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Initial State Radiation Corrections to Off-Shell W-Pair Production in e^+e^--Annihilation †

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ABSTRACT

We calculate the gauge-invariant initial-state lowest order, $O(\alpha)$ radiative corrections for on- and off-shell W-pair production. The soft photon inclusive exponentiation is included. This result generalizes the convolution formula, which is know from the description of the Z resonance, to the case of the production of two W-bosons. Including the Coulomb singularity, the dominant corrections are covered. We present numerical results in a large energy range from LEP 200 up to NLC, $\sqrt{s} = 1$ TeV.

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1 Introduction

We calculate the initial-state radiation (ISR) to order $\mathcal{O}(\alpha)$ with soft-photon exponentiation for on- and off-shell W-pair production:

$$e^+e^- \rightarrow W^+W^- + n\gamma, \tag{1}$$

$$e^+e^- \rightarrow (W^+W^-) \rightarrow 4f + n\gamma. \tag{2}$$

Problems related to a proper definition of ISR as well as those related to the W virtuality are discussed. Further, we draw numerical conclusions.

2 Born Cross Section

The virtuality of the W-pair is very influential near the resonance production threshold, and it may not be neglected at higher energies $\sqrt{s} >> 4M_W^2$; see Fig. 1.

The reaction (1) is described by the cross section [1]:

$$\sigma_B^{on}(s) = \sigma_0(s; M_W^2, M_W^2), \tag{3}$$

corresponding to the diagrams with γ and Z exchange in s- channel and ν-exchange in t-cahnnel. The virtuality of the W's may be taken into account as follows [2]:

$$\sigma_B^{off}(s) = \int_0^s ds_1\, \rho(s_1) \int_0^{(\sqrt{s}-\sqrt{s_1})^2} ds_2\, \rho(s_2)\, \sigma_0(s; s_1, s_2), \tag{4}$$

$$\rho(s_i) = \frac{1}{\pi} \frac{\sqrt{s_i}\,\Gamma_W(s_i)}{|s_i - M_W^2 + i\sqrt{s_i}\,\Gamma_W(s_i)|^2} \times \mathrm{BR}(i). \tag{5}$$

One could use also a constant width function: $\sqrt{s}\,\Gamma_W(s) \rightarrow M_W\Gamma_W$. Effectively, this corresponds to a re-definition of M_W [3]: $M_W \rightarrow \bar{M}_W = M_W + \frac{1}{2}\Gamma_W^2/M_W \approx M_W + 26$ MeV. We studied numerically the effect of the s-dependent width on the WW-excitation curve. It lowers the cross section in the region of its steep rise, reaching maximal value $\approx -1.7\%$ at $\sqrt{s} \approx 160GeV$. Above $\sqrt{s} \approx 180GeV$, the effect is positive and very small ($\leq 0.1\%$).

3 Background

The virtuality of the W bosons bares certain difficulties for a proper description of the W excitation curve. The reaction (1) (with n=0) proceeds via three diagrams, while (2) (with n=0) may proceed in addition through channels with only one or even zero W boson in the intermediate state – the so-called background processes. Table 1 contains the number of diagrams which contribute to certain 4f final states (in the unitary gauge and neglecting Higgs boson exchange).

Although it seems at first glance that a complete semi-analytical, non-MC treatment of the background is hopeless, we managed to perform such a calculation for the production of two different fermion doublets, if no electrons are produced [4] (numbers in bold face in the Table 1). Another class of background processes, if one of the fermions is an electron, is more involved as there are also diagrams of the Bhabha type (e.g., with t-channel photon exchange). More diagrams include the case when two charge-conjugated doublets are produced (numbers in italic in Table 1). The number of diagrams increases due to other virtual states (ZZ, $Z\gamma$, and so on) involved.

We show in [4] (see also [8]) that the background contributions are relatively small; nevertheless, their analytical calculation is an interesting problem since the Feynman diagrams are of the same type as for many other 4f production processes, e.g., the Higgs boson production via the Bjorken process. Further, one should mention that the background contributions are necessary for a gauge-invariant treatment of the off-shell W-pair production.

