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Photoproduction of Jets at HERA: Comparison of Next-to-leading Order Calculation with ZEUS Data

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Abstract

We have calculated inclusive single-jet production in low Q^2 ep-collisions at next-to-leading order superimposing direct and resolved contributions. The results are compared with recent experimental results from the ZEUS Collaboration at HERA.

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1 Introduction

The production of high tranverse energy (E_T) jets in collisions of almost real photons with protons has become one of the important research topics at HERA. Many experimental results by the H1 [1] and ZEUS [2] Collaborations have been published since these two experiments started taking data in 1991. The photoproduction is of interest since it allows to investigate the hadronic structure of the photon in a way that complements the study of the photon structure in deep inelastic $e\gamma$ scattering. The underlying theoretical framework is the QCD improved parton model in which the parton distribution functions of the colliding particles and the hard parton-parton scattering processes are calculated as a power series in the strong coupling constant α_s.

As is well known two types of processes contribute to large E_T photoproduction: The incoming photon can interact either directly with a parton in the proton (direct component) or the photon acts as a source of partons which collide with the partons in the proton (resolved component). At leading order (LO) of the strong coupling α_s the final state consists just of two partons with large transverse energies fragmenting into two jets of hadrons in addition to the proton remnant and the scattered electron which come out with small tranverse energies. In the resolved process the spectator parton of the photon produces also a remnant jet which also has small transverse energy. The resolved part looks very similar to jet production in high energy hadron-hadron collisions. Due to the presence of the pointlike photon-quark coupling, however, some substantial differences occur which makes high energy photoproduction particularly interesting.

In next-to-leading order (NLO) of α_s the distinction between direct and resolved contribution becomes ambiguous. Both components are related to each other through the factorization scale M_γ at the photon leg. The M_γ dependence of the NLO direct cross section cancels against the M_γ dependence in the resolved cross section via the photon structure function. This compensation of the M_γ dependence has been studied recently by Bödeker and two of us [3] for the inclusive single jet cross section. So, to perform a consistent calculation in the NLO formalism both components must be superimposed using the same M_γ scale in both. For the actual NLO calculation we need the NLO hard scattering parton-parton cross sections for the direct (here one of the partons is the photon) and the resolved process together with the two-loop evolved structure functions of the photon and the proton.

In earlier NLO work direct [4, 5] and resolved [4, 6, 7] photoproduction of jets were considered separately. Exceptions are [3] and a recent note by Aurenche et al. [8] in which direct and resolved processes were superimposed. At HERA energies and for low E_T (below 20 GeV) jets the resolved contribution is dominant if the scale M_γ is equal to E_T as is usually assumed [3]. Since so far all experimental data on jet production were in this low E_T range the neglect of the direct component over the resolved one was justified. This situation has changed now. The ZEUS collaboration has published experimental results also for $E_T > 20\ GeV$ [9], a region, where both components are definitely needed to interprete the data. When both parts are added we are also free to vary the M_γ scale in the low E_T region.

Before embarking upon rather involved NLO calculations one might try to compare the experimental data with LO predictions. This, however, is not sensible for the following reasons.

It is well known, that all perturbatively calculated cross sections are scale dependent, i. e. they depend on the chosen renormalization scale μ and the factorization scales M_γ and M_p (M_p stands for the scale of the proton structure function). This dependence is particularly strong for LO predictions even when $\mu = M_\gamma = M_p$ whereas in NLO results, because of compensation between LO and NLO contributions, this scale dependence is reduced [3]. Furthermore, as explained above, the compensation of the M_γ dependence is at work only when the NLO direct contribution is added at least to the LO resolved contribution. Another deficiency of LO predictions is the fact that in LO only two large E_T jets can be produced. Therefore these cross sections are independent of the cone size R which is used to define the extension of the jets in the experimental analysis, usually taken as $R = 1$. In the analysis with NLO corrections included the cone size dependence originates from the production of three partons when two of them are combined in one jet if their distance in the azimuth-rapidity space is less than R.

The purpose of this work is twofold. First, we present the NLO predictions for the inclusive one-jet cross sections $d\sigma/d\eta$ and $d\sigma/dE_T$ for a range of η and E_T values which are the same as in the recent ZEUS analysis [9] and where resolved and direct contributions are added consistently. The method of calculation is the same as in the earlier work [3] where we investigated in detail the compensation of the M_γ dependence between the resolved and direct contribution. Second, we compare with the experimental results from ZEUS [9]. For this comparison it is important that we choose the same kinematic contraints, in particular with respect to the equivalent photon spectrum, as in the experimental analysis. This way we want to find out how much the NLO QCD results correctly account for the η and E_T dependence of the experimental cross sections concerning shape and absolute normalization.

In sect. 2 we shall collect the theoretical input which is more or less the same as in [3]. The numerical results of our computation and the comparison with the ZEUS data are presented in sect. 3. Here we also give results for the scale dependence of the single-inclusive jet cross section under the ZEUS kinematical constraints.

2 Theoretical Input

We limit ourselves to the inclusive one-jet cross section in low Q^2 ep production at HERA. The spectrum of the virtual photons is approximated by the Weizsäcker-Williams formula

$$yF_{\gamma/e}(y) = \frac{\alpha}{2\pi}(1 + (1-y)^2)\log\left(\frac{\hat{Q}^2(1-y)}{m_e^2\, y^2}\right) \qquad (1)$$

where, as in the ZEUS analysis [9] $\hat{Q}^2 = 4 \ \ GeV^2$ and y is restricted to $0.20 \leq y \leq 0.85$. m_e is the electron mass.

