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$B^0 - \bar{B}^0$ MIXINGS AND RARE B-DECAYS

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Abstract

I present an update on the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements taking into account the current experimental and theoretical information on weak decays. The resulting fit is combined with the measured value of the B_d^0-$\overline{B_d^0}$ mixing ratio x_d and estimates of the pseudoscalar coupling constants to determine the allowed range of the B_s^0-$\overline{B_s^0}$ mixing ratio x_s in the Standard Model (SM). For the central values of the parameters used we find $8 \leq x_s \leq 24$. Flavour changing neutral current (FCNC) B-decays are briefly reviewed; in particular the role of such decays in determining the CKM matrix elements V_{td} and V_{ts} is emphasized on the example of radiative B-decays, $B \to (X_d + \gamma)$ and $B \to (X_s + \gamma)$.

INTRODUCTION AND OVERVIEW

In SM, mass mixings and FCNC decays have something in common, namely they are allowed only as higher order (loop) processes. For B-hadrons these transitions are dominated by the (virtual) top quark contribution. Hence, their measurements provide valuable constraints on the top quark mass m_t and its CKM couplings V_{td}, V_{ts} and V_{tb}. This information is complementary to the one from the electroweak radiative corrections to the neutral current (NC) and charged current (CC) processes which constrain m_t and (to a smaller extent) the Higgs boson mass through loop effects.

This contribution gives an update on the CKM matrix elements using the fits in ref. 1 and the modifications discussed below. The aim is to test both the consistency of the CKM framework and get constrained estimates for a number of FCNC transitions in B physics. To illustrate this, I review some recent theoretical work, carried out in the SM context, on B^0-$\overline{B^0}$mixings[1] and the FCNC rare B-decays $B \to X_s + \gamma$ [2] and $B \to X_d + \gamma$ [3]. The former will manifest themselves through exclusive decays such as $B \to K^* + \gamma$ (and higher resonances[4]) and the latter through the CKM-suppressed decay $B \to \rho + \gamma$ (and higher resonances). Also, FCNC B-decays involving dileptons are potentially very interesting [5,6,7]. While m_t will hopefully soon be measured directly, very probably FCNC transitions will be the only source of information for the CKM-matrix elements involving the top quark. In view of this, I emphasize here the quantitative role that such measurements will play in determining the elements of the CKM matrix, V_{ti}.

AN UPDATE ON THE CKM MATRIX

In the Wolfenstein parametrization[8], the CKM-matrix is characterized by the four parameters, λ, A, ρ, η, with η being a complex phase. The present status of these parameters is summarized below. The essential update compared to the values given in the PDG compilation[9] and the work reported in ref. 1 lies in the use of a significantly lower value, $V_{ub}/V_{cb} = 0.085 \pm 0.015$, which is an estimate of the world average of this ratio using the Altarelli et al. model[10]. The various input quantities used for the CKM-matrix fits are discussed below.

First of all, $|V_{us}|$ has been extracted with good accuracy from $K \to \pi e \nu$ and hyperon decays to be [8]

$$|V_{us}| = \lambda = 0.2205 \pm 0.0018 \ . \qquad (1)$$

This agrees quite well with the determination of $V_{ud} \simeq 1 - \frac{1}{2}\lambda^2$ from β-decay[8]:

$$|V_{ud}| = 0.9744 \pm 0.0010 \ . \qquad (2)$$

The parameter A is related to the CKM matrix element V_{cb}, which can be obtained from semileptonic B-decays. Its value has been determined from the inclusive semileptonic decay measurements, which give $|V_{cb}| = 0.042 \pm 0.007$, where the quoted error takes into account the uncertainty due to the semileptonic decay model [11]. Alternatively, and theoretically more reliably, one could use the heavy quark symmetry, relating the *heavy* $\to$ *heavy* hadron transitions, to determine the form factors in the semileptonic decays $B(v) \to (D, D^*)(v')\ l\nu$ (here v and v' are the four-velocities as indicated). Using the heavy quark symmetry one can show that all hadronic form factors describing these decays can be expressed in terms of a single function, the Isgur-Wise function [12], $\xi(v \cdot v')$, having the normalization $\xi(v \cdot v' = 1) = 1$. To extract $|V_{cb}|$, one has to extrapolate the data to the point $y = v \cdot v' = 1$, for which one needs an Ansatz for the Isgur-Wise function $\xi(y)$. Using a linear parametrization for $\xi(y)$ (which is consistent with data), the present measurements of the B-lifetime, $\tau_B = (1.35 \pm 0.05) \times 10^{-12} s$, and the semileptonic branching ratios $BR(B \to D^* l\nu) = (5.1 \pm 0.9)\%$, the ARGUS analysis yields $|V_{cb}| = 0.043 \pm 0.007$ and $|V_{cb}| = 0.045 \pm 0.005$, for the neutral and charged D^* state, respectively[13]. Using other parametrizations, the extracted values for $|V_{cb}|$ are significantly different. For the CKM fits I shall use a value

