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Abstract

Implications of the recent CLEO observation [1] of the rare decay mode $B \to K^* + \gamma$ having a combined branching ratio $BR(B \to K^* + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ and an improved upper limit on the inclusive branching ratio $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (95% $C.L.$) [2] are discussed in the context of the Standard Model (SM). Using the unitarity of the CKM-matrix and taking into account QCD radiative corrections in the decay rate and the inclusive photon energy spectrum we obtain an improved upper limit on the inclusive branching ratio $BR(B \to X_s + \gamma) < 4.8 \times 10^{-4}$ (95% $C.L.$). This can be used to constrain possible non-SM contributions to the inclusive branching ratio, giving $BR(B \to X_s + \gamma)$ (non-SM) $< 3.0 \times 10^{-4}$ for $m_t \geq 108\ GeV$. Within the SM, we show that the resulting experimental upper limit can be interpreted as a corresponding limit on the CKM-matrix element ratio yielding $|V_{ts}|/|V_{cb}| < 1.67$, with the top quark mass assumed to weigh less than 200 GeV. We calculate the relative exclusive to inclusive branching ratio $R(K^*/X_s) \equiv \Gamma(B \to K^* + \gamma)/\Gamma(B \to X_s + \gamma)$, based on the inclusive hadronic invariant mass distribution in $B \to X_s + \gamma$. Estimating the K^*-contribution from this distribution in the threshold region $(m_K + m_\pi) \leq m_{X_s} \leq 0.97\ GeV$ and using experimental measurements from the semileptonic D-decays $D \to K + \pi + \ell\nu_\ell$ in the same mass interval, we obtain $R(K^*/X_s) = 0.13 \pm 0.03$. This enables us to put a lower bound on the ratio $|V_{ts}|/|V_{cb}|$ from the 95% C.L. lower limit on the branching ratio $BR(B \to K^* + \gamma) > 1.6 \times 10^{-5}$ [1, 2]. Combining the exclusive and inclusive decay rates, we determine $0.50 \leq |V_{ts}|/|V_{cb}| \leq 1.67$ (at 95% $C.L.$).

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1 Introduction

In a series of papers devoted to the QCD-improved studies of rare B-decays in the Standard Model (SM) [3]-[6], we have argued that measurements of the inclusive photon energy spectra in B-decays and exclusive radiative B-decays provide new means to determine the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [7] and test the consistency of SM involving the top quark and its CKM couplings. In particular, in ref. [5] we had elaborated this point on the examples of the inclusive decay $B \to X_s + \gamma$ and the exclusive decay $B \to K^* + \gamma$, whose measurements would determine the CKM matrix element $|V_{ts}|$, using the CKM-unitarity. Likewise, measurements of the CKM-suppressed decays $B \to X_d + \gamma$ and the exclusive decays $B \to \rho + \gamma$, $B \to \omega + \gamma$, will measure the CKM-matrix element $|V_{td}|$ [4].

Recently, the CLEO collaboration has announced the measurements of the decay mode $B \to K^* + \gamma$ having a combined branching ratio $BR(B \to K^* + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$ [1]. Moreover, an upper limit based on the CLEO data on the inclusive branching ratio $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (95% $C.L.$) has also been reported [2]. The CLEO measurement for the exclusive decay mode $B \to K^* + \gamma$ is in comfortable agreement with our estimates of $(3.1 - 7.8) \times 10^{-5}$ for the same quantity, which was based on the QCD-improved SM-effective Hamiltonian and K^*-dominance of the hadronic invariant mass recoiling against the photon in the decay $B \to X_s + \gamma$ in the mass range $(m_K + m_\pi) \leq m_{X_s} \leq 1.0\ GeV$ [3, 6]. We further refine these estimates in this paper, taking into account additional experimental constraints from the semileptonic $D_{\ell 4}$ decays $D \to (K + \pi) + \ell\, \nu_\ell$ and argue that heavy quark symmetry relates the rare decay $B \to (K + \pi) + \gamma$ and $D_{\ell 4}$ decays. We also note that the present experimental upper limit for the inclusive radiative rare decay reported in [2] is marginally above the SM-estimates, $BR(B \to X_s + \gamma) = (1.8 - 4.2) \times 10^{-4}$, presented earlier and refined in this report. While more precise determination of the SM parameters will be possible only after the measurement of the inclusive rare decays $B \to X_s + \gamma$, we argue that already the present CLEO measurements allow a meaningful determination of the matrix element ratio $|V_{ts}|/|V_{cb}|$, using the CKM-unitarity.

In this paper we present an analysis of the CLEO measurements in the context of the Standard Model. We give QCD-improved SM-expressions for the branching ratios for the inclusive and exclusive decays $B \to X_s + \gamma$ and $B \to K^* + \gamma$, respectively, making the dependence of the decay rates on the various input parameters explicit. In particular, we discuss the photon energy spectrum in the decays $B \to X_s + \gamma$ emphasizing the effect of the QCD radiative corrections, since they have a direct bearing on the extracted value of the upper bound and eventually on the comparison of experiment and theory in the inclusive decays $B \to X_s + \gamma$. Our analysis reported here yields an improved upper limit on the inclusive branching ratio, $BR(B \to X_s + \gamma) < 4.8 \times 10^{-4}$ (95% $C.L.$). This can be used to constrain possible non-SM contributions to the inclusive branching ratio, yielding $BR(B \to X_s + \gamma)$ (non-SM) $< 3.0 \times 10^{-4}$ for $m_t \geq 108\ GeV$, the present lower bound on m_t from direct searches [8].

The branching ratio $BR(B \to K^* + \gamma)$ provides a (model dependent) normalization of the electromagnetic penguins in B-decays. Since the FCNC signal has so far been established only in the exclusive decay mode $B \to K^* + \gamma$, we attempt to estimate this branching ratio in a phenomenologically well founded framework, getting $R(K^*/X_s) \equiv \Gamma(B \to K^* + \gamma)/\Gamma(B \to X_s + \gamma) = (13 \pm 3)\%$. This allows us to put a lower bound on $|V_{ts}|/|V_{cb}|$ from the 95% C.L. lower limit on the branching ratio $BR(B \to K^* + \gamma) > 1.6 \times 10^{-5}$ [1, 2]. Combining the exclusive and inclusive decay rates, we determine $0.50 \leq |V_{ts}|/|V_{cb}| \leq 1.67$ (at 95% $C.L.$), giving a quantitative content to the compatibility of the CLEO measurements with the Standard Model.

2 Estimates of $BR(B \to X_s + \gamma)$ in the Standard Model

We recall here that in the lowest order (i.e., in 1-loop approximation) the SM-based branching ratios for both the inclusive and exclusive decays $B \to X_s + \gamma$ and $B \to K^* + \gamma$ depend crucially on the top quark contribution in the electromagnetic-penguin dominated amplitudes. The matrix element for the process $b \to s + \gamma$ in the lowest order can be written as:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \sum_i V_{ib} V_{is}^* F_2(x_i)\, q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b \quad (1)$$

where G_F and e are, respectively, the Fermi and electromagnetic coupling constant, $L = (1 - \gamma_5)/2$, $R = (1 + \gamma_5)/2$, $x_i = m_i^2/m_W^2$, q_μ and ϵ_μ are, respectively, the photon four-momentum and polarization vector, the sum is over the quarks, u, c, and t, and V_{ij} are the CKM matrix elements. The Inami-Lim function $F_2(x_i)$ derived from the (1-loop) penguin diagrams is given by [9]:

$$F_2(x) = \frac{x}{24(x-1)^4} \left[6x(3x-2)\log x - (x-1)(8x^2 + 5x - 7)\right] \quad (2)$$

where in writing the expression for $F_2(x_i)$ above we have left out constant terms, since on using the unitarity constraint they sum to zero. It is instructive to write the unitarity constraint for the decays $B \to X_s + \gamma$ in full:

$$V_{tb} V_{ts}^* + V_{cb} V_{cs}^* + V_{ub} V_{us}^* = 0 \quad (3)$$

Now, since the last term is completely negligible compared to the others (by direct experimental measurements), we set it to zero. In the discussion of the decays $B \to X_s + \gamma$ that follows, we shall interpret the CKM-constraints in the form

$$V_{cb} V_{cs}^* = -V_{tb} V_{ts}^*, \ \textit{i.e.,}\ \lambda_c = -\lambda_t \quad (4)$$

Since, we set $\lambda_u = 0$ consistently in these calculations, one can express the one-loop penguin amplitude as follows:

$$\mathcal{M}(b \to s + \gamma) = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} \lambda_t \left(F_2(x_t) - F_2(x_c)\right) q^\mu \epsilon^\nu \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b \quad (5)$$

The GIM mechanism [11] at the one-loop level is manifest, as well as, the CKM-matrix element dependence, which factorizes in the approximation $\lambda_u = 0$.

A measurement of the radiative decays $B \to X_s + \gamma$ would be readily interpreted as a *direct measurement* of the CKM- matrix element product $\lambda_t/|V_{cb}|$ or equivalently $|V_{ts}|/|V_{cb}|$, provided m_t is known or sufficiently well bounded. It is well known through the leading order QCD corrections [10] that the operator-mixing brings in additional contributions which stem from the charm quark and they are proportional to the CKM factor $\lambda_c \equiv V_{cs} V_{cb}^*$. Thus, QCD effects alter the CKM-matrix element dependence of the decay rates for both $B \to X_s + \gamma$ and $B \to X_d + \gamma$ in a non-trivial fashion. However, with the help of the unitarity condition (4), the CKM matrix dependence in the effective Hamiltonian incorporating the QCD corrections for the decays $B \to X_s + \gamma$ factorizes, and one can write this Hamiltonian as [2]:

$$H_{eff}(b \to s) = -\frac{4 G_F}{\sqrt{2}} \lambda_t \sum_{j=1}^{8} C_j(\mu)\, O_j(\mu) \quad (6)$$

[2]Note that in addition to the penguins with the u-quark intermediate state there are also non-factorizing contributions due to the operators $(\bar{u}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{s}_{L\beta} \gamma_\mu u_{L\beta})$, which like the u-quark contribution to the 1-loop electromagnetic penguins are proportional to the CKM-factor $\lambda_u \equiv V_{us} V_{ub}^*$, and hence are consistently set to zero.

