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An Application of Modular Inclusion to Quantum Field Theory in Curved Space-Time

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Abstract

Applying recent results by Borchers connecting geometric modular action, modular inclusion and the spectrum condition, earlier results by Kay and Wald concerning the temperature of physically significant states of the linear Hermitean scalar field propagating in the background of a space-time with a bifurcate Killing horizon are generalized.

1 Introduction

In this note we shall address properties of certain states in quantum field theories on a class of curved space-times, more specifically, in theories where a linear Hermitean scalar field propagates in the background of a space-time with a bifurcate Killing horizon. Recently Kay and Wald [5] have studied thermal and uniqueness properties of isometry-invariant Hadamard states in such a situation. Space-times with a bifurcate Killing horizon can be viewed as a generalization of black-hole space-times, and one natural question of physical interest is under which conditions thermal equilibrium states of quantum fields propagating in such a background are forced to assume the Hawking temperature; the Hawking temperature itself provides information

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about the geometry of the background space-time, since it is in one-one correspondence with the mass of the black hole and the surface gravity of the bifurcate Killing horizon of the underlying space-time. Kay and Wald restricted their attention to Hadamard states; here, however, we do not make this restriction and concentrate on the issue of whether the modular objects associated with certain states on (subalgebras of) the Weyl algebra of the linear Hermitean scalar field in a space-time with a bifurcate Killing horizon contain information about the underlying space-time geometry, in particular in this class of examples, about the mass of the black hole.

In studying physically significant states in terms of the geometric content of their modular objects, we are motivated by a program outlined in [3] that hopes to characterize such states by their geometric modular action. Though this is our motivation, the tools that we shall be employing here are due to Borchers [1] and Wiesbrock [8], who established the intimate interconnection of a weak form of geometric modular action, the spectrum condition and the notion of modular inclusion (to be discussed below).

In the next two sections we shall introduce the geometrical setting in space-times with a bifurcate Killing horizon, prove a crucial relationship between two geometric actions on this horizon, and define the net of observable algebras associated with a linear Hermitean scalar field on such a space-time, following [5]; since their exposition is quite detailed, we shall be brief in describing these matters. In Section 4 we shall state and prove our main results, which is essentially that even without the assumption that the state be Hadamard (or even Hadamard in a neighborhood of the horizon), the geometric modular action forces any KMS-state on the horizon to adopt the Hawking temperature.

2 Space-times With a Bifurcate Killing Horizon

A space-time with a *bifurcate Killing horizon* is characterized by a quintupel $(M, g, \tau_t, \Sigma, \mathbf{h})$, where (M, g) is a globally hyperbolic space-time, $\{\tau_t\}$ is a (nontrivial) one-parameter group of isometries of (M, g), Σ is a two-dimensional spacelike submanifold, contained in a spacelike Cauchy-surface of M, which is left pointwise invariant under the action of $\{\tau_t\}$, i.e. $\tau_t(p) = p$ for all $t \in \mathbf{R}$, $p \in \Sigma$, and $\mathbf{h}$ is the bifurcate Killing horizon, i.e. the three-dimensional C^0-manifold in M formed by the lightlike geodesics emanating from Σ ; it is assumed that a choice of two continuous, linearly independent, lightlike, future-directed vector fields χ_A^a, χ_B^a along Σ can be made.

Let γ_{Ap} and γ_{Bp} be the maximal geodesics defined by $\chi_A^a(p)$ and $\chi_B^a(p)$, respectively, for $p \in \Sigma$. These are lightlike geodesics which are left invariant under the action of $\{\tau_t\}$. This means, in particular, that the corresponding Killing field ξ^a is tangent to these geodesics, and geodesics starting at different points $p \in \Sigma$ cannot cross. One defines the following pieces of $\mathbf{h}$: $\mathbf{h}_X$ is the subset of $\mathbf{h}$ generated by γ_{Xp}, $p \in \Sigma$, and $\mathbf{h}_X^{\pm} \equiv \mathbf{h}_X \cap I^{\pm}(\Sigma)$ for $X \in \{A, B\}$. Then we set

$$\mathbf{h}_A^R \equiv \mathbf{h}_A^+ , \quad \mathbf{h}_A^L \equiv \mathbf{h}_A^-$$

and

$$\mathbf{h}_B^R \equiv \mathbf{h}_B^- , \quad \mathbf{h}_B^L \equiv \mathbf{h}_B^+ .$$

