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ABSTRACT

In this lecture we analyze the Minimal Supersymmetric Standard Model with radiatively induced breaking of the $SU(2)_L \times U(1)_Y$ sector. An underlying Grand Unification of the strong and electroweak symmetries is assumed. The mass spectra of the supersymmetric particles is studied also in the case of heavy top quark masses and big values of $\tan\beta = v_2/v_1$.

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ABSTRACT

In this lecture we analyze the Minimal Supersymmetric Standard Model with radiatively induced breaking of $SU(2)_L \times U(1)_Y$. An underlying Grand Unification of the strong and electroweak symmetries is assumed. The mass spectra of the supersymmetric particles are studied also in the case of heavy top quark masses and large values of v_2/v_1. In addition, a comparison with some of the existing lower bounds on these masses is given.

1. Introduction

The topic of this lecture is the analysis of the low-energy spectrum of an explicitly broken globally supersymmetric theory, as obtained after the spontaneous breaking of a local (N=1) Supersymmetry (SUSY) [1].

There is no need to give a motivation for the introduction of such a theory, already treated in great detail in the first two lectures of this workshop. It is enough only to recall that this theory fulfills the requirement of protecting the scalar fields from jumping to higher scales, like the Planck scale M_P or a possible Grand Unification scale M_X. The local Supersymmetry is able to cure the problem of quadratic divergences [2], and the spontaneous breaking of this symmetry [3] guarantees that this feature is not lost after the breaking. After decoupling of gravity, i.e. once the limit $M_P \to \infty$ is taken, we are left with a globally supersymmetric theory plus some terms of "soft breaking" of this symmetry. There is a very precise technical meaning of these words [4], which we will not investigate here.

The theory we are dealing with is the supersymmetrized version of the most *minimal* extension of the Standard Model (SM) compatible with Supersymmetry. That is, a two Higgs doublet model (2HDM) considered as embedded in an underlying Grand Unified Theory (GUT).

One of the reasons for the success of this theory is the observation that if the effective SUSY breaking is $\mathcal{O}(M_Z)$, then the logarithmic corrections to the mass parameters of the scalar field potential are able to induce the breaking of the electroweak gauge symmetry. The problem of the $SU(2)_L \times U(1)_Y$ symmetry breaking is then "solved" by being linked to the mechanism of breaking of the local Supersymmetry at the Planck scale. The dynamic of this mechanism is unknown, but it can be parametrized by a finite

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number of soft breaking terms, resulting in a finite number of new free parameters to be added to the ones already present in the globally supersymmetric Lagrangian, i.e. μ plus the usual SM parameters. The *minimal* choice of only three new parameters, m, A and M, can be justified by a criterion of economy and makes this model, called then Minimal Supersymmetric Standard Model, particularly suitable for phenomenological studies.

The rich range of possibilities offered by the supersymmetric mass spectrum has made this a vital model capable of evading experimental detection to date. Only a relatively small restriction of the allowed SUSY parameter space has been achieved so far through experimental searches. The recent precision measurements at LEP of the electroweak gauge coupling constants and the observation of their compatibility with the supersymmetric Grand Unification scenario [5], are far from providing any evidence for this model. Nevertheless, they may be suggestive that there is more to it than just theoretical speculation.

In this lecture we shall analyse in detail the mass spectrum of this model and we shall compare it, in a few cases, to the existing experimental lower bounds on supersymmetric masses. We shall give particular emphasis to the unification of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions, to the corrections of the mass parameters during the evolution from the GUT scale [§] to the electroweak scale and finally to the mechanism of breaking of $SU(2)_L \times U(1)_Y$.

Several analyses of these issues exist in the literature. In the earlier papers [6,7], not very high values of the top mass were considered and the supersymmetric parameter μ was taken to be small. In a more recent series of papers [8–12], the higher constraints on squark and gluino masses are implemented and the possibility of heavy top masses is considered. As a consequence, the increase of the value of $\tan\beta$ no longer allows the neglect of the Yukawa couplings of the bottom quark, h_b, and of the tau lepton, h_τ. In some of these papers the correct constraints on the GUT scale Yukawa couplings imposed by Grand Unified Theories are also implemented [10–12] and some uncertainty in the on-shell value of the bottom quark mass is allowed [11]. In some cases also the consequences of high values of $\tan\beta$ are studied [10,12]. In others, particular care is taken in the correct trading of low-energy parameters with high-energy ones when the radiative breaking of the electroweak sector is imposed [11,12]. The correspondence between high- and low-energy parameters is in general not one-to-one.

We shall discuss these issues in the following sections. The numerical results presented are taken from different sources in the literature and may be, at times, not in complete agreement because of different choices of α_S, $\sin^2\theta_W$ or m_b. They have therefore to be taken as indicative and some variation has to be considered possible due to changes of these input parameters.

The lecture is organized as follows. In section 2 we present the Lagrangian for this model. Section 3 explains the procedure followed in selecting from between all the possible points of the supersymmetric parameter space the ones allowing a radiatively induced breaking of $SU(2)_L \times U(1)_Y$. In sections 4, 5 and 6, a detailed description of the one-loop renormalization effects on different sectors of the model is given. Sections 7, 8 and 9 deal with the theoretical predictions for the allowed range of supersymmetric masses and their comparison to the existing experimental lower bounds. Some conclusions will be finally drawn in section 10.

[§] Renormalization effects between M_P and M_X are neglected here as well as in the existing literature.

2. Lagrangian

The request of minimality mentioned in the introduction, strongly constrains the structure of the Lagrangian of our model. The requirements that only the fields due to the supersymmetrization of the SM are present and that matter parity is preserved, immediately dictate the expression of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant superpotential:

$$W = h_U^{ij} Q_i U_j^c H_2 + h_D^{ij} Q_i D_j^c H_1 + h_E^{ij} L_i E_j^c H_1 + \mu H_1 H_2 \tag{1}$$

where the chiral matter superfields Q, U^c, D^c, L, E^c, H_1 and H_2 transform as follows under $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$\begin{array}{lll} Q \equiv (3,2,1/6); & U^c \equiv (\bar{3},1,-2/3); & D^c \equiv (\bar{3},1,1/3); \\ L \equiv (1,2,-1/2); & E^c \equiv (1,1,1); & \\ H_1 \equiv (1,2,-1/2); & H_2 \equiv (1,2,1/2)\,. & \end{array} \tag{2}$$

Isospin and colour indices are contracted in the usual way. The couplings h_U, h_D and h_L are 3×3 matrices in the generation space $(i,j = 1,2,3)$.

The expression for the soft Supersymmetry breaking terms turns out to be quite simplified by the assumption of having a flat Kähler metric. At the Grand Unified scale M_X, they appear as:

- A cubic gauge invariant polynomial in the complex scalar fields:

$$S = mA\left[h_U \tilde{Q}\tilde{U}^c H_2 + h_D \tilde{Q}\widetilde{D}^c H_1 + h_E \tilde{L}\tilde{E}^c H_1\right] + Bm\mu H_1 H_2 + h.c. \tag{3}$$

  where the tilde denotes the *scalar* component of the chiral matter superfields Q, U^c, D^c, L and E^c, while for simplicity we denote by H_1 and H_2 also the scalar components of the Higgs superfields H_1 and H_2. Later on, we shall often indicate with $\tilde{q}_L$ $(\tilde{u}_L, \tilde{d}_L, \ldots, \tilde{t}_L, \tilde{b}_L)$ the components of the three scalar $SU(2)_L$ doublets $\tilde{Q}$ and similarly, with $\tilde{q}_L^c$ or $\tilde{q}_R$ the components of the three scalar $SU(2)_L$ singlets $\tilde{U}^c$ and $\widetilde{D}^c$. We shall also (improperly) refer to $\tilde{q}_L$ and $\tilde{q}_R$ as the "left-handed" and "right-handed" component of the squark $\tilde{q}$. A similar convention will be adopted for the scalar leptons. The coefficients A and B in (3) are c-numbers and, in the presence of a flat Kähler metric, the equality $B = A - 1$ holds.

- A universal mass term for the scalar components y_i of the chiral superfields:

$$\mathcal{M}^2 \equiv m^2 \Sigma_i |y_i|^2\,. \tag{4}$$

- Gaugino Majorana mass terms:

$$\widehat{M} \equiv \frac{M}{2}(\lambda_1\lambda_1 + \lambda_2\lambda_2 + \lambda_3\lambda_3) + h.c. \tag{5}$$

where λ_1, λ_2 and λ_3 denote the two-component gaugino fields of $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. Notice that in (5), the Grand Unification constraint of equal gaugino masses at M_X is added to the usual requirement of minimality.

Hence, the Minimal Supersymmetric Standard Model considered here is described at the Grand Unification scale by the Lagrangian

$$L = W + S + \mathcal{M}^2 + \widehat{M} + \text{kinetic terms} . \tag{6}$$

In order to discuss the physical implications of this Lagrangian at low-energy (i.e. at the electroweak scale), we need to renormalize the relevant parameters from M_X down to M_Z. A detailed discussion of this procedure will be given in the following sections. For the time being, for the sake of setting our notation, we shall focus on the expression of the scalar potential at M_Z. The correct $SU(2)_L \times U(1)_Y$ breaking down to $U(1)_{em}$ is achieved by the vacuum

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}; \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}; \quad \langle \tilde{q} \rangle = \langle \tilde{\ell} \rangle = 0, \tag{7}$$

where the last two equalities have to be satisfied by all the scalar quarks and leptons of the model. It is possible to redefine the phases of H_1 and H_2 so that v_1 and v_2 turn out to be real and non-negative. The low-energy Higgs potential along the neutral direction is then

$$V = \mu_1^2 |H_1^0|^2 + \mu_2^2 |H_2^0|^2 - \mu_3^2 (H_1^0 H_2^0 + h.c.) + \frac{1}{8}(g^2 + g'^2)(|H_1^0|^2 - |H_2^0|^2)^2, \tag{8}$$

where g and g' denote the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants, respectively, and $\mu_{1,2,3}$ are running mass parameters. At M_X they read

$$\mu_1^2 = \mu_2^2 = m^2 + \mu^2 \tag{9}$$
$$\mu_3^2 = -Bm\mu . \tag{10}$$

The minimization of the Higgs potential (8) yields the two constraints

$$v_1^2 + v_2^2 = \frac{2\left[\mu_2^2 - \mu_1^2 - (\mu_1^2 + \mu_2^2)\cos 2\beta\right]}{(g^2 + g'^2)\cos 2\beta} \tag{11}$$
$$\sin 2\beta = \frac{2 v_2 v_1}{v_1^2 + v_2^2} = \frac{2\mu_3^2}{\mu_1^2 + \mu_2^2} . \tag{12}$$

By recalling that $2M_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)$, one obtains for (11) the more convenient form

$$\tan^2 \beta = \left(\frac{v_2}{v_1}\right)^2 = \frac{\mu_1^2 + M_Z^2/2}{\mu_2^2 + M_Z^2/2} . \tag{13}$$

For the above desired minimum (7) to occur we must enforce a stability condition which ensures that the potential is bounded from below together with the condition that the origin $\langle H_1 \rangle = \langle H_2 \rangle = 0$ is a local maximum. They read:

$$\mathcal{S} \equiv \mu_1^2 + \mu_2^2 - 2|\mu_3^2| > 0 \tag{14}$$
$$\mathcal{B} \equiv \mu_1^2 \cdot \mu_2^2 - \mu_3^4 < 0 , \tag{15}$$

respectively. From these last two conditions it is clear that the tree-level potential, where μ_1, μ_2, μ_3 take the GUT scale values provided in (9) and (10), cannot yield the desired minimum. The renormalization effects are however big enough to ensure that the stability and breaking conditions (14) and (15) can be simultaneously satisfied together with the two minimization conditions (12) and (13).