As discussed in [10] for the partonic decays $b \to s + \gamma$ and in [3, 4] for the decays $b \to s + \gamma + g$, the dominant contributions in the radiative decays $B \to X_s + \gamma$ arise from the operators O_1, O_2, O_7 and O_8, whereas the operators $O_3, ..., O_6$ get coefficients through operator mixing only, which numerically are negligible. The dominant operators read as follows:

$$\begin{aligned} O_1 &= (\bar{c}_{L\beta} \gamma^\mu b_{L\alpha})(\bar{s}_{L\alpha} \gamma_\mu c_{L\beta}) \\ O_2 &= (\bar{c}_{L\alpha} \gamma^\mu b_{L\alpha})(\bar{s}_{L\beta} \gamma_\mu c_{L\beta}) \\ O_7 &= (e/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) b_\alpha F_{\mu\nu} \\ O_8 &= (g_s/16\pi^2)\, \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) T^A_{\alpha\beta} b_\beta G^A_{\mu\nu} \end{aligned} \quad (7)$$

Here g_s denotes the QCD coupling constant. The effects of QCD corrections, contained in the Wilson coefficients $C_j(\mu)$, have been evaluated to leading logarithmic accuracy and summed using renormalization group methods [10]. At the scale $\mu = O(m_b) \ll m_W$, which we shall assume to vary in the range $m_b/2 \le \mu \le 2m_b$, these coefficients read as follows:

$$\begin{aligned} C_1(\mu) &= \frac{1}{2}\left[\eta^{-6/23} - \eta^{12/23}\right] C_2(m_W) \\ C_2(\mu) &= \frac{1}{2}\left[\eta^{-6/23} + \eta^{12/23}\right] C_2(m_W) \\ C_7(\mu) &= \eta^{-16/23} \left\{C_7(m_W) - \frac{58}{135}\left[\eta^{10/23} - 1\right] C_2(m_W) - \frac{29}{189}\left[\eta^{28/23} - 1\right] C_2(m_W)\right\} \\ C_8(\mu) &= \eta^{-14/23} \left\{C_8(m_W) - \frac{11}{144}\left[\eta^{8/23} - 1\right] C_2(m_W) + \frac{35}{234}\left[\eta^{26/23} - 1\right] C_2(m_W)\right\} \end{aligned} \quad (8)$$

with $\eta = \frac{\alpha_S(\mu)}{\alpha_S(m_W)}$ being the ratio of the QCD coupling constants. At the scale $\mu = m_W$, where the matching conditions from the lowest order results are imposed [9], we have

$$\begin{aligned} C_j(m_W) &= 0 \ , \quad j = 1, 3, 4, 5, 6 \\ C_2(m_W) &= 1 \\ C_7(m_W) &= F_2(x_t) - F_2(x_c) \\ C_8(m_W) &= C_8(m_W, x_t) - C_8(m_W, x_c) \\ C_8(m_W, x_i) &= -\frac{x_i}{8(x_i - 1)^4}\left[6x_i \log x_i + (x_i - 1)(x_i^2 - 5x_i - 2)\right] \end{aligned} \quad (9)$$

For the two-body decays $b \to s + \gamma$, only the magnetic moment operator O_7 contributes and hence the QCD corrected rate can be represented by formulæ similar to the ones given above for the lowest order, with the function $(F_2(x_t) - F_2(x_c))$ replaced by the QCD corrected function $C_7(\mu)$. Taking into account only the two-body decays $b \to s + \gamma$, QCD-improvements boil down to a scaling of the Wilson coefficient of the magnetic moment operator, $C_7(m_W) \to C_7(\mu)$.

A number of remarks are in order at this stage:

- The net result of the QCD corrections for the two-body radiative decay width is very significant. For $\mu = m_b = 5.0\ GeV$, they increase the rate by ~ 4.5 for $m_t = 100\ GeV$ to ~ 2.5 for $m_t = 200\ GeV$, assuming $\Lambda_{QCD} = 200\ MeV$.
- The term proportional to $\lambda_t((F_2(x_t) - F_2(x_c))$ in the one-loop penguin amplitude is, however, numerically less important after including the QCD effects since $\eta > 1$.

- The electromagnetic penguins in the decay $b \to s+\gamma$, after including the QCD corrections, are dominated by the intermediate charm quark contribution due to the mixings of the operators O_2 and O_7. A corollary of this and the previous observation is that the QCD corrected branching ratio $BR(B \to X_s + \gamma)$ is less sensitive to m_t.
- We have eliminated λ_c from the expression for $BR(B \to X_s + \gamma)$ using the CKM-unitarity and neglecting λ_u. Equivalently, one could have eliminated λ_t and expressed the result in terms of λ_c. This provides another means to determine the matrix element $|V_{cs}|$ using the electromagnetic penguins in B-decays.

Concerning the last point we remark that the matrix element $|V_{cs}|$, determined from the decay $D \to K\ell\nu_\ell$, is quoted in the Particle Data Group compilation as $|V_{cs}| = 1.0 \pm 0.2$ [12]. The present theoretical uncertainty (in particular the scale dependence discussed below) and the experimental errors on the decays $BR(B \to K^* + \gamma)$ are expected to give a value for $|V_{cs}|$ which will not be competitive with the direct decays of the charmed hadron. However, the electromagnetic B-penguin decays offer a complementary approach for the determination of the CKM matrix elements, as the experiment and related theory improve.

In the estimates for the $BR(B \to X_s + \gamma)$ presented below, we have included the contribution of the QCD bremsstrahlung process $b \to s + \gamma + g$ and the virtual corrections to $b \to s + \gamma$, both calculated in $O(\alpha\alpha_s)$. We shall present a two-parameter fit of the penguin-based data in terms of the parameters (λ_t, m_t), fixing $m_c = 1.68\ GeV$ (though the dependence of the rates on m_c is rather modest as shown by us in [3, 4]) and varying the QCD scale parameter, μ. We have chosen to express the branching ratio $BR(B \to X_s + \gamma)$ in terms of the (well measured) inclusive semileptonic branching ratio $BR(B \to X\ell\nu_\ell)$, which is calculable on firmer theoretical grounds [13]-[15]. We remark that both the semileptonic and radiative rare B-decay rates and the lepton and photon energy spectra have been calculated theoretically in the leading order in α_s. The inclusive branching ratio for $B \to X_s + \gamma$ can be expressed as:

$$BR(B \to X_s + \gamma) = 6\frac{\alpha}{\pi}\frac{|\lambda_t|^2}{|V_{cb}|^2}\frac{|C_7(x_t,\mu)|^2 K(x_t,\mu)}{g(m_c/m_b)(1 - 2/3\frac{\alpha_s}{\pi}f(m_c/m_b))} \times (0.11) \qquad (10)$$

where $C_7(x_t,\mu) \equiv C_7(\mu)$, defined in H_{eff} earlier. The phase space function $g(r)$ for $\Gamma(b \to c\ell\nu_\ell)$ is defined as:

$$g(r) = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln(r) \qquad (11)$$

and the function $f(m_c/m_b)$ can be seen, for example, in ref. [13]. We recall here that $f(r)$ is a slowly varying function of r, and for the quark mass ratio relevant for the decay $b \to c + \ell\ \nu_\ell$, namely $r = 0.35 \pm 0.05$, it has the value $f(r) = 2.37 \mp 0.13$. [3] In the above expression for $BR(B \to X_s + \gamma)$, we have neglected the contribution from the decays $b \to u + \ell\ \nu_\ell$ in the denominator since it is numerically inessential (more so due to the recent trend of data giving $|V_{ub}|/|V_{cb}| = 0.06 - 0.08$ [16]). The factor (0.11) above represents the measured semileptonic branching ratio $BR(B \to X\ell\nu_\ell)$.

The inclusive decay width for $B \to X_s + \gamma$ is dominantly contributed by the magnetic moment term $C_7(x_t,\mu)O_7(\mu)$, hence the rationale for factoring out this coefficient in the expression for $BR(B \to X_s + \gamma)$ in Eq. (10) above. Including $O(\alpha_s)$ corrections brings to fore other operators with their specific Wilson coefficients. The effect of these additional terms can be expressed in terms of the function $K(x_t,\mu)$, which is indeed a K-factor in the sense of QCD corrections. Note that since the anomalous dimensions of the operators contributing to the

[3] $f(r)$ increases as r decreases.

K-factor are different from the ones contributing to the decay $b \to s + \gamma$, the μ-dependence of the K-factor is also in general different. Numerically, this dependence is rather mild, however. In [3, 4], we have calculated the $O(\alpha_s)$ real and bremsstrahlung corrections to the process $b \to s + \gamma$, from which it is easy to compute $K(x_t,\mu)$. As numerically worked out out in ref. [5], the scale ($= \mu$) dependence of the coefficient functions, in particular that of $C_7(x_t,\mu)$, is very strong in the presently available calculations of the anomalous dimension matrix. This inherent uncertainty of the present theoretical framework has to be included in a quantitative discussion of the QCD-improved decay rates.

To quantify this dependence, we plot the functions $C_7(x_t;\mu)^2$ and $K(x_t,\mu)$ in Fig. 1a) and 1b), respectively, where they are shown as functions of m_t in the range $100\ GeV \le m_t \le 200\ GeV$ and we have made explicit the μ-dependence of these functions in the range $2.5\ GeV \le \mu \le 10.0\ GeV$. For the scale dependence of the strong coupling constant, which is needed for the estimates of these functions, we use the expression based on the two loop β-function:

$$\alpha_s(\mu) = \frac{12\,\pi}{(33 - 2\,N_f)\log(\mu^2/\Lambda^2)}\left(1 - \frac{6\,(153 - 19N_f)}{(33 - 2N_f)^2}\frac{\log\log(\mu^2/\Lambda^2)}{\log(\mu^2/\Lambda^2)}\right) \qquad (12)$$

We set $N_f = 5$ and the corresponding $\Lambda^{(5)} = 225$ MeV, which is the central value obtained from a recent compilation [17].

We note that the m_t-dependence of the function $C_7(x_t,\mu)^2$ in the indicated m_t-range is rather modest, $O(40\%)$, while the function $K(x_t,\mu)$ is practically independent of m_t. The coefficient function $C_7(x_t,\mu)^2$ and the K-factor are both μ-dependent of which the former shows marked dependence on the choice of the scale parameter, μ. The reason for the μ-dependence lies in the fact that at present only the first term in the perturbative expansion of the anomalous dimension matrix has been calculated [10]. The next-to-leading order terms in this matrix together with the consistent calculation of the matrix elements of the operators entering in $H_{eff}(b \to s)$ (often called matching condition) are expected to reduce the μ-dependence of the branching ratio. For the time being, it is prudent to estimate the theoretical error by varying μ in the range $m_b/2 \le \mu \le 2m_b$. The result for the branching ratio $BR(B \to X_s + \gamma)$ obtained by setting the CKM-matrix element ratio $|\lambda_t|^2/|V_{cb}|^2 = 1$, expected in the Standard Model [4], is shown in Fig. 2, giving

$$BR(B \to X_s + \gamma) = (3.0 \pm 1.2) \times 10^{-4} \qquad (13)$$

for m_t and μ in the range $100\ GeV \le m_t \le 200\ GeV$ and $2.5\ GeV \le \mu \le 10.0\ GeV$. Thus, the SM-estimates are quantitative to only about appromiately a factor 2 due to the indicated parametric dependence. It is worthwhile to emphasize this uncertainty in the SM-based expectations for the inclusive decay rate in $B \to X_s + \gamma$. Improvements of this will come only if the top quark mass has been measured and the next-to-leading order corrections in the decay rate have been consistently carried out. We give below a simple parametrization of our calculations for the branching ratio $BR(B \to X_s + \gamma)$, valid in the range $100\ GeV \le m_t \le 200\ GeV$. We have (in units of 10^{-4}):

$$BR(B \to X_s + \gamma)(m_t;\mu) = a_0(\mu) + a_1(\mu)(\frac{m_t\ (GeV)}{100} - 1)^{a_2(\mu)} \qquad (14)$$

and the various parameters are given in Table 1.

[4] This, for example, can be readily seen in the Wolfenstein representation [18] of the CKM matrix.