By convention, it will be assumed that ξ^a is future oriented on $\mathbf{h}_A^R$. A point $q \in \mathbf{h}_A$ ($\mathbf{h}_B$) can be coordinatized by a pair (U, p) (resp. (V, p)), where the point $p \in \Sigma$ determines on which geodesic q lies and the affine parameter U (resp. V) indicates where on the specified geodesic q lies, so that we have $\gamma_{Ap}(U) = q$ (resp. $\gamma_{Bp}(V) = q$). We assume that the affine parameters are chosen such that $\gamma_{Ap}(U = 0) = p$ (resp. $\gamma_{Bp}(V = 0) = p$). As we have already mentioned, ξ^a is tangent to the geodesics γ_{Ap} and γ_{Bp}, whose tangent vector fields will be denoted by χ_{Ap}^a and χ_{Bp}^a, and it can be shown that there exists a smooth function f_A^R, defined on $\mathbf{R}^+ \times \Sigma$, positive and strictly increasing with U, such that

$$\xi^a(U, p) \; = \; f_A^R(U, p)\, \chi_{Ap}^a(U) \, , \tag{1}$$

for points $(U, p) \in \mathbf{h}_A^R \equiv \mathbf{R}^+ \times \Sigma$, and the quantity

$$\kappa \equiv \xi^a \nabla_a \ln(f_A^R) \; > \; 0 \tag{2}$$

is a constant, i.e. independent of U and p (see [5]). κ is called the *surface gravity* of $\mathbf{h}_A$. Similar arguments apply with functions f_A^L, f_B^R and f_B^L for the other parts of $\mathbf{h}$, yielding the same κ [1]. This implies that the action of $\{\tau_t\}$ on points of the bifurcate Killing horizon is of the following form, which relates the action of the Killing flow on the horizon to the action of the affine dilatations:

Lemma 1: Under the stated assumptions, one has

$$\tau_t(U, p) \; = \; (e^{\kappa t} U, p) \quad \text{and} \quad \tau_t(V, p) \; = \; (e^{-\kappa t} V, p).$$

Proof. Choose arbitrary $U > 0$ and $p \in \Sigma$ and set $\tau_t(U, p) \equiv r(t)$ for $t \in \mathbf{R}$. From (1) one deduces

$$\frac{d}{dt} r(t) \; = \; f_A^R \circ r(t), \tag{3}$$

and from (2) it follows that

$$\frac{d}{dt} \ln(f_A^R \circ r(t)) \; = \; \frac{1}{f_A^R \circ r(t)} (f_A^R)' \circ r(t) \frac{d}{dt} r(t) \; = \; \kappa \, .$$

Hence one has

$$\frac{d}{dt}(f_A^R \circ r(t)) \; = \; \kappa (f_A^R \circ r(t)) \, ,$$

which implies

$$f_A^R \circ r(t) \; = \; e^{\kappa t + C} \, ,$$

and thus, with (3)

$$r(t) \; = \; \frac{1}{\kappa} e^{\kappa t + C} + \tilde{C} \, .$$

Now one has $r(t) = (\hat{\tau}_t(U), p)$, and hence

$$\hat{\tau}_t(U) \; = \; \frac{1}{\kappa} e^{\kappa t + C(U)} + \tilde{C}(U) \, .$$

By the group property $\hat{\tau}_{t'} \hat{\tau}_t(U) = \hat{\tau}_{t'+t}(U)$ for all $t', t \in \mathbf{R}$, one easily obtains $\tilde{C}(U) = 0$. Then $U = \hat{\tau}_0(U) = \frac{1}{\kappa} e^{C(U)}$, implying $\hat{\tau}_t(U) = e^{\kappa t} U$. The argument for $\mathbf{h}_A^L$ ($U < 0$) and for $\mathbf{h}_B$ (i.e. $\tau_t(V, p) = (e^{-\kappa t} V, p)$) is similar. □

3 The Weyl Algebra of the Linear Hermitean Scalar Field in a Space-time with a Bifurcate Killing Horizon

We next introduce a net of local algebras corresponding to the linear Hermitean scalar field, following Kay and Wald [5]. The relevant field

[1] the $f^L \ldots$ are negative and so one has to take $-f^L \ldots$ in the argument of ln in (2)

equation on (M,g) is the Klein-Gordon equation:

$$(\nabla^a \nabla_a + m^2)\varphi = 0\,, \qquad (4)$$

for $m \geq 0$. As explained in the "Note added in proof" in [5], it is necessary to consider special spaces of solutions of (4), since we wish to view certain Weyl algebras associated with symplectic spaces formed by characteristic data of (4) on the bifurcate Killing horizon as subalgebras of the Weyl algebra over the symplectic space of solutions of (4) whose restrictions to Cauchy surfaces have compact support.

