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Fluctuations in Quantum Electrodynamics with Boundary Conditions at Finite Temperature

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Abstract

Using standard techniques of Quantum Field Theory at finite temperature (Real-Time Technique) we determine correlation functions of observables. It turns out that for the photon field simple analytical expressions can be derived. They depend strongly on the given geometry. Its structure is reflected by the fluctuations of the electromagnetic field-strength as well as the components of the energy momentum tensor. Even though the allowed frequencies are fixed by the boundary conditions, the weight function of the spectrum is changed by the temperature. Thereby the temperature dependent contributions superpose with the zero point contributions. For the space in-between two plates the resonance structure of the zero temperature fluctuations is superposed by smooth T-dependent fluctuations. As for $T = 0$ for special conditions of the measuring process the fluctuations of the Casimir pressure depend on the model of the plates. The leading contributions result from the vacuum fluctuations.

1 Introduction

In Quantum Theory the physical state is characterized by the expectation values of observables and by its fluctuations. A surprising example are the vacuum expectation values of electromagnetic field strength. As to be expected the vacuum expectation values of the electromagnetic field strength vanish, however its fluctuations are different from zero [1]. Thereby local fluctuations are divergent whereas

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investigations with the help of correlation functions lead to satisfactory results [2]. Besides these observables it is interesting to consider expectation values and its fluctuations for other physical observables for the vacuum state. Such quantities are the components of the energy momentum tensor, i.e. pressure densities and energy densities. In the case of free fields the expectation values itself are infinite and should be subtracted carefully on the other hand its correlation functions are well defined and should be taken seriously. More difficult is this procedure in the case of boundaries. Let us consider for example a two plate system with superconducting plates. In this case the vacuum expectation values of physical quantities should be split into a finite boundary dependent physical part and a boundary independent divergent part (subtraction procedure). In this way one can determine the Casimir force as the difference between the pressure on both sides of the plate [2]. On the other hand the fluctuation of the pressure, the energy density and the Casimir pressure are well defined quantities [3], [4].

These investigation of the fluctuations of the electromagnetic field strength and the components of the energy-momentum tensor for the vacuum state at zero temperature should be extended to the case of finite temperature. Expectation values of components of the energy- momentum tensor at finite temperature are determined using different methods. The best known example is the Casimir pressure at finite temperature [16], [10], [7]. Our aim here is the investigation of fluctuations at finite temperature.

The first problem concerns the correct definition of the correlation functions. These function must depend on the space-time points and on the temperature. Therefore, for its determination we have to apply a real-time technique of the Quantum Field Theory at finite temperature. We apply two procedures. First we determine the Wightman-like functions according to standard relations from the given path-dependent functions [5]. As second approach we use the operator formalism [6]. Both approaches lead to the same functions. Because of the masslessness of the photon field it is possible to derive reasonable explicit analytical expressions. It turns out that the correlation functions consists of the contributions resulting from the zero point fluctuations and the temperature induced fluctuations. Even though the allowed frequencies are fixed by the boundary conditions, the weight functions of the spectrum is changed by the temperature. The important point is, that the temperature dependent part is exponentially damped for very high frequencies. Consequently this part of the correlation functions is smooth and does not contribute to singularities, produced by the temperature independent zero point fluctuations.

Consequently in the case of the two plate system the known resonance structure of the correlations functions at $T = 0$ remains. Such resonances appeared if the distances between the considered events correspond to a classical light signal n-times reflected at the plates. This structure concerns the fluctuations of the electromagnetic field strength, the pressure and the Casimir force.

As examples we derive typical correlation functions for cases of the free space, one plate or two plates. Already in the case of vanishing temperature it turned out that the fluctuations depend in an essential manner on the external conditions, i.e. whether there are one or two plates, or no plate at all. Consequently the vacuum fluctuations at vanishing or nonvanishing temperature constitute an important indication of the physical situation.

Finally we show that analogously to the case of zero temperature for a special measuring process the fluctuations of the Casimir force depend on the model of the plates only and the essential contributions are temperature independent.

2 Green Functions at Nonvanishing Temperature and in the Presence of Plates

For the determination of temperature dependent expectation values we need a formulation of Quantum Field theory at finite temperature. There are in principle two approaches, the imaginary and the real-time technique [5]. Here we are interested in the real-time technique only because we expect a temperature and a time-dependence of the correlation functions.

In our case we have the additional difficulties due to the presence of boundary conditions. However these problems are already solved for vanishing temperature, the generalization for nonvanishing temperature are straightforward [7].

In most cases it is sufficient to have a technique on the levels of Feynman diagrams. Then it is quite natural to construct the Wightman- like functions from the propagators as it is known from standard Quantum Field Theory.

To be sure we look for a independent calculation of these functions. As in the case of nonvanishing temperature it seems to be helpful to have an operator formalism [15], [4] for controlling the used construction. Although there are doubts concerning the connections between both Hilbert spaces rotated to each other by Bogoliubov-transformations we apply this formalism and obtain the same results for the correlation functions.

2.1 Green Functions in Diagram Technique

We formulate the necessary technique for a massless scalar field first and later we give the corresponding expressions for the photon field. We apply the real-time technique based on the functional integral in the formulation of [5]. The typical feature of all real time techniques are the doubling of the involved fields. Additionally to the field operator $\phi(x)$ we have the ghost field $\hat{\phi}(x)$ or in short notation $\phi_a(x) = (\phi(x), \hat{\phi}(x)$. Then according to the real time technique the temperature dependent propagators form a 2x2-matrix.

$$< T\phi_a(x), \phi_b(x') > = \frac{1}{i} D^c_{ab}(x, x'), \quad a, b = 1, 2. \tag{2.1}$$

To simplify the notations we write most of the expressions in a Fourier representation using

$$D^c_{ab}(x, x') = \int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0)} D^c_{ab}(k_0, \vec{x}, \vec{x}'). \tag{2.2}$$

The temperature dependent functions D^c_β are given with the help of Green functions at vanishing temperature according to

$$D^c_{\beta,ab}(k_0, \vec{x}, \vec{x}') = R_{ac} D^c_{cd} R_{db} \tag{2.3}$$

with

$$R_{ab} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \tag{2.4}$$

and

$$D^c_{ab} = \begin{pmatrix} D^c(k_0, \vec{x}, \vec{x}') & 0 \\ 0 & -D^{c*}(k_0, \vec{x}, \vec{x}') \end{pmatrix}. \tag{2.5}$$

The rotation matrix is determined by the temperature dependent angle

$$\cosh^2\theta = \frac{1}{1 - e^{-\beta|k_0|}}, \quad \sinh^2\theta = \frac{1}{e^{\beta|k_0|} - 1}, \quad \beta = 1/(kT). \tag{2.6}$$

