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On the renormalization of nonrenormalizable interactions

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Abstract

We show that supersymmetric QCD with additional nonrenormalizable interaction $\delta L = (h/M)\int d^2\theta (XY)^2 + h.c.$ (here X and Y are superquark fields) and QCD with additional four-fermion interaction of the colour quarks can be renormalized. We find that the considered models are equivalent to the fixed-point solutions of the renormalization group equations for the corresponding renormalized analogs of the nonrenormalizable models.

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1 Introduction

It is well known that the requirement of the renormalizability restricts considerably the arbitraireness of the Lagrangian's choise. That leads to the big predictive power of the renormalizable theories. Naturally the question arises: are the nonrenormalizable theories of any meaning? In refs.[1-4] it was shown that nonlinear σ-model and four-fermion models are renormalizable in $d = 3$ within $1/N$ expansion. Moreover these models are renormalizable in dimensions $d < 4$ and they are equivalent to the infrared fixed points of the corresponding superrenormalizable analogs [5,6]. Certainly, the four-dimensional case is the most interesting. In our previous papers [7,8] we have shown that the four-dimensional supersymmetric QCD with additional nonrenormalizable interaction of superquarks and the standard QCD plus nonrenormalizable four-fermion Nambu interaction of colour quarks can be renormalized and moreover the considered models are equivalent to the fixed-point solutions of the corresponding renormalized analogs. Note that in refs.[6,9,10] the four-fermion interaction of the Nambu type has been studied in the limit $d \to 4$. The main conclusion of the refs.[6,9] is that the four-fermion interaction is trivial in the limit of the regularization removing. The main difference between the models of the refs.[6,9] and the models considered in our papers [7,8] is that we consider the four-fermion interaction plus QCD, whereas in refs.[6,9,10] only the four-fermion interaction without any additional nonabelian interaction has been studied. It appears that an account of the nonabelian gauge interaction is crusial, namely, the asymptotic freedom of QCD makes some logarithmically ultraviolet divergent integrals ultraviolet convergent that allows to make sence to the nonrenormalizable interaction.

This paper is an extended version of our previous short notes [7,8]. We clarify some subtle points of refs.[7,8]. The organization of the paper is as follows. In section 2 we consider four-fermion interaction of the vector⊗vector type in $d = 4 - 2\epsilon$ space-time and show that it is equivalent to the infrared fixed point of quantum electrodynamics with massless fermions. We find that both QED and vector⊗vector four-fermion interaction are trivial in the limit of the regularization removing. In section 3 we study the supersymmetric QCD with additional nonrenormalizable interaction of superquarks. In section 4 we study QCD with additional Nambu type four-fermion interaction of coloured quarks. We show that both supersymmetric and nonsupersymmetric nonrenormalizable models can be renormalized and moreover they are equivalent to the fixed-point solutions of the corresponding renormalized models. Section 5 contains concluding remarks.

2 Four-fermion vector⊗vector interaction in $d = 4 - 2\epsilon$

Consider the four-fermion Lagrangian

$$L = i\sum_{k=1}^{N}\overline{\psi}_k\gamma^\mu\partial_\mu\psi_k - (1/2MN)j^\mu j_\mu\,, \qquad (2.1)$$

$$j_\mu = \sum_{k=1}^{N}\overline{\psi}_k\gamma_\mu\psi_k$$

The Lagrangian (2.1) can be rewritten in the form

$$L = i\sum_{k=1}^{N}\overline{\psi}_k\gamma^\mu\partial_\mu\psi_k + N^{-1/2}j^\mu A_\mu + \frac{M}{2}A^\mu A_\mu\,, \qquad (2.2)$$

where A_μ is an auxilliary field. To restore the $U(1)$ gauge invariance let us introduce additional scalar field $\phi(x)$. The Lagrangian

$$L_1 = i\sum_{k=1}^{N}\overline{\psi}_k\gamma^\mu\partial_\mu\psi_k + N^{-1/2}j^\mu A_\mu + \frac{M}{2}(A_\mu - \partial_\mu\phi)(A^\mu - \partial^\mu\phi) \qquad (2.3)$$

is invariant under the gauge transformations

$$A_\mu \to A_\mu + \partial_\mu\alpha\,,$$

$$\phi \to \phi + \alpha\,,$$

$$\psi_k \to \exp(i\alpha N^{-1/2})\psi_k$$

In the unitare gauge $\phi = 0$ the Lagrangians (2.2) and (2.3) coincide. We shall work in the transverse gauge $\partial_\mu A^\mu = 0$. The free propagator for the auxilliary vector field A_μ is

$$D_{\mu\nu}(k) = i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{M} \qquad (2.4)$$