We adopt the jet definition of the snowmass meeting [10] defining a jet as a bunch of outgoing particles contained in a cone of radius R in the plane of pseudorapidity and azimuthal angle around the jet momentum. Within this jet algorithm the jet multiplicity is ambiguous in some regions of phace space. Then, two partons may be viewed as a single jet or as two separate ones. In this case we adopted the convention of [11] by taking their first choice.

The theoretical framework of our calculation is the same as in [3]. The resolved photoproduction including the NLO corrections is computed with the procedure described in [6]. For very small cone radii $R \ll 1$ the inclusive single-jet cross section has been calculated analytically by Aversa et al. [12], neglecting contributions which vanish in the limit $R \to 0$. For large cone sizes $R \propto O(1)$ the corresponding corrections are calculated numerically from the $2 \to 3$ parton scattering cross sections. Further details can be found in [6]. The same method was applied in [4, 7, 8]. The modifications needed for varying the factorization scales M_γ and M_p independently are described in [3].

The NLO corrections for the direct process have been calculated with the program developed by Bödeker based on the formulas given in [5]. In this work the NLO corrections are calculated with the so-called subtraction method and not with the phace spase slicing method as in [12]. This has the advantage that the analytical results can be used for jet radii $R \propto O(1)$. The results in [5] are given in the $\overline{MS}$ factorization scheme for the photon and the proton. This scheme is also applied in the calculation of the resolved contribution.

Concerning structure functions we used for the proton the MRS(D-) of Martin, Stirling and Roberts [13] which is a NLO parametrization with $\overline{MS}$ factorization. This structure function reproduces also the new structure function measurements at small x from HERA [14]. For the photon structure function we have chosen the NLO parametrization of Glück, Reya and Vogt [15] in the so-called DIS_γ scheme. The necessary transformation of the NLO direct photon contribution to the DIS_γ scheme had been added already by Bödeker [16]. The running coupling constant $\alpha_s(\mu)$ is computed from the two-loop formula with five flavours and $\Lambda^4_{\overline{MS}} = 215\ MeV$. We define the rapidity η to be positive in the direction of the incoming proton and take $p_e = 27.0\ GeV$ and $p_p = 820\ GeV$ giving $\sqrt{S} = 298\ GeV$.

3 Results and Comparison with ZEUS Data

First we have calculated the one-jet cross section $d^2\sigma/d\eta dE_T$ as a function of η with E_T integrated over the range $E_T \geq E_T^{min}$ where E_T^{min} was chosen as in the ZEUS analysis, i. e. $E_T^{min} = 8$, 11 and 17 GeV. This cross section is denoted $d\sigma/d\eta$ and yields the rapidity distribution for large enough E_T. The results are shown in Fig.1, 2 and 3, where $d\sigma/d\eta$ is plotted for the direct, resolved and complete contribution. The jet radius is $R = 1$ as in the experimental analysis. For comparison we show also the results in lowest order. In this calculation the same NLO structure functions as in the NLO calculation have been applied, only the higher order corrections of the hard scattering cross sections have been removed and α_s is calculated from the one-loop formula with the same Λ value instead of the two-loop formula applied in the NLO calculation.

As to be expected the direct contribution has its maximum in the electron direction, i. e. $\eta < 0$ and the maximum is shifted towards positive η the larger the E_T^{min} is. In Fig. 3 where the curve with $E_T^{min} = 17\ GeV$ is shown the maximum of the direct contribution occurs at $\eta \simeq 0.5$ and is comparable to the resolved contribution for $\eta < 1.0$. For simplicity we have chosen all scales to be equal: $\mu = M_\gamma = M_p = E_T$. It is clear that in all three distributions the direct contribution is not negligible over the whole η range. It is mostly contributing in electron direction, of course. We also show the data points of the ZEUS Collaboration [9].

The agreement between theoretical and experimental results with respect to shape and absolute normalization is quite reasonable. The experimental errors include the statistical and the systematic errors - not associated with the energy scale of the jets - added in quadrature. An error approximately 1-2 times the size of the error given in the figures must be added if the uncertainty due to the energy of the jets is taken into account. Further details are found in [9]. Due to the still large experimental errors we cannot claim that the full curves which include the resolved and direct contribution give a better account of the data than the resolved contribution alone. Furthermore by changing M_γ we can shift the relation between the resolved and direct part so that the question whether the resolved contribution is already sufficient for describing the data is not very meaningful. In Fig. 1 where $E_T^{min} = 8\ GeV$ there seems to be a larger discrepancy between theory and data at $\eta \simeq 2$, i. e. near the proton remnant direction. In Fig. 2 and 3 a discrepancy near $\eta \simeq 2$ is not so apparent, may be due to the larger experimental errors.

In Fig. 4 and 5 we present $d\sigma/dE_T$ as a function of E_T. This is the cross section $d^2\sigma/d\eta dE_T$ integrated over two different η intervals: $-1 \leq \eta \leq 2$, plotted in Fig. 4, and $-1 \leq \eta \leq 1$ shown in Fig. 5. We show again three curves as in Fig. 1, 2, 3 for the direct, the resolved and the complete cross section. These curves are compared to the data from [9], which were integrated over the same η regions. We see that the agreement of the theoretical curve with the experimental data points which extend in E_T up to almost 40 GeV is excellent. Looking at the whole E_T range it seems that the full curve which is the sum of direct and resolved cross sections agrees better with the data than the resolved contribution alone. In Fig. 5 where $d\sigma/dE_T$ with η integrated over the smaller interval is shown the theoretical curve agrees with the data points only for small $E_T \leq 11\ GeV$, at larger E_T the theoretical cross section is somewhat larger than the experimental one. This discrepancy is visible already in Fig. 3 where the resolved cross section alone agrees better with the data for $\eta < 1$. Also in Fig. 5 the data points for $E_T > 11\ GeV$ follow better the dashed curve which is the resolved cross section. This might indicate that the direct contribution is overestimated in the theory. On the other hand the direct term is much better determined compared to the resolved term since the latter one depends on the not so very well known photon structure function. Whether this discrepancy makes a modification of the photon structure function necessary perhaps can be decided when data for $d\sigma/d\eta$ with larger E_T^{min} are available.