Inserting the rotations we obtain

$$D^c_{\beta,ab}(k_0, \vec{x}, \vec{x}') = \begin{pmatrix} \cosh^2\theta D^c(k_0) - \sinh^2\theta D^{c*}(k_0) & \cosh\theta\sinh\theta(D^c(k_0) - D^{c*}(k_0)) \\ \cosh\theta\sinh\theta(D^c(k_0) - D^{c*}(k_0)) & -\cosh^2\theta D^{c*}(k_0) + \sinh^2\theta D^c(k_0) \end{pmatrix}. \tag{2.7}$$

The diagonal elements of the matrix of propagators can be rewritten using $\cosh^2\theta = \sinh^2\theta + 1$ in the following way

$$\begin{aligned} &< T\phi_1(x)\phi_1(x') >_\beta = \\ &\frac{1}{i}\int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0)} (D^c(k_0, \vec{x}, \vec{x}') + \sinh^2\theta[D^c(k_0, \vec{x}, \vec{x}') - D^{c*}(k_0, \vec{x}, \vec{x}')]) \\ &= \frac{1}{i} D^c(x, x') + \frac{1}{i}\int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0)} \sinh^2\theta[D^c(k_0, \vec{x}, \vec{x}') - D^{c*}(k_0, \vec{x}, \vec{x}')] \end{aligned} \tag{2.8}$$

and analogously

$$\begin{aligned} &< T\phi_2(x)\phi_2(x') >_\beta = \\ &-\frac{1}{i} D^{c*}(x, x') + \frac{1}{i}\int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0} \sinh^2\theta(D^c(k_0, \vec{x}, \vec{x}') - D^{c*}(k_0, \vec{x}, \vec{x}'). \end{aligned} \tag{2.9}$$

The important point for our procedure is the identification of

$$< T\phi(x)\phi(x') >_\beta = < T\phi_1(x)\phi_1(x') >_\beta \tag{2.10}$$

and

$$< T^*\phi(x)\phi(x') >_\beta = < T\phi_2(x)\phi_2(x') >_\beta . \tag{2.11}$$

Then we are able to apply the standard relations for the construction of the Wightman-like functions for $T \neq 0$. These definitions are

$$< \phi(x)\phi(x') >_\beta = \Theta(x-x') < T\phi(x)\phi(x') >_\beta + \Theta(x'-x) < T^*\phi(x)\phi(x') >_\beta, \qquad (2.12)$$

whereby

$$< T\phi(x)\phi(x') >_\beta = \frac{1}{i} D^c{}_\beta(x,x') \qquad (2.13)$$

and

$$< T^*\phi(x)\phi(x') >_\beta = -\frac{1}{i} D^{*c}{}_\beta(x,x') \qquad (2.14)$$

are the propagators satisfying all necessary boundary conditions. According to (2.12) and (2.8) - (2.11) we get for the Wightman-like function

$$\begin{aligned} < \phi(x)\phi(x') >_\beta &= < \phi(x)\phi(x') > \\ &+ \frac{1}{i} \int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0)} \sinh^2\theta (D^c(k_0, \vec{x}, \vec{x}') - D^{c*}(k_0, \vec{x}, \vec{x}')) \end{aligned} \qquad (2.15)$$

or

$$\begin{aligned} D^-_\beta(x,x') &= D^-(x,x') + D^1_\beta(x,x') \\ D^1_\beta(x,x') &= \int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0)} \sinh^2\theta (D^c(k_0, \vec{x}, \vec{x}') - D^{c*}(k_0, \vec{x}, \vec{x}')) \end{aligned} \qquad (2.16)$$

It results a representation which consists of two parts. The first contribution is the Wightman function for vanishing temperature. The second part is temperature dependent and does not have the standard analytic properties of the Wightman functions. For this reason it may be more useful here to consider directly the correlation functions which are symmetrized and do not possess the analytical properties of Wightman functions. In the case of boundary conditions the temperature dependent Wightman-like function satisfying boundary conditions ${}^sD^-_\beta(x,x')$ can be obtained by replacing in eq.(2.16) the free space functions D^c and D^{c*} by the functions ${}^sD^c$ and ${}^sD^{*c}$ fulfilling the necessary boundary conditions.

The generalization to Quantum Electrodynamics is not trivial because we want to take care of boundary conditions [7] of ideal conductors. Moreover we use a covariant gauge condition. For the case of ideal conductors the boundary conditions $E_t = B_n = 0$ can be written in terms of the electromagnetic potentials A_μ by

$$\epsilon_{\mu\nu\rho\sigma} n^\rho \partial^\sigma A^\nu|_S = 0 \qquad (2.17)$$

where n^ρ denotes the normal vector of the surface S.

As usual, we expand the free photon field A_μ in terms of four polarization vectors e^i_μ

$$A_\mu(x) = \sum_{i=0}^{3} e^i_\mu f_i(x) \qquad (2.18)$$

In order to find the boundary conditions to be fulfilled by the wave functions $f_i(x)$ one has to choose a suitable basis e^i_μ. It follows from eq.(2.17) that the boundary conditions act actually in the space perpendicular to the vectors n_ρ and ∂_σ at the surface S. Because of the triviality of the surface considered here we are able to introduce globally polarization vectors which satisfy the necessary conditions at the surface. For the case that the plates are perpendicular to the x_3-axis we choose [4], [8]

$$e^1_\mu = \frac{1}{\sqrt{\Delta_\perp}} \begin{pmatrix} 0 \\ -\partial_2 \\ \partial_1 \\ 0 \end{pmatrix}, \quad e^2_\mu = \frac{1}{\sqrt{\Delta_\perp \tilde{\Delta}}} \begin{pmatrix} \Delta_\perp \\ \partial_0\partial_1 \\ \partial_0\partial_2 \\ 0 \end{pmatrix} \qquad (2.19)$$

with $\tilde{\Delta} = \tilde{\partial}^2 = \partial_0^2 - \partial_1^2 - \partial_2^2$ and $\Delta_\perp = \partial_\perp^2 = \partial_1^2 + \partial_2^2$. The remaining orthogonal polarization vectors are

$$e^3_\mu = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad e^0_\mu = \frac{1}{\sqrt{\tilde{\Delta}}} \begin{pmatrix} \partial_0 \\ \partial_1 \\ \partial_2 \\ 0 \end{pmatrix}. \qquad (2.20)$$

The boundary condition (2.17) leads to

$$\epsilon_{\mu\nu\rho\sigma} n^\rho \partial^\sigma A^\nu|_S = \frac{1}{\sqrt{\Delta_\perp}} \begin{pmatrix} \Delta_\perp \\ -\partial_0\partial_1 \\ -\partial_0\partial_2 \\ 0 \end{pmatrix} f_1|_S + \frac{1}{\sqrt{\Delta_\perp \tilde{\Delta}}} \begin{pmatrix} 0 \\ \partial_2\tilde{\partial}^2 \\ -\partial_1\tilde{\partial}^2 \\ 0 \end{pmatrix} f_2|_S = 0 \qquad (2.21)$$

on the plates. All the derivatives act in the (x_0, x_1, x_2)- subspace so that we have the Dirichlet conditions

$$f_i|_S = 0 \quad (i = 1, 2) \qquad (2.22)$$

whereas f_0 and f_3 are free of boundary conditions. We do not discuss possible modifications further.

