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DETERMINATION OF THE $\pi\pi$ SCATTERING PHASE SHIFTS UP TO 1.3 GEV CMS ENERGY

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DETERMINATION OF THE $\pi\pi$ SCATTERING PHASE SHIFTS UP TO 1.3 GEV CMS ENERGY

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Summary: The experimental data on elastic $\pi\pi$ scattering were combined and analysed in terms of the $\pi\pi$ scattering phase shifts up to cms energies of the $\pi\pi$ system of 1.3 GeV. Most of these data come from the application of the one-pion exchange model to reactions of the type $\pi N \rightarrow \pi\pi N$. The analysis resulted in a set of phase shifts which give a good fit to the existing data. For the width of the ρ meson a value of 170 MeV was obtained. The behaviour of the asymmetry parameter $R = (F-B)/(F+B)$ in $\pi^+\pi^-$-scattering implies the existence of a $T = 0$, $J = 0$ $\pi\pi$ resonance. A mass of .74 GeV and a width of 90 MeV were found for this resonance.

There is now considerable information available on various aspects of the elastic $\pi\pi$ interaction, such as angular distributions, cross sections and S wave contributions at low energies. Most of this information comes from the application of the one pion exchange (OPE) model [1,2] to the experimental results on single and double pion production in πN collisions [3-9]. However, one has to modify the OPE model in order to get a quantitative description of the experimental data, as the production processes appear to be far more peripheral than is predicted by the uncorrected OPE model. Such modifications are the introduction of form factors [10] and/or the inclusion of absorptive effects in the initial and final states [11]. The experimental results on the $\pi\pi$ interaction have been obtained mainly by using the form factor and off-shell correction functions of Ferrari and Selleri [10]. This approach yielded consistent results from a variety of reactions and with different momenta of the incoming pion [6,7,9]. In particular, the cross sections for elastic $\pi\pi$ scattering which have been determined in this way reach the unitary limits at the masses of the ρ mesons (in $\pi^-\pi^0$ and $\pi^+\pi^-$ scattering) and the f mesons (in $\pi^+\pi^-$ scattering). These results give some confidence in the data on $\pi\pi$ scattering which have been obtained with this model.

In the study reported here we tried to combine all experimental data on elastic $\pi\pi$ scattering and to analyse them in terms of the $\pi\pi$ scattering phase shifts. A set of phase shifts was obtained which fits the data well. A short account of this analysis has already been published.[12]

<u>$\pi\pi$ scattering:</u> The elastic $\pi\pi$ scattering amplitude $A\ (\omega,\ \cos\Theta)$ (ω total $\pi\pi$ cms energy, Θ scattering angle) decomposes into the isospin amplitudes according to [13]

$$A(\omega,\cos\Theta) = \sum_T C(T,T_3,t_3)\ A^T(\omega,\cos\Theta) \qquad (1)$$

where T,T_3 denote the total isospin and its 3rd component, and where t_3 is the 3rd isospin component of one of the pions. The coefficients $C(T,T_3,t_3)$ are tabulated in Table I:

Table I: Coefficients $C(T,T_3,t_3)$

	T = 0	T = 1	T = 2
$\pi^+\pi^+ \to \pi^+\pi^+$			1
$\pi^+\pi^o \to \pi^+\pi^o$		1/2	1/2
$\pi^+\pi^- \to \pi^+\pi^-$	1/3	1/2	1/6
$\pi^+\pi^- \to \pi^o\pi^o$	-1/3		1/3
$\pi^o\pi^o \to \pi^o\pi^o$	1/3		2/3

The isospin amplitude $A^T(\omega, \cos\Theta)$ can be expressed by the phase shifts δ_J^T :

$$A^T(\omega, \cos\Theta) = \omega \sum_J (2J+1) \frac{e^{i\delta_J^T}\sin\delta_J^T}{q} P_J(\cos\Theta) \qquad (2)$$

where q is the pion momentum in the $\pi\pi$ rest system and the $P_J(\cos\Theta)$ are the Legendre polynomials. The differential cross section for $\pi\pi$ scattering is given by

$$\frac{d\sigma}{d\cos\Theta} = \frac{8\pi}{\omega^2} \left| \sum_T C(T,T_3,t_3)\, A^T(\omega, \cos\Theta) \right|^2 \qquad (3)$$

A priori little is known on how the different $\pi\pi$ phase shifts should behave. Near threshold one can use the effective range expansion of the scattering amplitude [14]. However, the experimental data are at present not accurate enough to allow for a determination of the expansion coefficients. Therefore, one usually applies the effective-range approximation of Chew and Mandelstam which is derived from the S-wave dominant solutions of the $\pi\pi$ equations [15]

$$\left(\frac{\nu}{\nu+1}\right)^{1/2} \operatorname{ctg} \delta_0^T = \frac{1}{a_0^T} + \frac{2}{\pi}\left(\frac{\nu}{\nu+1}\right)^{1/2} \ln(\sqrt{\nu} + \sqrt{\nu+1}) \qquad (4)$$

with $\nu = (q/\mu)^2$, μ the pion mass and a_0^T the scattering length which has to be determined experimentally.