3. Numerical Procedure

The Lagrangian described in the previous section has still to undergo renormalization down to the weak scale. These evolution effects will be analysed in the following sections for each sector of the theory. In this section, following the analysis of [11], we shall present the kind of procedure one uses to restrict the SUSY parameter space to those regions where the requirement of radiative breaking of the electroweak sector can be correctly implemented.

The free parameters present in the Lagrangian (6) are the SUSY breaking parameters m, M, A plus the supersymmetric parameter μ. Gauge couplings and Yukawa couplings appear also in (6). The former ones are quite well known at low energy. As far as the latter ones are concerned, only the third generation fermions are considered here as massive and we take the limit of Cabibbo-Kobayashi-Maskawa matrix as being equal to the unit matrix $\mathbb{1}$ ¶. The matrices h_U, h_D and h_L are diagonal and reduce at low-energy to the elements $h_t(M_Z)$, $h_b(M_Z)$ and $h_\tau(M_Z)$ which can be linked to $m_t(M_Z)$, $m_b(M_Z)$ and $m_\tau(M_Z)$ through the vacuum expectation values v_1 and v_2. The low-energy unknowns are then $m_t(M_Z)$ and $\tan\beta$. In principle, there is some uncertainty also in the value of m_b. As we shall see in section 5, the limited freedom for this parameter is partially cancelled by a further Grand Unification condition which also simultaneously reduces the range of allowed values of m_t and $\tan\beta$. For the time being, in this section, we shall neglect this complication and consider m_b as known.

Contact has to be made between the low-energy and high-energy parameters of the theory. This will be achieved through the renormalization group equations (RGE) and the two minimization conditions (12) and (13). These two conditions will enable us to fix two of the high-energy parameters, for example A and μ ‖.

In practice we shall proceed as follows. We shall evolve the low-energy gauge couplings $\alpha_S(M_Z)$, $\sin^2\theta_W$ and $\alpha_{em}(M_Z)$ to higher values of energy until a unification point is found. For an effective SUSY mass at the weak scale ($M_{SUSY} = M_Z$) and for

$$\alpha_S(M_Z) = 0.114, \quad \sin^2\theta_W = 0.233, \quad \alpha_{em}(M_Z) = 1/128 \tag{16}$$

we obtain the following values for the unification mass and for the common gauge coupling constant at the unification point:

$$M_X = 1.5 \times 10^{16}, \quad \alpha_X^{-1}(M_X) = 24.4 . \tag{17}$$

¶The results presented in Fig. 1 include also some generational mixing. This and some of the following figures were obtained in [11] with the aim of studying the effects of flavour change induced by SUSY. These results are obviously still valid in the approximation considered here. The values of the squark masses used for these figures, in fact, are not sizeably affected by the small inter-generational mixing elements in the squark mass matrices.

‖A different procedure of choosing the value of A and solving for $\tan\beta$ is used in [12].

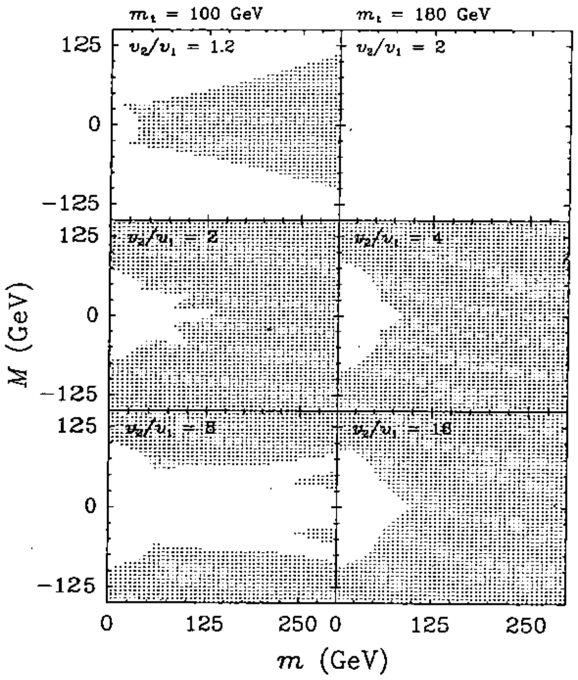

Figure 1: Possible configurations in the (m, M) space allowed by the conditions of radiative breaking of the electroweak sector, for different values of m_t and $\tan\beta$. The range considered here is $|m|$ and $m < 150\,\mathrm{GeV}$. Each allowed point in this plot can have up to a fourfold degeneracy in (A, μ).

A change of $M_{\rm SUSY}$ up to the TeV range affects mildly these results. For $M_{\rm SUSY} > M_Z$, the gauge couplings have to be evolved in two steps, according to the 2HDM-RGE up to the $M_{\rm SUSY}$ scale and then up again according to the SUSY-RGE. This type of analysis is obviously a simplified one since a sharp transition between the two regions is assumed. No threshold effects are included when the energy becomes high enough to allow the appearence of a new degree of freedom in the evolution of the gauge coupling constants and of all the other parameters in the Lagrangian (6). The same kind of sharp transition is assumed at the scale M_X.

Once M_X and α_X are known, for a fixed choice of $(m_t, \tan\beta)$, the Yukawa couplings can be evolved up to M_X in the same way. The values obtained will be used as high-scale input in the RGE for the remaining parameters whose high-scale boundary conditions are given by (10) and by:

$$m^2_{\tilde Q} = m^2_{\tilde U^c} = m^2_{\tilde D^c} = m^2_{\tilde L} = m^2_{\tilde E^c} = m^2$$
$$A_t = A_b = A_\tau = A \; . \qquad (18)$$

A formal solution for all the low-energy parameters, in terms of m, A, M and μ and their products, can be easily obtained by inspection of the relevant RGE. Once a pair (m, M) is chosen, the two minimization conditions (12) and (13) will provide solutions for A and μ. These are in general not unique solutions. Therefore a choice of the four parameters $(m, M, m_t, \tan\beta)$ can correspond to different possible supersymmetric realizations as observed in [11] and [12].

Now, among the "multiple" points of the four-dimensional space (m, M, m_t, $\tan\beta$) the ones giving the correct realization of SUSY have to be selected. For each point, it has to be checked for example, that the conditions (14) and (15) are verified, that no parameter of the theory is growing too much during the evolution from M_Z to M_X, and so on. Moreover, it has especially to be checked that no further unphysical vacua breaking charge and/or colour are accidentally generated. This can still happen, even after the removal of the points (m, M, m_t, $\tan\beta$) where negative values for the squared masses of squarks and sleptons are obtained. The minimization of the Higgs potential (8) is, in fact, not sufficient to guarantee that the vacuum (7) is the absolute minimum of the full scalar low-energy potential. In practice, the minimization of the full potential cannot be achieved through simple means. In order to avoid dangerous charge and colour breaking absolute minima, one imposes further constraints on the allowed range of the SUSY parameters [13–15]. These conditions are only necessary to avoid such minima [14]. They are given by:

$$A_t m^2 < 3(m^2_{\tilde t_L} + m^2_{\tilde t^c_L} + \mu_2^2) \qquad (19)$$
$$A_b m^2 < 3(m^2_{\tilde b_L} + m^2_{\tilde b^c_L} + \mu_1^2) \qquad (20)$$
$$A_\tau m^2 < 3(m^2_{\tilde \tau_L} + m^2_{\tilde \tau^c_L} + \mu_1^2) \qquad (21)$$

In [15] one finds a detailed discussion on the conditions for which (19)-(21) may become sufficient. In general, no necessary and sufficient conditions can be derived analytically, and the absence of stable colour and charge breaking vacua has to be checked numerically point by point in the parameter space. The unphysical vacua become stable in the direction of those charged and/or coloured scalars whose fermion partners have small Yukawa couplings (compared to the $SU(2)_{\rm L}$ gauge constant). This is certainly the case for the bottom and tau fields, when one considers not too large values of $\tan\beta$. On the other hand, precisely in the limit of small Yukawa couplings, (19)-(21) become also sufficient. This does not apply to (19), which remains only necessary. However, for large enough Yukawa couplings it is possible to show that the unwanted minima lie above the "correct" ones.

Finally, the existing experimental lower bounds on the supersymmetric masses have to be considered and an analysis of the cuts that these impose on the SUSY parameter space has to be made. Care has to be taken to use bounds as model independent as possible not conflicting with the main assumptions of the model we consider. These bounds are not taken into account in Figs. 1, 6, 7 and 8. The aim there is mainly to show the kind of restrictions imposed by the condition of radiatively induced electroweak breaking in the region $m < 300\,\mathrm{GeV}$ and $|M| < 150\,\mathrm{GeV}$ for several values of m_t and $\tan\beta$.