According to standard counting rules the model (2.3) is nonrenormalizable for $d > 2$. However it appears that the model is renormalizable in the framework of the 1/N expansion [11] . Really, in the leading order on 1/N only bubble one loop diagrams with fermions inside of the loop contribute to the auxilliary vector propagator and in the leading order on 1/N it has the form

$$D^{eff}_{\mu\nu}(k) = i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{M + c(\epsilon)(-k^2)^{1-\epsilon}}\,, \qquad (2.5)$$

where

$$c(\epsilon) = 2^{1-d}\frac{d-2}{d-1}\pi^{-d/2}\Gamma(2-d/2)\frac{\Gamma^2(d/2-1)}{\Gamma(d-2)}$$

The 1/N perturbation theory is in fact equivalent to the standard perturbation theory and the single difference consists in the use of the effective propagator (2.5) instead of free propagator (2.4). An account of bubble one loop graphs leads to the power falloff of the vector propagator

$$D_{\mu\nu}(k) \sim (k^2)^{\epsilon-1} \qquad (2.6)$$

From the ultraviolet behaviour (2.6) of the vector propagator we find that the dimension of A_μ field is $(d-2)/2$. The dimension of the fermion field $\psi_k(x)$ remains canonical, $(d-1)/2$. Thus, the $j_\mu A^\mu$ interaction is dimensionless, and as a consequence, the model (2.3) is renormalizable within 1/N perturbation theory for $d < 4$. Moreover, it appears that the model is ultraviolet finite in each order of the $1/N$ perturbation expansion [4] . For instance, L loop correction to the propagator of the vector field A_μ schematically is determined by the integral

$$\int d^{L(4-2\epsilon)}(p^2 + f(k^2))^{-\epsilon-L(4-2\epsilon)}$$

which is ultraviolet convergent as a whole and the single ultraviolet subdivergences of the fermion field propagator and the vertex function are eliminated by the introduction of the counterterm

$$\delta L = \delta Z_F(i\sum_{k=1}^{N}\overline{\psi}_k\gamma^\mu\partial_\mu\psi_k + (N)^{-1/2}j^\mu A_\mu) \qquad (2.7)$$

The δZ_F depends on the gauge like in standard QED and it is possible to find a gauge where it vanishes. So we have found that the four-fermion interaction (2.1) is renormalizable and ultraviolet finite within 1/N perturbation theory. It appears that it is very easy to understand the obtained results using the renormalization group technique and the minimal subtraction scheme. Let us consider QED with N identical massless fermions in $d = 4 - 2\epsilon$ space-time with the Lagrangian

$$L_{QED} = -\frac{1}{4e_B^2}F_{\mu\nu}F^{\mu\nu} + i\sum_{k=1}^{N}\overline{\psi}_k\gamma^\mu\partial_\mu\psi_k + (N)^{-1/2}j^\mu A_\mu \qquad (2.8)$$

The relation between bare and renormalized charges in the minimal subtraction scheme is [12]

$$\alpha_B = \alpha_r\mu^{2\epsilon}[1 + \sum_{n=1}^{\infty}\frac{c_n(\alpha_r, N)}{\epsilon^n}]\,, \qquad (2.9)$$

where $\alpha_B = \frac{e_B^2}{4\pi}$ and $\alpha_r = \frac{e_r^2}{4\pi}$.

In $d = 4 - 2\epsilon$ QED is superrenormalizable theory and all Green's functions are ultraviolet finite in each order of the perturbation theory. The β-function has the form [12]

$$\beta(\alpha_r, \epsilon, N) = -\epsilon\alpha_r + \beta_4(\alpha_r, N)\,, \tag{2.10}$$