If there exists a $\pi\pi$ resonance in the (T,J) state, the corresponding phase shift can be expressed near the resonance in terms of the position and width of the resonance [16]:

$$\mathrm{tg}\,\delta_J^T = \frac{\omega_r}{\omega_r^2-\omega^2}\,\Gamma(\omega) \qquad (5)$$

Here ω_r is the resonance energy and $\Gamma(\omega)$ the width. For broad resonances the width is energy dependent. Following Dürr and Pilkuhn one obtains [17]:

$$\Gamma(\omega) = \frac{q\,V_J(rq)}{q_r\,V_J(rq_r)}\,\Gamma_r \qquad (6)$$

where q_r is the pion momentum at the resonance, Γ_r is the full width at half maximum and

$$V_J(x) = \left[x^2\left(j_J^2(x) + n_J^2(x)\right)\right]^{-1} \qquad (7)$$

$j_J(x)$, $n_J(x)$ are the spherical Bessel and Neumann Functions. r is the "radius of the interaction" and is of the order of $1/3\mu$ For $q \simeq q_r$ (6) reduces to the familiar formula $\Gamma \simeq (q/q_r)^{2J+1}\Gamma_r$.

<u>The OPE model:</u> The OPE process for single pion production is illustrated by the diagram in Fig. 1. With the modifications by Ferrari and Seeleri the OPE model makes the following prediction for the differential cross section of the reactions $\pi N \to \pi\pi N$ [10]:

$$\frac{d\sigma_{\pi N\to\pi\pi N}}{dt\,d\omega\,d\cos\Theta} = \frac{f^2}{\pi}\,\frac{m^2}{p^2 s}\,\frac{-t}{\mu^2}\,\frac{F^2(t)}{(\mu^2-t)^2}\,\omega^2\,q^{\mathrm{off}}\,\frac{d\sigma_{\pi\pi\to\pi\pi}(\omega,t,\cos\Theta)}{d\cos\Theta} \qquad (8)$$

with m nucleon mass

s square of the total energy in the overall cms system

p momentum of the incoming nucleon in the overall cms system t square of the momentum transfer between incoming and outgoing nucleon

$$q^{off} = \left[\left(\frac{\omega^2+\mu^2-t}{2\omega}\right)^2 - \mu^2\right]^{1/2}$$ momentum of the exchanged pion in the $\pi\pi$ rest system

f πN coupling constant; $f^2=0.08$ if the exchanged pion is neutral, $f^2=0.16$ if the exchanged pion is charged.

F (t) is a product of three form factors, which correct the $\pi\pi$ and the πN vertex and the pion propagator. F(t) has been determined empirically:

$$F(t) = .28 + \frac{.72}{1 + \frac{\mu^2-t}{4.73\mu^2}}$$

$\frac{d\sigma_{\pi\pi\to\pi\pi}(\omega,t,\cos\theta)}{d\cos\theta}$ is defined as the off-shell differential cross section for $\pi\pi$ scattering, which in the approach of Ferrari and Selleri is given by

$$q^{off}\frac{d\sigma_{\pi\pi\to\pi\pi}}{d\cos\theta} = 8\pi q\left|\sum_T C(T,T_3,t_3)\sum_J(2J+1)\frac{e^{i\delta_J^T}\sin\delta_J^T}{q}\left(\frac{q^{off}}{q}\right)^J P_J(\cos\theta)\right|^2 \quad (9)$$

The correction function $\left(\frac{q^{off}}{q}\right)^{J-1/2}$ has originally been obtained under the assumption that the partial wave with angular momentum J is in a resonant state. We will use it also for the non-resonant case. Provided the OPE model is applicable, Formula (8) allows to determine the scattering parameters from the measured values of $\frac{d\sigma_{\pi N\to\pi\pi N}}{dt\,d\omega\,d\cos\theta}$. In particular, if the $\pi\pi$ scattering takes place in a definite angular momentum state J, Formula (8) yields directly the on-shell differential cross section for elastic $\pi\pi$ scattering.

There are special corrections to be made for the nucleon vertex[18] when the OPE model is compared with the reaction $\pi^- p\to\pi^-\pi^- N^{*++}_{33}$.

Formulas (8),(9) have been used to determine the total cross sections for $\pi^\pm \pi^\pm$, $\pi^-\pi^o$ and $\pi^+\pi^-$ scattering from the reactions $\pi^+ p \rightarrow \pi^+\pi^+ n$ (4.o[9]), $\pi^- p \rightarrow \pi^-\pi^- N^{*++}_{33}$ (4.o[19], see remark made above), $\pi^- p \rightarrow \pi^-\pi^o p$ (1.59[6a], 2.75[6b], 4.o[7b]), $\pi^- p \rightarrow \pi^+\pi^- n$ (1.59[6a], 2.75[6b], 4.o[7]). The figures in brackets give the laboratory momenta of the incident pion at which the reactions have been studied [21].

Further information on $\pi\pi$ scattering has been obtained by measuring

a) the $\pi\pi$ angular distribution W(cos Θ), where Θ is defined, for off-shell scattering as well as for on-shell scattering as the angle between the incoming and the outgoing pion of the same charge in the $\pi\pi$ rest system;

b) the asymmetry parameter R = (Forward - Backward)/(Forward + Backward). Both W(cos Θ) and R have been measured for low momentum transfers. In addition to the experiments cited above there are data available from the reactions $\pi^- p \rightarrow \pi^+\pi^- N^{*++}_{33}$ (2.75[6c]), $\pi^- p \rightarrow \pi^-\pi^o p$ (3.0[22], 3.3[23]), $\pi^- p \rightarrow \pi^+\pi^- n$(3.0[22], 3.3[23], 6.o[24]), and $\pi^+ n \rightarrow \pi^+\pi^- p$ (6.0[25]).