In Fig. 1-5 we have also shown the LO results which agree in average as well with the experimental data as the NLO results. But this is fortuitous. The experimental data are obtained with a jet cone radius $R = 1$ as well as the NLO results. The LO results do not depend on R, i. e. the same predictions in LO would be obtained for any R. From the earlier work on resolved photoproduction alone it is known, however, that the LO results, as we have defined them, agree approximately with the NLO calculations just when $R \simeq 1$, a result, as we see it in Fig. 1-5, that is obtained also when the direct contribution is included.

From the comparison of the NLO results with the ZEUS data above we conclude that the E_T distributions $d\sigma/dE_T$ (see Fig. 4 and 5) agree reasonably well inside the experimental errors. But the η distributions $d\sigma/d\eta$ show deviations between theory and data, in particular in the vicinity of the proton and the electron directions.

We must keep in mind, however, that the NLO predictions have uncertainties which must be taken into account. The major uncertainty comes from the scale dependence of the results,

which is ultimately connected to the fact, that our results are obtained in finite order perturbation theory. This scale dependence was already investigated in [3]. It might be interesting to see how the cross sections under kinematical conditions as in the ZEUS analysis are influenced by such scale changes. First let us look at the M_γ dependence separately. M_γ is the factorization scale of the photon structure function. It should largely cancel if NLO direct and resolved contributions are superimposed. As an example we show in Fig. 6 how the η distribution with $E_T^{min} = 11\ GeV$ changes with M_γ. In the upper part of Fig. 6 the LO and in the lower part the NLO results are plotted as a function of η and for $\xi = M_\gamma/E_T$ equal to 0.25 and 4.0, respectively. Varying M_γ in this range we observe that the M_γ dependence is very much reduced in NLO compared to the LO curves shown in the upper part of Fig. 6. In the NLO curves there is still some M_γ dependence left, in particular in the region $\eta > 0$ but it is much smaller now than in the LO curves. This remaining dependence is to be expected since we work in a fixed order of perturbation theory. In addition the compensation works only between the leading logarithmic terms of the LO resolved contribution and the NLO direct terms. Fig. 6 also shows that varying M_γ in the assumed interval of ξ does not improve the agreement between theoretical and experimental results (compare Fig. 6 with Fig. 2). The other cross sections $d\sigma/d\eta$ for $E_T^{min} = 8$, 17 GeV show a similar behaviour. Of course, the variation of just one scale does not give us an error estimate of our NLO results. For this we must vary all scales simultaneously. We have done this in the form that $\mu = M_p = M_\gamma = \xi E_T$ and ξ was chosen equal to 0.25 and 4.0, respectively. In Fig. 7 the $d\sigma/d\eta$ distribution for $E_T^{min} = 11\ GeV$ is plotted for these two scales and compared to the ZEUS data. If we compare with Fig. 2 we see that the curve with the larger scale equal to 4.0 E_T agrees somewhat better with the data than the curve with scale equal to E_T. In general the cross sections decrease when the overall scale is increased. In the upper part of Fig. 7 the equivalent plot in LO is shown. Here the scale variation is much larger, in particular for $\eta > 0.5$. This is reduced in the NLO curves shown in the lower part of Fig. 7. From this comparison we see directly how higher order QCD corrections stabilize the predictions with respect to scale variation and give us an error estimate of the NLO prediction.

Since the contribution of the resolved part is dominant, in particular for the smaller E_T range we have to consider that our results depend on the assumed photon structure function. The GRV photon structure function [15] has been fitted to the available data for F_2 in deep inelasic $e\gamma$ scattering. This determines the quark content of the photon structure function but much less the gluon part which enters only through the evolution to higher Q^2. To see where the gluon structure function of the photon influences our results we have split up the cross section into the direct part, the part originating from the quark distribution and from the gluon distribution of the photon for the case $d\sigma/d\eta$ and $E_T^{min} = 8\ GeV$, where the resolved part is the largest. The corresponding curves are in Fig. 8 for LO (upper part) and NLO (lower part). The gluon distribution of the photon contributes mostly in the positive η region. Increasing the gluon contribution there could improve the agreement with the data in the proton direction. Similarly by decreasing the quark distribution for negative η 's could make the theory agree better with the data in the electron direction. Whether such adjustments are consistent with the information of F_2 of the photon is unclear and needs further work.

Before we would envisage such a detailed analysis we must ask whether other effects must be considered that influence the comparison with the experimental data. First, we remember that the virtual photon is treated in the Weizsäcker-Williams approximation where one neglects the dependence of the γp cross section on the virtuality of the photon. Although $\hat{Q}^2 = 4\ GeV^2$ is not very small we expect only a few percent corrections since $\hat{Q} \ll E_T$ for the E_T's of the ZEUS data. Work of Bawa and Stirling [19] for the case of Compton scattering at HERA indicates that the error produced by the Weizsäcker-Williams approximation at such small $\hat{Q}^2$ agrees with this estimate. The same conclusion applies to the effect of a transverse momentum of the photon with respect to the electron direction, which should be of the order of $\hat{Q}$ also. Such effects have been studied by Aurenche et al. [17] for inclusive one-jet production in $\gamma\gamma$ reactions at TRISTAN, where the kinematical conditions are different, however.