where

$$\beta_4(\alpha_r, N) = \frac{\alpha_r^2}{3\pi} + O(\alpha_r^3) \tag{2.11}$$

Due to the Ward identities we have the relation

$$\beta_4(\alpha_r, N) = \alpha_r\gamma_A(\alpha_r, N) \tag{2.11}$$

between the β-function and the anomalous dimension $\gamma(\alpha_r, N)$ of the electromagnetic field A_μ. As a consequence we find that at the infrared fixed point ($\beta(\alpha_r^f, \epsilon, N) = 0$) the anomalous dimension of the photon field $\gamma(\alpha_r^f, N) = \epsilon$ and the photon propagator at the fixed point is proportional to $(k^2)^{-1+\epsilon}$. One can check that the infrared fixed point α_r^f corresponds to the infinite value of the bare coupling constant α_B. Really, the dimensional analysis leads for $M = 0$ to the following general formula for the vector propagator:

$$D_{\mu\nu}(k) = i(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\frac{(-k^2)^\epsilon}{k^2}F(\alpha_B(-k^2)^{-\epsilon})\,, \tag{2.12}$$

where $F(x)$ is some function. From the expression (2.12) we find that the infrared limit $k^2 \to 0$ is equivalent to the limit $\alpha_B \to \infty$. So we find that the infrared fixed point corresponds to the infinite bare coupling constant or to the QED without kinetic term for photon field in full correspondence with the ϕ^4 model where the infrared fixed point describes nonlinear σ-model [5]. In other words, our nonrenormalizable model (2.3) for $M = 0$ is equivalent to the infrared fixed point of the superrenormalizable model, QED in $d = 4 - 2\epsilon$, and it is the main lesson from this section.

3 Nonrenormalizable supersymmetric interaction

Consider the Lagrangian

$$L_t = L_{SQCD} + \delta L\,, \tag{3.1}$$

where

$$L_{SQCD} = \int d^4\theta[\overline{X}_i \exp(gV)X_i + \overline{Y}_i exp(-gV)Y_i] + \frac{1}{64C_2(G)}Tr\int d^2\theta W^\alpha W_\alpha - \int d^2\theta m_i X_i Y_i + \tag{3.2}$$

is the Lagrangian of $N = 1$ supersymmetric QCD and

$$\delta L = (h/M)\int d^2\theta (X_i Y_i)^2 + h.c. \tag{3.3}$$

According to standard counting rules the interaction δL is nonrenormalizable. The model contains a gauge field supermultiplet [13] $W_\alpha \sim (\lambda_\alpha, F_{\mu\nu})$ in the adjoint representation of the $SU(N)$ gauge group and M (flavours) chiral matter supermultiplets,

$$X_i = \phi_i + 2^{1/2}\theta\psi_i + \theta\theta F_i\,,$$

$$Y_i = \overline{\phi}_i + (2)^{1/2}\theta\overline{\psi}_i + \theta\theta\overline{F_i}$$

in the N and $\overline{N}$ fundamental representations respectively. The Lagrangian δL can be rewritten in the equivalent form

$$\delta L = \int d^2\theta(hX_iY_i\Phi - \frac{1}{4}M\Phi^2) + h.c. \tag{3.4}$$

Here $\Phi(x,\theta)$ is an auxilliary chiral superfield

$$\Phi(x,\theta) = \Phi(x) + (2)^{1/2}\theta\psi(x) + \theta\theta F(x)$$

The Lagrangian L_t can be obtained as the limit $Z \to 0$ of the renormalized Lagrangian

$$L_Z = L_t + Z\int d^4\theta\overline{\Phi}\Phi \tag{3.5}$$

In fact, to prove the "renormalizability" of the nonrenormalizable Lagrangian L_t we have to prove that it is not necessary to introduce the ultraviolet divergent counterterm $\sim \int d^4\theta\overline{\Phi}\Phi$, in other words, we have to prove that the radiative corrections to the propagator $< \overline{\Phi}\Phi >$ are ultraviolet finite. We shall be interested in the ultraviolet behaviour of the model (3.1), therefore we can neglect the masses. The h^2-correction to the propagator of the chiral superfield is proportional to the correlator

$$K(p^2) = i\int < 0 \mid T((\phi_i(x)\overline{\phi}_i(x)), (\phi_j(0)\overline{\phi}_j(0))^*) \mid 0 > \exp(ipx)d^4x \tag{3.6}$$

