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The Pionic Form Factor ~~and~~ of the First πN Resonance

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The Pionic Formfactor of the First πN Resonance

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Abstract:

Off-shell πN-scattering in the $^3/2$, $^3/2$-state is treated by the Omnès-Muskhelishvili-method. The left hand cut is represented by nucleon exchange and by approximate double spectral functions coming from iterations of nucleon exchange, and on the right hand cut some inelasticity is assumed. The off-shell amplitude is roughly determined by the behavior of the Born term taken at resonance energy, but is not insensitive to the absorption at high energies. If the formfactor is approximated by a single pole in momentum transfer, the relevant mass is 1.5 nucleon masses.

I. Introduction

The formfactors of stable particles and resonances with respect to exchanged pions are of great importance in peripheral models for processes like

$$p + p \to N^{*++} + n$$
$$\pi^+ + p \to N^{*++} + \rho^0$$

which one tries to interprete by the diagrams shown in Fig. 1. The absorptive version of the peripheral model [1] predicts too high absolute values of cross-section in some cases[2],[3], while the formfactor version proposed by Ferrari and Selleri[4] can reproduce these values fairly well in terms of one unknown function, which is fitted in one experiment[4],[5]. A non-relativistic model for off-shell scattering has been treated by Dürr and Pilkhun[6], who show how formfactors of resonances arise in potential theory. It is the aim of this paper to investigate the relativistic problem by dispersion methods, specializing strictly to the first pion-nucleon resonance N*. There we know from the success of the Chew-Low effective range formula[7] that the main attractive force responsible for the resonance is provided by nucleon exchange (NE) together with a smaller contribution from exchange of a scalar object in the t-channel[8]. Furthermore the phase and the inelasticity are known up to energies well beyond the resonance, which will help us to fix some parameters we need in setting up a model for the resonance.

Usually the electromagnetic formfactors of the nucleon are studied using dispersion relations in the momentum transfer variable. The analytic properties of the πN-scattering amplitude in the corresponding variable, the virtuality of the pion, have been investigated for special perturbation diagrams(9), but their application is at least inconvenient in practice because complex anomalous thresholds are present. Also the question of intermediate states is much more complicated than in the nucleon case. Therefore it is perhaps more powerful to use analytic properties in the subenergy variable*) $s = (p_2+q_2)^2$ with fixed virtuality Δ^2. From dispersion relations in s for fixed Δ^2 and $t = (p_1-p_2)^2$, Ferrari and Selleri(1o) derived an integral equation in s for the off-shell scattering amplitude $f_{1+}(s,\Delta^2)$ (i.e. the formfactor of the N*)

$$f_{1+}(s,\Delta^2) = f_{1+}^{B}(s,\Delta^2) + \frac{k_1 k_2}{\pi} \int_{(M+\mu)^2}^{\infty} \frac{\operatorname{Im} f_{1+}(s',\Delta^2)\, ds'}{k_1' k_2' (s'-s-i\varepsilon)} \tag{1}$$

where $f_{1+}^{B}(s,\Delta^2)$ is the Born term arising from NE, and $k_1 = |\vec{p}_1|$, $k_2 = |\vec{p}_2|$. From this equation and the off-shell unitarity condition (δ is the on-shell phase shift)

$$\operatorname{Im} f_{1+}(s,\Delta^2) = e^{-i\delta} \sin\delta\, f_{1+}(s,\Delta^2)$$

the authors deduced for $s \sim M^{*2}$

$$\frac{f_{1+}(s,\Delta^2)}{f_{1+}(s,\mu^2)} = \frac{k_1^*}{k_2^*} \left\{ \frac{1 - 3\,\frac{\Delta^2-\mu^2}{2M(M^*-M)}}{1 - \frac{\Delta^2-\mu^2}{2M(M^*-M)}} \right\} \tag{2}$$

Here M* means the mass of the N*, M = nucleon mass, μ = pion mass, and k_1^* (k_2^*) means $k_1(k_2)$ taken for $s=M^{*2}$.

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*) See Fig. 2 for kinematical notations. The metric is

$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1.$

A more careful treatment of the Born projection was carried through in Ref. (11) with the result that the bracket in (2) should be replaced by a factor decreasing less rapidly with Δ^2. Neither in Ref. (1o) nor in (11) the analytic solution of (1) was explicitly evaluated, but the authors guessed from it that $f_{1+}(s,\Delta^2)$ should be proportional to the Born term, taken at $s = M^{*2}$.

This approach was questioned by Jackson(12), who arrived at

$$\frac{f_{1+}(s,\Delta^2)}{f_{1+}(s,\mu^2)} = \frac{k_1^*}{k^*} \tag{3}$$

starting from the same integral equation and from the same approximation for $f^B_{1+}(s,\Delta^2)$ as in Ref. (1o), but his analysis contains an unjustified contour deformation and a diverging integral. One might clarify this discrepancy by a numerical solution of (1), but this is not very instructive as can be seen by considering (1) for $\Delta^2 = \mu^2$:

It is known(13) that a one-particle exchange force together with one-shell unitarity condition for partial waves with angular momentum $\ell \geq 1$ needs a cutoff, if the correct threshold behavior is enforced on the amplitude. If the same cutoff is not included in the off-shell integral equation, its solution differs for $\Delta^2 = \mu^2$ from the on-shell amplitude. Now the cutoff parameters may be functions of the virtuality of the pion, and to determine these, a physical interpretation of the cutoff must be found. This will be achieved by a model, in which double spectral functions are roughly included in computing the left hand cut in a partial wave dispersion relation. The motivation for this and the location of these extra singularities will be taken from analogy with potential theory.

Section II contains kinematical relations and the off-shell integral equation, which allows to include inelasticity. In Section III it is explained how the left hand cut of the integral equation can be approximated, while in Section IV the details of our model for the N* are discussed. In Section V the solution of the off-shell integral equation is investigated numerically and results for the formfactor are given. Section VI contains conclusions.

II Kinematics and integral equations

The momenta and the masses of the in- and outgoing particles are shown in Fig. 2. Δ^2 is the squared mass of the virtual pion. Since both nucleons are on-shell, the T-matrix element can be written as usual in the form

$$T(s,t,\Delta^2) = \bar{u}(P_2)\left\{A + \gamma\cdot\frac{q_1+q_2}{2}B\right\}u(P_1) \tag{4}$$

where A and B depend on Δ^2 and the variables*)

$$\begin{aligned} s &= (P_2+q_2)^2 \\ t &= (P_1-P_2)^2 \\ u &= (P_1-q_1)^2 \end{aligned} \tag{5}$$

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*) The variables are connected by $s+t+u = 2M^2+\mu^2+\Delta^2$

The partial wave amplitude for angular momentum j = 3/2 and positive parity is given by

$$f_{1+}(s,\Delta^2) = \frac{1}{2}\int_{-1}^{+1} dx \left\{ P_1(x) f_1(s,t) + P_2(x) f_2(s,t) \right\}$$

$$f_1(s,t) = \frac{\pi}{W} \sqrt{(E_1+M)(E_2+M)} \left\{ A + (W-M)B \right\} \tag{6}$$

$$f_2(s,t) = \frac{\pi}{W} \sqrt{(E_1-M)(E_2-M)} \left\{ -A + (W+M)B \right\}$$

with following notations:

$W^2 = s$,

$E_1 = (s + M^2 - \Delta^2)/2W$ = c.m.- energy of incoming nucleon,

$k_1 = \sqrt{E_1^2 - M^2}$ = c.m.- momentum of " " ,

$E_2 = (s + M^2 - \mu^2)/2W$ = c.m. - energy of outgoing " ,

$k_2 = \sqrt{E_2^2 - M^2}$ = c.m. - momentum of " " ,

$\varepsilon_2 = W - E_2$ = c.m. - energy of outgoing pion ,

$x = \cos\vartheta$, ϑ = c.m. - scattering angle of nucleons,

$u = M^2 + \mu^2 - 2(E_1\varepsilon_2 + k_1 k_2 x)$.

The normalisation is such that for $\Delta^2 = \mu^2$

$$f_{1+}(s,\mu^2) = \frac{e^{i\delta}\sin\delta}{k_2} \tag{7}$$

(δ = 3/2, 3/2 pion-nucleon phase shift).

