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Increased Sensitivity for Asymmetry Measurements at Linear Colliders with Polarized Electron and Positron Beams

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Introduction

In a linear collider it is relatively easy to collide longitudinally polarized beams once polarized particle sources are available. While in a storage ring electrons and positrons polarize transversely due to a spin flip asymmetry in the synchrotron radiation process (Sokolov-Ternov-effect) it is comparatively difficult to rotate the spin into the longitudinal direction. Moreover, one cannot take full advantage of the polarization since the sum of the polarization vectors of interacting electrons and positrons is zero as long as they are stored in the same ring and pass through the same pair of spin-rotators. Hence one has to depolarize one beam in order to make use of the polarization as has been pointed out by A. Blondel [1].

In a linear collider on the other hand the polarization vectors of both beams are not coupled, so that all spin combinations can be provided if polarized particle sources are available.

Polarized electrons can be produced by means of photoemission from a photocathode illuminated by circularly polarized laser light. A source of this kind is in operation at SLAC and has proven reliable operation for extended periods of run time [2]. Even though the multi-bunch operation of a polarized electron source is somewhat more difficult and needs further R&D we assume that a polarized electron source will be available for a future linear collider.

Polarized positrons can be produced by means of high energy circularly polarized photons generated in a long helical undulator and converted into polarized pairs in a thin target. An electron beam energy of about 200 GeV is necessary to produce highly polarized photons of about 20MeV energy [3, 4, 5]. This energy is in reach at linear collider projects currently under investigation. However, it seems to be impractical to produce polarized positrons at lower energy with this method.

A number of problems can be attacked by means of polarized beams in linear colliders like the precise determination of the weak mixing angle by means of the measurement of the left-right asymmetry at the Z^0-pole or the investigation of SUSY particles [6]. A detailed analysis of the left-right asymmetry measurement at the Z^0-pole can be found in [7] for the case of polarized electrons only. Some implications of asymmetry measurements at linear colliders with both electrons and positrons polarized will be outlined in the following.

As an example we will concentrate on effects associated with the additional polarization of the positron beam for the case of the measurement of the left-right asymmetry at the Z^0-pole. Similar results should also be expected in case of asymmetry measurements at higher energies.

General definitions

We consider longitudinally polarized beams. A transverse polarization can lead to an azimuthal asymmetry but will not contribute to the total reaction rate. The longitudinal polarization of an electron beam is defined by:

$$P_e = \frac{N_e^+ - N_e^-}{N_e^+ + N_e^-}$$

where and N_e^+ and N_e^- denote the number of electrons with spin parallel or anti-parallel to the direction of motion (positive or negative helicity), respectively.

It is convenient to define the polarization of the positron beam with inverse sign as:

$$P_p = \frac{N_p^- - N_p^+}{N_p^+ + N_p^-}$$

where the index p refers to the positrons.

With these definitions equal signs of the polarization of both beams leads to an increased reaction rate via the left-handed coupling (negative polarization) or right-handed coupling (positive polarization), respectively, while unequal signs lead to a general reduction of the reaction rate.

The fraction of left-handed and right-handed particles in the beams is found to be:

$$N_e^L = \frac{N_e^-}{N_e^+ + N_e^-} = \frac{1}{2}(1 - P_e) \qquad N_e^R = \frac{N_e^+}{N_e^+ + N_e^-} = \frac{1}{2}(1 + P_e)$$

$$N_p^L = \frac{N_p^-}{N_p^+ + N_p^-} = \frac{1}{2}(1 + P_p) \qquad N_p^R = \frac{N_p^+}{N_p^+ + N_p^-} = \frac{1}{2}(1 - P_p)$$

One can write neutral current interactions in the form of vector g_V and axial vector g_A coupling

$$g_V = g_R + g_L$$
$$g_A = g_R - g_L$$

g_L and g_R refer to the left-handed and the right-handed part of the coupling, respectively. The rate induced by left-handed electrons interacting with right-handed positrons is proportional to g_L^2, and the rate of right-handed electrons interacting with left-handed positrons is proportional to g_R^2.