Let $\mathcal{C}$ be a Cauchy surface for (M,g) and let, for C^1-functions ψ on M, $\rho_0\psi \equiv \psi|\mathcal{C}$ and $\rho_1\psi \equiv n^a\nabla_a\psi|\mathcal{C}$, with n^a denoting the future-directed unit-normal field of $\mathcal{C}$. We define S as the space of all *real-valued* C^2-solutions φ of (4) such that $\rho_0\varphi \in C_0^5(\mathcal{C})$ and $\rho_1\varphi \in C_0^4(\mathcal{C})$. S will be endowed with the symplectic form

$$\sigma(\varphi,\psi) \equiv \int_{\mathcal{C}} (\varphi\,\nabla_a\psi - \psi\,\nabla_a\varphi)n^a\, d\eta_{\mathcal{C}}\,,$$

where $d\eta_{\mathcal{C}}$ denotes the induced measure on $\mathcal{C}$. That σ is indeed a symplectic form and independent of $\mathcal{C}$ follows from standard theorems on existence and uniqueness of initial-value solutions of (4) in globally hyperbolic space-times (cf. [6]) and from Green's formula.

We next introduce some symplectic subspaces of (S,σ). Let S_A consist of all solutions φ in S such that there is a function $f \in C_0^\infty(\mathbf{h}_A)$ so that the characteristic data $\varphi|\mathbf{h}_A$ of φ on $\mathbf{h}_A$ have the form

$$\varphi|\mathbf{h}_A(U,p) = U^5 \frac{\partial^5}{\partial U^5} f(U,p)\,, \qquad (5)$$

for all $U \in \mathbf{R}$ and all $p \in \Sigma$ (by the results of the "Note added in proof" in [5] one obtains that (5) indeed implies $\varphi \in S$). Moreover, we shall say that φ is in the set S_A^R if the f in (5) lies in $C_0^\infty(\mathbf{h}_A^R)$. The subspaces S_B and S_X^Y for $X = A,B$ and $Y = R,L$ are defined analogously. The symplectic form $\sigma(\varphi,\psi)$ for elements $\varphi,\psi \in S_A$ takes the form

$$\sigma(\varphi,\psi) = \int_{\mathbf{h}_A} \left(\tilde\varphi(U,p)\frac{\partial}{\partial U}\tilde\psi(U,p) - \tilde\psi(U,p)\frac{\partial}{\partial U}\tilde\varphi(U,p)\right) dU\, d\eta_\Sigma(p)\,,$$

where $\tilde\varphi, \tilde\psi$ denote the restrictions of φ,ψ to $\mathbf{h}_A$ in the coordinatization of $\mathbf{h}_A$ which we have chosen; note that we shall henceforth maintain

this notation. The expression for $\sigma(\varphi,\psi)$ when $\varphi,\psi \in S_B$ is analogous. Therefore one can show that S_A, S_B and S_X^Y $(X = A,B;\, Y = R,L)$ are symplectic subspaces of (S,σ) (see [5] for further details). If we write

$$T_t\varphi \equiv \varphi \circ \tau_{-t}\,,$$

then $\{T_t\}$ is a symplectomorphism group on (S,σ). It is also clear from Lemma 1 that the action of $\{T_t\}$ on S_X^Y $(X = A,B;\, Y = R,L)$ and on S_A and S_B leaves these symplectic subspaces of (S,σ) invariant. Also, one can define on S_A (and likewise on S_B) an antisymplectic involution I by

$$(\widetilde{I\varphi})(U,p) \equiv \tilde\varphi(-U,p)\,;$$

note that on S_A, and also on S_B, I and $\{T_t\}$ commute, i.e. $T_t \circ I = I \circ T_t$. Notice furthermore that I maps S_X^R onto S_X^L, for $X = A,B$. Another group of symplectomorphisms on S_A, and on S_B, is given by the *affine translations*,

$$(\widetilde{\Lambda_a\varphi})(U,p) \equiv \tilde\varphi(U-a,p)\,,$$

for $a \in \mathbf{R}$. Observe that S_X^Y $(X = A,B;\, Y = R,L)$ are not left invariant under the action of $\{\Lambda_a\}$, but are left *half-sided invariant:*

$$\Lambda_a\varphi \in S_A^R \quad \text{for} \quad \varphi \in S_A^R,\, a \in \mathbf{R}^+ \qquad (6)$$

$$\Lambda_a\varphi \in S_A^L \quad \text{for} \quad \varphi \in S_A^L,\; a \in \mathbf{R}^-\,,\ \text{etc.}$$

