DEUTSCHES ELEKTRONEN – SYNCHROTRON

DESY 92-181
December 1992

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ISSN 0418-9833

NOTKESTRASSE 85 · D-2000 HAMBURG 52

ISSN 0418-9833

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ARCLUS
A new jet clustering algorithm inspired by the Colour Dipole Model

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Abstract

A new jet clustering algorithm - ARCLUS - is presented. The main difference between this and conventional algorithms is that while the latter in each step join two clusters into one, ARCLUS joins three clusters into two. The performance of ARCLUS in terms of the size of hadronization corrections is studied for some jet-reconstruction tasks in e^+e^- and ep collisions, and is found to be as good as or, for some tasks in ep collisions, better than conventional clustering algorithms.

1 Introduction

Jet reconstruction algorithms have been proven to be useful tools when comparing predictions from perturbative QCD with experimental data. The idea behind it is to associate the large amounts of hadronic energy in small angular regions found in experiments [1], with corresponding partonic energy, calculable within the framework of perturbative QCD. In this way the comparison becomes less sensitive to the non-perturbative hadronization process, of which we today have very poor theoretical knowledge.

The comparisons are however not completely independent of the hadronization process and one has to resort to phenomenological models to estimate the systematical errors introduced. It is found that size of these errors are very sensitive to the details in the definition of the jet algorithm [2], and it is of course a requirement for a "good" jet reconstruction algorithm that these hadronization corrections are as small as possible.

Another equally important requirement is that the algorithm is suitable for use in perturbative QCD calculations. One important issue here is that the algorithm should specify a resolution scale which can be used as a cutoff in fixed order matrix element calculations to avoid singularities.

For e^+e^- physics the most commonly used jet algorithms are the so called cluster algorithms. The procedure used in these is to define a distance measure d_{ij} (usually denoted by the scaled quantity $y_{ij} = d_{ij}^2/W^2$) between two clusters (where a cluster initially is just a particle found in the detector). The distance between all combinations of two clusters is calculated and the two which have the minimum distance is are joined into one cluster. This is then repeated until no two clusters are closer than some distance d_{cut}, and the remaining clusters are considered to be reconstructed jets, which can be used in comparison with fixed order QCD calculations with the resolution scale taken to be d_{cut}.

JADE	$d_{ij}^2 = 2E_iE_j(1 - \cos\theta_{ij})$
LUCLUS	$d_{ij} = \frac{2\lvert\vec{p}_i\rvert\lvert\vec{p}_j\rvert\sin(\theta_{ij}/2)}{\lvert\vec{p}_i\rvert+\lvert\vec{p}_j\rvert}$
$k_\perp$-algorithm	$d_{ij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$

Table 1: *The three "conventional" clustering algorithms used for comparisons in this paper, together with their corresponding distance measures.*

The exist a number of such clustering algorithms, which differ from each other in the definition of d_{ij} and in the procedure used for joining two clusters into one [2]. Table 1 presents three algorithms, JADE [3], LUCLUS [4] and the $k_\perp$-algorithm [5], together with their definition of d_{ij} (the joining procedure in all three cases is to simply assigning the sum of the two four-momenta to the new cluster). These will be used in this paper when comparing to "conventional" algorithms.

One problem that occurs when using these algorithms in fixed order QCD calculations, is that for small resolution scales d_{cut} the higher order corrections become very large. Recently it has been shown that these corrections can be dealt with by using a technique of resummation of leading and next-to-leading logarithms of $y_{cut} = d_{cut}^2/W^2$ [6]. This technique may be applied if the distance measure corresponds to the $p_\perp$ of a soft cluster w.r.t. a harder one as is the case for the $k_\perp$-algorithm[1] and LUCLUS[2]. On the contrary, in algorithms like JADE, the definition of the distance measure (approximately the invariant mass of two clusters) is shown to introduce strong kinematical correlations between partons which makes this resummation impossible [6].

It should be noted that the $k_\perp$-algorithm is closely related to the angular ordered parton cas-

[1] In fact, the $k_\perp$-algorithm was invented to conform to this requirement.

[2] LUCLUS however uses by default a special reassignment procedure [4] which makes it difficult to apply in analytical calculations.

cade implemented in the HERWIG [7] event generator program (in some sense the $k_\perp$-algorithm can be viewed as the inverse of an angular ordered parton cascade). This is no coincidence, as the resummation technique above relies on the Sudakov exponentiation of soft gluon emissions which is possible within the framework of an angular ordered cascade [8].

