$$\hat{R}_\mu = R_\mu + \epsilon(\Box\delta_{\mu\nu} - \partial_\mu\partial_\nu)\gamma^\nu\gamma\cdot R \tag{29}$$

The constraint $\partial\cdot\hat{R} = 0$ is manifestly satisified; the constraint $\gamma\cdot\hat{R} = 0$ requires

$$\epsilon = \frac{-1}{3m^2} \tag{30}$$

A little computation shows that $\hat{R}_\mu$ satisfies the Klein-Gordon equation and is thus a massive spin-$3/2$ field

$$(\Box - M^2)\hat{R}_\mu = 0, \quad \partial\cdot\hat{R} = \gamma\cdot\hat{R} = 0, \; M^2 = \frac{-\gamma}{2\beta} \tag{31}$$

In order not to produce tachyons, when one starts from the Einstein action where $\gamma = 1$, one must have that α is positive and β is negative.

The spin-1 equation reads

$$-8\alpha\,\partial_\mu\partial\cdot A - \frac{4\beta}{3}\partial_\nu F_{\mu\nu} + \frac{2}{3}\gamma A_\mu = 0 \tag{32}$$

Contracting with ∂_μ one obtains the Proca equation for a massive spin-1 field

$$(\Box - m^2)\,\partial\cdot A = 0, \qquad m^2 = \frac{\gamma}{12\alpha} \tag{33}$$

The remainder $\hat{A}_\mu = A_\mu + \eta\partial_\mu\partial\cdot A$ satisfies the Lorentz condition $\partial\cdot\hat{A} = 0$ if η is chosen properly

$$\hat{A}_\mu = A_\mu - \frac{12\alpha}{\gamma}\partial_\mu\partial\cdot A \tag{34}$$

From the original equation for A_μ and the equation for $\partial\cdot A$ it follows that $\hat{A}_\mu$ is a massive spin-1 (Proca) field

$$(\Box - M^2)\hat{A}_\mu = 0 \quad , \quad \partial\cdot\hat{A} = 0, \quad M^2 = \frac{-\gamma}{2\beta} \tag{35}$$

Finally we turn to the fields S and P. Their field equations are given by

$$(\Box - m^2)S = (\Box - m^2)P = 0, \qquad m^2 = \frac{\gamma}{12\alpha} \tag{36}$$

We see thus that a supersymmetric action of the form

$$\frac{1}{\kappa^2}\beta R_{\mu\nu}^2 + \frac{1}{\kappa^2}\left(\alpha - \frac{\beta}{3}\right)R^2 - \frac{\gamma}{2\kappa^2}R + \dots\dots \tag{37}$$

contains the following:

i) One massive $(2, \frac{3}{2}, \frac{3}{2}, 1)$ multiplet with mass $M^2 = -\gamma/2\beta$, given by $e_{a\mu}$, $\hat{R}_\mu$, $\hat{A}_\mu$. If there are no tachyons ($\alpha > 0$, $\beta < 0$), all excitations are ghosts.

ii) One massive $(\frac{1}{2}, 0^+, 0^-)$ multiplet with mass $\gamma/(12\alpha)$, given by S, P, and one of the two solutions of $(\Box - m^2)\gamma\cdot R = 0$. These fields are physical if there are no tachyons.

iii) Another massive $(\frac{1}{2}, 0^+, 0^-)$ multiplet with the same mass, given by the other spin-$\frac{1}{2}$ solution of $(\Box - m^2)\gamma\cdot R = 0$, $\partial\cdot A$ and the scalar mode in $R + R^2$. These particles are physical if all masses are real. For example, in a suitable gauge $R_\mu \sim \not{\partial}\psi_\mu$ and decomposing $\left[(\not{\partial} + m)(\not{\partial} - m)\not{\partial}\right]^{-1}$ one finds that both residues of the massive spin-$\frac{1}{2}$ excitations are positive.

iv) The usual massless spin $(2, \frac{3}{2})$ multiplet.

