$$\hat{g}_i\hat{g}_{i+1}\hat{g}_i = \hat{g}_{i+1}\hat{g}_i\hat{g}_{i+1}$$

$$\hat{g}_i\hat{g}_j = \hat{g}_j\hat{g}_i, |i-j| \geq 2$$

$$\hat{g}_i^2 = (q-1)\hat{g}_i + q \tag{46}$$

For generic q the representations of $H_n(q)$ are classified by Young diagrams (as for the permutation group , obtained from (46)by setting $q = 1$). If $\lambda = [\lambda_1, \ldots \lambda_k]$ is a tableau with n boxes ; λ_1 boxes in the first row etc. , we can associate a representation of $H_n(q)$ to each λ. When q is an l-th root of unity , most of this representations are singular. Wenzl [18] has studied which among all the tableaux furnish regular representations. His conclusion is that (k,l)-tableaux with $k \leq l+1$, at most k rows and $\lambda_1 - \lambda_k \leq l-k$ define regular irreducible representations $\pi_{(k,l)}$ of $H_N(q), q^l = 1$ and that the truncation to this subset gives a consistent representation theory. From the point of view of Hecke algebras , there is no a priori restriction on the values of $k \leq l+1$. However if we are interested in $C_N^{1/2}$ (to understand the restrictions on the representations of $SL(2,q)$) we obtain the constraint $k = 2$. The origin of this constraint is as follows : $C_N^{1/2}$ is isomorphic to Jones algebra , which in turn is a quotient of H_{N+1} by a cubic polynomial relation between the g_i's ([19] equation 11.6) , and only representations of H_{N+1} with at most two rows satisfy this constraint [19]. For $q^l = 1$ the representations of $C_N^{1/2}$ are reduced to tableaux with two rows , and such that $\lambda_1 - \lambda_2 \leq l-2$. To get some feeling for these restrictions , we analyze the example with $l = 5$.

The $(2,5)$-Young tableaux with $3,4$ and 5 boxes appear in fig.7. From the point of view of $SU(2)$ these are representations with spins $j = 1/2, 3/2, 0, 1, 1/2, 3/2$ respectively. Hence $(2,5)$ diagrams correspond to $j \leq 3/2$. In general $(2,l)$ tableaux satisfy $j \leq (l-2)/2$. In conclusion , for $q^l = 1$ the only irreps. of the centralizer are given by W_j's with $j \leq (l-2)/2$. (There are similar generalizations for $SU(N)$). This restriction is the same as the constraint on the possible integrable highest weight representations describing the $SU(2)_k$ WZW theory (see [15] for details). In particular the conformal blocks of these

theories define representations of $H_N(q)$ with $q^{k+2} = 1$ something already known [6]. To determine the dimensions of the spaces W_j and the fusion rules requires more work. By considering explicitly the sequence of embeddings :

$$\ldots \pi_{(k,l)}H_n(q) \subset \pi_{(k,l)}H_{n+1}(q) \subset \ldots, q^l = 1 \tag{47}$$

it is possible to show that the dimensions of the W's are exactly the ones one would obtain using the fusion rules for $SU(2)$ level k. More precisely , to compute $dim W_j$ for different tableaux , it suffices to look at the integers appearing in the Bratteli diagram associated to the inclusion (47)computed in [18, 17]. Details will be presented elsewhere [13].

5. In conclusion , we have shown that the representation theory of $SL(2,q)$ with q a root of unity provides solutions to the polynomial equations for RCFT [3, 4] , and it also provides a rather efficient way to compute the duality matrices (2). Furthermore, there seems to be a rather close connection between the modular properties of the Kac-Moody charaters , and the behavior of the q-characters under the transformation $q \to q^{-1}$. Similar conclusions also hold for other quantum groups , and preliminary results show that one can extend the construction to q-homogeneous spaces [13] as well. There is an interesting puzzle if we use the quantum groups to obtain solutions to the polynomial equations. Even when q has unit modulus , but it is not a root of unity, the hexagon and pentagon identities still hold , however it does not seem easy to construct a local CFT whose C, N matrices are determined by the correponding $6j$ symbols. It is not clear yet however whether the one loop part of the polynomial equations will still be satisfied. If this where the case , this might imply that for irrational Conformal Field Theories extra conditions should be required in order to have modular invariance and factorization for all genera. This may be similar to the potential modular anomalies that one should find in the Coulomb gas approach to the minimal theories [5] contructed by Dotsenko and Fateev [20] when the background charge α_0 does not have a rational square. Another possibility is that

what fails is the possibility of reconstruction the Hilbert space of conformal blocks with reasonable properties. When q is a root of unity , the relevant factors of the centralizer are C^*-algebras , and for these one can use the GNS reconstruction theory to reconstruct a Hilbert space with a positive definite metric. We find very attractive the fact that a second quantized two dimensional field theory (which could represent a string moving on a classical space , the group manifold) can be described rather accurately in terms of a quantum system with a finite number of degrees of freedom "moving" on a quantum space : the quantum group. These questions are currently being investigated.