As physical situation we assume that the plates are infinitely thin and characterized by the boundary conditions (2.17) only. Slight modifications necessary for the treatment of "thick" plates in a covariant quantization procedure can be found in [4]. For the case of parallel plates perpendicular to the x_3-direction we get the following general structure of all Green functions

$$< 0|TA_\mu(x)A_\nu(y)|0 >= \tag{2.23}$$
$$i(g_{\bar\mu\bar\nu} - \frac{\partial_{\bar\mu}\partial_{\bar\nu}}{\bar\partial^2})^s D^c(\bar x - \bar y, x_3, y_3) + i \begin{pmatrix} \frac{\bar\partial_\mu\bar\partial_\nu}{\bar\partial^2} & 0 \\ 0 & -1 \end{pmatrix} D^c(x-y)$$

whereby it has been used

$$\sum_{i=1,2} e^i_\mu e^j_\nu g_{ij} = - (g_{\bar\mu\bar\nu} - \frac{\partial_{\bar\mu}\partial_{\bar\nu}}{\bar\partial^2}), \tag{2.24}$$

$$\sum_{i=0,3} e^i_\mu e^j_\nu g_{ij} = + \begin{pmatrix} \frac{\bar\partial_\mu\bar\partial_\nu}{\bar\partial^2} & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.25}$$

${}^sD^c(x,y)$ is the scalar Feynman propagator satisfying the Dirichlet boundary condition and $D^c(x-y)$ the free space Feynman propagator. Formally the same tensor representations are valid for all other Green functions in electrodynamics, especially for the Wightman functions too. With the help of these formulae we can now write down immediately the corresponding expressions for the Wightman-like functions.

$$\begin{aligned} < A_\mu(x)A_\nu(x') >_\beta &= i^sD^-_{\beta\,\mu\nu}(x,x') \\ &= +i \begin{pmatrix} \frac{\bar\partial_\mu\bar\partial_\nu}{\bar\partial^2} & 0 \\ 0 & -1 \end{pmatrix} D^-_\beta + i(g_{\bar\mu\bar\nu} - \frac{\partial_{\bar\mu}\partial_{\bar\nu}}{\bar\partial^2})^s D^-_\beta \end{aligned} \tag{2.26}$$

Here we use the notations with $\bar z = (z^0, z^1, z^2) = \bar x - \bar y$, The scalar temperature dependent Green functions are constructed according to the rules (2.16 given above. For clarity we write down eq.(2.16) for the case of boundary conditions

$$\begin{aligned} {}^sD^-{}_\beta(x,x') &= {}^sD^-(x,x') \\ &+ \int \frac{dk_0}{2\pi} e^{ik_0(x_0 - x'_0}\sinh^2\theta({}^sD^c(k_0,\vec x,\vec x') - {}^sD^{c*}(k_0,\vec x,\vec x')). \end{aligned} \tag{2.27}$$

2.2 Green Functions in the Operator Formalism

Our aim is to derive the representations (2.16) and (2.26) in an alternative way. Again the field degrees of freedom are doubled, we have the field operator A_μ and the "ghost" field $\hat A_\mu$ which can be described by the field doublet $A_{\mu a}$. The mode expansion of the field operators looks like

$$A_\mu(x) = \int \frac{d^3\vec k}{(2\pi)^3} \sum e^i_\mu(a_i(k)f_i^-(k,x) + a^+{}_i(k)f_i^+(k,x) \tag{2.28}$$

where $f_i^\pm$ describes the field modes and a_i resp. $a^*{}_i$ are the destruction resp. creation operators. An analogous representation is valid for the "ghost" field operators

$$\hat A_\mu(x) = \int \frac{d^3\vec k}{(2\pi)^3} \sum e^i_\mu(\hat a_i(k)f_i^-(k,x) + \hat a^+{}_i(k)f_i^+(k,x)). \tag{2.29}$$

The essential point of the operator formalism are the rotated states. In opposite to the standard formalism the ground state satisfies the conditions

$$\alpha_i(k,\beta)|0>_\beta = 0, \quad \hat\alpha_i(k,\beta)|0>_\beta = 0, \tag{2.30}$$

with destruction operators which are connected by the original creation and destruction operators by a Bogoliubov rotation

$$\begin{pmatrix} a_i(k) \\ \hat a_i^+(k) \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \alpha_i(k,\beta) \\ \hat\alpha_i^+(k,\beta) \end{pmatrix} \tag{2.31}$$

and

$$\begin{pmatrix} a_i^+(k) \\ \hat a_i(k) \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} \alpha_i^+(k,\beta) \\ \hat\alpha_i(k,\beta) \end{pmatrix} \tag{2.32}$$

Substituting the creation and destruction operators a_i and a_i^+ according the relations (2.31) and (2.32) and forming the vacuum expectation values $< A_{\mu a}A_{\nu b} >_\beta$ we obtain for the Wightman-like functions

$$\begin{aligned} < A_\mu(x)A_\nu(x') >_\beta &= - \int d^3\vec k \sum e^i_\mu e^i_\nu g_{ij} \\ &(f_i^-(k,x)f_j^+(k,x') + \sinh^2\theta(f_i^-(k,x)f_j^+(k,x') + (f_i^+(k,x)f_j^-(k,x')) \end{aligned} \tag{2.33}$$

$$\begin{aligned} < \hat A_\mu(x)\hat A_\nu(x') >_\beta &= - \int d^3\vec k \sum e^i_\mu e^j_\nu g_{ij} \\ &(f_i^+(k,x)f_j^-(k,x') + \sinh^2\theta(f_i^+(k,x)f_j^-(k,x') + (f_i^-(k,x)f_j^+(k,x')) \end{aligned} \tag{2.34}$$

$$\begin{aligned} &< \hat A_\mu(x)A_\nu(x') >_\beta = < A_\mu(x)\hat A_\nu(x') >_\beta = \\ &- \int d^3\vec k \sum e^i_\mu e^j_\nu g_{ij} \sinh\theta\cosh\theta((f_i^-(k,x)f_j^+(k,x') + (f_i^+(k,x)f_j^-(k,x')). \end{aligned} \tag{2.35}$$

Finally we have to introduce the standard notations, for vanishing temperature the mode summation leads to the well-known Green functions

$$iD^-_{\mu\nu}(x,x') = - \int d^3\vec k \sum e^i_\mu e^j_\nu g_{ij} f_i^-(k,x)f_j^+(k,x'), \tag{2.36}$$

$$iD^1_{\mu\nu}(x,x') = - \int d^3\vec k \sum e^i_\mu e^j_\nu g_{ij}[(f_i^-(k,x)f_j^+(k,x') + (f_i^+(k,x)f_j^-(k,x')]. \tag{2.37}$$

In the case of boundary conditions, the modes $f_i^\pm$ have to satisfy the appropriate conditions (2.22). Taking into account these notations eq.(2.33) is in accordance with the corresponding eq.(2.10) - eq.(2.11) and eq.(2.26) derived in the foregoing subsection.

