For the first moment one has to include also conservation of probability :

$$f^{(1)}(0,\underline{1};t)+f^{(1)}(1,\underline{1};t)+f^{(1)}(0,\underline{8};t)+f^{(1)}(1,\underline{8};t)=1$$

and finds :

$$\frac{d}{dt}\begin{pmatrix} f^{(1)}(1,\underline{1};t) \\ f^{(1)}(0,\underline{8};t) \\ f^{(1)}(1,\underline{8};t)\end{pmatrix} \simeq \frac{\alpha_s}{2\pi}\left[\begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{12} \\ \frac{1}{4} & -1 & \frac{2}{9} \\ -\frac{1}{4} & -\frac{1}{9} & -\frac{4}{3}\end{pmatrix}\begin{pmatrix} f^{(1)}(1,\underline{1};t) \\ f^{(1)}(0,\underline{8};t) \\ f^{(1)}(1,\underline{8};t)\end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1\end{pmatrix}\right] \qquad (2.3)$$

Before we solve these equations we have to discuss at which scale the coupling α_s should be taken. Naively one might think that $\alpha_s = \alpha_s(Q^2)$ is the appropriate choice, Q^2 being the only scale in the process as in deep inelastic scattering. But this is not quite true, since the extension of the $Q\bar{q}$ object provides another scale, namely the form factor or decay constant. Again this question cannot be answered rigorously without a real understanding of the bound state dynamics. We then propose to use a fixed or "frozen" effective coupling for two reasons :

i) annihilation requires the co-operation of both Q and $\bar{q}$. Therefore one is reminded of the occurrence of higher twist terms in deep inelastic scattering, which also lead to powerlike scale breaking effects ;

ii) there is not too much difference between powerlike and logarithmic scale breaking in the functions $f(i,\underline{j};t)$, at least as long as one compares charm with bottom decays. As discussed at the end of the Introduction we want to take a pragmatic attitude in the semi-quantitative analysis presented here. It will be obvious how a "running" coupling would modify the results.

We want to add a further remark : we include diagrams as shown in Fig. 2a while ignoring the one of Fig. 2b although that could also lead to weak annihilation (Fig. 3). We expect such contributions to be unimportant since only for soft gluon propagators can one hope to obtain a non-negligible result. In that case, however, the gluon travels a long distance before it fragments into $q\bar{q}$ and we consider it to be very unlikely that such a $\bar{q}$ or q has sufficient overlap with the Q quark to undergo weak annihilation which at these energies is still local.

Equation (2.3) can most easily be solved by choosing appropriate linear combinations :

$$\frac{d}{dt}\Sigma_1(t) = -\frac{\alpha_s}{2\pi}\frac{5}{4}\Sigma_1(t) \qquad (2.5a)$$

$$\frac{d}{dt}\Sigma_8(t) = -\frac{\alpha_s}{2\pi}\frac{10}{9}\Sigma_8(t) \tag{2.5b}$$

$$\frac{d}{dt}\Sigma_R(t) = -\frac{\alpha_s}{2\pi}\frac{35}{36}\Sigma_R(t) \tag{2.5c}$$

with

$$\Sigma_1(t) \equiv f^{(1)}(1,\underline{1};t) + f^{(1)}(1,\underline{8};t) - \frac{4}{5}$$

$$\Sigma_8(t) \equiv f^{(1)}(1,\underline{8};t) + f^{(1)}(0,\underline{8};t) - \frac{9}{10}$$

$$\Sigma_R(t) \equiv f^{(1)}(1,\underline{1};t) + \frac{8}{9}f^{(1)}(0,\underline{8};t) + \frac{7}{9}f^{(1)}(1,\underline{8};t) - \frac{4}{5}$$

Solving Eq. (2.5a-c) we find

$$\Sigma_1(t) = A\exp\left(-\frac{5\alpha_s}{8\pi}(t-t_o)\right) \tag{2.6a}$$

$$\Sigma_8(t) = B\exp\left(-\frac{5\alpha_s}{9\pi}(t-t_o)\right) \tag{2.6b}$$

$$\Sigma_R(t) = C\exp\left(-\frac{35\alpha_s}{72\pi}(t-t_o)\right) \tag{2.6c}$$

with three integration constants A, B and C. In principle we could use three different annihilation decay widths as inputs to fix these constants, analogous to what is done in inelastic scattering. Unfortunately, as we will see below, there are presumably not that many different hadronic weak decays which one could clearly compare with each other. Therefore in order not to jeopardize predictive power we suggest an intrinsic argument for fixing A, B and C : so far we have discussed the situation where $M_Q^2 \gg m_q^2$. Now we let M_Q decrease towards m_q assuming our ansatz still to make sense. Then we expect for $M_Q = m_q$, i.e., onia states that they are pure quark-antiquark states. Thus for $t_o = (\log(M_Q^2/m_q^2)) = 0$ we demand

$$f(1,\underline{1};0) \simeq f(0,\underline{8};0) \simeq f(1,\underline{8};0) \simeq 0 \qquad \text{for pseudoscalar}$$

and

$$f(0,\underline{1};0) \simeq f(0,\underline{8};0) \simeq f(1,\underline{8};0) \simeq 0 \qquad \text{for vector states}$$

We state three reasons for these boundary conditions :

i) if vector onia were not very pure $(Q\bar{Q})$ configurations, one would have no understanding of the OZI rule, i.e., why the width of ϕ, ψ, Υ, etc., is so small, since these decays should then be $\sim O(\alpha_s)$ effects (Fig. 4) ;

ii) furthermore in that case one would expect the decay widths of vector and pseudoscalar onia to be of roughly the same order of magnitude ; yet one knows experimentally that whereas $\Gamma(\psi/J) = 67$ keV one found for the η_c candidate $\Gamma(\text{"}\eta_c\text{"}) \sim 20\text{-}40$ MeV ;

iii) $(Q\bar{Q})$ systems possess only one scale, namely M_Q ; thus one cannot resolve their inner structure in their totally inclusive decays.