Electrons and positrons interact via photon or Z^0 exchange. Photons and Z^0-bosons have spin 1, therefore the annihilation cross section for left-handed electrons interacting with left-handed positrons or right-handed electrons interacting with right-handed positrons is zero within the Standard Model, except for small radiative corrections where the helicity of the incoming particle is changed due to the radiation of a photon.

The reaction rate of the annihilation of a polarized electron beam interacting with a polarized positron beam is hence proportional to:

$$R^+ \sim N_e^L \cdot N_p^R \cdot g_L^2 + N_e^R \cdot N_p^L \cdot g_R^2 =$$

$$\frac{1}{4}(1 - P_e - P_p + P_e \cdot P_p) \cdot g_L^2 + \frac{1}{4}(1 + P_e + P_p + P_e \cdot P_p) \cdot g_R^2$$

$$\sim \frac{1}{2}(1 + P_e \cdot P_p) \cdot (g_V^2 + g_A^2) + (P_e + P_p) \cdot (g_V \cdot g_A) \qquad (1)$$

where multiplicative constants have been dropped, since they will cancel later. Here a positive polarization of both beams according to the above definition has been assumed. In the case of a negative polarization of both beams one finds:

$$R^- \sim \frac{1}{2}(1 + P_e \cdot P_p) \cdot (g_V^2 + g_A^2) - (P_e + P_p) \cdot (g_V \cdot g_A)$$

Here we assume that the absolute value of the polarization is on the average not changed if the direction of the spins of the particles is reversed. In order to achieve this it is important to change the direction of the spins randomly from pulse to pulse as it is done at SLC. Then randomly and systematically fluctuations of the polarization and the machine performance cancel to a high degree.

Both electron and positron sources produce longitudinally polarized beams. However, the longitudinal polarization would be destroyed in the damping rings of a linear collider due to the spin precession. Hence, spin rotators have to rotate the spin before the damping ring into the transverse direction parallel or anti-parallel to the guiding field of the ring. A second spin rotator brings the spin back into the longitudinal direction after the extraction out of the ring. In case of the positrons two spin rotators in parallel beam lines served by two kicker magnets might be used to change the spin direction at the IP from pulse to pulse. The polarization of the electrons can be reversed simply by reversing the polarization of the laser light used for the electron production.

The experimental left-right asymmetry A_{LR}^{exp} is defined by:

$$A_{LR}^{exp} = \frac{R^+ - R^-}{R^+ + R^-} = \frac{P_e + P_p}{1 + P_e \cdot P_p} \cdot \frac{2(g_V \cdot g_A)}{g_V^2 + g_A^2}$$

As has been pointed out in [7] the entire sample of hadronic Z^0 decays as well as the muon and tau lepton final states can be used to measure A_{LR}^{exp}. In order to exclude t-channel processes the electron final state has to be excluded.

Following the notation of M. Swartz we call $P_g = \frac{P_e + P_p}{1 + P_e \cdot P_p} \leq 1$ the generalized polarization.

The theoretical left-right asymmetry A_{LR} can be written as:

$$A_{LR} = A_{LR}^{exp} / P_g$$

Fig. 1 shows P_g as function of P_e for different ratios P_p / P_e. Note that $\frac{\partial P_g}{\partial P_{e,p}} = 0$ for $P_e = P_p = 1$. For an electron polarization of 60-70% the generalized polarization becomes ~90% if the positron beam is polarized to the same level as the electron beam.

Error analysis

The dominating error ΔA_{LR} on the measurement of A_{LR} is given by :

$$\Delta A_{LR} = \left[A_{LR}^2 \left(\frac{\Delta P_g}{P_g}\right)^2 + \frac{1}{P_g^2 \cdot N_{tot}} \right]^{1/2}$$

N_{tot} = total Number of events

here $(P_g \cdot A_{LR})^2 << 1$ is assumed, thus the error is overestimated.

The first term describes the error on the asymmetry in the limit of large statistics due to the uncertainty of the measured polarization P_g, while the rate of convergence to the asymptotic limit is described by the second term.

We consider the two terms separately.