	$a_0(\mu)$	$a_1(\mu)$	$a_2(\mu)$
$\mu = 2.5\ GeV$	3.22	1.01	0.845
$\mu = 5.0\ GeV$	2.36	1.05	0.859
$\mu = 10.0\ GeV$	1.72	1.05	0.873

Table 1: Fit parameters obtained for the inclusive branching ratio $BR(B \to X_s + \gamma)$ (including the QCD virtual and bremsstrahlung corrections) valid for the range $100\ GeV \le m_t \le 200\ GeV$. The QCD parameter has been set to $\Lambda_{QCD} = 0.225\ GeV$.

Suppose the inclusive branching ratio $BR(B \to X_s + \gamma)$ has been measured experimentally. What could one learn from it for the SM-parameters? First of all, it would provide a more quantitative test of the electromagnetic penguins in B-decays than the exclusive branching ratio $BR(B \to K^* + \gamma)$ already measured, as the latter depends on (in principle model-dependent) form factors. A much desired goal at present is to determine m_t either directly or indirectly through the loop corrections in which the top quark contributes. Unfortunately, the measurement of $BR(B \to X_s + \gamma)$ would not be a big help here as the m_t-dependence of the branching ratio $BR(B \to X_s + \gamma)$ is rather mild. Numerically, we find:

$$\frac{(BR(B \to X_s + \gamma);\ m_t = 200\ GeV)}{(BR(B \to X_s + \gamma), m_t = 100\ GeV)} \simeq 1.4 \qquad (15)$$

In contrast, the μ-dependence of the branching ratio $BR(B \to X_s + \gamma)$ is significantly more pronounced. For $m_t = 140\ GeV$, we determine:

$$\frac{(BR(B \to X_s + \gamma);\ \mu = 2.5\ GeV)}{(BR(B \to X_s + \gamma), \mu = 10.0\ GeV)} \simeq 1.7 \qquad (16)$$

Hence, we don't expect a more stringent bound on m_t from a measurement of $BR(B \to X_s + \gamma)$ than what is already known from the precision electroweak analysis based on the combined LEP data [19]:

$$m_t = 148^{+22}_{-26}\ {}^{+19(M_H = 1 TeV)}_{-23(M_H = 50 GeV)}\ \mathrm{GeV} \qquad (17)$$

with $\alpha_s(M_Z^2) = 0.125 \pm 0.005 \pm 0.002(M_H)$.

Instead of attempting to determine m_t from $BR(B \to K^* + \gamma)$ and $BR(B \to X_s + \gamma)$, we advocate to focus on the determination of the CKM-matrix element ratio λ_t, given a measurement of the inclusive decay rate for $B \to X_s + \gamma$. To that end we show in Figs. 3 the branching ratio $BR(B \to X_s + \gamma)$ as a function of the CKM-matrix element ratio $|V_{ts}|^2/|V_{cb}|^2$, in the range $100\ GeV \le m_t \le 200\ GeV$.[5] In Fig. 3a, we vary the scale μ in the range $2.5\ GeV \le \mu \le 10.0\ GeV$, while in Fig. 3b we have fixed it to the value $\mu = 5.0\ GeV$. A measurement of the inclusive branching ratio can be immediately turned into a determination of the indicated matrix-element ratio, using Figs. 3. However, we point out that the present CLEO upper limit on this quantity is already good enough to get a non-trivial upper limit on the CKM-matrix element ratio. We turn next to the discussion of the upper and lower bounds on the CKM-ratio λ_t from the present CLEO data.

[5] since $|V_{tb}| \simeq 1$ to a very good approximation, we have set $\lambda_t = |V_{ts}|$.

3 Photon energy spectrum in $B \to X_s + \gamma$ and an improved upper bound on $BR(B \to X_s + \gamma)$

We emphasize here that in order to reliably calculate the photon energy spectrum in the inclusive decay $B \to X_s + \gamma$, one has to implement both the real and virtual corrections to the lowest order process $b \to s + \gamma$. Their effect in the integrated decay rate is given by $K(x_t, \mu)$, which has a value between 0.79 and 0.86, hence reducing the inclusive branching ratio $BR(B \to X_s + \gamma)$ by typically $(15 - 20)\%$. However, their effect on the photon energy spectrum is quite marked. They give a radiative tail to the photon energy spectrum and depopulate its high frequency part. While the low energy end of the photon energy spectrum does not have much direct experimental use, the depletion of the high energy end of the spectrum has important experimental consequences. Qualitatively, this effect is simple to understand: The emission of a gluon (or gluons) results in reduced energy for the residual system thereby depleting the high energy photon radiation. An equivalent way of testing the QCD radiative corrections would be to measure the inclusive hadron mass distribution recoiling against the photon in the decays $B \to X_s + \gamma$ [3, 4]. This distribution will be quite appreciably different in the low-mass region with the radiatively corrected spectrum being softer. As also emphasized in [20], this spectrum plays an essential role in deriving an upper limit or measuring the inclusive decay $B \to X_s + \gamma$.

We recall that there are three important ingredients that have gone in the calculation of the photon energy spectrum in the FCNC B-decays, $B \to X_s + \gamma$.

- First, the leading order bremsstrahlung process $b \to s + \gamma + g$ being a three-body decay gives a non-trivial spectrum away from the discrete spectrum resulting from the 2-body decay $b \to s + \gamma$. Perturbative consistency requires to also include the virtual corrections to the decay $b \to s + \gamma$.
- Near the end point, i.e., as $E_\gamma \to E_\gamma^{max}$, these $O(\alpha_s)$ terms have to be exponentiated à la Sudakov. We have explained this exponentiation in [3, 4].
- Finally, one has to implement the B-meson wave function effects which modify the underlying photon energy and hadron mass spectra.

The model we use for this purpose has been specified by us in ref [4, 5]. In this model the b-quark in the B-hadron is given a non-zero momentum having a Gaussian distribution represented by an à priori free (but adjustable) parameter, p_F:

$$\phi(p) = \frac{4}{\sqrt{\pi} {p_F}^3} \exp(\frac{-p^2}{{p_F}^2}) \quad ; \quad p = |\vec{p}| \qquad (18)$$

with the obvious normalization

$$\int_0^\infty dp p^2 \phi(p) = 1 \qquad (19)$$

The energy-momentum constraint is imposed in the form:

$$W^2 = {m_B}^2 + {m_q}^2 - 2 m_B \sqrt{p^2 + {m_q}^2} \qquad (20)$$

where m_B is the B-meson mass, W, the effective momentum dependent mass of the b-quark, and m_q, the constant mass of the spectator quark in the B-meson, $B = \bar{b}q$. This model has previously been used in the derivation of the inclusive lepton energy spectrum in the semileptonic decays of the B and D hadrons [13, 14], where it is known to give a good description of the measured lepton energy spectra. For p_F, we take the best value obtained from fits of the ARGUS and CLEO inclusive lepton energy spectra in B-decays giving $p_F = 0.30 \pm 0.09\ GeV$ [21, 22]. The dependence of the inclusive branching ratios for $B \to X_s + \gamma$ on the wave-function parameter, p_F, is negligible. However, the shape of the photon energy spectrum (equivalently the invariant mass distribution of the final state hadrons in the decay $B \to X_s + \gamma$) is sensitive to the value of p_F assumed, as shown previously by us [4].

The resulting photon energy spectra from the FCNC process $B \to X_s + \gamma$ in the rest frame of the B-meson are shown in Fig. 5, where we have taken $m_t = 140\ GeV, \mu = 5\ GeV$ and fixed the Fermi momentum parameter to its central value $p_F = 0.3\ GeV$. Note that the photon energy spectrum from the two-body decay $b \to s + \gamma$ is entirely determined by the B-meson wave-function, shown through the dashed curve. The QCD-improved spectrum (calculated in $O(\alpha_s)$ and Sudakov exponentiated) is shown through the solid line. We draw attention to the considerable depletion of the high energy photon region due to the radiative QCD effects. This depletion survives the wave function smearing and we argue that in a good statistics experiment, it should be possible to test the shape of the QCD-improved photon energy spectrum in radiative rare B-decays.

To help implement the experimental constraints on our analysis that have been used in ref. [2] in obtaining an upper limit on the inclusive decay branching ratio $BR(B \to X_s + \gamma)$, we Lorentz boost the photon energy spectra from the B-meson rest frame (shown in Fig. 5) to the lab frame, in which the B-mesons are given a uniformally distributed momentum of 0.3 GeV. This would then correctly implement the experimental situation $\Upsilon(4S) \to B\bar{B}$. With the help of this spectrum it is now an easy matter to calculate the fraction of events that we expect in the photon energy interval $2.2\ GeV \leq E_\gamma \leq 2.7\ GeV$ (used in the analysis reported in [2]) from the decays $B \to X_s + \gamma$. The fraction of events from the partonic decay $b \to s + \gamma$, i.e., not including the QCD bremsstrahlung corrections, is estimated by us to be as high as 93% (for $p_F = 0.3\ GeV$). However, we don't use this number in working out the improved bound for $BR(B \to X_s + \gamma)$.

The rest frame spectrum (solid curve) and its Lorentz boosted version (dashed curve) from the decays $B \to X_s + \gamma$, including the QCD radiative corrections, are shown in Fig. 6. The resulting fractions of the events in the stated photon energy interval are shown in Table 2, for three values of m_t and p_F. For the central value of $p_F = 0.3\ GeV$, our calculations predict that 85 % of all the photons from the decays $B \to X_s + \gamma$ are expected to have an energy in the interval $2.2\ GeV \leq E_\gamma \leq\ 2.7\ GeV$. This fraction, which is independent of m_t could become as low as 0.78 for $p_F = 0.39\ GeV$, but that would amount to pushing all the parameters to their extreme values. We recall that the corresponding number assumed for this fraction in the analysis reported in [2] is 0.70 [6], yielding an upper limit for the branching ratio $BR(B \to X_s + \gamma) < 5.4 \times 10^{-4}$ (95% $C.L.$). Using the QCD corrected spectrum, this upper limit in our analysis goes down to 4.5 (4.8) $\times 10^{-4}$ assuming a fraction 0.85 (0.78) for the photons in the indicated energy range. In our theoretical analysis for the CKM parameter λ_t and in putting an upper limit on possible non-SM contributions in $BR(B \to X_s + \gamma)$, we use the number 4.8×10^{-4}, being the more conservative of the two. Our derived upper limit for $BR(B \to X_s + \gamma)$ is so tantalizingly close to the SM predictions $(2-4) \times 10^{-4}$ that we predict that a positive measurement in this inclusive decay channel is imminent!

We now show the implications of the upper bound $BR(B \to X_s + \gamma) < 4.8 \times 10^{-4}$ (95%$C.L.$) for the upper bound on the CKM-matrix element ratio $|V_{ts}|/|V_{cb}|$ by varying μ in the range $2.5\ GeV \leq \mu \leq\ 10.0\ GeV$ (Fig. 3a) and by fixing $\mu = 5\ GeV$ (Fig. 3b). This gives at 95 %

[6] We thank Ed Thorndike for explaining to us the selection procedures and efficiencies used in his analysis.

$m_t(GeV) =$	100	140	200
$p_F = 0.21\ GeV$	0.89	0.89	0.89
$p_F = 0.30\ GeV$	0.85	0.85	0.85
$p_F = 0.39\ GeV$	0.78	0.78	0.78

Table 2: Fraction of events from the decay $B \to X_s + \gamma$ (including the QCD virtual and bremsstrahlung corrections) with the photons having an energy in the interval (2.2-2.7) GeV calculated in the lab frame, estimated using the B-meson wave function model described in the text.