Therefore we put for pseudoscalar states

$$A_{PS} = -\frac{4}{5} \; ; \; B_{PS} = -\frac{9}{10} \; ; \; C_{PS} = -\frac{4}{5}$$

and obtain after some trivial algebra :

$$f_{PS}^{(1)}(1,\underline{1};M_Q^2) = -\frac{2}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{5\alpha_s}{8\pi}} + \frac{18}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{5\alpha_s}{9\pi}} - \frac{18}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{35\alpha_s}{72\pi}} + \frac{2}{25} \qquad (2.7a)$$

$$f_{PS}^{(1)}(1,\underline{8};M_Q^2) = -\frac{18}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{5\alpha_s}{8\pi}} - \frac{18}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{5\alpha_s}{9\pi}} + \frac{18}{25}\left(\frac{m_q^2}{M_Q^2}\right)^{\frac{35\alpha_s}{72\pi}} + \frac{18}{25} \qquad (2.7b)$$

For vector states, one follows the analogous procedure.

The picture which emerges from our treatment of the evolution equations for $(Q\bar{q}...)$ mesons is the following : for $M_Q = m_{\bar{q}}$, the meson is still described as a pure quark-antiquark system. For M_Q increasing with m_q fixed, i.e., when one "hops" from D to B and then to T mesons, etc., one finds the gluonic component to be more and more important.

3. - DISCUSSION OF DECAY CONSTANTS AND COLOUR FACTORS

What we have calculated so far is just the first moment of the various distribution functions. Since we are really interested in the annihilation contributions, we had in principle to fold in the annihilation amplitude before integrating over x. But it is presumably a reasonable approximation just to multiply the integrated distribution by the annihilation cross-section, since the $Q\bar{q}$ system will very likely carry most of the mass of the meson.

We need three ingredients to calculate annihilation contributions :

i) The $Q\bar{q}$ overlap wave function, which depends on the bound state dynamics. Therefore one does not know how to calculate it rigorously. Usually one calculates it in a non-relativistic $Q\bar{q}$ bound state model. Yet it is not clear whether such an ansatz can be applied when $m_{\bar{q}}$ is very light, say < 1 GeV ; indeed it is impossible to explain the large lifetime ratio $\tau(D^+)/\tau(D^0)$ as due to annihilation when one uses wave functions calculated in such a fashion [12),13)]. Furthermore we need the overlap for $Q\bar{q}$ in the presence of gluons, where one has to deal, in principle at least, with a three-body problem and where $Q\bar{q}$ is in a colour singlet or octet state. Therefore we will make two different choices which we consider typical :

$$f_M^2 \propto \frac{\mu^2 \text{MeV}}{M} \tag{3.1a}$$

$$f_M^2 \propto \mu^2 \tag{3.1b}$$

with μ = reduced mass of the meson. We will fix the constants of proportionality, assumed to be flavour-independent in D decays and extrapolate them to other decays.

ii) The second ingredient is the annihilation amplitude for $(Q\bar{q})$ in a $(1,\underline{1})$ and $(1,\underline{8})$ configuration computed in a quark parton model before strong radiative corrections are included. The real issue involved here is the occurrence of colour factors. If there were no colour, i.e., $N_c = 1$, one would obtain :

$$\Gamma_{ann.}^{N_c=1}(Spin=1) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3$$

where it does not matter whether the annihilation occurs in the s or t channel. When one includes colour it does matter and we obtain for colour singlet configurations

$$\Gamma_{ann.,s}(1,\underline{1}) = \frac{N_f}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \qquad (3.2a)$$

$$\Gamma_{ann.,t}(1,\underline{1}) = \frac{1}{N_c}\frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \qquad (3.2b)$$

with $N_f = N_c$ + number of weak lepton doublets. Equation (3.2a,b) just reflects a Fierz transformation on the product of quark currents, e.g. :

$$\bar{s}_L \gamma_\mu c_L \, \bar{u}_L \gamma_\mu d_L \xrightarrow{\text{Fierz}} \frac{1}{3} \bar{u}_L \gamma_\mu c_L \, \bar{s}_L \gamma_\mu d_L + 2 \, \bar{u}_L \gamma_\mu \frac{\lambda_i}{2} c_L \, \bar{s}_L \gamma_\mu \frac{\lambda_i}{2} d_L$$

Therefore we obtain for colour octet configurations in charmed meson decays :

$$\Gamma_{ann.,s}(1,\underline{8}) = 0 \qquad (3.3a)$$

$$\Gamma_{ann.,t}(1,\underline{8}) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \qquad (3.3b)$$

iii) In the above expressions, i.e. (3.1)-(3.3), colour was introduced as a quantum number derived from a global symmetry. Yet it is generally believed to stem from a gauge symmetry with the accompanying gauge bosons, the gluons. Their exchange further modifies the effective coupling, which usually goes under

the heading of "short-distance effects". According to the conventional wisdom the effects of strong interactions on weak non-leptonic decays can be included by using an effective coupling [14]

$$\mathcal{L}_{\text{eff.}} = \frac{G_F}{\sqrt{2}} \left[\frac{c_-}{2} L_- + \frac{c_+}{2} L_+ \right] \tag{3.4}$$

where, for simplicity, we have dropped all references to mixing angles. L_- [L_+] are products of quark currents which are odd [even] under Fierz transformations, e.g., for charm decays

$$L_{\mp} \equiv \bar{s}_L \gamma_\mu c_L \bar{u}_L \gamma_\mu d_L \mp \bar{u}_L \gamma_\mu c_L \bar{s}_L \gamma_\mu d_L$$