This last point is the original motivation behind the ARCLUS clustering algorithm [9] described in this paper. In the same way as the $k_\perp$-algorithm relates to the HERWIG parton cascade, ARCLUS is inspired by the Colour Dipole Model [10] and the dipole cascade implemented in the Ariadne [9] program.

The dipole cascade (see section 2) differs very much from a conventional parton cascade in that it treats gluon emissions as radiation from colour dipoles between partons, instead of emission from individual partons. In the same way the ARCLUS algorithm in each step, joins *three* clusters into *two* instead of two into one as in conventional clustering algorithms. The distance measure is a Lorentz invariant $p_\perp$ of one cluster w.r.t. *two* other clusters.

Although it has not been proven one would expect that ARCLUS would allow for the Sudakov exponentiation of soft gluon emission necessary for the resummation procedure above, as the distance measure is the transverse momentum and due to the close relation to the dipole cascade which exhibits angular ordering.

Due to the relative complexity of the ARCLUS algorithm it is not clear if there exist a simple way of applying it in analytical fixed order QCD calculations, although it is certainly possible to used it in numerical calculations.

This paper will however only deal with the more practical issue of hadronization corrections. It will be shown that the corrections in ARCLUS in the case of e^+e^--annihilation is comparable with the conventional algorithms used as reference. And that in the case of ep collisions the corrections are smaller in ARCLUS due to a better treatment of the remnant jet.

The outline of this paper is as follows. In section 2 a detailed description of the algorithm is presented together with some comparisons with the Colour Dipole Model. In section 3 the performance of the ARCLUS algorithm in e^+e^--annihilation is studied and compared to conventional algorithms. In section 4 the same is done for deep inelastic ep collisions, and in section 5 some conclusions are presented.

2 The algorithm

As ARCLUS is very closely related to the Colour Dipole Model (CDM) it is instructive to first look at some details of the latter.

The CDM is based on the fact that a gluon emitted from a $q\bar{q}$ pair in an e^+e^- collision can be treated as radiation from the colour dipole between the q and $\bar{q}$, and that to a good approximation, the emission of a second softer gluon can be treated as radiation from two independent dipoles, one between the q and g and one between the g and $\bar{q}$. In the CDM this is generalized so that the emission of a third, still softer gluon, is given by three independent dipoles etc.

The fundamental scale for an emission in the CDM, is the Lorentz invariant $p_\perp$ defined as

$$p_\perp^2 = S_{dip}(1 - x_1 + \frac{m_1^2 - (m_2 + m_3)^2}{S_{dip}})$$
$$(1 - x_3 + \frac{m_3^2 - (m_2 + m_1)^2}{S_{dip}}) \qquad (1)$$

where x_1 and x_3 denotes the final state energy fractions $= 2E_i/\sqrt{S_{dip}}$ of the emitting partons in the dipoles center of mass system. This $p_\perp$ is used both as argument in α_s and as the ordering variable in the Sudakov form factor. Also the cutoff in the cascade is defined in terms of a minimum $p_\perp$ of an emission. It can be shown that the ordering in $p_\perp$ in the CDM implies that the emissions also are ordered in angle.

The CDM only predicts the energy fractions x_i in each emission, leaving e.g. the azimuth angle of the emitted gluon, and polar angle of parton

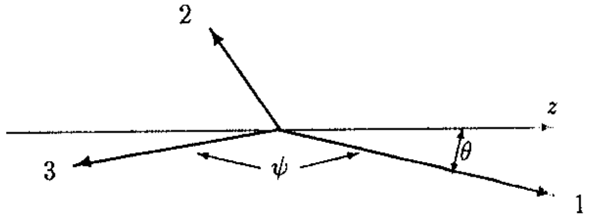

Figure 1: The orientation of a dipole after emission.

1 (see figure 1) undetermined. In the Ariadne program [9] the azimuth angle is assumed to be evenly distributed between 0 and 2π. For the polar angle a couple of different alternatives are given, but the standard procedure is to chose it so that the sum of the $p_\perp^2$ of the emitting partons is minimized. More precisely

$$\theta = \frac{x_3^2}{x_1^2 + x_3^2}(\pi - \psi) \qquad (2)$$

In the ARCLUS algorithm the whole dipole cascade is basically reversed. In each step three jets are combined into two, with the invariant $p_\perp$ of the three jets increasing in each step.

Initially each final state particle is considered to be a cluster. Then a two step procedure is iterated as follows:

(1) For all combinations of three clusters, the one is selected that gives the smallest invariant $p_\perp$ according to equation 1. If this $p_\perp$ is larger than some given resolution scale $p_{\perp min}$, the iteration is interrupted and the remaining clusters are considered to be jets.