C.L. ,

$$|V_{ts}|/|V_{cb}| < 1.42\ (1.67) \quad \text{for } \mu = 5.0\ GeV\ (2.5\ GeV \leq \mu \leq\ 10.0\ GeV) \tag{21}$$

The information given in Fig. (3) is sufficient to determine the CKM-matrix element ratio once the inclusive branching ratio $BR(B \to X_s + \gamma)$ has been measured.

We can use the improved experimental bound on $BR(B \to X_s + \gamma)$ derived above to constrain possible non-SM contributions in the FCNC radiative decays. We recall that the minimum value of the SM-branching ratio $BR(B \to X_s + \gamma)$ in our calculation is obtained for the choice $\mu = 10.0\ GeV$ and $m_t = 108\ GeV$, with the number for m_t set to its present lower bound coming from the Fermilab collider experiments [8]. From our Fig. 2, we get:

$$BR(B \to X_s + \gamma)(SM) > 1.8 \times 10^{-4}. \tag{22}$$

With the improved upper bound $BR(B \to X_s + \gamma) < 4.8 \times 10^{-4}$ (95 %$C.L.$) obtained from the inclusive data on $BR(B \to X_s + \gamma)$, we put a lower bound on the non-SM contribution to the decay $BR(B \to X_s + \gamma)$ getting,

$$BR(B \to X_s + \gamma)\ (\text{non-SM}) < 3.0 \times 10^{-4}. \tag{23}$$

The upper bound on $BR(B \to X_s + \gamma)$ (non-SM) can be seen as a function of m_t in Fig. 4. Constraints on the parameters of the SM-extensions can meaningfully be discussed only in well defined scenarios. We hope to come back to this issue in a subsequent publication.

4 The branching ratio $BR(B \to K^\star + \gamma)$ and determination of $|V_{ts}|$

In this section, we present our analysis for determining the CKM ratio $|V_{ts}|/|V_{cb}|$ from the CLEO measurements of the exclusive branching ratio $BR(B \to K^\star + \gamma) = (4.5 \pm 1.5 \pm 0.9) \times 10^{-5}$, and the corresponding (95% $C.L.$) upper and lower bounds on this branching ratio [2]:

$$1.6 \times 10^{-5} \leq\ BR(B \to K^\star + \gamma)\ \leq\ 7.4 \times 10^{-5} \tag{24}$$

To get the desired bounds on the CKM-matrix element ratio, we have to specify a model to calculate the exclusive branching ratio $BR(B \to K^\star + \gamma)$. For that purpose we introduce the following exclusive to inclusive B-decay width ratio:

$$R(K^\star/X_s) \equiv \frac{\Gamma(B \to K^\star + \gamma)}{\Gamma(B \to X_s + \gamma)} \tag{25}$$

We shall tabulate below the values for the ratio $R(K^*/X_s)$, obtained by somewhat modifying the method used by us previously [4, 5], in which we had used the inclusive hadronic invariant mass distribution discussed in the previous section as an input and assumed K^*-dominance of the spectrum in $B \to X_s + \gamma$, in the invariant mass region $m_K + m_\pi \leq m_{X_s} \leq 1.0\ GeV$. In this update, we have reduced the upper integration limit of m_{X_s} to $0.971\ GeV$, so as to conform to the definition of K^* used in the CLEO analysis [1] and, more importantly, have now estimated the ratio of the resonating p-wave (i.e., K^*) and a non-resonating (i.e. non-K^*) contribution in the indicated interval of m_{X_s}. To that end, we note that in the decays $D \to (K+\pi) + \ell + \nu_\ell$, the corresponding quantity (i.e., the ratio of the resonating $(K\pi)$ intermediate state) has been measured in three independent experiments, E691 [23], MARKIII [24] and WA82 [25]. Defining $P(K^*)$ as the fraction of events due to the K^*-resonance in the indicated range of m_{X_s}, their results are:

$$P(K^*) = 0.7 \pm 0.2 \quad (\text{WA82}) \tag{26}$$

$$\begin{aligned} P(K^*) &= 0.79^{+0.15}_{-0.17}\ {}^{+0.09}_{-0.03} \quad (\text{MKIII}) \\ P(K^*) &= 0.91 \pm 0.06 \pm 0.06 \quad (\text{E691}) \end{aligned} \tag{27}$$

We argue that it is reasonable to expect a similar number for the fraction of K^*-events in the radiative decays $B \to X_s + \gamma$. In fact, it has been argued in literature that one could use heavy quark symmetry (only for the c- and b-quarks) to relate the radiative B-decays $B \to K^* + \gamma$ and the semileptonic D-decays $D \to K^* \ell \nu_\ell$ [26]. In particular, it has been shown in [27] that the heavy quark symmetry gives relation at a specific point in the Dalitz plot in the decay $D \to K^* \ell \nu_\ell$ and $B \to K^* + \gamma$. A corrolary of this result is that the heavy quark symmetry implies that the form factors in the decays $B \to K + \pi + \gamma$ and $D \to K + \pi + \ell + \nu_\ell$ are also related, implying an approximate equality over a certain kinematic range. This symmetry works best when the variables $v.p/m_D$ and $v.p/m_B$ are small (here v is the 4-velocity of the B or D-meson and p is the momentum of the light hadrons). This corresponds to the kinematic point where $q^2 = (p_\ell + p_\nu)^2$ is close to its maximum value. However, there also exist arguments extending this equality in the entire kinematic domain [28].

Integrating the m_{X_s}-distribution in the decay $B \to X_s + \gamma$ in the range, $(m_K + m_\pi) \leq m_{X_s} \leq 0.971\ GeV$, and setting $P(K^*) = 0.8$, the results for the exclusive to inclusive ratio $R(K^*/X_s)$ are presented in Table 3, giving

$$R(K^*/X_s) = (13 \pm 3)\% \tag{28}$$

We note that in our model $R(K^*/X_s) = 0.12$ represents the central value, corresponding to $p_F = 0.3\ GeV$, independent of m_t and μ.

$m_t(GeV) =$	100	140	200
$p_F = 0.21\ GeV$	0.16	0.16	0.16
$p_F = 0.30\ GeV$	0.12	0.12	0.12
$p_F = 0.39\ GeV$	0.10	0.10	0.10

Table 3: The ratio of exclusive-to inclusive branching ratios $R(K^*/X_s)$ in the model described in the text.

The resulting branching ratios for the exclusive decay $BR(B \to K^* + \gamma)$ with the scale parameter $\mu = 5.0\ GeV$ are shown in Table 4. This result can be expressed as

$$BR(B \to K^* + \gamma) = (4.0 \pm 1.6) \times 10^{-5} \tag{29}$$

with $R(K^*/X_s)$ in the range $(13 \pm 3)\%$, and m_t in the range $100\ GeV \leq m_t \leq 200\ GeV$. Varying the parameter μ in the range $2.5\ GeV \leq \mu \leq 10.0\ GeV$ would introduce a theoretical uncertainty of $\pm 1.0 \times 10^{-5}$. The estimates presented here and earlier [4, 5, 6] for $BR(B \to K^* + \gamma)$ are in very comfortable agreement with the CLEO measurements, $BR(B \to X_s + \gamma) = (4.5 \pm 1.0 \pm 0.9) \times 10^{-5}$ [1].

Estimates for the ratio $R(K^*/X_s)$ in literature are mostly in the ball park $(5-15)\%$ [29]-[33], hence we are consistent with them. However, there are some others, mostly based on the QCD sum rule approach, which give values in the range $(25-40)\%$ [34, 35]. On face value such high estimates are almost in conflict with the CLEO measurement of $BR(B \to K^* + \gamma)$. However, from a theoretical point of view, the reason of the high values of $R(K^*/X_s)$ in the QCD sum rule approach deserves further theoretical studies [36]. We wait for the inclusive measurements for a verdict on these estimates.

In Figs. 7a), we plot the branching ratio $BR(B \to K^* + \gamma)$ as a function of the CKM-matrix element ratio $|V_{ts}|/|V_{cb}|^2$ by varying the QCD-scale parameter μ in the range $2.5\ GeV \leq \mu \leq 10\ GeV$, with the top quark mass in the range $100\ GeV \leq m_t \leq 200\ GeV$ and for three indicated values of the exclusive to inclusive ratio $R(K^*/X_s) = 0.10, 0.12$ and 0.16. The result of the corresponding analysis done by setting $\mu = 5\ GeV$ is shown in Figs. 7b). The 95% $C.L.$ upper and lower bound from the CLEO measurements are also shown on these figures. It is easy to see that more stringent upper bounds on the ratio $|V_{ts}|/|V_{cb}|$ are obtained from the 95% $C.L.$ upper limit on the inclusive branching ratio $BR(B \to X_s + \gamma)$. These figures, however, provide at present the lower bounds on the matrix element ratio, since the corresponding experimental lower bound on the inclusive rates $BR(B \to X_s + \gamma)$ is still being awaited.

$m_t(GeV) =$	100	140	200
$p_F = 0.21\ GeV$	3.8	4.6	5.6
$p_F = 0.30\ GeV$	2.8	3.4	4.1
$p_F = 0.39\ GeV$	2.3	2.8	3.3

Table 4: Branching ratio for the decay $B \to K^* + \gamma$ in units of 10^{-5}

Based on our analysis presented here and by others [29]-[33], we find that the value $R(K^*/X_s) \leq 0.16$ is a reasonable upper bound, with $R(K^*/X_s) = 0.12$ as our best value. We use the above estimate $R(K^*/X_s)^{max} = 16\%$ and the 95% $C.L.$ lower bound $BR(B \to K^* + \gamma) > 1.6 \times 10^{-5}$ [1, 2] to put a 95% $C.L.$ lower bound on the inclusive branching ratio:

$$BR(B \to X_s + \gamma) \geq 1.0 \times 10^{-4}. \tag{30}$$

This lower bound is approximately a factor 1.8 below the minimum value of the inclusive branching ratio $BR(B \to X_s + \gamma)$ in the Standard Model, calculated above by us.