As soon as strong radiative corrections are included, i.e., $c_- > 1 > c_+$, the notion of a t channel or s channel annihilation becomes ambiguous. We will nevertheless still use it and call those decays t or s channel annihilation where this is true in the context of the quark model, i.e., for $c_- = c_+ = 1$. Thus s channel annihilation contributes to F^+ and the Cabibbo disfavoured D^+ decays, whereas t channel annihilation contributes to D^0 decays. With this notation we obtain :

$$\Gamma_{\text{ann.,s}}(1,\underline{1}) = \frac{N_f}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \left(\tfrac{2}{3} c_+ + \tfrac{1}{3} c_-\right)^2 \tag{3.5a}$$

$$\Gamma_{\text{ann.,t}}(1,\underline{1}) = \frac{N_c}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \left(\tfrac{2}{3} c_+ - \tfrac{1}{3} c_-\right)^2 \tag{3.5b}$$

$$\Gamma_{\text{ann.,s}}(1,\underline{8}) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \tfrac{1}{4} (c_+ - c_-)^2 \tag{3.5c}$$

$$\Gamma_{\text{ann.,t}}(1,\underline{8}) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \tfrac{1}{4} (c_+ + c_-)^2 \tag{3.5d}$$

The coefficients $c_\mp$ were calculated perturbatively as a short-distance effect and one obtained [14] :

$$c_{\pm} = \left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_W^2)}\right)^{\gamma_{\pm}} + O\left(\frac{1}{\log \frac{M_W^2}{\Lambda^2}}\right) \tag{3.6}$$

$$\gamma_- = -\gamma_+ = \frac{12}{33-2N_f} \tag{3.7}$$

Thus

$$\begin{aligned} c_+(m_c^2) &= 0.68 \\ c_-(m_c^2) &= 2.15 \end{aligned} \tag{3.8}$$

Since the original analysis based on the dominance of cascade decays and on the short-distance renormalizations, Eq. (3.8), led to predictions which were not borne out by experiment, it was suggested [3], as a minimal modification, to change the ratio c_-/c_+ drastically from the value three as derived from Eq. (3.6) and Eq. (3.7). We do not believe that the whole puzzle could be "explained" by such an (ad hoc) assumption [5]. Nevertheless it is quite possible that something like it occurs and contributes to the decay phenomena. One can envision it to happen in two ways, either as a genuine short-distance effect due to higher loops and/or non-leading contributions or it could be faked by soft-gluon exchanges as proposed by Deshpande et al. [4].

For our numerical analysis given below we have used two different values for the ratio c_-/c_+ - namely $c_-/c_+ = 1$ or 3 - to see which quantities are sensitive to this ratio and which are not.

4. - SEMI-QUANTITATIVE SCENARIO OF HADRONIC WEAK DECAYS

As was said above the bound state characteristics enter the problem of decay widths via the decay constant f_M. Since a potential model approach has so far been very successful in the ψ and Υ family [15] we feel confident to apply the same concepts to other non-relativistic states, like $(b\bar{c})$, $(t\bar{c})$ and

$(t\bar{b})$ mesons. There we will use for f_M the expression given in (3.1a) [13]. The situation is different when one quark is light - u, d or s quark : then a non-relativistic ansatz might not be a useful approximation. Nevertheless we still believe that the bound state character of such systems should be the same as those having the same reduced mass. Thus

$$f_{c\bar{s}} \simeq f_{b\bar{s}} \simeq f_{t\bar{s}} \tag{4.1a}$$

$$f_{c\bar{u}} \simeq f_{b\bar{d}} \simeq f_{t\bar{u}} \tag{4.1b}$$

Of course, it is tempting to compare, say, also F^+ and D^o decay widths, although this requires more knowledge on the decay constants.

If one uses reduced mass arguments, then

$$f_{F^+} \sim 1.5\, f_D$$

If, on the other hand, one argues that for such light quarks another scale determines the bound state, maybe $\Lambda \sim 0.5$ GeV, used in the definition of α_s, then one would expect

$$f_{F^+} \sim f_D$$

A similar problem arises in the evolution equations with the terms $\log(M^2/m_q^2)$. For $(b\bar{c})$, $(t\bar{c})$, $(t\bar{b})$ mesons one puts $m_q^2 = m_c^2$ or $m_q^2 = m_b^2$. For s, u, d quarks it is not so clear which mass scale to insert : constituent masses or even current masses or something of the order of Λ. A leading log ansatz as ours does not allow an answer to this question. But, for the same reason why we argued above that for mesons with the same reduced mass the decay constants should be the same, we assume that for them m_q should be inserted with the same value. Thus we have the u/d class of mesons - D^o, D^+, B_u^-, B_d^o, T_u, T_d - the s class - F^+, B_s^o, T_s - etc.

Of course, annihilation will contribute to the different decays with a strongly varying strength depending on mixing angles. For example D^+ decays receive contributions from annihilation only for the Cabibbo disfavoured modes, while in D^o decays both Cabibbo favoured and disfavoured modes are affected.

Therefore to make full use of the machinery developed so far, we have to consider not only global rates like lifetimes, but also inclusive Cabibbo (etc.) suppressed rates. Thus we will compare

$$\frac{\Gamma(D^0)}{\Gamma(D^+)}\,,\ \frac{\Gamma(D^0\to S=0)}{\Gamma(D^0\to S=-1)}\quad \text{vs.}\ \frac{\Gamma(D^+\to S=0)}{\Gamma(D^+\to S=-1)}\,,\ \frac{\Gamma(B^0)}{\Gamma(B^-)}\,,\ \ldots$$

For our discussion we will mainly use a "reduced" annihilation width, i.e., one where the obvious M^3 dependence is already taken out and which is normalized to unity for D^0 decay :

$$R(M) \equiv \frac{\Gamma_{ann}(M)}{\Gamma_{ann}(D^0)}\left(\frac{M_D}{M}\right)^3 \tag{4.2}$$