It may be possible to relate the three dimensional topological Chern-Simons theory [11] with the results given here. If one considers three manifolds with boundaries and three dimensional Wilson lines going through them , it was argued by Witten [11] that the constraints on the possible gauge connections were equivalent to having a quantum group. In his study of the WZW theory , Fröhlich (see [14]) finds a connection satisfing the classical Yang-Baxter equation and giving the monodromy properties of the solutions to the Knizhnik-Zamalodchikov differential equations. We believe that if one constructed explicitly the global holonomy for this connection , one would be led to the braiding matrices constructed in this paper. This might provide an indirect way to solve these contraints in the Chern-Simons theory.

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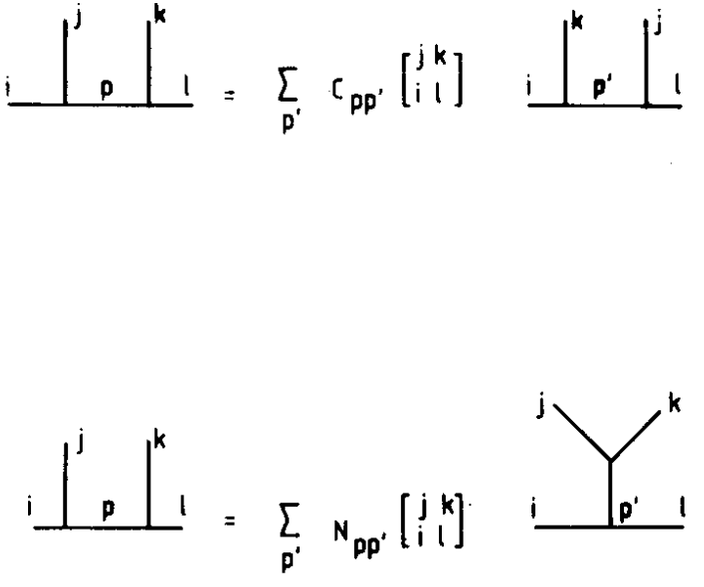


Fig. 1

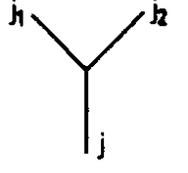


Fig. 2

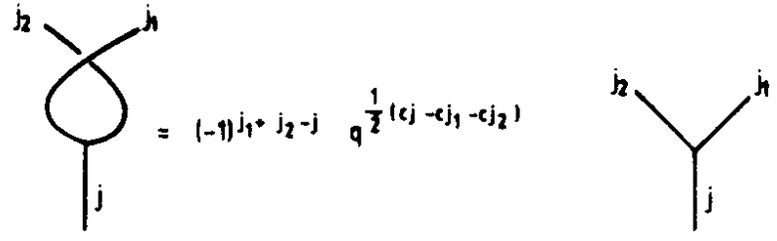
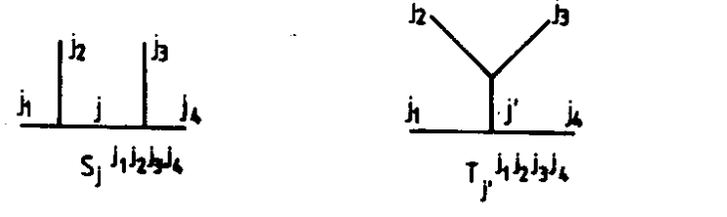


Fig. 3



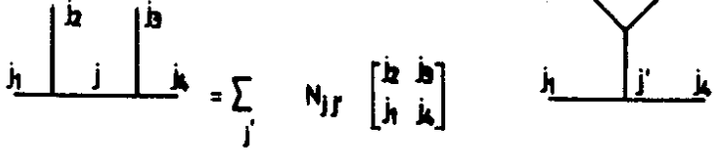


Fig. 4

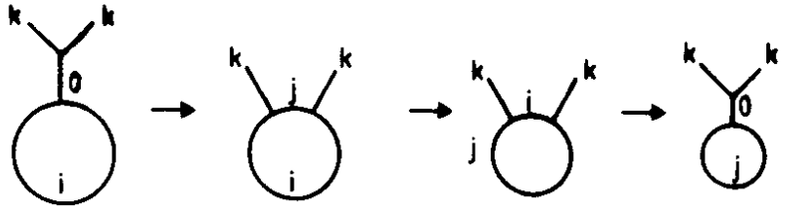


Fig. 5

Fig. 6

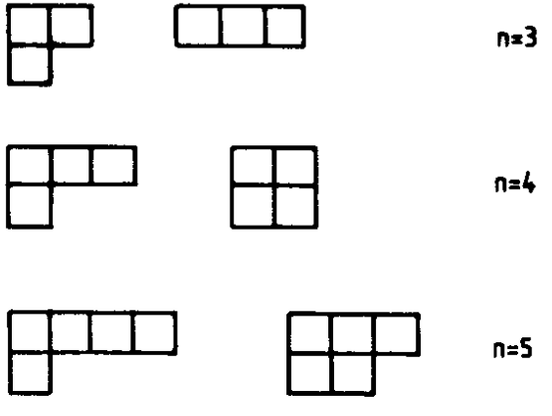


Fig. 7