I Statistical error

From: $$\Delta A_{LR} = \frac{1}{P_g \cdot \sqrt{N_{tot}}}$$

one obtains: $$N_{tot} = \frac{1}{P_g^2 \cdot \Delta A_{LR}^2} \qquad (2)$$

Assuming the same time is spent for the measurement of R^+ and R^-, respectively, the number of events N is given by:

$$N_{tot} = L \cdot t \cdot \sigma_{unpol}(1 + P_e \cdot P_p) \qquad (3)$$

where $L \cdot t$ refers to the integrated luminosity and σ_{unpol} is the cross section for unpolarized beams. Note that (3) holds even for a pure vector coupling, i.e. $g_R = g_L$.

From (2) and (3) we get: $$L \cdot t \sim \frac{1}{P_g^2 (1 + P_e \cdot P_p)}$$

where $L \cdot t$ denotes the integrated luminosity which is necessary to measure A_{LR} within a given statistical error ΔA_{LR}. Fig. 2 shows $L \cdot t$ versus P_e for different ratios P_p / P_e. More instructive is fig. 3 where the ratio of $L \cdot t$ for two polarized beams to $L \cdot t$ with only one beam polarized is shown. At P_e=60% a factor of 3 can be gained by polarizing the positron beam.

We can write the luminosity of a linear collider as:

$$L \cdot t \sim P \cdot \sqrt{\frac{dp/p}{\gamma\varepsilon_y}} \cdot t$$

P = beam power

dp/p = momentum spread due to beamstrahlung

$\gamma\varepsilon_y$ = normalized vertical emittance

Hence, the factor 3 in $L \cdot t$ corresponds to nearly an order of magnitude in $\gamma\varepsilon_y$ or dp/p or a factor of 3 in beam power P or wall plug power ~ P.

II Systematic error

The asymmetry error in the limit of large statistics:

$$\frac{\Delta A_{LR}}{A_{LR}} = \frac{\Delta P_g}{P_g}$$

is given by the error of the measurement of P_g. With:

$$\Delta P_g = \sum_{i=e,p} \left| \frac{\partial P_g}{\partial P_i} \right|_{(P_e,P_p)} \cdot \Delta P_i$$

one finds:

$$\Delta P_g = \frac{1-P_p^2}{(1+P_e \cdot P_p)^2} \Delta P_e + \frac{1-P_e^2}{(1+P_e \cdot P_p)^2} \Delta P_p \qquad (4)$$

Here the maximum error has been calculated since this result will be needed later. In case of independent measurements of P_e and P_p the average error can be calculated which is up to a factor $1/\sqrt{2}$ smaller.

Note, that P_e and P_p denote the average of the absolute values of the two spin directions used in the measurement, respectively.

For simplicity we assume $P_e = P_p = P$ and $\Delta P_e = \Delta P_p = \Delta P$, yielding:

$$\Delta P_g = 2 \cdot \frac{1-P^2}{(1+P^2)^2} \Delta P$$

or:

$$\frac{\Delta P_g}{P_g} = \frac{1-P^2}{1+P^2} \cdot \frac{\Delta P}{P}$$

The ratio $\left(\frac{\Delta P_g}{P_g}\right) / \left(\frac{\Delta P}{P}\right)$ is plotted in fig. 4.

In case that both beams have a different polarization, the error of the measurement of P_g is dominated by the error in the measurement of the beam with the higher polarization.

For $P_e = 1$:

$$\frac{\Delta P_g}{P_g} = \frac{1-P_p^2}{(1+P_p)^2} \cdot \frac{\Delta P_e}{P_e}$$

Again a plot of the ratio $\left(\frac{\Delta P_g}{P_g}\right) / \left(\frac{\Delta P_e}{P_e}\right)$ can be found in fig. 4.

It can be seen that the measurement error of P_g can be significantly decreased if both beams are polarized. Note that the error becomes zero for $P_p = 1$ corresponding to $\frac{\partial P_g}{\partial P_{e,p}} = 0$ for $P_e = P_p = 1$ (see fig. 1). This is of course only true for $\Delta P_e / P_e \ll 1$.

