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The Analytic Structure of the Anomalous Dimension of the Four-Gluon Operator in Deep Inelastic Scattering

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Abstract In the double logarithmic approximation of perturbative QCD we show that the anomalous dimension of the four-gluon operator in DIS has a rich analytic structure. In addition to a pole on the physical sheet we find singularities on second and other nonphysical sheets. We attempt to interprete these singularities as bound states and resonances.

1.First HERA data[1] strongly support the $1/\sqrt{x}$ growth of the gluon density at small x_B, predicted by the leading-log(1/x) approximation in perturbative QCD[2]. A precise study of the small-x behaviour of the DIS amplitude is therefore of particular interest.

It is well known that because of unitarity this power-like increase of the cross section cannot continue down to arbitrarily small values of x and has to be tamed by absorptive corrections. These absorptive corrections are intimately connected with nonleading-twist (4,6,...) operators, and in the simplest approximation they are given by the GLR equation[3]. If one wants to be more precise one has to study the Q^2-evolution of the higher-twist operators, i.e. calculate their anomalous dimensions and coefficient functions. For the twist-four four-gluon operator the anomalous dimension has been calculated in the double logarithmic aproximation (DLA) in ref.[4] [5]. It has a rather complicated analytical structure, and recently [6] it was found that the correct treatment may change the numerical estimates of the GLR-screening by as much as 70%. We therefore feel that a proper understanding of the rather rich analytic structure is a matter of real importance. Our discussion will be based upon the study of the four-gluon amplitude performed in [5].

2.The leading-logarithmic behaviour of the four-gluon amplitude is determined by the system of Feynman graphs, where four t-channel gluons interact with each other through the one gluon (s- channel) exchange. It is convenient to order the summation over all these diagrams in a particular way: consider two pairs of t-channel gluons - e.g. gluons '1' and '2', and gluons '3' and '4' - and perform the sum over all interactions inside each pair. This

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leads to the formation of two reggeons, which are in one of the possible colour states: singlet (0), f- or d-octets ($8_A, 8_S$), decuplets ($10,\bar{10}$) and 27-plet.

Next we have to take into account the interaction between these reggeons: in DLA it is given by graphs, where the s-channel gluon is emitted by one of the reggeons but absorbed by another one. In our example, gluon '1' emits a s-channel particle which is absorbed by gluon '4'. Subsequent interactions will take place inside the pairs (1,4) and (2,3) and again give rise to a pair of reggeons. Such a "switch" from one pair of reggeons to another is accompanied by a numerical coefficient which depends on the colour degrees of freedom of the reggeons before and after the "switch". Putting all these coefficients together, we obtain a 3×3-matrix M, where the elements consist of five-dimensional, orthogonal block matrices Λ_i which describe the transitions between the three systems of gluon pairs - (1,2)(3,4), (1,3)(2,4) and (1,4)(2,3). These matrixes Λ_i differ from each other only in the sign of those elements which correspond to the antisymmetric states (8_A) and ($10,\bar{10}$). So one can write the Λ_i as Λ, $P\Lambda$ and $P\Lambda P$, resp., where $P=diag(1,-1,1,-1,1)$ and the elements refer to the representations $1, 8_A, 8_S, 10+\bar{10}, 27$, resp. As the product of all 3 matrixes should be equal to the identity, we have

$$\Lambda_1\Lambda_2\Lambda_3 = \Lambda P \Lambda P \Lambda P = 1. \tag{1}$$

This implies that $\Lambda P \Lambda = P \Lambda P$ ($\Lambda = \Lambda^T = \Lambda^{-1}$). For the calculation of the matrix Λ it is convenient to use projectors P_i (($i = 0, 8_A, 8_S, 10, \bar{10}, 27$)) and the analogue of the Fierz identity

$$\begin{aligned} i^2 f_{ade} f_{cbe} &= \frac{1}{2}\{\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}\} + \frac{3}{2} d_{abe} d_{cde} + \frac{i^2}{2} f_{abe} f_{cde} \\ &= 3P_0 + \frac{3}{2}P_{8_A} + \frac{3}{2}P_{8_S} - P_{27} \\ &= \sum \lambda_i P_i \end{aligned} \tag{2}$$