The Kallen-Lehmann representation for the correlator (3.6) has the form

$$K(p^2) = \int_0^\infty \frac{\rho(t)dt}{(t - p^2 - i\epsilon)} \tag{3.7}$$

For the free superquark fields in $d = 4 - 2\epsilon$ the spectral density $\rho(t) \sim NM(16\pi^2)^{-1}t^{-\epsilon}$, the integral (3.7) is ultraviolet convergent and it is proportional to $K(p^2) \sim \frac{1}{\epsilon}p^2$. In the limit $\epsilon \to 0$ of the regularization removing $K(p^2)$ is divergent and to make it convergent we have to make subtraction in the Kallen-Lehmann representation that at the language of counterterms corresponds to the introduction of the wave function counterterm $\delta Z\int d^4\theta\overline{\Phi}\Phi$. However, if we take into account SQCD corrections in the squark loop the situation changes entirely. Due to the property of asymptotic freedom of SQCD we can reliably calculate the ultraviolet behaviour of the spectral density $\rho(t)$. The spectral density in $d = 4 - 2\epsilon$ for the renormalized chiral superfield $\Phi_r(x,\theta) = Z^{1/2}\Phi(x,\theta)$ obeys the renormalization group equation

$$(\mu d/d\mu + \beta(g,\epsilon) + 4\gamma(g))\rho(t) = 0 \tag{3.8}$$

In one loop approximation

$$\beta(g,\epsilon) = -g\epsilon - \beta_0 g^3 ,$$

$$\gamma(g) = -\gamma_0 g^2 ,$$

$$\beta_0 = (1/16\pi^2)(3N - M) ,$$

$$\gamma_0 = (1/16\pi^2)(N^2 - 1)/N$$

The solution for the renormalization group equation (3.8) for the spectral density $\rho(t)$ is

$$\rho(t) = c(\epsilon)NM(16\pi^2)^{-1}(t/\mu^2)^{-\epsilon}(\overline{g}^2 + \epsilon/\beta_0)^b(g^2 + \epsilon/\beta_0)^{-b} . \tag{3.9}$$

$$\overline{g}^2 = (-\beta_0/\epsilon + (\beta_0/\epsilon + 1/g^2)(t/\mu^2)^\epsilon)^{-1} , \tag{3.10}$$

$$b = \frac{2(N^2 - 1)}{N(3N - M)} ,$$

$$c(0) = 1$$

In the limit $\epsilon \to 0$ the spectral density $\rho(t) \sim \ln^{-b}(t)$. One can check that for $b > 1$ the dispersion relation (3.7) without subtraction is ultraviolet finite in the limit of the regularization removing $\epsilon \to 0$. At the language of counterterms it means that it is not necessary to introduce the wave function renormalization counterterm. In other words we have found that in the approximation when inside of the squark loop only SQCD corrections are taken into account the quantum corrections to the propagator of chiral superfield are ultraviolet finite in the limit of the regularirization removing. Note that we have used dimensional regularization which does not preserve the supersymmetry in higher loops. However we worked in fact in leading log approximation which corresponds to the summation of the one loop bubble type diagrams, so our approximation preserves supersymmetry. Due to the property of asymptotic freedom our approximation is reliable in the ultraviolet region so an account of higher loops does not change our main result about ultraviolet convergency of the dispersion representation (3.7). However in general we have to take into account the corrections due to the interaction $h \int d^2\theta XY\Phi + h.c.$ term in the nonrenormalizable Lagrangian (3.1). In this case unlike to the case of the four-fermion interaction in $d = 4 - 2\epsilon$ it is very difficult to develop $1/N$ perturbation theory and we have to look for indirect ways for the investigation of the model.

To understand the relation between the nonrenormalized Lagrangian (3.1) and the renormalized Lagrangian $L_{Z=1}$ consider the renormalization group equations for the effective coupling constants $\overline{g}(t)$ and $\overline{h}(t)$ for the renormalized lagrangian $L_{Z=1}$. In one loop approximation in $d = 4$ the equations read

$$\mu d\overline{g}/d\mu = -\beta_0 g^3 , \tag{3.11}$$

$$\mu d\overline{h}/d\mu = a\overline{h}^3 - 2\gamma_o\overline{g}^2\overline{h} , \tag{3.12}$$

where $a = (2 + NM)(16\pi^2)^{-1}$, $\gamma_0 = (1/16\pi^2)(N^2 - 1)/N$, $\beta_0 = (1/16\pi^2)(3N - M)$ and $t = \ln(\mu/\mu_0)$.