If we assume A and B to satisfy a Mandelstam representation **) in s, t and u also for $\Delta^2 \leq \mu^2$, then $f_{1+}(s,\Delta^2)$ is an analytic

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**) The fourth order box graph has no anonalous threshold in s and t, if $\Delta^2 \leq \mu^2$ (14).

function in the cut s-plane with some kinematical singularities. If $\Delta^2 = \mu^2$, the amplitude $W^2 f_{1+}(W,\Delta^2)$ has no kinematical singularites in the W-plane, but for $\Delta^2 \neq \mu^2$ the square roots in Eq.(6) produce new short cuts which we neglect. Further singularities which will not be considered arise from the D-wave amplitude which is coupled with the P-wave through the reflection symmetry(15)

$$f_{\ell+}(W,\Delta^2) = -f_{\ell+1-}(-W,\Delta^2) \tag{8}$$

The threshold branch points at $k_1 = 0$ and $k_2 = 0$ are removed by considering the amplitude

$$g(W,\Delta^2) = \frac{f_{1+}(W,\Delta^2)}{k_1 k_2}$$

Now we can write down a dispersion relation for $g(W,\Delta^2)$ ($\int_L \Delta g$ means integral over the "left hand" cut with the discontinuity Δg):

$$g(W,\Delta^2) = \frac{1}{\pi}\int_L \frac{\Delta g(W',\Delta^2)\,dW'}{W'-W} + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{\operatorname{Im} g(W',\Delta^2)\,dW'}{W'-W-i\varepsilon} \tag{9}$$

To account in a rough manner for inelasticity, we split $\operatorname{Im} g(W,\Delta^2)$ into its elastic part

$$\Delta g_{el}(W,\Delta^2) = g(W,\Delta^2)\, e^{-i\delta^*(W)} \sin \delta^*(W) \tag{10}$$

and its inelastic part $\Delta g_{in}(W,\Delta^2)$, which we assume to be a given quantity. In Eq.(1o), $\delta^*(W)$ is the complex conjugate

of the complex scattering phase shift in the 3/2, 3/2 state.
The well known solution of (9) and (1o) is[16]

$$g(W,\Delta^2) = m(W,\Delta^2) + \frac{1}{\pi \mathcal{D}(W)} \int_{M+\mu}^{\infty} \frac{e^{i\delta^*(W')} \sin\delta^*(W')\mathcal{D}(W')m(W',\Delta^2)\,dW'}{W'-W-i\varepsilon} \qquad (11)$$

$$\mathcal{D}(W) = \exp\left\{-\frac{W}{\pi}\int_{M+\mu}^{\infty} \frac{dW'\,\delta^*(W')}{W'(W'-W-i\varepsilon)}\right\} \qquad (12)$$

$$m(W,\Delta^2) = \int_L \frac{\Delta g(W',\Delta^2)\,dW'}{W'-W} + \int_{M+2\mu}^{\infty} \frac{\Delta g_{in}(W',\Delta^2)\,dW'}{W'-W-i\varepsilon} \qquad (13)$$

It is reasonable to assume $g(W,\Delta^2) = O\left(\frac{1}{W^3}\right)$ also for $\Delta^2 \neq \mu^2$.
Then the existence and uniqueness of (11) is guaranteed if

$$m(W,\Delta^2) = O\left(\frac{1}{W}\right), \quad \mathrm{Re}\,\delta \underset{W\to\infty}{\longrightarrow} \mathrm{const} < \pi, \quad \mathrm{Im}\,\delta = -\frac{\log\eta}{2} \underset{W\to\infty}{\longrightarrow} \mathrm{const} > 0$$

(η is the absorption parameter), but the convergence of (11) can be also prooved if $\eta \sim \frac{1}{W}$ (note $e^{i\delta^*}\sin\delta^* \sim \frac{1}{2i\eta}$, if $\eta \to 0$).
For practical purposes this is not suitable because the integrand of (11) oscillates at infinity if $\eta \to 0$. Therefore we assume $\lim\limits_{W\to\infty} \eta > 0$.

It has been common practice to approximate the left hand integrand by the Born term. That this is an unjustified assumption will be discussed in the next section.

III. The left hand cut

The left hand integral (l.h.i.) in (13) contains the projection of the Born term which is primarily given by NE. We have

$$m_{NE}(W,\Delta^2) = \frac{g^2}{8\pi}\;\frac{1}{W k_1^2 k_2^2}\Big\{\sqrt{(E_1+M)(E_2+M)}\,(W-M)\,Q_1(z) + \\ + \sqrt{(E_1-M)(E_2-M)}\,(W+M)\,Q_2(z)\Big\}, \tag{14}$$

$$z = \frac{\mu^2 - 2\varepsilon_2 E_1}{2k_1 k_2},$$

$Q_\ell(z)$ = Legendre function of the second kind.

If we approximate the complete l.h.i. by $m_{NE}(W,\Delta^2)$, choose a reasonable phase shift $\delta(W)$ and evaluate (11) and (12) for $\Delta^2=\mu^2$, $g(W,\mu^2)$ differs drastically from $g_0(W) \equiv \frac{e^{i\delta}\sin\delta}{k_2^3}$ (examples are given in Section V).

This is necessary since if $g(W,\mu^2) = g_0(W)$, then $\Delta g(W,\mu^2) = k_2^3 |g(W,\mu^2)|^2 \rho(W)$ with $\rho(W) \geq 1$, and (11) would provide a solution of the integral equation

$$g(W) = m_{NE}(W) + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{|g(W')|^2 k_2'^3 \rho(W')\,dW'}{W'-W-i\varepsilon}, \\ m_{NE}(W) = m_{NE}(W,\mu^2), \tag{15}$$

which is known to have no solution[13]. One can avoid this difficulty by considering the amplitude

$$\tilde{g}(W) = (W^2 + W_c^2)\,g(W) \tag{16}$$

instead of $g(W)$, where W_c is an arbitrary constant. Then instead of Eq.(15) we have

$$\tilde{g}(W) = (W^2+W_c^2)m_{NE}(W) + \frac{1}{\pi}\int_{M+\mu}^{\infty} \frac{|\tilde{g}(W')|^2 k_2'^3 \rho(W')\, dW'}{(W'^2+W_c^2)(W'-W-i\varepsilon)} \qquad (17)$$

If we solve (17) by the N/D-method, insert the resulting phase shift into (11) and (12) together with $m(W) = (W^2+W_c^2)m_{NE}(W)$, the resulting $g(W)$ of course coincides with $g_0(W)$. By using the ansatz (16) we have introduced poles at $W=\pm iW_c$ with unspecified residues, and the variation of the position of these poles and of these residues as a function of Δ^2 is unknown, since the physical nature of these extra singularities is unclear. In the following we want to construct a model, which allows us to determine the position and the amount of left hand singularities other than those poles, which bring the solution (11) in agreement with $g_0(W)$. First we note, that the discrepancy between $g(W,\mu^2)$ and $g_0(W)$ would equally well be present in potential scattering, since apart from inelasticity, Eqs. (9)-(13) are common the nonrelativistic and relativistic problem. It is therefore natural to cure the trouble with the same methods in both cases, namely the inclusion of iterations of the potential rather than the modifications of the potential at short distance, (i.e. exchange of particles with higher masses). That means we have to include double spectral functions (d.s.f.) to some extent, where the boundary curves of the d.s.f. are determined by the iteration of the NE-diagram, the box graph shown in Fig. 3.

For this diagram, the d.s.f. ρ_A' and ρ_B' belonging to the functions A and B have been calculated by Mandelstam[17]:

$$\rho_A'(s,t) = \frac{\gamma g^4}{W^2 k_2^3} M(W^2 - M^2)\varepsilon_2\left(1 - \frac{2X}{1+Z}\right)K(X,X,Z)$$

$$\rho_B'(s,t) = \frac{\gamma g^4}{W^2 k_2^3}(W^2 - M^2)\left(\varepsilon_2 - \frac{\mu^2 W}{W^2 - M^2} + \frac{2X E_2}{1+Z}\right)K(X,X,Z) \qquad (18)$$

with $X = 1 + \frac{M^2 + 2\mu^2 - s}{2k_2^2}, \quad Z = 1 + \frac{t}{2k_2^2},$

$$K(x,y,z) = \begin{cases} 0 & \text{if } x > \sqrt{(y^2-1)(z^2-1)} + yz \\ -(x^2+y^2+z^2-1-2xyz)^{-1/2} & \text{if } x < \sqrt{(y^2-1)(z^2-1)} + yz, \end{cases}$$

and γ is a numerical factor.

