strange quark mass one finds

$$\frac{\langle (g G^a_{\mu\nu})^2 \rangle_{m_s=0}}{\langle (g G^a_{\mu\nu})^2 \rangle_{real}} \simeq \frac{\langle \bar{s}s \rangle \rho_c^2}{\langle \bar{s}s \rangle \rho_c^2 - 3m_s/2\pi^2} \simeq 0.6 \qquad (43)$$

in good agreement with (41). It means, that if strange quark to be massless, instanton density is two times smaller! It is very instructive to observe this fact in connection with our discussion in section 4 of the origin of small packing fraction f. Let us also add, that with m_s=0 the chiral symmetry breaking by the mechanism discussed in section 5 becomes questionable. We therefore conclude, that even such operation as putting m_s to zero may significantly change the physics of QCD.

Now let us look in other direction and increase u,d quark masses in order to discuss gluodynamics. This theory is now intensively studied in its lattice formulation in "theoretical laboratory" by Monte-Carlo simulation.

It is not easy to extrapolate from our picture of QCD vacuum to that of gluodynamics. Obviously, the absence of small fermionic factor F in (4) increases the instanton density, but other factors like $(\rho\Lambda)^{\frac{2}{3}N_f}$ suppress small instantons. Still our estimates show that small parameter f (22) hardly is present in gluodynamics and therefore the whole instanton language as applied to it becomes questionable.

Now we pass to multicolour QCD [15] discussed in many theoretical papers. It was suggested that $N_c=3$ is in some sense large number and $N_c \to \infty$ limit is close to reality. Some facts like relatively nerrow resonances provide some support to it.

However, as emphasized by Witten [27], this limit hardly can be smoth for instanton-induced effects. Later it was discussed in papers [28,29]. The basis of the discussion is the free instanton density (4). Fortunately, large factors $(N_c)^{N_c}$ cansel among themselves and what is left can be written as

$$\frac{dn}{d\rho} \propto \exp\left[N_c \left(2.91 + \frac{5}{11} \ln \ln \frac{1}{\rho \Lambda_{pv}} - \frac{11}{3} \ln \frac{1}{\rho \Lambda_{pv}}\right)\right] \qquad (44)$$

At N_c going to infinity all fluctuations for which the bracket in (44) is negative are completely suppressed. According to [28,29] it is always negative in Pauli-Villars regularization, but change sign in lattice one, so physics depends on scheme used! The resolution is that in fact one has to use two-loop expression for the charge, not one-loop one, in order to answer this question. This was done in (44) and the point where the fluctuations are not suppressed exists, it is $\rho_0 = 1/2.1\Lambda_{pv} \simeq 1\,fm$.

What happens in this point depends on instanton interaction and is not clear. The core in form (15) leads to density stabilization. However, on more general ground [30] the gluon condensate $\langle (gG^a_{\mu\nu})^2 \rangle \propto N_c^1$ which can not be reproduced by finite at $N_c \to \infty$ instanton density [29]. Most possible, instantons are completely "melted" in multicolour QCD, in contrast with real case.

Some phenomenological observations, pointing to the same directions, were given by Novikov et al [25]. For example, all mixings are small in $1/N_c$, but in some cases they are not small.

They propose that new mass scales enter and their ratio can compete with N_c. We propose to relate these scales with different instanton-induced effects and, finally, with $\rho_c = 1/600$ Mev which is much smaller than $1/\Lambda$. As explained above, this difference of ρ and $1/\Lambda$ becomes much smaller in $N_c \to \infty$ limit and, presumably, "all hadrons are alike" in this case.

In conclusion, our real world turns out to be very different from various imaginary ones, invented by theorists to make their life simpler. What is found in our work, in some sense the real case is the best one.

8. Conclusions and discussion

The main conclusion from the present work is that the component of the QCD vacuum of instanton nature is dominant in producing so important its characteristics as the gluon and quark condensates. We definitely has some instanton liquid, not gas. Still it turns to be rather dilute liquid so that the constituents - the instantons - are not deformed too much. As a result, one has approximate but valuable method of calculations. It predicts definite numerical values for different vacuum properties. Two of the results: the value of the quark condensate and the factorization of its square are already known empirically, and the agreement is very encouraging. Among other nontrivial predictions not tested so far we may mention strong violation of factorization for gluonic operators.

The existence of small parameter - the packing fraction (22)- is not understood completely. We feel that light quarks are important here. In this connection it is important that even turning the strange quark mass to zero may significanly change the numbers involved by the order of unity, and transition to gluodynamics leads to completely different physics. The rapid convergence of Monte-Carlo simulations for gluodynamics also suggests the absence of scales, essentially exceeding Λ . However, as we argue in this and forthcoming works, such scale does exist in real world.