One remark has to be made concerning the measurements of the $\pi\pi$ angular distribution W(cos Θ). According to the OPE model (with the corrections of Ferrari and Selleri), W(cos Θ) is altered if one of the pions is off the mass shell and if partial waves of different angular momentum states are contributing. This can be seen from the Formulas (8), (9). The $\pi\pi$ angular distribution measured in reactions such as $\pi N \rightarrow \pi\pi N$ may therefore differ from the angular distribution for on-shell $\pi\pi$ scattering. For example, the corrections for R are typically of the order of 10 - 30 o/o for momentum transfer squared $-t = 1o^{2}$; they were taken into account in this analysis.

Determination of the phase shifts: Using the relation $J_{max} \simeq qr$ one expects for $\omega < 1.3$ GeV partial waves up to J = 4 to be contributing. Consequently the phase shifts $\delta^0_0, \delta^0_2, \delta^0_4, \delta^1_1, \delta^1_3$ and $\delta^2_0, \delta^2_2, \delta^2_4$ were considered. As no 4π decays of the ρ and f mesons have been observed, the δ^T_J were assumed to be real in this energy region.

The phase shifts δ^T_J were determined by first analysing $\pi^{\pm}\pi^{\pm}$ scattering which involves only the T = 2 isospin amplitude. After fitting the T = 2 phase shifts the experimental results on $\pi^-\pi^0$ scattering were used to obtain the T = 1 phase shifts. The $\pi^+\pi^-$ scattering then allowed to fit the T = 0 phase shifts and served as a check for the T = 1 and T = 2 phase shift values obtained before [26].

$\pi^{\pm}\pi^{\pm}$ scattering: The angular distribution is isotropic below 0.6 GeV[7c,9], and consequently pure S-wave scattering was assumed in this energy region. As the experimental data on $\sigma_{\pi^{\pm}\pi^{\pm}}$ are rather poor at low energies, $\sigma_{\pi^{\pm}\pi^{\pm}}$ was fitted for $\omega < 0.5$ GeV to the effective range formula (4). The fit gave $a^2_0 = -\frac{1}{3\mu}$ (for the sign of a^2_0 see below). In the energy range 0.6 GeV $< \omega < 1.3$ GeV the cross section and the angular distribution can well be described assuming S and D waves only. The relative sign of δ^2_0 and δ^2_2 turned out to be positive. The final determination of δ^2_0 and δ^2_2 was made by combining the data on $\pi^{\pm}\pi^{\pm}$ and $\pi^-\pi^0$ scattering. The asymmetry parameter $R_{\pi^-\pi^0}$ for $\pi^-\pi^0$ scattering, which is negative at low energies, determined the negative sign of δ^2_0 (it was assumed, that δ^1_1 stays positive at low energies).

$\pi^-\pi^0$ scattering: To $\pi^-\pi^0$ scattering the T = 1 and T = 2 isospin amplitudes can contribute and hence the phase shifts δ^2_0, δ^2_2 and δ^1_1, δ^1_3 were considered. As δ^1_1 is dominated by the ρ meson, a resonance form given by (5) with J = 1 was adopted for the energy variation of δ^1_1 in the ρ region. With $\omega_\rho = .760$ GeV the fit of the width to the behaviour of $R_{\pi^-\pi^o}$ gave a full width at half maximum of 170 MeV. Above 0.6 GeV δ_1^1 calculated in this way together with δ_o^2, δ_2^2 and a small positive δ_3^1 fits the experimental data well. Below 0.6 GeV δ_1^1 was determined from the values of $R_{\pi^-\pi^o}$ and $\sigma_{\pi^-\pi^o}$ directly.

<u>$\pi^+\pi^-$ scattering:</u> Using the $\pi^+\pi^-$ scattering data the T = 0 phase shifts were fitted. At very low energies ($\omega < 0.4$ GeV) the T = 0 $\pi\pi$ interaction has been measured by studying the reaction $p+d \to He^3 + 2\pi$[27]. It is safe to assume S-wave contributions only because the $\pi\pi$ mass is small. It was then found that δ_o^o can be fitted by the effective-range Formula (4) with $a_o^o = (2\pm1)\mu^{-1}$. Taking this result for δ_o^o, the observed low energy behaviour of $\pi^+\pi^-$ scattering agrees well with that calculated from δ_o^o, δ_1^1 and δ_o^2 (the experimental data are, however, not accurate in this energy region). At higher $\pi\pi$ masses ($\omega \gtrsim 1$ GeV) there is a strong D-wave contribution which comes from the f-meson. δ_2^o was therefore calculated from (5) with J = 2, using for the mass and the width of the f-meson the values $\omega_f = 1.25$ GeV and $\Gamma_f = 140$ MeV.