$$|V_{cb}| = 0.044 \pm 0.006 \qquad (3)$$

giving the CKM parameter A

$$A = 0.90 \pm 0.12 \ . \qquad (4)$$

The other two CKM parameters ρ and η are constrained by the measurements of $|V_{ub}/V_{cb}|$, $|\epsilon|$ (the CP-violating parameter in the kaon system), x_d (B^0_d-$\overline{B^0_d}$ mixing) and (in principle) ϵ'/ϵ ($\Delta S = 1$ CP-violation in the kaon system). I shall not use the constraint from the ϵ'/ϵ-analysis but discuss the rest in turn, presenting a fit in which the allowed region of ρ and η and the CKM-unitarity triangle are shown.

So far $|V_{ub}/V_{cb}|$ has been obtained by analyzing the endpoint of the inclusive lepton spectrum in semileptonic B decays. The resulting values are in general model dependent. Using the Altarelli et al. model [10], the earlier ARGUS and CLEO results were compatible with the range $\left|\frac{V_{ub}}{V_{cb}}\right| = 0.12 \pm 0.02$. The new analysis presented by the CLEO collaboration at this conference, however, gives a significantly lower value for this ratio, with the CLEO II measurements being compatible with $|V_{ub}/V_{cb}| \simeq 0.07$ in the same model[11,14].

With the experimental situation in flux, probably it is best to average the old and new CLEO and ARGUS results, yielding

$$\left|\frac{V_{ub}}{V_{cb}}\right| = 0.085 \pm 0.015 \ . \qquad (5)$$

in the Altarelli et al. model [10]. This gives

$$\sqrt{\rho^2 + \eta^2} = 0.39 \pm 0.07 \ . \qquad (6)$$

which is significantly lower than the value $\sqrt{\rho^2 + \eta^2} = 0.63 \pm 0.23$, used earlier [1].

The experimental value of $|\epsilon|$ is

$$|\epsilon| = (2.26 \pm 0.02) \times 10^{-3} \ . \qquad (7)$$

Theoretically, $|\epsilon|$ is essentially proportional to the imaginary part of the box diagram for K^0-$\overline{K^0}$ mixing, and is given by

$$|\epsilon| = \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} B_K \left(A^2\lambda^6\eta\right) \left(y_c \left\{\eta_{ct} f_3(y_c, y_t) - \eta_{cc}\right\} + \eta_{tt} y_t f_2(y_t) A^2\lambda^4(1-\rho)\right). \qquad (8)$$

Here, the η_i are QCD correction factors, $\eta_{cc} \simeq 0.82$, $\eta_{tt} \simeq 0.62$, $\eta_{ct} \simeq 0.35$ for $\Lambda_{QCD} = 200$ MeV $y_i \equiv m_i^2/M_W^2$, and the functions f_2 and f_3 are given in ref. 1.

