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Generalized Parton Distributions and Nucleon Structure at Large x from Lattice QCD

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Abstract. Recent progress in lattice calculations of generalized parton distributions is discussed, with emphasis on the large-x behavior of nucleon structure functions.

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INTRODUCTION

Understanding the internal structure of hadrons in terms of quarks and gluons (partons), and in particular how quarks and gluons provide the binding and spin of the nucleon, is one of the outstanding problems in particle physics. The region of large fractional parton momentum x, being at the borderline between inclusive and exclusive processes, is of particular interest in this respect, as it provides valuable information about the dynamics of quark confinement.

Generalized parton distributions [1] (GPDs) are a powerful tool to address these questions. While ordinary parton distributions measure the probability of finding a parton with fractional longitudinal momentum x in the fast moving nucleon at a given resolution $1/Q$, and no information on the transverse distribution of partons is provided, GPDs describe the coherence of two different hadron wave functions, one where the parton carries fractional momentum $x+\xi$ and one where this fraction is $x-\xi$, from which information on the transverse motion and binding of partons can be gathered [2].

Moments of GPDs are amenable to lattice calculations [3, 4]. In this talk I will present some selected results obtained by the QCDSF collaboration [5].

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[1] Talk given at HiX2004, Marseille, France.

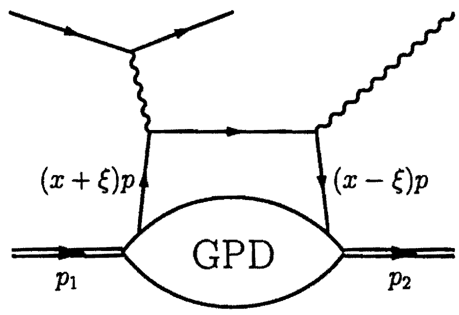

FIGURE 1. The leading diagram for deeply virtual Compton scattering.

GENERALIZED PARTON DISTRIBUTIONS

Let me introduce some notation first and briefly comment on the lattice calculation. Consider the deeply virtual Compton scattering process depicted in Fig. 1, where $p = p_1 + p_2$, $\Delta = p_1 - p_2$, $q = (q_1 + q_2)/2$ and $\xi = q\Delta/qp$. At high energies and large $Q^2 = -q^2$ an operator product expansion (OPE) becomes effective. For $\xi = 0$ the OPE can be mapped onto a parton model description [6]. In this case the momentum transfer of the struck quark is purely transverse, i.e. $\Delta = (0, \Delta_\perp)$. Throughout this talk I shall consider the case $\xi = 0$ only.

The soft part of this process can be parameterized by unpolarized and polarized GPDs [1], $H_q(x,\Delta^2,Q^2)$, $E_q(x,\Delta^2,Q^2)$ and $\tilde{H}_q(x,\Delta^2,Q^2)$, $\tilde{E}_q(x,\Delta^2,Q^2)$, respectively. At zero momentum transfer, $\Delta = 0$, they coincide with the ordinary parton distributions. For example

$$\begin{aligned} H_q(x,0,Q^2) &= q(x,Q^2), \\ \tilde{H}_q(x,0,Q^2) &= \Delta q(x,Q^2). \end{aligned} \tag{1}$$

The GPDs are not directly calculable, but only their moments:

$$\begin{aligned} \int_0^1 dx x^n H_q(x,\Delta^2,Q^2) &= A^q_{n+1}(\Delta^2), \\ \int_0^1 dx x^n E_q(x,\Delta^2,Q^2) &= B^q_{n+1}(\Delta^2), \\ \int_0^1 dx x^n \tilde{H}_q(x,\Delta^2,Q^2) &= \tilde{A}^q_{n+1}(\Delta^2), \\ \int_0^1 dx x^n \tilde{E}_q(x,\Delta^2,Q^2) &= \tilde{B}^q_{n+1}(\Delta^2), \end{aligned} \tag{2}$$

where A^q_n, B^q_n, $\tilde{A}^q_n$ and $\tilde{B}^q_n$ are generalized form factors, which can be related to (off-forward) nucleon matrix elements of certain operators. The lowest form factors (for