(2) If the selected combination has a $p_\perp$ below $p_{\perp min}$, the three clusters are boosted to their c.o.m. frame and rotated according to figure 1 and equation 2. The three clusters are then replaced by two (mass-less) clusters with opposite momenta ($= 0.5\sqrt{S_{dip}}$) along the z-axis. The two new clusters are then transformed back to the original Lorentz frame and the iteration is continued from (1).

A few things should be noted:

- The algorithm is, just like the dipole cascade, completely Lorentz invariant.
- The resulting jets are mass-less, except for possible initial clusters that may have "survived" the algorithm.
- The minimum number of jets that can be reconstructed is two.
- No hadron can be said to belong to one jet. Rather its momenta is shared by two jets.

The last point may at first sight seem a bit unintuitive. It obviously fits very well into the CDM picture, where a gluon is emitted from a dipole between two partons and takes momentum from both of them. But it also fits well with hadronization models like the cluster- or string fragmentation, where hadrons are produced in the field between partons.

Ideally the ARCLUS algorithm would really reconstruct each step of the dipole shower, at least when applied to a partonic state produced by Ariadne. This is however not the case, as in Ariadne emissions only take place between colour connected partons, while no colour information is available in ARCLUS.

In figure 2, the effect of this lack of colour information in ARCLUS is shown by comparing the n-parton rates obtained by Ariadne as a function of the cutoff $p_{\perp cut}$ and the n-jet rate obtained by ARCLUS as a function of $p_{\perp min}$ when applied to the same events. The effect is best seen in the three jet rates. In Ariadne it is possible to radiate a gluon with a $p_\perp$ as high as $0.5E_{cm}$ while for three jets, the minimum $p_\perp$ found by ARCLUS is always below $E_{cm}/3$.

It should be noted that the problem with lack of colour information also is present in algorithms like the $k_\perp$-algorithm and LUCLUS, which in some sense tries to reconstruct an angular ordered parton cascade. As the angular ordering is determined by the colour flow, these algorithms should ideally only consider colour connected pairs of jets for clustering.

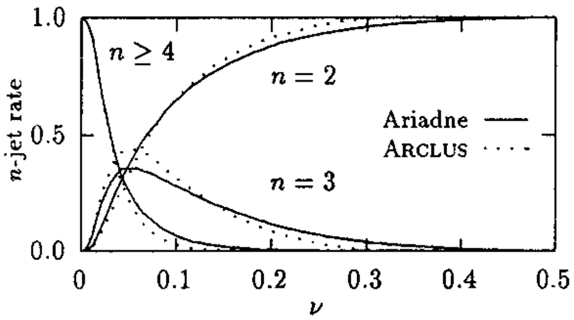

Figure 2: *The n-jet rates for e^+e^- collisions at 91.2 GeV, as obtained by varying the cutoff $p_{\perp cut}$ in Ariadne and by varying the $p_{\perp min}$ of* ARCLUS *applied to the same events after having continued the dipole cascade in Ariadne down to $p_{\perp cut} = 0.6 GeV$. ν is the corresponding $p_\perp$ scaled with the c.o.m. energy.*

$< r_2 >$ (r.m.s.)	(SF)	(IF)
ARCLUS	0.046 (0.14)	0.13 (0.19)
$k_\perp$-algorithm	0.006 (0.18)	0.11 (0.21)
LUCLUS	-0.023 (0.13)	0.04 (0.18)
JADE	-0.007 (0.18)	0.07 (0.23)

Table 2: *Mean value and r.m.s. of r_2 for different algorithms with string (SF) and independent (IF) fragmentation.*

3 ARCLUS at LEP

One of the most important property of a jet algorithm is that it gives approximately the same result when applied to an event, on the parton level and on the hadronic level. However since our knowledge of the non-perturbative hadronization process is very poor we have to use different phenomenological models to estimate how well the algorithms handles this task.

In this section the size of the hadronization corrections of ARCLUS is compared with a few other algorithm (the $k_\perp$-algorithm, LUCLUS and JADE) by applying them to simulated LEP events produced by JETSET (with default parameters). The algorithms were first applied to the partonic state just before fragmentation, then to the hadronic final state after string fragmentation and decay of unstable particles.