Thus we can write down

$$R(B^0) = f_B^2\left[3\left(\tfrac{2}{3}c_+ - \tfrac{1}{3}c_-\right)^2 f^{(1)}(1,\underline{1};M_B^2) + \tfrac{1}{4}(c_+ + c_-)^2 f^{(1)}(1,\underline{8};M_B^2)\right]\Big/ K_{D^0} \tag{4.3a}$$

$$R(T^0) = f_T^2\left[3\left(\tfrac{2}{3}c_+ - \tfrac{1}{3}c_-\right)^2 f^{(1)}(1,\underline{1};M_T^2) + \tfrac{1}{4}(c_+ + c_-)^2 f^{(1)}(1,\underline{8};M_T^2)\right]\Big/ K_{D^0} \tag{4.3b}$$

$$R(D^+\to S=0) = f_D^2\left[N_f\left(\tfrac{2}{3}c_+ + \tfrac{1}{3}c_-\right)^2 f^{(1)}(1,\underline{1};M_D^2) + \tfrac{1}{4}(c_+ - c_-)^2 f^{(1)}(1,\underline{8};M_D^2)\right]\left(\tan^2\theta_c\right)\Big/ K_{D^0} \tag{4.3c}$$

$$K_{D^0} \equiv f_D^2\left[3\left(\tfrac{2}{3}c_+ - \tfrac{1}{3}c_-\right) f^{(1)}(1,\underline{1};M_D^2) + \tfrac{1}{4}(c_+ + c_-)^2 f^{(1)}(1,\underline{8};M_D^2)\right] \tag{4.3d}$$

In Fig. 5, we have plotted $f^{(1)}(1,\underline{1};(M^2/m_q^2))$ and $f^{(1)}(1,\underline{8};(M^2/m_q^2))$ versus the heavy mass M for $m_q = 0.33$ GeV and $\alpha_s = 0.5$. We see that in this mass range the ratio octet over singlet - $f^{(1)}(1,\underline{8})/f^{(1)}(1,\underline{1})$ - is still much larger than the asymptotic value nine.

In Fig. 6 we have plotted the mass dependence of $R(M)$ for t channel annihilation, i.e., D^0, B^0, T^0, etc. decays, using the two different choices for f_M, namely $f_M \propto \mu^2/M$ (Fig. 6a) or $f_M \propto \mu^2$ (Fig. 6b) (μ is the reduced mass). One can make the obvious observation that with our choice of the strong

coupling α_s, i.e., $\alpha_s = 0.5$, the functions $f(1,\underline{1};M^2)$ and $f(1,8;M^2)$ rise more slowly than linearly in M. Therefore R(M) reflects mainly the mass dependence of f_M, which, as said above, is not well known at the moment. This ignorance will hopefully diminish in the future and then a quantity like R(M) will be more useful. Attributing $\tau(D^+)/\tau(D^\circ) \sim 5$ to annihilation effects one can obtain as a ball park estimate

$$\frac{\tau(B^-)}{\tau(B^\circ)} \sim 1.2 - 1.7$$

with $M_b \sim 4.5$ GeV.

Nevertheless even with our present knowledge we can derive interesting results from our evolution equations by comparing t and s channel annihilations at (almost) the same mass, like $\Gamma(D^\circ)$ and $\Gamma(D^+ \to S = 0)/tg^2\theta_c$ or, cum grano salis, $\Gamma(D^\circ)$ and $\Gamma(F^+)$. It has been claimed that $\Gamma_{ann}(F^+) \lesssim \Gamma_{ann}(D^\circ)$ for two reasons mainly [6] :

i) the wave function overlap or the decay constant f_F is supposedly larger than f_D, since $m_s > m_u$;

ii) the amplitude for s channel annihilation contains N_f as a factor, i.e., the number of effective weak doublets whereas the amplitude for the t channel process does not [Eqs (3.2a,b)].

We believe that these arguments are incomplete since two other points are not included :

iii) as we have argued above the probability to find $Q\bar{q}$ in a $J = 1$ state should decrease as m_q increases ; we expect this effect to compensate at least partially for $f_F > f_D$;

iv) since due to our statistical weighting of transition rates $(Q\bar{q})$ is much more frequent in a colour octet than in a colour singlet state, the t channel process (in the language of the parton model) $\Gamma_{ann}(D^\circ)$ is enhanced relative to $\Gamma_{ann}(F^+)$; for the same reason

$$\frac{\Gamma_{ann.}(D^+ \to S=0)}{\Gamma_{ann.}(D^\circ) \times tg^2\theta_c} < 1$$

To illustrate this point more clearly, we write down what one expects for asymptotic masses [(Eqs. (2.7a,b)] :

$$f_{PS}^{(1)}(1,\tfrac{1}{-};\infty) = \tfrac{1}{9} f_{PS}^{(1)}(1,\tfrac{8}{-};\infty) \tag{4.4}$$

Then

$$\left.\frac{\Gamma_{ann.,s}(Q\bar{q})}{\Gamma_{ann,t}(Q\bar{q})}\right|_{M\to\infty} = \frac{\frac{N_f}{10}\left(\frac{2}{3}c_+ + \frac{1}{3}c_-\right)^2 + \frac{9}{40}(c_+ - c_-)^2}{\frac{N_c}{10}\left(\frac{2}{3}c_+ - \frac{1}{3}c_-\right)^2 + \frac{9}{40}(c_+ + c_-)^2}$$

$$\simeq \begin{cases} 0.53 \;, & c_-/c_+ = 1 \\ 0.63 \;, & c_-/c_+ = 3 \\ 0.9 \;, & c_-/c_+ = 10 \end{cases} \tag{4.5}$$

i.e., the t channel process possesses a larger transition rate than the s channel reaction if one takes colour singlet and octet $Q\bar{q}$ configurations into account.

For sub-asymptotic masses the ratio of Eq. (4.5) is even smaller as can be read off from Fig. 7, where we have plotted $(\Gamma_{ann,s}(M)/\Gamma_{ann,t}(M))$ for two choices of c_-/c_+, namely $(c_-/c_+) = 1$ and 3. We want to make three observations :

i) $(\Gamma_{ann,s}(M)/\Gamma_{ann,t}(M))$ is much smaller than unity ;

ii) it depends significantly on c_-/c_+, and

iii) it rises with an increasing mass.