The region where ρ_A' and ρ_B' are nonzero, is shown in Fig. 4. The quantity $K(X,X,Z)$ behaves for fixed s like $(t - t(s))^{-1/2}$ near the boundary curve C_1 ($t(s)$ is the boundary value of t on C_1) and for $t \to \infty$ ρ_A' and ρ_B' decrease like t^{-1}. Their contributions to $g(W,\mu^2)$ coming from high values of t are further suppressed by the fact that the projection into the P-wave introduces a factor $Q_1\left(1+\frac{t}{2k_2^2}\right)$ which goes like t^{-2} for $t \to \infty$. Therefore in order to calculate the l.h.i., it seems to be a good approximation to assume the d.s.f. to be concentrated in a narrow strip along the curve C_1. Now, if we represent the d.s.f. by a finite number of δ-functions in the form (forgetting the nucleon spin for a moment)

$$\rho(s,t) = \sum_{i=1}^{n} c_i\,\delta(s-s_i)\,\delta(t-t_i) \qquad (19)$$

where the points (s_i, t_i) lie on C_1, we can determine the constants c_i in the following way:

Denote the contribution of $\rho(s,t)$ to the l.h.i. by $m_\rho(W)$ and that of Δg_{in} to $m(W,\mu^2)$ by $m_{in}(W)$.
Then on the right hand cut

$$\mathrm{Im}\, g(W,\mu^2) = \mathrm{Im}\left\{ \frac{1}{\pi \mathcal{D}(W)} \int_{M+\mu}^{\infty} \frac{e^{i\delta^*(W')} \sin\delta^*(W') \mathcal{D}(W')}{W' - W - i\varepsilon} \times \right.$$

$$\times \left(m_{NE}(W') + m_\rho(W') + m_{in}(W') \right) dW' + \tag{20}$$

$$+ \Delta g_{in}(W,\mu^2) .$$

From (19) we should have approximately

$$\int_{\overline{W}_i}^{\overline{W}_{i+1}} \mathrm{Im}\, g(W',\mu^2)\, dW' \approx \frac{c_i}{\pi} Q_1\left(1 + \frac{t_i}{2 k_{2i}^2}\right) ,$$

$$\overline{W}_i^2 < s_i < \overline{W}_{i+1}^2 \tag{21}$$

Comparing (2o) and (21), we get n linear equations for n constants c_i.

The spin of the nucleon allows for 2n constants, since we have two d.s.f. ρ_A and ρ_B , and to determine these would perhaps be possible by treating more partial waves simultaneously.
This is too complicated and not adequate in view of the unprecise knowledge of the on-shell phase and absorption. In practice only two points (s,t) have been retained, and for one of them ρ_A/ρ_B has been taken from the perturbation theoretic expression (18), while for the other the ratio has been varied until $g(W,\mu^2) \approx g_0(W)$ in the resonance region.

The details will be given in the next section, which starts with a simple parametrisation of the phase shift.

IV. Model for on-shell scattering

In order to have a phase shift and an absorption parameter consistent with dispersion relations, we first describe the on-shell problem by a N/D calculation, which contains a cutoff. We do not regard this as a real solution of the on-shell problem, but only as a phenomenological description of the phase shift. We write

$$\frac{W^2+W_c^2}{M^2+W_c^2}\, g_0(W) = \frac{N(W)}{D(W)} \tag{22}$$

and prescribe the singularities of N(W) instead of calculating them from given forces. They are

a) the static pole at W = M, (23)

b) a square-root branch point at $W = \pm iW_s$ with a discontinuity of the form $\frac{\lambda_s}{\sqrt{W \pm iW_s}}$, $\lambda_s > 0$, (24)

c) a pair of poles on the imaginary axes at $W = \pm iW_p$ with free residue λ_p. (25)

Terms coming from exchange of nucleon isobars and the scalar meson(8) are neglected.

Thus we have (normalizing D(M) = 1)

$$N(W) = \frac{4}{3}\,\frac{f^2}{W-M} + 2\,\mathrm{Re}\left(\frac{\lambda_s}{\sqrt{W-iW_s}} + \frac{\lambda_p}{W-iW_p}\right), \tag{26}$$

$$f^2 = 0.088 .$$

The square root singularity (24) was introduced because $\mathrm{Im}\, D(W)$ should behave like $\sqrt{W}$ for $W \to \infty$, if the total phase of $g_o(W)$ is to approach $\pi/2$ at infinity+). This can be easily seen(18) from the dispersion relation for $D(W)$. Since λ_s must be positive (otherwise $N(W)$ had a zero, the phase of $g_o(W)$ would tend to $\frac{3\pi}{2}$, and a CDD-Pol would be present), the contribution of (24) acts like a short range attraction, which is compensated by the poles (25) in order to produce a resonance at the correct energy.

Inelasticity is taken into account by fixing the ratio $R(W) = \Delta g_{el}/\Delta g_{in}$ with $R(W) = 0$ for $W \leqq 1o{,}7\mu$ (this corresponds to a kinetic energy of 570 MeV in the laboratory). At infinity R is taken < 1 in order to get a definite Re δ . Three choices of $R(W)$ are plotted in Fig. 5a)-c), together with Reδ and η , which are obtained by integrating

$$D(W) = 1 - \frac{W-M}{\pi}\int_{M+\mu}^{\infty} \frac{k_2'^3\, N(W')(M^2+W_c^2)(1+R(W'))\,dW'}{(W'-M)(W'^2+W_c^2)(W'-W-i\varepsilon)} \tag{27}$$

Now we can evaluate (11) - (13), where we insert

$$m(W,\Delta^2) = m_{NE}(W,\Delta^2) + \int_{M+2\mu}^{\infty} \frac{\Delta g_{in}(W',\Delta^2)\,dW'}{W'-W-i\varepsilon}$$
$$+ \sum_{i=1,2} \frac{c_i}{\pi}\,\bigl(L_i(W) - L_i(W_i)\bigr)/(W-W_i) \tag{28}$$

+) Complete absorption is likely to occur at high energies, which requires an imaginary amplitude.

with $$L_i(W) = \frac{\sqrt{(E_1+M)(E_2+M)}}{2W k_1^2 k_2^2}(d_i + W + M)\,Q_1(z_i),$$

$$z_i = \frac{2E_1E_2 - 2M^2 + t_i}{2k_1k_2}$$

and $$\Delta g_{in}(W,\mu^2) = R\,k_2^3 \left|\frac{N(W)}{D(W)}\right|^2 .$$

The constants d_i determine the ratio $\rho_A(s_i,t_i)/\rho_B(s_i,t_i)$. From the projection formula (6), Eq.(28) should also contain terms with $Q_2(z_i)$, which can be omitted safely because of $t_i \geq 4M^2$. One point(s_i,t_i) will be located at the resonance position, while the second one cannot be fixed by simple arguments. Fortunately it turns out that the formfactor is rather insensitive to its position. The same is true for the interval, over which the imaginary part of the amplitude has to be integrated according to Eq.(21). If $i = 1$, we integrate from $W = M + \mu$ to $\overline{W}$, and for $i = 2$ from $\overline{W}$ to infinity.