For the initial conditions $\overline{g}(0) = g$, $\overline{h}(0) = g$ the solution of the system (3.11-12) is

$$\overline{g}^2(t) = \frac{g^2}{1 + 2\beta_o g^2 t} , \tag{3.13}$$

$$\overline{h}^2(t) = \frac{(t + 1/2\beta_0 g^2)^{-2\gamma_0/\beta_0}}{C + [2a\beta_0/(2\gamma_0 - \beta_0)](t + 1/2\beta_0 g^2)^{1-2\gamma_0/\beta_0}} , \tag{3.14}$$

where

$$C = \frac{2(2\gamma_O - \beta_O)\beta_0 g^2 - 2\beta_0 a h^2}{2h^2 g^2 \beta_0(2\gamma_0 - \beta_0)(2\beta_0 g^2)^{-2\gamma_0/\beta_0}}$$

For $2\gamma_0 - \beta_0 < 0$ the effective coupling $\overline{h}^2(t)$ develops a ghost pole, that means the inconsistency of the model at quantum level. The existence of the ghost pole manifests in the logarithmical divergence of the correlator $K(p^2)$ after the regularization removing. For the case $2\gamma_0 - \beta_0 > 0$ we have three regions on the phase plane ($\overline{h}^2 \geq 0, \overline{g}^2 \geq 0$) describing qualitatively different ultraviolet behaviour:

$$\overline{h}^2(t) < k\overline{g}^2 , \tag{3.15}$$

$$\overline{h}^2(t) = k\overline{g}^2 , \tag{3.16}$$

$$\overline{h}^2(t) > k\overline{g}^2 , \tag{3.17}$$

where $k = (2\gamma_0 - \beta_0)/a$.

In the region (3.17) the effective coupling constant $\overline{h}^2(t)$ is not asymptotically free and moreover it has ghost pole that means the inconsistency of the theory for $h^2 > kg^2$. In the regions (3.15), (3.16) we have asymtotic freedom for both $\overline{g}^2(t)$ and $\overline{h}^2(t)$ coupling constants so the theory is consistent. The ultraviolet asymptotics of the Φ propagator in the region (3.15) coincides with the free propagator

$$D_{\Phi\Phi^*} \equiv -i \int < 0 \mid T(\Phi(x), \Phi^*(0)) \mid 0 > \exp(ipx) d^4x \sim 1/p^2 \tag{3.18}$$

From the equal-time commutation relation for the renormalized scalar field $\Phi(x)$

$$[\partial_0\Phi^*(x_0,\overline{x}),\Phi(x_0,\overline{y})] = (1/i)Z^{-1}\delta^{(3)}(\overline{x}-\overline{y})\,. \tag{3.19}$$

(Z is a wave function renormalization of the scalar $\Phi(x)$ field) and the Kallen-Lehmann representation for the $\Phi(x)$ field it follows that the ultraviolet asymptotics for the scalar propagator is

$$D_{\Phi\Phi^*}(p^2) \sim \frac{Z}{p^2} \tag{3.20}$$

Thus we conclude that for the region (3.15) we have $Z = const \neq 0$ (the finite wave function renormalization). For the fixed-point solution (3.16) the ultraviolet asymptotics for the scalar propagator is

$$D_{\Phi\Phi^*} \sim (1/p^2)\ln^g(p^2)\,, \tag{3.21}$$

$$g = (2\frac{\gamma_0}{\beta_0}-1)\frac{NM}{2+NM}$$

and it decreases in the ultraviolet region more slowly than $1/p^2$. Therefore we find that for the special renormalization-group solution (3.16) the wave function renormalization Z of the Φ field goes to zero in the limit of the regularization removing. It means that the chiral superfield $\Phi(x,\theta)$ becomes the auxilliary field (kinetic term for the $\Phi(x,\theta)$ field vanishes) in the limit of the regularization removing. So we conjecture that the fixed-point solution (3.16) is equivalent to the nonrenormalizable model (3.1). The precise statement is that for the fixed-point solution (3.16) the Schwinger's equations for the renormalized Green's functions coincide with formal Schwinger's equations for the non-renormalizable Lagrangian (3.1). For instance, the Schwinger's equation for the D_{F,F^*} propagator has the form

$$D^{-1}_{F,F^*}(p^2) - Z = \frac{ih}{(2\pi)^4}\int \Gamma_{FXY}(p,k)D_{\Phi,\Phi^*}(k^2)D_{\Phi,\Phi^*}((p-k)^2)d^4k\,, \tag{3.22}$$

where Γ_{FXY} is the vertex function for the fields F,X,Y. One can check the ultraviolet consistency of the left and right handsides of the eq.(3.22). So we find that the Schwinger's equations for both models coincide for the fixed point solution, that means that the fixed-point solution (3.17) of the renormalized model is simultaneously the solution of the nonrenormalizable model.

