DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 66/7
März 1966
Theorie

What if anything is SU(6) Symmetry

by

P. Stichel
Physikalisches Staatsinstitut der Universität Hamburg
and Deutsches Elektronen-Synchrotron (DESY), Hamburg

Talk presented at the
V. Internationale Universitätswochen für Kernphysik,
Schladming, Styria, Austria,
February 24 – March 9, 1966

2 HAMBURG 52 · NOTKESTIEG 1

In this seminar I want to talk about the violation of some basic physical concepts caused by applying formal group theoretical methods to elementary particle physics. What I want to present is a critical review of the so called SU (6) - symmetry. I will neither discuss group theoretical details nor numbers and experiments. What I want to talk about are the basic ideas and the main defects of the different approaches within the SU (6) business. In particular, I will discuss the connections between the different approaches which are of a logical, quasi logical or nonlogical nature. As we have learned in Prof. Källen's seminar talk, such connections may be at best illustrated by a diagram containing boxes and lines.

So, let us start with the SU (6) box

SU (6)

Now we will look at this box by using a microscope in order to discover its fine-structure. What we will see is illustrated by the following diagram.

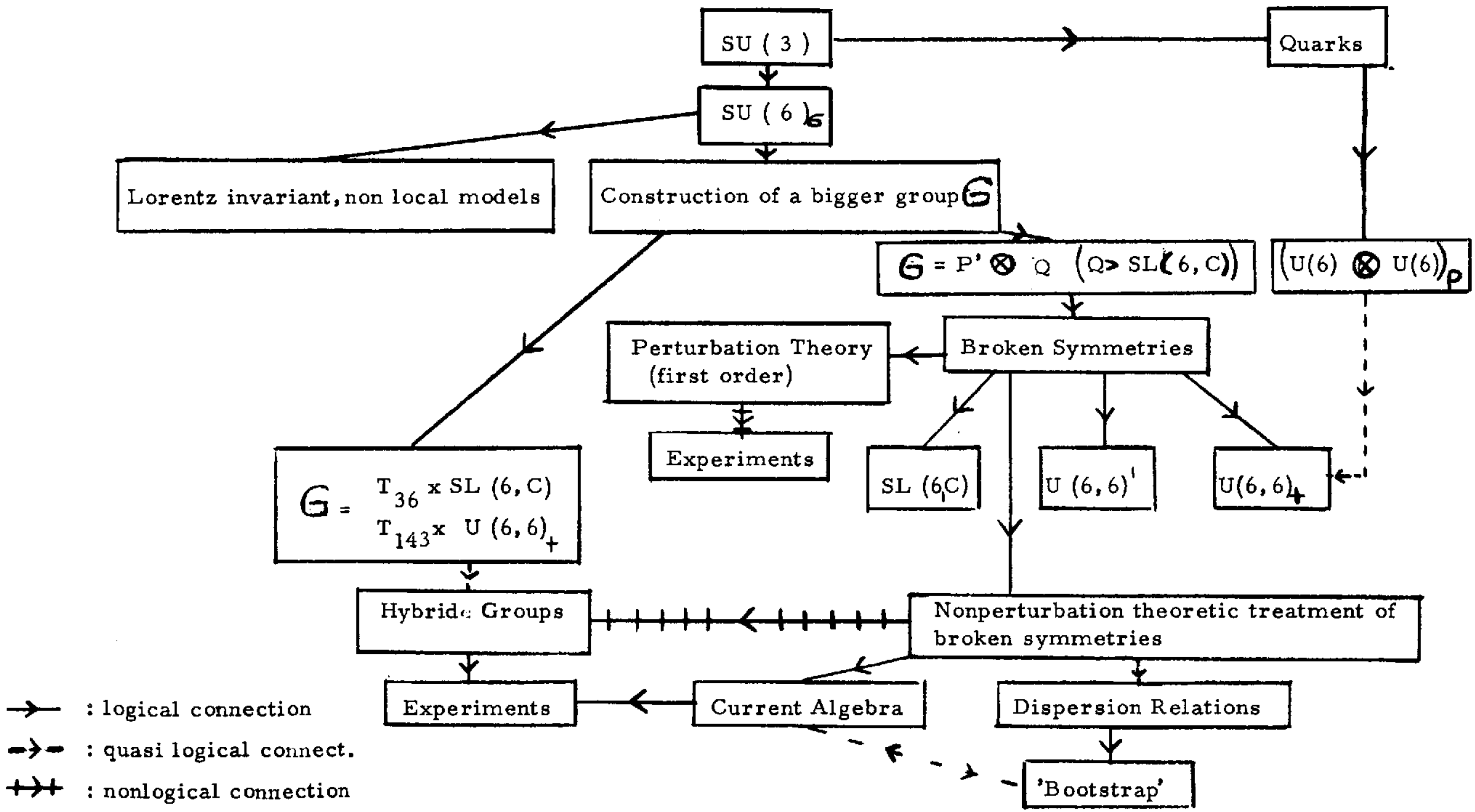
You see different boxes connected by unbroken, broken or forbidden lines expressing the logical, quasi logical or nonlogical nature of the considered connection.