The actual positions of the points and of the interval choosen are shown in Fig. 4. We take $d_2 = \rho_A'(s_2,t_2)/\rho_B'(s_2,t_2) \approx 0.7\,\mu$ and vary d_1, until $g(W,\mu^2)$ as defined by (11) and (28) agrees with $g_0(W)$ near the resonance. The constants C_i in (28) are determined "selfconsistently" as described in Section III. The values of d_1 obtained thus depend strongly on the behavior of η, and some values for specific phases are listed in Table 1. The perturbation theoretic value is $d_1 = 1.85$.
In Fig. 6 we compare the imaginary parts of $g_0(W)$, $g(W,\mu^2)$ and $g_{NE}(W,\mu^2)$, the latter of which is obtained from (11) by approximating the l.h.i. by $m_{NE}(W)$. The curves drawn corres-

pond to phase shift II of Fig. 5b). It should be noted, that $\mathrm{Im}\, g(W,\mu^2)$ does not vanish necessarily at threshold, since, in our treatment of inelastic effects, $\mathcal{D}(W)$ is not real there. The odd behavior of $g_{NE}(W)$ above the resonance has been obtained by several authors[19].

The agreement between $g_0(W)$ and $g(W,\mu^2)$ is not excellent, but it is hardly justified to try an improvement by taking more poles into account, since the unknown phase at high energies plays a too important role even at low energies. It will be shown in the next section, how different models for the phase affect the predictions for the formfactor.

V. Formfactor of the Resonance

After having specified the on-shell model, we can apply Eqs.(11) and (28) for $\Delta^2 \neq \mu^2$, thus calculating the formfactor defined by

$$F(\Delta^2) = \left| \frac{g(M^*,\Delta^2)}{g_0(M^*)} \right| \tag{29}$$

This is not the complete formfactor of the resonance, since $m_{NE}(W)$ has to be multiplied by the pionic formfactor[10] of the nucleon, $K(\Delta^2)$, which cannot be calculated presently. Hence $F(\Delta^2)$ describes, apart from the factor k_1/k_2, the ratio of the πNN^* - vertex to the πNN - vertex.

We have to discuss, how the parameters s_i, t_i, d_i and $\overline{W}$ vary if $\Delta^2 \neq \mu^2$. One finds, that the curve C_1 changes only slightly for $\Delta^2 \geqslant -200\mu^2$, and the corresponding change of t_i has no effect on the formfactor. We make the assumption that d_i and $\bar{W}$ remain constant independently of Δ^2, and again moderate changes do not alter $F(\Delta^2)$.

The behavior of $g_{im}(W,\Delta^2)$ for $\Delta^2 \neq \mu^2$ is completely unknown. In order to get an idea of its influence, we calculate the formfactors for two cases, namely

a) $\Delta g_{im}(W,\Delta^2) = \Delta g_{im}(W,\mu^2)$ for all W and Δ^2,

b) $\Delta g_{im}(W,\Delta^2) \equiv 0$ for $\Delta^2 \neq \mu^2$.

The results, summarised for $\Delta^2 = -100\,\mu^2$ in Table I, indicate that the theoretic uncertainty due to the unknown variation of the inelasticity is not very large. However, different on-shell absorptions bring changes in $F(\Delta^2)$, which are not negligible.

It may be interesting to ask how the formfactor changes if we modify the Born approximation by introducing a formfactor into the NE-term, i.e. by substituting

$$\frac{1}{u - M^2} \cdot \frac{1}{1 - \frac{u - M^2}{\lambda^2}} \tag{30}$$

for single u-pol. The resulting $F(\Delta^2)$ for $\lambda^2 = 30\mu^2$ is shown by a dashed line in Fig. 7, together with $F(\Delta^2)$ obtained for $\lambda^2 = \infty$, both corresponding to phase shift II. Obviously no drastic change occurs. In Fig. 7 also the old formfactor calculated by Ferrari and Selleri[10] is plotted (it is given by the bracket in Eq.(2)). The result of Jackson[12] is $F(\Delta^2) = 1$.

VI. Discussion

Although the projection of the NE-graph gives a bad approximation to the l.h.i. in Eq.(11), its value at the resonance energy largely influences the formfactor (see Table I). The reason for this can be found in Eqs. (2o) and (21), which couple the strength of the extra left hand singularities to the Born term. This is certainly adequate in potential theory, and that it is not unreasonable here is indicated by the photoproduction of the N*. There the situation is quite similar: For the reasonant magnetic dipol amplitude M_{33} one has the same integral equation as in πN-off-shell scattering and, up to a constant factor, nearly the same contribution from the NE-graph. The CGLN-formula[2o]

$$M_{33}(W) \approx \frac{m_{NE,\gamma}(M^*)}{m_{NE}\ (M^*)}\ g_o(W) \tag{31}$$

where $m_{NE,\gamma}(W)$ is the Born term for photoproduction coming from NE, agrees excellently with experiment[21], but cannot be obtained[19] as an exact solution of Eq.(9). If we again introduce the d.s.f. representing iterations of the potential, Eq.(31) becomes clear, since the potential generates (up to the constant factor) the same iterations. If we would produce agreement between $g(W,\mu^2)$ and $g_o(W)$ by adding a short range potential term to $m_{NE}(W)$, the appearance of the same potential in photoproduction would be a mere accident.

A different way to make the Omnès method work has been discussed in connection with photoproduction(19). It uses a real phase shift δ with the property $\delta(\infty) = \pi$, and this makes the solution of Eq.(9) nonunique or, expressed alternatively, allows for a CDD-pol.

The parameters of this pol may be adjusted to bring the solution of (9) in agreement with $g_\delta(W)$ near the resonance. Besides the fact, that such a phase would make the dynamical calculation of the N* impossible, the variation of the CDD-pol parameters with Δ^2 seems difficult to estimate.

Of course our results differ from those of Ferrari and Selleri(10) (see Fig. 7) due to the approximate technique used by these authors in calculating the NE-projection(11). In principle, their statement, that the formfactor is proportional to the Born term at resonance, is not in strong contradiction with the present calculations.

If we want to fit $F(\Delta^2)$ by a pol-formula, writing

$$F(\Delta^2) = \frac{1}{1 - \frac{\Delta^2 - \mu^2}{\Lambda^2}}$$

we obtain $\Lambda^2 \approx 2{,}2\, M^2 \pm 20\,\%$, where the error reflects our imcomplete knowledge of the phase shift. By looking at the diagram shown in Fig. 8a), which represents the coupling between π, N and N* in our model, one would expect $\Lambda^2 \approx 4\, M^2$ due to the normal cut in Δ^2, starting at $\Delta^2 = 4$ M.

Intermediate states with masses smaller than 4 M^2 are not completely neglected. If they can be represented by diagrams of the type shown in Fig. 8b), they contribute to the inelasticity in the W-channel and are contained in $g_{in}(W,\Delta^2)$. Diagrams like those of Fig. 8c) are expected to give rise to the formfactors K (Δ^2) of the nucleon. Perhaps the most dangerous approximation was made by neglecting the exchange of a low mass scalar-meson, whose contribution to the 3/2, 3/2-state has been estimated by Donnachie et al(8) to give an attractive force with a strength relative to the NE-term of 25 $^o/o$. Due to the small mass of the hypothetical σ-meson the corresponding Born term drops rapidly with decreasing Δ^2: Assuming $m_\sigma = 3\mu$, the projection of the Born term is reduced by a factor 3 for $\Delta^2 = -15\ \mu^2$, and if we could multiply it by the same formfactor $K(\Delta^2)$ as $m_{NE}(W,\Delta^2)$, then $F(\Delta^2)$ should be lowered by 16 $^o/o$ (and by 25 $^o/o$ at $\Delta^2=-1oo\mu^2$). This of course depends strongly on the mass of the σ-meson and on the high energy behavior of the exchange graph, and both are unknown at present.

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Phase	λ^2	i	d_1	$F(-100\ \mu^2)$	$F_{NE}(-100\ \mu^2)$	$R_{NE}(-100\ \mu^2)$
I	∞	1	6.0	0.55	0.65	0.429
		0	6.0	0.61	0.81	
II	∞	1	2.0	0.48	0.60	
		0	2.0	0.41	0.74	
	$30\mu^2$	1	9.0	0.44	0.48	0.395
III	∞	1	2.0	0.51	0.68	0.429
		0	2.0	0.55	0.87	

Table I. Variation of the formfactor $F(\Delta^2)$ for different phase shifts. λ^2 is defined in Eq.(30). If i=1(0), the inelastic part $g_{in}(W,\Delta^2)$ in Eq.(28) is taken equal to its on-shell value (equal to zero). The quantity d_1 is defined in Eq. (28). By $F_{NE}(\Delta^2)$ we mean $\left|\frac{g_{NE}(M^*,\Delta^2)}{g_{NE}(M^*,\mu^2)}\right|$, and $R_{NE}(\Delta^2)$ is the ratio of the Born terms at resonance energy, $R_{NE}(\Delta^2) = \frac{m_{NE}(M^*,\Delta^2)}{m_{NE}(M^*,\mu^2)}$.

