DEUTSCHES ELEKTRONEN – SYNCHROTRON

DESY 92-090
June 1992

B-Decays – An Overview

A. Ali
Deutsches Elektronen-Synchrotron DESY, Hamburg

ISSN 0418-9833

NOTKESTRASSE 85 · D-2000 HAMBURG 52

DESY 92-090
June 1992
ISSN 0418-9833

B-Decays–An Overview*

A. Ali
Deutsches Elektronen Synchrotron - DESY
Hamburg, Germany

ABSTRACT

We review some selected aspects of B-decays in the context of the Standarad Model and the Cabibbo-Kobayashi-Maskawa theory of weak mixing. The calculational framework used is based on perturbative QCD. The topics discussed include an update of the CKM matrix parameters, estimates of the inclusive B-decay rates in the context of the QCD improved spectator model, and inclusive rare B-decays $B \to X_s + \gamma$ and $B \to X_d + \gamma$. Exclusive branching ratios $BR(B \to K^* + \gamma)$ and $BR(B \to \rho + \gamma)$ are also presented. The importance of rare B-decays in determining the CKM matrix elements is emphasized.

*To be published in the Proceedings of the Rencontres de Moriond: QCD and High Energy Hadronic Interactions, Les Arcs, France, March 22-28, 1992.

1 The CKM Matrix and the Unitarity Triangle

In the Standard Model (SM) [1], the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2,3], described by three angles and one complex phase, provides the framework to discuss flavour physics. Some time ago Wolfenstein noticed [4] that the elements of this matrix exhibited a hierarchy in terms of λ, the Cabibbo angle. In this parametrization the CKM matrix can be written approximately as

$$V_{CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - iA^2\lambda^4\eta & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \quad (1)$$

The present status of the four CKM matrix parameters, λ, A, ρ, η is discussed in this section. First of all, $|V_{us}|$ has been extracted with good accuracy from $K \to \pi e \nu$ and hyperon decays [5] to be

$$|V_{us}| = \lambda = 0.2205 \pm 0.0018 . \quad (2)$$

This agrees quite well with the determination of $V_{ud} \simeq 1 - \frac{1}{2}\lambda^2$ from β-decay:

$$|V_{ud}| = 0.9744 \pm 0.0010 . \quad (3)$$

The parameter A is related to the CKM matrix element V_{cb}, which can be obtained from semileptonic decays of B mesons. There are two classes of models which describe such decays. First of all, in the spectator quark model of ref. [6,7], the semileptonic decay of a B meson is described at the quark level in a manner completely analogous to that of muon decay, with QCD and phase space effects taken into account. The main uncertainty in this approach is in the value of m_b. The second class of models are form factor models, in which the branching ratios for exclusive final states are predicted. In particular, for $b \to c$ transitions, the rates for $B \to Dl\nu$ and $B \to D^* l\nu$ are given as functions of $|V_{cb}|^2$. The models of Isgur et. al. (ISGW) [8], Wirbel, Stech and Bauer (WBS) [9] and Körner and Schuler (KS) [10], have been used to determine $|V_{bc}|$ from the ARGUS and CLEO data, giving $|V_{bc}| = 0.036 \pm 0.007$, $|V_{bc}| = 0.041 \pm 0.007$, and $|V_{bc}| = 0.041 \pm 0.007$, respectively [11].

Recently, there has been a further development in such models [12]. By taking the formal limit of infinite quark masses, one obtains what is known as the heavy-quark effective theory (HQET) [13]. The HQET approach has also been used in the exclusive $B(v) \to D^*(v')l\nu$ decay to determine $|V_{bc}|$, since in this case the decay at the symmetry point $v \cdot v' = 1$ is governed by the Isgur-Wise function $\xi(v \cdot v')$, having the normalization $\xi(v \cdot v' = 1) = 1$ (here v and v' are the four-velocities as indicated). Since $O(1/m_Q)$ corrections to the Isgur-Wise function $\xi_{A_1}(v \cdot v' = 1)$ determining the rate for the decay $B \to D^* \ell \nu_\ell$ at the symmetry point are absent [14,15], any deviation from the relation $\xi_{A_1}(v \cdot v' = 1) = 1$ is dominantly of perturbative QCD origin. The differential decay rate is given by:

$$\lim_{v \cdot v' \to 1} \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \frac{d\Gamma(B \to D^* \ell \nu_\ell)}{d(v \cdot v')} = \frac{G_F^2}{4\pi^3} M_{D^*}^3 (M_B - M_{D^*}) |V_{cb}|^2 \eta_{QCD}^2 . \quad (4)$$

The QCD-correction factor turns out to be essentially 1, namely $\eta_{QCD} \simeq 0.99$. However, one has to extrapolate the data to the point $y = v \cdot v' = 1$, for which one needs an Ansatz for the Isgur-Wise function $\xi(y)$. To extract $|V_{cb}|$ from data, the following parametrization has been employed in ref. [16]:

$$\xi(y) = \frac{2}{1+y} exp\{-(2{y_0}^2 - 1)\frac{y-1}{y+1}\}. \tag{5}$$

A fit of the data on $B \to D^* \ell \nu_\ell$ was then attempted in terms of the parameters y_0 and $|V_{cb}|$ [16]. This fit has since been updated using the present average B-hadron lifetime (including the LEP results), $\tau_B = 1.36 \pm 0.05$ ps, and the branching ratio $BR(B \to D^* \ell \nu_\ell) = (5.1 \pm 0.9)\%$, giving $|V_{cb}|\left(\frac{\tau_B}{1.36\ ps}\right)^{1/2} = 0.046 \pm 0.007$ [11]. With the perturbative corrections being at the level of 1%, the dominant theoretical uncertainty lies in extrapolation of the data. For the purpose of the CKM parameter fit, a value $|V_{cb}| = 0.044 \pm 0.006$ has been used in ref. [17], which we use, giving

$$A = 0.90 \pm 0.12\ . \tag{6}$$

The other two CKM parameters ρ and η are constrained by the measurements of $|V_{ub}/V_{cb}|$, $|\epsilon|$ (the CP-violating parameter in the kaon system), x_d (B_d^0-$\overline{B_d^0}$ mixing) and (in principle) ϵ'/ϵ ($\Delta S = 1$ CP-violation in the kaon system). Concerning $|V_{ub}/V_{cb}|$, we note that its value can be obtained by the analysis of the endpoint of the inclusive lepton spectrum in semileptonic B decays. The latest values for the ARGUS and CLEO results using the quoted models are: $|V_{ub}/V_{cb}| = 0.11 \pm 0.01, 0.13 \pm 0.02, 0.11 \pm 0.01$, and 0.20 ± 0.02 for the ACCMM[7], WBS [9], KS [10], and ISGW [8] model, respectively. It is clear that, although there is quite good evidence for a non-zero $|V_{ub}/V_{cb}|$, its value is quite uncertain. We shall take