The tensor on the l.h.s. of eq.(2) describes the exchange of one s-channel gluon while the r.h.s. reflects the probability amplitude to form a definite colour state. One obtains:

$$\Lambda = \begin{pmatrix} \frac{1}{8} & \sqrt{\frac{1}{8}} & \sqrt{\frac{1}{8}} & \frac{\sqrt{5}}{4} & \frac{3\sqrt{3}}{8} \\ \sqrt{\frac{1}{8}} & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2}\sqrt{\frac{3}{2}} \\ \sqrt{\frac{1}{8}} & \frac{1}{2} & -\frac{3}{10} & -\sqrt{\frac{2}{5}} & \frac{3}{10}\sqrt{\frac{3}{2}} \\ \frac{\sqrt{5}}{4} & 0 & -\sqrt{\frac{2}{5}} & \frac{1}{2} & -\frac{1}{4}\sqrt{\frac{3}{5}} \\ \frac{3\sqrt{3}}{8} & -\frac{1}{2}\sqrt{\frac{3}{2}} & \frac{3}{10}\sqrt{\frac{3}{2}} & -\frac{1}{4}\sqrt{\frac{3}{5}} & \frac{7}{40} \end{pmatrix} \tag{3}$$

where the columns and rows belong to the representations $1, 8_A, 8_S, 10+\bar{10}, 27$, resp. (note that Λ is othogonal and symmetric). Finally, the full transition matrix is:

$$M = \begin{pmatrix} 0 & \Lambda & P\Lambda P \\ \Lambda & 0 & P\Lambda \\ P\Lambda P & \Lambda P & 0 \end{pmatrix} \tag{4}$$

where the elements correspond to the different pairings of reggeons (12)(34), (13)(24), and (14)(23), resp.

For the full four-gluon amplitude it is convenient to use a double Mellin transform and to write the amplitude in terms of the "anomalous dimension" γ and complex angular momentum $\omega = j-1$:

$$A(x,Q^2) = \int \frac{d\omega d\gamma}{(2\pi i)^2} e^{\omega ln\frac{1}{x} + \gamma ln Q^2} A(\omega,\gamma) \tag{5}$$

Our main interest are the singularities of $A(\omega,\gamma)$ in the complex γ-plane. The sum of all interactions of the four-gluon system can be written as an inhomogeneous integral equation. However, as far as the singularities in γ (and also ω) are concerned, we are interested in solutions to the following homogeneous equation:

$$A_j(\omega,\gamma) = \sum_k M_{j,k}\frac{1}{2}\left(\frac{1}{\sqrt{1-\frac{4\alpha_s\lambda_j}{\pi\omega\gamma}}} - 1\right) A_k(\omega,\gamma) \tag{6}$$

Here the indices j and k refer to both the colour of the reggeons $(0, 8_A, 8_S, 27)$ and the different ways of pairing the gluon lines ((12)(34), (13)(24), (14)(23)). We omit the decuplets, since on the r.h.s. of eq.(2) $\lambda_{10} = \lambda_{\bar{10}} = 0$.

3. Because of statistics we need to consider only those states which are completely symmetric under the exchange of any pair of gluon lines; for the symmetric representations $1, 8_S$, and 27 we therefore require symmetry under the exchange of the (12)(34), (13)(24) and (14)(23). As it is seen from eq.(6), the amplitude has branch cuts in the γ-plane: they belong to noninteracting ladders in the singlet channel (from $\omega\gamma = \frac{12\alpha_s}{\pi} = a$ to $\omega\gamma = 0$, referred to as "Pomerons"), two octets (from $\omega\gamma = \frac{6\alpha_s}{\pi} = a/2$ to $\omega\gamma = 0$), and two 27-plets (from $\omega\gamma = 0$ to $\omega\gamma = -a/3$). In addition to these cuts, we find several poles. First, as the rightmost singularity, there is a pole very close to the two-Pomeron cut at

$$\omega\gamma = a\cdot(1+\delta) = a \cdot 1.0095 \quad \cdot \tag{7}$$

on the first, physical, sheet. The eigenvector A_j is symmetric under the exchange of the pairings (12)(34), (13)(24), (14)(23), and it has the form:

$$A^{(P)} = \begin{pmatrix} 1 & 0 & 0.1164 & -0.090 \\ 1 & 0 & 0.1164 & -0.090 \\ 1 & 0 & 0.1164 & -0.090 \end{pmatrix} \tag{8}$$

(the first horizontal vector belongs to the pairing (12)(34), the second and third one to (13)(24) and (14)(23), resp.). This state represents mainly a bound state of two Pomerons. On the r.h.s of (6), the matrix element $\Lambda_{11} = \frac{1}{N_c^2-1} = \frac{1}{8}$ is small, but this smallness is compensated by the cut factor $1/\sqrt{1-\frac{4\alpha_s\lambda_0}{\pi\omega\gamma}}$ which becomes large in the vicinity of the tip of the cut. We also mention that the nonsinglet ladders serve as a renormalization of the four

Pomeron interaction: their total contribution is of about the same order as the matrixelement Λ_{11}.