Also the algorithms were applied to the hadronic final state obtained by applying the independent fragmentation model built in to JETSET (again with default parameters) to the same partonic states. It should be noted that the independent fragmentation model in JETSET does not reproduce data very well, and the hadronization corrections obtained in this case should be thought of as "maximum" corrections, while the ones obtained with string fragmentation are more "probable" corrections.

In figure 3 the results for the n-jet rates are shown for the four algorithms as a function of the scaled cutoff $\nu_{cut} = d_{cut}/E_{cm}$ (E_{cm} is the invariant mass of the particles included in the clustering).

It is clear that all algorithms have very small hadronization corrections for sting fragmentation, especially for high resolution scales. ARCLUS has somewhat smaller corrections at large cutoffs, but clearly larger corrections at small cutoffs and for the case of independent fragmentation.

Another way of investigating the hadronization corrections is to look at which scale $\nu_2 = d_2/E_{cm}$ an event, previously clustered into a 3-jet configuration, is clustered into a 2 jets. Looking at a scatter plot where every event is entered as a point in the (ν_2^p, ν_2^h) plane (where ν_2^p is the value at the parton level and ν_2^h on the hadron level), this should ideally just give a straight line. In figures 4 and 5 such plots are shown for the different algorithms for the case of string fragmentation. A more quantitative description of the same thing is obtained by looking at the mean value and r.m.s. of e.g. $r_2 = (\nu_2^h - \nu_2^p)/(\nu_2^h + \nu_2^p)$. The result for the different algorithms for independent and string fragmentation is given in table 2.

It is clear that ARCLUS and LUCLUS have narrower distributions, but especially ARCLUS is more shifted from zero than the others. Also

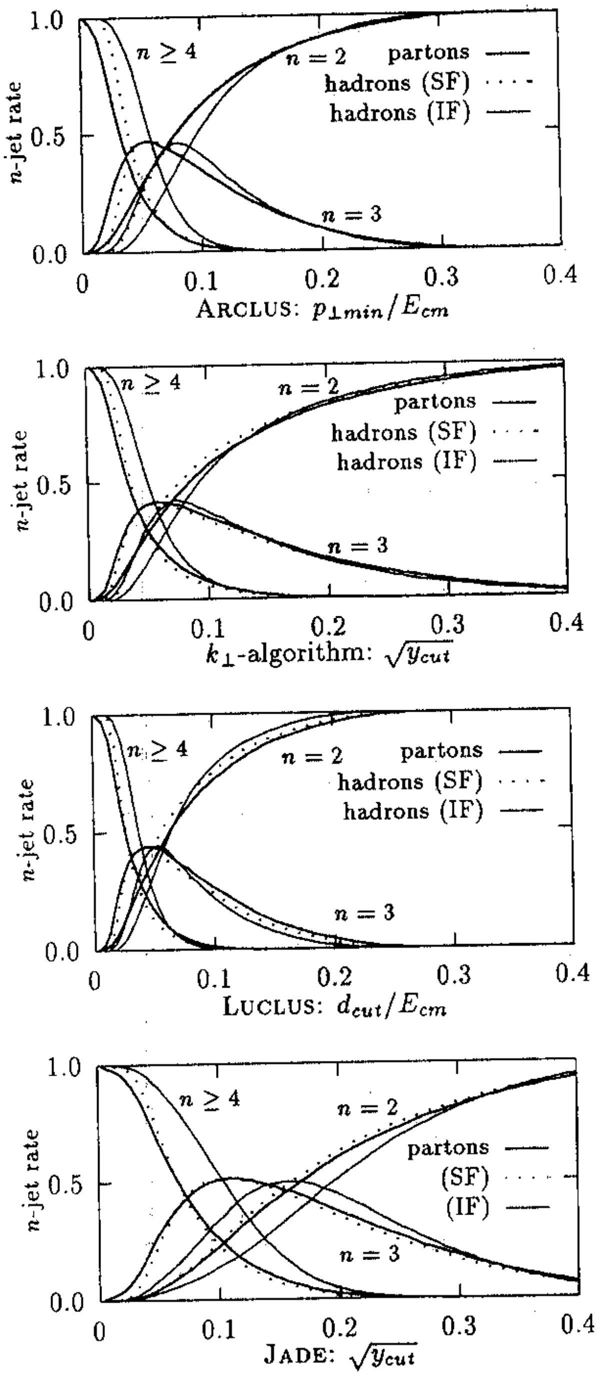

Figure 3: *The n-jet rates for different algorithms as a function of the cutoff, before hadronization and after string (SF) and independent (IF) fragmentation.*