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.14 \pm 0.05\ . \tag{7}$$

This gives

$$\sqrt{\rho^2 + \eta^2} = 0.63 \pm 0.23\ . \tag{8}$$

The experimental value of $|\epsilon|$ is $|\epsilon| = (2.26 \pm 0.02) \times 10^{-3}$ [5]. Theoretically, $|\epsilon|$ is essentially proportional to the imaginary part of the box diagram for K^0-$\overline{K^0}$ mixing, and is given by [18]

$$\begin{aligned}|\epsilon| \;=\; & \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} B_K \left(A^2\lambda^6\eta\right)\left(y_c\left\{\eta_{ct} f_3(y_c, y_t) - \eta_{cc}\right\}\right. \\ & \left. + \eta_{tt} y_t f_2(y_t) A^2 \lambda^4 (1-\rho)\right).\end{aligned} \tag{9}$$

Here, the η_i are QCD correction factors, $\eta_{cc} \simeq 0.82$, $\eta_{tt} \simeq 0.62$, $\eta_{ct} \simeq 0.35$ for $\Lambda_{QCD} = 200$ MeV $y_i \equiv m_i^2/M_W^2$, and the functions f_2 and f_3 are given by

$$\begin{aligned} f_2(x) &= \frac{1}{4} + \frac{9}{4}\frac{1}{(1-x)} - \frac{3}{2}\frac{1}{(1-x)^2} - \frac{3}{2}\frac{x^2 \ln x}{(1-x)^3}, \\ f_3(x,y) &= \ln\frac{y}{x} - \frac{3y}{4(1-y)}\left(1 + \frac{y}{1-y}\ln y\right). \end{aligned} \tag{10}$$

One of the unknowns in this analysis is the top quark mass. Direct searches at the Tevatron [19] and the phenomenology of electroweak radiative corrections [20] give the bounds: 91 GeV $< m_t <$ 180 GeV.

The final parameter in the expression for $|\epsilon|$ is B_K, which represents our ignorance of the matrix element $\langle K^0|(\bar{d}\gamma^\mu(1-\gamma_5)s)^2|\overline{K^0}\rangle$. The evaluation of this matrix element has been the subject of much work. The results are summarized in ref. [17]. Although the entire range of B_K is $1/3 \le B_K \le 1$, the $1/N_C$ and lattice approaches are generally considered more reliable, which give

$$B_K = 0.8 \pm 0.2. \tag{11}$$

We now turn to B_d^0-$\overline{B_d^0}$ mixing. The latest value of x_d, which is a measure of this mixing, is [11]

$$x_d = 0.67 \pm 0.10\ . \tag{12}$$

The mixing parameter x_d is calculated from the B_d^0-$\overline{B_d^0}$ box diagram. Unlike the kaon system, where the contributions of both the c- and the t-quarks in the loop were important, x_d is dominated by t-quark exchange:

$$x_d \equiv \frac{(\Delta M)_B}{\Gamma} = \tau_B \frac{G_F^2}{6\pi^2} M_W^2 M_B \left(f_{B_d}^2 B_{B_d}\right) \eta_B y_t f_2(y_t) |V_{td}^* V_{tb}|^2\ , \tag{13}$$

where, $|V_{td}^* V_{tb}|^2 = A^2\lambda^6\left[(1-\rho)^2 + \eta^2\right]$. Here, η_B is the QCD correction. In ref. [18], this correction is analyzed in great detail, including the effects of a heavy t-quark. They find that η_B depends sensitively on the definition of the t-quark mass, and that, strictly speaking, only the product $\eta_B(y_t)f_2(y_t)$ is free of this dependence. However, in comparison to the enormous uncertainty in $f_{B_d}^2 B_{B_d}$, these details are rather minor. Just like B_K, the evaluation of $f_{B_d}^2 B_{B_d}$ has been the subject of much work, summarized in ref. [17]. Until very recently, the scaling law, $f_B(m_B)\sqrt{m_B}$ = constant, was thought to be valid for the B system. This led to rather small values for $f_{B_d}^2 B_{B_d}$ in the range 100 MeV $\le f_{B_d}\sqrt{B_{B_d}} \le$ 170 MeV. However, recent lattice calculations have indicated that there are scaling violations. These have led to larger estimates for $f_{B_d}^2 B_{B_d}$, roughly in the range 200 MeV $\le f_{B_d}\sqrt{B_{B_d}} \le$ 300 MeV. A fit of the CKM parameters with the product coupling constant $C_d \equiv f_{B_d}\sqrt{B_{B_d}\eta_B}$ which enters in x_d has been done for the two ranges for C_d [17]:

$$\begin{aligned} (\text{old}): \quad C_d &= 125 \pm 20 \text{ MeV}, \\ (\text{new}): \quad C_d &= 225 \pm 30 \text{ MeV} \end{aligned} \tag{14}$$

Because the CKM matrix is unitary, one has the relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$. Using the form of the CKM matrix in Eq. 1, this can be recast as

$$\frac{V_{ub}^*}{\lambda V_{cb}} + \frac{V_{td}}{\lambda V_{cb}} = 1\ , \tag{15}$$

that is, a triangle relation in the complex plane (i.e. ρ-η space). The allowed unitarity triangles have been estimated recently in ref. [17] using the computer program MINUIT to fit the CKM parameters A, ρ and η to the experimental values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|\epsilon|$ and x_d. For m_t, three different values: m_t = 100, 140, 180 GeV were used and, to take into account the uncertainties in the hadronic matrix elements, the ranges for B_K and C_d defined in Eqs. 11 and 14 were used. The resulting fits can be seen in ref. [17].