One of the unknowns in Eq. (8) is the top quark mass. I shall use a value $100\ GeV \leq m_t \leq 180\ GeV$, which is compatible with the present bounds from electroweak radiative corrections and direct top quark searches. The final parameter in the expression for $|\epsilon|$ is B_K, which represents our ignorance of the matrix element $\langle K^0|\left(\overline{d}\gamma^\mu(1-\gamma_5)s\right)^2|\overline{K^0}\rangle$. The evaluation of this matrix element has been the subject of much work. In what follows I shall take

$$B_K = 0.8 \pm 0.2. \qquad (9)$$

Turning now to B^0_d-$\overline{B^0_d}$ mixing, the quantitity x_d, defined below, has been measured to be

$$x_d = 0.67 \pm 0.10 \ . \qquad (10)$$

In the SM, x_d is calculated from the B^0_d-$\overline{B^0_d}$ box diagram, which is dominated by t-quark exchange:

$$x_d \equiv \frac{(\Delta M)_{B_d}}{\Gamma_{B_d}} = \tau_B \frac{G_F^2}{6\pi^2} M_W^2 M_B \left(f_{B_d}^2 B_{B_d}\right) \eta_B y_t f_2(y_t) |V_{td}^* V_{tb}|^2 \ , \qquad (11)$$

where $(\Delta M)_{B_d}$ and Γ_{B_d} are, respectively, the mass difference and the average width of the two mass eigenstates, and $|V_{td}^* V_{tb}|^2 = A^2\lambda^6\left[(1-\rho)^2 + \eta^2\right]$. Here, η_B is the QCD correction, which has been analyzed in great detail in ref. 15, including the effects of a heavy t-quark. They find that η_B depends sensitively on the definition of the t-quark mass, and that, strictly speaking, only the product $\eta_B(y_t)f_2(y_t)$ is free of this dependence. I will use the next-to-leading order QCD-improved value, $\eta_B = 0.55$. For the B system, the hadronic uncertainty is given by $f_{B_d}^2 B_{B_d}$, analogous to B_K in the kaon system, except that in this case, also f_{B_d} is not measured. And, just like B_K, the evaluation of $f_{B_d}^2 B_{B_d}$ has been the subject of much work. For the CKM fits two ranges for $f_{B_d}^2 B_{B_d}$ are used:

$$(I): \ f_{B_d}\sqrt{B_{B_d}} = 135 \pm 25 \text{ MeV},$$
$$(II): \ f_{B_d}\sqrt{B_{B_d}} = 200 \pm 20 \text{ MeV} \qquad (12)$$

which represent, respectively, the estimates of the older vintage and more modern Lattice-QCD results [16].

The information regarding the allowed region in ρ-η space can be displayed quite elegantly using the so-called unitarity triangle. Because the CKM matrix is unitary, one has the following relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \qquad (13)$$

This can be (approximately) recast as

$$\frac{V_{ub}^*}{\lambda V_{cb}} + \frac{V_{td}}{\lambda V_{cb}} = 1 \ , \qquad (14)$$

that is, a triangle relation in the complex plane (in ρ-η space).

In order to find the allowed unitarity triangles, the computer program MINUIT has been used to fit the CKM parameters A, ρ and η to the experimental values of $|V_{cb}|$, $|V_{ub}/V_{cb}|$, $|\epsilon|$ and x_d in ref. 1. This fit is updated in this contribution and the results are shown in Fig. 1 and 2 for the coupling constant choice (I) and (II), respectively. Note that the graph for $f_{B_d}\sqrt{B_{B_d}} = 135 \pm 25$ MeV and $m_t = 100$ GeV is a bad fit of the data ($\chi^2/d.o.f. = 2.17$). In all these graphs, the solid line has $\chi^2 = \chi^2_{min} + 1$. For comparison, we include the dashed line, which is the 90% c.l. region ($\chi^2 = \chi^2_{min} + 4.6$). The "best values" of the parameters (ρ, η), together with their $(\chi^2/d.o.f.)$, are given in Table 1.

THE UNITARITY TRIANGLE AND x_s

We now discuss the estimates for $x_s \equiv \frac{(\Delta M)_{B_s}}{\Gamma_{B_s}}$. Mixing in the B^0_s-$\overline{B^0_s}$ system follows quite closely that of the B^0_d-$\overline{B^0_d}$ system. The B^0_s-$\overline{B^0_s}$ box diagram is again dominated by t-quark exchange, and the mixing parameter x_s is given by a formula analogous to that of Eq. (11):

$$x_s = \tau_{B_s} \frac{G_F^2}{6\pi^2} M_W^2 M_{B_s} \left(f^2_{B_s} B_{B_s}\right) \eta_{B_s} y_t f_2(y_t) |V^*_{ts} V_{tb}|^2 \qquad (15)$$