By $\mathcal{A}$, $\mathcal{A}_A$, $\mathcal{A}_B$, $\mathcal{A}_X^Y$ we shall denote the Weyl algebras corresponding to the symplectic spaces (S,σ), $(S_A, \sigma\,|_{S_A})$, $(S_B,\sigma\,|_{S_B})$, $(S_X^Y, \sigma\,|_{S_X^Y})$, and by α_t, $\mathcal{I}$, λ_a the induced actions of τ_t, I, Λ_a on the appropriate Weyl algebras.

4 Modular Inclusion and the Hawking Temperature

Beginning with [1] and continuing with [8], [9], [10], [3] and [2], interesting connections between the spectrum condition and the 'geometric' action of modular objects have been established. We recall the first result of this nature.

Theorem 2 [1]: Let $\mathcal{M}$ be a von Neumann algebra acting on some Hilbert space $\mathcal{H}$ and assume that $\Omega \in \mathcal{H}$ is cyclic and separating for $\mathcal{M}$. Then let Δ, J be the modular operator and modular conjugation corresponding to $(\mathcal{M}, \Omega)$. Let $\mathsf{U}(a)$, $a \in \mathbf{R}$, be a continuous one-parametric group with positive generator leaving Ω invariant. If, in addition, $\mathsf{U}(a)\mathcal{M}\mathsf{U}(a)^* \subset \mathcal{M}$ for $a \geq 0$, then it follows that

$$\Delta^{it}\mathsf{U}(a)\Delta^{-it} = \mathsf{U}(e^{-2\pi t}a) \text{ and } \mathrm{J}\mathsf{U}(a)\mathrm{J} = \mathsf{U}(-a),$$

for all $t, a \in \mathbf{R}$; if, instead, $\mathsf{U}(a)\mathcal{M}\mathsf{U}(a)^* \subset \mathcal{M}$ for $a \leq 0$, then it follows that

$$\Delta^{it}\mathsf{U}(a)\Delta^{-it} = \mathsf{U}(e^{2\pi t}a) \text{ and } \mathrm{J}\mathsf{U}(a)\mathrm{J} = \mathsf{U}(-a),$$

for all $t, a \in \mathbf{R}$.

(For the proof, see Theorem II.9 in [1].)

Remark: In [8] Wiesbrock has proven an interesting converse to Borchers' result. He showed that if $\mathsf{U}(a)$ is a continuous unitary group such that $\mathsf{U}(a)\mathcal{M}\mathsf{U}(a)^* \subset \mathcal{M}$ for $a \geq 0$, and if

$$\Delta^{it}\mathsf{U}(a)\Delta^{-it} = \mathsf{U}(e^{-2\pi t}a) \text{ and } \mathrm{J}\mathsf{U}(a)\mathrm{J} = \mathsf{U}(-a),$$

for all $t, a \in \mathbf{R}$, then it follows that the generator of $\mathsf{U}(a)$ is positive.

We shall use Theorem 2 to show that any ground state on $\mathcal{A}_{A/B}$ with respect to $\{\lambda_a\}$ which is also a KMS-state with respect to $\{\alpha_t\}$ on $\mathcal{A}^R_{A/B}$ and on $\mathcal{A}^L_{A/B}$ *must* have the Hawking temperature.

Proposition 3: Let ω be a state on $\mathcal{A}_A$ ($\mathcal{A}_B$) and suppose that it is a ground state with respect to $\{\lambda_a\}$. Then ω is faithful on $\mathcal{A}^R_A$ and $\mathcal{A}^L_A$ ($\mathcal{A}^X_B$, $X = R, L$). Suppose also that it restricts to a KMS-state with inverse temperature $\beta \in \mathbf{R}\backslash\{0\}$ with respect to $\{\alpha_t\}$ on (a) $\mathcal{A}^R_A$ ($\mathcal{A}^R_B$) and (b) $\mathcal{A}^L_A$ ($\mathcal{A}^L_B$).
Then $\beta = 2\pi/\kappa$ for (a) and $\beta = -2\pi/\kappa$ for (b).