Another problem is the question whether multiple interactions which are not included in the NLO calculations affect the comparison with the data [18]. There is general agreement that the inclusive single-jet cross section is hardly influenced, except perhaps for forward-going jets, where some discrepancy in the jet shape for $\Delta\eta > 1$ has been seen in the comparison with the PYTHIA Monte Carlo results [9]. However, the core of the jet which determines the data used for comparison in this work is not affected.

Finally we have to keep in mind that the experimental cross sections are for jets made of hadrons whereas the theoretical cross sections are for the inclusive production of jets consisting of partons. They would be equal if exact parton-hadron duality was satisfied. This is not expected at least not at these moderate E_T's. The corrections can be obtained from Monte Carlo simulations. Such simulations have been done by Glasman and Terron [20]. For example, for the $E_T^{min} = 8\ GeV$ rapidity distributions they compared the pure LO (this corresponds to the upper curves in Fig. 1) with the PYTHIA model results using GRV and MRS(D-) parton distributions for the photon and the proton, respectively. The PYTHIA model generates multihadronic final states through resolved and direct processes. The partonic processes are simulated with LO parton cross sections including initial and final state parton showers which fragment at a low scale into hadrons on the basis of the LUND string model. The jet analysis is done in the same way as with the experimental data (see [9]). Their results show that the combined effect of parton showering and fragmentation is largest near $\eta \simeq 2$ (appproximately 40%) and almost negligible for $\eta \leq -0.5$. At $\eta \simeq 0$ the correction decreased to 20%. These shower and fragmentation corrections increase the cross section as compared to the pure LO result. This means that the discrepancy between our NLO results and the ZEUS data in the region $\eta > 1$ could easily be due to hadronisation effects, which we expect to diminish with increasing E_T. This is consistent with the comparison of the $E_T^{min} = 17\ GeV$ data in Fig. 3, although the experimental errors are larger there, and with the satisfactory agreement between data and predictions for $d\sigma/dE_T$ in Fig. 4. Of course, this would not explain the discrepancies in the $\eta < 0$ region, where the NLO predictions (see lower curves in Fig. 1, 2 and 3) are systematically above the data points outside the errors. Actually in this region the data agree well with the LO curves (upper curves in Fig. 1 and 2), which, however, is not significant, as was explained above. Instead of correcting for hadronisation plus initial and final state showering we prefer to correct for the effect of hadronisation only since the effect of the parton shower should, to a large extent, be contained in the NLO corrections. Unfortunately a Monte Carlo study, which shows the hadronisation corrections only, was not available.

In conclusion, we find reasonable agreement between the NLO predictions of jet photoproduction and the 1993 ZEUS data. The agreement is better for the $d\sigma/dE_T$ averaged over η bins than for the $d\sigma/d\eta$ distributions integrated over $E_T \geq E_T^{min}$. Here the discrepancies are largest in the $\eta < 0$ region which could easily be due to hadronisation effects and due to a soft underlying event contribution which was not subtracted from the data. We expect that the parton-hadron duality, which was the basis of our comparison, will work better for larger E_T's than measured so far. Then we expect that the discrepancies observed in the rapidity distributions will have become much smaller.

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Figure Captions

Fig. 1: Inclusive single-jet cross section $d\sigma/d\eta$ integrated over E_T above $E_T^{min} = 8\ GeV$ and $R = 1$. The ZEUS data [9] are compared with LO and NLO calculations of the resolved part (dashed lines), the direct part (dotted lines) and the complete photoproduction (full lines) as a function of η.

Fig. 2: Same as Fig. 1 with $E_T^{min} = 11\ GeV$.

Fig. 3: The same as Fig. 1 with $E_T^{min} = 17\ GeV$.

Fig. 4: Inclusive single-jet cross section $d\sigma/dE_T$ as a function of E_T with rapidity integrated over $-1 < \eta < 2$ and $R = 1$. The ZEUS data [9] are compared with LO and NLO calculations of the resolved part (dashed lines), the direct part (dotted lines) and the complete photoproduction (full lines).

Fig. 5: The same as Fig. 4 with integration over η in the interval $-1 < \eta < 1$.

Fig. 6: Inclusive single-jet cross section $d\sigma/d\eta$ integrated over E_T above $E_T^{min} = 11\ GeV$ and $R = 1$. The ZEUS data [9] are compared with the complete photoproduction calculation in LO and NLO. The full line is for $M_\gamma/E_T = 0.25$ and the dashed line for $M_\gamma/E_T = 4.0$ with $\mu = M_p = E_T$.

Fig. 7: Inclusive single-jet cross section $d\sigma/d\eta$ integrated over E_T above $E_T^{min} = 11\ GeV$ and $R = 1$. The ZEUS data [9] are compared with the complete photoproduction calculation in LO and NLO. The full line is for a common scale $\mu = M_p = M_\gamma = 0.25\ E_T$ and the dashed line for the scale $\mu = M_p = M_\gamma = 4.0\ E_T$.

Fig. 8: Inclusive single-jet cross section $d\sigma/d\eta$ integrated over E_T above $E_T^{min} = 8\ GeV$ and $R = 1$. The ZEUS data [9] are compared with the LO and NLO calculations of the complete photoproduction (full lines). The short-dashed line is the direct part, the dotted line is the contribution of the quark distribution function and the long-dashed line of the gluon distribution function of the photon.