One can now try to determine δ_o^o for $\omega > 0.5$ GeV as δ_o^o is the only free parameter left. As the contribution of δ_o^o to $\sigma_{\pi^+\pi^-}$ is small compared with that of δ_1^1, $\sigma_{\pi^+\pi^-}$ is rather insensitive to the value of δ_o^o. On the other hand the value of $R_{\pi^+\pi^-}$ depends critically on δ_o^o and is therefore well suited for a determination of δ_o^o. Fitting δ_o^o to $R_{\pi^+\pi^-}$ we found that the large asymmetry ($R_{\pi^+\pi^-} \cong 0.4 - 0.6$) observed from 0.6 GeV up to 0.9 GeV can only be explained by assuming the existence of a resonance in the T = 0, J = 0 state with a mass of about that of the ρ meson. Best agreement was obtained for a mass of 0.74 GeV and a width of 90 MeV.

This result has been suggested by several authors [28] or has been found in similar investigations. Islam and Pinon [29], Patil [30] and Durand and Chiu [31] studied the $\pi^-\pi^o$ and $\pi^+\pi^-$ angular distribution and proposed the existence of a T = 0, J = 0 resonance, the so-called ε meson. Recently several experiments have been made to observe the decay of the ε directly. Feldmann et al.[32] and Corbett et al.[33] have investigated the reaction $\pi^- p \to \pi^0 \pi^0 n$. Hagopian et al.[34] have analysed the $\pi^+\pi^-$ system in the reaction $\pi^- p \to \pi^+\pi^- n$ in a region of the $\pi^+\pi^-$ scattering angle where ρ production should not predominate. In all experiments an enhancement in the $\pi\pi$ system at about 0.7 GeV was found.

Fig. 2 gives the phase shift values obtained with this procedure. In Fig. 3a, b the asymmetry parameters $R_{\pi^-\pi^0}$ and $R_{\pi^+\pi^-}$ are compared with the measurements. Both parameters were calculated

a) for on-shell $\pi\pi$ scattering

b) for a square of the momentum transfer $-t = 15\mu^2$. As has been proposed by Dürr and Pilkuhn [17], the factor $(q^{off}/q)^J$ in Formula (9) has been replaced by $\left[v_J(rq^{off})/v_J(rq)\right]^{1/2}$, where v(x) is defined in (7). A value of $10\mu^2$ was used for $1/r^2$. Figs. 4a - c show a comparison between the measured and the calculated $\pi^\pm\pi^\pm$, $\pi^-\pi^0$ and $\pi^+\pi^-$ cross sections. In Figs. 5a, b the cross sections for the reactions $\pi^+\pi^- \to \pi^0\pi^0$ and $\pi^0\pi^0 \to \pi^0\pi^0$ are also given. As can be seen from Figs. 3 and 4 the agreement between the experimental results and those obtained from the phase shift values is satisfactory.

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References

1) C. Goebel, Phys. Rev. Lett. 1, 337 (1958)

2) G.F. Chew, F.E. Low, Phys.Rev. 113, 1640 (1959)

3) I.A. Anderson, V.X. Bang, P.G. Burke, D.D. Carmony, and N. Schmitz, Rev. Mod. Phys. 33, 431 (1961)

4) A.R. Erwin, R. March, W.D. Walker, and E. West, Phys. Rev. Lett. 6, 628 (1961)

5) E. Pickup, D.K. Robinson and E.O. Salant, Phys. Rev. Lett 7, 192 (1961)

6) Saclay - Orsay - Bari - Bologna - Collaboration

a) Nuovo Cimento, 25, 365 (1962)
b) Nuovo Cimento, 35, 713 (1965)
c) Nuovo Cimento, 35, 1 (1965)

7) Aachen-Birmingham-Bonn-Hamburg-London-München-Collaboration

a) Phys. Lett. 5, 153 (1963)
b) Nuovo Cimento 31, 729 (1964)
c) Nuovo Cimento 31, 485 (1964)

8) N. Schmitz, Nuovo Cimento 31, 255 (1964)

9) Aachen-Berlin-Birmingham-Bonn-Hamburg-London-München-Collaboration, Phys. Rev. 138, B 897 (1965)

10) E. Ferrari and F. Selleri, Phys. Rev. Lett. 7, 387 (1961) and F. Selleri, Phys. Lett. 3, 76 (1962)

11) see J.D. Jackson, J.T. Donohue, K. Gottfried, R. Keyser, and B.E.Y. Svensson, Phys. Rev. 139, B 428 (1965)

12) G. Wolf, Phys. Lett., to be published

13) see J. Hamilton, The Theory of Elementary Particles, Oxford at the Clarendon Press, 1959

14) W. Zimmermann, Nuovo Cimento 21, 249 (1961)

15) G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

16) J.D. Jackson, Nuovo Cimento 34, 1644 (1964)

17) H.P. Dürr and H. Pilkuhn, CERN preprint 65/1009/5 -TH 581 (1965)

18) E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963)

19) In Ref. (8) $\sigma_{\pi^-\pi^-}$ was determined without the use of off-shell corrections for the $\pi\pi$ vertex. The corresponding influence on the values of $\sigma_{\pi^-\pi^-}$ was estimated and the corrected values were used in the analysis.

20) G. Wolf, University of Hamburg, thesis (1964), unpublished

21) The data from the reaction $\pi^+ p \to \pi^+ \pi^0 p$ were not used since they depend strongly on how one subtracts the large background which is due to the production of the N^{*++}_{33}.