Next we look for poles on nonphysical sheets; they are reached by analytic continuation through one of the cuts. There imaginary part signals the possibility of a "decay" into "lighter" states (we are using the terminology employed for the analytic structure of the scattering matrix in the complex energy plane: note, however, that the "lightest" state, in our context, has the rightmost singularity in the γ-plane). As an example, we consider the pole on the second sheet of the symetric octet. It has $\delta = -0.407 \pm i0.215$ and belongs to a symmetric state:

$$A^{(d)} = \begin{pmatrix} -.17 \pm i0.33 & 0 & 1.0 & -0.037 \pm i0.050 \\ -.17 \pm i0.33 & 0 & 1.0 & -0.037 \pm i0.050 \\ -.17 \pm i0.33 & 0 & 1.0 & -0.037 \pm i0.050 \end{pmatrix} \tag{9}$$

It is mainly a bound state of two reggeons in the 8_S representation. Another pole which belongs to a symmetric state lies on the second sheet of the 27-representation, and it is bult mainly from reggeons of the 27-representation. It is situated close to the end of the corresponding cut at ($\delta = -1.3343$), and it is purely real since there are no cuts at $\delta < -4/3$.

$$A^{(27)} = \begin{pmatrix} -0.286 & 0 & -0.109 & 1 \\ -0.286 & 0 & -0.109 & 1 \\ -0.286 & 0 & -0.109 & 1 \end{pmatrix} \tag{10}$$

Finally we discuss poles which can be reached by passing through two or even more cuts. First there is a pole associated with the 8_S and 27 cuts. It lies at $\delta = -1.41$, and its eigenvector is

$$\begin{pmatrix} -0.114 & 0 & -1.096 & 1 \\ -0.114 & 0 & -1.096 & 1 \\ -0.114 & 0 & -1.096 & 1 \end{pmatrix} \tag{11}$$

Two other poles - one associated with the 1 and 27 representations, the other one with the 1, 8_S, and 27 representations - are located at $\delta = -1.56$ and $\delta = -1.86$, resp. Their eigenvectors have the color components $(-0.856 \;\; 0 \;\; -0.0193 \;\; 1)$ and $(-0.616 \;\; 0 \;\; -0.582 \;\; 1)$, resp.

4. So far we have discussed states which are completely symmetric under the exchange of (12)(34), (13)(24), and (14)(23). As it can be seen from the structure of the matrix M (see eq.(4)), such symmetric solutions can be obtained only by putting the contribution of the antisymmetric f-octet equal to zero. It is, however, instructive to discuss a more general class of solution of eq.(6), which has also non-zero contributions of the f-octet. The main features of such eigenfunctions are obtained most easily by taking the large N_c (number of colours) limit. The only elements of the matrix Λ which remain nonzero when $N_c \to \infty$ are those which belong to the two octet representations: $\Lambda_{8_A8_A} = \Lambda_{8_S8_A} = \Lambda_{8_A8_S} = \Lambda_{8_S8_S} = 1/2$.

It is easy to check that the eigenvector takes the form:

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \qquad (12)$$

The eigenvalue $\omega\gamma = (12\alpha_s/\pi)\cdot 0.5625$ (i.e. $\delta = -0.4375 = -7/16$) has a threefold degeneracy: the other solutions are

$$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}, \qquad (13)$$

thus restoring the symmetry of the four gluon system.

Any of these eigenvectors describes a so-called "cylinder" configuration, where, e.g. in the second case, gluon 1 interacts with gluon 2, 2 with 3, 3 with 4 and 4 with 1, but not 1 with 3 or 2 with 4 (that the cylinder configurations give the main leading contribution in the large $N_c \to \infty$ limit, is, of course, a well-known fact).