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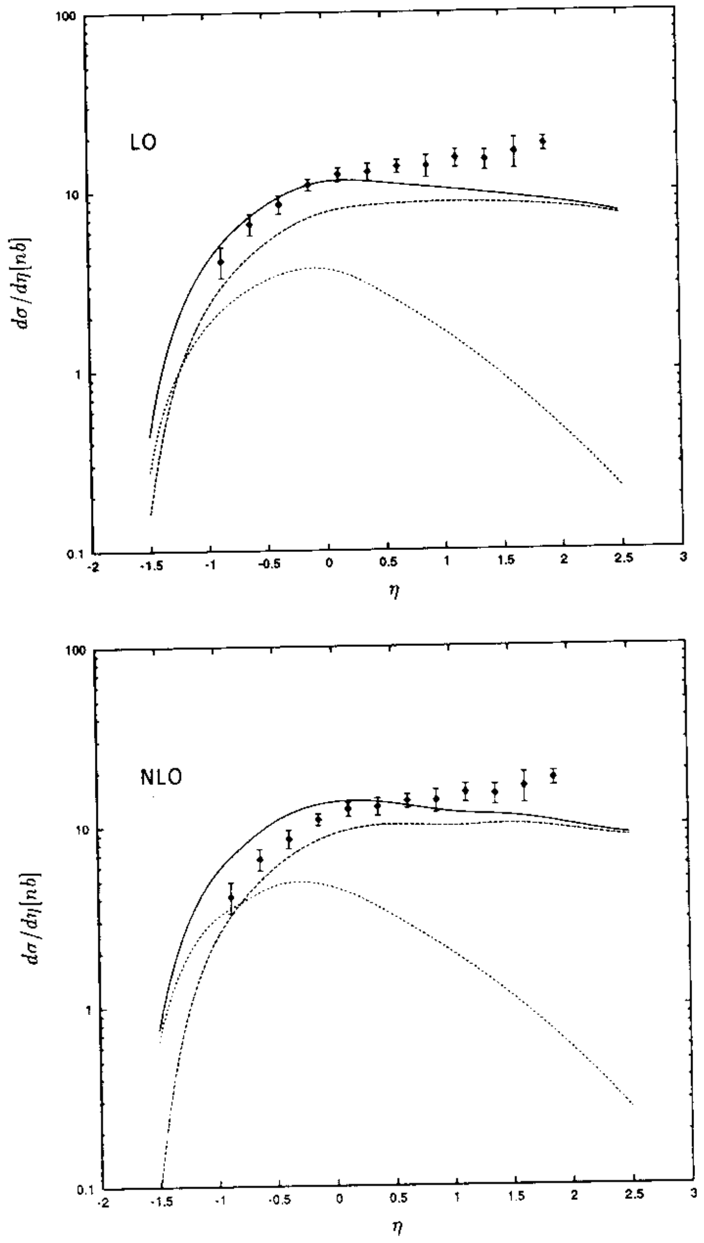
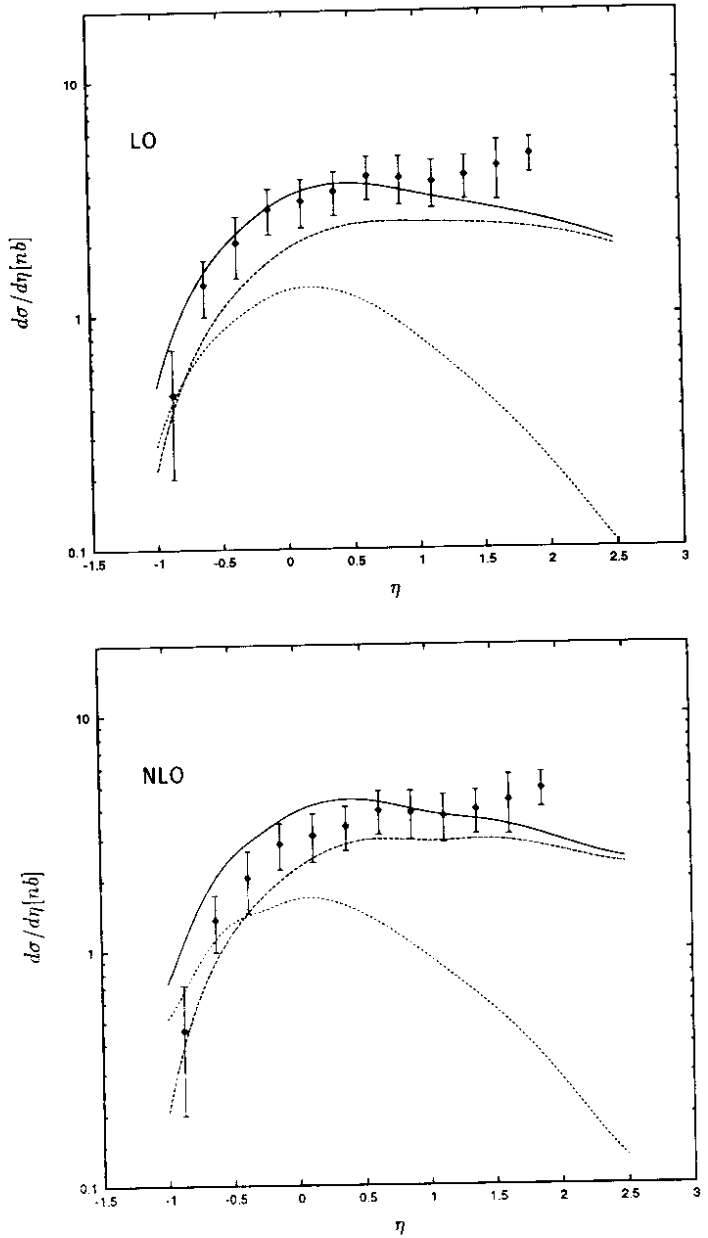