22) V. Hagopian and W. Selove, Phys. Rev. Lett. 10, 533 (1963)

23) Z.G.T. Guiragossian, UCRL 10731 (1963), unpublished

24) I.J. Veillet, J. Hennessy, H. Bingham, M. Bloch, D. Drijard, and P. Negri, Phys. Rev. Lett. 10, 29 (1963)

25) CERN-Ecole Polytechnique Collaboration, Phys. Lett. 17, 354 (1965)

26) A procedure to determine the ππ phase shifts similar to ours has been proposed by I. Butterworth, private communication

27) A. Abashian, N.E. Booth, and K. Crowe, Phys. Rev. Lett. 5, 258 (1960); N.E. Booth and A. Abashian, Phys. Rev. 132, 2314 (1964)

28) V. Hagopian and W. Selove, Phys. Rev. Lett. 10, 533 (1963)
J.P. Baton, J. Reignier, Nuovo Cimento, to be published

29) M.M. Islam and R. Piñon, Phys. Rev. Lett. 12, 310 (1964)

30) S.H. Patil, Phys. Rev. Lett. 13, 261 (1964)

31) L. Durand and Y.T. Chiu, Phys. Rev. Lett. 14, 329 (1965)

32) M. Feldmann, W. Frati, J. Halpern, A. Kanofsky, M. Nussbaum, S. Richert, P. Yamin, A. Choudry, S. Devons, and J. Grunhaus, Phys. Rev. Lett. 14, 869 (1965)

33) I.F. Corbett, C.J.S. Damerell, N. Middlemas, D. Newton, A.B. Clegg, W.S.C. Williams, and A.S. Caroll, Rutherford Laboratory, preprint RPP/H/7, May 1965

34) V. Hagopian, W. Selove, J. Alitti, J.P. Baton, M. Neveu-René, R. Gessaroli, and A. Romano, Phys. Rev. Lett. 14, 1077 (1965)

Captions for figures

Fig. 1 OPE diagram for the reaction $\pi N \rightarrow \pi\pi N$

Fig. 2 $\pi\pi$ phase shifts δ_J^T as a function of the $\pi\pi$ cms energy ω.

Fig. 3 Energy dependence of the asymmetry parameter R. The data were taken from the compilation given in Ref.(6b) and from Ref.(25). The curves shown were calculated from the δ_J^T for on-shell $\pi\pi$ scattering (solid line) and for the scattering of a real pion on a pion with a mass squared of $-t = 15\,\mu^2$ (dashed line)

a) $\pi^-\pi^o$ system

b) $\pi^+\pi^-$ system

Fig. 4 Elastic $\pi\pi$ scattering cross sections as a function of the $\pi\pi$ cms energy ω

a) $\pi^-\pi^o$ scattering. The data come from $\pi^-p \rightarrow \pi^-\pi^o p$ (1.59[6b], 2.75[6b]);

b) $\pi^+\pi^-$ scattering. The data come from $\pi^-p \rightarrow \pi^+\pi^-n$ (•1.59 and 2.75[6b], ⫯4.0[20])

c) $\pi^\pm\pi^\pm$ scattering. The data come from $\pi^+p \rightarrow \pi^+\pi^+n$ (▽4.0[9]) and from π^-p $\pi^-\pi^-N^{*++}_{33}$ (▼4.0[19]).

Fig. 5 Cross sections for the reactions $\pi^+\pi^- \rightarrow \pi^o\pi^o$ (a) and $\pi^o\pi^o \rightarrow \pi^o\pi^o$ (b) as calculated from the phase shifts.