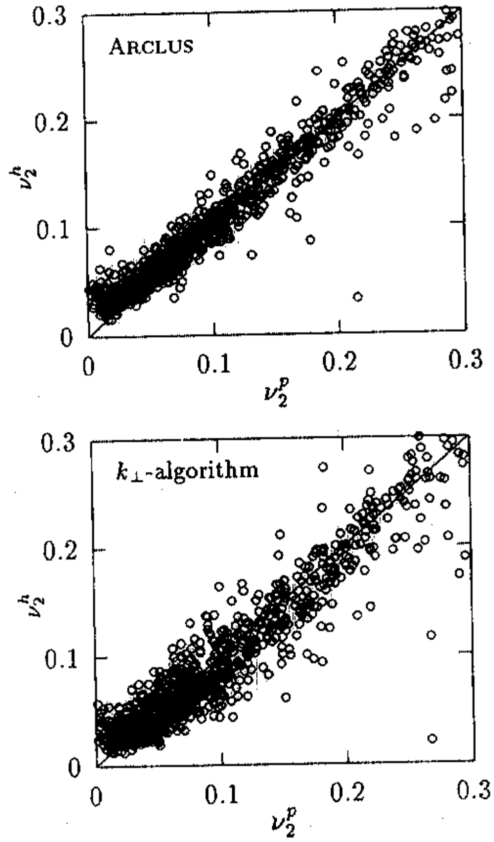

Figure 4: *Scatter plot of* ν_2^p *and* ν_2^h *for* ARCLUS *and* $k_\perp$*-algorithm.*

in the figures, one notes that JADE is more symmetric around the $\nu_2^p = \nu_2^h$ line, while the others deviates more at small scales.

In some situations it is of interest to be able to reconstruct not only the number of jets in an event, but also their kinematical properties. To investigate the hadronization corrections in this case, the algorithms were applied to the same events as above and iterated until a three-jet configuration was found. Then three measures was studied. The first two was the the scaled differences between the energy fractions ($x_j = 2E_j/E_{cm}$) of the two largest jets on the parton level compared to the corresponding jets on the hadron level $r_{xj} = (x_j^h - x_j^p)/(x_j^h + x_j^p)$. The third was the difference between the angles between the two largest jest on the parton level and between the corresponding jets on the hadron level

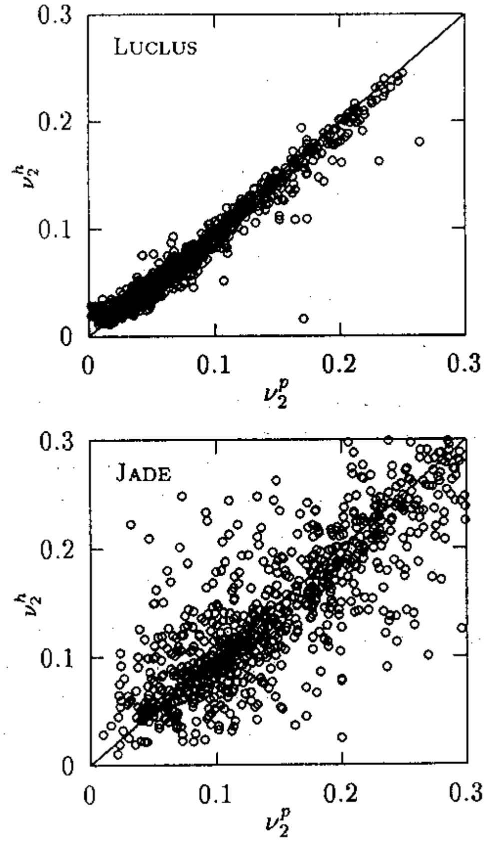

Figure 5: *Scatter plot of ν_2^p and ν_2^h for* LUCLUS *and* JADE.

$$\Delta\theta_{12} = \theta_{12}^h - \theta_{12}^p.$$

The results for string fragmentation are shown in table 3 where it is clear that the different algorithms performs very well in all cases. There are however differences and although LUCLUS in the cases studied here seems to give the overall smallest corrections, one should of course try out different algorithms to find the one with the smallest corrections for the specific reconstruction task in question.

4 ARCLUS at HERA

In deep inelastic *ep* collisions, the situation is somewhat more complicated, both theoretically and experimentally.