We close this section with a comment. It has been observed that one-loop corrections to the Higgs potential can unexpectedly play a major role [16–18]. Only for certain ranges of the scale at which one stops the renormalization of the parameters are we allowed to neglect the one-loop corrections and use the *RG improved* tree-level potential [16]. For the results presented here we have not tried to determine, case by case, the correct range of the renormalization scale. We have, however, a posteriori verified the stability of our results for different choices of the effective low-energy scale in the $M_Z - 2M_Z$ range. A more careful procedure was followed only in the particular case of $m_t = 150\,\mathrm{GeV}$ and $\tan\beta = 40$ described in section 6.

4. Gauge Couplings

We shall verify in this section that SUSY does allow the unification of the three gauge interactions without predicting too short a lifetime for the proton. To this end, we start by normalizing the hypercharge Y, in order to guarantee that the generators of $SU(3)_\mathrm{C}$, $SU(2)_\mathrm{L}$ and $U(1)_\mathrm{Y}$ have the same normalization once embedded in a Grand Unifying group. The one-loop evolution equations for coupling constants, g_3, g_2, g_1 (or g_S, g and $\sqrt{5/3}\, g_Y$) look like [19]

$$\frac{d\,\tilde\alpha_i}{d\,t} = -b_i\,\tilde\alpha_i^2 \tag{22}$$

where $\tilde\alpha_i$ is defined as $\tilde\alpha_i \equiv \alpha_i/(4\pi) = g_i^2/(16\pi^2)$, the variable t is given by $t = \log(\mu/M_Z)$ and b_i are the coefficients of the beta function of the couplings g_i. These equations can be easily integrated to give, in terms of the low-energy parameters $\alpha_S(M_Z)$, $\alpha_{em}(M_Z)$ and the Weinberg angle θ_W, the following relations:

$$\begin{aligned} \tilde\alpha_S^{-1}(M_Z) &= \tilde\alpha_3^{-1}(\mu) - b_3\, t \\ \sin^2\theta_W\ \tilde\alpha_{em}^{-1}(M_Z) &= \tilde\alpha_2^{-1}(\mu) - b_2\, t \\ \frac{3}{5}\cos^2\theta_W\ \tilde\alpha_{em}^{-1}(M_Z) &= \tilde\alpha_1^{-1}(\mu) - b_1\, t\ . \end{aligned} \tag{23}$$

The unification of the three interactions is then possible if there exists a scale $\mu = M_X$, such that

$$\tilde\alpha_3(M_X) = \tilde\alpha_2(M_X) = \tilde\alpha_1(M_X) = \tilde\alpha_X(M_X)\ . \tag{24}$$

The system (23) is a system of three equations and two unknowns, $\tilde\alpha_X(M_X)$ and $t_X \equiv \log(M_X/M_Z)$ which may not necessarily be solvable. Moreover, even if a solution is found, one still has to verify that it is not in conflict with the existing lower bound for the proton lifetime. It is easy to see from (23) that a unifying point for the three couplings can be found if $\alpha_S(M_Z)$, $\alpha_{em}(M_Z)$ and $\sin^2\theta_W$ satisfy the consistency condition

$$\frac{5}{3}\ \frac{\sin^2\theta_W\alpha_S(M_Z) - \alpha_{em}(M_Z)}{\left(1-\frac{8}{3}\sin^2\theta_W\right)\alpha_S(M_Z)} = \frac{b_2-b_3}{b_1-b_2}\ . \tag{25}$$

The SM does not fulfill this requirement. We recall that the one-loop expression for the coefficients b_i is given by

$$(b_3,b_2,b_1) = \left(-11+\frac{4}{3}\mathrm{N_G},\ -\frac{22}{3}+\frac{4}{3}\mathrm{N_G}+\frac{1}{6}\mathrm{N_H},\ \frac{4}{3}\mathrm{N_G}+\frac{1}{10}\mathrm{N_H}\right) \tag{26}$$

where $\mathrm{N_G}$ is the number of generations and $\mathrm{N_H}$ is the number of light Higgs doublets. For $\mathrm{N_G} = 3$ and $\mathrm{N_H} = 1$, the right hand side of (25) gives a value which is not possible to match on the left hand side, not even when the experimental uncertainty on $\alpha_S(M_Z)$ and $\sin^2\theta_W$ is taken into account. A careful analysis of these issues is contained in[20] and in [5] from where the Fig. 2 is borrowed. The results shown in Fig. 2a are based on the two-loop evolution equations for the running coupling constants. The same qualitative features can be obtained also by simply using the coefficients in (26).

A quick inspection of (26) shows that the unification of the electroweak and strong couplings cannot be achieved in the two simplest possible extensions of the SM:

- An increase in the number of generations, for example, gives the same result as the SM with $N_G = 3$. The reason is that $\mathrm{N_G}$ enters with the same weight in all the coefficients b_i and cancels on the right hand side of (25).
- An increase in the number of Higgs doublets is in principle enough to achieve unification. By plugging (26) in (25) one gets in fact a solvable equation in $\mathrm{N_H}$ with possible solution $\mathrm{N_H} = 7$. The two-loop analysis lowers this value to $\mathrm{N_H} = 6$ [5]. Unfortunately these values do not give viable unification points, as it can be seen in Fig. 2b. A generic increase of $\mathrm{N_H}$, in fact, while not touching the slope of the strong coupling, slows down the increase of α_2^{-1} more than it speeds up the falling of α_1^{-1}. If one keeps increasing the value of $\mathrm{N_H}$, eventually the unification becomes possible, but for values of energy which give too fast a proton decay.

In contrast, SUSY introduces a drastic change in the values of the coefficients b_i, here compared with the ones for the SM and the two Higgs doublet model (2HDM):

$$(b_3,b_2,b_1) = \begin{cases} (-7,\ -19/6,\ 41/10) & \text{SM} \\ (-7,\ -3,\ 21/5) & \text{2HDM} \\ (-3,\ 1,\ 33/5) & \text{SUSY.} \end{cases} \tag{27}$$

In particular, it manages to slow down the strong coupling constant evolution and to achieve unification at the scale given in (17) if $M_\mathrm{SUSY} = M_Z$ and the input values (16) are used. The results obtained in [5] are displayed in Fig. 2c.

Before closing this session, we should mention that there still exist other non-supersymmetric possibilities to achieve unification, by allowing larger groups than $SU(5)$ to break to the SM in at least two stages [20]. An ordinary $SO(10)$, for example, can break first to a left-right symmetric $SU(3)_\mathrm{C} \times SU(2)_\mathrm{L} \times SU(2)_\mathrm{R} \times U_\mathrm{B-L}$ at a scale M_X and then to the SM at M_R. The unification cannot be achieved though if one tries to fix M_R in the TeV range. On the contrary, by leaving M_R as a free parameter, one obtains solvable equations with viable unification point for $M_R = 10^{10}\,\mathrm{GeV}$. While SUSY still allows a one-step unification and can fill the so called desert between M_Z and M_X in the lower corner, these models seem to be offering a complementary alternative.

5. Yukawa Couplings

The Yukawa couplings form the other sector of the model where the influence of SUSY is simply felt through the increased number of degrees of freedom and the value

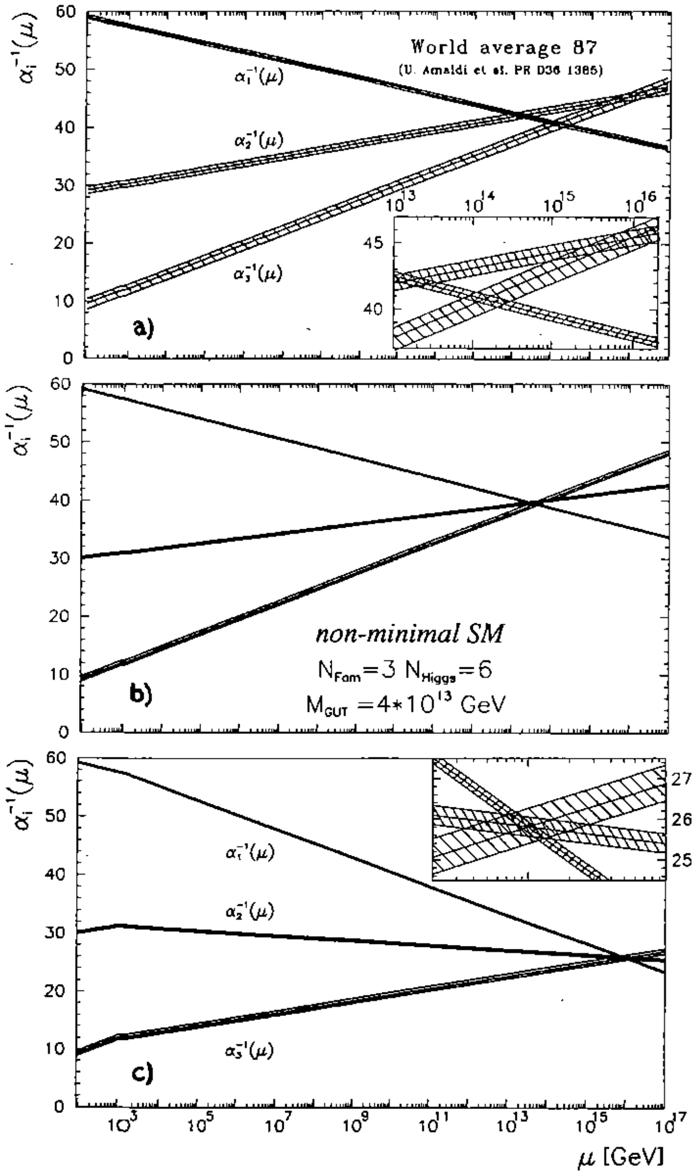

Figure 2: Evolution of the strong and electroweak gauge couplings in the Standard Model, an extension of the Standard Model with six Higgs doublets and the minimal supersymmetric model. The input values of $\alpha_S(M_Z) = 0.108 \pm 0.005$ and $\sin^2\theta_W(\overline{MS}) = 0.2336 \pm 0.0018$ are used here.