Table 1: The "best values" of the CKM parameters (ρ,η) obtained by a minimum χ^2 fit of the experimental values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|\epsilon|$ and x_d for the indicated values of m_t and the coupling constant product C_d. The resulting minimum χ^2 values from the MINUIT fits are also given. The table is based on ref. [17].

m_t (GeV)	C_d (MeV)	(ρ,η)	$\chi^2_{\min}$
100	125 ± 20	$(-0.6,\ 0.27)$	0.05
140	125 ± 20	$(-0.47,\ 0.25)$	0.55
180	125 ± 20	$(-0.34,\ 0.25)$	2.1
100	225 ± 30	$(0.22,\ 0.54)$	0.20
140	225 ± 30	$(0.42,\ 0.42)$	0.10
180	225 ± 30	$(0.51,\ 0.34)$	0.02

It was found that, as we pass from the value $C_d \equiv f_{B_d}\sqrt{B_{B_d}\eta_B} = 125\pm20$ MeV with $m_t = 100\ GeV$ to $C_d = 225\pm30$ MeV with $m_t = 180\ GeV$, the "most likely" unitarity triangles become more and more acute. Quantitatively, this can be seen in Table 1, where the best fits for the CKM parameters (ρ,η) for the assumed values of m_t and the coupling constant product C_d are given, together with the resulting $\chi^2_{\min}$. Based on this, we conclude that the present experimental data constrain the CKM parameters ρ and η in the range:

$$\begin{aligned} 0.25 \le \eta \le 0.54 \\ -0.60 \le \rho \le 0.51 \end{aligned} \tag{16}$$

This leaves a large range for the angles of the unitarity triangle which determine CP-violating asymmetries.

2 QCD Corrected Inclusive B-decay Widths

In this section we study the $\Delta B = 1$ transitions taking into account the QCD effects. We express the Lagrangian as a sum of the semileptonic and non-leptonic parts:

$$\begin{aligned} \mathcal{L}(\Delta B=1) &= \mathcal{L}_{SL}(\Delta B=1) + \mathcal{L}_{NL}(\Delta B=1) \\ \mathcal{L}_{SL}(\Delta B=1) &= \frac{G_F}{\sqrt{2}}\{V_{bc}(\bar{b}c)_L(\bar{\ell}\nu_\ell)_L + V_{bu}(\bar{b}u)_L(\bar{\ell}\nu_\ell)_L\} \\ \mathcal{L}_{NL}(\Delta B=1) &= \frac{G_F}{\sqrt{2}}\{\sum_{q=u,c}\sum_{q'=d,s} V_{qb}^{*}V_{qq'}(\bar{b}q)_L(\bar{q}q')_L\} \end{aligned} \tag{17}$$

where the notation is $(\bar{b}q)_L \equiv \bar{b}\gamma^\mu(1-\gamma_5)q$. Writing out the non-leptonic Lagrangian in full one gets altogether eight terms. However, a glance at the CKM factors shows that in the leading order, $O(\lambda^2)$, there are only two terms that contribute to $\mathcal{L}_{NL}(\Delta B=1)$, inducing the transitions $b \to c(\bar{u}d)$ and $b \to c(\bar{c}s)$, and we shall concentrate on them.

The leading order QCD corrections to the semileptonic and non-leptonic decays are known for quite some time. In the case of the semileptonic decays, the decay width can

be written as (neglecting the small contribution due to the decay $b \to u\ell\nu_\ell$),

$$\Gamma_{SL} = \Gamma_0\ I(m_c/m_b, m_\ell/m_b, 0)\{1 - 2\frac{\alpha_s}{3\pi}f(m_c/m_b)\} \tag{18}$$

where $\Gamma_0 = G_F^2 m_b^5 |V_{bc}|^2/192\pi^3$, and the phase space function $I(x_1,x_2,x_3)$, normalized as $I(0,0,0)=1$, is given in ref. [21]. The function $f(m_c/m_b)$ specifies the QCD correction to the semileptonic width and has been calculated in ref. [22]. For the case of massless fermions it is given by $f(0) = \pi^2 - 25/4$, slowly decreasing to $f(0.5) \simeq 2$. The QCD correction reduces the semileptonic decay rate by typically $O(10\%)$.

Let us now consider the QCD corrections to the non-leptonic B-decay width, concentrating first on the $(\bar{b}c)_L(\bar{u}d)_L$ part in $\mathcal{L}_{NL}(\Delta B=1)$ above, since under QCD it has a simpler renormalization. The effect on the relevant decay width can be expressed as [23]:

$$\Gamma(b \to c\bar{u}d) = 3\Gamma_0 \frac{(2L_+^2 + L_-^2)}{3} \cdot J \cdot I(m_c/m_b, m_d/m_b, m_u/m_b) \tag{19}$$

The factor $(2L_+^2 + L_-^2)/3 \cdot J$ is the QCD correction factor. Here $L_{+,-} = (\alpha_s/\alpha_s(m_W))^{d_{+,-}}$, with $d_{+,-} = (-6/23, 12/23)$, obtained by summing the leading logs, and J represents the next-to-leading order QCD corrections the explicit form of which can be seen in ref. [23]. While the exact QCD enhancement depends somewhat on the values of the indicated parameters, a typical number for the QCD factor is $\simeq 1.2$.

In the case of the decay $b \to c(\bar{c}s)$, one has to take into account the mixing of the four-fermi operator in $\mathcal{L}_{NL}(\Delta B=1)$ with the so-called penguin operators, since they mix under QCD renormalization. It is customary to carry out these renormalizations in an effective Hamiltonian framework obtained by integrating out the heavy degrees of freedom, which in the present context means the $W^\pm$ and top quark. The penguin graphs bring into fore also QCD and QED magnetic moment operators which are crucial in the study of rare B-decays, in particular involving a photon or a dilepton in the final state. It is, however, known that their contribution to the total B decay rate is negligible. Leaving them out for the time being, $\mathcal{H}_{\rm eff}\,(\Delta B=1)$ can be written as:

$$H_{eff}(\Delta B=1) = -\frac{G_F}{\sqrt{2}}|V_{bc}|\sum_{j=1}^{6} C_j(\mu)\,\hat{O}_j(\mu) \tag{20}$$

with $C_j(\mu)$ being the Wilson coefficients evaluated at the scale μ. The various operators are defined as:

$$\begin{aligned} \hat{O}_1 &= (\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha}\gamma_\mu c_{L\beta}) \\ \hat{O}_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta}) \\ \hat{O}_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\beta})\right] \\ \hat{O}_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + \cdots + (\bar{b}_{L\beta}\gamma_\mu b_{L\alpha})\right] \\ \hat{O}_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\beta})\right] \\ \hat{O}_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})\left[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + \cdots + (\bar{b}_{R\beta}\gamma_\mu b_{R\alpha})\right] \end{aligned} \tag{21}$$