	$< r_{x1} >$ (r.m.s.)	$< r_{x2} >$ (r.m.s.)	$\Delta\theta_{12}$ (r.m.s.)
ARCLUS	-0.022	-0.001	0.9°
	(0.07)	(0.10)	(3.0°)
$k_\perp$-algorithm	-0.029	-0.006	0.2°
	(0.08)	(0.12)	(3.1°)
LUCLUS	-0.018	-0.004	0.2°
	(0.06)	(0.09)	(2.6°)
JADE	-0.016	0.008	0.5°
	(0.06)	(0.08)	(3.2°)

Table 3: *Mean value and r.m.s. of r_x and $\Delta\theta_{12}$ for different algorithms.*

Due to the presence of initial state QCD radiation and the presence of the proton remnant it is not necessarily adequate to take the cluster algorithms used in e^+e^- and apply them directly to *ep* events. One problem is which scale to choose for algorithms like JADE and the $k_\perp$-algorithm. In the case of e^+e^- there is only one scale involved ($Q^2 = W^2 = S$), but in *ep* there are basically two scales involved - Q^2 and W^2, and any combination of them could be used in the definition of the scaled distance measure y_{ij}. Also there is the question of in which Lorentz frame the algorithms should be applied.

For the $k_\perp$-algorithm there exist a special version developed for deep inelastic *ep* collisions [11], to deal with these problems. It involves a two step procedure: first a pre-clustering phase to find a beam-jet and "final state macro-jets", then only the hadrons belonging to the "macro-jets" are considered for the next step which follows the same procedure as in e^+e^-. The scale used in the y_{ij} definition is called E_t and should satisfy $Q^2 \geq E_t^2 >> \lambda_{QCD}^2$. The whole procedure is carried out in the Breit-frame of the initial state hadron and the exchanged vector boson.

In the Colour Dipole Model for deep inelastic *ep* collisions [12] the concept of initial-state radiation is not necessary. Instead it is assumed that all radiation can be described in terms of a dipole cascade from the colour dipole formed between the struck quark and the proton remnant. The only difference w.r.t. the e^+e^- case is that the the radiation in the target region is suppressed due to the transverse extension of the remnant. The extension of the remnant also implies the emission of so-called recoil gluons which can be show to correspond to initial state radiation in a conventional parton shower. Another feature of the CDM is that the only relevant scale is W.

As the ARCLUS algorithm again is supposed to reconstruct the dipole shower there is no conceptual problem with applying the same scheme as in e^+e^- to deep inelastic ep collisions. Also there is no problem with choosing the Lorentz frame as ARCLUS is completely Lorentz invariant.

Experimentally there is the problem that a large fraction of the hadronic energy is undetectable as it is too close to the proton beam and disappears "down the beam-pipe". The standard procedure for dealing with this problem is to introduce a pseudo particle along the proton direction carrying the missing longitudinal momentum. This pseudo particle is then included among the initial clusters, and the clustering proceeds as in the e^+e^- case [13].

In the investigation below the effect of the beam-pipe is included in the hadronization effects, and the procedure using a pseudo particle above is used for all algorithms (also for the $k_\perp$-algorithm where it in principle is not necessary due to its special treatment of the remnant jet). The scale in the case of the JADE and LUCLUS algorithms was taken to be W^2. For LUCLUS and JADE, where the choice of Lorentz frame is not prescribed, the investigation was performed both in the hadronic c.o.m. and in the lab frame and the frame which had the least hadronization corrections was chosen in each case. Finally, for the $k_\perp$-algorithm, only the first step of the procedure was used, varying the scale E_t, to facilitate the comparisons.

As in the case of e^+e^- one of the main usage of jet algorithms is to measure the n-jet cross section. To estimate the corrections due to hadronization events was produced at HERA energies in the kinematical region $x > 0.005$, $W^2 > 8000\,GeV^2$, using the LEPTO [14] program (with default parameters and the default structure function parametrization = EHLQ1 [15])

	$< r_2 >$ (r.m.s.)
ARCLUS	0.001 (0.14)
$k_\perp$-algorithm	-0.036 (0.18)
LUCLUS	-0.028 (0.15)
JADE	-0.003 (0.26)

Table 4: *Mean value and r.m.s. of r_2 for different algorithms in ep collisions*

The algorithms were first applied to the partonic state, just before the hadronization, then to the hadronic final state after string fragmentation, decays of unstable particles and after the introduction of the pseudo particle, assuming a beam-pipe cut of 4°.

The results for the n-jet rates (where n includes the beam jet) is shown in figure 6 and the mean and r.m.s. of the r_2 variable defined in section 3 is presented in table 4. It is clear that ARCLUS is as good as or, in the case of r_2, better than the three other algorithms.