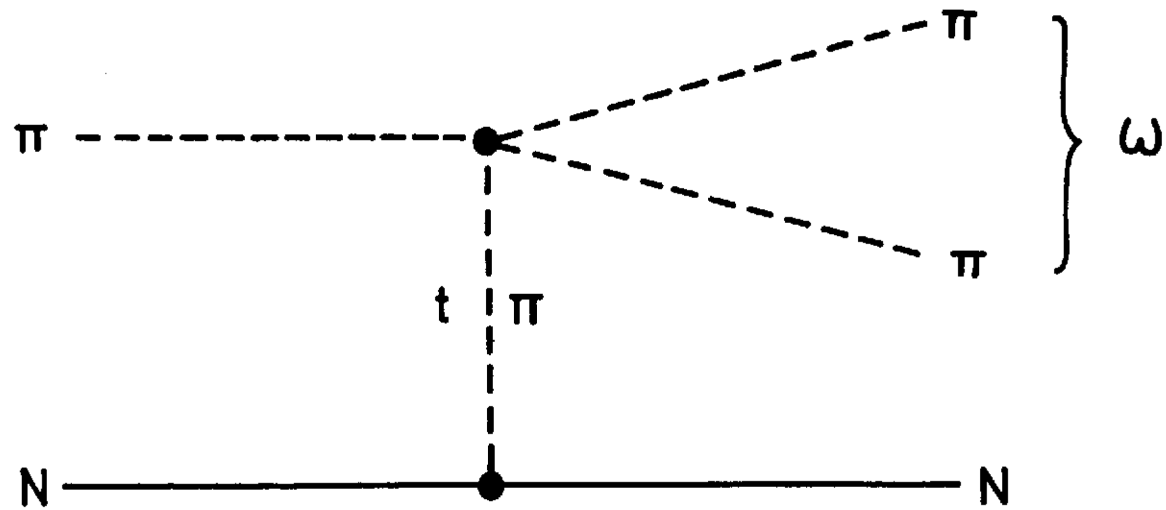

fig 1

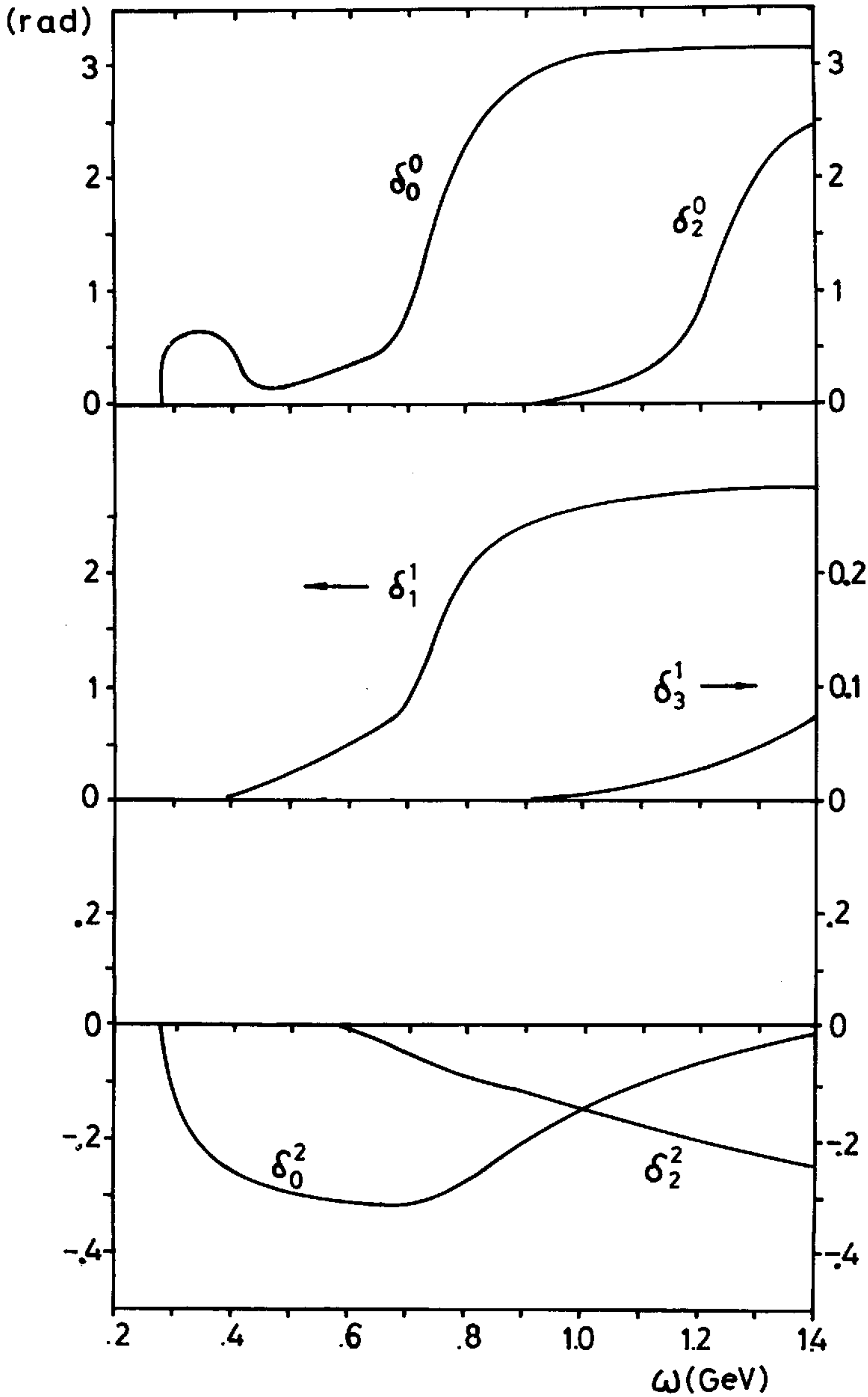
(rad)
3
2
1
0
δ_0^0
δ_2^0
δ_1^1
δ_3^1
0.2
0.1
.2
-.2
-.4
δ_0^2
δ_2^2
.2
.4
.6
.8
1.0
1.2
1.4
ω(GeV)