Perturbative QCD corrections, contained in the Wilson coefficients $C_j(\mu)$, have been evaluated to leading logarithmic accuracy, and more recently also in the next-to-leading order [24]. For the dominant opeartors $\hat{O}_1$ and $\hat{O}_2$, the Wilson coefficients have the values: $C_1 = -0.27(-0.29)$ and $C_2 = 1.12(1.13)$ in the LLA(NLLA) approximation, for $\mu = m_b = 4.8\ GeV$ and $\Lambda^{(4)}_{\overline{MS}} = 0.25\ GeV$ [24]. The QCD corrections enhance the phenomenologically relevant combination $3C_2 - C_1$ from its free quark model value 3.0 to 3.62 in the LLA, and to 3.68 in the NLLA approximation. Moreover, the Wilson coefficients of the operators $\hat{O}_3, ..., \hat{O}_6$ are found numerically small and their effect on Γ_{NL} is not significant. The overall QCD effects boil down to an enhancement of the non-leptonic B-decay width by $O(20\%)$.

3 BR_{SL} in the QCD-improved Spectator Model

Given the QCD corrections in the previous section, the question is if the inclusive B-decay widths are correctly estimated by the QCD-improved spectator model. This can be answered indirectly by calculating the inclusive semileptonic B branching ratio BR_{SL}, as has been done by Altarelli and Petrarca[23]. The result of their analysis is, however, inconclusive since an ambivalent theoretical dispersion in their estimates of BR_{SL} remains due to the dependence on the quark masses. Using their jargon, one could classify the solutions in two classes *light masses* and (*heavy masses*) characterized by the values (in GeV):

$$\begin{aligned} m_{u,d} = 0(0.16) \quad &, \quad m_s = 0.15(0.30) \\ m_c = 1.2(1.7) \quad &, \quad m_b = 4.6(5.0). \end{aligned} \tag{22}$$

giving,

$$\begin{aligned} BR_{SL} &= [12.2 \pm 1.25]\% \ \ (light\ masses) \\ &= [14.4 \pm 1.25]\% \ \ (heavy\ masses) \end{aligned} \tag{23}$$

to be compared with the branching ratio measured at the $\Upsilon(4S)$, $BR_{SL}[\Upsilon(4S)] = (10.3\pm0.2)\%$ [11]. The corresponding LEP numbers measured from the $Z^0 \longrightarrow b\bar{b}$ decays are consistent with $BR_{SL}[\Upsilon(4S)]$ [11]. We remark that the quark mass ambiguity in the appraoch of Altarelli and Petrarca can be resolved (or at least this dependence reduced) if in addition to the branching ratio BR_{SL} one also uses the shape of the lepton energy spectrum in semileptonic B-decays, as was advocated by Rückl some time ago [25]. The present best fits of the model parameters using the CLEO and ARGUS data gives values for $(m_b - m_c)$ which are closer to the so-called heavy mass choice of Altarelli and Petrarca, giving $BR_{SL} \simeq 13\%$, which is $O(25\%)$ higher than the present experimental measurements. On the other hand, the experimental branching ratio quoted above requires a certain extrapolation in the soft part of the lepton energy spectrum. Hence, mundane issues such as the contribution of higher D^* resonances may play a significant role. The final word on the experimental branching ratio is not yet in. QCD corrections have the desired effect; whether they are completely adequate remains to be settled. We mention here *en passant* that experimental evidence in favour of $\tau_{B^\pm} \simeq \tau_{B^0}$, which follows naturally from the dominance of the spectator model, is now slowly mounting. The most convincing measurement to back this up is due to ARGUS and CLEO [11]:

$$\frac{\tau_{B^\pm}}{\tau_{B^0}} = \frac{BR(B^\pm \to D^{(*)0}\ell^\pm\nu)}{BR(B^0 \to D^{(*)-}\ell^+\nu)} = 0.96 \pm 0.14 \tag{24}$$

4 RADIATIVE RARE B-DECAYS

FCNC processes are forbidden at the tree level in the SM Lagrangian but are allowed at the one-loop order, where their amplitudes are governed by the GIM mechanism [26]. This may not be so in extensions of SM, where already at the tree level one may have FCNC transitions. Their measurements would therefore serve both as SM precision tests and windows on nearby new physics. In the SM, FCNC B-decay rates are dominated by the top quark. This is also the case for the $\Delta B = 2, \Delta Q = 0$ transitions giving rise to the mixing ratios x_d and x_s, with $x_i = (\Delta M/\Gamma)_i$. Hence, FCNC B-physics provides unmatched oppurtunities to determine the CKM matrix elements, V_{td}, V_{ts}, and V_{tb}.

In terms of the quark transitions, the main FCNC B-decays are the following:

1. $b \to (s,d) + \gamma$
2. $b \to (s,d) + \gamma + g$
3. $b \to (s,d) + g$
4. $b \to (s,d) + \ \ell^+\ell^- \ (\ell = e, \mu, \tau)$
5. $b \to (s,d) + \ \nu\bar{\nu}$

Of these, the most difficult to measure experimentally are the ones involving the hadronic final states $b \to (s,d) + g$, since they can be easily cofused with the dominating CC decays $b \to u\bar{u}d$ and $b \to u\bar{u}s$. So, we shall neglect them and concentrate on the rest in the above list. The relevant decay rates have been calculated both at the one-loop level, and in the LLA limit of QCD. The QCD-improved effective Hamiltonian incorporating the leading order RG-improvement is the basis of the present phenomenological studies in rare B-decays [30].

Experiments involved in B-decays, in particular the CLEO collaboration, are on the verge of measuring the electromagnetic penguins through the radiative decays $b \to s + \gamma + (g)$. The penguin contributions give rise to an enhancement in the high-E_γ tail of the inclusive decays $B \to X + \gamma$, some of which may be measured in the exclusive decay $B \to K^* + \gamma$ [27]. In view of the anticipated experimental development in the near future, we will concentrate here only on radiative B-decays. Remaining within the SM, the principal questions in the interpretation of such measurements are the following:

- What are the uncertainties in the rates and distributions given the various unknown parameters involved?
- What is the least model dependent way to compare theory and experiment in the FCNC sector?
- Could one quantify the improvements in the measurements of the CKM parameters and m_t, using the impending measurements of the FCNC B-decays?