2.3 Special Green Functions

It remains to study some explicit expressions for temperature dependent Wightman-like functions. We study these functions for the free space, the space with one plate and the space with two plates. For simplicity we consider here the case of infinitely thin plates only. Taking into account the general structure (2.16) an investigation of the scalar Green function is sufficient.

Free space

At first we study the Green functions for the free space. The basic Green functions [17] are

$$< 0|T\phi(x)\phi(x')|0> \;=\; \frac{1}{i}D^c(x-x') = \frac{1}{i}\int \frac{d^4k}{(2\pi)^4} e^{ik(x-x')}\frac{(-1)}{k^2+i\epsilon} \quad (2.38)$$

$$< 0|\phi(x)\phi(x')|0> \;=\; \frac{1}{i}D^-(x-x') = \int \frac{d^4k}{(2\pi)^3} e^{ik(x-x')}\delta(k^2)\Theta(-k_0) \quad (2.39)$$

$$< 0|T\phi(x)\phi(x')|0> + < 0|T^*\phi(x)\phi(x')|0> = \frac{1}{i}D^1(x-x') = \int \frac{d^4k}{(2\pi)^3} e^{ik(x-x')}\delta(k^2) \quad (2.40)$$

According to the general considerations in the subsections 2.1 and 2.2 we have to investigate the expression (2.16) where the temperature dependent contribution has the explicit form

$$D^1_\beta \;=\; i\int \frac{d^4k}{(2\pi)^3} e^{ik(x-x')}\delta(k^2)\frac{e^{-\beta|k_0|}}{1-e^{-\beta|k_0|}} \quad (2.41)$$

$$=\; i\int \frac{d^3\vec{k}}{2k_0(2\pi)^3}(e^{ik_0(x_0-x'_0)} + e^{-ik_0(x_0-x'_0)})e^{i\vec{k}(\vec{x}-\vec{x}')}\frac{e^{-\beta k_0}}{1-e^{-\beta k_0}}. \quad (2.42)$$

In eq.(2.42) k_0 is the fixed positive expression $k_0 = \sqrt{\vec{k}^2}$. For large β the denominator in (2.42) can be expanded in a Taylor series with respect to $e^{-\beta k_0}$, the angle integrations can be carried out and finally also the remaining $|\vec{k}|$-integrations. As result we obtain:

$$D^1_\beta(x,x') = -\frac{i}{4\pi^2}{\sum_{n=-\infty}^{\infty}}'\left(\frac{1}{(z_0 - in\beta - i\epsilon)^2 - \vec{z}^2}\right), \quad z = x - x'. \quad (2.43)$$

By an inclusion of the temperature independent part we have to add the term with $n = 0$, where $-i\epsilon$ defines its Wightman-type singularity. If in this expression we carry out the formal substitution $x_0 = ix_4$ then we obtain directly the well-known Matsubara construction of imaginary time Green functions.

It is now possible to investigate D^-_β further. The summation can be performed using a Sommerfeld - Watson transformation:

$$D^-_\beta(x,x') \;=\; \frac{-i}{4\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{1}{(z_0 - i\epsilon - in\beta)^2 - \vec{z}^2}\right) \quad (2.44)$$

$$=\; -\frac{i}{4\pi^2}\frac{\pi}{2|\vec{z}|\beta}[\coth\frac{\pi}{\beta}(z_0 - i\epsilon - |\vec{z}|) - \coth\frac{\pi}{\beta}(z_0 - i\epsilon + |\vec{z}|)]. \quad (2.45)$$

The light-cone singularity is defined by the $i\epsilon$ prescription in accordance with the free space Wightman function. This expression is well suited for a study of the high temperature limit i.e. $\beta \to 0$. As result we obtain

$$D^-_\beta(x,x')|_{\beta\to 0} = \frac{i}{4\pi^2}\frac{(\pi)}{2|\vec{z}|\beta}[\epsilon(|\vec{z}| - z_0) + \epsilon(|\vec{z}| + z_0)]. \quad (2.46)$$

This is in accordance with calculation starting from eq.(2.42) in momentum space using $\frac{e^{-\beta|k_0|}}{1-e^{-\beta|k_0|}} \to \frac{1}{\beta k_0}$. Note that

$$[\epsilon(|\vec{z}| - z_0) + \epsilon(|\vec{z}| + z_0)] = 2\Theta(-z^2). \quad (2.47)$$

At very high temperature there is no causal propagation!

The low temperature limit can be studied without difficulties. Note that because of (2.16) we have clear separation of the temperature dependent part where the small temperature corrections are obtained by taking the first terms of the Taylor expansion (performed in (2.42)) and in the series-representation (2.44).

Presence of one Plate

All functions for the case of the one plate system can be obtained by an application of the reflection principle using the corresponding formulae of the free space. For this reason we write down only one representation as an example:

$$^sD^-_{1,\beta}(x,x') = \quad (2.48)$$

$$= -\frac{i}{4\pi^2}\frac{\pi}{2|\vec{z}|\beta}[\coth\frac{\pi}{\beta}(z_0 - i\epsilon - |\vec{z}|) - \coth\frac{\pi}{\beta}(z_0 - i\epsilon + |\vec{z}|)]$$
$$+\frac{i}{4\pi^2}\frac{\pi}{2|\vec{z}_1|\beta}[\coth\frac{\pi}{\beta}(z_0 - i\epsilon - |\vec{z}_1|) - \coth\frac{\pi}{\beta}(z_0 - i\epsilon + |\vec{z}_1|)].$$

where

$$|\vec{z}_1| = [(x_1 - x'_1)^2 + (x_2 - x'_2)^2 + (x_3 + x'_3)^2]^{1/2}.$$

For the high-temperature limit we get

$$D^-_\beta(x,x') \;=\; \frac{i}{4\pi^2}\frac{(\pi)}{2|\vec{z}|\beta}[\epsilon(|\vec{z}| - z_0) + \epsilon(|\vec{z}| + z_0)]$$
$$-\frac{i}{4\pi^2}\frac{(\pi)}{2|\vec{z}_1|\beta}[\epsilon(|\vec{z}_1| - z_0) + \epsilon(|\vec{z}_1| + z_0)].$$

Other expressions can be constructed using the corresponding formulae of the free space case.