With this lower bound on the inclusive branching ratio $BR(B \to X_s + \gamma)$, we extract from Figs. 3(a) and 3(b) the following lower bounds on the CKM-matrix element ratio:

$$|V_{ts}|/|V_{cb}| > 0.55\ (0.50) \quad \text{for } \mu = 5.0\ GeV\ (2.5\ GeV \leq \mu \leq 10.0\ GeV) \tag{31}$$

for the indicated values of the QCD-parameter μ. Taking the lower of the two limits, we determine using, respectively, the 95% $C.L.$ lower and upper bounds on the branching ratios $BR(B \to X_s + \gamma)$ and $BR(B \to K^* + \gamma)$:

$$0.50 \leq |V_{ts}|/|V_{cb}| \leq 1.67. \quad (32)$$

The upper bound obtained is mildly model dependent, since it assumes prior knowledge of the photon energy spectrum and the inclusive branching ratio $BR(B \to X_s + \gamma)$. We have tried to get an educated profile of the inclusive final state but, of course, this remains to be tested experimentally. The lower bound depends on $R(K^*/X_s)^{max}$ which is a more model dependent enterprise. While these estimates are not yet completely quantitative, we have demonstrated that the electromagnetic penguins in B-decays provide a determination of the matrix element $|V_{ts}|/|V_{cb}|$, using the CKM-unitarity constraints. We are confident that with the measurements of the inclusive branching ratio $BR(B \to X_s + \gamma)$ this model dependence can be largely removed and the route involving the exclusive decay $BR(B \to K^* + \gamma)$ that we have undertaken here will no longer be necessary. We compare our determination of the CKM-matrix element ratio from the electromagnetic penguins in B-decays gotten above with the indirect bounds due to the CKM-unitarity that can be worked out from the fits reported in the Particle data Group [12],

$$0.55 \leq |V_{ts}|/|V_{cb}| \leq 1.68 \quad (33)$$

The two bounds are quite comparable, though they are derived by using very different considerations, which can be taken as a consistency check for the SM-normalization of the electromagnetic penguins in B-decays. On the same point, we note that the number obtained by us for the ratio $\lambda_t/|V_{cb}| \simeq |V_{ts}|/|V_{cb}|$ in Eq. (31) is equivalent to a bound on $|V_{cs}|$, by virtue of the unitarity relation (4).

5 Conclusions and Summary

From the foregoing analysis it is evident that the CLEO measurement of the exclusive decay $B \to K^* + \gamma$ and improved upper limit on $BR(B \to X_s + \gamma)$ provide new and independent constraints on the CKM-matrix elements and the physics of the FCNC interactions. These decay modes amount to a measurement of the flavour non-diagonal $b \to s + \gamma$ magnetic moment couplings, with the QCD corrections playing an essential role. In SM, these couplings are predominantly determined by the so-called electromagnetic penguin diagrams, which in turn depend on m_t, m_c and the CKM matrix element λ_t and λ_c. Using unitarity, one could express the result in terms of the ratio λ_t. We have done a QCD-based analysis of the CLEO data obtaining first estimates of the CKM-matrix element ratio $|V_{ts}|/|V_{cb}|$ from the electromagnetic penguins in B-decays. The value determined for this quantity is in agreement with the phenomenological CKM-unitarity fits performed in the Particle Data Group, elucidating the SM-description of the electromagnetic $b \to s + \gamma$ penguins quantitatively. With more data, in particular measurements of the inclusive decay $B \to X_s + \gamma$, the model dependence due to $R(K^*/X_s)$ will be removed. Equivalently, one would be able to determine the form factor in the decay $B \to K^* + \gamma$ to test various competing models. Our analysis also gives a bound on possible non-SM contribution to the branching ratio $BR(B \to X_s + \gamma)$. Finally, we emphasize the importance of calculating the next-to-leading order corrections in the Wilson coefficients and the relevant matrix elements, without which the scale dependence of the branching ratio $BR(B \to X_s + \gamma)$ will not be attenuated.

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Figure captions

Figure 1

a)-The Wilson coefficient function $C_7^2(x_t,\mu)$, evaluated in the leading logarithmic approximation as a function of the top quark mass for three indicated values of the scale parameter μ. ($x_t = m_t^2/m_W^2$).
b)-The QCD K-factor $K(x_t,\mu)$ for the inclusive decay rate for $B \to X_s + \gamma$, calculated from the $O(\alpha_s)$ bremsstrahlung and virtual corrections to the decay $b \to s + \gamma$, as a function of the top quark mass for three indicated values of the scale parameter μ.

Figure 2

Inclusive branching ratio $BR(B \to X_s + \gamma)$ in the Standard Model assuming $|V_{ts}|/|V_{cb}| = 1$ as a function of the top quark mass for three indicated values of the scale parameter μ.

Figure 3

Inclusive branching ratio $BR(B \to X_s + \gamma)$ in the Standard Model as a function of the CKM-matrix element ratio $(|V_{ts}|/|V_{cb}|)^2$, with the top quark mass in the range $100\ GeV \le m_t \le 200\ GeV$, a)- the QCD-scale parameter μ in the range $2.5\ GeV \le \mu \le 10\ GeV$ and b)- setting $\mu = 5\ GeV$. The derived upper and lower bounds (95% $C.L.$) from the CLEO measurements are also shown.

Figure 4

Upper bound on the non-SM contribution to the branching ratio $BR(B \to X_s + \gamma)$ (non-SM) (95% $C.L.$) as a function of the top quark mass, obtained from the corresponding (95% $C.L.$) experimental upper limit from the improved CLEO measurements [2] and estimates of the inclusive branching ratio $BR(B \to X_s + \gamma)$ in the Standard Model, given in Fig. 2.

Figure 5

Inclusive photon energy spectra in the rest frame of the B-meson from the FCNC decay $B \to X_s + \gamma$ for $m_t = 140\ GeV, \mu = 5.0\ GeV$ and Fermi momentum $p_F = 0.3\ GeV$. The normalization of the curves is determined by the expression given in Eq. 10. The dashed curve corresponds to the partonic decay $b \to s + \gamma$ (evaluated with $K = 1$) and the solid curve includes the $O(\alpha_s)$ QCD- virtual and bremsstrahlung corrections.

Figure 6

Inclusive photon energy spectra from the FCNC decay $B \to X_s + \gamma$ including the QCD-bremsstrahlung and virtual corrections calculated in the rest frame of the B-meson (solid curve) and in the lab frame of $\Upsilon(4S)$ with a boost $|p_B| = 0.3\ GeV$ (dashed curve). The spectrum corresponds to $m_t = 140\ GeV, \mu = 5.0\ GeV$ and Fermi momentum $p_F = 0.3\ GeV$.

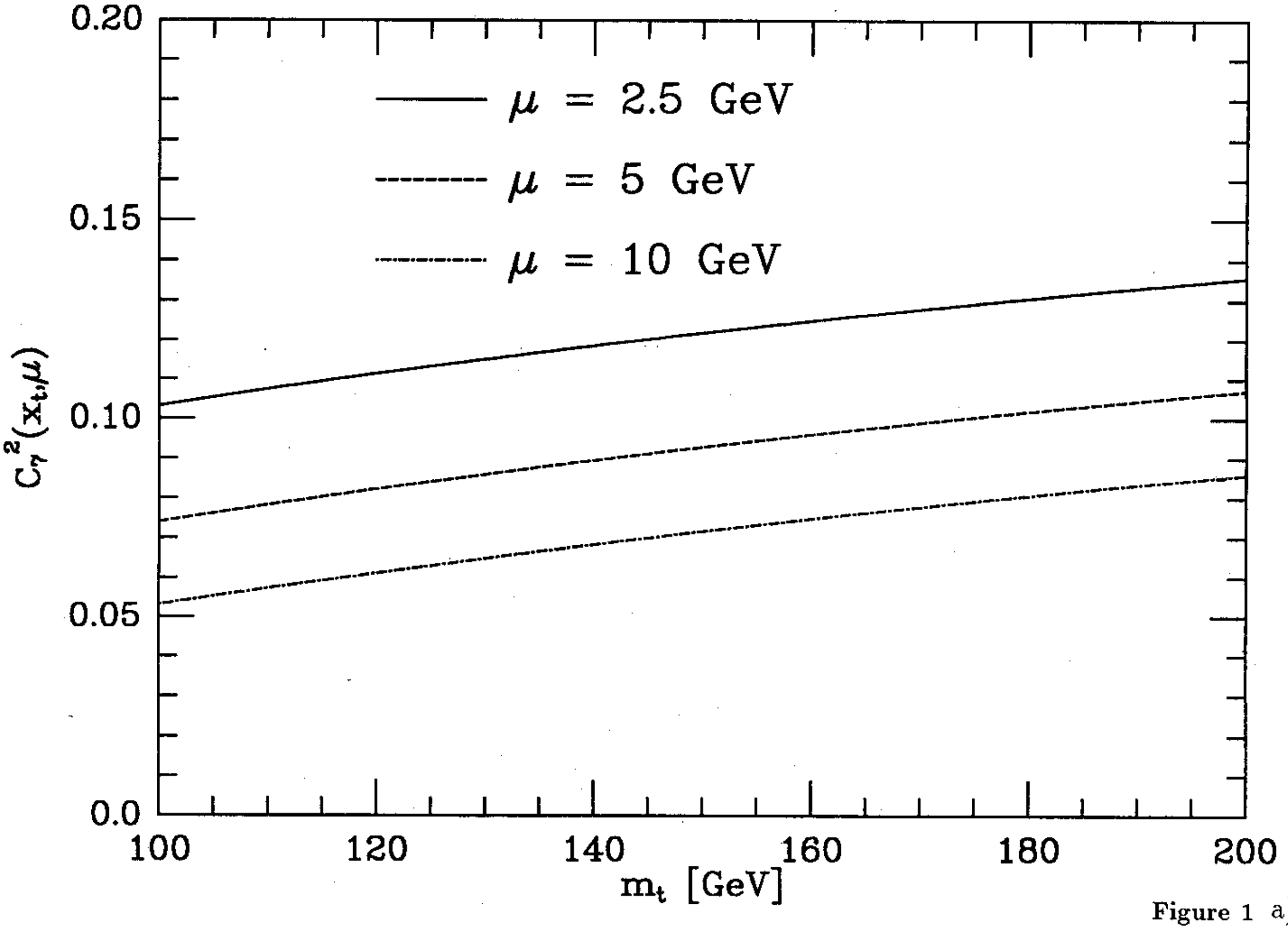

Figure 1 a)

Figure 7

Inclusive branching ratio $BR(B \to K^* + \gamma)$ as a function of the CKM-matrix element ratio $(|V_{ts}|/|V_{cb}|)^2$, with the top quark mass in the range $100\ GeV \leq m_t \leq 200\ GeV$ and for three indicated values of the exclusive to inclusive ratio $R(K^*/X_s) = 0.10, 0.12$ and 0.16. a)- the QCD-scale parameter μ in the range $2.5\ GeV \leq \mu \leq 10\ GeV$ and b)- setting $\mu = 5\ GeV$. The 95% *C.L.* upper and lower bound from the CLEO measurements are also shown.