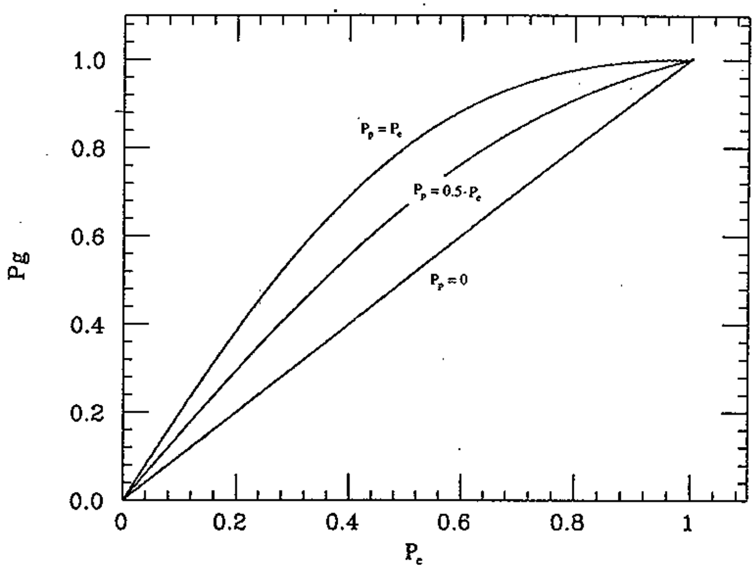

Fig. 1 Generalized polarization P_g as function of P_e for different ratios P_p / P_e.

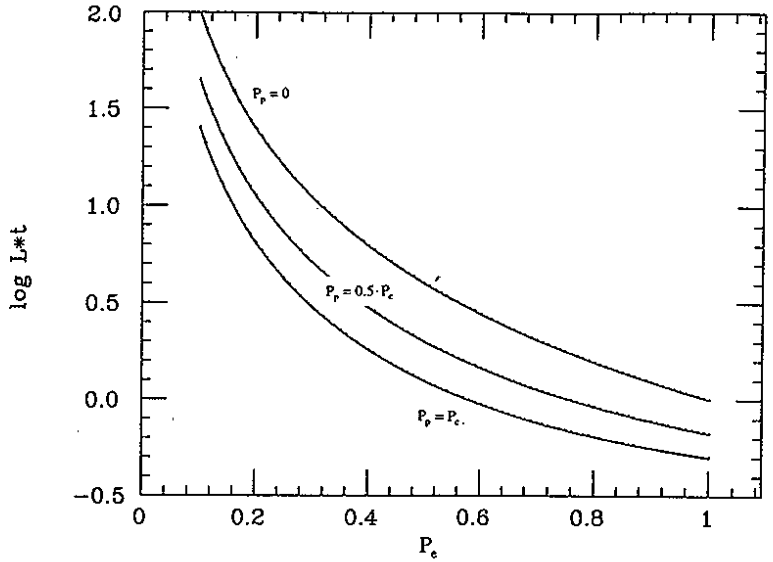

Fig. 2 Integrated luminosity necessary to measure the asymmetry to a given statistical error as function of P_e for different ratios of P_p / P_e.

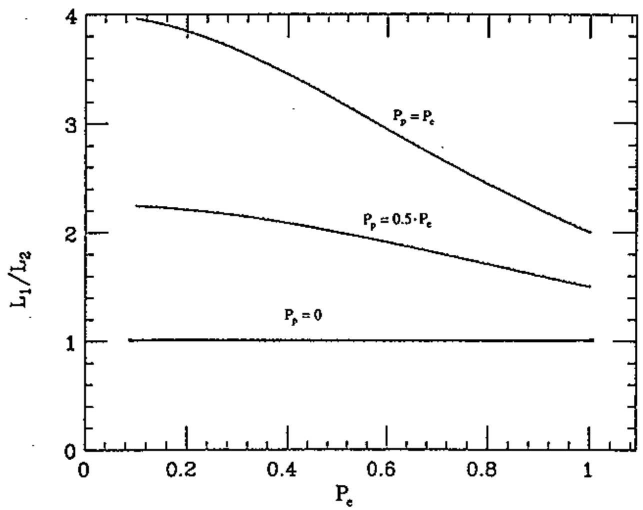

Fig. 3 Gain due to polarization of the positron beam. L_1 refers to the integrated luminosity which is necessary to measure the asymmetry if only one beam is polarized, while L_2 refers to both beams polarized.