In our real world with $N_c = 3$ these solutions have $\delta = -0.423 \pm i0.000956$ and lie on the second sheet of the two-pomeron cut. It still has a threefold degeneracy, and its eigenvectors are

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -.080 + i0.090 & 0.5 & 0.86 - i0.049 & 0.123 + i0.004 \\ 0.080 - i0.090 & 0.5 & -0.86 + i0.049 & -0.123 - i0.004 \end{pmatrix},$$
$$\begin{pmatrix} -.080 + i0.090 & 0.5 & 0.86 - i0.049 & 0.123 + i0.004 \\ 0 & 1 & 0 & 0 \\ 0.080 - i0.090 & -0.5 & -0.86 + i0.049 & -0.123 - i0.004 \end{pmatrix},$$
$$\begin{pmatrix} 0.080 - i0.090 & 0.5 & -0.86 + i0.049 & -0.123 - i0.004 \\ -.080 + i0.090 & -0.5 & 0.86 - i0.049 & 0.123 + i0.004 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \qquad (14)$$

(note that, although δ is not very diffent from its value in the large-N_c limit, the eigenvectors are not very close to those given in eqs.(12) and (13): this is due to $\Lambda_{8_S8_S}$ which, in this limit, chnages from $-3/10$ to $1/2$).The small imaginary part $Im\delta \simeq 10^{-3}$ reflects the fact that these states couple only weakly to the two-pomeron state. [2]

Another nonsymmetric state (on the second sheet of the 27-cut) has $\delta = -1.363$ and the eigenvector

$$\begin{pmatrix} 0.143 & -0.87 & 0.048 & 1 \\ -0.143 & 0.87 & -0.048 & -1 \\ 0 & -.174 & 0 & 0 \end{pmatrix} \qquad (15)$$

[2] A reasonable estimate of $Im\delta$ is $Im\delta \sim 2Re(\delta - 1/2)\Lambda^2_{18_S}\sqrt{2(\delta-1/2)/\delta}\cdot\frac{|A_{(1)}|}{\sqrt{A^2}} \sim 10^{-3}$.

It is built mainly from two 27 reggeons and has no imaginary part as $\delta < -4/3$. There are two more eigenstates with the same eigenvalue. They are obtained from (15) by a simple replacement: one interchanges the first and the second line (or the first and the third line) and, at the same time, changes the sign of the 8_A component in the third (or the second) line.

Other nonsymmetric states lie on the second 8_A or 8_S sheet, and they are built mainly from the octets. They have $\delta = -0.468 \pm i0.126$, and the eigenvectors (for the case of the positive imaginary part of δ) are:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -0.28 - i0.466 & 0.5 & -0.886 + i0.0394 & 0.0388 - i0.0381 \\ 0.28 + i0.466 & 0.5 & 0.886 - i0.0394 & -0.0388 + i0.0381 \end{pmatrix}. \qquad (16)$$

The other two eigenvectors are obtained by the permutation rules which have been described before.

5. We have thus shown that the γ-transform of the four-gluon amplitude has a rich analytic structure, even in the DLA. Let us phrase our results in terms of operator product expansions. The four-gluon amplitude of [5] on which the discussion of this paper has been based, has been derived within an approximation which is completely analogous to that of the BFKL-Pomeron [2]. The latter has been shown [7] to possess a short distance expansion:

$$\langle\phi^{a_1}(\rho_1)\phi^{a_2}(\rho_2)\phi^{a_1'}(\rho_1')\phi^{a_2'}(\rho_2')\rangle = \sum_{(ln)} C^1_{(ln)}(\rho_1-\rho_2)\langle O^1_{(ln)}(\frac{\rho_1+\rho_2}{2})\phi^{a_1'}(\rho_1')\phi^{a_2'}(\rho_2')\rangle \qquad (17)$$

where $\phi^a(\rho)$ denotes the field of a reggeized gluon which lives in the two-dimensional transverse space and carries the color index "a". The superscript "1" on the rhs indicates that the BFKL Pomeron is a color singlet, and the sum on the right hand side extends over the conformal dimension l and the conformal spin n (the classification in terms of the conformal group is possible since the underlying BFKL-equation has been shown to be conformal invariant). The short distance behavior of the coefficient functions is governed by the anomalous dimensions

$$\gamma_{l,n} = \frac{4N_c\alpha_s}{\omega\pi} + O((\frac{\alpha_s}{\omega})^2) \qquad (18)$$