The same conclusion can be repeated in connection with multicolour QCD as well. The limit $N_c \to \infty$ is not so harmless operation as it was assumed by many theorists, and in studying it we go rather far from realistic case $N_c=3$.

Comparing our results with those of other authors, in particular by Callan, Dashen and Gross, we observe that our instanton density is generally much smaller and shifted to smaller instanton dimensions. As a result, our value of vacuum permeability is not far from unity and there is no place for the first order transition claimed by them. At the same time the charge renormalization becomes even in better agreement with strong coupling results, for so large effect is not needed empirically. Our discussion is more quantitative, in particular in discussion of quark effective mass we find reasonable numerical value which was not done previously. The same comment is valid for several vacuum averages of different operators.

Of course, a lot of questions remain open. The most general problem is the explicite calculation of core-type effects and its role in formation of the "instanton liquid".

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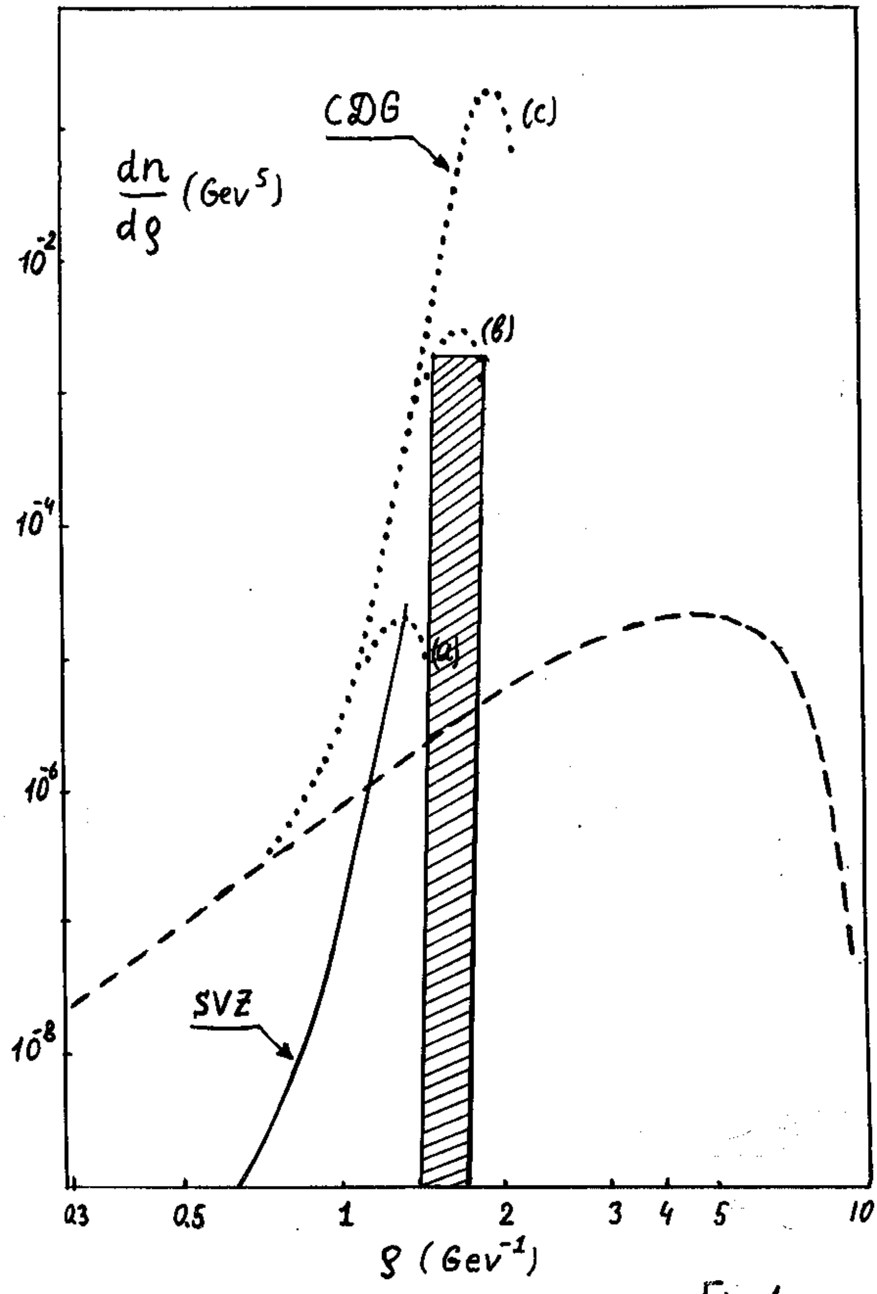
Figure captions

1. Instanton density $dn/d\rho$ (Gev^5) versus instanton radius ρ (Gev^{-1}). Free instanton density is given for gluodynamics by the dashed curve. Solid curve marked SVZ correspond to works [7,8] with the account of quark and gluon condensates. The three dotted curves correspond to CDG work [4] for: (a) dilute phase, (b) instability point, (c) meron ionization. The histogramm is our approximation as discussed in the text.