Figure 1



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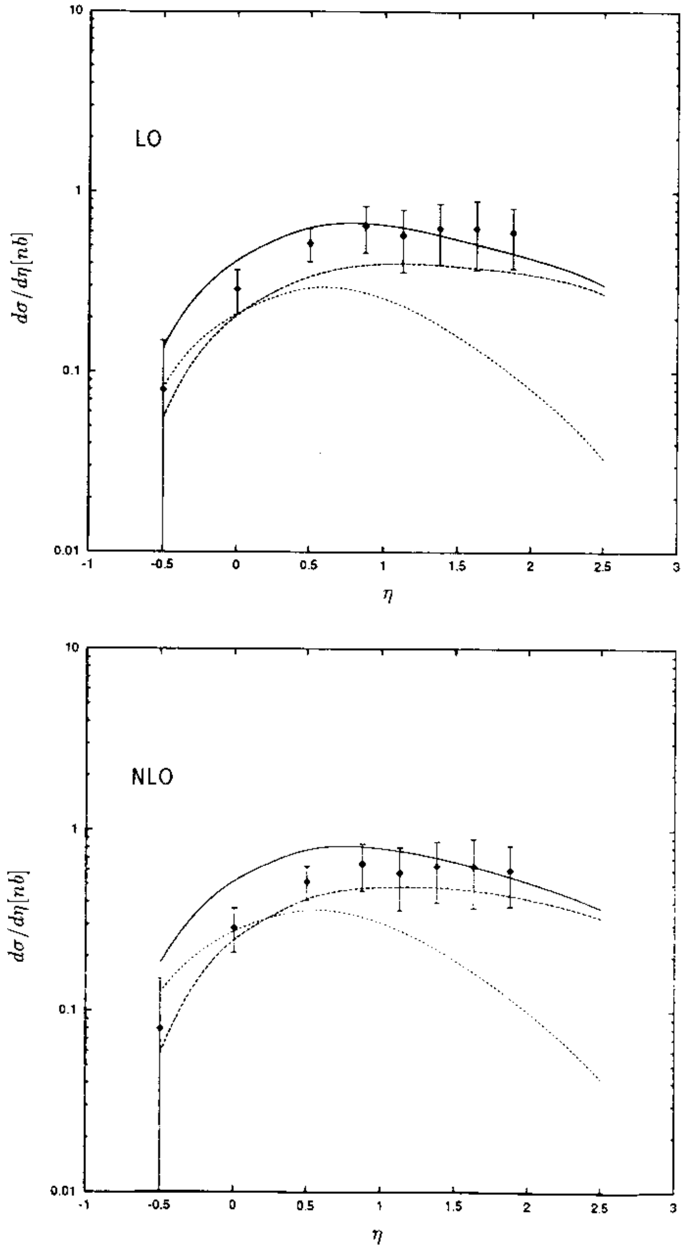

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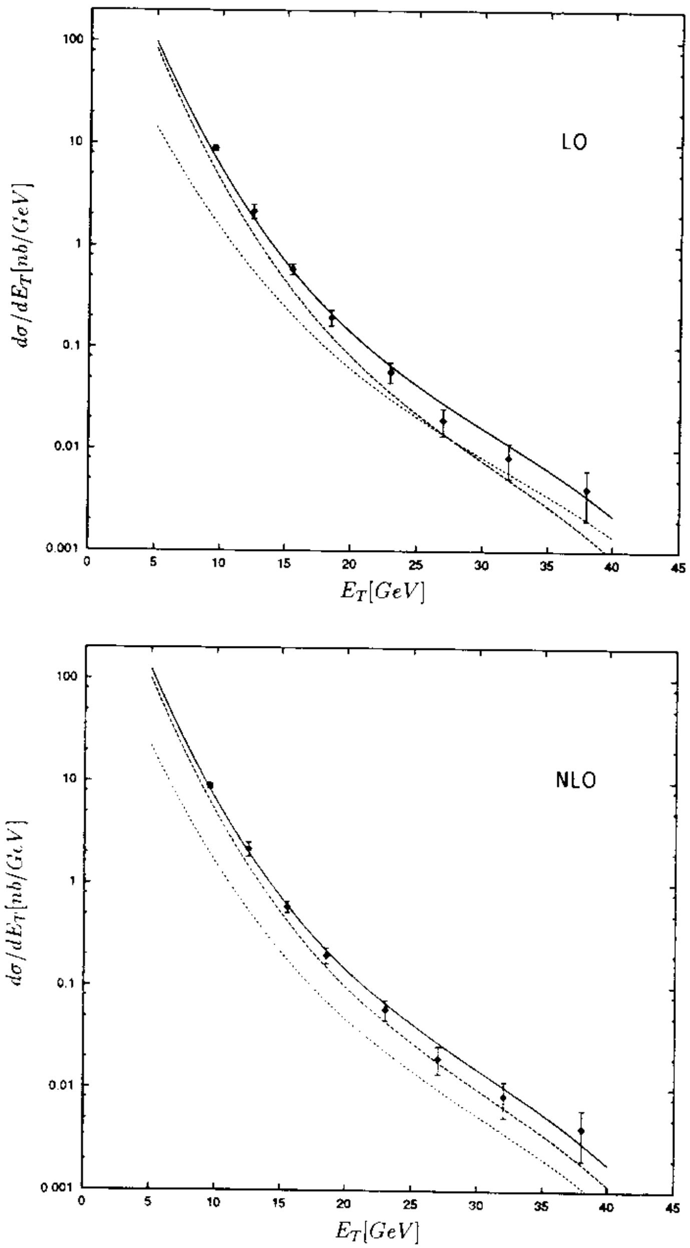