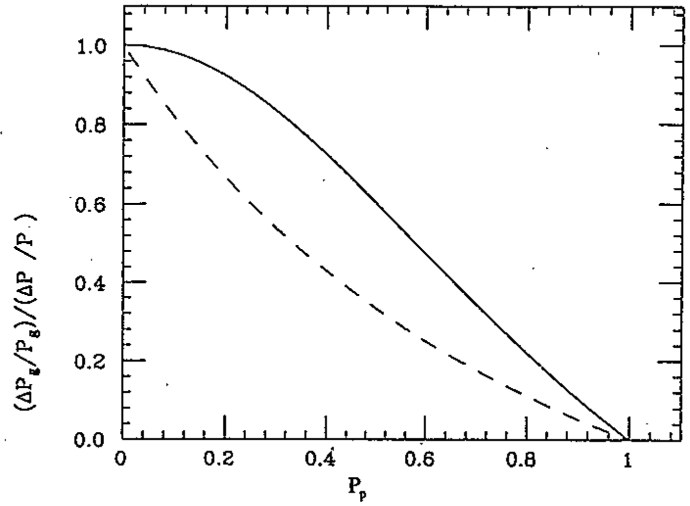

Fig. 4 Accumulated error of P_g as function of P_p. Solid line: $P_e = P_p$; $\Delta P_p = \Delta P_e$.

Dashed line: $P_e = 1$; $\Delta P_e / P_e \ll 1$.

Polarization measurement

So far only two of four possible spin combinations have been used in the analysis. By means of the remaining combinations the polarization of the individual beams as well as the generalized polarization can be measured. From (1) one gets by reversing the sign of one polarization only

$$R^{\pm} \sim \frac{1}{2}(1 - P_e \cdot P_p)\cdot(g_V^2 + g_A^2) + (P_e - P_p)\cdot(g_V \cdot g_A)$$

and

$$R^{\mp} \sim \frac{1}{2}(1 - P_e \cdot P_p)\cdot(g_V^2 + g_A^2) - (P_e - P_p)\cdot(g_V \cdot g_A)$$

Defining the ratios B and C by:

$$B = \frac{R^{\pm} + R^{\mp}}{R^+ + R^-} = \frac{1 - P_e \cdot P_p}{1 + P_e \cdot P_p}$$

$$C = \frac{R^{\mp} - R^{\pm}}{R^+ - R^-} = \frac{P_p - P_e}{P_p + P_e}$$

the polarization of the two beams can be written as:

$$P_e^2 = \frac{(1-B)(1-C)}{(1+B)(1+C)}$$

$$P_p^2 = \frac{(1-B)(1+C)}{(1+B)(1-C)}$$

Again P_e and P_p denote the averages of the absolute values of the two spin directions used in the measurement.

The error of B and C is given by:

$$\Delta B = \sqrt{\frac{B+B^2}{N}}$$

$$\Delta C = \sqrt{\frac{B+C^2}{(A_{LR}^{exp})^2 \cdot N}}$$

Here, N denotes the total number of events gathered with the polarization states defined as R^+ and R^-, respectively. Note, that while the measurement of B and C is independent of any coupling constant, the error of C depends on the experimental left-right asymmetry A_{LR}^{exp}. The error of the measurement of P_e and P_p is given by:

$$\frac{\Delta P}{P} = \frac{1}{(1-B^2)(1-C^2)}\left[\left|1-B^2\right| \cdot \Delta C + \left|1-C^2\right| \cdot \Delta B\right] \qquad (5)$$

or, in the limit of $P_e = P_p = 1$:

$$B = C << 1$$

$$\frac{\Delta P}{P} = \Delta B + \Delta C$$

As an reasonable example we assume $P_e = 0.8$, $P_p = 0.6$, $P_g = 0.95$. One finds:

$$A_{LR}^{exp} \cong 0.15 \cdot P_g = 0.14$$

$$B = 0.351$$

$$C = -0.143$$

$$\frac{\Delta P}{P} = \frac{5.23}{\sqrt{N}}$$

With N = 10^6 events an absolute calibration of the polarimeters with a statistical error of $\Delta P/P = 5 \cdot 10^{-3}$ can be performed. This kind of analysis measures directly the polarization available during interaction, therefore depolarizing effects due to beamstrahlung and disruption are included. The high number of events would require a calibration run at the Z^0-pole with high luminosity. At higher energies other processes like Bhabha scattering might be used for a polarization measurement.

The generalized polarization P_g can be written as:

$$P_g^2 = \frac{1-B^2}{1-C^2}$$

with the error:

$$\frac{\Delta Pg}{Pg} = \frac{1}{(1-B^2)(1-C^2)}\left[\left|C(1-B^2)\right|\Delta C + \left|B(1-C^2)\right|\Delta B\right] \qquad (6)$$

Note, that the error of the direct measurement of Pg is in general somewhat smaller than the error of P_g calculated by means of (4) and (5). The measurements of the ratios B and C contain mixed terms of P_e and P_p, thus the errors of P_e and P_p are correlated. Equation (4) is

a worst case treatment of the errors of P_e and P_p, while the direct measurement includes a precise calculation of the error. With the numbers quoted above one gets $\Delta Pg/Pg = 1.8 \cdot 10^{-3}$ from (4) and (5), and $\Delta Pg/Pg = 9.6 \cdot 10^{-4}$ using (6). Note, that the error from (6) has a minimum at $P_e = P_p$ and $C = 0$.

Conclusion

Compared to a storage ring a linear collider offers the advantage that all possible spin combinations can be provided if polarized particle sources are available.

The analyzing power of asymmetry measurements with electron-positron beams can be significantly improved if both electrons and positrons are polarized as compared to measurements with only one beam polarized. The integrated luminosity necessary to measure an asymmetry within a given statistical error is reduced by a factor 2-3.

The systematic error from the measurement of the polarization can be reduced by a factor of ~3 if both beams are polarized to 70-80%. This is due to the fact that the system is overdetermined if both beams are highly polarized.

An absolute, high precision calibration of the polarimeters is possible. With ~10^6 events a calibration with a statistical error of $5 \cdot 10^{-3}$ of both polarimeters can be performed.

A direct measurement of the generalized polarization yields an even smaller error of ~10^{-3} for ~10^6 events.

This analysis is restricted to the dominating error sources identified so far. A number of effects which might contribute to the error are not included and need a further careful investigation.

We concentrated on the left-right asymmetry in the standard model which, however, might be considered as an example for a measurement with polarized beams. At higher energies new effects are expected which can be investigated by means of polarized beams. We assume that in this case the polarization of both beams is also advantageous.

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References

1 ' A Scheme to Measure the Polarization Asymmetry at the Z Pole in LEP ' A. Blondel, Physics Letters B 202, 145, 1988

2 ' The Polarized Electron Source of the Stanford Linear Accelerator Center ' D. Schultz et. al. Proceedings of the 1994 International Linac Conference, Tsukuba, 1994

3 ' The Conversion System for Obtaining high polarized Electrons and Positrons ' V. E. Balakin, A. A. Mikhailichenko, Preprint INP 79-85, 1979

4 ' Conversion System for Obtaining high polarized Electrons and Positrons at high Energy ' A. A. Mikhailichenko, Novosibirsk INP 1986, (In Russian)

5 ' Investigations Toward the Development of Polarized and Unpolarized High Intensity Positron Sources for Linear Colliders ' K. Flöttmann, DESY 93-161, 1993

6 ' JLC-I ' JLC Group KEK Report 92-16, 1992

7 ' Physics With Polarized Electron Beams ' M. L. Swartz, SLAC-PUB-4656, 1988