It should be noted that it is possible to renormalize also the nonlinear supersymmetric σ-model with the lagrangian

$$L_\sigma = L_{SQCD} + \delta L_\sigma\,, \tag{3.23}$$

$$\delta L_\sigma = \int d^2\theta(h\Phi(XY-C)) + h.c. \tag{3.24}$$

In the ultraviolet region we can neglect the linear term in (3.24) and the model (3.23) coincides with the model (3.1) for $M=0$.

Note also that the fixed-point solution of the type (3.16) has been considered in standard Weinberg-Salam model in an attempt to predict the masses of the top quark and Higgs boson [14-17].

4 Four-fermion interaction of colour quarks

Consider the Lagrangian

$$L_t = L_{QCD} + \delta L \tag{4.1}$$

Here L_{QCD} is the standard QCD Lagrangian [18,19]

$$L_{QCD} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \sum_{n=1}^{6} i\overline{q}_n\gamma^\mu D_\mu q_n\,, \tag{4.2}$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\nu + gf^{abc}A^b_\mu A^c_\nu\,,$$

$$D_\mu = \partial_\mu - igA^a_\mu T^a\,,$$

$$q_n = u,d,s,c,b,t$$

and

$$\delta L = (h^2/m^2)[\bar{t}_L t_R \bar{t}_R t_L + \bar{b}_L t_R \bar{t}_R b_L]\,, \tag{4.3}$$

$$t(b)_{L,R} = \frac{(1\mp\gamma_5)}{2}t(b)$$

By the introduction of the auxilliary scalar isodoublet $\phi_i (i=1,2)$ one can rewrite the nonrenormalizable four-fermion interaction δL in the equivalent form

$$\delta L = h(\bar{t}_L t_R\phi_1 + \bar{b}_L t_R\phi_2 + h.c.) - m^2\phi_i^*\phi_i \tag{4.4}$$

The Lagrangian (4.1) can be obtained as $Z \to 0$ limit of the renormalized Lagrangian

$$L_Z = Z\partial_\mu\phi_i^*\partial^\mu\phi_i + L_t \tag{4.5}$$

The h^2-correction to the scalar propagator is proportional to the correlator

$$I(p^2) = i\int <0\mid T(\bar{t}_L(x)t_R(x),\bar{t}_R(0)t_L(0)\mid 0> \exp(ipx)d^4x \tag{4.6}$$

To calculate the correlator $I(p^2)$ it is convenient to use the Kallen-Lehmann representation

$$I(p^2) = \int \rho(t)[t-p^2-i\epsilon]^{-1}dt - subtractions \tag{4.7}$$

For the case of free quarks in $d = 4-2\epsilon$ space-time the spectral density is $\rho(t)\sim(t)^{1-\epsilon}$ and to make the integral (4.7) ultraviolet convergent we have to make one subtraction which corresponds to the introduction of the mass renormalization counterterm $\delta m^2 \phi_i^* \phi_i$. After the subtraction the integral is proportional to $I(p^2) \sim (1/\epsilon)(p^2)^{1-\epsilon}$. In order to make $I(p^2)$ finite in the limit $\epsilon \to 0$ we have to make additional subtraction in the dispersion relation (4.7) which corresponds to the introduction of the infinite wave function counterterm $\delta Z(\partial_\mu \phi_i^* \partial^\mu \phi_i)$. However like in the case of supersymmetric QCD the situation changes drastically when we take into account QCD corrections for the correlator $I(p^2)$. The renormalization group equation for the spectral density $\rho(t)$ for the renormalized $\phi_{i,r}$ field ($\phi_{i,r} = Z^{1/2}(\epsilon)\phi_i$) in the minimal subtraction scheme in one loop approximation is

$$(\mu d/d\mu + \beta(g,\epsilon)d/dg + 2\gamma(g))\rho(t,\epsilon) = 0\,, \tag{4.8}$$

where [12,18,19]

$$\beta(g,\epsilon) = -g\epsilon - \beta_0 g^3\,,$$
$$\beta_0 = (11 - (2/3)n_f)/16\pi^2\,,$$
$$\gamma(g) = -g^2/2\pi^2\,.$$