	$\bar{d}u$	$\bar{s}c$	$\bar{e}\nu_e$	$\bar{\mu}\nu_\mu$	$\bar{\tau}\nu_\tau$
$d\bar{u}$	*43*	**11**	20	**10**	**10**
$e\bar{\nu}_e$	20	20	*56*	18	18
$\mu\bar{\nu}_\mu$	**10**	**10**	18	*19*	**9**

Table 1: *Number of Feynman diagrams contributing to the production of two fermion doublets.*

4 Universal Part of the Initial-State Radiation

For the cross section corresponding to the diagrams with γ and Z exchange in the s channel, the ISR can be treated in the same way as for the Z line shape [5]. The leading ISR corrections to the cross section due to diagram with ν exchange in the t channel (and st-interference) must also be the same in order to ensure that the gauge cancellations between the Born diagrams are not spoiled:

$$\sigma^{\rm off}_{\rm univ}(s) = \int_{(\sqrt{s_1}+\sqrt{s_2})^2}^{s} ds' \left[\beta_e v^{\beta_e-1}(1+\bar{S}) + \bar{H}\right] \sigma^{\rm off}_{\rm B}(s'). \qquad (6)$$

For a calculation of the on-shell cross section, one has to replace the $\sigma^{\rm off}_{\rm B}(s')$ by $\sigma^{\rm on}_{\rm B}(s')$ under the integral in (6). The numerical influence of the universal part of the ISR may be seen in Fig. 1, and the notational details are introduced in [6].

A correct treatment of ISR is faced with two problems, both of them being related to the t-channel exchange diagram:
(a) The factorized form (6) does not hold, even for the 'naive' ISR corresponding to the diagrams with a photon (real or virtual) attached to the external electronic lines. There are additional nonfactorizable (nonuniversal) photonic corrections.
(b) Such 'naive' treatment of the ISR is not gauge invariant.

5 The Current Splitting Technique

In earlier calculations, the problem of gauge invariance for the initial state $\mathcal{O}(\alpha)$ corrections has been circumvented by a complete numerical calculation of *all* the $\mathcal{O}(\alpha)$ corrections, including also final state radiation, and others. (see [7] for the on-shell, and [8] for the off-shell cases).

We tried to take advantage of the fact that ISR yields a large fraction of the net correction. For that purpose, we define a gauge-invariant ISR expression. To do so, we split the electrically neutral neutrino into two oppositely flowing leptons, each with charge one. One of them is combined with the 'naive' ISR diagrams to build a continuous flow of electric charge in the initial state; the other part of the split neutrino propagator then will be combined with final state photon emission and is neglected here. We call this technique, which is explained in more detail in [6], the *Current Splitting Technique.*

For the ISR, which is gauge-invariant now to order $\mathcal{O}(\alpha)$, we performed three angular integrals for on-shell and seven angular integrals for off-shell W-pair production. The cross section has then the following structure:

$$\frac{d^2\sigma^{\rm off}_{\rm a}}{ds_1 ds_2} = \int_{(\sqrt{s_1}+\sqrt{s_2})^2}^{s} ds' \rho(s_1)\,\rho(s_2) \left[\beta_e v^{\beta_e-1} \mathcal{S}_{\rm a} + \mathcal{H}_{\rm a}\right], \qquad (7)$$

$$\mathcal{S}_{\rm a}(s,s';s_1,s_2) = \left[1+\bar{S}(s)\right]\sigma^{\rm a}_0(s';s_1,s_2) + \sigma^{\rm a}_{\hat{S}}(s';s_1,s_2), \qquad (8)$$

$$\mathcal{H}_{\rm a}(s,s';s_1,s_2) = \bar{H}(s,s')\sigma^{\rm a}_0(s';s_1,s_2) + \sigma^{\rm a}_{\hat{H}}(s,s';s_1,s_2). \qquad (9)$$

Here, a=s,t,st denote the s,t-channel and st-interference contributions. For $a = t, st$, the functions $\mathcal{S}_{\rm a}(s,s';s_1,s_2)$ and $\mathcal{H}_{\rm a}(s,s';s_1,s_2)$ contain extra cross-section pieces with deviations from the structure of the s-channel case. They are *nonuniversal* with respect to s,t,st and *do not factorize* into a structure which contains the Born cross section as an explicit factor. Further, these extra pieces are *screened*, i.e., they have a damping overall factor,

$$\sigma^{\rm st,t}_{\hat{S},\hat{H}}(s';s_1,s_2) \sim \frac{s_1 s_2}{s^2}. \qquad (10)$$

It is this factor which ensures the unitary behavouir of the nonuniversal terms at high energy even for individual (st and t) contributions.