Presence of two Plates

The case of two plates is much more complicated. We write down several alternative expressions valid for the space in-between the two plates. Outside the plates the Green functions are those of the case for one plate. In-between the plates according to the representation (2.26) we need the scalar Green function ${}^{s}D^{-}_{2,\beta}$ satisfying the Dirichlet boundary condition at the positions of the plates at $x_3 = 0; d$. For $T = 0$ these functions are given in [4]

$$ {}^{s}D^{-}_{2}(x,y) = \frac{2i}{(2\pi)^2 d}\sum_{n=1}^{\infty}\int \frac{d^2k_\perp}{2k_0} e^{-i\widetilde{k(x-y)}} \sin\frac{n\pi}{d}x_3 \sin\frac{n\pi}{d}y_3 \qquad (2.49)$$

$$ = -\frac{1}{8\pi d\zeta}\{\frac{1}{e^{\frac{i\pi}{d}(\zeta-x_3-y_3)}-1} + \frac{1}{e^{\frac{i\pi}{d}(\zeta+x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta-x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta+x_3-y_3)}-1}\}, \qquad (2.50)$$

$$\zeta^2 = (x_0 - x'_0 - i\epsilon)^2 - (x_1 - x'_1)^2 - (x_2 - x'_2)^2.$$

From these expressions follow the corresponding representations for $T \neq 0$. It is quite simple to write down a mixed representation where the $T = 0$ contributions are written explicitly in coordinate space whereas the temperature dependent part is given by a Fourier representation

$$ {}^{s}D^{-}_{2,\beta}(x,y) = \frac{2i}{(2\pi)^2 d}\sum_{n=1}^{\infty}\int \frac{d^2k_\perp}{2k_0}\frac{e^{-\beta k_0}}{1-e^{-\beta k_0}} \sin\frac{n\pi}{d}x_3 \sin\frac{n\pi}{d}y_3 \left(e^{-i\widetilde{k(x-y)}} + e^{i\widetilde{k(x-y)}}\right) \qquad (2.51)$$

$$ = -\frac{1}{8\pi d\zeta}\{\frac{1}{e^{\frac{i\pi}{d}(\zeta-x_3-y_3)}-1} + \frac{1}{e^{\frac{i\pi}{d}(\zeta+x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta-x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta+x_3-y_3)}-1}\}.$$

This representation shows the presence of the singularities resulting from the vacuum fluctuations and the less singular contribution resulting from the temperature dependent part. Consequently the known resonance structure of the correlation functions at $T = 0$ remains. Such resonances appeared if the distances between the considered events correspond to a classical light signal n-times reflected at the plates. On the other hand it is interesting to consider a Fourier representation which explicitly shows the frequency distribution. This is most easily obtained for the the correlation function (symmetrized Wightman-like function)

$${}^{s}W_{2,\beta} = {}^{s}D^{-}_{2,\beta}(\tilde{x}-\tilde{y}, x_3, y_3) - {}^{s}D^{-}_{2,\beta}(\tilde{y}-\tilde{x}, y_3, x_3)$$

$$ {}^{s}W_{2,\beta}(\tilde{x}-\tilde{y}, x_3, y_3) = \frac{2i}{(2\pi)^2 d}\sum_{n=1}^{\infty}\int\frac{d^2k_\perp}{2k_0}\left(e^{-ik_0(x_0-x'_0)} + e^{+ik_0(x_0-x'_0)}\right) e^{-ik_\perp(\vec{x}_\perp-\vec{x}'_\perp)} \sin\frac{n\pi}{d}x_3 \sin\frac{n\pi}{d}y_3 [1 + \frac{e^{-\beta k_0}}{1-e^{-\beta k_0}}]. \qquad (2.52)$$

Even though the allowed frequencies are fixed by the boundary conditions, the weight functions of the spectrum is changed by the temperature. The important point is, that the temperature dependent part is exponentially damped for very high frequencies. Consequently this part of the correlation functions is smooth and does not contribute to singularities produced by the temperature independent zero point fluctuations. Additionally we are interested in representations in coordinate space. The simplest possibility is to write an expression based on the reflection principle:

$$ {}^{s}D^{-}_{2,\beta}(x,x') = \sum_{l=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} [D^{-}(x_0 - x'_0 + in\beta, (x-x')_\perp, x_3 - x'_3 + 2ld) - D^{-}(x_0 - x'_0 + in\beta, (x-x')_\perp, x_3 + x'_3 + 2dl)]. \qquad (2.53)$$

Here we are able to perform partial summations. It is possible to write a formula relying on the reflection principle using the correct Green functions at $T = 0$ (2.50) satisfying the Dirichlet boundary conditions

$$ {}^{s}D^{-}_{2,\beta}(x,x') = \sum_{-\infty}^{+\infty} -\frac{1}{8\pi d\zeta_n}\{\frac{1}{e^{\frac{i\pi}{d}(\zeta_n-x_3-y_3)}-1} + \frac{1}{e^{\frac{i\pi}{d}(\zeta_n+x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta_n-x_3+y_3)}-1} - \frac{1}{e^{\frac{i\pi}{d}(\zeta_n+x_3-y_3)}-1}\} \qquad (2.54)$$

where

$$\zeta_n^2 = (x_0 - x'_0 - i\epsilon - in\beta)^2 - (x_1 - x'_1)^2 - (x_2 - x'_2)^2.$$

The other possibility is, to use the summed up temperature dependent free field function and apply then the reflection principle for fulfilling the Dirichlet boundary conditions

$$ {}^{s}D^{-}_{2,\beta}(x,x') = \sum_{l=-\infty}^{+\infty} [D^{-}_{\beta}(x_0 - x'_0, (x-x')_\perp, x_3 - x'_3 + 2ld) - D^{-}_{\beta}(x_0 - x'_0, (x-x')_\perp, x_3 + x'_3 + 2dl)]. \qquad (2.55)$$