To study the kinematical properties of the jets, the same method as in section 3 was used, i.e. the algorithms were applied to the same events as above and iterated until a three-jet (again including the remnant jet) configuration was found. Then the jet with largest $E_\perp$ in the partonic state, and the corresponding jet after hadronization and beam-cut was selected and the angle between them ($\Delta\theta$) and the scaled difference between their $E_\perp$, $\Delta E_\perp = (E_\perp^h - E_\perp^p)/(E_\perp^h + E_\perp^p)$ was measured.

The results presented in table 5 shows that ARCLUS is better for reconstructing the angle, while the reconstruction of the $E_\perp$ is better for LUCLUS and JADE. The comment in the end of section 3, about choosing the algorithm that gives the smallest correction for the particular reconstruction task in question, applies of course also here. But one would be inclined to recommend anyone doing analysis involving the reconstruction of jet-angles to try out the ARCLUS algorithm.

The reason that LUCLUS and JADE reconstructs angles better when applied in the lab-system as

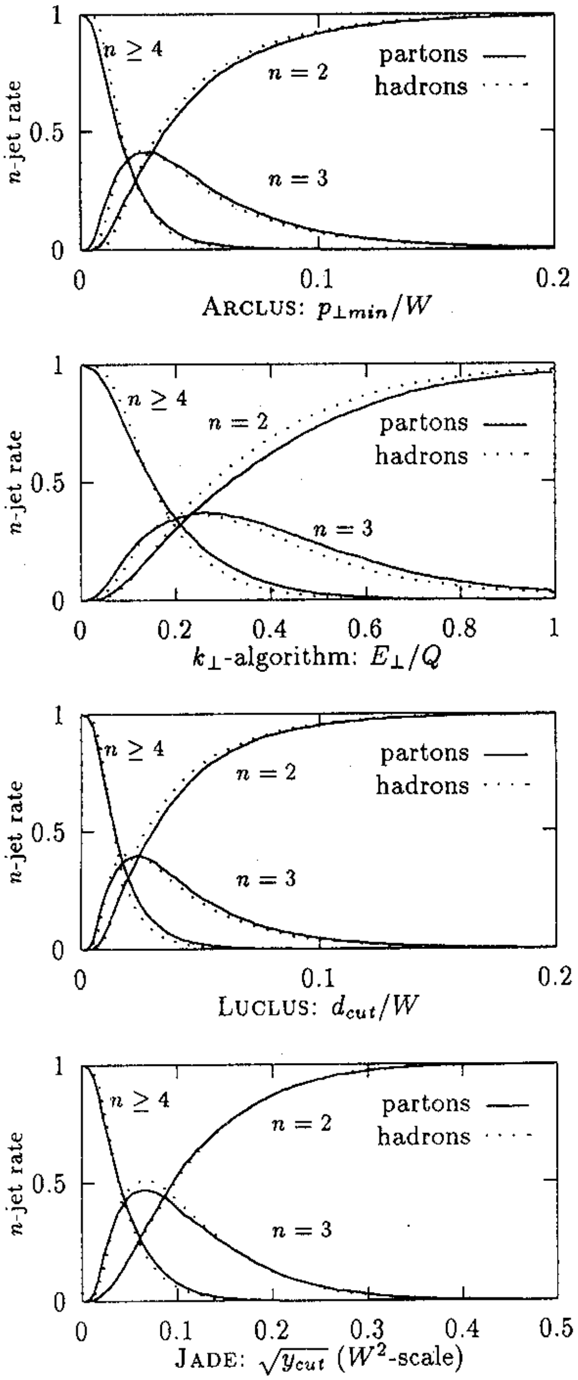

Figure 6: *The n-jet rates for different algorithms as a function of the cutoff, at HERA energies before and after string fragmentation and introduction of pseudo particle.*

	$<\Delta\theta>$ (r.m.s.)	$<\Delta E_\perp>$ (r.m.s.)
ARCLUS	0.5° (13°)	-0.08 (0.19)
$k_\perp$-algorithm	−1.9° (12°)	-0.11 (0.20)
LUCLUS (cms)	−8.3° (14°)	-0.04 (0.18)
LUCLUS (lab)	−2.8° (11°)	-0.06 (0.16)
JADE (cms)	−7.6° (16°)	-0.03 (0.20)
JADE (lab)	−3.8° (12°)	-0.05 (0.17)

Table 5: *Mean value and r.m.s. of $\Delta\theta$ and $\Delta E_\perp$ for different algorithms in ep collisions. For* LUCLUS *and* JADE *the clustering was performed both in the lab-system (lab) and in the hadronic center of mass system (cms). In all cases the measurements was made in the lab-system.*

compared with when applied in the c.o.m. system is that the angles is affected by the boost. Although the angles in the c.o.m. system may be well reconstructed, the jet-mass are generally larger for the jets reconstructed from the hadrons, which are then "dragged" more closely to the beam direction in the boost back to the lab-system than the jets reconstructed from the partons.