Remarks: (1) Of course, this quantity is, up to sign, exactly the (inverse) Hawking temperature (cf. [5] for further discussion).
(2) Note that this is analogous to the 'Rindler-Fulling-scenario', where the Minkowski vacuum, which is a ground state with respect to the

usual time-translations and which satisfies the spectral condition with respect to lightlike affine translations, restricts to a thermal equilibrium state on the algebra of local observables localized in the "right Rindler wedge" with respect to the Lorentz boosts leaving this region invariant. In fact, this scenario is simply an *example* of a space-time with a bifurcate Killing horizon (cf. [5]).
(3) Kay and Wald prove a related result with different hypotheses in Theorem 4.2 of [5]. Note, however, that we do not assume that the state is Hadamard (even in a neighborhood of the horizon) nor that there exists a wedge reversal isometry i of the space-time which commutes with τ_t, leaves Σ invariant, and reflects points on each of the horizon generators about Σ, as do Kay and Wald.

We shall now apply Theorem 2 to prove Proposition 3.

Proof. 1. Consider the case where ω is a state on $\mathcal{A}_A$. Let $(\mathcal{H}_\omega, \pi_\omega, \Omega_\omega)$ be the GNS-representation of ω. The first step is to show that Ω_ω is cyclic and separating for the von Neumann algebras

$$\mathcal{M}^R_A \equiv \pi_\omega(\mathcal{A}^R_A)'' \text{ and } \mathcal{M}^L_A \equiv \pi_\omega(\mathcal{A}^L_A)''.$$

Since ω is invariant under the induced action of $\{\lambda_a\}$, there is a unitary implementation $\{\mathsf{U}(a)\}$ of $\{\lambda_a\}$ on $\mathcal{H}_\omega$, i.e.

$$\mathrm{ad}\mathsf{U}(a) \circ \pi_\omega = \pi_\omega \circ \lambda_a,$$

for all $a \in \mathbf{R}$, such that the unitaries $\mathsf{U}(a)$ leave Ω_ω invariant. Moreover, since ω is a ground state, $\{\mathsf{U}(a)\}$ has a generator with positive spectrum. Now assume that $\Psi \in \mathcal{H}_\omega$ is such that $\langle \Psi, A\Omega_\omega \rangle = 0$, for all $A \in \mathcal{M}^R_A$. Then also

$$0 = \langle \Psi, \mathsf{U}(a)A\mathsf{U}(a)^*\Omega_\omega \rangle = \langle \Psi, \mathsf{U}(a)A\Omega_\omega \rangle; \qquad (7)$$

for all $A \in \mathcal{M}^R_A$ and all $a \geq 0$. The function $a \mapsto \langle \Psi, \mathsf{U}(a)A\Omega_\omega \rangle$ is continuous on $\mathbf{R}$ and possesses an analytic extension to the upper complex half-plane $\mathrm{Im}z > 0$. By Schwarz' reflection principle, it can be extended to an analytic function in the region $\mathrm{Re}z > 0$ where it is equal to zero by (7), and hence one may conclude that

$$\langle \Psi, \mathsf{U}(a)A\mathsf{U}(a)^*\Omega_\omega \rangle = 0,$$

for all $a \in \mathbf{R}$ and all $A \in \mathcal{M}_A^R$. But $\bigcup_{a\in\mathbf{R}} \mathsf{U}(a)\mathcal{M}_A^R\mathsf{U}(a)^*$ generates $\pi_\omega(\mathcal{A}_A)''$, and thus it follows that $\Psi = 0$, i.e. Ω_ω is cyclic for $\mathcal{M}_A^R$. The proof of the cyclicity for $\mathcal{M}_A^L$ is similar. Since

$$\mathcal{M}_A^R \subset (\mathcal{M}_A^L)' \quad \text{and} \quad \mathcal{M}_A^L \subset (\mathcal{M}_A^R)',$$

one finds that Ω_ω is also separating for $\mathcal{M}_A^R$ and $\mathcal{M}_A^L$. The arguments are the same when the index A is replaced by B.