of M_{SUSY}. The evolution equations for these couplings are

$$\begin{aligned} 2\dot{\bar{h}}_t &= \left[(GY)_t - 6\bar{h}_t^2 - \bar{h}_b^2\right]\bar{h}_t & & (GY)_t = \tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{13}{15}\tilde{\alpha}_1 \\ 2\dot{\bar{h}}_b &= \left[(GY)_b - 6\bar{h}_b^2 - \bar{h}_t^2 - \bar{h}_\tau^2\right]\bar{h}_b & \text{with} \quad & (GY)_b = \tfrac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \tfrac{7}{15}\tilde{\alpha}_1 \\ 2\dot{\bar{h}}_\tau &= \left[(GY)_\tau - 4\bar{h}_\tau^2 - 3\bar{h}_b^2\right]\bar{h}_\tau & & (GY)_\tau = 3\tilde{\alpha}_2 + \tfrac{9}{5}\tilde{\alpha}_1 \end{aligned} \tag{28}$$

and with $\bar{h}_i \equiv h_i/4\pi$. The convention $\dot{\bar{h}}_i = \mathrm{d}\bar{h}_i/\mathrm{d}t$ has been adopted in order to have a more compact form of the equations. Since the 2HDM-RGE have exactly the same structure as the supersymmetric ones, we give for comparison only the evolution equations for the Yukawa couplings in the SM

$$\begin{aligned} 2\dot{\bar{h}}_t &= \left[(GY)_t - \tfrac{9}{2}\bar{h}_t^2 - \tfrac{3}{2}\bar{h}_b^2 - \bar{h}_t a u^2\right]\bar{h}_t & & (GY)_t = 8\tilde{\alpha}_3 + \tfrac{9}{4}\tilde{\alpha}_2 + \tfrac{17}{20}\tilde{\alpha}_1 \\ 2\dot{\bar{h}}_b &= \left[(GY)_b - \tfrac{9}{2}\bar{h}_b^2 - \tfrac{3}{2}\bar{h}_t^2 - \bar{h}_\tau^2\right]\bar{h}_b & \text{with} \quad & (GY)_b = 8\tilde{\alpha}_3 + \tfrac{9}{4}\tilde{\alpha}_2 + \tfrac{1}{4}\tilde{\alpha}_1 \\ 2\dot{\bar{h}}_\tau &= \left[(GY)_\tau - 3\bar{h}_t^2 - 3\bar{h}_b^2 - \tfrac{5}{2}\bar{h}_\tau^2\right]\bar{h}_\tau & & (GY)_\tau = \tfrac{9}{4}\tilde{\alpha}_2 + \tfrac{9}{4}\tilde{\alpha}_1 \, . \end{aligned} \tag{29}$$

Due to the presence of only one Higgs doublet, the equations (29) differ from (28) in the contribution of the top quark to the renormalization of the tau Yukawa coupling.

The nonlinearity of the equations (28) makes the increase of the Yukawa couplings with energy quite fast. This feature, present also in the SM and the 2HDM equations, is in (28) quantitatively enhanced by the presence of new degrees of freedom. Given the value of the coefficients of $\tilde{\alpha}_3$ and $\bar{h}_t$ in (28), the Yukawa coupling relative to the existing lower bound on the top mass of 89 GeV [21],

$$h_t = \frac{m_t}{v_2} > \frac{m_t}{v} \sim 0.5 \tag{30}$$

is still not quite competitive with the strong coupling g_3, but it becomes dangerously so as soon as m_t approaches $180 - 190$ GeV. The requirement of applicability of (28) throughout the evolution of the low-energy parameters up to the GUT scale, is then the origin of the existence of an upper bound on the top mass in supersymmetric models. The approximate value of 196 GeV is obtained in the limit $\tan\beta \gg 1$, but lower upper bounds are obtained for intermediate values of $\tan\beta$. The too fast growth of h_t while approaching the GUT scale M_X is also the reason why sufficiently high values of m_t may not give acceptable solutions of the equations (28) for $\tan\beta$ too close to 1 **.

A few more observations are in order regarding the boundary conditions of the evolution equations (28):

- The Yukawa couplings entering in (28) are related to the third generation fermion masses and to $\tan\beta$ by

$$\frac{m_t(M_Z)}{m_b(M_Z)} = \frac{h_t(M_Z)}{h_b(M_Z)}\tan\beta, \qquad \frac{m_\tau(M_Z)}{m_b(M_Z)} = \frac{h_\tau(M_Z)}{h_b(M_Z)} \, . \tag{31}$$

**We restrict ourselves here to values of $\tan\beta > 1$ as predicted by SUSY before the introduction of radiative corrections to the Higgs potential. This modification, while allowing smaller values of $\tan\beta$ would not modify the discussion for values greater than 1.

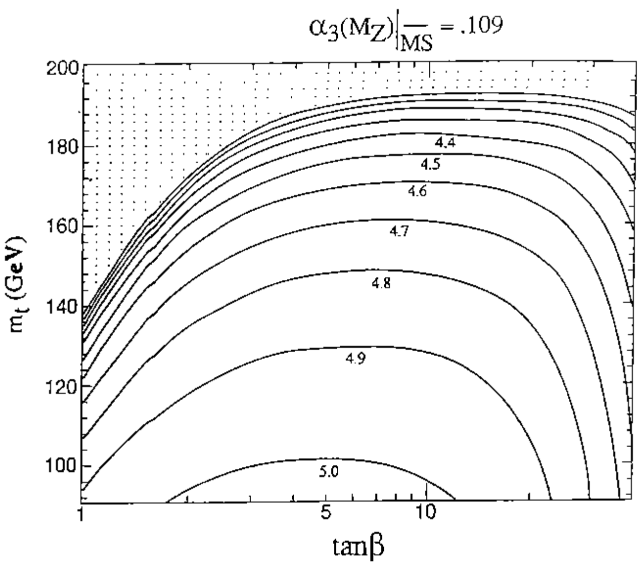

Figure 3: Restrictions imposed upon the values of m_t, m_b and $\tan\beta$ by the condition $h_b(M_X) = h_\tau(M_X)$ and the request of validity of perturbative calculations.

Besides m_t and $\tan\beta$ being unknown, also the value of m_b is plagued by uncertainties. We can only restrict ourselves to a range of values $4.2 - 5.2$ for $m_b(2m_b)$. The values of $m_b(M_Z)$ can then be obtained by QCD evolution.

- The GUT scale values of the Yukawa couplings, while unknown, are nevertheless linked to each other by relations depending on the particular Grand Unified model considered. For an underlying $SU(5)$ and $SO(10)$ symmetry we have respectively

$$\bar{h}_b(M_X) = \bar{h}_\tau(M_X) \qquad (SU(5)) \tag{32}$$

$$\bar{h}_t(M_X) = \bar{h}_b(M_X) = \bar{h}_\tau(M_X) \qquad (SO(10)) \; . \tag{33}$$

Both conditions reduce the region of parameter space, $\tan\beta$, m_t, m_b [11,22,23].

We choose to impose the condition (32). For fixed values of m_t, for the input parameters (16) and for the GUT scale parameters (17), we scan the allowed values of m_b and $\tan\beta$ until solutions of the evolution equations (28) satisfying the unification conditions (32) are found. The results shown in Fig. 3 (taken from [22]) are quite obvious:

- For fixed values of $\tan\beta$ an increase in the top mass favours smaller values of m_b in the phenomenologically allowed range. Higher values of h_b would further increase the growth of h_t, as it can be seen from (28), spoiling (32) first and then giving values of h_t not acceptable in the perturbative regime. This explains why in Fig. 3 the curves with smaller values of m_b are always above the ones with higher m_b.

- On the other hand, for fixed values of m_t and for small values of $\tan\beta$, an increase in $\tan\beta$ gives naturally higher values of h_b and of m_b for $h_b \ll h_t$. Further increases of $\tan\beta$ require still higher values of h_b without influencing very much m_b. All the curves display in fact a central plateau. Moreover, smaller values of h_t have to be used in order to mantain m_t unchanged. Eventually, the region $h_b \sim h_t$ is approached. Further increases of $\tan\beta$ can start affecting the condition (32) explaining therefore the sharp drop of the curves for high values of $\tan\beta$.

- The lower possible value of $m_b(M_Z)$, compatible with a fixed value of $m_t(M_Z)$ gives a top dependent upper bound of $\tan\beta$

$$(\tan\beta)_{\max} \sim \frac{m_t(M_Z)}{m_b(M_Z)} \; . \tag{34}$$

This value is obviously obtained in the limit $h_t \sim h_b$ [ll]. It was also shown in a different context [12] that values exceeding this bound would be in disagreement with (14).

These features are also implicitly studied in [11] and used for Figs. 1, 6, 7 and 8. It can be observed in Fig. 1, for example, that for $m_t = 100\,\text{GeV}$, a numerical solution of (28) is found for $\tan\beta$ as small as 1.2, while for $m_t = 180\,\text{GeV}$, a solution is obtained only for $\tan\beta > 2$. For this same m_t, values of $\tan\beta$ in the range $2 - 2.5$ violate the condition (32).

We conclude this section by observing that it is quite hard to satisfy the condition (32) in the SM. The presence of the top Yukawa coupling in the evolution equation for h_τ and the lack of α_3, makes the ratio h_b/h_τ too big for a top mass $\sim 140\,\text{GeV}$ [24].

6. Higgs Potential Parameters and $SU(2)_L \times U(1)_Y$ Breaking

As already mentioned, one of the reasons for the success of this theory was due to the realization that the logarithmic radiative corrections to the mass parameters of the scalar potential were big enough to induce the breaking of the electroweak sector. This result, initially obtained for an effective SUSY breaking of $\mathcal{O}(M_Z)$, remains unchanged if the SUSY breaking scale moves up to the TeV range. This mechanism, besides being appealing, has the clear advantage of avoiding additional Higgs fields and therefore of keeping the number of new free parameters of the theory quite small.