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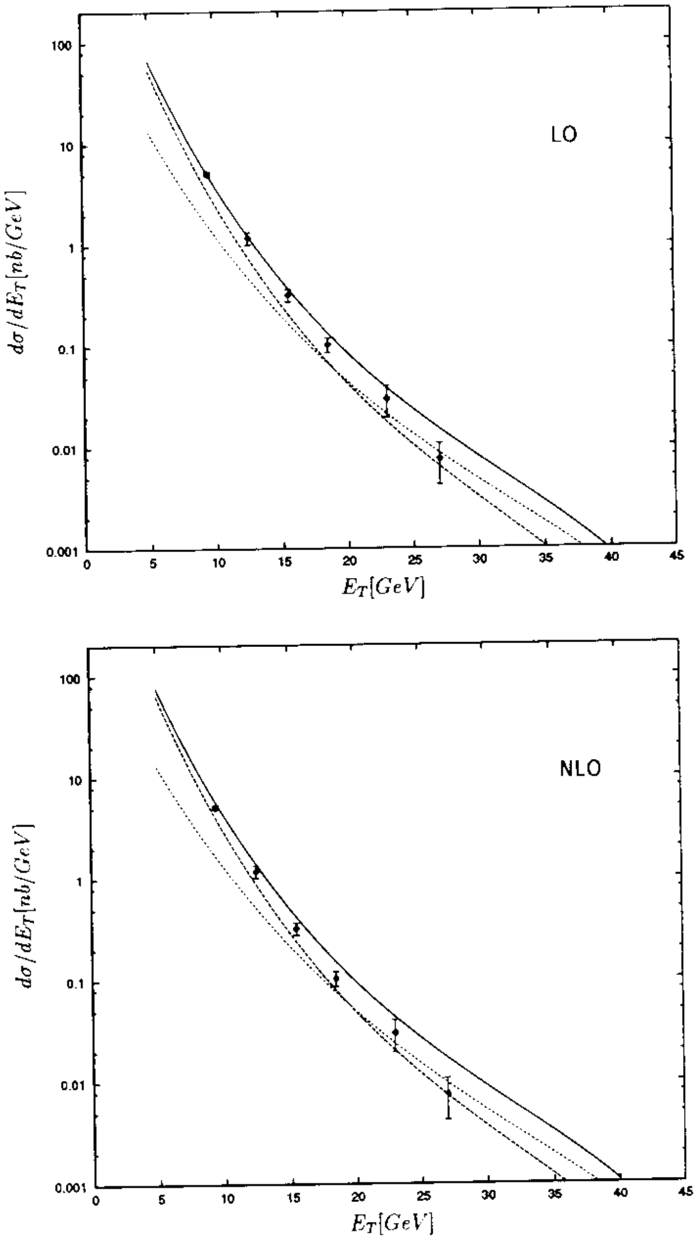