As discussed above,

$$\left.\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}\right|_{M=M_D} = \frac{\Gamma_{ann}(D^+ \to S=0)}{(tg^2\theta_c)\,\Gamma_{ann}(D^0)} \tag{4.6a}$$

$$\left.\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}\right|_{M\sim M_D} \lesssim \frac{\Gamma_{ann}(F^+)}{\Gamma_{ann}(D^0)} \tag{4.6b}$$

where the ">" sign in Eq. (4.6b) allows for $f_F > f_D$ and $M_F > M_D$. Since for the "canonical" value $c_-/c_+ = 3$ we read off

$$\left.\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}\right|_{M=M_D} \sim \frac{1}{3} \tag{4.7}$$

we conclude that Cabibbo disfavoured D^+ decays are enhanced and that the lifetime of the F^+ is shorter than that of the D^+, but not by as much as one expects naively [8]:

$$BR(D^+ \to S=0) \sim (tg^2\theta_c)\,\frac{\tau(D^+)}{\tau(D^0)}\,\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}$$

$$\sim 0.09 \div 0.18 \tag{4.8a}$$

$$\frac{\tau(F^+)}{\tau(D^0)} \lesssim 3 \tag{4.8b}$$

where we have used $\tau(D^+)/\tau(D^0) \sim 5\text{-}10$. A similar test can be performed by comparing $\Gamma_{ann}(B^0)$ with B^- decays into final states without charm :

$$\left.\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}\right|_{M=M_B} = \frac{\Gamma_{ann}(B^- \to C=0)}{\Gamma_{ann}(B^0)\times(\text{mixing angles})^2} \sim \frac{1}{3} \tag{4.9}$$

Mesons where both constituent quarks are heavy, e.g., B_c, T_c, etc., can be treated also in a different fashion. It seems to be a reasonable ansatz to treat them as non-relativistic bound states, for which one can more reliably calculate the decay constant. B_c will probably decay into $B_s + \pi$ most of the time due to the presumably small $b \to c$ mixing angle. For T_c, T_b, etc. decays, one relies on a non-relativistic potential model [15] to obtain

$$f_M^2 \sim \frac{900\ \text{MeV}\ \mu^2}{M}$$

Thus, apart from T_b^* decays, one expects very small annihilation contributions [16]. Alternatively one can apply the non-relativistic analysis of Bander et al. to such a system to obtain

$$\frac{\Gamma(T^+)}{\Gamma(T_c)} \sim 1 + \left(\frac{m_{T_c}}{m_t}\right)^5\left(\frac{f_{T_c}}{m_c}\right)^2 \frac{16\pi\alpha_s}{27}\,\frac{(c_+ + c_-)^2}{4(2+2c_+^2+c_-^2)} \tag{4.10}$$

For $m_t \sim 20$ GeV one finds $f_{T_c} \sim 240$ MeV and thus for $m_c \sim 1.5$ GeV

$$\frac{\Gamma(T^+)}{\Gamma(T_c)} \simeq 1 \tag{4.11}$$

Therefore we conclude that although the same analysis, based on evolution equations as discussed above, can be applied to non-relativistic systems, we expect all annihilation effects to be very small for non-relativistic pseudoscalar mesons. At the same time we also expect non-relativistic vector mesons T_c^*, etc., to decay as suggested in a parton model approach [13].

5. - CONCLUSIONS

We have stated in the Introduction that our aim is not to propose a precision test of QCD. We wanted to show that :

i) a semi-quantitative description can be formulated despite our still considerable ignorance regarding bound state dynamics, meson form factors, the impact of short-distance phenomena and their separation from long-distance phenomena, etc., and

ii) that this description can be refined step by step.

We want to summarize the important lessons which can come out from such an analysis : if the salient features of the proposed description were confirmed by experiment, one would learn :

i) gluons play a very important rôle in $(Q\bar{q})$, etc., hadrons, unlike the situation for onia states $(Q\bar{Q})$;

ii) although the assumed non-Abelian gauge nature of gluons (and QCD as a whole) will very likely not become apparent in studies as discussed here, their spin and colour quantum numbers affect the analysis in a very significant way as we have discussed in the comparison between s and t channel annihilations.

Therefore the issues involved are well worth the experimental effort which is required and on which we want to make a few comments. In this paper we have discussed the annihilation decay width Γ_{ann}, but it is not obvious how to extract it from the observed width Γ_{obs}, since it is quite likely that Γ_{obs}

receives further contributions not only from cascade decays, but also from strong final state interactions, say resonances. This problem could presumably be avoided by comparing decays of bottom and top mesons. Yet since the annihilation width increases presumably only like $\sim M^3$ (or even less), while cascade decays rise like $\sim M^5$ we expect the latter to be dominant in top decays, assuming $m(\text{top}) > 17.5$ GeV. Therefore it will be very difficult to pin down annihilation effects in top decays, with the exception of T_b^* decays, as has been discussed in the literature [16].

In charm decays one can disentangle the contributions from different mechanisms by studying exclusive channels and assuming them to reflect the same pattern as inclusive decays. Since D^+ decays into an exotic final state it is affected neither by annihilation nor by resonances. Therefore one can study the ratio c_-/c_+ from the effective weak Lagrangian by comparing $D^+ \to \bar{K}^0\rho^+$ and $D^+ \to (\bar{K}^0)^*\pi^+$. The annihilation effects on the other hand can be studied most clearly in a comparison of $D^0 \to \bar{K}^0\phi$ with $D^0 \to \bar{K}^0\rho^0$ and $D^0 \to (\bar{K}^0)^*\pi^0$ [4),17)], of $D^0 \to \bar{K}^0\eta$ with $D^0 \to \bar{K}^0\pi^0$ [17)] and of $F^+ \to \rho^+\pi^0$, $\omega\pi^+$ with $F^+ \to \eta\pi^+$ [4),17),18)].

It was noted before that annihilation enhances the $\Delta I = \frac{1}{2}$ mode in K decays [19)]; it was even claimed that it could explain the $\Delta I = \frac{1}{2}$ rule altogether [6)]. We are not convinced by this latter claim. It involves an extrapolation from charm decays down to K and hyperon decays which depend crucially on the parameters used : one has to use a current quark mass $m_s \lesssim 100$ MeV instead of the constituent quark mass $m_s \sim 500$ MeV, which seems more reasonable to us. We share the belief of Finjord [9)] that other mechanisms like Penguin diagrams are still needed to explain the $\Delta I = \frac{1}{2}$ rule.

ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Figure 1 $(Q\bar{q})$ seen by a W boson at two different scales.

Figure 2a Diagrams included in the evolution equations.

Figure 2b Diagrams not included in the evolution equations.

Figure 3 Annihilation diagram for contributions as shown in Fig. 2b.

Figure 4 T decays in the presence of spectator gluons

Figure 5 Singlet and octet moments $f^{(1)}(1,\underline{1};(M^2/m_q^2))$ and $f^{(1)}(1,\underline{8};(M^2/m_q^2))$ as a function of the heavy mass M for m_q = 330 MeV and α_s = 0.5.

Figure 6 Normalized t channel annihilation R(M) as function of the heavy mass M with $f_v^2 \propto (\mu^2/M)$ (Fig. 6a) and $f_v^2 \propto \mu^2$ (Fig. 6b).

Figure 7 $\Gamma_{ann,s}(M)/\Gamma_{ann,t}(M)$ as a function of M for $c_-/c_+ = 3$ (solid curve) and $c_-/c_+ = 1$ (broken curve).

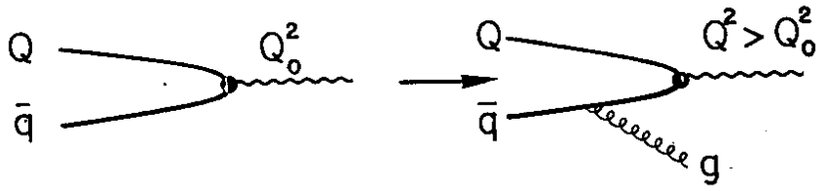

Fig. 1

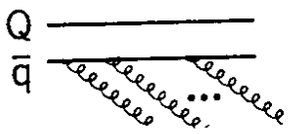

Fig. 2a

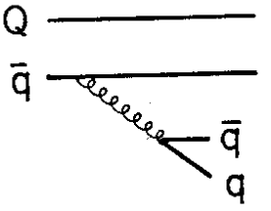

Fig.2b

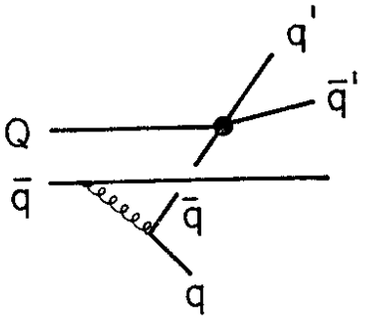

Fig. 3

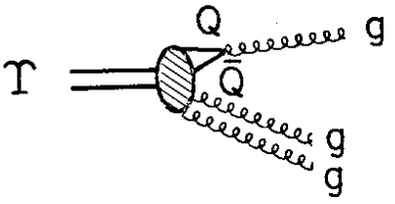

Fig. 4

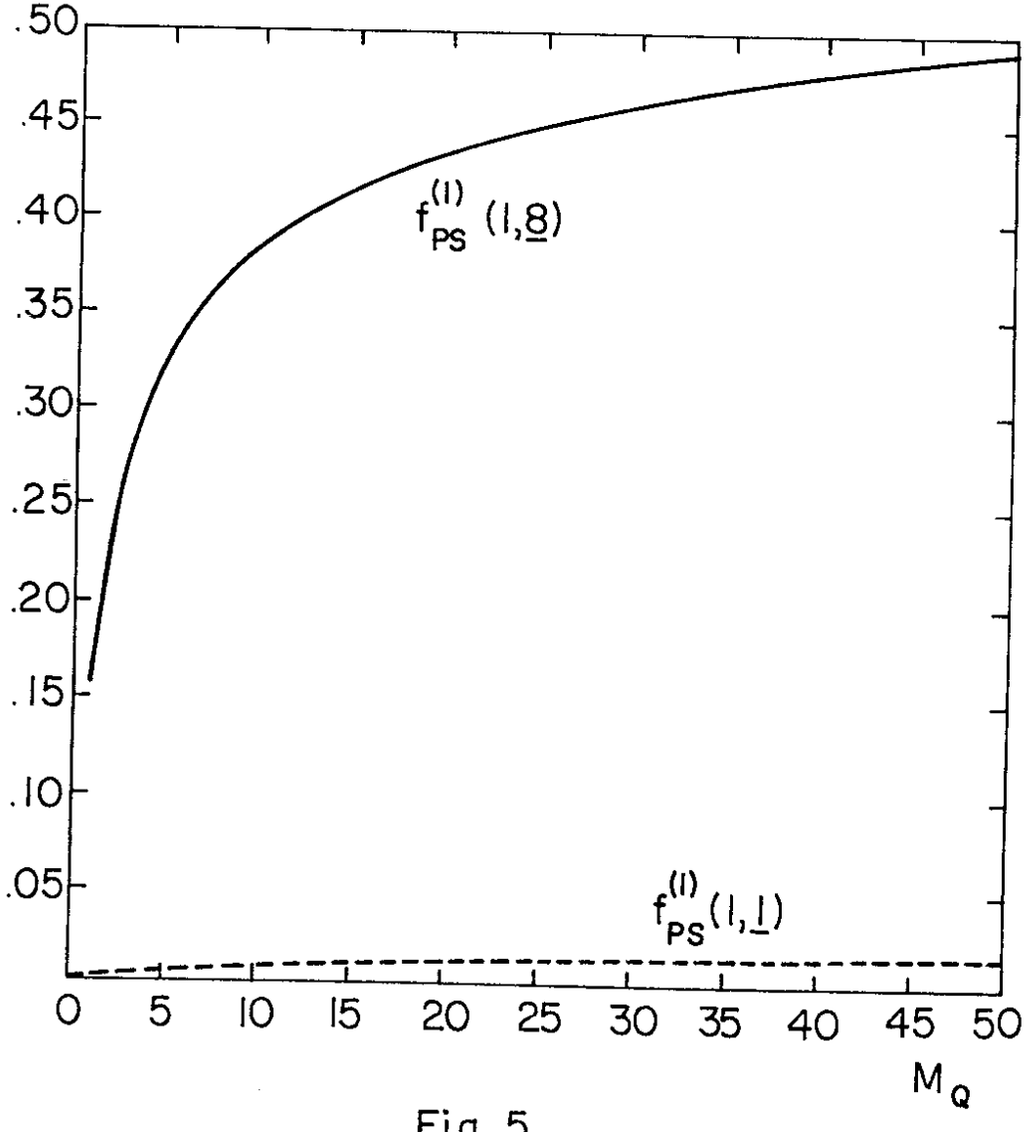
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
$f^{(1)}_{PS}(1,\underline{8})$
$f^{(1)}_{PS}(1,\underline{1})$
0
5
10
15
20
25
30
35
40
45
50
M_Q