It is now very interesting to study the high-temperature limit. Inserting the result for this limit in the free space case eq.(2.46) into eq.(2.55) we get:

$$D^-_\beta(x,x')|_{\beta\to 0} = \sum_{-\infty}^{+\infty} \frac{i}{4\pi^2}\frac{\pi}{2|\vec z_{-,l}|\beta}[\epsilon(|\vec z_{l,-}| - z_0) + \epsilon(|\vec z_{l,-}| + z_0)] - \sum_{-\infty}^{+\infty} \frac{i}{4\pi^2}\frac{\pi}{2|\vec z_{+,l}|\beta}[\epsilon(|\vec z_{+,l}| - z_0) + \epsilon(|\vec z_{+,l}| + z_0)]. \tag{2.56}$$

where

$$z_{-,l} = [z_\perp^2 + z_{3,-,l}^2]^{1/2},\quad z_{+,l} = [z_\perp^2 + z_{3,+,l}^2]^{1/2},\quad z_\perp^2 = z_1^2 + z_2^2,$$
$$z_{3,-,l} = x_3 - x'_3 + 2ld \quad z_{3,+,l} = x_3 + x'_3 + 2ld.$$

This alternate series must not be absolutely convergent. We try to perform a $\frac{1}{d}$-expansion. We apply it for simultaneous times of x_0 and x'_0 or $z_0 = 0$. The last restriction eliminates the ϵ-functions in eq.(2.56). We expand

$$\frac{1}{|\vec z_{l,\pm}|} = \frac{1}{[z_\perp^2 + (x_3 \pm x'_3 + 2ld)^2]^{1/2}} = \frac{1}{|2ld|}(1 - \frac{x_3 \pm x'_3}{2ld} + \frac{(x_3 \pm x'_3)^2 - \frac{1}{2}z_\perp^2}{(2ld)^2} - \ldots).$$

In this way we obtain

$$D^-_\beta(x,x')|_{\beta\to 0} = \frac{i\pi}{4\pi^2\beta}\{(\frac{1}{\sqrt{z_\perp^2 + (x_3 - x'_3)^2}} - \frac{1}{\sqrt{z_\perp^2 + (x_3 + x'_3)^2}}) + \sum_{-\infty}^{\prime\, +\infty}\frac{1}{|2ld|}([1 - \frac{x_3 - x'_3}{2ld} + ((x_3 - x'_3)^2 - \frac{1}{2}z_\perp^2)\frac{1}{(2ld)^2} + \ldots] - [1 - \frac{x_3 + x'_3}{2ld} + ((x_3 + x'_3)^2 - \frac{1}{2}z_\perp^2)\frac{1}{(2ld)^2} + \ldots])\}. \tag{2.57}$$

The infinite sum can be carried out using

$$\sum_1^\infty (\frac{1}{l})^3 = \zeta(3)$$

so that we obtain

$$D^-_\beta(x,x')|_{\beta\to 0} = \frac{i\pi}{4\pi^2\beta}(\frac{1}{\sqrt{z_\perp^2 + (x_3 - x'_3)^2}} - \frac{1}{\sqrt{z_\perp^2 + (x_3 + x'_3)^2}}) + \frac{i}{4\pi\beta d^3}(x_3 x'_3)\zeta(3). \tag{2.58}$$

This expansion is correct for $2ld > \min(|\vec z|, \sqrt{z_\perp^2 + (x_3 + x'_3)^2})$ The the d-independent part of this expression represents the result valid for one plate, the following terms constitute the $\frac{1}{d^3}$ corrections.

3 Fluctuations

As observables $T(x)$ we consider here the field strength, components of the energy-momentum tensor or the Casimir pressure. Local observable are measured over a finite space and finite time intervals $T = \int f(x)T(x)dx$, whereby the function $f(x)$ describes the measuring process. The essential information for the fluctuation of an observable is contained in the expectation value

$$\begin{aligned} W(x,x') &= \frac{1}{2}(<0|T(x)T(x')|0>_\beta + <0|T(x')T(x)|0>_\beta) \\ &\quad - <0|T(x)|0>_\beta <0|T(x')|0>_\beta \\ &= <0|T(x)T(x')|0>'_\beta. \end{aligned} \tag{3.1}$$

3.1 Field strength Fluctuations

The Green functions for the field strength are determined by the Green functions for the electromagnetic potentials.

$$<0|F_{\mu\nu}(x)F_{\mu'\nu'}(x')|0>_\beta = \tag{3.2}$$
$$\begin{aligned} &\partial^x_\mu\partial^{x'}_{\mu'} <0|A_\nu(x)A_{\nu'}(x')|0>_\beta + \partial^x_\nu\partial^{x'}_{\nu'} <0|A_\mu(x)A_{\mu'}(x')|0>_\beta \\ -\ &\partial^x_\nu\partial^{x'}_{\mu'} <0|A_\mu(x)A_{\nu'}(x')|0>_\beta - \partial^x_\mu\partial^{x'}_{\nu'} <0|A_\nu(x)A_{\mu'}(x')|0>_\beta \\ =\ &O^{\rho\rho'}_{\mu\nu;\mu'\nu'} <0|A_\rho(x)A_{\rho'}(x')|0>_\beta . \end{aligned} \tag{3.3}$$

where

$$O^{\rho\rho'}_{\mu\nu,\mu'\nu'} = +g^\rho_\nu g^{\rho'}_{\nu'}\partial^x_\mu\partial^{x'}_{\mu'} + g^\rho_\mu g^{\rho'}_{\mu'}\partial^x_\nu\partial^{x'}_{\nu'} - g^\rho_\mu g^{\rho'}_{\nu'}\partial^x_\nu\partial^{x'}_{\mu'} - g^\rho_\nu g^{\rho'}_{\mu'}\partial^x_\mu\partial^{x'}_{\nu'} \tag{3.4}$$

From this expression it is clear that the properties of the correlation functions are directly reflected by the fluctuations. The essential points are

- The allowed frequencies are those of the $T = 0$ case. However the shape of the frequency spectrum is changed.
- For large frequencies in any case the zero point fluctuations become dominant
- The high frequency behaviour determines the singularity structure in x-space. Therefore the possible singularities are those of the vacuum case.

- The temperature dependent contributions are damped exponentially for high frequencies. therefore we expect in the x-space only relatively smooth contributions.