6 Numerical Results and Discussion

Numerical results have been obtained with the Fortran program GENTLE [9]. In Figure 1, we show the cross sections $\sigma^{\rm on}_{\rm B}$ and $\sigma^{\rm off}_{\rm B}$ for Born on- and off-shell W-pair production, and also $\sigma^{\rm off}_{\rm univ}$ for the off-shell production with universal ISR. Compared to $\sigma^{\rm on}_{\rm B}$, the cross section $\sigma^{\rm off}_{\rm B}$ develops a tail. The other tail phenomenon is due to the universal ISR. It is much more pronounced, but weaker than one observed for the narrow Z-resonance due to the broader shape of the W-pair production cross section. In the figure, the radiatively corrected on-shell cross section $\sigma^{\rm on}_{\rm univ}$ is not shown. We mention that the relative differences between $\sigma^{\rm on}_{\rm B}$ and $\sigma^{\rm off}_{\rm B}$, and between $\sigma^{\rm on}_{\rm univ}$ and $\sigma^{\rm off}_{\rm univ}$, are similar.

From Figure 2, one may see that the difference between the universal and the complete ISR corrections [1] is relatively small over a large energy region. The nonuniversal corrections $\sigma_{\hat{S}}$ and $\sigma_{\hat{H}}$ increase the total cross section by +0.4%; +0.8%; +1.5% at $\sqrt{s} =$ 165; 500; 1000 GeV, respectively. In this figure, we show also the important part of the final state corrections – the so-called Coulomb singularity. It yields a positive correction, which has its maximum value of about 6 percent at the threshold and vanishes at high energies. For instance at $\sqrt{s}$=1000 GeV it amounts .75 percent. The Coulomb correction is taken into account according to Eq.(5) of Ref. [11]. We would like to note, however, that at high energies other final state corrections are important [7].

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[1] The numerical results for the complete ISR in [6] were overestimated due to a sign error in the analytical calculations [10].

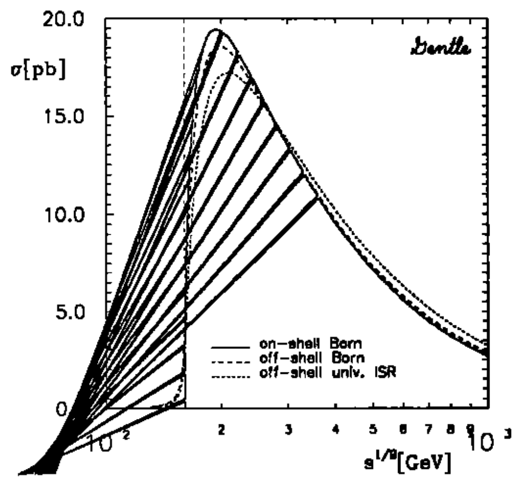

Figure 1: *Total cross section for W-pair production: σ_B and universal ISR corrections.*

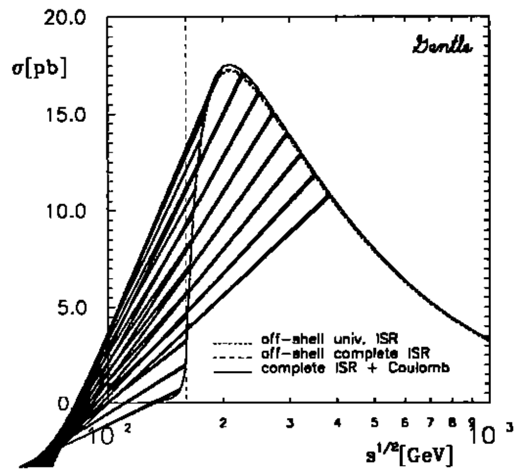

Figure 2: *Total cross section for W-pair production with complete ISR and Coulomb corrections [6].*