It was observed in section 2 that the GUT scale values of μ_1, μ_2, μ_3 are quite far from satisfying the stability and breaking conditions (14) and (15) and the two

[ll] Some attempts have been made in the past to obtain an absolute upper bound on $\tan\beta$. It was argued in [6] and [8] that for increasing values of $h_t(m_Z) \sim h_b(M_Z)$, h_b would eventually evolve to non-perturbative values. Some non-admissible value of $h_b(M_Z)$ was then used to constrain the value of $\tan\beta$ independently of m_t.

minimization conditions (12) and (13). Their low-energy values can be obtained from the evolution equations

$$\begin{aligned}(\dot{\bar{\mu}}_1^2) &= (GH)_H - 3\bar{h}_b(SS)_b - \bar{h}_\tau(SS)_\tau \\ (\dot{\bar{\mu}}_2^2) &= (GH)_H - 3\bar{h}_t(SS)_t \end{aligned} \tag{35}$$

$$\begin{aligned}(\dot{\mu}_{\rm R}^2) &= \left[(GH)_\mu - \left(3\bar{h}_t^2 + 3\bar{h}_b^2 + \bar{h}_\tau^2\right)\right]\mu_{\rm R}^2 \\ m\dot{B}_{\rm R} &= -\left[(GH)_B - \left(3\bar{h}_t^2 A_t + 3\bar{h}_b^2 A_b + \bar{h}_\tau^2 A_\tau\right) m\right] \end{aligned} \tag{36}$$

where the definition $\bar{\mu}_{1,2}^2 \equiv \mu_{1,2}^2 - \mu_{\rm R}^2$ has been used. The subscript R in $\mu_{\rm R}^2$ and $B_{\rm R}$ distinguishes these renormalized parameters from the high-scale ones μ and B. The relation (10) at low-energy then reads $\mu_3^2 = -B_{\rm R} m \mu_{\rm R}$. From this and (36) one can easily obain the evolution equation for μ_3^2. The gaugino and scalar masses contribution to (35) and (36) are

$$\begin{aligned}(GH)_H &= 3\tilde{\alpha}_2 M_2^2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1^2 & \qquad (SS)_t &= (m_{\tilde Q}^2 + m_{\tilde U}^2)_{33} + \bar{\mu}_2^2 + A_t^2 m^2 \\ (GH)_\mu &= 3\tilde{\alpha}_2 + \tfrac{3}{5}\tilde{\alpha}_1 \qquad \text{and} & (SS)_b &= (m_{\tilde Q}^2 + m_{\tilde D}^2)_{33} + \bar{\mu}_1^2 + A_b^2 m^2 \\ (GH)_B &= 3\tilde{\alpha}_2 M_2 + \tfrac{3}{5}\tilde{\alpha}_1 M_1 & (SS)_\tau &= (m_{\tilde L}^2 + m_{\tilde E}^2)_{33} + \bar{\mu}_1^2 + A_\tau^2 m^2 \ . \end{aligned} \tag{37}$$

It is easy to see from the previous equations that μ_2^2 decreases faster than μ_1^2 since its evolution is driven by the top quark, and both μ_1^2 and μ_2^2 decrease faster than μ_3^2. These three quantities start evolving down from the initial conditions (9) and (10) to a situation where

$$\left(\mu_3^2\right)^2 < \left(\mu_2^2\right)^2 < \mu_1^2 \cdot \mu_2^2 < \left(\frac{\mu_1^2 + \mu_2^2}{2}\right)^2 < \left(\mu_1^2\right)^2 \ . \tag{38}$$

It is still $B > 0$, which also implies $S > 0$. Afterwards, μ_2^2 decreases even further, while evolving down to lower energies and eventually equals μ_3^2. The $SU(2)_{\rm L} \times U(1)_{\rm Y}$ breaking becomes possible when μ_3^2 is located between the geometrical and arithmetical mean of μ_1^2 and μ_2^2 without μ_1^2 and μ_2^2 having necessarily to be negative. An example is given in Fig. 4 where suitable values of A and μ are taken after having tested that they do satisfy the correct breaking pattern.

A few more considerations can be drawn here on the possible values of $\tan\beta$ with respect to the SUSY parameters:

- If no correction to the Higgs potential (8) are added, at the electroweak scale one generally has $\mu_2^2 < \mu_1^2$. The minimization condition (13) then implies $\tan\beta > 1$.
- For top masses above the existing experimental lower bound, values of $\tan\beta$ too close to one (i.e. $h_t \gg h_b$) but still compatible with the requirements of perturbation theory, may admit only a very limited region of SUSY parameters. This is the case of $\tan\beta = 1.2$ for $m_t = 100\,$GeV depicted in Fig. 1. We see there in fact, that too big values of the gaugino parameter M would bring μ_2^2 too far from μ_1^2 through the indirect effect of the term $(SS)_t$ in (35).

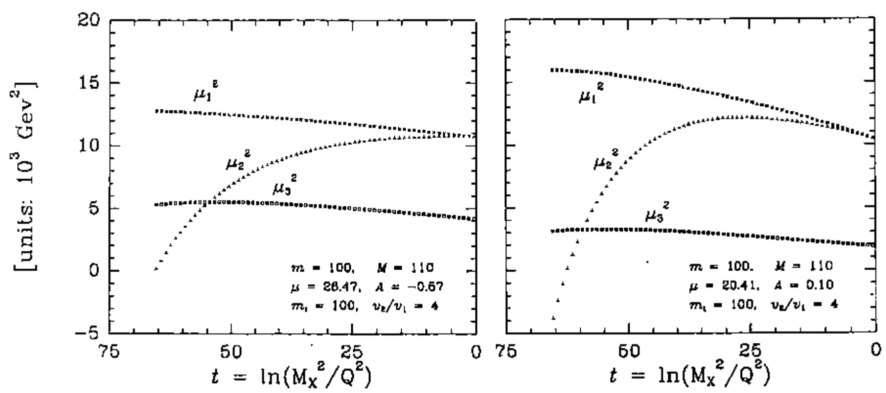

Figure 4: Evolution of μ_1^2, μ_2^2 and μ_3^2, from the unification scale to the weak scale. The two figures correspond to the same value (m, M), but different solutions of A and μ.

- Heavy enough values of m_t cannot allow an arbitrary increase of $\tan\beta$. Since it is in this case $h_b \sim h_t$, the evolution of μ_1^2 and μ_2^2 becomes more and more similar limiting therefore the value of $\tan\beta$ in (13). As already mentioned, the upper bound (34) was also derived in [12] making use of the boundedness of the Higgs potential and the conditions imposed by radiative breaking.
- For $h_b \sim h_t$ the value of μ_3^2 giving the correct electroweak breaking, forced to lie between μ_1^2 and μ_2^2, becomes smaller and smaller the more h_b approaches h_t. A small value of μ_3^2 implies either a small value of $\mu_{\rm R}$ or a small value of $B_{\rm R}$. We find this second possibility for $m_t = 150\,$GeV and $\tan\beta = 40$ throughout the full range (m, M) considered, i.e. $0 < m < 250\,$GeV and $0 < M < 470\,$GeV. This solution is shown in Fig. 5 where for each point in the two-dimensional space (m, M), also the values of $\mu_{\rm R}$ and $B_{\rm R}$ compatible with the radiative breaking of $SU(2)_{\rm L} \times U(1)_{\rm Y}$ are displayed. Negative values of M, also allowed, are not shown for simplicity. This situation is quite "extreme" since it requires a value of m_b at the lower limit of the range we allow. It is nevertheless quite instructive. The narrow distribution of the values of $B_{\rm R}$ around zero shows, however, that some fine-tuning among all the parameters is needed. For this reason, the approximation $M_{\rm SUSY} = \mathcal{O}(M_Z)$ is not possible here. This solution was obtained by stopping the downward renormalization of squarks and gluinos at the average value of their masses obtained for each pair (m, M).

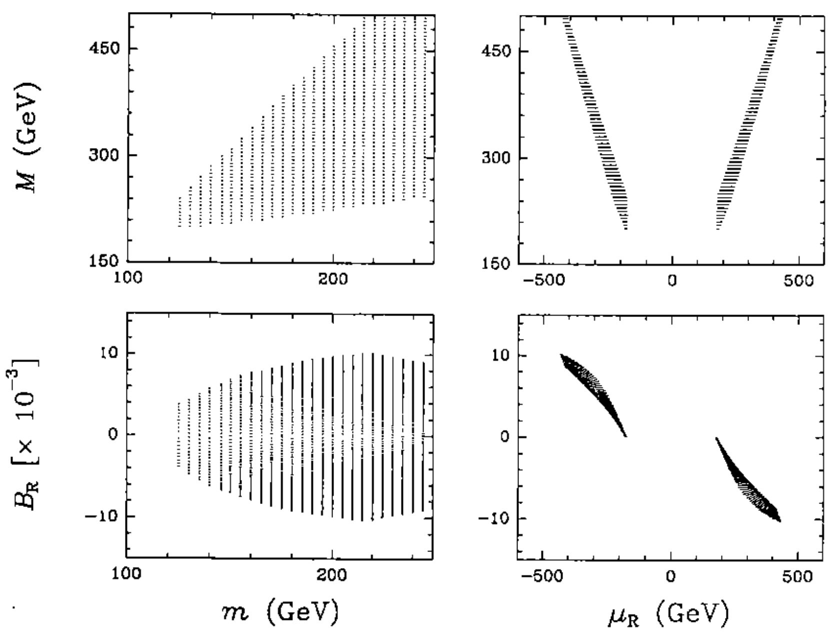

Figure 5: Possible supersymmetric realizations for the "extreme" choice of $m_t = 150$ GeV and $\tan\beta = 40$. For simplicity, only positive values of M are plotted here.

7. Gaugino Masses

Starting from this section we shall discuss the type of supersymmetric particle spectra one can obtain in the Minimal Supersymmetric Standard Model. We start here with the gaugino masses determined by the parameter M. This common parameter in (5) renormalizes down to three distinct values M_1, M_2 and M_3 according to the equations

$$\frac{d\,M_i}{d\,t} = -b_i\,\tilde{\alpha}_i M_i \tag{39}$$

with the coefficients b_i already given in (27). For $M_{\rm SUSY} = M_Z$ and the usual input parameters (16), the ratios M_3/M, M_2/M and M_1/M are 2.79, 0.82 and 0.42, respectively. M_1 and M_2 enter in the chargino and neutralino mass matrices. M_3 gives the gluino mass $m_{\tilde{g}}$, once a possible negative sign allowed for the parameter M is removed. Except for some particular values of m_t and $\tan\beta$ and only in a limited region of m (see the cases $m_t = 100$ GeV and $\tan\beta = 1.2$ and 8), no lower limit on $m_{\tilde{g}}$ can be inferred in this model. A comparison with the experimental lower bound for $m_{\tilde{g}}$ is given in section 9.