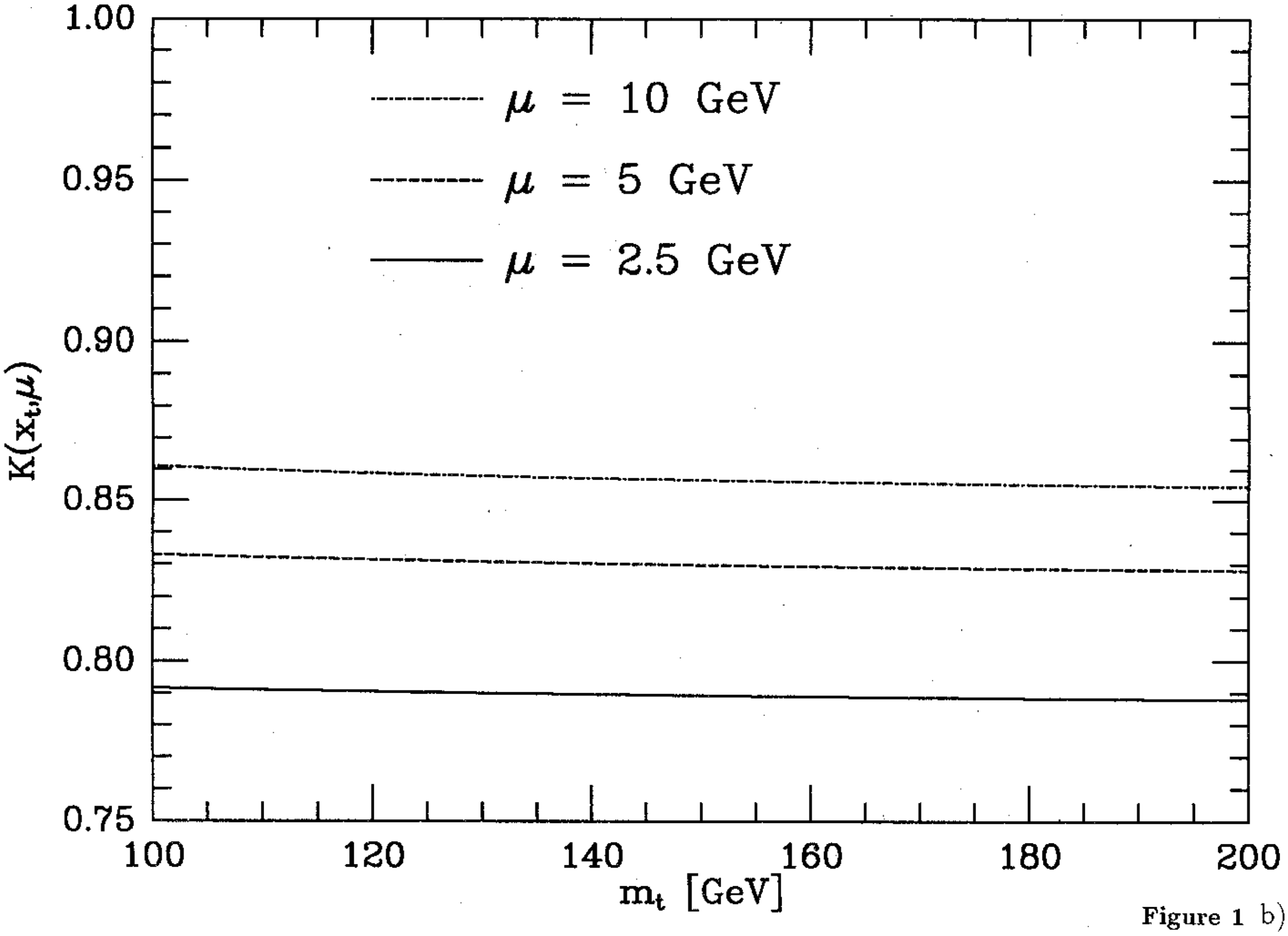

Figure 1 b)

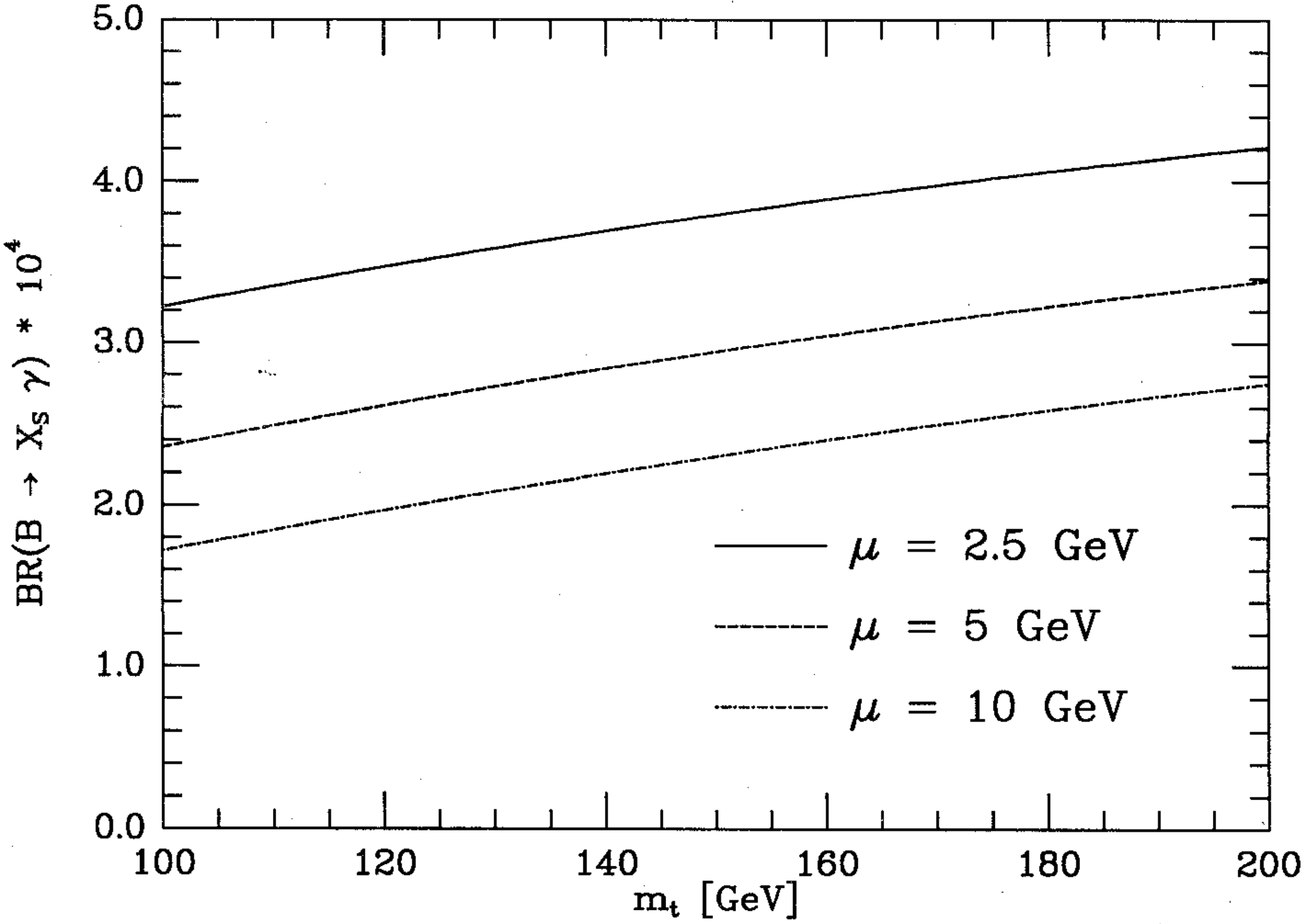

Figure 2

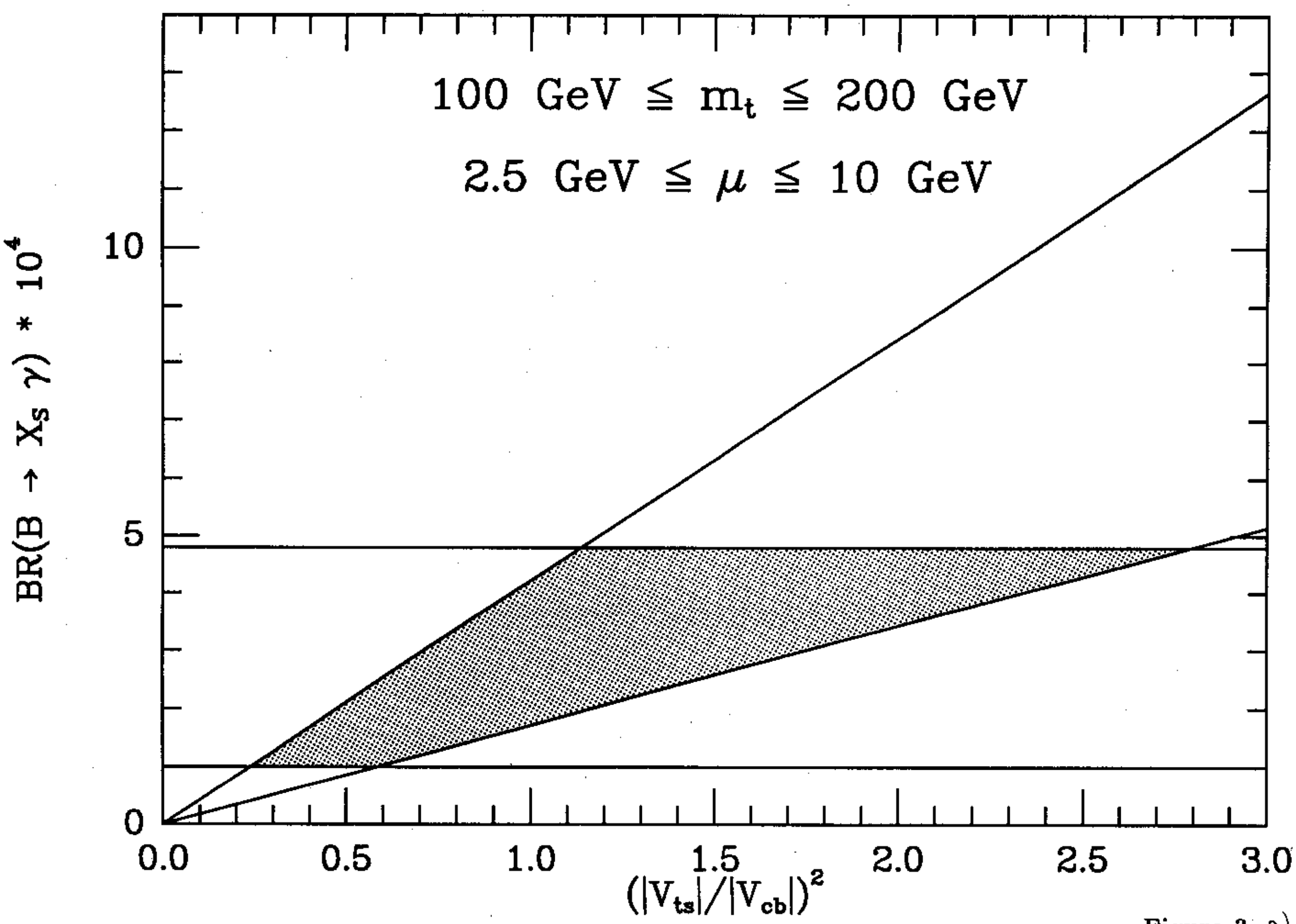

Figure 3 a)