where the leading terms have been found to be independent of l and have $n = 0$. The first term on the rhs of (17) coincides with the two-gluon operator in DIS. According to [5], the four-gluon amplitude seems to allow for a slightly more complicated but quite analogous expansion. First, the amplitude A is written as a sum of three terms:

$$A = A_{(12)(34)} + A_{(13)(24)} + A_{(14)(23)} \qquad (19)$$

Here $A_{(ij)(kl)}$ denotes the sum of all those ladder diagrams (organized as described before eq.(1)) where the last interaction has taken place in the subsystems (ij) and (kl). In analogy with (17), we write each of these terms, say $A_{(12)(34)}$, as the expectation value of a product of ϕ-fields:

$$A^{a_1...a_4;a'_1...a'_4}_{(12)(34)}(k_1,k_2,k_3,k_4;k'_1,k'_2,k'_3,k'_4;\omega) = \int \prod_i d^2\rho_i d^2\rho'_i \exp(i\sum k_i\rho_i - i\sum k'_i\rho'_i)$$
$$\langle\phi^{a_1}(\rho_1)...\phi^{a_4}(\rho_4)\phi^{a'_1}(\rho'_1)...\phi^{a'_4}(\rho'_4)\rangle \quad (20)$$

Taking the limit $|\rho_1-\rho_2| \sim |\rho_3-\rho_4| \ll |\frac{\rho_1+\rho_2}{2} - \frac{\rho_3+\rho_4}{2}|$ we have the expansion:

$$\langle\phi^{a_1}(\rho_1)...\phi^{a_4}(\rho_4)\phi^{a'_1}(\rho'_1)...\phi^{a'_4}(\rho'_4)\rangle = \sum_{(ln)(l'n')} C^i_{(ln)}(\rho_1-\rho_2)C^i_{(l'n')}(\rho_3-\rho_4)$$
$$\langle O^i_{(ln)}(\frac{\rho_1+\rho_2}{2})O^i_{(l'n')}(\frac{\rho_3+\rho_4}{2})\phi^{a'_1}(\rho'_1)...\phi^{a'_4}(\rho'_4)\rangle \quad (21)$$

where the superscript "i" refers to the color representation of the composite operators (two-reggeon system). For the singlet case, the coefficient functions $C^1_{(ln)}$ and the composite operators $O^1_{(ln)}$ are the same as in the BFKL Pomeron (17). Finally, in the limit $\frac{\rho_1+\rho_2}{2} - \frac{\rho_3+\rho_4}{2} \to 0$, we have the expansion:

$$\langle O^i_{(ln)}(\frac{\rho_1+\rho_2}{2})O^i_{(l'n')}(\frac{\rho_3+\rho_4}{2})\phi^{a'_1}(\rho'_1)...\phi^{a'_4}(\rho'_4)\rangle = \sum_{(l''n'')} \tilde{C}^i_{(ln)(l'n')(l''n'')}(\frac{\rho_1+\rho_2}{2} - \frac{\rho_3+\rho_4}{2})$$
$$\langle \tilde{O}^i_{(ln)(l'n')(l''n'')}\phi^{a'_1}(\rho'_1)...\phi^{a'_4}(\rho'_4)\rangle (22)$$

What we have considered in this paper (using the DLA) are the leading terms in the sums over l and n in (21) and (22), allowing for all color representations "i". The cuts in the γ-plane signal that when considering the Q^2-evolution we have to include a continous spectrum of operators (somewhat analogous to the analytic continuation in angular momentum in Regge theory), and the poles correspond to the anomalous dimension of the various composite operators. It would be interesting to see whether the underlying dynamics, in particular the transition vertex 2-gluons $\to$ 4-gluons, is conformally invariant, and what the symmetry properties of the coefficients are. Further work along these lines is in progress.

6. Finally, let us return to a more intuitive interpretation. We have used the terms "bound states", "resonances", for the poles and cut singularities in the complex γ-plane, and we now have to give some justification for this analogy, making use of the results obtained in [6]. Within the parton picture, the poles reflect the strong correlations during the evolution of the two branches of parton cascades. We shall show that, as a result of these correlations, the two branches behave like a bound state system: they remain close to each other, and the behaviour of the system is independent of how it was formed ("factorization").