The solution of the renormalization group equation (4.8) reads

$$\rho(t) \sim t^{1-\epsilon}(\bar{g}^2 + \frac{\epsilon}{\beta_0})^b\,, \tag{4.9}$$

where

$$\bar{g}^2 = (-\beta_0/\epsilon + (\beta_0/\epsilon + (1/g^2)(t/\mu^2)^\epsilon)^{-1}\,,$$
$$b = 8(11 - (2/3)n_f)^{-1}$$

and the n_f is the number of quark flavours (in our model $n_f = 6$). One can check that for $n_f \geq 5$ the dispersion relation (4.7) with a single subtraction is ultraviolet finite in the limit of the regularization removing $\epsilon \to 0$. At the language of the counterterms it means that it is not necessary to introduce the wave function renormalization counterterm. The selfinteraction of the scalar field ϕ_i in the h^4-approximation is proportional to

$$\int < 0 \mid T(\bar{t}_L(x)t_R(x), \bar{t}_L(y)t_R(y), \bar{t}_R(z)t_L(z), \bar{t}_R(0)t_L(0)) \mid 0 > \exp(ipx+iky+iqz)d^4x d^4y d^4z \tag{4.10}$$

and the corresponding one loop integral for free quarks is logarithmically divergent that corresponds to the ultraviolet divergent scalar field self-interaction counterterm $\delta\lambda(\phi_i^*\phi_i)^2/2$. However as in the previous case an account of the QCD effects leads to the ultraviolet convergence of the corresponding integral.

To analyze the ultraviolet properties of the nonrenormalizable model (4.1) in general case when we take into account Yukawa interaction inside of the quark loop consider the renormalized analog of the model (4.1). Namely, consider the renormalized model with the Lagrangian

$$L_{ren} = L_{QCD} + \partial_\mu \phi_i^* \partial^\mu \phi_i - m^2 \phi_i^* \phi_i - \lambda(\phi_i^*\phi_i)^2/2 + h(\bar{t}_L t_R \phi_1 + \bar{b}_L t_R \phi_2 + h.c.) \tag{4.11}$$

The renormalization group equations for the Lagrangian (4.11) in one loop approximation read

$$16\pi^2 d\bar{g}/dt = -(11 - 2n_f/3)\bar{g}^3\,, \tag{4.12}$$

$$16\pi^2 d\bar{h}/dt = (9\bar{h}^2/2 - 8\bar{g}^2)\bar{h}\,, \tag{4.13}$$

$$16\pi^2 d\bar{\lambda}/dt = 12(\bar{\lambda}^2 + \bar{h}^2\bar{\lambda} - \bar{h}^4) \tag{4.13a}$$

Since we are interested in the investigation of the ultraviolet properties of the theory we have written down the renormalization group equations for massless case. Consider at first the g^2, h^2 phase plane. As in the supersymmetric case there are three different regions on the phase plane which correspond to the qualitatively different ultraviolet behaviour:

$$h^2 > kg^2\,, \tag{4.14}$$

$$h^2 = kg^2\,, \tag{4.15}$$

$$h^2 < kg^2 \tag{4.16}$$

The koefficient $k = (4n_f/3-6)/9$ is positive for $n_f > 4.5$ and for the most interesting case $n_f = 6$ it is equal to 2/9. In regions (4.15) and (4.16) the effective coupling constant $\bar{h}^2(t)$ is asymptotically free. In the region (4.14) the coupling constant $\bar{h}^2(t)$ is not asymptotically free and moreover it has ghost pole that means the inconsistency of the model at least in the leading log approximation. The anomalous dimension of the scalar field ϕ_i in one loop approximation is $\gamma_\phi = bh^2(16\pi^2)^{-1}$, $b = 6$. From the renormalization group equation

$$(\mu d/d\mu + \beta_g d/dg + \beta_h d/dh + \beta_\lambda d/d\lambda + 2\gamma_\phi)D(p^2) = 0 \tag{4.17}$$