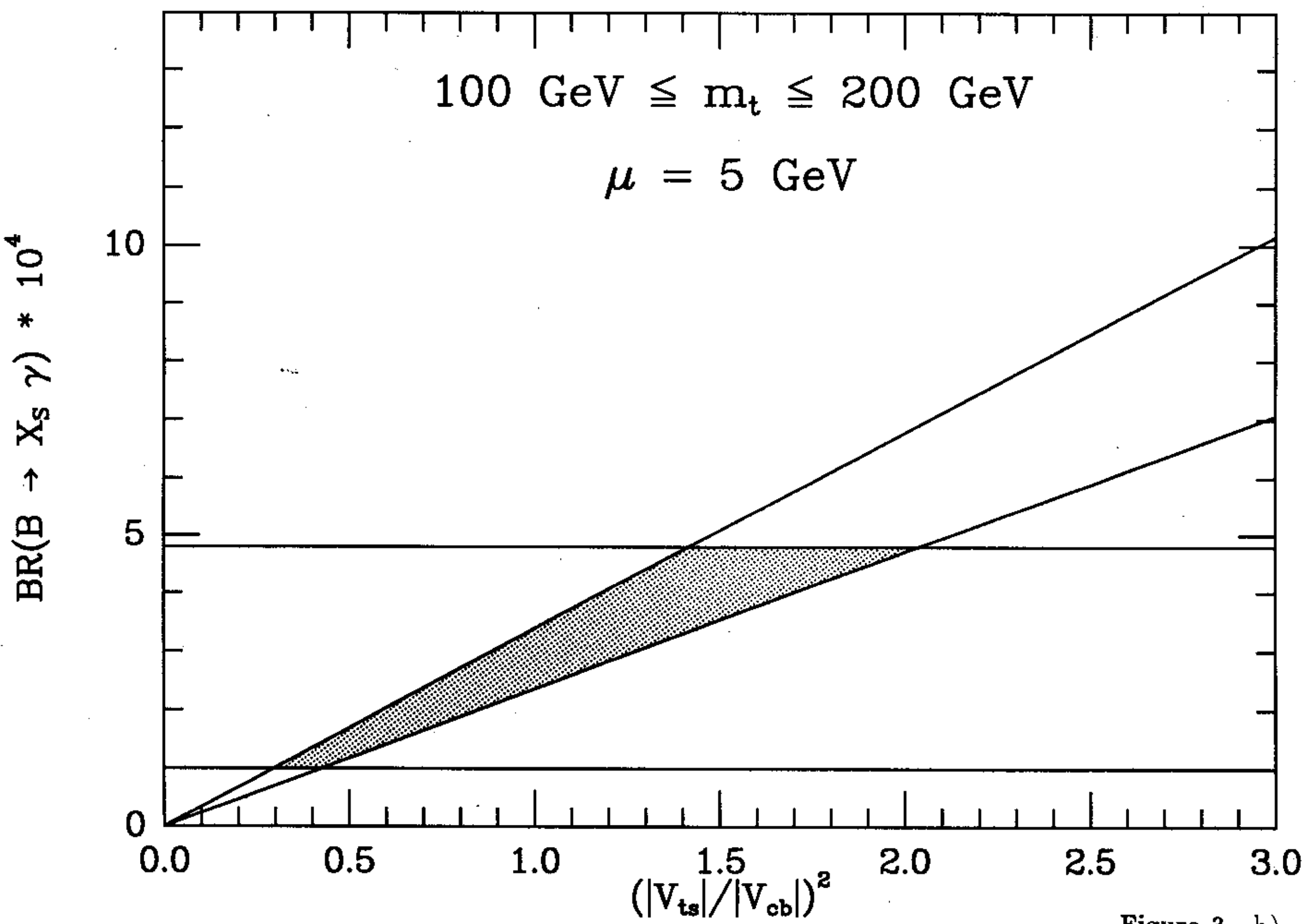

Figure 3 b)