It is however very important that the frequencies are influenced by the boundary conditions (the presence of conductors) and only these frequencies are contribute to the fluctuations. Therefore also the pure temperature dependent fluctuations are typical for the given physical situations. The investigations of enhanced or inhibited transitions in a resonator are not disturbed by temperature effects. The effects of the different behaviour of thin or thick plates is not influenced by the temperature. Here we have considered only the case of thin plates. It is interesting that the large T and large d approximation of the correlation function (2.58) leads directly to the Casimir pressure in the same approximation

$$p(d,\beta)|_{\frac{d}{\beta}\to\infty} = -\frac{\zeta(3)}{4\pi\beta d^3}. \tag{3.5}$$

3.2 Pressure Fluctuations

As a typical quantity we consider the diagonal $T_{\mu\mu}$ elements of the energy-momentum tensor [7], [10], [11], [12], [13], [14]. For a discussion of the Casimir pressure we need the 33-component. From this quantity the Casimir pressure on a plate located at $x_3 = a$ can be obtained as the difference of T_{33} across the plates

$$p(x) = T_{33}(x_3 = a+\epsilon) - T_{33}(x_3 = a-\epsilon) \tag{3.6}$$

For the energy- momentum tensor we use the symmetric tensor

$$T_{\mu\nu} = F^{\rho}_{\mu}F_{\rho\nu} - 1/4 g_{\mu\nu}F_{\sigma\tau}F^{\sigma\tau} \tag{3.7}$$

with the field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

As regularization procedure we use the point splitting technique. For the calculation of the correlation in free field theory we can simply apply the Wick theorem, we obtain (see [4])

$$\begin{aligned} &< 0|T_{\mu\mu}(x)T_{\mu\mu}(x')|0>'_\beta = \\ &\quad -\partial^{\overline{xy}}\partial^{\overline{x'y'}}[(D^-_\beta(x,x') + \bar{D}^-_{\beta-}(x,x'))(D^-_\beta(y,y') + \bar{D}^-_{\beta-}(y,y')) \\ &\quad +\bar{D}^-_{\beta+}(x,x')\bar{D}^-_{\beta+}(y,y')]|_{y\to x, y'\to x'} . \end{aligned} \tag{3.8}$$

whereby we take into account the general structure of Green functions in the presence of plates [4], [8].

$$< 0|A_\mu(x)A_\nu(y)|0>_\beta = i g_{\mu\nu} D^-_\beta(x-y) + i(\tilde{g}_{\mu\nu} - \frac{\tilde{\partial}^x_\mu \tilde{\partial}^y_\nu}{\tilde{\partial}^x\tilde{\partial}^y})\bar{D}^-_\beta(x,y) \tag{3.9}$$

The function $\bar{D}^-_\beta$ is defined as the difference term between the Green function ${}^sD^-_\beta$ satisfying the Dirichlet boundary condition and the free space function

$$\bar{D}^-_\beta = {}^sD^-_\beta - D^-_\beta . \tag{3.10}$$

One further specification of the function $\bar{D}^-$ follows from the structure of the x_3, x'_3 dependence

$$\bar{D}^-_\beta(x,y) = \bar{D}^-_{\beta-}(\tilde{x}-\tilde{y}, x_3-y_3) + \bar{D}^-_{\beta+}(\tilde{x}-\tilde{y}, x_3+y_3). \tag{3.11}$$

The indexes $\mu\mu$ are suppressed in part, they are included in the following definitions

$$\partial^{xy} = g^{\rho\lambda}\partial^x_\rho\partial^y_\lambda, \quad \partial^{\overline{xy}} = {}^{(\mu)}h^{\rho\lambda}\partial^x_\rho\partial^y_\lambda,$$

whereby the matrix ${}^{(\mu)}h_{ij}$ reads

$${}^{(\mu)}h_{\alpha\beta} = \begin{cases} -g_{\alpha\beta}\, g_{\mu\mu} & \alpha \neq \mu \text{ or } \beta \neq \mu \\ +g_{\alpha\beta}\, g_{\mu\mu} & \alpha = \beta = \mu . \end{cases}$$

As it should be the formula for the pressure fluctuations (3.8) is bilinear in the two-point Green functions. There we observe a superposition of zero point fluctuations and temperature fluctuations. In fact the Green functions is split according to

$$\begin{aligned} {}^sD^-_\beta(x,x') &= {}^sD^-_{\beta,-}(x,x') + {}^sD^-_{\beta,+}(x,x'), \\ {}^sD^-_{\beta,-}(x,x') &= D^-_\beta(x,x') + \overline{D}^-_{\beta,-}(x,x') \end{aligned}$$

If we look for an example to the Green functions for the case of the space between the two plates then we observe this splitting in eq.(2.53) as well as in the approximated representation for high temperature (2.58). Of course during the limiting procedure the zero point fluctuations are dropped out completely, but this may not be correct because of the singularities contained in the vacuum part.

The fluctuation of the Casimir pressure on a plate located at $x_3 = a$ can be reduced to the correlation function (3.8) due to the relation (3.6). One obtains

$$\begin{aligned} < 0|p(y)p(x')|0>'_\beta| \,|_{x_3=y_3=a} = &< 0|T_{33}(x)T_{33}(x')|0>'_\beta |_{x_3=x'_3=a+\epsilon} + \\ &< 0|T_{33}(x)T_{33}(x')|0>'_\beta |_{x_3=x'_3=a-\epsilon} \end{aligned} \tag{3.12}$$

for ideally conducting plates. The reason for the absence of mixed terms originates from the fact, that physical modes cannot propagate across the plates for ideal conductors.

As to be expected all expressions are reduced to the two-point correlation functions of the potentials, so that also here all properties of these correlation functions are reflected by the corresponding properties of the stress fluctuations. The general features of the field strength fluctuations are also valid here.

We give only a Fourier representation for the fluctuation of the Casimir pressure on the inner side of the plate

$$W_{2,\beta}(\zeta,d) \equiv <0|T_{33}(x)T_{33}(x')|0>'_\beta|_{x_3=x'_3=0_+} =$$
$$\frac{1}{2d^2}\sum_{n=1}^{\infty}\sum_{n'=1}^{\infty}\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{d^2p'_\perp}{(2\pi)^2}\frac{1}{2}(e^{-i(p_0+p'_0)(x_0-x'_0)}+e^{+i(p_0+p'_0)(x_0-x'_0)})e^{+i(p+p')_\perp(x-x')_\perp}$$
$$[(\frac{\pi n}{d})^2(\frac{\pi n'}{d})^2+(\widetilde{pp'})^2]\frac{1}{p_0p'_0}[1+\frac{e^{-\beta p_0}}{1-e^{-\beta p_0}}][1+\frac{e^{-\beta p'_0}}{1-e^{-\beta p'_0}}] \quad (3.13)$$

Up to the modifications due to the temperature dependence this result is identical the expression derived in [4]. As in [2] we expect for thick plates simple an extension of the summations to $n=0$ and $n'=0$. For the case of vanishing temperature we have shown that for special measuring processes the presence or absence of the $n=0$ terms in the summation (as characteristics of thin or thick plates) leads to different results [4] for the fluctuations. We show that the same conclusion can be drawn in the presence of temperature, moreover it turned out the temperature dependent part is nonleading in this special case.