4. THE MAXWELL-EINSTEIN SYSTEM

The Maxwell-Einstein system was obtained by coupling the spin-$(1, \frac{1}{2})$ vector multiplet to the spin-$(2, \frac{3}{2})$ gauge action of Poincaré supergravity and was the first example of a matter coupling[6]. Action and transformation rules were obtained by adding successively at each order in κ extra terms to action and transformation rules such that at each order in κ invariance was obtained. It was also shown that this process must stop after a finite number of steps if no spin-0 fields are present[6]. The action obtained reads

$$\mathcal{L} = \mathcal{L}(\mathrm{S.G.}) - \frac{e}{4} F_{\mu\nu}(V)^2 - \frac{e}{2} \bar{\lambda} \gamma^\mu D_\mu \lambda + \frac{3}{64} e\kappa^2 (\bar{\lambda} \gamma_5 \gamma_\rho \lambda)^2$$
$$+ \frac{e\kappa}{2} \bar{\psi}_\mu \sigma^{\alpha\beta} \gamma^\mu \lambda \, F_{\alpha\beta}(V) + \frac{e\kappa^2}{4} (\bar{\psi}_\mu \sigma^{\rho\sigma} \gamma^\mu \lambda)(\bar{\psi}_\rho \gamma_\sigma \lambda) \tag{38}$$

We repeat that D_μ always denotes the gravitational covariant derivative with gauge torsion but not matter torsion. The transformation rules were found to be

$$\delta e^a{}_\mu = \kappa \bar{\epsilon} \gamma^a \psi_\mu \, , \qquad \delta V_\mu = -\bar{\epsilon} \gamma_\mu \lambda$$
$$\delta \psi_\mu = \frac{2}{\kappa} D_\mu \epsilon + \frac{\kappa}{4} (\bar{\lambda} \gamma_5 \gamma^\rho \lambda)(\gamma_\rho \gamma_\mu - \sigma_{\rho\mu}) \gamma_5 \epsilon$$
$$\delta \lambda = [F_{\mu\nu}(V) + \kappa \bar{\psi}_\mu \gamma_\nu \lambda] \sigma^{\mu\nu} \epsilon \tag{39}$$

We will now introduce auxiliary fields such that the commutator algebra closes; this will simplify these results and clarify their structure.

The first point to be observed is that the Poincaré supergravity gauge action with auxiliary fields $\mathcal{L}(\mathrm{SG}, e, \psi, S, P, A)$ is separately invariant under the gauge transformation of Section 2, and thus the gauge action need no longer be considered. This is because we expect the gauge algebra to be model-indepedent and, in particular, the gauge transformation rules to be independent of the matter fields. Their separation of the action into separately invariant parts is one of the great advantages of introducing auxiliary fields. The second point to be stressed is that globally supersymmetric matter systems have in general their own auxiliary fields needed to close the global algebra in flat space. Obviously, these matter auxiliary fields must be carried over to the locally supersymmetric extension of the theory. In the case of the vector multiplet there is one pseudoscalar auxiliary field D, which enters into the action simply as

$$\mathcal{L} = \frac{e}{2} D^2 \tag{40}$$

and into the flat-space transformation rules as

$$\delta \lambda = F_{\alpha\beta}(V) \sigma^{\alpha\beta} \epsilon + i D \gamma_5 \epsilon \; ; \; \delta V_\mu = -\bar{\epsilon} \gamma_\mu \lambda \; ; \; \delta D = i \bar{\epsilon} \gamma_5 \not{\partial} \lambda \tag{41}$$