Concerning the reliability of the decay rates, we remark that in the FCNC B-decays inclusive measurements are expected to be theoretically better calculable. In particular, a direct application of the HQET appraoch to the exclusive rare decays is wrought with power corrections. Concentrating on radiative rare B-decays, probably the least model dependent comparison of theory and experiment will be achieved by measuring the inclusive photon spectra in the decays $B \to X_s + \gamma$ and $B \to X_d + \gamma$ (here and henceforth we use the quark flavour in the transition $b \to q$ to specify the resulting hadronic state, X_q). These spectra have been calculated in perturbative QCD with the non-perturbative effects due to the B-hadron wave function modelled after the inclusive lepton energy spectrum in B-decays [31,32]. Recently, the inclusive E_γ-spectra in the CC decays $B \to X_c + \gamma$ and $B \to X_u + \gamma$ have also been calculated in the same approach [33]. We discuss the main results from these studies below.

4.1 The Decays $b \to (s,d) + \gamma$: Lowest Order Contributions

We discuss first the lowest order (1-loop) calculations for the FCNC radiative decays $b \to (s,d) + \gamma$. The matrix element for the process $b \to s + \gamma$ in the lowest order can be written as:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \sum_i V_{ib} V_{is}^* F_2^i(x_i)\, q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b \tag{25}$$

where $x_i = m_i^2/m_W^2$, q_μ and ϵ_μ are, respectively, the photon four-momentum and polarization vector and the sum is over the quarks, u, c, and t. The Inami-Lim function $F_2(x_i)$ derived from the penguin diagrams is given by [28]:

$$F_2(x) = \frac{x}{24(x-1)^4} \left[6x(3x-2)\log x - (x-1)(8x^2+5x-7)\right] \tag{26}$$

where we have dropped the superscript on F_2, and in writing the expression for $F_2(x_i)$ above we have left out $O(m_s/mb)$ terms. Retaining only the top quark contribution in the amplitude, we have:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_t F_2(x_t)\, q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b \tag{27}$$

with $\lambda_t \equiv V_{tb} V_{ts}^*$. The amplitude for the CKM-suppressed FCNC radiative transition $b \to d + \gamma$ is obtained by replacing the s-quark variables by the d-quark ones. The CKM factor in this case is $\xi_t \equiv V_{tb} V_{td}^*$. The rates for rare B-decays may be expressed in terms of BR_{SL} (for which a value 0.12 is used below):

$$BR(b \to s + \gamma) = 6 \frac{\alpha}{\pi} \frac{|\lambda_t|^2}{|V_{bc}|^2} \frac{|F_2(x_t)|^2}{f(m_c/m_b)} (0.12) \tag{28}$$

$$\frac{BR(b \to d + \gamma)}{BR(b \to s + \gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \tag{29}$$

where the function $f(m_c/m_b) \simeq 0.44$ is the phase space factor in the CC semileptonic decay $b \to c + \ell \nu_\ell$. The branching ratio for $b \to s + \gamma$ as a function of m_t is shown in Fig. 1, where $|V_{ts}|^2/|V_{bc}|^2 = 1$ has been used. To quote a number from this figure, one gets BR($b \to s + \gamma) \simeq 1.5 \times 10^{-4}$ for $m_t = 150\ GeV$, for the lowest order (i.e. no QCD corrections). The ratio $BR(b \to d + \gamma)/BR(b \to s + \gamma)$ is expected to be $O(\lambda^2) \simeq 0.05$ (more on this later). We now discuss the leading order perturbative QCD corrections in the two-body decays $b \to (s,d) + \gamma$ (i.e. QCD renormalization of the Wilson coefficients) and the gluon bremsstrahlung contributions from the decays $b \to (s,d) + \gamma + g$, to calculate the photon energy spectra.

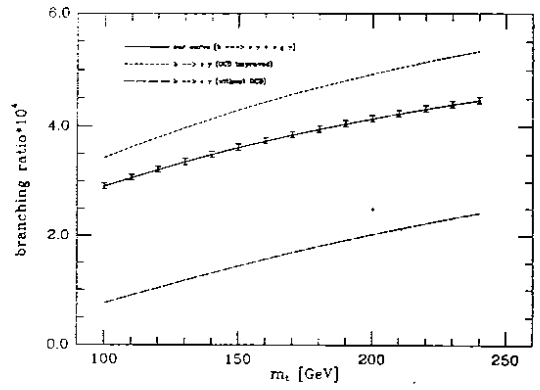

Figure 1: Inclusive branching ratio for the decays $B \to X_s + \gamma$ as a function of the top quark mass m_t. The charm quark mass dependence for $m_c = 1.5 \pm 0.2\ GeV$ is indicated by error bars. Also shown are the lowest order (i.e. without QCD corrections) and the leading order QCD-improved results for the two-body decays $b \to s + \gamma$ (from [31]).

4.2 QCD Corrections to the Decay $b \to s + \gamma$ and $b \to d + \gamma$

To implement QCD corrections, an appropriate operator basis has to be defined first. To leading order in the small (weak)-mixing angles, a complete set of dimension-6 operators relevant for the processes $b \to s + \gamma$ and $b \to s + \gamma + g$ is contained in the effective Hamiltonian

$$H_{eff}(b \to s + \gamma) = -\frac{4G_F}{\sqrt{2}} \lambda_t \sum_{j=1}^{8} C_j(\mu)\, \hat{O}_j(\mu) \tag{30}$$

with $C_j(\mu)$ being the Wilson coefficients evaluated at the scale μ. The four-fermion operators $\hat{O}_1, ..., \hat{O}_6$ have already been defined earlier and the two magnetic moment operators are:

$$\begin{aligned} \hat{O}_7 &= (e/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) b_\alpha F_{\mu\nu} \\ \hat{O}_8 &= (g_S/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) T^A_{\alpha\beta} b_\beta G^A_{\mu\nu} \end{aligned} \tag{31}$$

where e and g_S denote the QED and QCD coupling constant, respectively. Perturbative QCD corrections, contained in the Wilson coefficients $C_j(\mu)$, obtained by integrating out the top quark and the W-boson, have been evaluated to leading logarithmic accuracy [29]. For the two-body decays $b \to s + \gamma$, only the magnetic moment operator $\hat{O}_7$ contributes and hence the QCD corrected rate can be represented by formulæ similar to the ones given above for the lowest order, with the function $F_2(x)$ replaced by the QCD corrected function $C_7(m_b)$ [29]:

$$C_7(m_b) = \eta^{-16/23} \left\{ F_2(x) - \frac{58}{135}\left[\eta^{10/23} - 1\right] - \frac{29}{189}\left[\eta^{28/23} - 1\right] \right\} \tag{32}$$

with $x = m_t^2/m_W^2$ and $\eta = \alpha_S(m_b)/\alpha_S(m_W)$ being the ratio of the QCD coupling constants. The branching ratio for the decay $b \to s + \gamma$ (leading order QCD-improved) as a function of m_t is also shown in Fig. 1, as the topmost curve. The net result of these QCD corrections is very significant, increasing the rate by ~ 6 for $m_t = 100\ GeV$ to ~ 3 for $m_t = 200\ GeV$, using $\mu = m_b = 5.0\ GeV$ and $\Lambda_{QCD}^{(5)} = 200\ MeV$.