fig 2

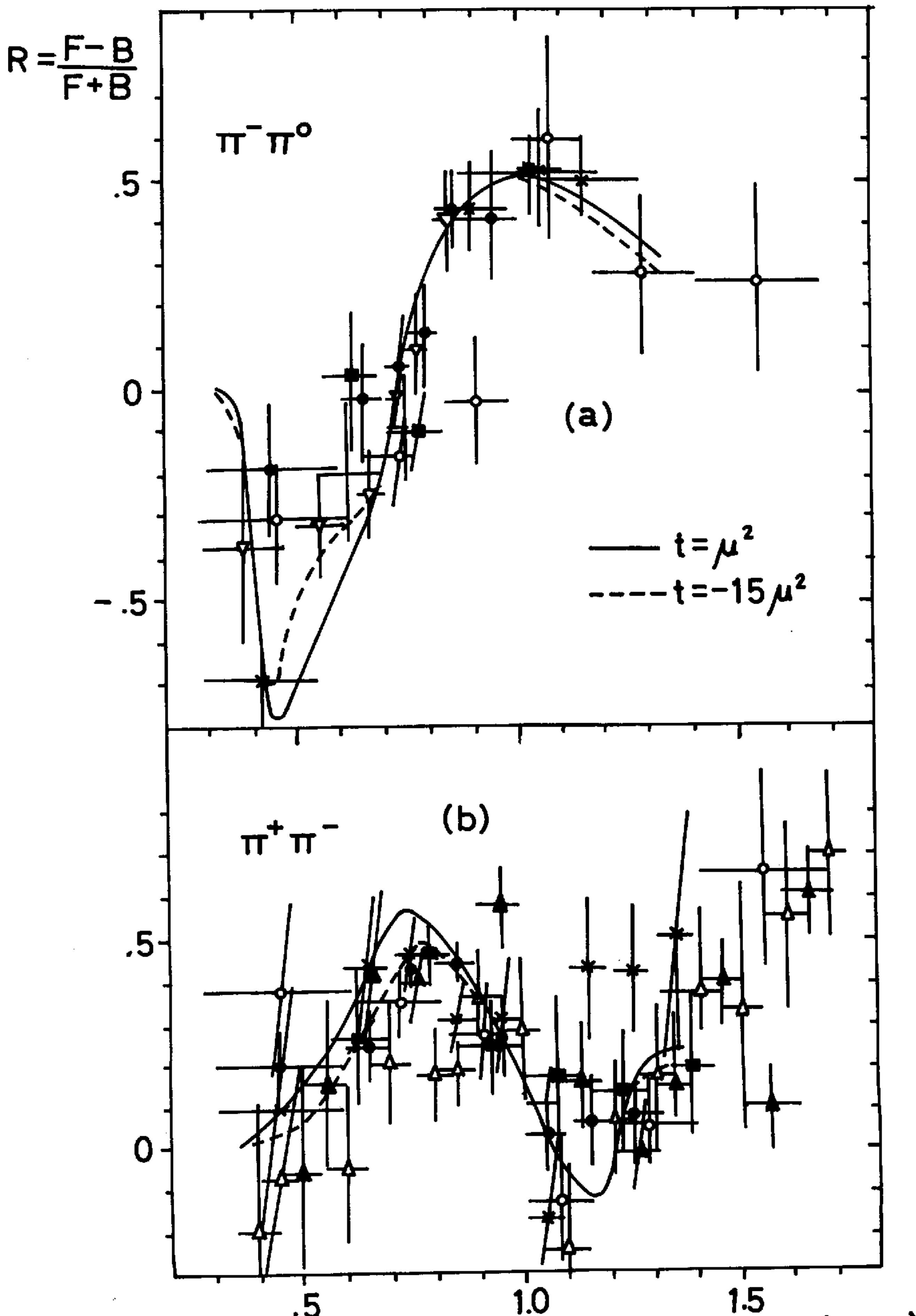

fig 3