A very important remark is that the Maxwell-Einstein system is locally scale invariant (Weyl invariant). It is thus also globally conformal invariant, and it seems plausible that it can be made locally superconformally invariant and be coupled to the superconformal gauge action. (This we will indeed do below.) Since the superconformal gauge action only contains the axial-vector auxiliary field A_μ, it follows that one only needs the auxiliary field A_μ and not S or P in the action and transformation rules for the coupling of the vector multiplet to the conformal supergravity gauge action. We will now argue that also the coupling of the vector multiplet to the Poincaré supergravity gauge action will only involve A_μ. Since the Poincaré transformation rules are a linear combination of the conformal Q- and S- supersymmetry rules, one must find these conformal Q- and S- rules for the matter multiplet. But since the superconformal gauge fields are inert under local conformal boosts K_μ, the matter fields will be locally K-inert as well, and from $[K_\mu, Q_a] = (\gamma_\mu)_a{}^b S_b$ it follows that the matter fields λ, A_μ, and D are also S-inert. [Note that this argument only applies to the vector multiplet, since it transforms according to the group; but it does not apply to the gauge fields, since their transformation rules have been modified by constraints 4).] Since the auxiliary fields S and P only entered into the coefficients of the S-supersymmetry laws, the transformation rules of λ, A_μ and D under local Poincaré supersymmetry will be independent of S and P. We conclude that only A_μ appears in the coupling of the vector multiplet to Poincaré supergravity. This field A_μ must be coupled in a locally chiral invariant way, since the superconformal gauge action is chirally gauge invariant. (We note that S and P still appear in the Poincaré supergravity gauge action and rules, but can be eliminated consistently, if desired.)

Finally, it should be stressed that field equations transform into field equations according to 1)

$$\delta I_{,j} = -I_{,k}\,\xi^{\alpha} R^{k}{}_{\alpha,j} \tag{42}$$

If D = 0, the λ field equation rotates under a local supersymmetry transformation into the V_μ field equation without a derivative on ε (since $\delta V_\mu = -\bar{\varepsilon}\gamma_\mu\lambda$; note that since $\delta\lambda = F\cdot\sigma\varepsilon + iD\gamma_5\varepsilon$, the opposite is not true). Thus, the λ-field equation must be supersymmetrically covariant, i.e. it does not contain $\partial\varepsilon$ if it is subjected to a local supersymmetry variation. This observation even dispenses one from explicitly constructing the Noether current of the matter action, since one can at once replace $\partial_\mu\lambda$ by its covariantization. [In the varied action $\partial\varepsilon$ terms of course do appear, since $\delta\mathscr{L} = \partial_\mu\kappa^\mu$ with κ^μ ε-dependent].

With these four preceding observations one can at once write down the action and transformation rules without any algebra

$$\mathcal{L} = \mathcal{L}(PSG; e, \psi, S, P, A) - \frac{e}{4} F_{\mu\nu}(V)^2 + \frac{e}{2} D^2 - \frac{e}{2} \bar{\lambda} \gamma^\mu \tilde{\hat{D}}_\mu \lambda$$
$$+ \frac{e\kappa}{4} \left(\bar{\psi}_\mu \sigma^{\alpha\beta} \gamma^\mu \lambda \right) F_{\alpha\beta}(V)$$

$$\delta\lambda = \hat{F}_{\mu\nu} \sigma^{\mu\nu} \epsilon + i D \gamma_5 \epsilon \; ; \quad \delta A_\mu = - \bar{\epsilon} \gamma_\mu \lambda$$
$$\delta D = i \bar{\epsilon} \gamma_5 \gamma^\mu \tilde{\hat{D}}_\mu \lambda - \frac{\kappa}{2} \bar{\epsilon} \gamma \cdot \psi D \equiv i \bar{\epsilon} \gamma_5 \gamma^\mu \bar{D}_\mu \lambda \tag{43}$$