Figure 5

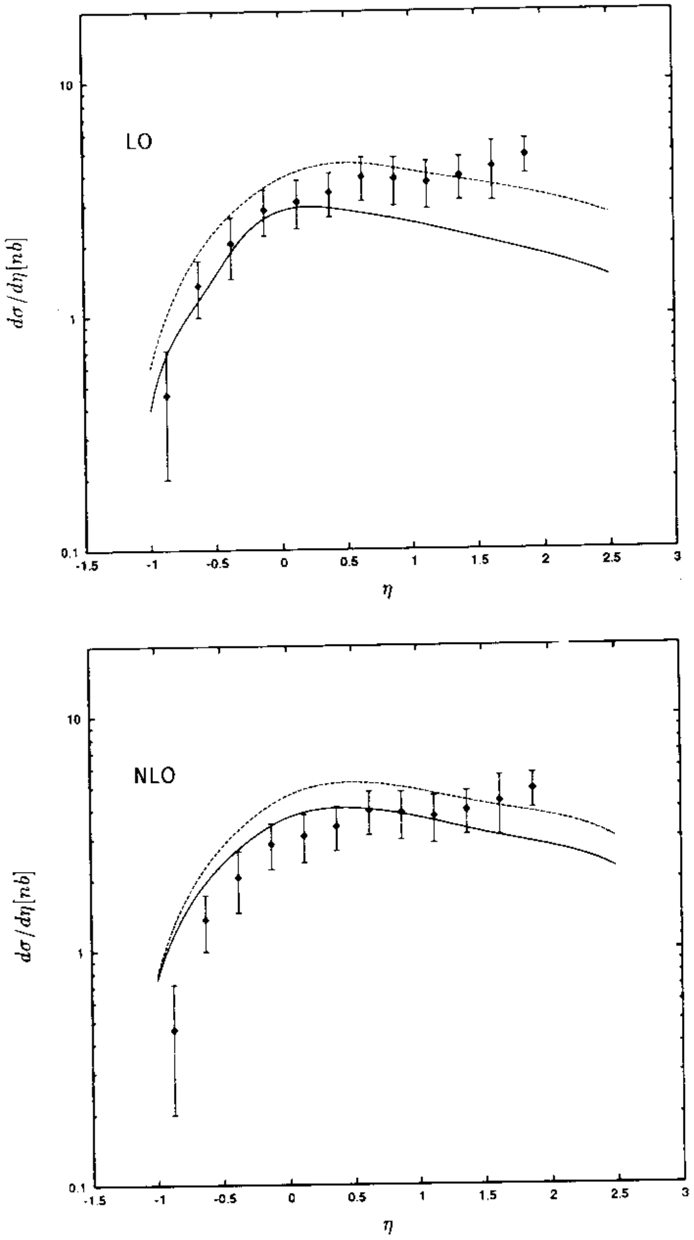

Figure 6

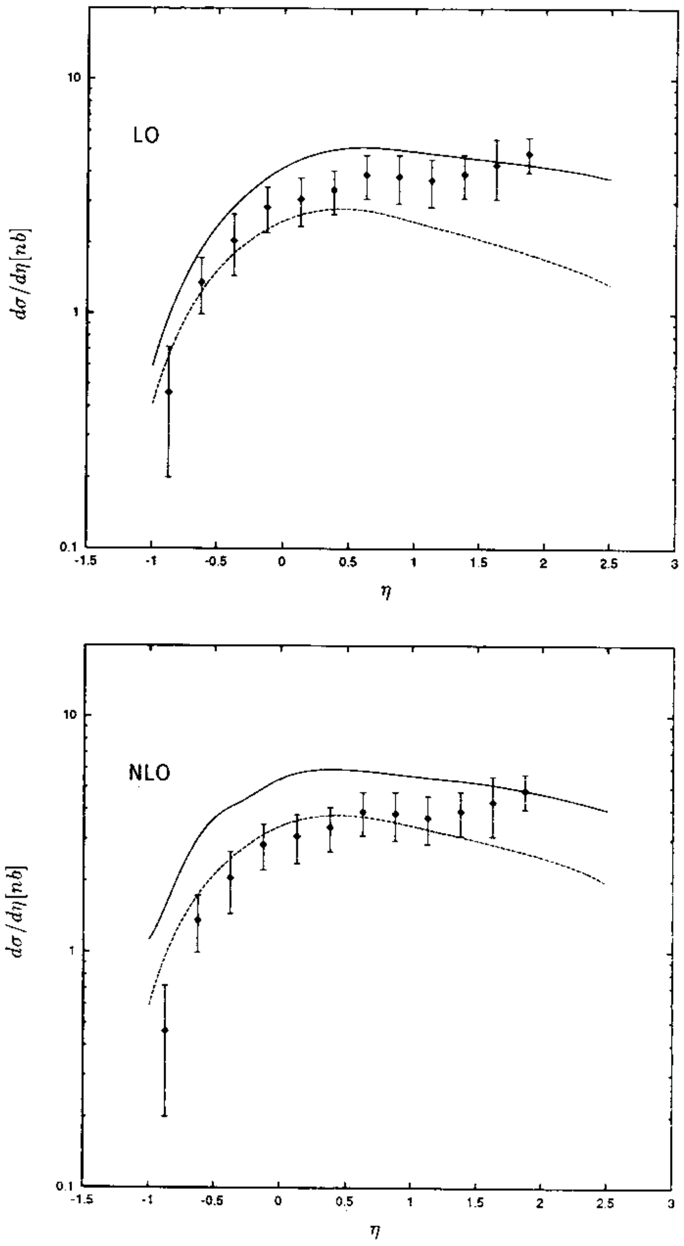

Figure 7

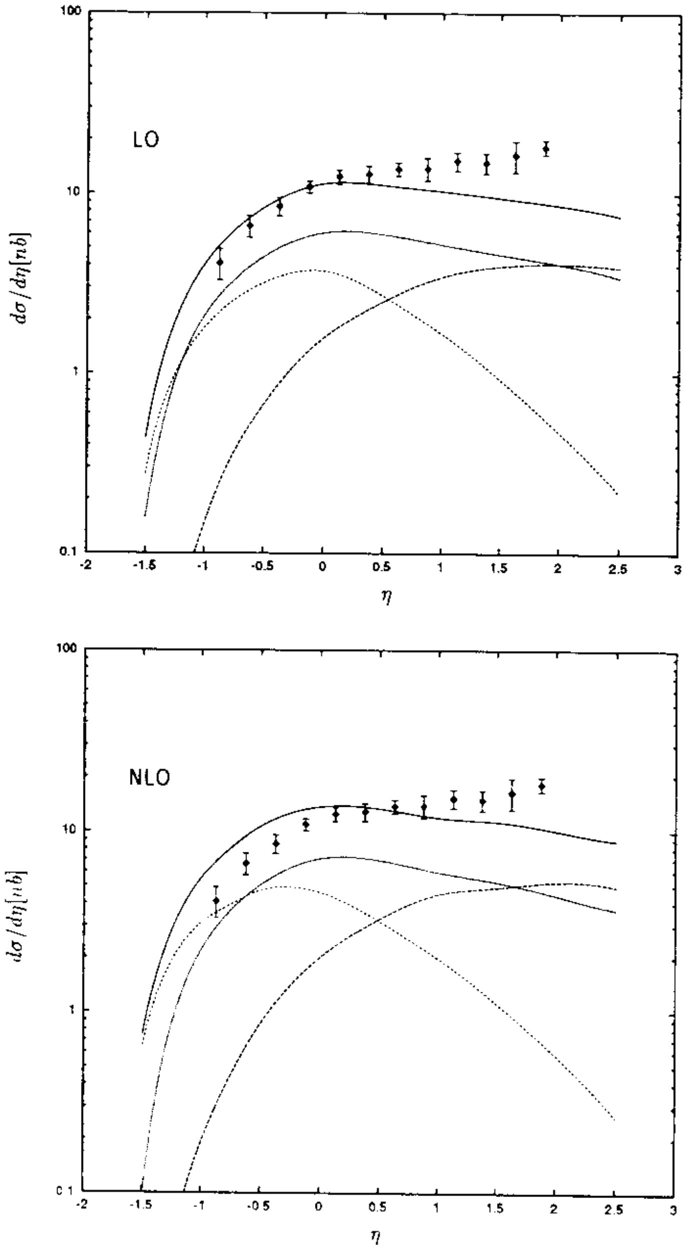

Figure 8