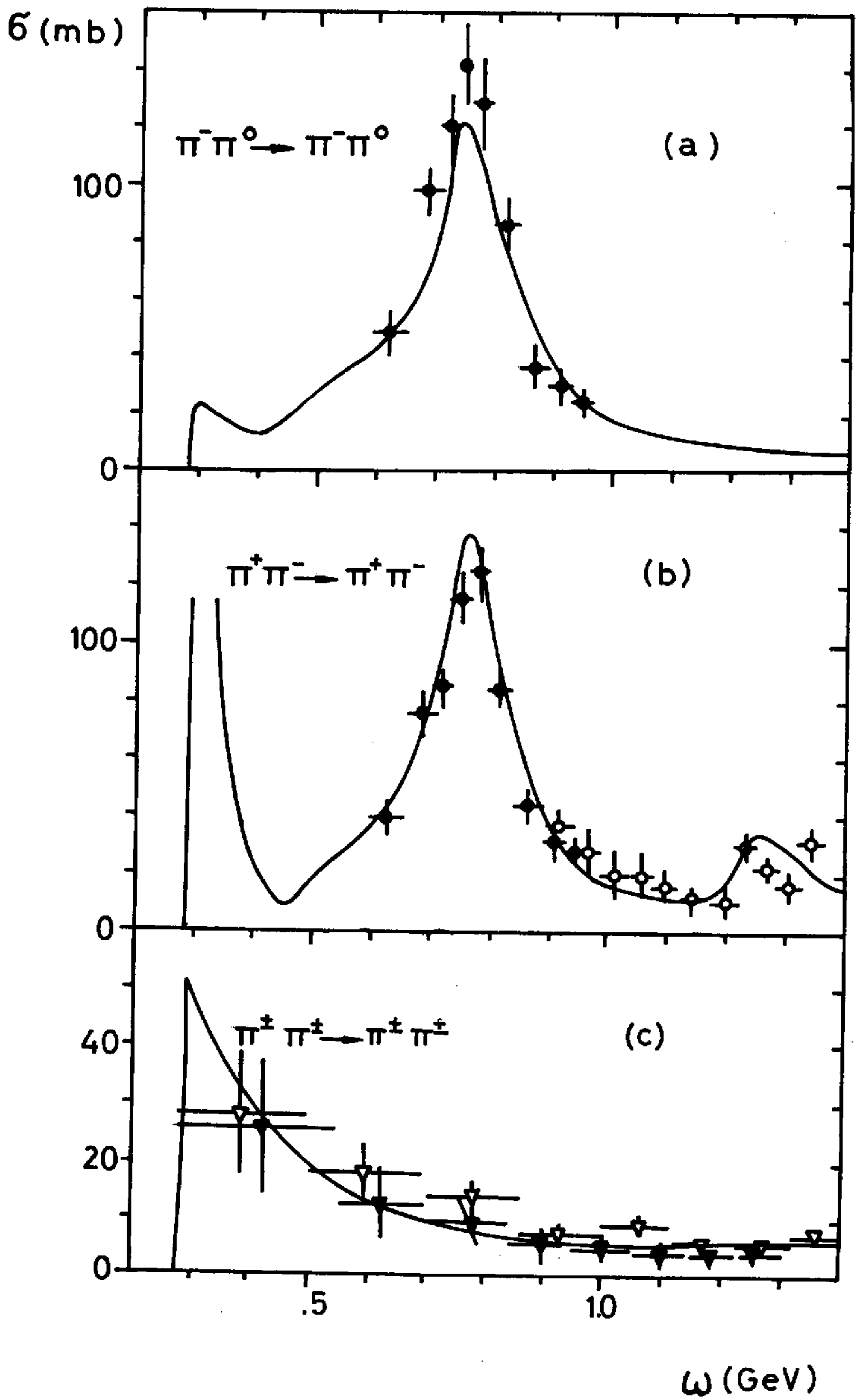

Fig 4

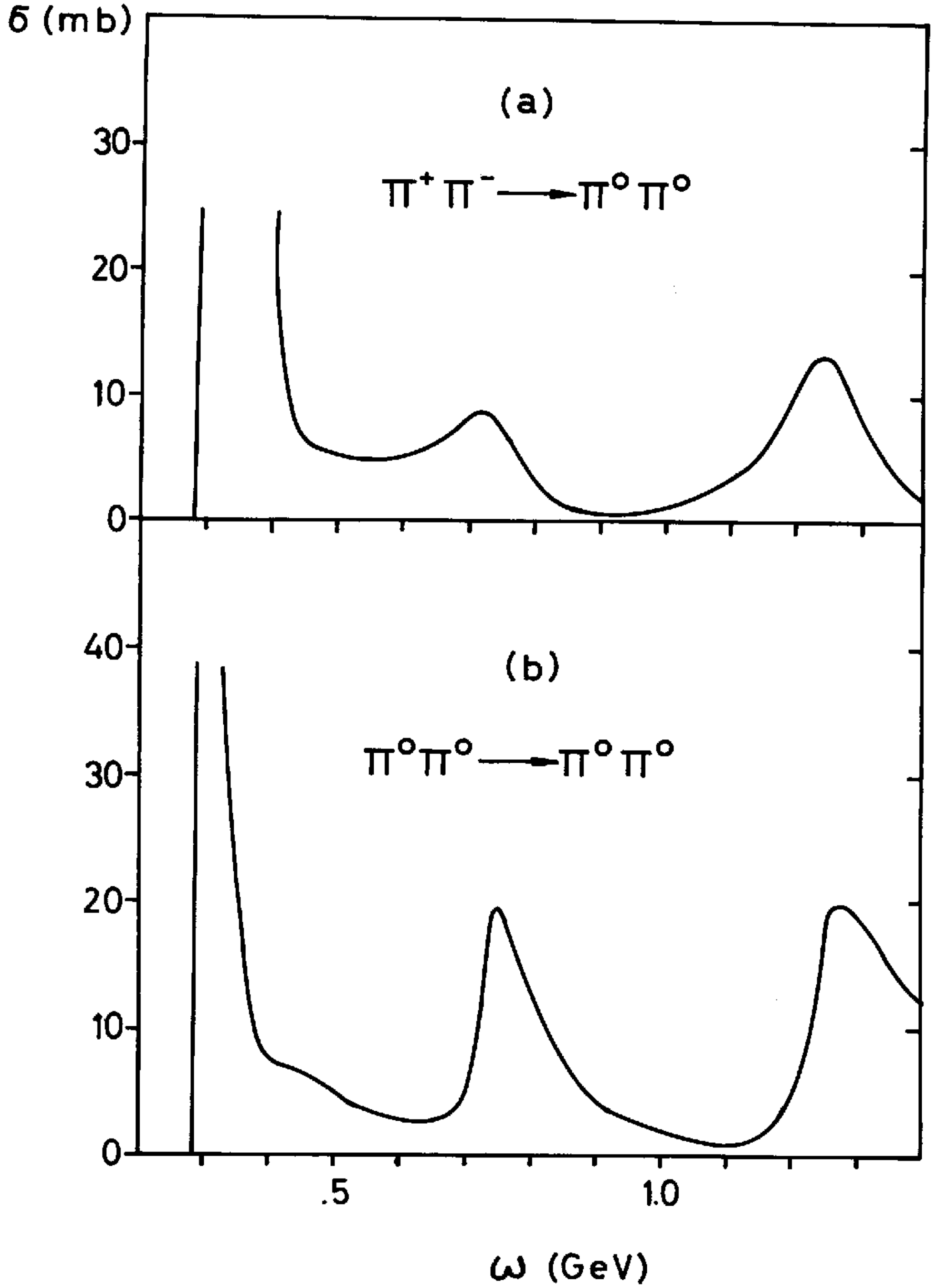

fig 5