Here $\hat{F}_{\mu\nu}$ is the covariant field strength

$$\hat{F}_{\mu\nu} = \left(\partial_\mu V_\nu + \frac{\kappa}{2} \bar{\psi}_\mu \gamma_\nu \lambda \right) - \left(\mu \leftrightarrow \nu \right) \tag{44}$$

while $\bar{D}_\mu \lambda$ is the completely covariant derivative*) i.e. gravitationally, supersymmetric and chirally covariant

$$\bar{D}_\mu \lambda = D_\mu \lambda + \frac{i\kappa}{2} A_\mu \gamma_5 \lambda - \frac{\kappa}{2} \hat{F}_{\alpha\beta} \sigma^{\alpha\beta} \psi_\mu - \frac{i\kappa}{2} D \gamma_5 \psi_\mu \tag{45}$$

The connection terms are of course the Noether couplings arising from the universal coupling

$$\bar{\psi}_\mu J^\mu_{SS} + A_\mu J^\mu_5 + e_{a\mu} T^{a\mu} \tag{46}$$

which have been further covariantized. The last term in the action is a consequence of the requirement that the λ field equation be covariant but not the action. Terms with one λ appear in the field equation with a weight ½ with respect to terms quadratic in λ, and thus one must give the terms linear in λ an extra factor two in the action. A final simplification occurs by replacing eD^2 by $(D')^2$ in the action and transformation rules. Then the D' law becomes

$$\delta D' = i e^{1/2} \bar{\epsilon} \gamma_5 \gamma_\mu \tilde{\hat{D}}_\mu \lambda \tag{47}$$

and the whole theory is truly minimal**).

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*) Note that Eq. (42) requires that the D-term in the derivative on λ be absent in the action.

**) We thank Dr. M. Roček for this observation.

It is thus clear that the complete coupling is simply a minimal coupling with respect to gravity, supersymmetry and chirality; or, equivalently, that replacing ordinary derivatives by completely covariant derivatives yields at once the total action and transformation rules.

Also clear is that replacing in the total action the Poincaré supergravity gauge action $\mathscr{L}(PSG, \bar{e}, \psi, S, P, A)$ by the conformal supergravity gauge action $\mathscr{L}(CSG, e, \psi, A)$ yields again an invariant action, such that the sum is again invariant.

We observe that these above results remain unchanged for the Yang-Mills case[6]. One can also add separately invariant actions, such that the sum is again invariant. For example, one can add a term $\alpha S + \beta P$ plus corresponding $\psi\psi$ terms to the Poincaré or conformal supergravity action, plus the vector multiplet. The sum of all these terms is again invariant since the auxiliary field equations do not interfere.

The interplay between the conformal and Poincaré supergravity should have important consequences for other matter couplings as well. For instance, one can immediately couple the scalar multiplet in improved form to Poincaré supergravity. Recently, the improved (superconformal) scalar multiplet was coupled to conformal supergravity[13]. In that theory the scalar multiplet is unphysical and by fixing the local axial, dilatational and S-supersymmetry gauge, one regains Poincaré supergravity. Using our techniques, one can instead couple a physical scalar multiplet to Poincaré supergravity. The corresponding action is the sum of the Poincaré action with auxiliary fields[1] and the action for the improved (superconformal) multiplet. The Poincaré supersymmetry transformation laws, which leave this action invariant, are the superconformal Q laws plus a field-dependent uniform S transformation with parameter $\varepsilon_S = \frac{1}{3}(i\gamma_5 P + i\not{A}\,\gamma_5 - S)\varepsilon$. This reproduces the Poincaré transformation rules for the gauge fields e_μ^a, ψ_μ, S, P, A_μ while only the matter field χ gets modified by a term $\delta\chi = (A - iB\gamma_5)\varepsilon_S$. The transformation rules for S supersymmetry can be obtained by using Jacobi identities and $[K_\lambda, Q] = \gamma_\lambda S$, while $[Q,\chi]_+ \sim \not{\partial}(A + iB\gamma_5)$ and $\partial_\mu A \sim [P_\mu, A]$. Since all fields are K-inert, the desired rules follow immediately.