At first sight it may look strange that the position of the pole in the anomalous dimension γ depends upon ω. However in the double log approximation there is only one parameter – the product $\omega\gamma$ – which controls the double log $-\alpha_s \cdot ln\frac{1}{x}lnq^2$ – behaviour of the amplitude. This parameter plays the role of the energy during the evolution "time", and it is in this variable where we have a pole. In a certain sense the DLA seems to loose one longitudinal

dimension. However this is not the case. The second variable shows up when one calculates the reggeon cut contribution. Namely due to this second variable one gets branch cuts in the $A(\omega,\gamma)$ amplitude, and not poles only, as one would expect in a truly one-dimensional space.

In order to understand how this second dimension reveals itself we have to follow the whole evolution history (i.e. not only to ask for the probability of finding the partons at the end of their evolution or, in other words, ask for the total cross section). To this end we introduce the new variables t and z as it was done in ref.[6]:

$$r' = \frac{T}{2}\frac{lnq^2}{lnQ^2} \quad , \quad y' = \frac{T}{2}\frac{ln1/x}{ln1/x_B}, \quad (23)$$

where Q^2 and Bjorken-x_B denote the final point of the evolution trajectory while q^2 and x are coordinates of some intermediate point of the evolution. Furthermore,

$$T = \sqrt{\frac{4N_c\alpha_s}{\pi}ln\frac{1}{x}ln\frac{q^2}{q_0^2}}. \quad (24)$$

More convenient are the coordinates

$$t = y' + r' \quad and \quad z = y' - r' \quad (25)$$

which play the role of a "time" and a "space" coordinate, resp. The latter variable z denotes the deviation from the main ("classical") evolution trajectory.

In these terms the DLA equation for the two-gluon pomeron

$$\frac{\partial^2 A}{\partial t^2} - \frac{\partial^2 A}{\partial z^2} = A \quad (26)$$

looks like a wave equation for the particle with the "mass" equal to 1. Its nonrelativistic solution $A = e^t\Psi(t,z)$

$$\Psi(t,z) = \frac{e^{-z^2/2t}}{\sqrt{2\pi t}} \quad (27)$$

can be obtained from the usual DLA expression $A = e^{2\sqrt{y'r'}}/\sqrt{4\pi\sqrt{y'r'}}$ in the limit $z << t$, expanding the $2\sqrt{y'r'} = \sqrt{t^2-z^2} \simeq t - z^2/2t$, or from the nonrelativistic equation for the Ψ -function

$$\frac{\partial\Psi(t,z)}{\partial t} = \frac{1}{2}\frac{\partial\Psi}{\partial z^2}. \quad (28)$$

The behaviour eq.(28) that we have obtained for the wave function Ψ is nothing but Lipatov's $logq^2$ diffusion law [2] for the DLA case. It implies that the system, whose center ($z=0$) moves along the classical trajectory, becomes broader and broader as time increases. The mean square of the distance of the parton grows linearly in t: $z^2 \sim 2t$.

Now turning to the twist four operator – i.e. the evolution of two partons – one has two possibilities: i) independent evolution of two branches of the parton cascade with the same initial and final points, or ii) the complete evolution which takes into account also the interaction between the two branches and considers the two branches as parts of one bigger system. In the first case, the amplitude is simply proportional to the product of two Ψ functions (28), and the probability of finding partons of the two branches at a relative distance $\Delta = z_1 - z_2$ is given by:

$$w(z_1 - z_2 = \Delta) = \frac{1}{2\pi t}\int e^{-z_1^2/2t} \cdot e^{-z_2^2/2t} dz_1 dz_2 \delta(z_1 - z_2 - \Delta) = \frac{1}{2\sqrt{\pi t}} e^{-\Delta^2/4t}. \qquad (29)$$

In other words, the mean square separation grows linearly in time:

$$\langle (z_1 - z_2)^2 \rangle \propto t \qquad (30)$$

When translated into γ-space, it leads to the cut contribution $A(\omega,\gamma) \propto 1/(\omega\gamma\sqrt{1 - \frac{4N_c\alpha_s}{\pi\omega\gamma}})$ or

$$A_{cut} \;=\; A_{tw=4}(t = T, z = 0) \;=\; \frac{1}{2\pi T} e^{2T}. \qquad (31)$$