Assuming $V_{cb} = V_{ts}$, it follows that one of the sides of the unitarity triangle, $|V_{td}/\lambda V_{cb}|$ can be obtained from the ratio of x_d and x_s:

$$\frac{x_d}{x_s} = \frac{\tau_{B_d} \eta_{B_d} M_{B_d} \left(f^2_{B_d} B_{B_d}\right)}{\tau_{B_s} \eta_{B_s} M_{B_s} \left(f^2_{B_s} B_{B_s}\right)} \left|\frac{V_{td}}{V_{ts}}\right|^2 . \qquad (16)$$

Conversely, the ratio on the l.h.s. can be predicted from the unitarity triangle constraints, as all dependence on the t-quark mass drops out, and we are left with the square of the ratio of CKM matrix elements, multiplied by a

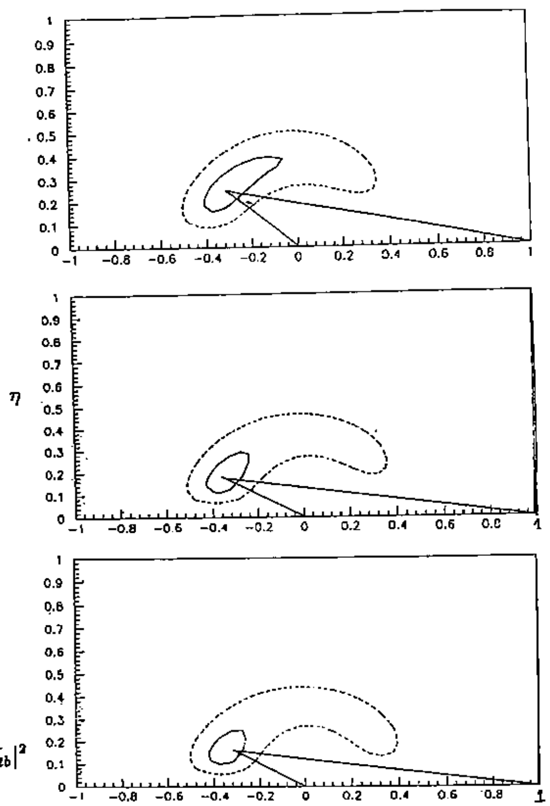

Figure 1. Allowed region in ρ-η space for different values of the standard model parameters. Figs. (a)-(c) have $f_{B_d}\sqrt{B_{B_d}} = 135 \pm 25$ MeV, with $m_t =$ 100, 140 and 180 GeV, respectively. The solid line represents the region with $\chi^2 = \chi^2_{min} + 1$; the dashed line denotes the 90% c.l. region. The triangles show the best fit (updated from ref. (1)).

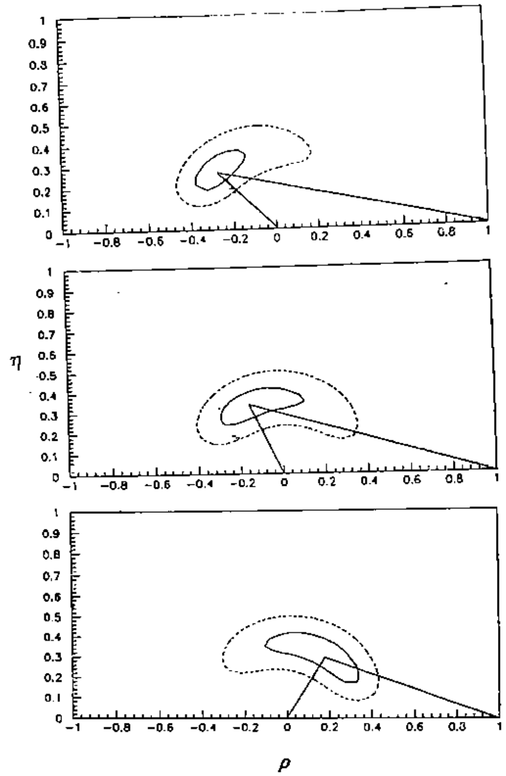

Figure 2. Allowed region in ρ-η space for different values of the standard model parameters. Figs. (a)-(c) have $f_{B_d}\sqrt{B_{B_d}} = 200 \pm 20$ MeV, with $m_t =$ 100, 140 and 180 GeV, respectively. The solid line represents the region with $\chi^2 = \chi^2_{min} + 1$; the dashed line denotes the 90% c.l. region. The triangles show the best fit (updated from ref. (1)).