Before going into the details let me make two statements which will be the main conclusions of this talk:

Statement 1:

It doesn't exist any 'SU (6)-symmetric' theory of strong interaction which fulfils all of the following requirements

a) Relativistic invariance
b) Locality (Crossing - symmetry),
c) Completeness of the physical states (Unitarity of the S-matrix)


SU (3)
Quarks
SU (6)$_S$
Lorentz invariant, non local models
Construction of a bigger group G
G = P' ⊗ Q (Q > SL(6, C))
(U(6) ⊗ U(6))$_P$
Perturbation Theory (first order)
Broken Symmetries
Experiments
G = T$_{36}$ x SL (6, C)
T$_{143}$ x U (6, 6)$_+$
SL (6,C)
U (6, 6)'
U(6, 6)$_+$
Hybrid Groups
Nonperturbation theoretic treatment of broken symmetries
Experiments
Current Algebra
Dispersion Relations
'Bootstrap'
: logical connection
: quasi logical connect.
: nonlogical connection

d) $S \neq 1$

Statement 2:

The approach of current algebras may be considered as a certain dynamical concept within the general framework of axiomatic quantum field theory.

Now I will discuss some of the details contained in the diagram:

I. Static SU (6) (SU $(6)_\sigma$):

In a rough description this is an extension of the nonrelativistic supermultiplet theory of Wigner (known in nuclear physics) by the substitution

$$SU(2)_I \rightarrow SU(3)$$

(This description is a rough one, because the SU (4) content of SU $(6)_\sigma$ is different from SU $(4)_{\text{Wigner}}$. This follows from the different physical interpretation of the fundamental representation of SU (4) in the two cases.)

Therefore SU $(6)_\sigma$ is a nonrelativistic theory of spin-independence of the interaction between elementary particles. I remind you on the meaning of spin-independence: Consider for example NN-forces, then spin independence means that you may have $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ terms, but no tensor-forces or spin-orbit couplings.

Defects of SU $(6)_\sigma$:

a) Nonrelativistic theory

b) Important p-wave couplings are forbidden:
$N \not\leftrightarrow N + \pi$, $N^* \not\leftrightarrow N + \pi$, ---
Consider the $NN\pi$-vertex. In the nonrelatavistic limit it is proportional to $X_2^+(\vec{\sigma} \cdot \vec{q})\, X_1$, i.e. it contains explicitly a spin-orbit coupling which fails to be SU $(2)_\sigma$ invariant.