The second case is more interesting. Due to the attractive interaction the parton branches may form a new bound state which shows up as a pole in the γ-plane. For the ground state with "energy"=1+δ (see ref.[6] and eq.(7) for the definition of δ) one finds the wave function

$$\Psi(z_1, z_2) \;=\; (\delta/4)^{1/4} e^{-\sqrt{\delta}|z_1 - z_2|} \qquad (32)$$

which describes a stable (i.e. non-decaying) state of two reggeons. The distance between the two reggeons (in our z, t-space) is limited by:

$$| z_1 - z_2 | \sim 1/\sqrt{\delta} \qquad (33)$$

and hence does not increase with time. In γ-space, this state leads to a pole. Alternatively,

$$A_{pole} = \frac{1}{\sqrt{\pi T}} e^{(2+\delta)T}. \qquad (34)$$

So far our discussion has been in terms of the variables z and t, but not in "real" space and time. Intuitively it seems rather obvious that the partons which belong to the two different parton branches can form a bound state only if they close to each other in both impact parameter and longitudinal space. A closer inspection of the Feynman diagrams shows that this is indeed the case. We illustrate this point in Fig.1 where the evolution of the two parton cascade branches are shown in (a) the longitudinal and (b,c) impact parameter coordinates. The picture corresponds to the target proton rest frame where the life time τ for each parton-$i(i')$ is given by its value of $x_i(x_{i'})$ (if M denotes the proton mass then $\tau_i = 1/Mx_i$). In Fig.1a, the vertical position of each line denotes its rapidity $y = ln1/x$: the parton with smallest x which is closest to the photon has the longest lifetime. In the

typical two-cascade situation, where one cascade is drawn inside the other, we observe, as the most probable situation, the "oscillations" of the two branches, one around the other. If, for example, $y_i > y_{i'} > y_{i-1} > y_{i'-1}...$, then $\tau_i > \tau_{i'} > \tau_{i-1} > \tau_{i'-1}...$. As a result, the two branches "live" at the same time. In impact parameter space (Fig.1b, c), on the other hand, the picture looks slightly more complicated. Instead of the usual diffusion in b-space which is typical for Regge dynamics, in DIS the motion in b-space slows down when rapidity increases and eventually "freezes" at the position of the point-like photon. Namely, inside each branch the stepsize $\Delta b_{ti} \sim 1/q_{ti}$ (see [8] for details) between two subsequent branching processes decreases with i, due to the strong ordering of $q_{ti} >> q_{t,i-1} >> ...$ in the leading-log approximation. In Figs. 1b and c we illustrate this situation for a two-branch process. A horizontal line denotes the region of b-space which is occupied by a parton of rapidity y. As the final b_t point of the evolution is fixed by the point-like photon (or large p_t parton in the general case), the branches must meet at the top. In the absence of correlations or interactions between the two branches (Fig.1b), they may evolve rather independently and, consequently, have little overlap in b-space. With correlations, on the other hand (Fig.1c), the two branches are forced to "meet" and, on the average, will stay close together. So we again obtain "oscillations" analogous to the Fig.1a.

Finally we have to stress that the pole contribution exhibits the very important property of factorization. Once the bound state has been formed, its wavefunction (i.e. the distribution in parton momenta and coordinates) "forgets" about its origin. This gives us the possibility of writing the pole amplitude as a product of the emission vertex and the pole propagator. Of course the poles we are discussing here are weakly bound states, and the distribution in the difference $logq_1^2 - logq_2^2$ is rather broad ($|r_1 - r_2| \sim 1/\sqrt{\delta} \sim N_c^2$). Thus one has to wait a very long "time" $t \sim 1/\delta$ (see eq.(34)) until the pole will finally be formed. For a shorter time $t << 1/\delta$ (which means: at smaller energies) the pole manifests itself only as a correction to the two-reggeon cut [3]. At asymptotically high energies, however, the poles will be formed in any case. In contrast to this, the cut contributions do not have this factorization property: the distributions over the momenta and coordinates of the partons coming from the two separated branches of the cascade do depend on the wave function of the target. So it is not possible to write the corresponding amplitude in a form as simple as for the pole.

Finally a remark on the poles formed by the coloured reggeons. They describe the correlations between the colours of the gluons in the two parton cascades, in the same way as, in the old reggeon field theory, the ρ^0 exchange in the Mueller-Kancheli[9] graphs describes the correlations between charged pions(hadrons). Of course, the colour correlation between the parton densities should be smaller than the absolute value of the density. Therefore, such a pole should be situated to the left of the leading singularity which is formed by the pomerons. This is indeed the case: the rightmost singularity is the pole in $A^{(P)}$ which correspods (mainly) to the two pomeron bound state (see eq.(8)) at $\omega\gamma = (12\alpha_s/\pi) \cdot 1.00955$.