$n = 0$) are

$$\begin{aligned} A_1^q(\Delta^2) &= F_1^q(\Delta^2), \\ B_1^q(\Delta^2) &= F_2^q(\Delta^2), \\ \tilde{A}_1^q(\Delta^2) &= G_A^q(\Delta^2), \\ \tilde{B}_1^q(\Delta^2) &= G_P^q(\Delta^2), \end{aligned} \tag{3}$$

where F_1^q, F_2^q are the Dirac and Pauli form factors and G_A^q, G_P^q the axial vector and pseudoscalar form factors of the nucleon. At $\Delta^2 = 0$ we have

$$A_{n+1}^q(0) = \langle x_q^n \rangle, \quad \tilde{A}_{n+1}^q(0) = \langle \Delta x_q^n \rangle. \tag{4}$$

When Fourier transformed to impact parameter space, both

$$\begin{aligned} H_q(x, \mathbf{b}_\perp^2, Q^2) &= \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \Delta_\perp} H_q(x, \Delta_\perp^2, Q^2), \\ \tilde{H}_q(x, \mathbf{b}_\perp^2, Q^2) &= \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \Delta_\perp} \tilde{H}_q(x, \Delta_\perp^2, Q^2), \end{aligned} \tag{5}$$

admit a probability interpretation. While $H_q(x, \mathbf{b}_\perp^2, Q^2)$ describes the probability of finding a quark of flavor q with fractional momentum x at impact parameter $\mathbf{b}_\perp$, $\tilde{H}_q(x, \mathbf{b}_\perp^2, Q^2)$ measures the fraction of the nucleon's spin carried by the quark q with fractional momentum x and at impact parameter $\mathbf{b}_\perp$.

The generalized form factors are obtained from nucleon matrix elements of local operators,

$$\begin{aligned} &\langle p_1 | \bar{\mathcal{O}}^q_{\{\mu_1 \cdots \mu_n\}}(a) | p_2 \rangle \\ &= \left(\frac{i}{2}\right)^{n-1} \langle p_1 | \bar{q} \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} q | p_2 \rangle \\ &= \left(\frac{1}{2}\right)^{n-1} \bar{u}(p_1) [\bar{A}_n^q(\Delta^2) \gamma_{\{\mu_1} \\ &+ \frac{i\Delta^\alpha}{2m_N} \bar{B}_n^q(\Delta^2) \sigma_{\alpha\{\mu_1}] \bar{p}_{\mu_2} \cdots \bar{p}_{\mu_n\}} u(p_2) + \cdots, \end{aligned} \tag{6}$$

which we compute on the lattice. Similar formulae hold for the polarized case. To match the Wilson coefficients, the (bare) lattice operators must be renormalized:

$$\mathcal{O}_{\{\mu_1 \cdots \mu_n\}}(\mu) = Z_{\mathcal{O}}(a\mu) \bar{\mathcal{O}}_{\{\mu_1 \cdots \mu_n\}}(a). \tag{7}$$

Due to bad convergence of lattice perturbation theory, this is gradually being done nonperturbatively [7].

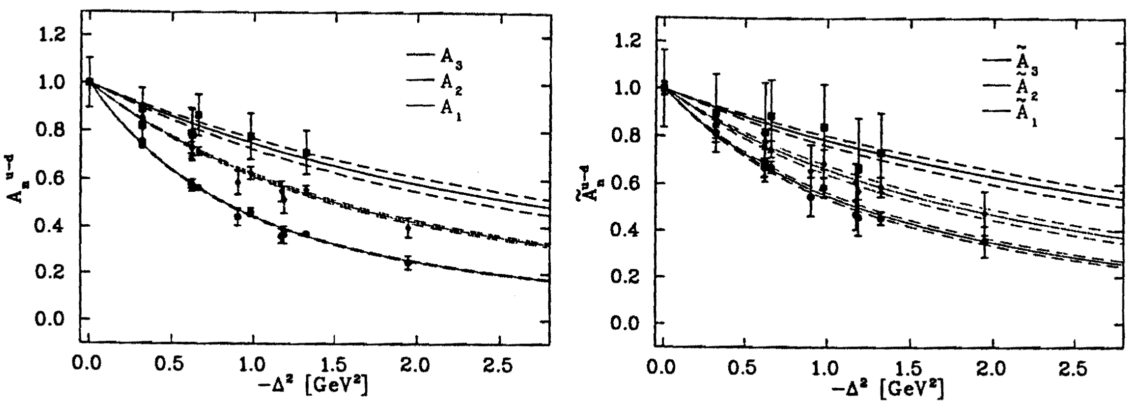

FIGURE 2. The nonsinglet generalized form factors A_n^{u-d} and $\tilde{A}_n^{u-d}$, together with a dipole fit.