Figure Captions:

Fig. 1 Examples for one-pion exchange graphs with production of N*.

Fig. 2 Kinematical notations for off-shell scattering.

Fig. 3 Fourth order diagram for πN-scattering.

Fig. 4 Boundary curve C_1 for the diagram of Fig. 3

Fig. 5a) Phase shift I The data marked by x are taken from Refs. (22) and (23).

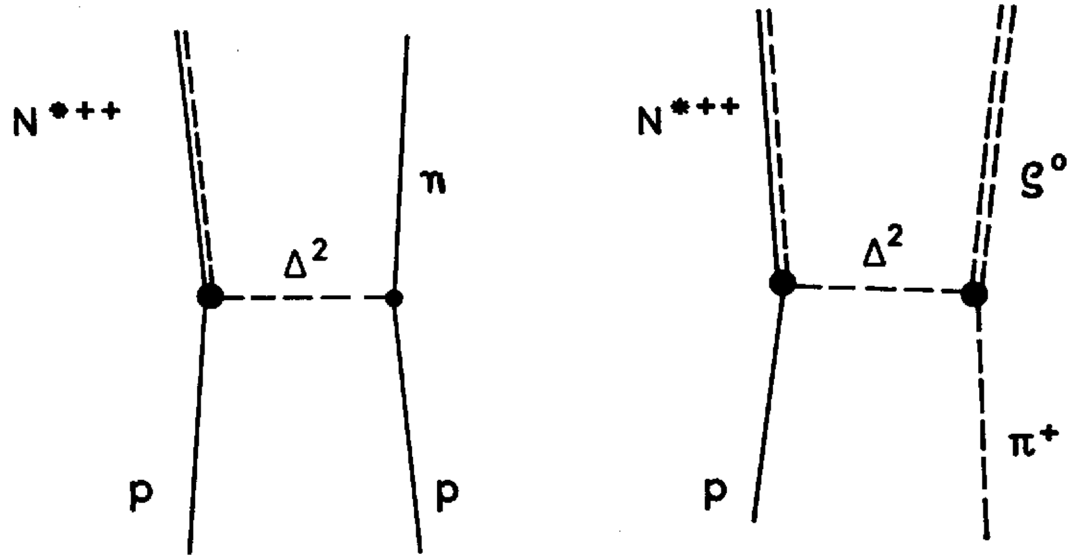
5b) Phase shift II

5c) Phase shift III

Fig. 6 Imaginary parts of $g(W,\mu^2)$, $g_0(W)$ and $g_{NE}(W,\mu^2)$, corresponding to phase shift II.

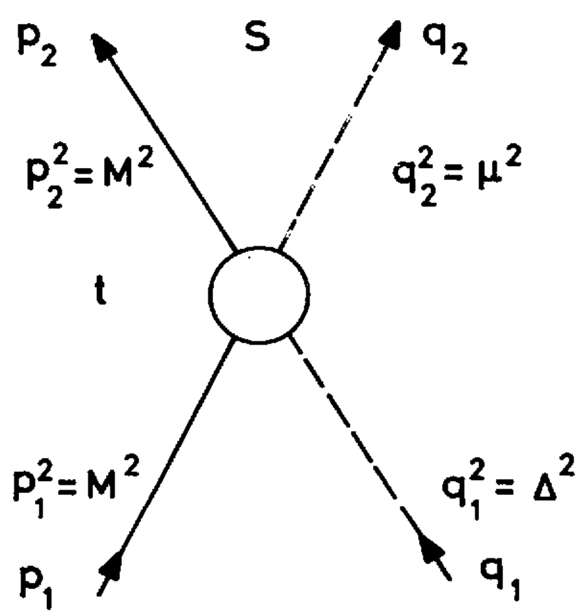
Fig. 7 Formfactor $F(\Delta^2)$, corresponding to phase shift II. λ^2 is defined in Eq. (30). The curve denoted by FS is the Ferrari-Selleri formfactor given in Eq. (2).

Fig. 8 Some diagrams with intermediate states of low mass in the Δ^2-channel.



N ***
n
Δ^2
p
p
N ***
ρ°
Δ^2
p
π⁺

Fig. 1



p_2
S
q_2
$p_2^2 = M^2$
$q_2^2 = \mu^2$
t
$p_1^2 = M^2$
$q_1^2 = \Delta^2$
p_1
q_1