8. Chargino, Neutralino and Higgs Masses

The chargino and neutralino sector is entirely determined by M, the value of parameter μ at the weak scale, which we indicate with μ_R, $\tan\beta$, the Yukawa couplings and the electroweak gauge couplings. We shall examine these three sectors in the following. The results are shown in Figs. 6 and 7.

Chargino Masses:

The relevant terms in the low-energy Lagrangian are [25]

$$\frac{1}{2}\left\{(\widetilde{W}^- \widetilde{H}_1^-)\; M_C \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_2^+ \end{pmatrix} + h.c.\right\} \qquad \text{with} \qquad M_C = \begin{pmatrix} M_2 & gv_2 \\ gv_1 & \mu_R \end{pmatrix} \tag{40}$$

where $\widetilde{W}^\pm \equiv -i\lambda^\pm = -i(\lambda_1 \mp i\lambda_2)/\sqrt{2}$. We denote by $\tilde{\chi}^+_{1,2}$ the mass eigenstates for the Lagrangian terms in (40). By using the relation $2M_W^2 = g^2(v_1^2+v_2^2)$ and by assuming M and μ to be real, it is easy to see that the chargino mass eigenvalues can be written as

$$m^2_{\tilde{\chi}^+_{1,2}} = \frac{M_2^2+\mu_R^2+2M_W^2}{2} \pm \frac{1}{2}\sqrt{(M_2^2+\mu_R^2+2M_W^2)^2 - 4\,(M_2\mu_R - M_W^2\sin 2\beta)^2}\;. \tag{41}$$

For $\tan\beta \gg 1$ one may expect to obtain the characteristic gaugino-higgsino spectrum. In this limit and for small μ_R in fact, the two eigenvalues are:

$$m^2_{\tilde{\chi}^+_2} \sim \frac{M_2^2\mu_R^2}{M_2^2+2M_W^2}\,, \qquad m^2_{\tilde{\chi}^+_1} \sim M_2^2 + 2M_W^2\;. \tag{42}$$

It was argued that high values of $\tan\beta$, favoured by a heavy top mass, should have led to the detection of a light chargino at LEP. In practice the situation is far from being so simple. High values of $\tan\beta$ do imply small values of μ_3^2, but this does not allow us to draw a definite conclusion on μ (see Fig. 5). The interplay between μ_R, M_2, $\tan\beta$ and indirectly also of all the other independent parameters of the theory, is more complex for $\tan\beta \geq 1$. In principle, the full range of masses from approximately zero up to the rough upper bound given by the chosen value of M is allowed for the lightest chargino $\tilde{\chi}^+_2$.

Neutralino Masses:

A similar situation holds also for the neutralino masses. The relevant term of the low-energy Lagrangian is in this case:

$$\frac{1}{2}(\widetilde{B},\widetilde{W}_3,\widetilde{H}^0_1,\widetilde{H}^0_2)\begin{pmatrix} M_1 & 0 & -g'v_1/\sqrt{2} & g'v_2/\sqrt{2} \\ 0 & M_2 & gv_1/\sqrt{2} & -gv_2/\sqrt{2} \\ -g'v_1/\sqrt{2} & gv_1/\sqrt{2} & 0 & -\mu_R \\ g'v_2/\sqrt{2} & -gv_2/\sqrt{2} & -\mu_R & 0 \end{pmatrix}\begin{pmatrix} \widetilde{B} \\ \widetilde{W}_3 \\ \widetilde{H}^0_1 \\ \widetilde{H}^0_2 \end{pmatrix} + \; h.c. \tag{43}$$

We indicate by $\tilde{\chi}^0_i$ the four-component (Majorana) spinors obtained in terms of the two-components mass eigenstates χ^0_i $(i=1,4)$, $(\tilde{\chi}^0_i \equiv (\chi^0_i, \bar{\chi}^0_i))$. As shown in Fig. 6, the lightest of these states, $\tilde{\chi}^0_1$ can have a mass from zero up to a maximum value in

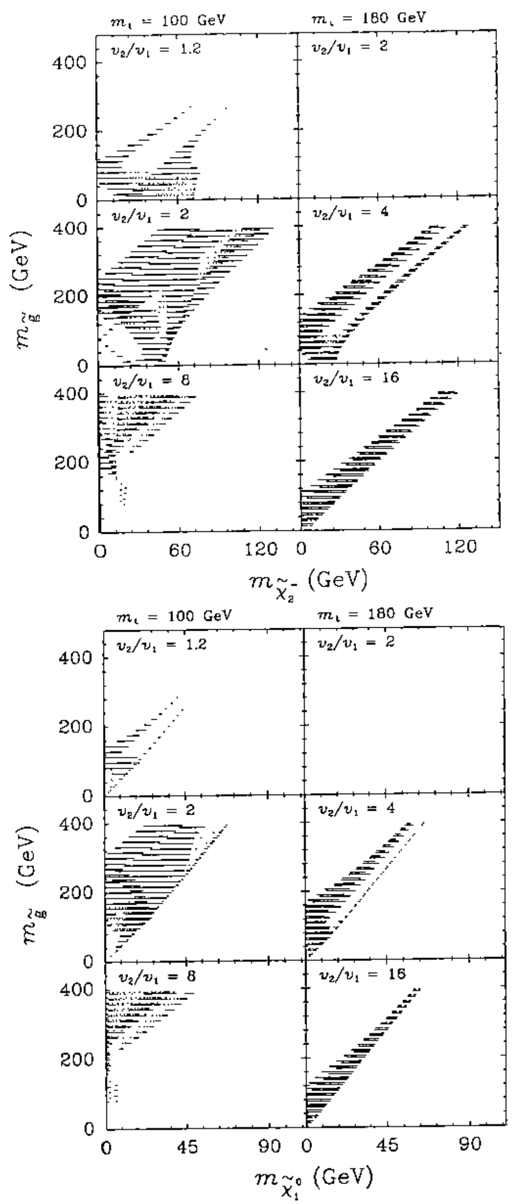

Figure 6: Lightest chargino and lightest neutralino masses obtained for the same values of the SUSY parameter space considered in Fig. 1.

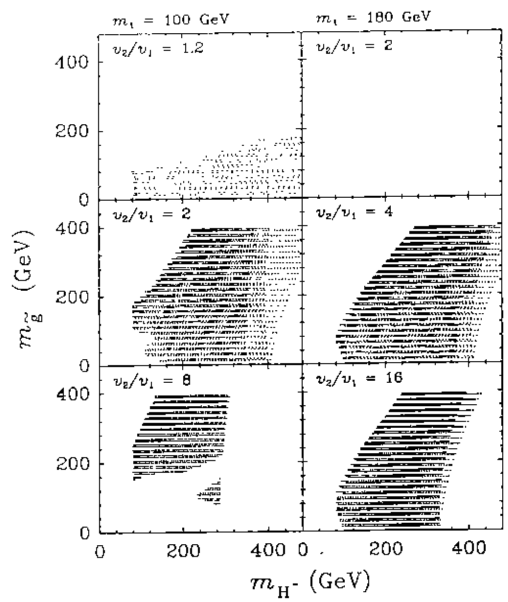

Figure 7: Charged Higgs masses obtained for the same values of the SUSY parameter space considered in Fig. 1.

general quite smaller than the corresponding maximum value allowed for $m_{\tilde{\chi}_2^+}$ in the same region of SUSY parameter space.

Higgs Masses:

The Higgs sector is the one which has received the greatest attention lately. Five physical scalars appear in this sector after the spontaneous breaking of $SU(2)_L \times U(1)_Y$. We denote by $H^0_{1,2}$ the two CP-even mass eigenstates, by H^0_3 the CP-odd state, and by $H^\pm$ the two charged ones. The tree-level effective potential (8) allows the following mass spectrum for these scalars:

$$m^2_{H^0_3} = \mu_1^2 + \mu_2^2 = \frac{2\mu_3^2}{\sin 2\beta} \tag{44}$$

$$m^2_{H^\pm} = M_W^2 + m^2_{H^0_3} \tag{45}$$

and

$$m^2_{H^0_1,H^0_2} = \frac{1}{2}\left[m^2_{H^0_3} + M_Z^2 \pm \sqrt{(m^2_{H^0_3} + M_Z^2)^2 - 4m^2_{H^0_3} M_Z^2 \cos^2 2\beta}\,\right] . \tag{46}$$

At the tree-level then, the following relations hold independently of the value of $\tan\beta$

$$m_{H^0_2} < m_{H^0_3} < m_{H^0_1} , \qquad m_{H^0_2} < M_Z . \tag{47}$$

Until recently, the second of these two relations was considered the main test of SUSY. The lack of observation of a Higgs particle below the Z mass was considered enough to completely rule out the model. The situation has changed drastically after the radiative corrections to the potential (8) have been added (more details will be given in the following lectures). The most affected of all the five Higgs is of course H_2^0 whose mass can be as small as zero if no corrections to the potential (8) are considered. Smaller is the effect on the remaining four scalars. Of these, the pseudoscalar is the one which sets the scale for $m_{H_1^0}$ and $m_{H^\pm}$. We have already observed that in the limit $h_t \sim h_b$, μ_3^2 can be quite small while $\tan\beta$ approaches the maximum value allowed by m_t and m_b (34) i.e. $\tan\beta_{\max} \sim m_t(M_Z)/m_b(M_Z)$. In this case, $m_{H_3^0}$ can be very light or nearly massless bringing therefore the values of $m_{H_1^0}$ and $m_{H^\pm}$ down to M_Z and M_W, respectively. The decrease of $m_{H^\pm}$ with increasing values of $\tan\beta$ can already be observed in Fig. 7, even for not so high values of $\tan\beta$.

We conclude this section by observing that for average values of $\tan\beta$, as shown in Fig. 6 and Fig. 7, this model indicates that the neutralino has the lightest possible spectrum among the particles considered in this section. For the same choice of SUSY parameters, the charged Higgs is *on average* heavier than the lightest chargino, although the situation may be different for $\tan\beta \gg 1$.

From the experimental point of view the possibility of imposing lower bounds on the masses of these particles is far from being obvious or assumption-free. Absolute lower bounds exist only for the charged particles, i.e. chargino [26] and charged Higgs [27]. They come from LEP and they are about 45 GeV.