for the scalar propagator we find that its ultraviolet asymptotics in the region (4.16) is proportional to $1/p^2$, whereas for the fixed point solution (4.15) it decreases in the ultraviolet region more slowly than the free scalar propagator, namely

$$D(p^2) \sim [\ln(-p^2/\mu^2)]^a/p^2, \tag{4.18}$$

where $a = bk(11 - 2n_f/3)^{-1}$. As it has been explained in section (3) it means that the wave function renormalization of the scalar field is finite in the limit of the regularization removing and $Z = 0$ for the fixed-point solution (4.15). So the fixed-point solution (4.15) corresponds to the case of the auxilliary scalar field ϕ_i, i.e. to the case of nonrenormalizable interaction (4.1). To be precise: For Z=0 the Schwinger's equations for the renormalized Green's functions coincide with the Schwinger's equations for the nonrenormalizable model. In this sence the models are equivalent. The renormalization group equation for the ratio $x = \frac{\bar{\lambda}}{\bar{h}^2}$ reads

$$16\pi^2 dx/dt = 12\bar{h}^2(x^2 + x/4 - 1 + \frac{4}{3}\bar{g}^2\bar{h}^{-2}x) \tag{4.19}$$

For the fixed point solution (4.15) the equation (4.19) takes the form

$$16\pi^2 dx/dt = 12\overline{h}^2(x - x_+)(x - x_-) , \tag{4.20}$$

where $x_\pm = (-25 \pm (689)^{1/2})/8$, $x_+ > 0$, $x_- < 0$. For $x > x_+$ the property of the asymptotic freedom for x and hence λ coupling constants is lost and they have Landau ghost pole that means the inconsistency of the model. For $x < x_+$ the effective coupling constant $\overline{\lambda}$ approaches $\overline{\lambda} \approx x_- \overline{h}^2 < 0$ in the ultraviolet region, i.e., the effective coupling constant $\overline{\lambda}$ is negative. It means that the effective potential is unbounded and hence there is no stable vacuum state in the model. For $x = x_+$ we have the fixed point and the self-interaction coupling constant $\overline{\lambda} = x_+ \overline{h}^2 > 0$, therefore the effective potential is positive in the ultraviolet region. So we conclude that on the $(\overline{h}^2, \overline{\lambda}$ phase plane only the line $\overline{\lambda} = x_+ \overline{h}^2$ corresponds to the physically reasonable solution. This solution corresponds to the bare coupling constant $\lambda_B = 0$. As it has been mentioned before the four-fermion interaction without additional color QCD interaction has been investigated in refs.[6,9,10]. The main conclusion of the refs.[6,9] is that the four-fermion interaction exists as a local field theory only in $d < 4$ and for $d = 4$ the ultraviolet cutoff can be removed only at the expence of making the model trivial. It appears that additional QCD interaction softens ultraviolet divergences that leads to the renormalizability of the four-fermion interaction.

5 Conclusion

In this paper we considered two examples of the four-dimensional nonrenormalizable interactions: supersymmetric QCD with an additional nonrenormalizable interaction and standard QCD with additional four-fermion interaction. We have found that the asymptotic freedom creates miracle and softens some ultraviolet infinities that allows not to introduce the wave function counterterm for the auxilliary field, i.e., the auxilliary field remains the auxilliary one at quantum level. Moreover, it appears that the considered nonrenormalizable models are equivalent (the Schwinger's equations coincide) to the fixed-point solutions of the corresponding renormalized models in full analogy with three dimensional model (2.1). This is rather dissapointing since we don't obtain new physics and just fix some parameters of the renormalizable models. Probably, there are exceptions, namely if we consider Yang-Mills theory in $d = 4 + 2\epsilon$ space-time the β-function in the minimal subtraction scheme is

$$\beta(g, \epsilon) = g\epsilon + Ag^3 + O(g^5) \tag{5.1}$$

Due to the property of the asymptotic freedom of the Yang-Mills theory [18,19] the coefficient $A < 0$, so for small ϵ we have the ultraviolet fixed point and the theory is consistent at quantum level [20]. For the most interesting case $\epsilon = 0.5$ (five-dimensional

Yang-Mills theory) all one loop diagrams are ultraviolet finite in the sence of the analytical continuation and the theory consistent in the "leading log" approximation [20] . However the situation in higher loops is not clear. The main obstacle here is that we can't use the $1/N$-expansion technique.

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