Fig. 2

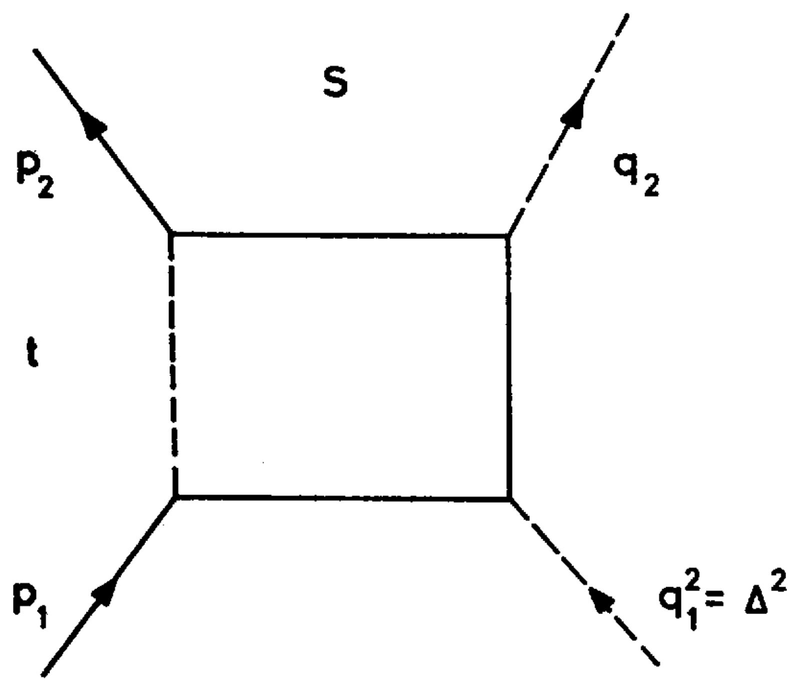


Fig. 3

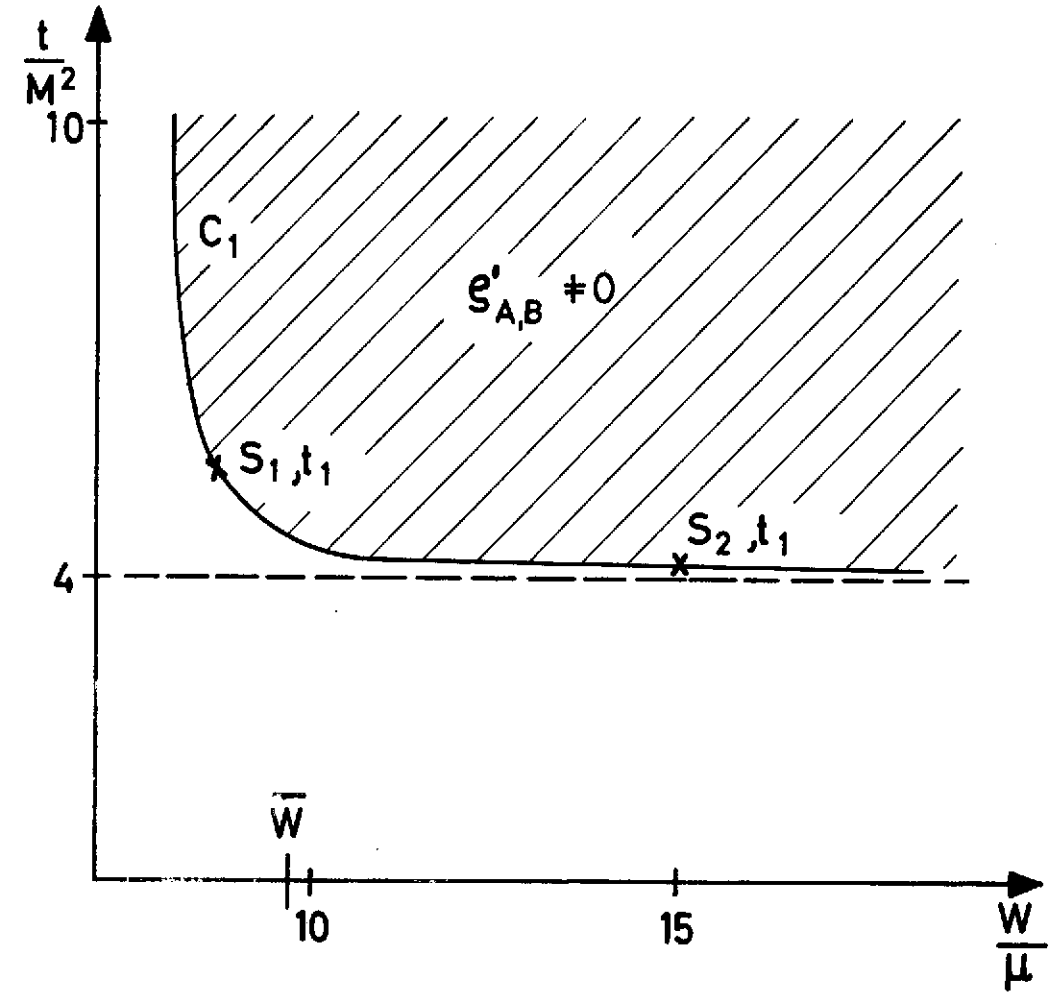
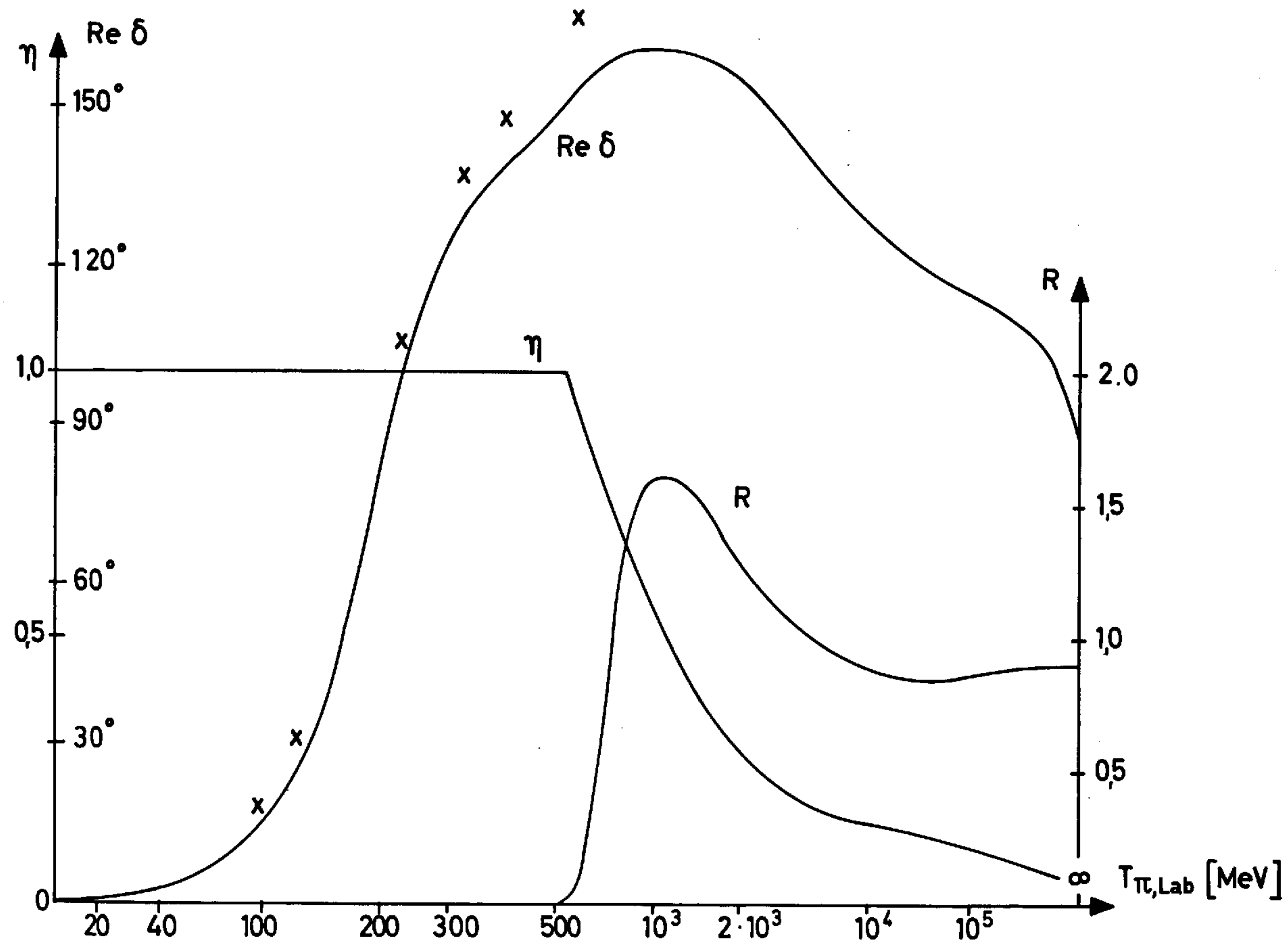
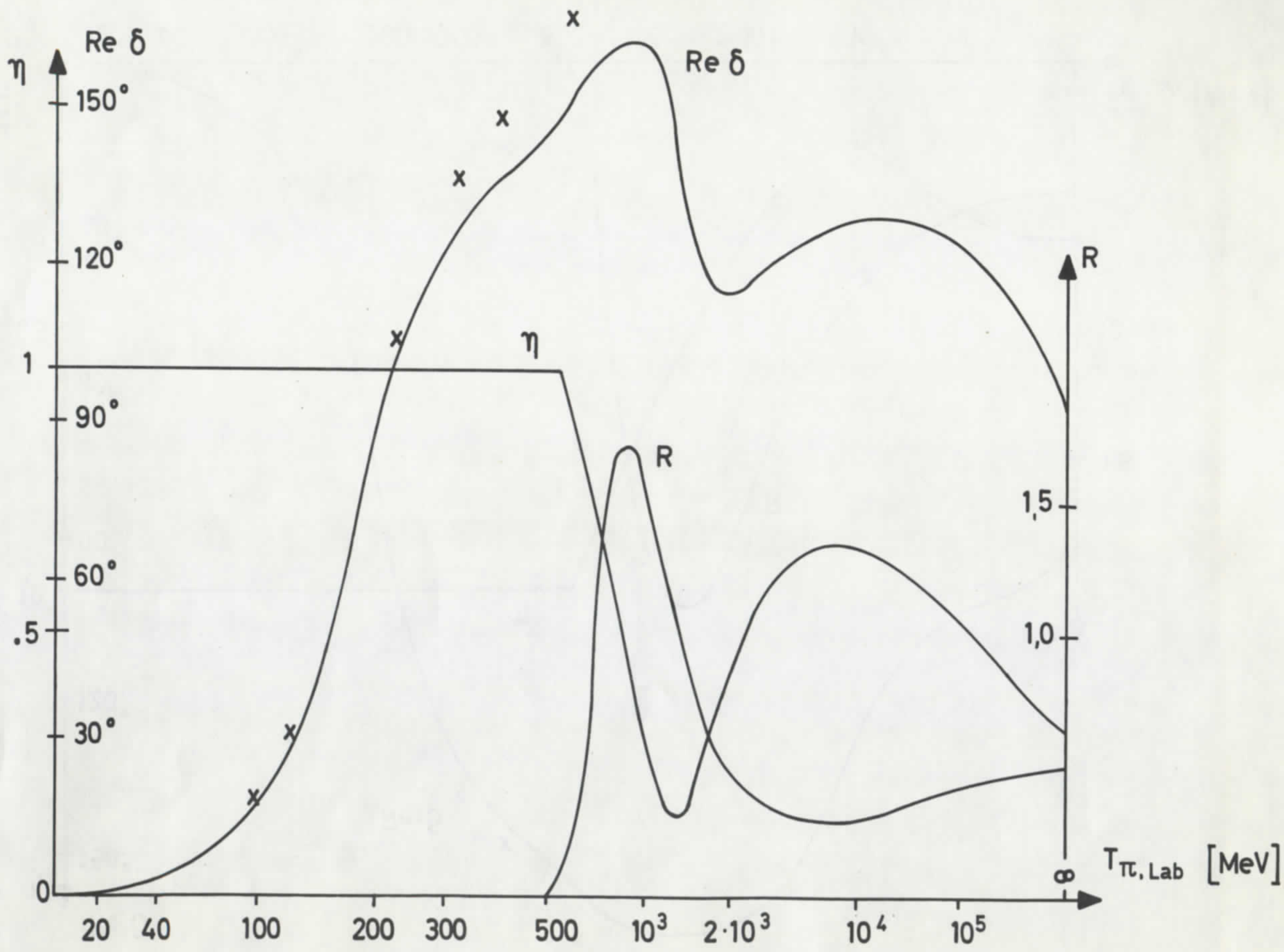