II. Relativistic invariant, nonlocal models (Radicatti Gürsey[1], Schroer[2])

This is an immediate extension of the concept of spin-independence to relativistic theories. This is a possible marriage between SU (6) and the Poincaré group P without constructing a larger group $\mathcal{G}$. The SU (2) content of SU (6) in this case is Wigner's little group for time-like p_μ with $p^2 > 0$. At first this little group is only defined in the subspace of fixed p_μ. But what we want is an operator-representation within the whole Hilbert-space H. It turns out that such an operator representation may be constructed only within a subspace for fixed particle number H_n. Therefore we need a theory with particle number conservation. This is necessarily a nonlocal theory: Consider a local field operator $A(x)$ acting on the vacuum $A(x) | 0 \rangle$. In nontrivial theories this state always contains components with an arbitrary particle number!

In the following consideration I restrict myself to the SU (2) spin-content of SU (6).

H_1:

The one-particle subspace contains particles of mass M. Then the operators of our SU (2) are defined as follows:

$$S_i^{(1)} = \frac{1}{M} \quad a_i^\mu \, (\Lambda_p^{-1}) \, T_\mu^{(1)}$$

where P is an operator. If p is a c-number, the Λ_p is a special Lorentz-transformation which transforms p rotational free from the rest system into the moving system. It is identic with the 'boost' -transformation introduced by Prof. Matthews in his lecture.

$T_\mu^{(1)}$ is the representation of the Bargman-Wigner spin operator in H_1, i.e.

$$T_\mu^{(1)} \equiv \frac{1}{2i} \quad \varepsilon_{\mu\nu\rho\sigma} \quad M_{(1)}^{\nu\rho} \quad P_{(1)}^{\sigma}$$

Then we get explicitly for the $S_i^{(1)}$

$$S_i^{(1)} = \frac{1}{M} \left(T_i^{(1)} - \frac{T_o^{(1)} \; P_i^{(1)}}{M + P_o^{(1)}} \right)$$

The operators $M_{\mu\nu}^{(1)}$, $P_\sigma^{(1)}$, $S_i^{(1)}$ do not constitute a closed commutator algebra. For this consider the following counter example:

$$\left[M_{oj}^{(1)} , S_1^{(1)} \right] = i \; \frac{\delta_{j1} \; \vec{S}^{(1)} \cdot \vec{P}^{(1)} - S_j^{(1)} \; P_1^{(1)}}{M + P_o^{(1)}}$$

Therefore, we have not constructed a larger group G which contains P and SU (6).

H_n:

I will not give an explicit construction of the $\vec{S}_{(n)}$ for $n \geq 2$. It should only be mentioned that the $\vec{S}_{(n)}$ are in general functions of the representation of the $M_{\mu\nu}$ and P_σ in all n one-particle subspaces, i.e.

$$\vec{S}_{(n)} = \vec{S}_{(n)} \left(M_{\mu\nu}^{(1)}, --- M_{\mu\nu}^{(n)} , \quad P_\sigma^{(1)}, --- P_\sigma^{(n)} \right)$$

This means: In each H_n we have a different SU (6) group! The relativistic generalization of spin-independence of the interaction may then be formulated as usual

$$\left[H^{(n)} , \vec{S}^{(n)} \right] = 0$$

where $H^{(n)}$ is the Hamilton-operator acting in H_n.

It may be that such nonlocal models ~~become~~ have some physical interest if one looks for a relativistic generalization of the static quark model discussed in the lectures by Prof. Thirring.

III. Construction of a larger group G .

In the following we denote by $\bar{P}$ the covering group of the connected part of the Poincaré group. We have now to discuss two cases.

IIIa. $\bar{P} \subset G$.

From a naive point of view one would look for a group G with the following properties:

1. $\bar{P} \subset G$
2. T is an invariant subgroup of G (T = translations)
3. The action of G on T preserves the Minkowski metric (i.e., if $g \in G$, then $g P^{\mu} P_{\mu} g^{-1} = P^{\mu} P_{\mu}$).

This is an extension of requirement 2.

4. The little group in G/T for a time-like translation is SU (6) (this means: we can the states for a given p classify according to SU (6)).

Michel and other people have shown that such a group does not exist, because according to requirements 1.-3. the little group cannot be a simple one.