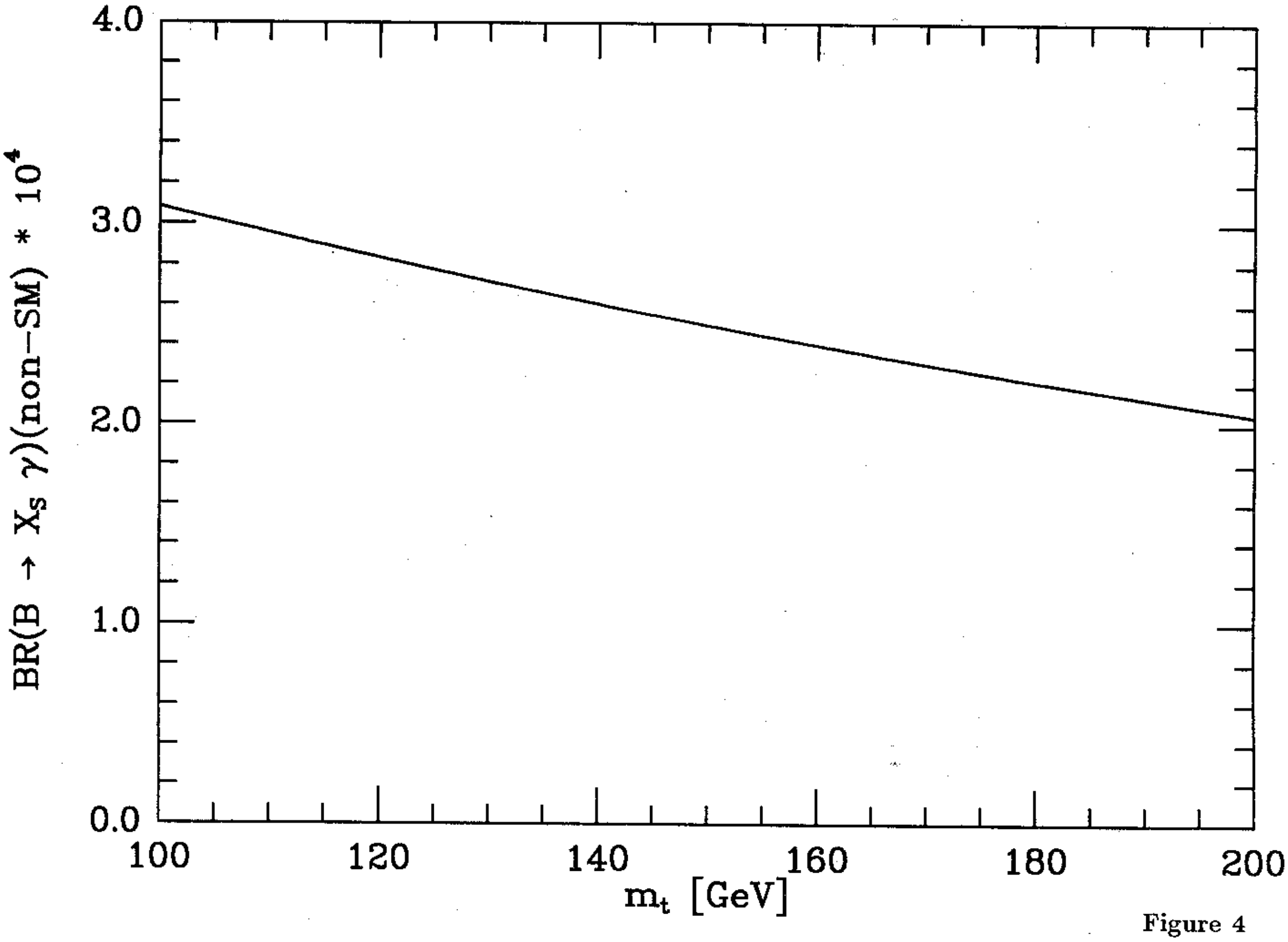

Figure 4

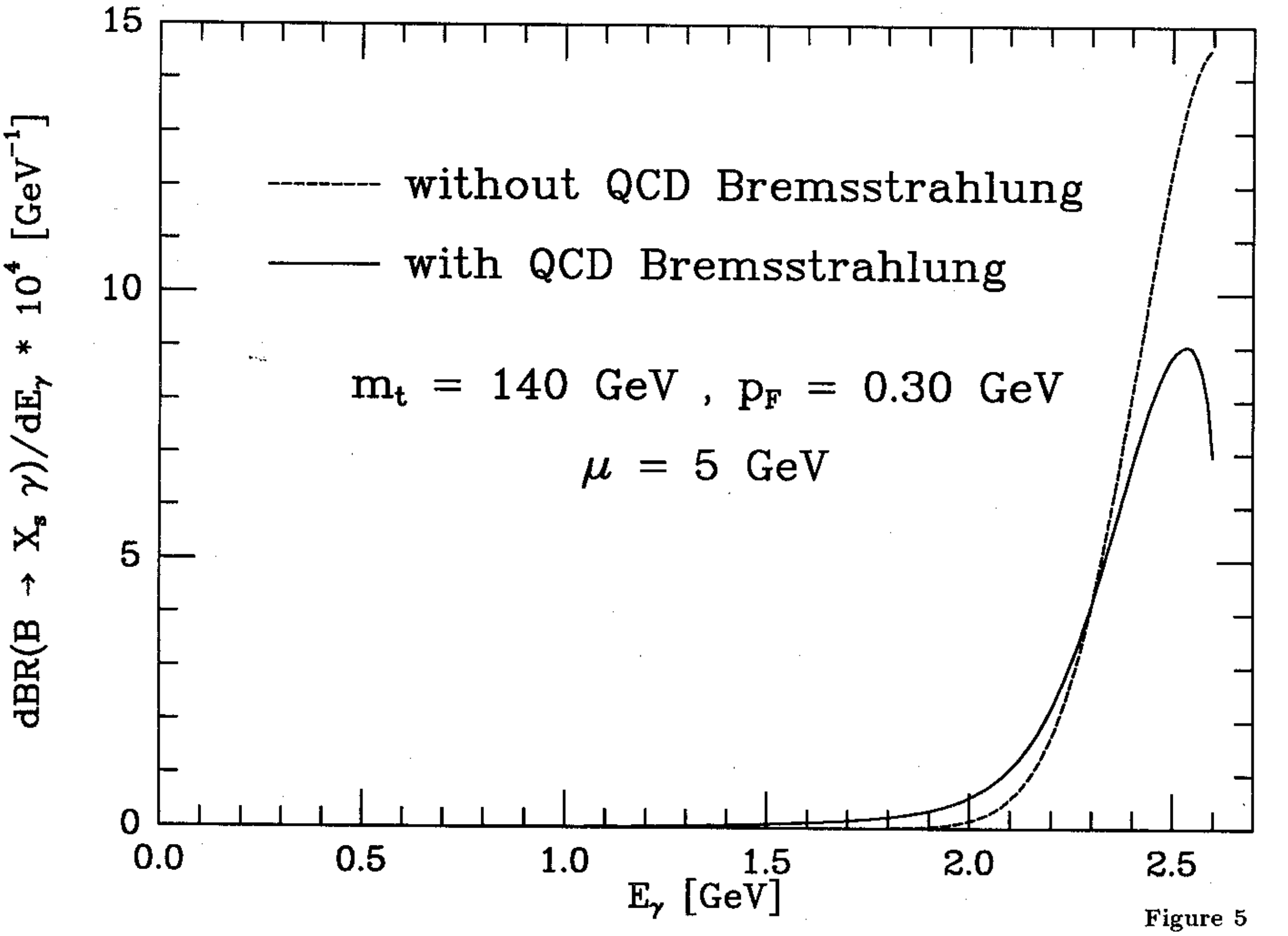

Figure 5

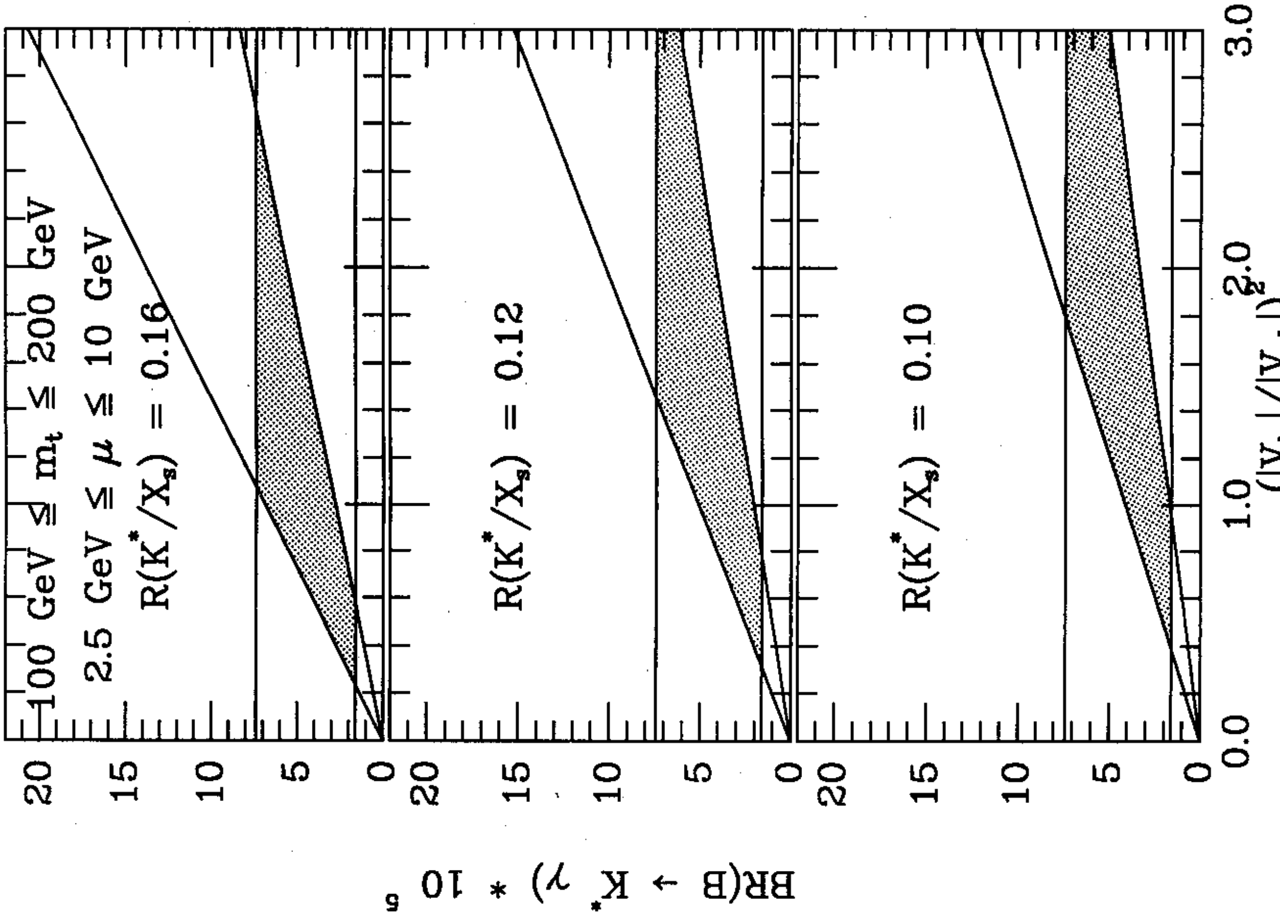

Figure 7 a)

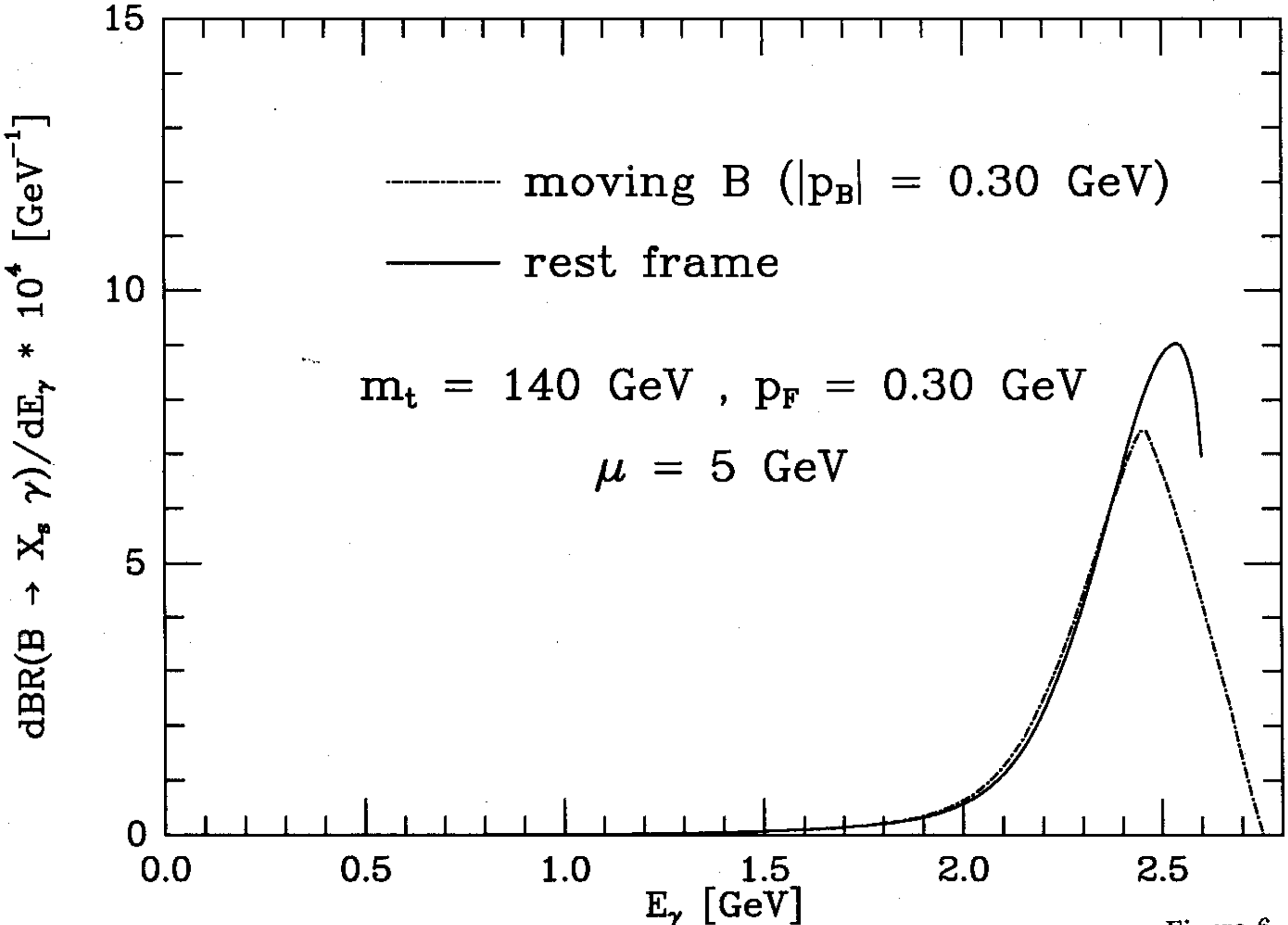

Figure 6

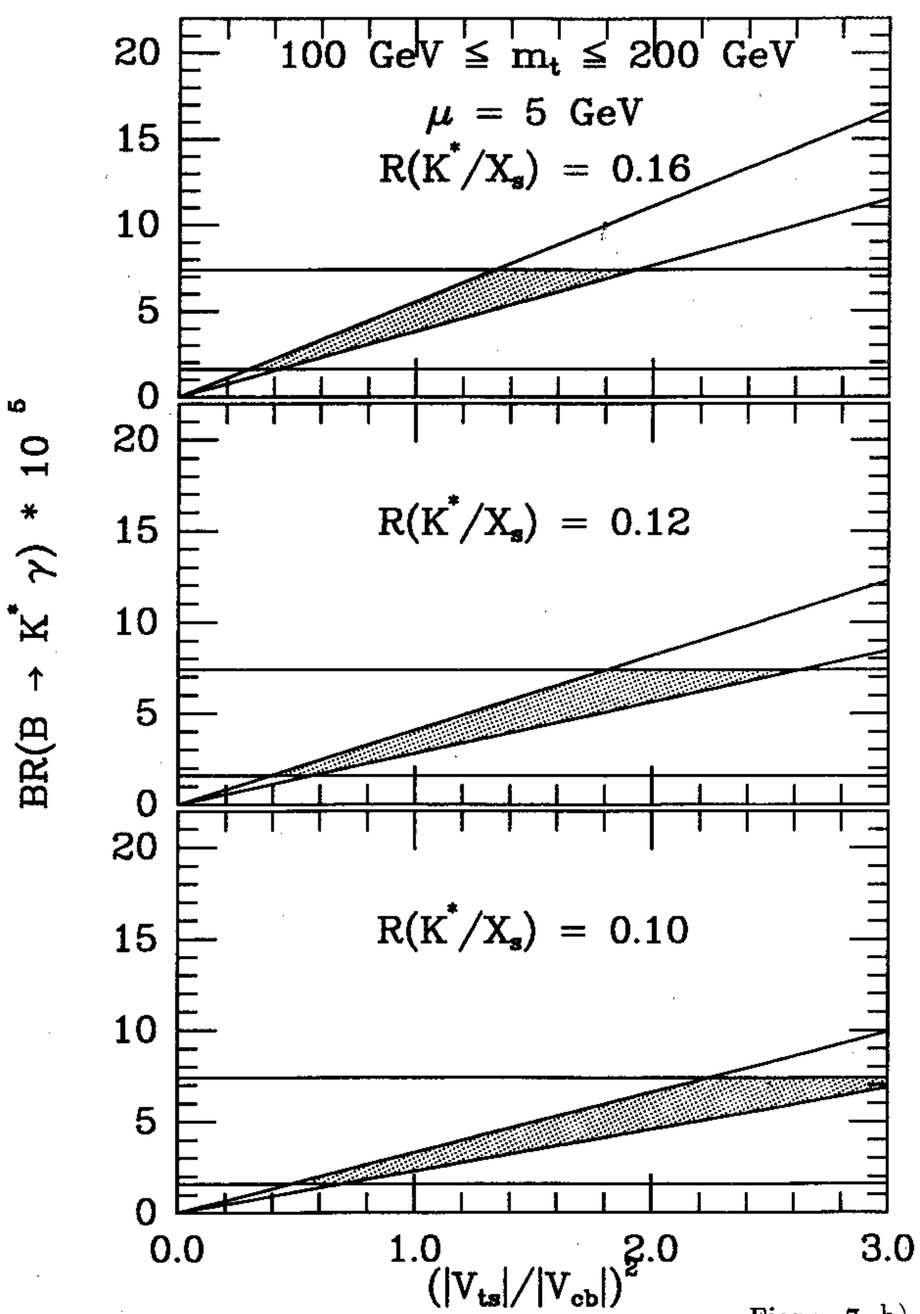
100 GeV ≤ m_t ≤ 200 GeV
μ = 5 GeV
$R(K^*/X_s)$ = 0.16
$R(K^*/X_s)$ = 0.12
$R(K^*/X_s)$ = 0.10
$BR(B \to K^* \gamma) * 10^5$
$(|V_{ts}|/|V_{cb}|)^2$
0.0
1.0
2.0
3.0
0
5
10
15
20

Figure 7 b)