We remark that for the decays $b \to s + \gamma$, the effective Hamiltonian was written in the approximation $\lambda_u = 0$, which is reasonable since one can easily see that $\lambda_u \ll (\lambda_c, \lambda_t)$ (where the CKM factors are defined as $\lambda_i \equiv V_{ib}V_{is}^*$). For the decays $b \to d + \gamma$ and $b \to d + \gamma + g$ the CKM factors λ_i are replaced by $\xi_i \equiv V_{ib}V_{id}^*$. Now ξ_u, ξ_c and ξ_t are all of the same order of magnitude, $O(\lambda^3)$, and therefore the corresponding approximation $\xi_u = 0$ cannot be made any longer. It has been shown in [32] that the differences due to the inclusion of the ξ_u terms needed for the $b \to d + \gamma$ and $b \to d + \gamma + g$ decays can be most easily built in the framework discussed for the decay $b \to s + \gamma$ by modifying the operators $\hat{O}_1$ and $\hat{O}_2$, given earlier. The dimension-6 operator basis in this case is:

$$H_{eff}(b \to d + \gamma) = -\frac{4G_F}{\sqrt{2}}\xi_t \sum_{j=1}^{8} C_j(\mu)\, O_j(\mu) \quad , \tag{33}$$

The two modified operators O_1 and O_2 are now defined as [32]:

$$\begin{aligned} O_1 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu u_{L\beta}) \\ O_2 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta}) - \frac{\xi_u}{\xi_t}(\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu u_{L\beta}). \end{aligned} \tag{34}$$

The other operators $O_3, ..., O_8$ are identical to their counterparts $\hat{O}_3, ..., \hat{O}_8$ encountered in the decays $b \to s + \gamma + g$, with the obvious replacement of the fields $s \to d$. With the operators defined in this basis, the matching conditions $C_i(m_W)$ and the corresponding Wilson coefficients at the scale $\mu = m_b$, $C_i(m_b)$, are precisely the same for the decays $b \to d + \gamma$ as for the $b \to s + \gamma$ case [33]. The consequence of this is that the leading order QCD-improvement for the two-body decay amplitude $\mathcal{M}(\ b \to d + \gamma)$ is identical to the one for the decay $b \to s + \gamma$, since both involve just the rescaling of the Wilson coefficient $C_7(m_W) \to C_7(m_b)$. It follows that the ratio $\Gamma(\ b \to d + \gamma)/\Gamma(\ b \to s + \gamma)$ given above in the lowest order remains unrenormalized due to the leading-order QCD-corrections to the two-body processes. Including QCD bremsstrahlung processes $b \to s + \gamma + g$ and $b \to d + \gamma + g$ will lead to non-factorizing corrections (in the CKM parameters) to the ratio of the inclusive decay rates $\Gamma(B \to X_d + \gamma)/\Gamma(B \to X_s + \gamma)$.

4.3 Inclusive Photon energy spectra in B-decays

The photon energy spectra in the decays $b \to (s, d) + \gamma + g$ have also been calculated in the effective Hamiltonian approach. It is to be noted that the resulting spectra in perturbative QCD have a nonintegrable infrared singularity for the gluon energy $E_g \to 0$, or equivalently as $E_\gamma \to E_\gamma^{max}$. Adding the contributions of the virtual QCD corrections to the processes $b \to (s, d) + \gamma$ with their respective bremsstrahlung part, the singularity cancels [31,32]. The end-point spectra, however, show sensitivity to the left over effects

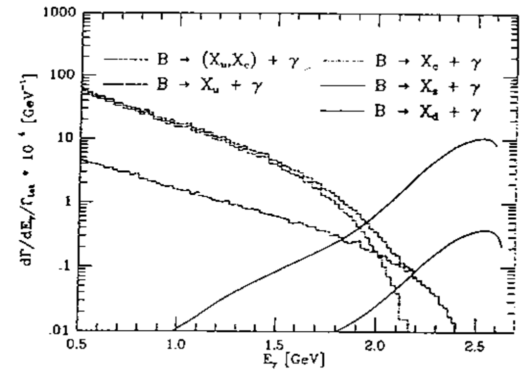

Figure 2: Inclusive photon energy spectra in B-decays from the indicated CC and FCNC processes discussed in the text. The parameters used are: $m_t = 140$, $m_c = 1.68$, $m_s = 0.5$, $m_d = m_u = 0.3$ (all in GeV), and $p_F = 0.3$ GeV. The CKM parameters are given in the text and $(\rho, \eta) = (0.2, 0.4)$ has been used to estimate the $B \to X_d + \gamma$ contribution (from [33]).

of the (cancelled) infrared singularity, with the photon-energy distribution $\frac{d\Gamma}{dx_\gamma}$ rising very steeply near the end-point, $x_\gamma \simeq 1$ (here x_γ is the fractional energy of the photon, $x_\gamma = E_\gamma/E_\gamma^{max}$). To remedy this, one often resorts to (an all order) resummation of the leading (infrared) logarithms.

To get the physical profile of the final states, one has to incorporate the B-meson wave function effects on the partonic distributions obtained in perturbative QCD, using a model. A simple model used in [6,7] to implement the D- and B-meson bound state effects on the inclusive lepton energy spectra was employed to calculate the wave function effects on the photon energy and hadron mass spectra in the decays $B \to X_s + \gamma$ and $B \to X_d + \gamma$ [31,32].