The calculations I am going to present have been performed using Wilson fermions with $N_f = 2$ flavors of light dynamical quarks. The lightest quark mass that has been reached so far corresponds to a pion mass of $O(500)$ MeV. Our smallest lattice spacing is $a \approx 0.07$ fm, which translates into a cut off of ≈ 2.8 GeV.

LATTICE RESULTS

Once we know the generalized form factors, we can reconstruct the GPDs. Consider $H_q(x,\Delta^2,Q^2)$. The generalized form factor can be written

$$\int_0^1 dx x^n C_q(x,\Delta^2) = A_{n+1}^q(\Delta^2)/A_{n+1}^q(0). \tag{8}$$

All we have to do is find $C_q(x,\Delta^2)$ by an inverse Mellin transform. The corresponding GPD is then given by the convolution integral

$$H_q(x,\Delta^2,Q^2) = \int_x^1 \frac{dy}{y} C_q(\frac{x}{y},\Delta^2)\, q(x,Q^2). \tag{9}$$

In Fig. 2 I show the Δ^2 dependence of the first three nonsinglet generalized form factors A_n^{u-d} and $\tilde{A}_n^{u-d}$, respectively, for some particular coupling and quark mass. The form factors are well described by the dipole ansatz

$$A_n^q(\Delta^2) = \frac{A_n^q(0)}{(1-\Delta^2/M_n^2)^2}. \tag{10}$$

The solid lines show a dipole fit, and the dashed lines indicate the error. We note here that the various form factors are well separated, and that their slopes decrease with increasing n. This does not come unexpectedly. For $x \longrightarrow 1$ we expect $H_q(x,\mathbf{b}_\perp^2,Q^2)$ to approach a δ-like function in $\mathbf{b}_\perp$. This requires that $A_n(\Delta^2) \longrightarrow const.$ as $n \longrightarrow \infty$. Fitting the form factors with a dipole is purely phenomenological. It provides us with a means to measure the change in slope by monitoring the extracted dipole masses. In Fig. 3 I show the dipole

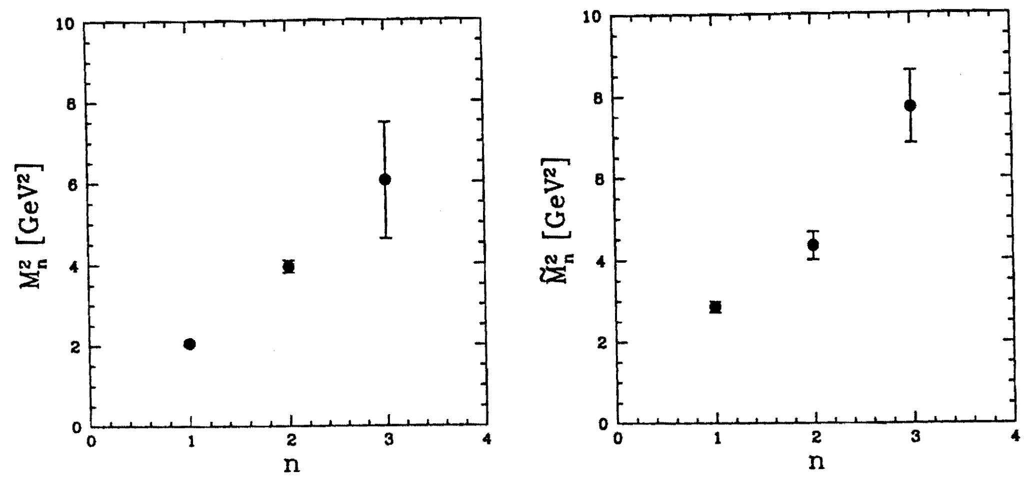

FIGURE 3. The dipole masses M_n and $\tilde{M}_n$ extracted from the generalized form factors shown in Fig. 2

masses M_n and (for the polarized case) $\tilde{M}_n$ as a function of n. Our results obtained so far support a Regge-like behavior

$$M_n^2 = c + n/\alpha'. \tag{11}$$

An important issue is the extrapolation to the chiral limit. In lack of any guidance from chiral perturbation theory, we have to rely on more or less phenomenological fits. At the physical point we find $c \approx -0.5$ GeV^2 and $1/\alpha' \approx 1.1$ GeV^2. The singlet form factors give generally different c's and α''s.