Fig. 5

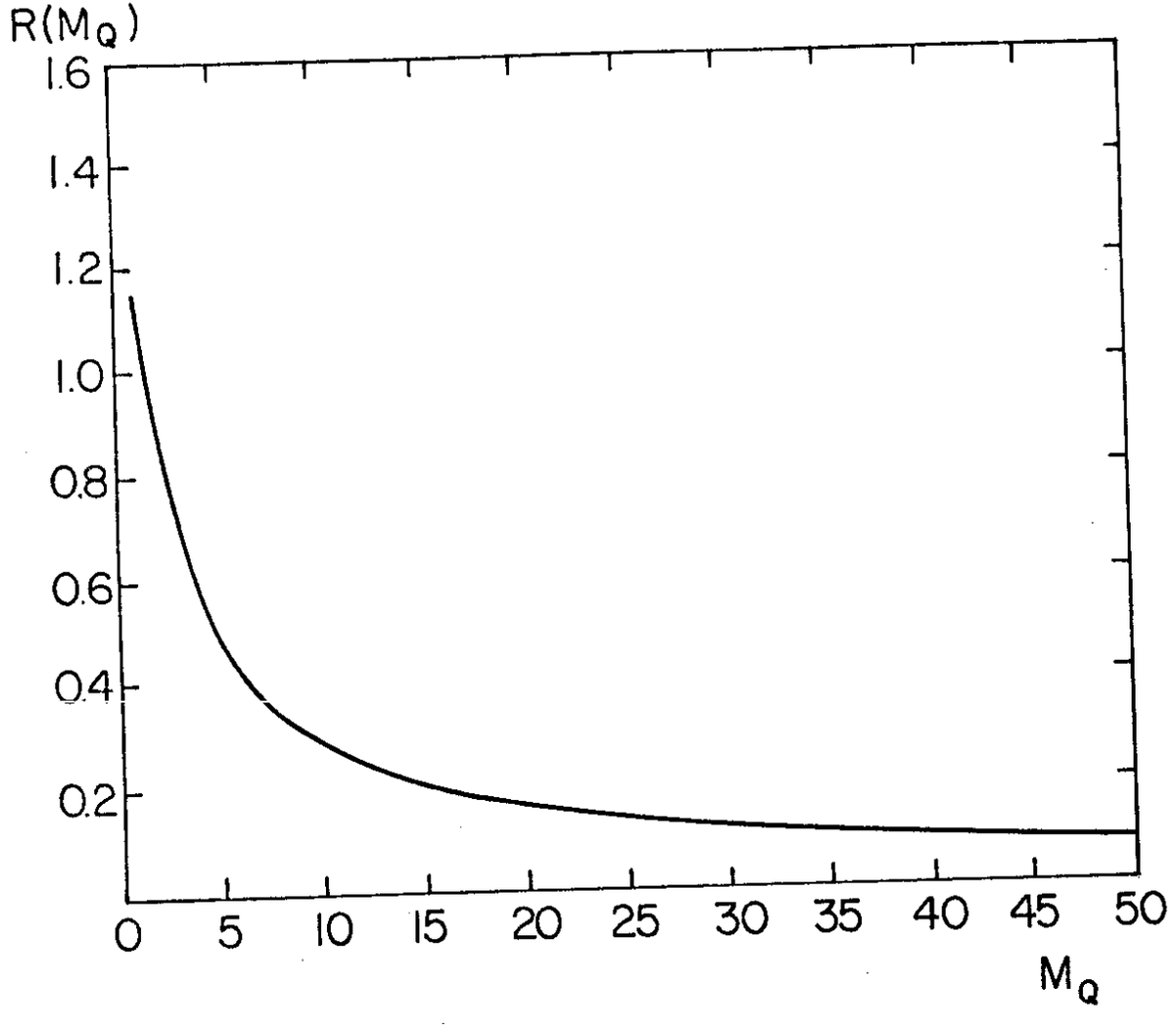

Fig. 6 a

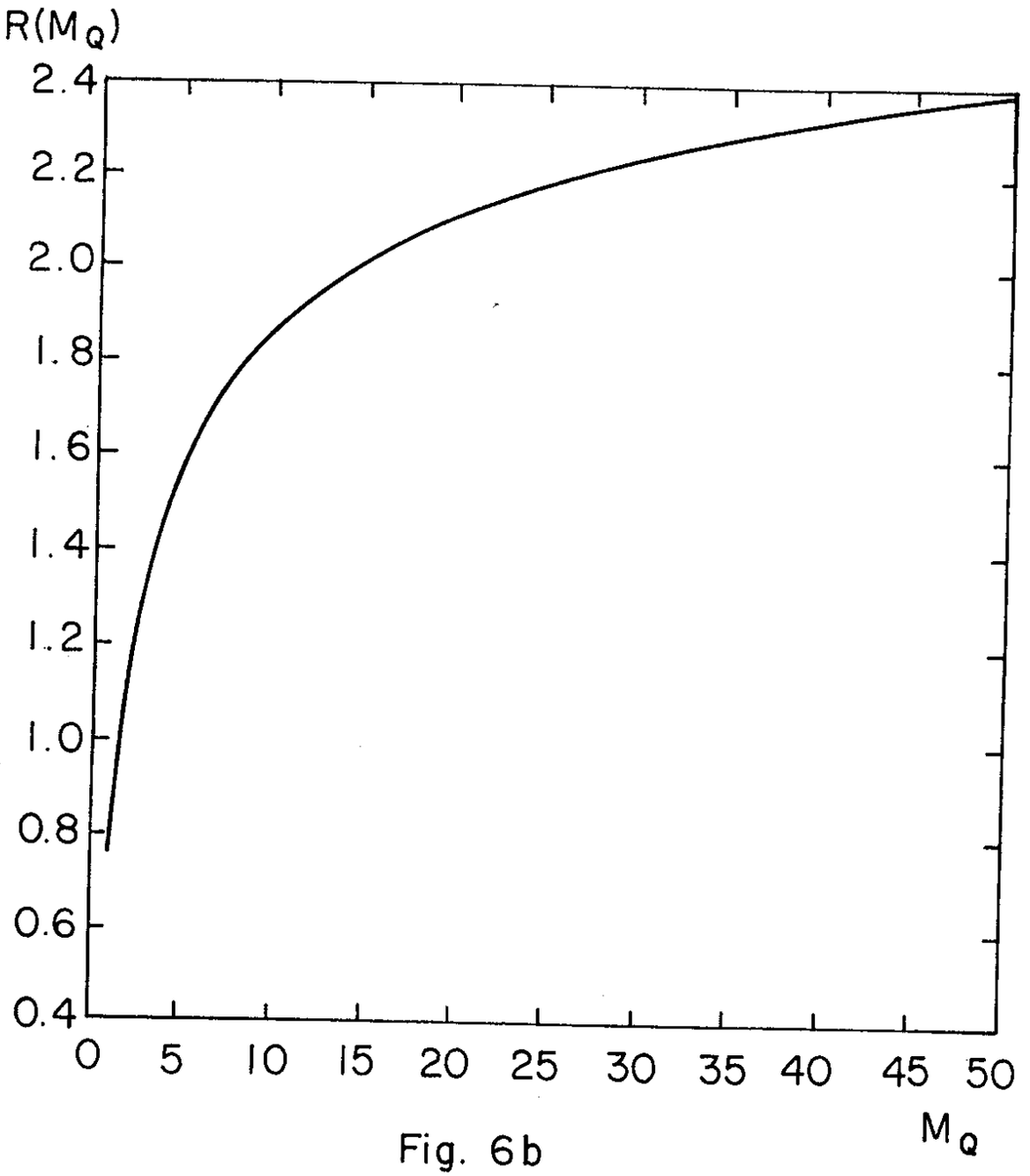

Fig. 6b

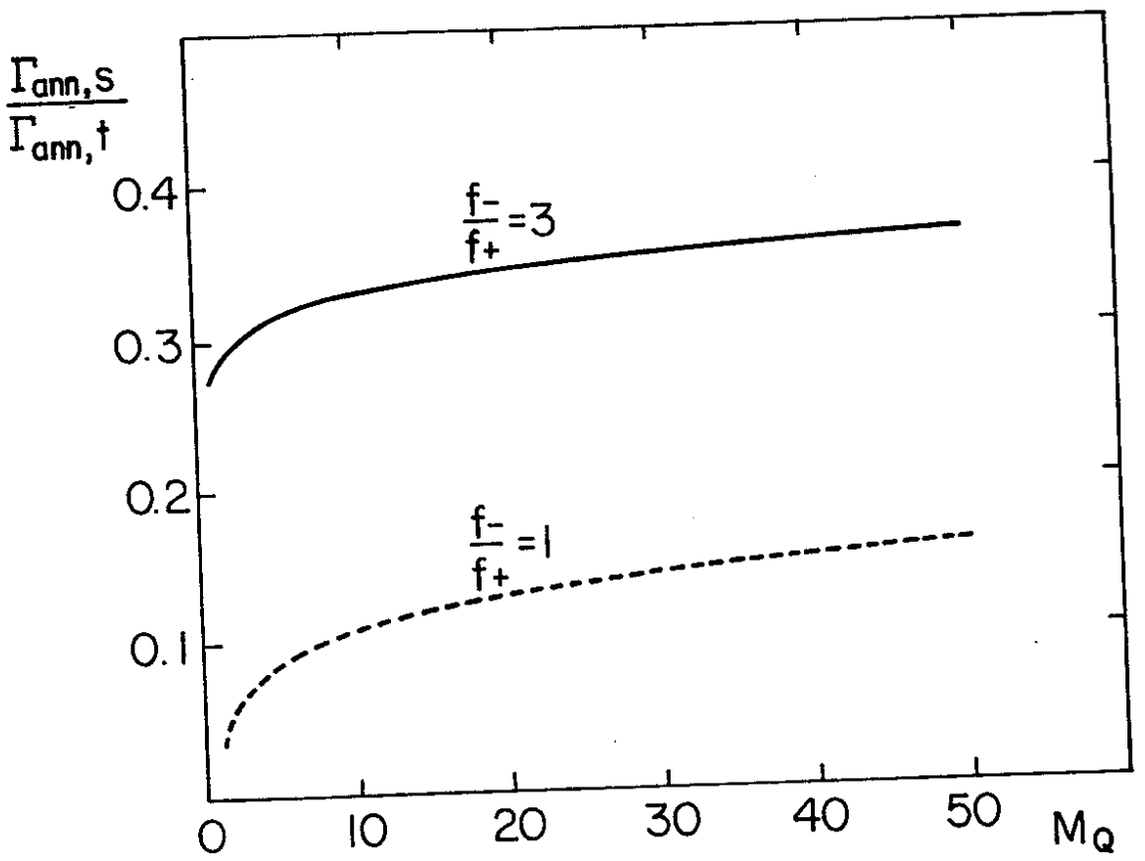
$\frac{\Gamma_{ann,s}}{\Gamma_{ann,t}}$
0.4
0.3
0.2
0.1
$\frac{f_-}{f_+}=3$
$\frac{f_-}{f_+}=1$
0
10
20
30
40
50
M_Q

Fig. 7