Recapitulating briefly, the b-quark in this model is given a non-zero momentum (due to its being bound in the B-hadron) having a Gaussian distribution:

$$\phi(p) = \frac{4}{\sqrt{\pi}{p_F}^3}\exp\left(\frac{-p^2}{{p_F}^2}\right) \quad ; \quad p = |\vec{p}| \tag{35}$$

The energy-momentum constraint is imposed in the form:

$$W^2 = {M_B}^2 + {m_q}^2 - 2M_B\sqrt{p^2 + {m_q}^2} \tag{36}$$

where M_B is the B-meson mass, W, the effective mass of the b-quark, and m_q, the mass of the spectator quark in the B-meson, $B = b\bar{q}$. There are two important parameters that influence the shape of the spectra, namely W and p_F, which have been experimentally constrained to be: $0.21\ GeV \leq p_F \leq 0.39\ GeV$ and $W \simeq 5.0$ GeV, using the CLEO and ARGUS analysis.

The inclusive photon energy spectra in the FCNC decays $B \to X_s + \gamma$ and $B \to X_d + \gamma$ and in the CC decays $B \to X_c + \gamma$ and $B \to X_u + \gamma$, calculated in the B-rest frame, are shown in Fig. 2 where the dependence on the assumed values of the parameters has been stated. A clear separation between the CC processes ($B \to$

$(X_c, X_u)+\gamma)$ and the FCNC processes $(B \to (X_s, X_d)+\gamma)$ is visible for energetic photons having energy in excess of 2.0 GeV. Separation of all the four spectra would require flavour tagging the hadrons X_q recoiling against the photon in B-decays. In principle, the prompt photon spectra in B-decays provide an alternative method to determine the CKM matrix elements, $|V_{cb}|, |V_{ub}|$ from $B \to X_c + \gamma$ and $B \to X_u + \gamma$, and the other two (matrix elements) involving the top quark $|V_{ts}|, |V_{td}|$ using the FCNC decays $B \to X_s + \gamma$ and $B \to X_d + \gamma$. This programme is well suited for an asymmetric B-factory. Concerning the shape of the spectra from the decays $B \to (X_s, X_d)+\gamma$, we note that they are not very sensitive to m_t (though their rates show more sensitivity). This can be attributed to the wave function effects which don't depend on m_t and dominate for large x_γ, overpowering the m_t-dependence of the perturbative QCD contribution.

4.4 Branching Ratios $BR(B \to X_s + \gamma)$ and $BR(B \to X_d + \gamma)$

The results for the branching ratio $BR(B \to X_s + \gamma)$, obtained including the leading real and virtual corrections, are shown as a function of the top quark mass in Fig. 1, where $\mu = m_b = 5.0\ GeV$ has been used. The dependence of the branching ratio on the charm quark mass in the range 1.3 GeV $\leq m_c \leq$ 1.7 GeV is indicated by the error bars. Comparison of this result with the leading order QCD improved results for the two-body decay $b \to s + \gamma$ in Fig. 1 shows that the additional corrections lower the previously discussed estimates of the inclusive branching ratio $BR(B \to X_s + \gamma)$ by about 15 % across the top quark mass range. This gives $BR(B \to X_s + \gamma) = 3.5 \times 10^{-4}$ for m_t = 140 GeV, and it rises to about 4.5×10^{-4} for $m_t =$ 250 GeV. Taking into account all the uncertainties, a firm prediction for the FCNC inclusive radiative B-decay in the Standard Model is:

$$BR(B \to X_s + \gamma) = (3 - 5) \times 10^{-4} \tag{37}$$

We now discuss the branching ratio for the CKM-suppressed inclusive decay $B \to X_d + \gamma$. As noted already, due to the contribution of the operator O_2 in $b \to d + g + \gamma$ the dependence of the rate on the CKM matrix-element product $|\xi_t|^2$ does not factorize any longer. Instead, one has [32]:

$$BR(B \to X_d+\gamma) = D_1|\xi_t|^2 \left\{1 - \frac{1-\rho}{(1-\rho)^2+\eta^2} D_2 - \frac{\eta}{(1-\rho)^2+\eta^2} D_3 + \frac{D_4}{(1-\rho)^2+\eta^2}\right\} , \tag{38}$$

where the coefficients D_i, given in ref. [32], do not depend on the CKM matrix elements. To get the inclusive branching ratio as a function of m_t, one has to vary the CKM parameters ρ and η over the presently allowed range. The resulting branching ratio $BR(B \to X_d + \gamma)$ is shown in Fig. 3 as a function of m_t. Taking into account the present uncertainties in ρ, η, and m_t, it has been estimated in [33] that the inclusive rate for the CKM-suppressed FCNC radiative decay in the SM is:

$$BR(B \to X_d + \gamma) = (0.5 - 3) \times 10^{-5} \tag{39}$$

We remark in passing that the ratio of the FCNC inclusive decay widths $\Gamma(B \to X_d + \gamma)/\Gamma(B \to X_s + \gamma)$ is less dependent on m_t and hence a better measure of the CKM

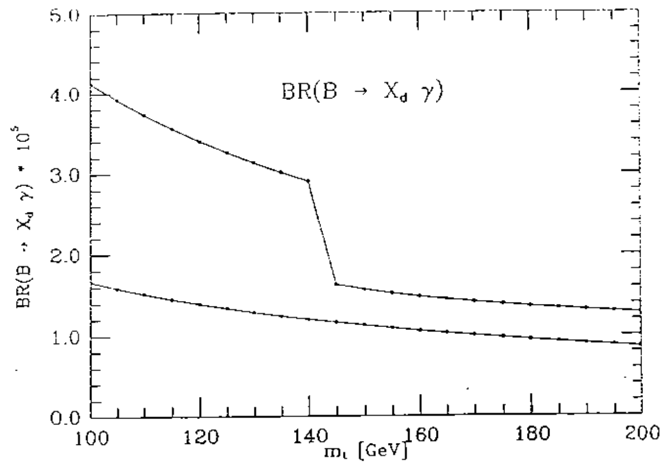

Figure 3: Upper and lower bounds on the branching ratio $BR(B \to X_d + \gamma)$ as a function of m_t, obtained by varying over the allowed range in the (ρ, η) plane, using the constraints discussed in the text. The full curve corresponds to $x_d = 0.67$ (from [32]).

parameters ρ and η. First measurements of the inclusive photon energy spectra in B-decays have been reported by the CLEO collaboration with some excees seen in the region 2.2 $GeV \leq E_\gamma \leq$ 2.7 GeV in the laboratory frame [27]. This can be used conservatively to put an upper bound on the inclusive branching ratios $BR(B \to (X_s + X_d) + \gamma) \leq 9.0 \times 10^{-4}$, which is about a factor 2 away from the SM predictions.