Assuming that the Regge behavior continues to hold for the higher moments as well, we can compute $C_q(x,\Delta^2)$. We find

$$C_q(x,\Delta^2) = \delta(1-x) + \alpha'\Delta^2\left(2+\alpha'\Delta^2 \ln x\right) x^{\alpha'(c-\Delta^2)}. \tag{12}$$

Taking $q(x,Q^2)$ from the literature and Fourier transforming (9) to impact parameter space, we finally obtain $H_q(x,\mathbf{b}_\perp^2,Q^2)$. In Fig. 4 I show the distribution of the valence u quark in the proton at the scale $Q^2 = 4$ GeV^2 as a function of x and the impact parameter $\mathbf{b}_\perp$. It appears that at small x the proton looks like a doughnut, being largely empty in the middle, while at larger values of x the hole closes and the distribution becomes increasingly narrow. Similar results are found for $\tilde{H}_q(x,\mathbf{b}_\perp^2,Q^2)$.

Let us discuss the behavior at large x in some more detail now. For $x \longrightarrow 1$ we find

$$H_q(x,\mathbf{b}_\perp^2,Q^2) \simeq const. \, e^{-\mathbf{b}_\perp^2/4\alpha'(1-x)} q'(x,Q^2). \tag{13}$$

Assuming that $q'(x,Q^2)$ remains finite as $x \longrightarrow 1$, we are then led to

$$\langle \mathbf{b}_\perp^2 \rangle \simeq 4 const. (1-x). \tag{14}$$

While the impact parameter is the transverse distance between the struck parton and the center of momentum of the nucleon, an estimate of the transverse size $\mathbf{r}_\perp$ of the nucleon

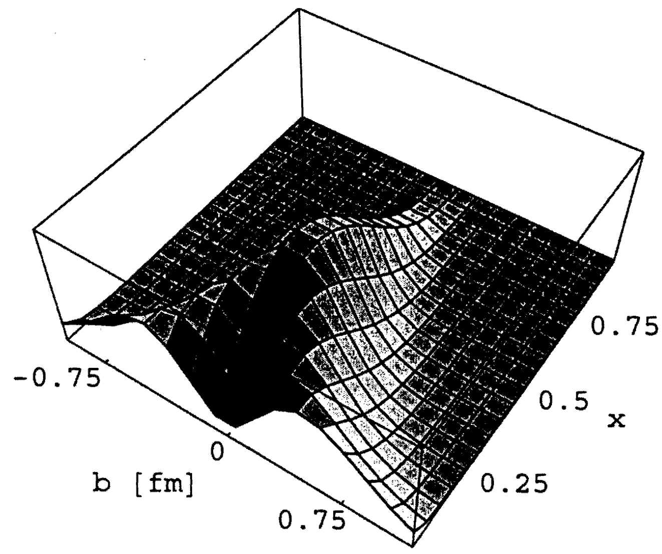

FIGURE 4. The generalized parton distribution $H_u(x, \mathbf{b}_\perp^2, Q^2)$ of the proton as a function of x and impact parameter $b = |\mathbf{b}_\perp|$ at $Q^2 = 4$ GeV^2.

is provided by the distance between the struck quark and the system of spectators. Its average square is given by [8]

$$\langle \mathbf{r}_\perp^2 \rangle = \frac{\langle \mathbf{b}_\perp^2 \rangle}{(1-x)^2}. \tag{15}$$

Hence, the average transverse size, $r = \sqrt{\langle \mathbf{r}_\perp^2 \rangle}$, of the nucleon diverges like $(1-x)^{-1/2}$ in the limit $x \longrightarrow 1$. This behavior can certainly not be reconciled with confinement. Perhaps our dipole ansatz for the generalized form factors is too crude. Or it was premature to assume a Regge-like behavior of the dipole masses. Future calculations will have to tell.

CONCLUSIONS

I have presented first lattice results for the lowest three moments of GPDs for $N_f = 2$ flavors of dynamical quarks. Our findings, though very crude yet, kill already many models of the spatial structure of the nucleon in the literature. There are still many problems to overcome. The most immediate challenges are to extend these calculations to more realistic quark masses, so that controlled extrapolations to the chiral limit can be performed, and to higher moments.

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