factor which reflects $SU(3)_{flavour}$ breaking effects. Whether or not x_s can be used to help constrain the unitarity triangle will depend crucially on the theoretical status of the ratio $f^2_{B_d} B_{B_d}/f^2_{B_s} B_{B_s}$. I assume that this ratio can be reliably calculated and use the recent Lattice-QCD update[16]

$$\frac{f^2_{B_s} B_{B_s}}{f^2_{B_d} B_{B_d}} = 1.19 \pm 0.10. \qquad (17)$$

Setting $\frac{\tau_{B_d} \eta_{B_d} M_{B_d}}{\tau_{B_s} \eta_{B_s} M_{B_s}} = 1$, which should hold to a good accuracy, gives

$$\frac{x_s}{x_d} = 1.19 \pm 0.10 \times \left|\frac{V_{ts}}{V_{td}}\right|^2 . \qquad (18)$$

The "best values" for the CKM ratio $|V_{td}/V_{ts}|$ following our fits are given in Table 1. It is interesting to note that this ratio is remarkably stable for the low-f_{B_d} solution (I), but varies considerably for the high-f_{B_d} solution (II), as one varies m_t. Using the central value for x_d in Eq. (18)together with the "best fit" for the CKM triangle gives:

$$8.0 \ \leq x_s \ \leq 24.0 \, . \qquad (19)$$

Thus, to measure x_s, one will have to undertake time-dependent experiments. They are discussed in ref. 1. In summary, a measurement of B^0_s-$\overline{B^0_s}$ mixing will allow a more accurate measurement of $|V_{td}/\lambda V cb|$, and will be an important further test of the unitarity triangle.

FCNC RARE B-DECAYS

The interest in the study of FCNC B-decays is reflected in the large number of inclusive and exclusive decays that have been investigated theoretically [2-7]. In particular, their measurements will provide a normalization of the magnetic moment operator matrix elements, which are dominated by

Table 1. The "best values" of the CKM parameters (ρ,η) and the ratio $|V_{td}/V_{ts}|^2$, obtained by a minimum χ^2 fit of the experimental data discussed in the text. Values of m_t and the coupling constant $f_{B_d}\sqrt{B_{B_d}}$ are stated. The resulting minimum χ^2 values from the MINUIT fits are also given (updated from ref. 1.)

m_t (GeV)	$f_{B_d}\sqrt{B_{B_d}}$ (MeV)	(ρ,η)	$\lvert V_{td}/V_{ts}\rvert^2$	$\chi^2_{\rm min}$
100	135 ± 25	$(-0.31,\ 0.25)$	0.086	2.17
140	135 ± 25	$(-0.35,\ 0.18)$	0.090	0.46
180	135 ± 25	$(-0.35,\ 0.16)$	0.089	0.0045
100	200 ± 20	$(-0.28,\ 0.27)$	0.083	0.25
140	200 ± 20	$(-0.16,\ 0.33)$	0.071	0.43
180	200 ± 20	$(0.19,\ 0.31)$	0.036	0.90

the penguin diagrams. For lack of space, I concentrate here on the radiative transitions $b\to s+(g)+\gamma$ and $b\to d+(g)+\gamma$ (where g stands for gluon). The CKM-suppressed rare B-decays of the type $b\to d+(g)+\gamma$ are of particular interest in determining the rather crucial matrix element V_{td}. A good prototype of such reactions is the decay $B\to\rho+\gamma$, which can be used with its CKM-allowed counterpart $B\to K^*+\gamma$ to determine the ratio V_{td}/V_{ts}, as argued in ref. 3 and below.