4.5 Estimates for $BR(B \to K^* + \gamma)$ and $BR(B \to \rho + \gamma)$

Having the hadronic invariant mass distributions for the decays $B \to X_d + \gamma$ and $B \to X_s + \gamma$, one could attempt to estimate the exclusive branching ratios using the assumption of vector meson dominance in the low-mass part of the invariant hadron mass spectrum. The motivation of doing this comes from experimental studies in semileptonic D-decays, in which the hadronic mass spectrum in the range $m_K + m_\pi \leq m_{X_s} \leq 1.0$ GeV is found to be completely saturated by the K^*-resonance. Although analogous information from the Cabibbo suppressed decays $c \to d\ell\nu_\ell$ and $b \to u\ell\nu_\ell$ is still not at hand, it is reasonable to assume that in the mass range between $2m_\pi$ and 1.0 GeV, the decays $B \to X_d + \gamma$ will be dominated by the ρ-resonance. Following this argument, we integrate the spectrum in the decays $B \to X_d + \gamma$ in the range $2m_\pi \leq M_{X_d} \leq 1$ GeV to estimate the branching ratio for $B \to \rho + \gamma$. This gives [32]:

$$\begin{aligned} BR(B \to K^* + \gamma) &= (3 - 8) \times 10^{-5} \\ BR(B \to \rho + \gamma) &= (2 - 4.6) \times 10^{-6} \times \left(\frac{|V_{td}|^2}{7.78 \times 10^{-5}}\right) , \end{aligned} \tag{40}$$

taking into account the uncertainties in the wave function model and with m_t in the range 100 $GeV < m_t <$ 200 GeV. A much firm prediction is however obtained on the relative branching ratios for the decays $B \to \rho + \gamma$ and $B \to K^* + \gamma$ [32]:

$$\frac{\Gamma(B \to \rho + \gamma)}{\Gamma(B \to K^* + \gamma)} = (0.062) \times \left(\frac{|V_{td}|^2}{7.78 \times 10^{-5}} \right). \quad (41)$$

With the above estimates, and the present experimental bound $BR(B \to K^* + \gamma) < 9.2 \times 10^{-5}$ (90% $C.L.$) [27], we feel that the discovery of this channel is just around the corner. It is conceivable that the CKM-suppressed rare decays discussed here, in particular $B \to \rho + \gamma$, can be measured in a first generation B-factory experiment, or perhaps at LEP and CLEO with $O(10^7)$ B-hadrons. This would then provide one of the most reliable estimates of the CKM matrix element $|V_{td}|$.

References

[1] S.L. Glashow, Nucl. Phys. **22** (1961) 579; S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264; A. Salam, in *Elementary Particle Theory*, Editor: N. Svartholm (Almqvist and Wiksell) (1968).

[2] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531.

[3] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[4] L. Wolfenstein, Phys. Rev. Lett. **51** (1983) 1945.

[5] M. Aguilar-Benitez et. al. (Particle Data Group), Phys. Lett. **B239**(1990) 1.

[6] A. Ali and E. Pietarinen, Nucl. Phys. **B154**(1979) 519.

[7] G. Altarelli, M. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, Nucl. Phys. **B208** (1982) 365.

[8] N. Isgur, D. Scora, B. Grinstein and M. Wise, Phys. Rev. **D39** (1989) 799.

[9] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29** (1985) 637.

[10] J.G. Körner and G.A. Schuler, Z. Phys. **C38** (1988) 511.

[11] H. Schröder, in *QCD–20 Years Later*, Aachen, June 9-13, 1992.

[12] N. Isgur and M. Wise, Phys. Lett. **B232** (1989) 113; **B237** (1990) 527.

[13] For an excellent review of the HQET approach and references see T. Mannel *Heavy Quark Effective Theory*, Habilitationsschrift, Technische Hochschule Darmstadt, IKDA 92/6 (1992).

[14] M.E. Luke, Phys. Lett. **252B** (1990) 447.

[15] C.G. Boyd and D.E. Brahm, Phys. Lett. **257B** (1991) 393.

[16] M. Neubert and V. Rieckert, Heidelberg Preprint HD-THEP-91-6 (1991); M. Neubert, Phys. Lett. **264B** (1991) 455, and Heidelberg Preprint HD-THEP-91-30 (1991).

[17] A. Ali and D. London, DESY Report 92-075 (1992).

[18] A.J. Buras, M. Jamin and P.H. Weisz, Nucl. Phys. **B347** (1990) 491.

[19] F. Abe et al. (CDF), Fermilab-Pub-91-352-E (1991), to appear in Phys. Rev. D.

[20] J. Carter, Proceedings of the *International Lepton-Photon Symposium and Europhysics Conference on High Eneregy Physics*, S. Hegerty, K. Potter and E. Quercigh eds. (Geneva, 1991).

[21] J.P. Cortes et al., Phys. Rev. **D25** (1982) 188.

[22] N. Cabibbo and L. Maiani, Phys. Lett **79B** (1978) 109;
M. Suzuki, Nucl. Phys. **B145** (1978) 420.

[23] G. Altarelli and S. Petrarca, Phys. Lett. **261B** 303 (1991).

[24] A.J. Buras and P.H. Weisz, Nucl. Phys. **B333** (1990) 66;
A.J. Buras et al., MPI-Muñchen Report, MPI-PAE/PTh 56/91 (1991).

[25] R. Rückl, Weak Decays of Heavy Flavours (Habilitationsschrift, Univ. München, 1983).

[26] S.L. Glashow, J. Illiopoulos, and L. Maiani, Phys. Rev. **D2** (1970) 1285.

[27] G. Eigen (CLEO), DESY Seminar, June 23, 1992.

[28] T. Inami and C.S. Lim, Progr. Theor. Phys. **65**(1981) 297.

[29] S. Bertolini, F. Borzumati and A. Masiero, Phys. Rev. Lett. **59** (1987) 180;
R. Grigjanis et al., Phys. Lett. **B213**(1988) 355;
B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. **B319** (1989) 271;
B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. **B339** (1990) 269;
G. Cella et al., Phys. Lett. **248B** (1990) 181;
M. Misiak, Phys. Lett. **B269** (1991) 161.

[30] A. Ali, DESY Report DESY 92-058 (1992).

[31] A. Ali and C. Greub , Z. Phys. **C49** (1991) 431; Phys. Lett. **B259** (1991) 182.

[32] A. Ali and C. Greub , DESY Report 92-048 (1992).

[33] A. Ali and C. Greub , DESY Report 92-089 (1992).