$\frac{t}{M^2}$
10
4
C_1
$\varrho'_{A,B} \neq 0$
S_1, t_1
S_2, t_1
$\overline{W}$
10
15
$\frac{W}{\mu}$

Fig. 4


η
Re δ
150°
120°
90°
60°
30°
1,0
0,5
0
20
40
100
200
300
500
10^3
$2\cdot10^3$
10^4
10^5
∞
$T_{\pi,Lab}$ [MeV]
R
2.0
1,5
1,0
0,5
x
x
x
x
x
x
x
Fig. 5a



η
Re δ
150°
120°
1
90°
60°
.5
30°
0
20
40
100
200
300
500
10^3
$2\cdot10^3$
10^4
10^5
∞
$T_{\pi,\,Lab}$ [MeV]
R
1,5
1,0
Re δ
η
R

Fig. 5b

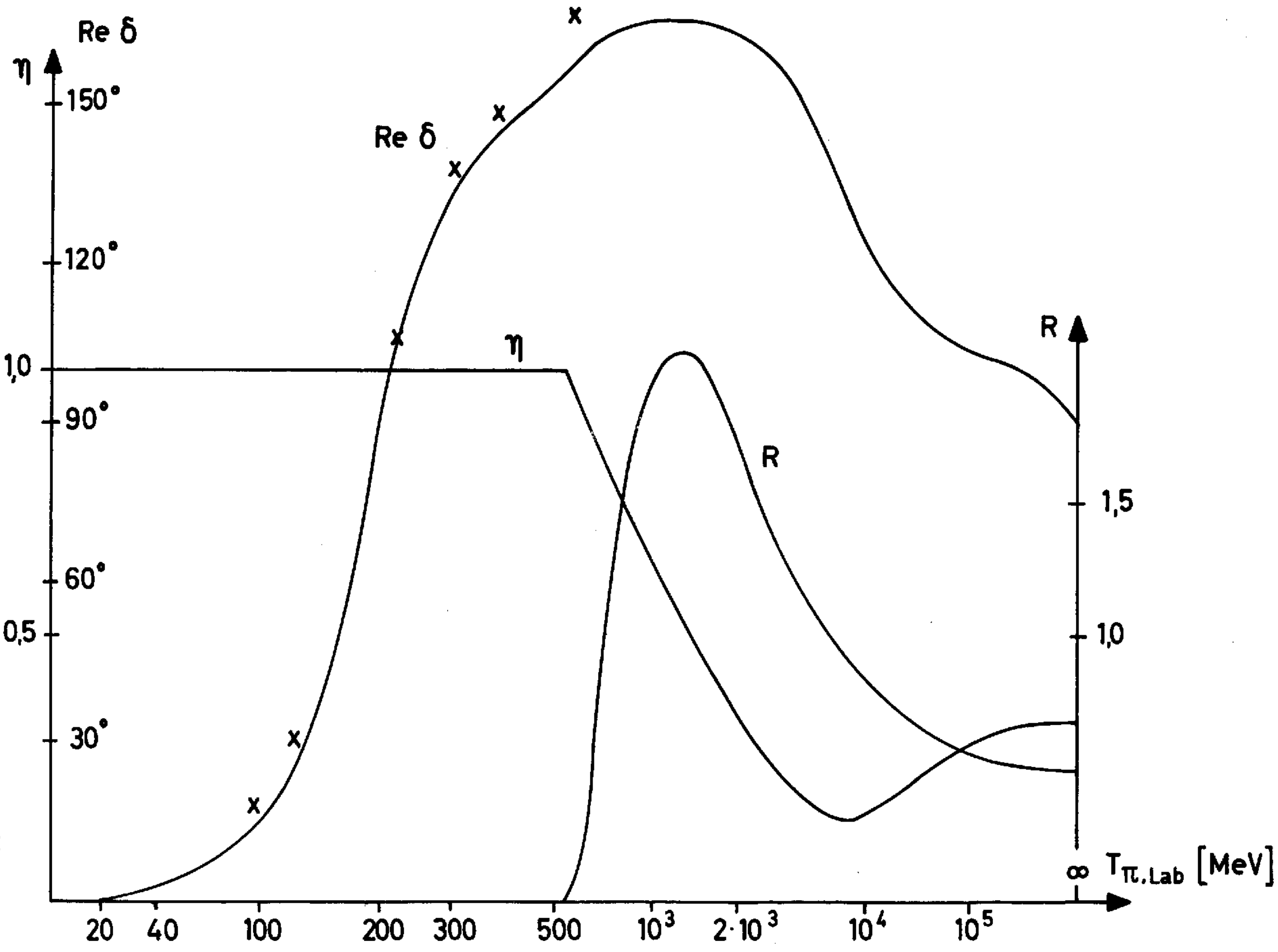

Re δ
η
150°
120°
1,0
90°
60°
0,5
30°
Re δ
η
R
R
1,5
1,0
∞ T_π,Lab [MeV]
20
40
100
200
300
500
10^3
2·10^3
10^4
10^5