FCNC Decays $B\to X_s+\gamma$ and $B\to X_d+\gamma$

The framework that has been employed in the calculations of the FCNC B-decays is that of an effective theory with five quarks, obtained by integrating out the heavier degrees of freedom (top quark and $W^\pm$ bosons). To leading order in the small (weak)-mixing angles, a complete set of dimension-6 operators relevant for the processes $b\to s+\gamma$ and $b\to s+\gamma+g$ is contained in the effective Hamiltonian

$$H_{eff}(b\to s\gamma) = -\frac{4G_F}{\sqrt{2}}\lambda_t\sum_{j=1}^{8}C_j(\mu)\,\hat{O}_j(\mu) \quad , \qquad (20)$$

where G_F is the Fermi coupling constant, $C_j(\mu)$ are the Wilson coefficients evaluated at the scale μ, and $\lambda_t = V_{tb}V_{ts}^*$. The definitions of the various operators, matching conditions $C_i(m_W)$, and the leading log pertubative QCD corrections giving $C_i(\mu)$ with $\mu << m_W$ can be seen in refs. 2-7, where also references to the original work can be found. It is known from earlier studies that the dominant contribution to $b\to s+\gamma$ and $b\to s+g+\gamma$ are due to the four-fermion operator $\hat{O}_2$ and the QED magnetic moment operator $\hat{O}_7$ which are defined as:

$$\begin{aligned}\hat{O}_2 &= (\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{s}_{L\beta}\gamma_\mu c_{L\beta})\\ \hat{O}_7 &= (e/16\pi^2)\,\bar{s}_\alpha\,\sigma^{\mu\nu}\,(m_b R + m_s L)\,b_\alpha\,F_{\mu\nu}\end{aligned}$$

$$L=\frac{1-\gamma_5}{2} \quad ; \quad R=\frac{1+\gamma_5}{2}$$

Here e denotes the QED coupling constant.

The inclusive rates, photon energy and hadron invariant mass spectra for the decays $b\to s+\gamma$ and $b\to s+\gamma+g$ have been calculated in refs. 2, from where the details can be seen. Here, I quote the inclusive branching ratio for these decays:

$$BR(B\to X_s+\gamma) = (2-5)\times10^{-4} \qquad (21)$$

The central value, $BR(B\to X_s+\gamma) = 3.5\times10^{-4}$ (corresponding to $\mu = 5.0\ GeV$ and $m_t = 140\ GeV$), is within a factor 3 away from the present experimental bound on this branching ratio; hence a measurement in this channel is in the wings!

As already stated, the overriding theoretical interest in the study of electromagnetic penguins in B-decays lies in the direct measurement of the CKM matrix elements $|V_{ts}|$ and $|V_{td}|$. The dependence of the branching ratio $BR(B\to X_s+\gamma)$ on the CKM ratio $(|V_{ts}|/|V_{cb}|)^2$ is being discussed by Greub in these proceedings [17]. Note that, despite considerable uncertainties in theoretical estimates, a measurement of the inclusive branching ratio for $B\to X_s+\gamma$ will provide the first direct measurement of the CKM matrix element $|V_{ts}|$.

The modifications that have to be incorporated for the CKM-suppressed radiative rare decays $b\to d+\gamma$ and $b\to d+\gamma+g$ have been discussed in ref. 3. We recall here that for the decays $b\to s+\gamma(+g)$ the effective Hamiltonian was written in the approximation where the CKM factor $\lambda_u = 0$, which is reasonable since $\lambda_u << \lambda_c, \lambda_t$ ($\lambda_i \equiv V_{ib}V_{is}^*$). For the decays $b\to d+\gamma$ and $b\to d+\gamma+g$ the CKM factors λ_i are replaced by $\xi_i \equiv V_{ib}V_{id}^*$. Now, ξ_u, ξ_c and ξ_t are all of the same order of magnitude $(A\lambda^3)$ and therefore the corresponding approximation $\xi_u = 0$ cannot be made any longer. The differences due to the inclusion of ξ_u to describe the decays $b\to d+\gamma$ and $b\to d+g+\gamma$ can be most easily built in the effective Hamiltonian framework by modifying the operators $\hat{O}_1$ and $\hat{O}_2$, encountered in the decay $b\to s+\gamma$. The dimension-6 operator basis again reads:

$$H_{eff}(b\to d\gamma) = -\frac{4G_F}{\sqrt{2}}\xi_t\sum_{j=1}^{8}C_j(\mu)\,O_j(\mu) \quad . \qquad (22)$$