As for the neutral Higgses H_0^2 and H_0^3, the situation has been considerably complicated by the addition of the one-loop corrections to the potential (8). The prediction $m_{H_2^0} < m_{H_3^0}$ is not valid anymore and the new decay $H_2^0 \to H_3^0 H_3^0$ becomes possible. Moreover, the search for neutral Higgses at LEP in the two channels $e^+e^- \to Z \to H_2^0 Z^*$ and $e^+e^- \to Z \to H_2^0 H_3^0$ is now sensitive to the top mass and the scalar quark masses, in particular the scalar partner of the top quark. The regions $m_{H_2^0} < 41$ GeV and $m_{H_3^0} < 31$ GeV have been excluded at 95% CL [28] for $m_t = 140$ GeV, $m_{\tilde t} = 1$ TeV and under the assumption that no mixing exists between the left-handed and right-handed component of the stop. As we shall see in the next section, the scalar partner of the top quark plays a particular role in the model we discuss here and these assumptions are quite far from being theoretically justified in this particular framework.

9. Scalar –Quark and –Lepton Masses

We start by considering the squark mass matrices. The 6×6 matrix of the $Q = 2/3$ sector is formally written in terms of the 3×3 submatrices $M^2_{U_{LL}}$, $M^2_{U_{RR}}$ and $M^2_{U_{LR}}$ as follows:

$$\widetilde{M}_U^2 = \begin{pmatrix} M^2_{U_{LL}} & M^2_{U_{LR}} \\ M^2_{U_{LR}} & M^2_{U_{RR}} \end{pmatrix} . \qquad (48)$$

$M^2_{U_{LL}}$ and $M^2_{U_{RR}}$ are the mass matrices of the left- and right-handed component of the up-type squarks, while $M^2_{U_{LR}}$ contains the mass terms mixing the two components. It

holds that:

$$\begin{aligned} M^2_{U_{LL}} &= \mathrm{diag}\left(m^2_{\tilde Q_{11}}, m^2_{\tilde Q_{22}}, m^2_{\tilde Q_{33}} + m_t^2\right) - |DT_U^L|\, \mathbb{1} \\ M^2_{U_{RR}} &= \mathrm{diag}\left(m^2_{\tilde U_{11}}, m^2_{\tilde U_{22}}, m^2_{\tilde U_{33}} + m_t^2\right) - |DT_U^R|\, \mathbb{1} \\ M^2_{U_{LR}} &= \mathrm{diag}\left(0, 0, (A_t m + \mu_R \cot\beta)\, m_t\right) \end{aligned} \qquad (49)$$

where $DT_U^{L,R}$ are the so-called "D-term" contributions given by

$$DT_U^{L,R} = M_Z^2 \cos 2\beta \left(T_U^3 - Q_U \sin\theta_W\right) . \qquad (50)$$

Similarly, the corresponding 3×3 submatrices $M_{D_{LL}}$, $M_{D_{RR}}$ and $M^2_{D_{LR}}$ for the $Q = -1/3$ sector are

$$\begin{aligned} M^2_{D_{LL}} &= \mathrm{diag}\left(m^2_{\tilde Q_{11}}, m^2_{\tilde Q_{22}}, m^2_{\tilde Q_{33}} + m_b^2\right) + |DT_D^L|\, \mathbb{1} \\ M^2_{D_{RR}} &= \mathrm{diag}\left(m^2_{\tilde D_{11}}, m^2_{\tilde D_{22}}, m^2_{\tilde D_{33}} + m_b^2\right) + |DT_D^R|\, \mathbb{1} \\ M^2_{D_{LR}} &= \mathrm{diag}\left(0, 0, (A_b m + \mu_R \tan\beta)\, m_b\right) . \end{aligned} \qquad (51)$$

The "D-term" contributions for the down-sector can be obtained from (50), once the correct quantum numbers are substituted for the ones for the up-sector. The same rules apply for building up the mass matrices of the lepton superpartners. The scalar neutrino mass matrix is only a 3×3 matrix, since neutrinos are here considered massless and do not have right-handed components.

A simple reshuffling of rows and columns in the matrices (49) and (51) gives a block-diagonal matrix with 2×2 diagonal submatrices for the first and second generation and 2 × 2 non-diagonal ones for the third generation. Moreover, since we neglect here the masses of the first two generations of quarks and leptons, the evolution of the masses of the first two generations of sfermions is determined by the same equations. Once we introduce the notation $m^2_{\tilde Q_{11}} = m^2_{\tilde Q_{22}} = m^2_{\tilde u_L} = m^2_{\tilde d_L}$, $m^2_{\tilde U_{11}} = m^2_{\tilde U_{22}} = m^2_{\tilde u_R}$, $m^2_{\tilde D_{11}} = m^2_{\tilde D_{22}} = m^2_{\tilde d_R}$ and similar definitions for the leptonic sector, these equations have the simple form

$$\begin{aligned} (\dot{m}^2_{\tilde u_L}) = (\dot{m}^2_{\tilde d_L}) &= (GG)_Q = \frac{16}{3}\tilde\alpha_3 M_3^2 + 3\tilde\alpha_2 M_2^2 + \frac{1}{5}\tilde\alpha_1 M_1^2 \\ (\dot{m}^2_{\tilde u_R}) &= (GG)_U = \frac{16}{3}\tilde\alpha_3 M_3^2 + \frac{16}{15}\tilde\alpha_1 M_1^2 \\ (\dot{m}^2_{\tilde d_R}) &= (GG)_D = \frac{16}{3}\tilde\alpha_3 M_3^2 + \frac{4}{15}\tilde\alpha_1 M_1^2 \\ (\dot{m}^2_{\tilde e_L}) = (\dot{m}^2_{\tilde \nu_L}) &= (GG)_L = \tilde\alpha_2 M_2^2 + \frac{3}{5}\tilde\alpha_1 M_1^2 \\ (\dot{m}^2_{\tilde e_R}) &= (GG)_E = \frac{12}{5}\tilde\alpha_1 M_1^2 . \end{aligned} \qquad (52)$$

The evolution of the first two generations of sfermion masses is driven only by the gaugino masses and their low-energy value can be expressed as $m_{\tilde q}^2 = m^2 + C_M(\tilde q) M^2 + DT(\tilde q)$. In particular, for the input values (16) and for $M_{\rm SUSY} = M_Z$, it holds that:

$$m^2_{\tilde u_L} = m^2 + 6.51\, M^2 - 0.35\, M_Z^2\, |\cos 2\beta|$$

$$
\begin{aligned}
m^2_{\tilde{u}_R} &= m^2 + 6.09\, M^2 - 0.16\, M_Z^2\, |\cos 2\beta| \\
m^2_{\tilde{d}_L} &= m^2 + 6.51\, M^2 + 0.42\, M_Z^2\, |\cos 2\beta| \\
m^2_{\tilde{d}_R} &= m^2 + 6.04\, M^2 + 0.08\, M_Z^2\, |\cos 2\beta| \\
m^2_{\tilde{e}_L} &= m^2 + 0.52\, M^2 + 0.27\, M_Z^2\, |\cos 2\beta| \\
m^2_{\tilde{e}_R} &= m^2 + 0.15\, M^2 + 0.23\, M_Z^2\, |\cos 2\beta| \\
m^2_{\tilde{\nu}_{eL}} &= m^2 + 0.52\, M^2 - 0.50\, M_Z^2\, |\cos 2\beta| \; .
\end{aligned} \tag{53}
$$

We observe here:

- The corrections to the initial value m^2 are quite substantial.
- The values of the coefficients $C_{\rm M}(\tilde{q})$ have to be considered as indicative. A change of α_S and $\sin^2\theta_W$ in (16), within the experimental error, induces small changes in in the gauge couplings contribution to the equations (52) which are only partially neutralized by the variations also induced in the unification scale M_X. Moreover, a change in $M_{\rm SUSY}$ from M_Z to $2M_Z$ can affect these coefficients up to a factor of 15%. Notice that for the same variation of the SUSY scale and for the initial value of α_S given in (16), the ratio $m_{\tilde{g}}/M$ changes from 2.79 to 2.61.
- The "D-term" contributions tend to increase the values of the down-type squarks and slepton masses, but to decrease the value of up-type ones. The effect is not very big for squarks, but is quite sizeable for sleptons. For $m \sim M \sim M_Z$ in fact, the decrease/increase for up/down squarks is about 5%, but the correction can be up to 20 and 30 % for scalar electrons and sneutrinos.

The spectrum of the third generation of squarks and sleptons is more complicated. To begin with, the 2×2 submatrices which can be obtained for this generation from (48), (49) and (51), are not diagonal. We give explicitly the ones for the squarks of the third generation

$$
M_t^2 = \begin{pmatrix} m^2_{\tilde{Q}_{33}} + m_t^2 - |DT_U^L|\, 1\!\!1 & (A_t m + \mu_{\rm R}\cot\beta)\, m_t \\ (A_t m + \mu_{\rm R}\cot\beta)\, m_t & m^2_{\tilde{U}_{33}} + m_t^2 - |DT_U^R|\, 1\!\!1 \end{pmatrix} \tag{54}
$$

$$
M_b^2 = \begin{pmatrix} m^2_{\tilde{Q}_{33}} + m_b^2 - |DT_D^L|\, 1\!\!1 & (A_t m + \mu_{\rm R}\tan\beta)\, m_b \\ (A_t m + \mu_{\rm R}\tan\beta)\, m_b & m^2_{\tilde{U}_{33}} + m_b^2 - |DT_D^R|\, 1\!\!1 \end{pmatrix} . \tag{55}
$$

As we can see, the non-diagonal pieces can have a quite substantial size. Moreover, the renormalization group equations for the diagonal terms are also different from the equations (52). Again, by using the notation $m^2_{\tilde{Q}_{33}} = m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L}$, $m^2_{\tilde{U}_{33}} = m^2_{\tilde{t}_R}$, $m^2_{\tilde{D}_{33}} = m^2_{\tilde{b}_R}$ and similar definitions for the leptonic sector, we can write them as

$$
\begin{aligned}
(\dot{m}^2_{\tilde{t}_L}) = (\dot{m}^2_{\tilde{b}_L}) &= (GG)_Q - \; \tilde{h}_t^2\,(SS)_t - \; \tilde{h}_b^2\,(SS)_b \\
(\dot{m}^2_{\tilde{t}_R}) &= (GG)_U - 2\, \tilde{h}_t^2\,(SS)_t \\
(\dot{m}^2_{\tilde{b}_R}) &= (GG)_D - 2\, \tilde{h}_b^2\,(SS)_b \\
(\dot{m}^2_{\tilde{\tau}_L}) = (\dot{m}^2_{\tilde{\nu}_{\tau L}}) &= (GG)_L - \; \tilde{h}_\tau^2\,(SS)_\tau \\
(\dot{m}^2_{\tilde{\tau}_R}) &= (GG)_E - 2\, \tilde{h}_\tau^2\,(SS)_\tau \; .
\end{aligned} \tag{56}
$$

The gauge terms $(GG)_i$ are given in (52) and the scalar field terms $(SS)_t$, $(SS)_b$ and $(SS)_\tau$ in (37). The evolution down to the weak scale is driven not only by the gaugino masses, but also by the third generation's Yukawa couplings, i.e. mainly by the top mass and the value of $\tan\beta$. The effect is that the values of the coefficients $C_{\rm M}(\tilde{q})$ for the third generation's sfermion masses are smaller than the ones listed in (53). We shall distinguish here the two situations $h_b \ll h_t$ and $h_b \sim h_t$.