5. PURE SUPERGRAVITY WITH COSMOLOGICAL CONSTANT

We next consider the gauge action of de Sitter supergravity. Action and transformation rules are given by

$$\mathcal{L} = \mathcal{L}(SG, e, \psi) + \frac{eg}{\kappa}\,\bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu + \frac{3eg^2}{\kappa^4} \tag{48}$$

$$\delta e^a_\mu = \kappa \bar{\epsilon} \gamma^a \psi_\mu \quad ; \quad \delta\psi_\mu = \frac{2}{\kappa}\left(D_\mu \epsilon + \frac{g}{2\kappa}\gamma_\mu \epsilon\right) \tag{49}$$

Hence it is the sum of the Poincaré supergravity gauge action plus a cosmological and a ψ-mass term; the latter is actually needed in a de Sitter space in order that ψ_μ remains massless. We repeat that $\omega_{\mu ab} = \omega_{\mu ab}(e) + K_{\mu ab}(e,\psi)$.

As noted in the last section we can put aside the action $\mathscr{L}$(SG), since it is separately invariant. From the term $\partial\psi_\mu \sim \gamma_\mu \varepsilon$ above, we see that only the term $\partial\psi_\mu \sim \gamma_\mu \varepsilon S$ survives in the full transformation law of ψ_μ after elimination of the auxiliary fields. Hence, only S appears in the matter action. The most general expression which contains a term linear in S, is given by

$$\mathcal{L}_S = a e \frac{S}{\kappa^2} + b \frac{e}{\kappa} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu + c \frac{e}{\kappa} \bar{\psi}\cdot\psi \tag{50}$$

Recalling the transformation laws of Section 2

$$\delta S = \frac{1}{2} \bar{\epsilon}\gamma\cdot R - \frac{\kappa}{2} \bar{\epsilon} \gamma\cdot\psi S + \cdots \tag{51}$$

$$\delta\psi_\mu = \frac{2}{\kappa} D_\mu \epsilon + \frac{1}{3}\gamma_\mu \epsilon S + \cdots \tag{52}$$

one finds from the $\varepsilon\psi S$ terms and the $\bar{\varepsilon}R$ terms in the action, using that $\sigma^{\mu\nu}D_\mu\psi_\nu =$ $= {}^1\!/\!_4\cdot R$, the result $c = 0$ and $a = 2b$, so that

$$\mathcal{L}_S = eb\left[\frac{1}{\kappa}\bar{\psi}_\mu \sigma^{\mu\nu}\psi_\nu + \frac{2}{\kappa^2} S\right] \tag{53}$$

Eliminating S from the S-dependent terms in the total action

$$\delta_S\left(-\frac{e}{3}S^2 + 2b\frac{e}{\kappa^2}S\right) = 0 \quad \longrightarrow \quad S = \frac{3b}{\kappa^2} \tag{54}$$

and substituting this result back into action and transformation rules, one obtains with $P = A_\mu = 0$

$$\mathcal{L} = 3\frac{e b^2}{\kappa^4} + \frac{eb}{\kappa}\bar{\psi}_\mu \sigma^{\mu\nu}\psi_\nu \;, \quad \delta\psi_\mu = \frac{2}{\kappa} D_\mu \epsilon + \frac{b}{\kappa^2}\gamma_\mu \epsilon \tag{55}$$

Clearly, with $b = g$ one obtains the desired equations.

In this example, the auxiliary field S was essential. In principle, one could also start from P; such a term would break parity. However, no new features arise since the chiral rotation $\psi_\mu \rightarrow \exp(-i\pi/4\ \gamma_5)\psi_\mu$ and $S \rightarrow P$ transforms this case back to the previous case. In fact, any $(\alpha S + \beta P)$ term leads to a cosmological term with positive coefficient. More interesting is the case of adding a term linear in D to the vector multiplet action, and extending this to a locally supersymmetric invariant. This might have important consequences for the elimination of unwanted cosmological terms.

Note added: After completion of this manuscript we received a preprint by K. Stelle and P.C. West, ICTP preprint 77-78/6, who have found the same results as in Ref. 1, except the matter coupling and relation to conformal supergravity.

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