The operators O_1 and O_2 are defined as:

$$\begin{aligned}O_1 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu c_{L\beta})\\ &\quad -\frac{\xi_u}{\xi_t}(\bar{u}_{L\beta}\gamma^\mu b_{L\alpha})(\bar{d}_{L\alpha}\gamma_\mu u_{L\beta})\\ O_2 &= -\frac{\xi_c}{\xi_t}(\bar{c}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu c_{L\beta})\\ &\quad -\frac{\xi_u}{\xi_t}(\bar{u}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{d}_{L\beta}\gamma_\mu u_{L\beta}) \qquad (23)\end{aligned}$$

With the operators defined in this basis, the matching conditions, $C_i(m_W)$, and the corresponding Wilson coefficients at the scale μ, $C_i(\mu)$, are precisely the same for the decays $b\to d+\gamma$ and $b\to s+\gamma$.

The decay rate for $B\to X_d+\gamma$ depends on the CKM parameters ρ and η which enter through the CKM factors ξ_t and ξ_c, as already discussed. In contrast to the $B\to X_s+\gamma$ case, the CKM-parametric dependence does not factorize in $B\to X_d+\gamma$, and to get the inclusive branching ratio one has to vary the CKM parameters ρ and η as a function of m_t over the presently allowed range. An estimate of the inclusive branching ratio $BR(B\to X_d+\gamma)$, modelled after the partonic decays $b\to d+\gamma$ and $b\to d+g+\gamma$ (i.e. only the so-called short distance contribution) has been obtained in this way in ref. 3. This analysis is updated here in view of the foregoing discussion in the previous section. The resulting branching ratio as a function of m_t is shown in Fig. 3, yielding

$$BR(B\to X_d+\gamma) = (0.8-3.0)\times10^{-5} \qquad (24)$$

for the top quark mass in the range $100\,GeV \le m_t \le 200\,GeV$. The branching ratios $BR(B\to\rho+\gamma)$, as well as the relative rate $BR(B\to\rho+\gamma)/BR(B\to K^*+\gamma)$, using vector meson dominance for the hadronic mass below 1 GeV, have also been estimated in ref. 3. This branching ratio factorizes to

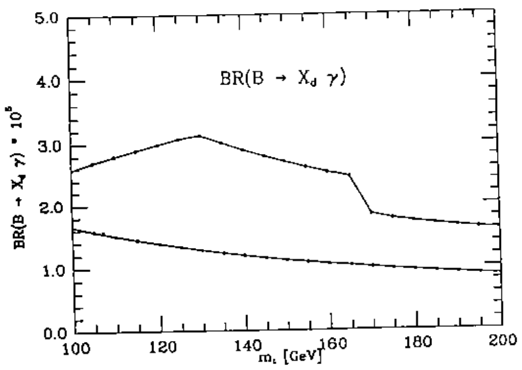

Figure 3. Upper and lower bounds on the branching ratio $BR(B \to X_d + \gamma)$ as function of m_t, obtained by varying over the allowed range in the (ρ, η) plane resulting from Fig. 2 (updated from ref. 3).

a very good approximation, and the updated estimate is:

$$BR(B \to \rho + \gamma) = (3.5\text{–}8.0) \times 10^{-6} \times \left(\frac{|V_{td}|^2}{1.37 \times 10^{-4}} \right) \quad (25)$$

A much firm prediction is however obtained on the relative branching ratios for the decays $B \to \rho + \gamma$ and $B \to K^* + \gamma$ [3]:

$$\frac{\Gamma(B \to \rho + \gamma)}{\Gamma(B \to K^* + \gamma)} = (0.1) \times \left(\frac{|V_{td}|^2}{1.37 \times 10^{-4}} \right) \quad (26)$$

where I have used the "best fit" CKM parameters" for $m_t = 140\ GeV$ and the choice (II) for the reference value of $|V_{td}|$. The role of FCNC B-decays in determining the matrix elements V_{ti} is abundantly clear from the analysis discussed here.

I would like to thank Christoph Greub and David London with whom the work reported here was carried out for updating some of the results shown. I also thank Sheldon Stone for a discussion.

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