2. By the stated assumption, ω restricts to a KMS-state at inverse temperature $\beta \neq 0$ with respect to $\{\alpha_t\}$ on $\mathcal{A}_A^R$. Therefore it follows that

$$\pi_\omega \circ \alpha_t | \mathcal{A}_A^R = \mathrm{ad}\Delta^{-it/\beta} \circ \pi_\omega | \mathcal{A}_A^R , \tag{8}$$

where Δ is the modular operator corresponding to $(\mathcal{M}_A^R, \Omega_\omega)$ (cf. Propositions 8.14.2-3 in [7]). Now notice that the assumptions of Borchers' Theorem are fulfilled, since $\mathrm{ad}\mathsf{U}(a)\mathcal{M}_A^R \subset \mathcal{M}_A^R$ for $a \geq 0$, using (6). Whence for all $\varphi \in S_A^R$ one concludes

$$\Delta^{it}\mathsf{U}(a)\Delta^{-it}(\pi_\omega \circ W)(\varphi)\,\Omega_\omega = \mathsf{U}(\mathrm{e}^{-2\pi t}a)(\pi_\omega \circ W)(\varphi)\,\Omega_\omega ,$$

where the $W(\varphi)$ are the Weyl operators of the CCR, and by (8) it now follows that

$$\pi_\omega \circ W(T_{t\beta} \circ \Lambda_a \circ T_{-t\beta}(\varphi))\Omega_\omega = \pi_\omega \circ W(\Lambda_{\exp(-2\pi t)a}(\varphi))\Omega_\omega ,$$

for all $t \in \mathbf{R}$, $a \geq 0$ and $\varphi \in S_A^R$. By the separability of Ω_ω and the injectivity of $\varphi \mapsto \pi_\omega \circ W(\varphi)$, this implies

$$T_{t\beta} \circ \Lambda_a \circ T_{-t\beta}(\varphi) = \Lambda_{\exp(-2\pi t)a}(\varphi) ,$$

and hence, using Lemma 1,

$$\tilde{\varphi}(\mathrm{e}^{-\kappa\beta t}(\mathrm{e}^{\kappa\beta t}U - a), p) = \tilde{\varphi}(U - \mathrm{e}^{-2\pi t}a, p)$$

for the characteristic data $\tilde{\varphi}$ on $\mathbf{h}_A^R$ of every $\varphi \in S_A^R$, and for all $U > 0$, $a \geq 0$, $t \in \mathbf{R}$ and $p \in \Sigma$. Differentiation with respect to t at $t = 0$ yields

$$\kappa\beta a\tilde{\varphi}'(U - a, p) = 2\pi a\tilde{\varphi}'(U - a, p)$$

and hence $\beta = 2\pi/\kappa$, as claimed. The proof of the other statements is similar. □

Remark: We comment that by employing the converse to Theorem 2 proven by Wiesbrock and by adapting the arguments in part (2) of the proof just presented, then one can show that if ω is affine translation invariant and restricts on the horizon algebras to a Killing flow KMS-state with the Hawking temperature, then ω is actually a vacuum state for the affine translations.

Of course, the natural question arises whether states on $\mathcal{A}_{A/B}$ satisfying the assumptions of Proposition 3 exist at all. But there do indeed exist such states. For example, we may take

$$w_\omega(\varphi, \psi) = -\frac{1}{\pi} \lim_{\epsilon \to 0+} \int \frac{\tilde{\varphi}(U_1, p)\,\tilde{\psi}(U_2, p)}{(U_1 - U_2 - i\epsilon)^2}\, d\eta_\Sigma(p)\, dU_1\, dU_2 \tag{9}$$

as the two-point function of a quasifree state ω on $\mathcal{A}_A$, which has in fact all the desired properties (cf. [5]). In connection with this it is interesting to note that Kay and Wald [5] proved that every quasifree Hadamard state which is invariant under the action of $\{\alpha_t\}$ on $\mathcal{A}$ must restrict to a quasifree state on $\mathcal{A}_A$ with the two-point function given in (9) [2]. A different question is, however, if states on the larger algebra $\mathcal{A}$ exist such that their restrictions to $\mathcal{A}_A$ and/or $\mathcal{A}_B$ satisfy the assumptions of Proposition 3. It may well be that this is not possible if the space-time contains more than one bifurcate Killing horizon which have different surface gravities. And, in fact, as in Section 6.3 in [5], if one considers as an example the Schwarzschild-deSitter space-time, where there is a pair of neighboring bifurcate Killing horizons with unequal surface gravities, then it is clear from Proposition 3 that there cannot exist a state on the horizons that is simultaneously a ground state for the affine translations and a KMS-state for the Killing flow. There are other such examples, and though a theorem analogous to Theorem 6.5 in [5] can be formulated, we leave it as an exercise for the reader.

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[2]In fact, with the new argument of Kay [4], the assumption that the state be quasifree (which, of course, we do not have to make) in [5] may be dropped, though the Hadamard condition is still crucial for their work.

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