2. Charge renormalization presented as Gell-Mann - Low function versus charge g in lattice regularization. Two dashed curves marked AF and SC correspond to asymptotic freedom [16] and strong coupling [13] respectively. The dash-dotted curve correspond to the work of CDG [5], the dotted one to [24], and solid one to our work.



$\frac{dn}{d\rho}$ (Gev^5)
CDG
(c)
(b)
(a)
SVZ
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0.3
0.5
1
2
3
4
5
10
ρ (Gev^{-1})

Fig. 1

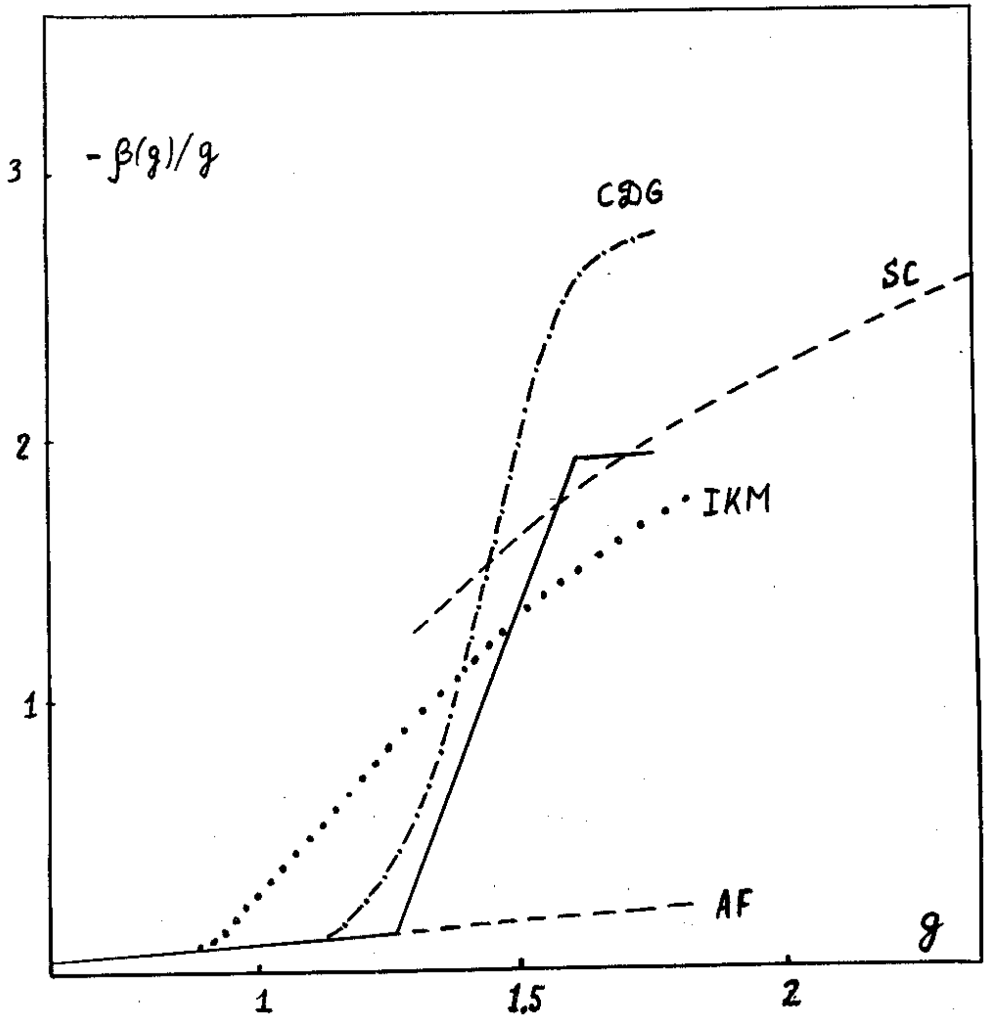


Fig. 2

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