Fig. 5c

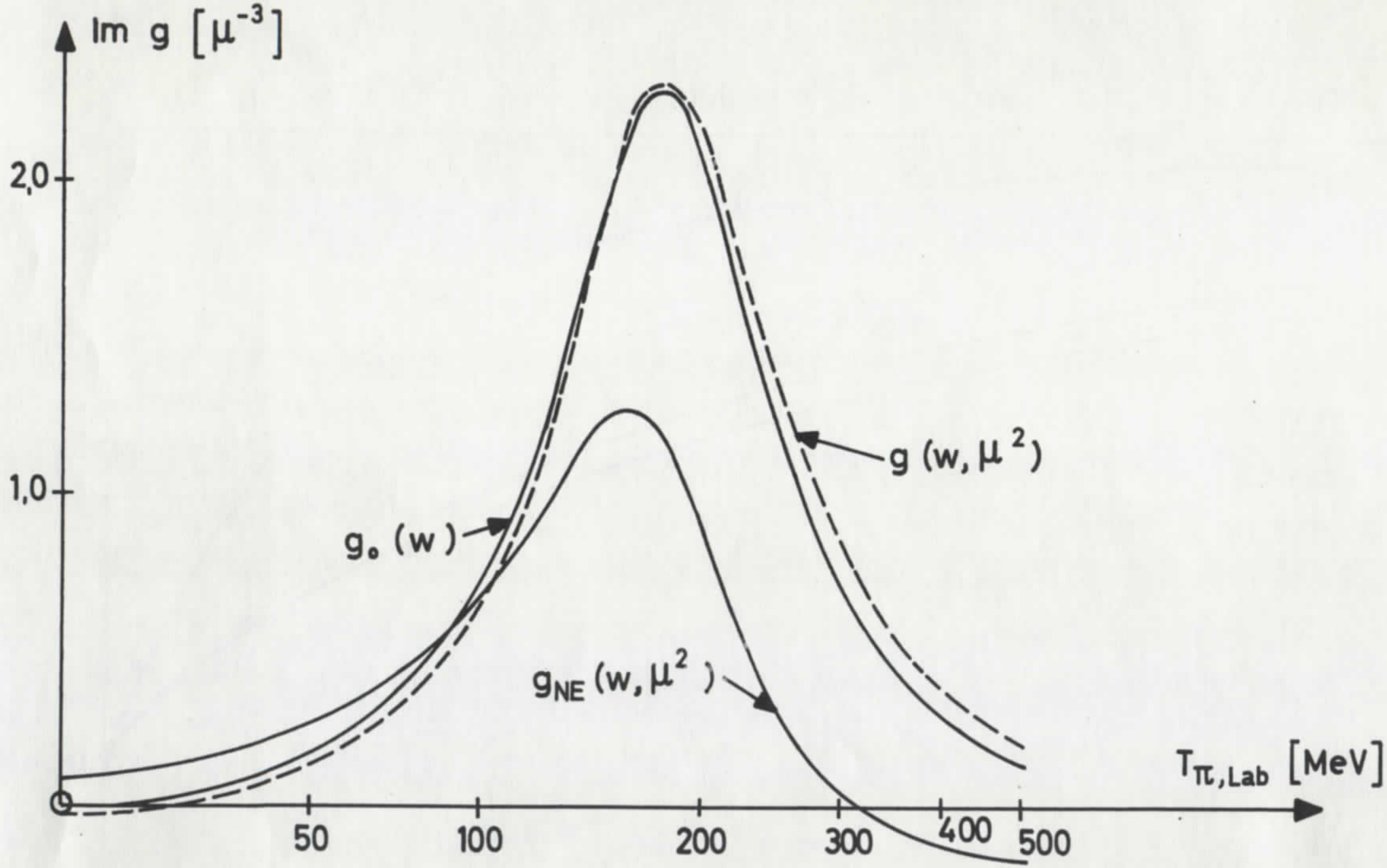


Fig. 6

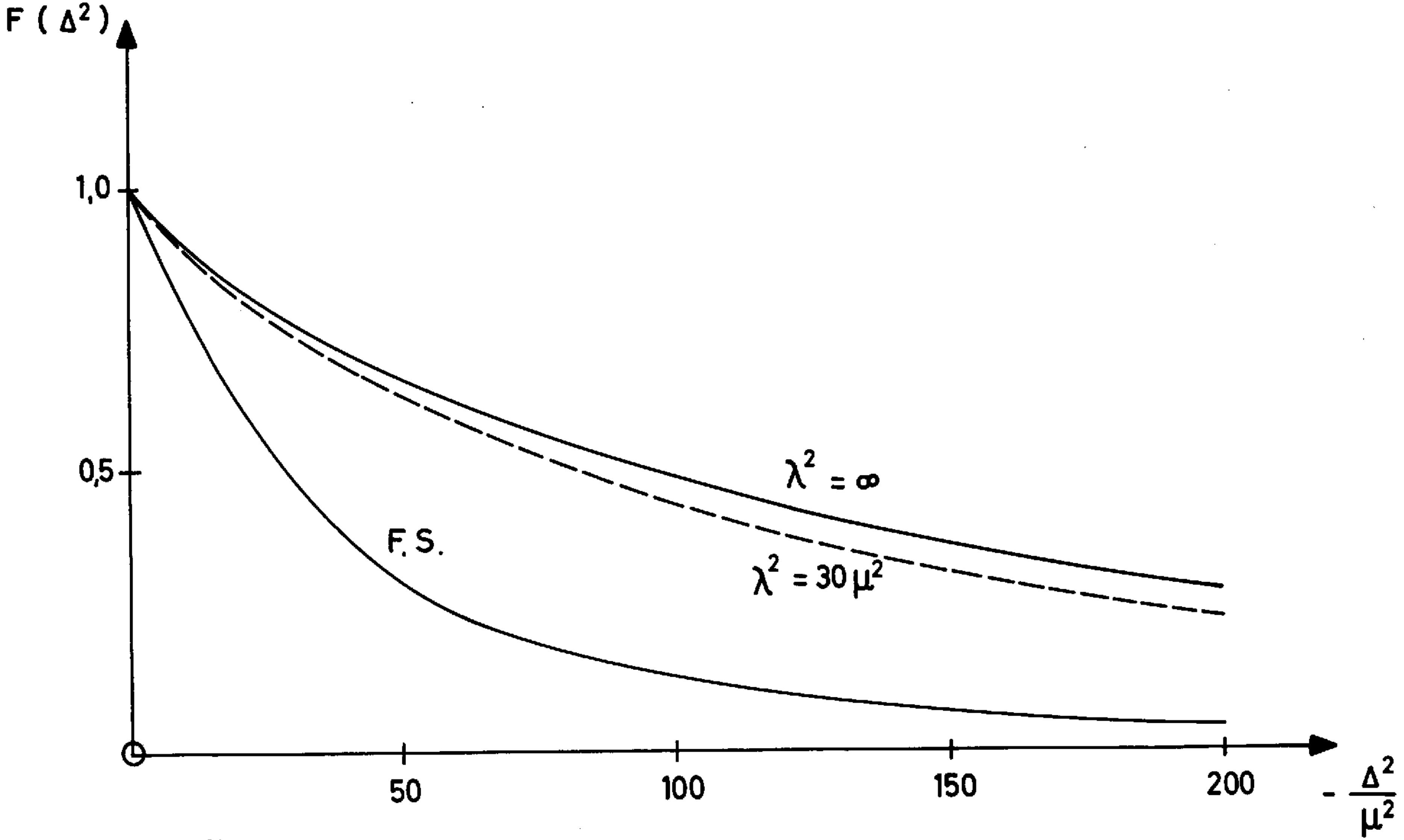


Fig. 7

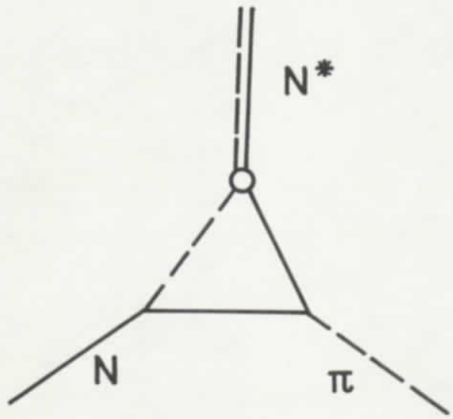


Fig. 8a

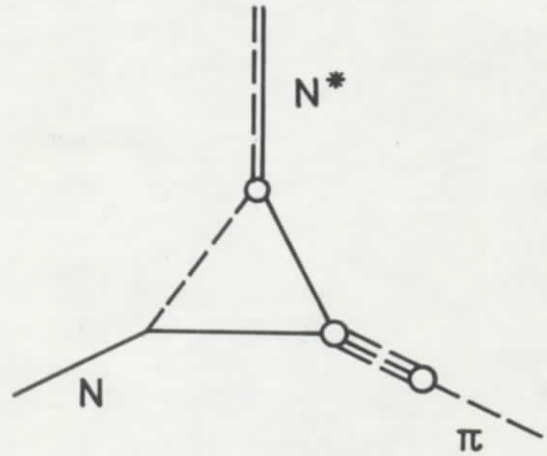


Fig. 8c

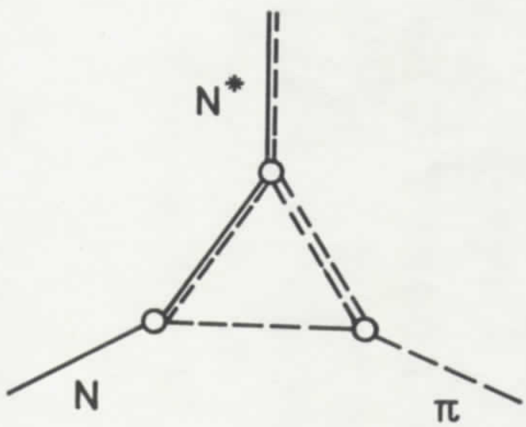


Fig. 8b