$h_b \ll h_t$

In the specific case of $m_t = 150\,{\rm GeV}$ and $\tan\beta = 3$ (the condition $h_b \ll h_t$ is here certainly verified), the decrease of the coefficients $C_{\rm M}(\tilde{q})$ with respect to the ones for the first two generations is about 15 % and 30 % for $m^2_{\tilde{Q}_{33}} = m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L}$ and $m^2_{\tilde{t}_R}$, respectively. It is negligible for $m^2_{\tilde{b}_R}$, $m^2_{\tilde{\tau}_L}$ and $m^2_{\tilde{\tau}_R}$. The conclusions which can be drawn for the stop, sbottom and stau mass eigenstates are therefore the following:

- Both left-handed and right-handed diagonal terms in the stop matrix (54), are below the corresponding values of the up-type squarks of the first two generations. As already observed, the decrease of the coefficient $C_{\rm M}(\tilde{t}_R)$ is twice as much as the decrease of $C_{\rm M}(\tilde{t}_L)$ (notice the factor of two multiplying h_t^2 in the evolution equation for $m^2_{\tilde{t}_R}$). Moreover, the off-diagonal terms in (54) are in this case quite big. The effect is that one of the two stop mass eigenstates, which we call $\tilde{t}_1$ has a mass smaller than the entry $m_{\tilde{t}_R}$ in (54). On the contrary, a partial/full compensation of the difference between the value of $m_{\tilde{t}_L}$ in (54) and the masses of the remaining four up-type squarks is obtained for the second mass eigenstate $\tilde{t}_2$. One mass eigenstate is then either comparable to the almost degenerate four up-type squarks, or moderately below them, according to the value of m_t and the region of SUSY parameter space considered. The second one $\tilde{t}_1$, can have a much smaller mass, in principle even compatible with zero as shown in Fig. 8 for $m_t = 100$. For some of the points already excluded in Fig. 8, the off-diagonal terms of the matrix (54) can be big enough to drive $m^2_{\tilde{t}_1}$ to negative values. The contribution of m_t to the diagonal entries of (54), up until now neglected, has the effect of reducing the splitting between the two stop mass eigenstates. For increasing values of m_t, but still such that $h_b \ll h_t$, the diagonal terms in the matrix (54) grow faster than the off-diagonal ones. Notice in fact, that no values close to zero are present in Fig. 8 for $m_t = 180$.
- As far as the sbottom mass matrix is concerned, practically no splitting is introduced by the off-diagonal terms in the matrix (55). Since $h_b \ll h_t$, the heavier of the two mass eigenstates, $\tilde{b}_2$, is roughly the right-handed component $\tilde{b}_R$, nearly degenerate with the other four down-type squarks of the first two generations. The other one, which we indicate with $\tilde{b}_1$, is smaller. The size of the splitting of

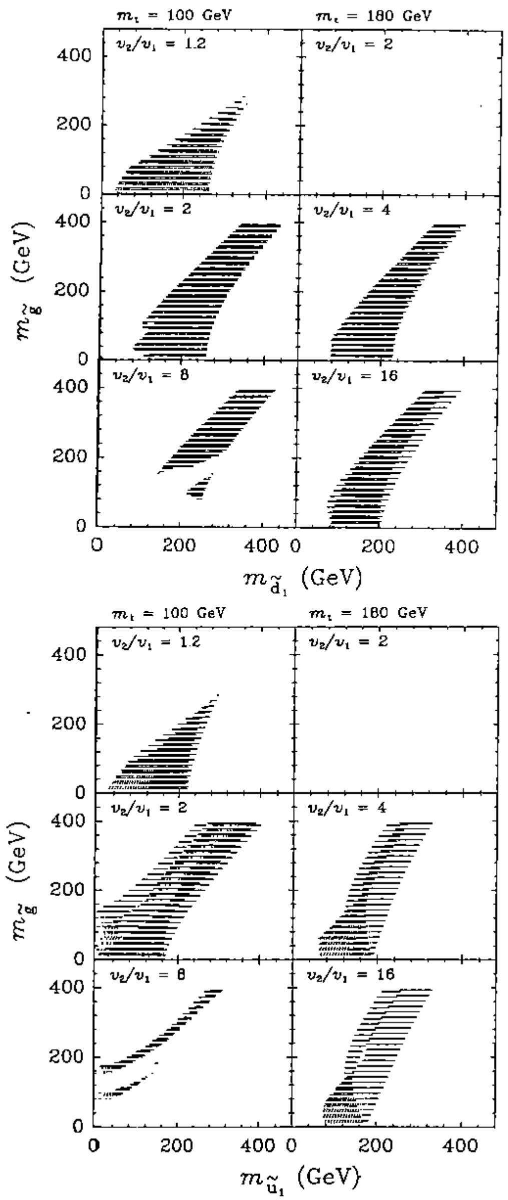

Figure 8: Masses of the lightest stop and sbottom quark mass eigenstates, here indicated as $\tilde{u}_1$ and $\tilde{d}_1$, obtained for the same values of the SUSY parameter space considered in Fig. 1.

these two eigenstates depends of course on the value of m and M. By inspection of Fig. 8a and 8b, we can conclude that as far as $h_b \ll h_t$, $\tilde{t}_1$ is *on average* lighter than $\tilde{b}_1$.

- The third generation of sleptons is a replica of the first two. The effect of the Yukawa coupling h_τ in the RGE for $m_{\tilde{L}}^2$ and $m_{\tilde{R}}^2$, is in fact almost negligible compared to the gaugino contribution. The lightest mass eigenstate $\tilde{\tau}_1$ is practically the $SU(2)_{\rm L}$-singlet component $\tilde{\tau}_R$.

$h_b \sim h_t$

In this case, the decrease of all diagonal entries of both stop and sbottom mass matrices is similar. For $m_t = 150$ and $\tan\beta = 40$, the gaugino contribution to these masses decreases with respect to the contribution it gives to the first two generation's squarks masses of about 25-30 %. In the leptonic case this decrease is sizeable only for $m^2_{\tilde{\tau}_R}$ (roughly 20 %) but negligible for $m^2_{\tilde{\tau}_L}$. The observations one can make in this case are:

- The off-diagonal terms are now quite relevant in both stop and sbottom mass matrices, either for the presence of m_t or for the presence of $\tan\beta$. The same considerations made before for the $\tilde{t}$ hold here also for $\tilde{b}$. Moreover, since the sbottom matrix does not have a direct dependence on m_t in the diagonal terms, $\tilde{b}_1$ can be in principle smaller than $\tilde{t}_1$ for heavy top masses and for big enough $\tan\beta$.

- Also in the case of $\tilde{\tau}$, a large splitting between the two mass eigenstates can be introduced by big off-diagonal terms in the mass matrix. The different renormalization pattern present in this case can further enlarge this splitting. Moreover, due to the smaller sensitivity to the value of M, the lightest stop $\tilde{\tau}_1$, can be lighter than $\tilde{t}_1$ and $\tilde{b}_1$ [12] in some regions of the SUSY parameter space

Some interesting phenomenological implications which can be obtained for large values of $\tan\beta$, are discussed in [12].

As for the case of chargino, neutralino and Higgs bosons, no experimental search has brought so far any evidence for the existence of squarks and sleptons. The best lower limits on the slepton masses coming from LEP [26] exclude values of about 43 GeV for photino masses up to $20 - 30$ GeV. The assumption made in these searches that the sleptons decay directly to the lightest supersymmetric particle (LSP), assumed to be the photino, are obviously acceptable for these values of masses. Moreover, a possible splitting between the left- and right-handed components $\tilde{l}_L$ and $\tilde{l}_R$ is also considered in these analyses. The situation is more complex as far as the bounds on squarks and gluino masses are concerned. The recent CDF limits [29] $m_{\tilde{g}} > 150$ GeV (independently of $m_{\tilde{q}}$) and $m_{\tilde{q}} > 150$ GeV (for $m_{\tilde{g}} < 400$ GeV), rely on at least two assumptions not supported by the Minimal Supersymmetric Standar Model. One of them is that all the squarks are considered as degenerate, the second is that squarks and gluinos are supposed to decay directly to the LSP withouth intermediate decays to charginos or neutralinos [30]. Obviously a more complete analysis which is able to relax these two assumptions, cannot be performed without specifying a particular theoretical framework. An attempt in this direction is made in [31] where the study of these cascade decays, as predicted by the Minimal Supersymmetric Standard Model, allows one to estimate that the CDF limits should be lowered by about 30 GeV.

10. Conclusions

In closing this lecture we would like to point out that the predictive power of this model strongly relies on the minimal choice of parameters made at some high-scale and on the fact the electroweak breaking can be induced by radiative effects. The possibility of reducing to only four the number of new parameters to be introduced in the theory, is certainly not theoretically motivated and may be considered as a drawback of this model. On the other hand, we have shown through a few examples that the experimental searches for supersymmetric particles are far from being assumption-free and are in many cases already inspired by this model. It might also become harder and harder to perform searches of supersymmetric particles heavier than the ones excluded up to now in a model-independent way. Moreover, in spite of its small number of parameters, this model still offers a very rich spectrum of masses and only a serious threat from experimental side could convince us to dismiss it.

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