Note that ARCLUS also involves a large amount of boosts to and from the lab-system. Here however, all jets are mass-less, and the angles are reproduced better after the boost.

To realize why ARCLUS is better for reconstructing angles it is instructive to look at a concrete example.

In figure 7, such an example is given for LUCLUS and ARCLUS. For simplicity an event was chosen with only two partons - one corresponding to the current-jet, and one to the beam-jet. When these are fragmented according to the string model, the hadrons are distributed flatly in rapidity in the hadronic c.o.m. system. When they are boosted back to the lab-system they therefore tend to lie between the two partons as in the figure.

If there was no beam-pipe cut, the jet directions would be reconstructed well for both algorithms. With a beam-pipe cut however some particles are

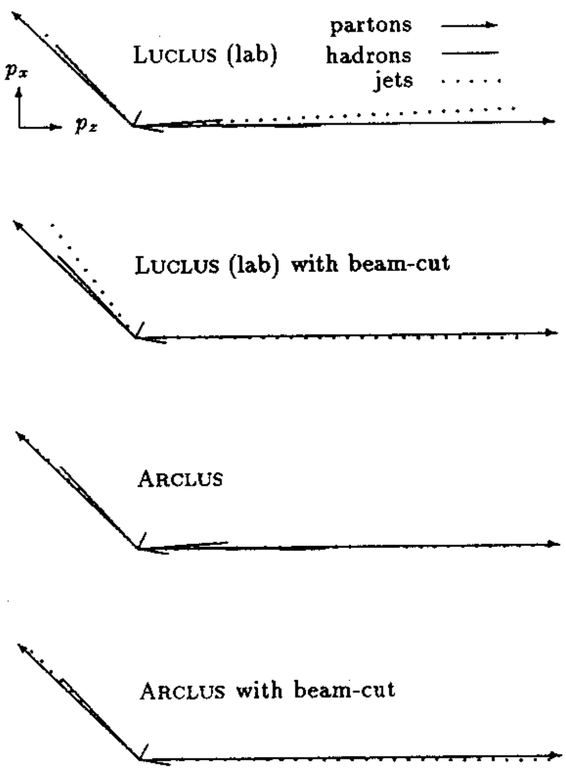

Figure 7: *x, z-view of a fictive ep event on the parton level, hadron level and after jet-reconstruction, with and without a beam-pipe cut of* $10°$. *(z is the proton direction.)*

not included in the clustering. In this example, without the cut, the particle going out in the middle between the jets would by LUCLUS be associated to a particle close to the beam and then in a later stage to the beam jet. With the cut, the particle close to the beam is not included and the particle in the middle would be closer to, and assigned to the current-jet.

In ARCLUS a particle is however always associated with two jets, and the momentum of our little friend in the middle would, with or without cut, be shared by both the current- and the beam-jet. This means that ARCLUS in general is less sensitive to the loss of particles close to the beam.

5 Conclusions

The definition of the ARCLUS algorithm presented in this paper is not the only one which could be derived from the Colour Dipole Model. In particular the prescription for the orientation of the two new clusters in the three-cluster center of mass (see figure 1 and equation 2) could be modified. One such modification, where the two new clusters are oriented along the direction of the largest of the three clusters, has been investigated, but the results were found to be very closed to the ones presented in this paper.

Another development of the ARCLUS algorithm could be to try to supply some kind of colour information in the cascade as discussed in section 2. One such method could involve the minimization of the so-called λ-measure [16] to find an approximate colour ordering of the particles detected. The clustering procedure would then be limited to joining colour connected clusters. However this minimization procedure turns out to be very complicated because of the non-local properties of the λ-measure, and is very difficult to apply to the high energy experiments of to day, where the number of final state hadrons can become very large.

In any case, the ARCLUS algorithm, as defined in this paper, seems to be a very good alternative to the conventional clustering algorithms. The hadronization corrections are generally of the same size as for the conventional ones. In addition, due to the procedure of dividing each particles momentum between two jets, ARCLUS is especially well suited for *ep* physics where it turns out to be less sensitive to the loss of particles close to the proton beam.

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