[3]This are the corrections which we have discussed in ref.[6]. For the HERA energy range they are already large enough – up to 70%.

We hope, the clear understanding of the analytical structure of higher twist amplitudes will be useful for the future study of the DIS processes in the small-x region.

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Figure caption:

Fig.1. The space-time structure of the parton cascade: a) in the longitudinal space (time) coordinates; b) in impact parameter space for two uncorrelated branches (two-reggeon cut); c) in impact parameter space for a bound state (pole).

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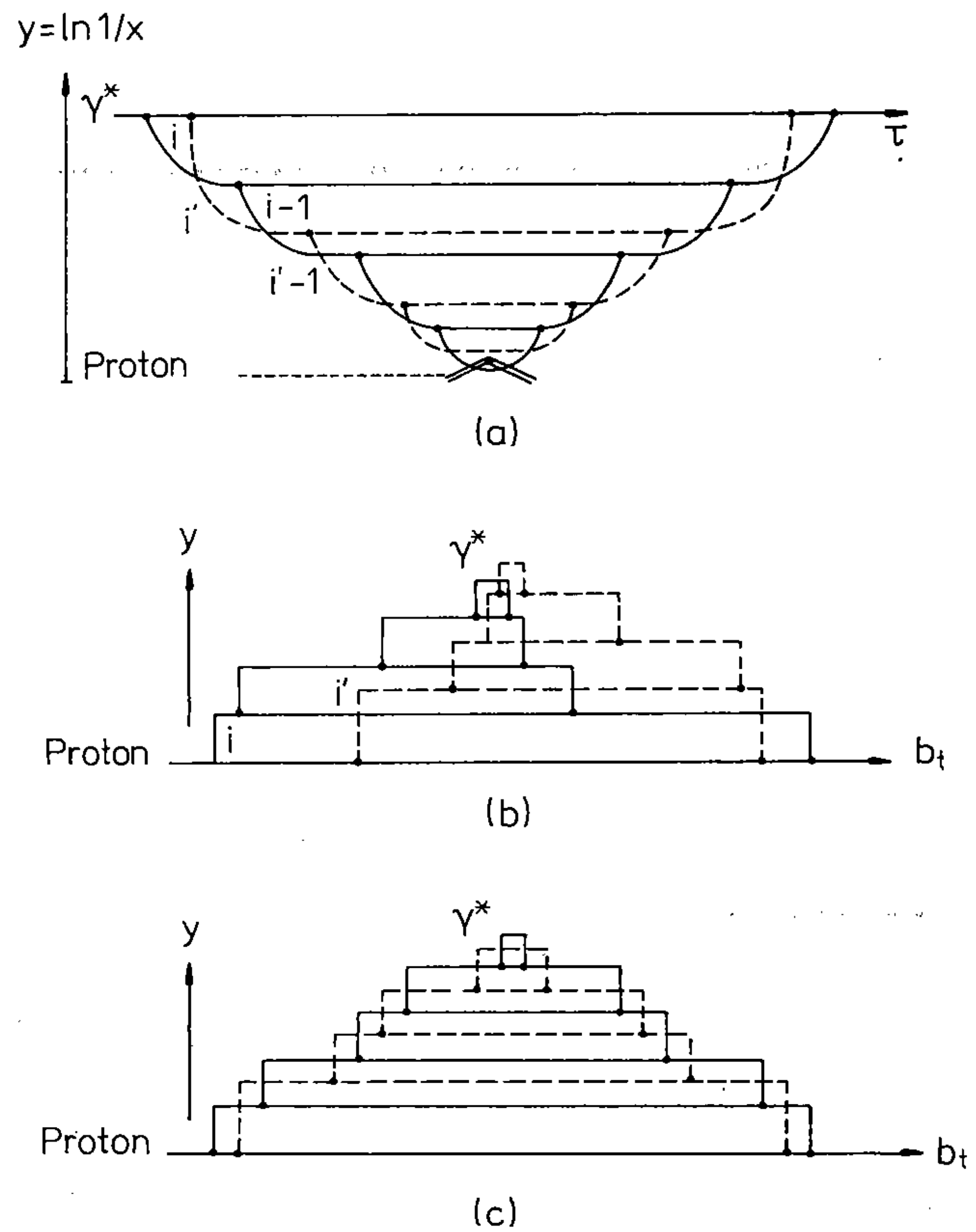

Fig.1