We combine our results on correlation functions with measuring processes which make recourse to specific functions which characterize the measuring procedure. We factorize the characteristic function $f(x_0,\vec{x}_\perp)$ according to $f(x_0,\vec{x}_\perp)=g(x_0)h(\vec{x}_\perp)$. As an example we choose

$$g(x_0)=\frac{\tau}{\pi}\frac{1}{x_0^2+\tau^2}, \qquad \int dx_0 e^{-ip_0x_0}g(x_0)=e^{-|p_0|\tau}$$

and $h(x_\perp)$ is implicitly defined by

$$\int dx_\perp e^{i\vec{p}_\perp\vec{x}_\perp}h(x_\perp)=e^{\frac{a\pi}{d}}e^{-a\sqrt{p_\perp^2+(\pi/d)^2}}$$

Both functions $g(x_0)$ and $h(x_\perp)$ are normalized to 1 and its Fourier transforms are dimensionless. By the help of this functions the fluctuation $(\Delta T)^2$ is expressed by means of the correlation function $W(\tilde{x},\tilde{x}')=<0|T_{33}(\tilde{x})T_{33}(\tilde{x}')|>'$ as

$$(\Delta T)^2=\int d\tilde{x}d\tilde{x}'f(\tilde{x})f(\tilde{x}')W(\tilde{x},\tilde{x}'), \quad \tilde{x}=(x_0,x_1,x_2). \quad (3.14)$$

At first we consider the case of two thin plates. Here we are interested in the fluctuations of T_{33} on the inner side of the plate at $x_3=0$. Combining the foregoing equations with eq.(3.13) we obtain

$$(\Delta T)^2 = \frac{1}{2d^2}e^{2a\pi/d}\sum_{n=1}^{\infty}\sum_{n'=1}^{\infty}\int\frac{d^2p_\perp}{(2\pi)^2p_0}\int\frac{d^2p'_\perp}{(2\pi)^2p'_0}((\tilde{p}\tilde{p}')^2+(\frac{n\pi}{d})^2(\frac{n'\pi}{d})^2)$$
$$\exp(-p_0\tau-p'_0\tau)\exp(-a(\sqrt{p_\perp^2+(\frac{\pi}{d})^2}+\sqrt{p'^2_\perp+(\frac{\pi}{d})^2}))$$
$$(1+\frac{e^{-\beta p_0}}{1-e^{-\beta p_0}})(1+\frac{e^{-\beta p'_0}}{1-e^{-\beta p'_0}}) \quad (3.15)$$

with $p_0=\sqrt{p_\perp^2+(\frac{n\pi}{d})^2}$ and $p'_0=\sqrt{p'^2_\perp+(\frac{n'\pi}{d})^2}$. According to realistic possibilities the characteristic time τ of a measuring process is large in comparison with the time interval necessary for a light signal to traverse the plate distance d

$$d \ll \tau. \quad (3.16)$$

Accordingly $(\Delta T)^2$ is dominated by the term with $n=n'=1$ with the corresponding modifications of p_0 and p'_0 This yields taking into account rotation invariance

$$(\Delta T)^2 = \frac{1}{2d^2}e^{2a\pi/d}\int\frac{d^2p_\perp}{(2\pi)^2p_0}\int\frac{d^2p'_\perp}{(2\pi)^2p'_0}$$
$$\{p_0^2p'^2_0+p_1^2p'^2_1+p_2^2p'^2_2+(\frac{\pi}{d})^4\}e^{-(\tau+a)(p_0+p'_0)}$$
$$(1+\frac{e^{-\beta p_0}}{1-e^{-\beta p_0}})(1+\frac{e^{-\beta p'_0}}{1-e^{-\beta p'_0}}). \quad (3.17)$$

In the limit (3.16) considered here the contributions from $p_1^2p'^2_1+p_2^2p'^2_2$ in the bracket are nonleading whereas the remaining contributions are equal. For the temperature dependent factors we apply a power series expansion. The final result reads

$$(\Delta T)^2=\frac{\pi^2}{4d^6(\tau+a)}e^{-\frac{2\pi\tau}{d}}\left(\frac{1}{\tau+a}+\frac{2}{\tau+a+\beta}e^{-\beta\frac{\pi}{d}}\right). \quad (3.18)$$

Its leading term coincides with the result of [4] because the high frequency temperature dependent fluctuations are suppressed. We underline that this result is based essentially on the absence of the modes with n=0. The presence of such modes would change (3.18) to a power-like behaviour, which coincides with the result obtained in reference [2]. These results are consequences of different models for the plates.

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References

[1] S. Hacyan et.al.,*J. Phys. A: Math. Gen.* **23** (1990) 2401.

[2] G. Barton, *J. Phys. A: Math. Gen.* **24** (1991) 991; **24** (1991) 5533;
New Aspects of the Casimir Effect: Fluctuations and radiative Reactions *in* "Cavity Quantum Electrodynamics" (P.R.Berman, Ed.), Academic Press.

[3] M.-Th. Jaekel and S. Reynoud, *Phys. Lett.* **A 167** (1992) 227.

[4] D. Robaschik, E. Wieczorek, "Fluctuations of the Casimir Pressure and the Quantization of Electrodynamics"
Preprint DESY 93-024, to be published in *Annals of Physics.*

[5] A. J. Niemi, G. W. Semenoff, *Annals of Physics* **152** (1984) 105 .

[6] H. Umezwawa, H. Matsumoto, M. Tachiki, " Thermo Field Dynamics and Condensed States ,
North-Holland Pub. Comp. Amsterdam 1982.

[7] K.Scharnhorst, D.Robaschik, E.Wieczorek, *Annalen der Physik* **44** (1987) 351

[8] M. Bordag, D. Robaschik, E. Wieczorek, *Ann. Phys. (N.Y.)* **165** (1985) 192.

[9] M.Bordag, Preprint Dubna P2-84-115, Dubna 1984.

[10] L. S. Brown and G.J. MacLay *Phys. Rev.* **184** (1969) 1272.

[11] G. Barton and N. S. J. Fawcett, *Phys. Rep.* **170** (1988) 1.

[12] V. M. Mostepanenko, N. N. Trunov, "The Casimir Effect and its Application", Atomizdat, Moscow, 1990 (in Russian);

[13] Yu. S. Barash, "Van der Waals Forces", Nauka, Moscow 1988 (in Russian).

[14] G. Plunien, B. Müller, W. Greiner, *Phys. Rep.* **134** (1986) 87.

[15] M. Babiker and G. Barton, *Proc. Roy. Soc. London* **A. 326** (1972) 277-288.

[16] M. Fierz, *Helv. Phys. Acta* **33** (1960) 855,
J. Mehra *Physica* **37** (1967) 145.

[17] N.N. Bogoliubov and D.V. Shirkov, "Introduction to Quantum Field Theory", Interscience Publishers, New York.