So let us drop requ. 3. (this seems to be the only possibility). Therefore the number of momenta has to be enlarged. According to Rühl the smallest group with the required properties is

$$G = T_{36} \times SL(6, C)$$

Because of T_{36} we are in a larger Hilbert space, and this is the crucial point, the physical Hilbert-space is not an invariant subspace with respect to G . Therefore S, i.e. the operator which transforms the physical outgoing into the physical ingoing states, is not an unitary operator.

The restriction to the so called hybrid groups for collinear or coplanar processes does not prevent the violation of unitarity. This has been explicitly demonstrated for q q and q $\bar{q}$ scattering by Alles and Amati. It turns out that unitarity and crossing symmetry only allow the trivial S-matrix

- 7 -

in this case.

III b. Group extension.

$\bar{P} = G/Q$

Q = internal symmetry group which is an invariant subgroup of G .

For central extensions it has been shown (Michel and other people) that

$$G = \bar{P} \otimes Q$$

If $Q \supset SU(6)$ then $Q \supset SL(6,c)$

This $\bar{P}$ must not be the physical Poincaré group, because group theory only tells us that this $\bar{P}$ is isomorphic to the physical $\bar{P}$, so let us write $G = \bar{P}' \otimes Q$.

To interpret this G as an relativistic generalization of the concept of spin independence, one may argue according to Michel as follows:

Suppose it exists an unitary representation of G in the physical Hilbert space with the following prescription:

Consider any many particle state in momentum space

$|p_1 --- p_n, \eta_1 --- \eta_n\rangle$ where the η_i are the internal degrees of freedom (spin, unitary spin), whereby the $\bar{P}'$ only acts on the momenta p_i and Q acts only on the η_i.

Now what is the physical Poincaré group $\bar{P}$: $\bar{P}$ consists of $\bar{P}'$ and the SL (2, c) content of SL (6, c).

IV. Broken symmetries.

It turns out, that the mentioned prescription given for G is in contradiction to any particle interpretation of field theory. This means: G has no unitary representation in the physical Hilbertspace.

To show this let us first consider old fashioned Langrangian field theory:

$$L = L_o + L_{int}$$

If the theory contains spin 1/2 particles, then L_o has the form

$$L_o = \bar{\Psi} (i\not{\partial} + M) \Psi + \cdots$$

This L_o cannot be invariant with respect to $\tilde{P}'$ because of the presence of the γ_μ, which is not transformed by $\tilde{P}'$. But the L_{int} may be formally invariant with respect to G .

Let us consider some examples within the Quark-model (restricting ourselves to four-fermion interactions without derivatives):

$$Q = SL(6,C) \quad : \quad L_{int} = g_1 \; (\bar{\Psi}\Psi)^2 + g_2 \; (\bar{\Psi}\gamma_5\Psi)$$
$$Q = U(6,6)_+ \quad : \quad L_{int} = g \; (\bar{\Psi}\Psi)^2$$
$$Q = U(6,6)' \quad : \quad L_{int} = g' \; (\bar{\Psi}\gamma_5\Psi)^2$$

It turns out, that this formal invariance of the interaction Langrangian is only a game with the insertion of certain functions depending on γ-matrices and Gell-Mann's λ_i. But this game has nothing to do with a symmetry, about which we speak only if there exists an unitary representation of the underlying symmetry group in the physical Hilbertspace.

This game with γ-matrices and λ_i leads to relations between certain unrenormalized coupling constants. Some theoretists like it to compare the predictions for the ratios of these unrenormalized coupling constants with the ratios of renormalized coupling constants as measured by experimental physicists. In some cases one gets a rather good agreement! Now we know that in general first order perturbation theory does not work for strong interaction physics. This means, if we get agreement of the mentioned game with experiments we have a situation which is by no means understood. It may be that it is possible to understand such partial successes by a careful study of broken symmetries within the framework of current algebras. I want to make one further remark to this game: It turns out, that the calculation of effective vertices in first order perturbation theory as mentioned violates the original idea of spin-independence, because of the spin-orbit coupling terms contained in the free field (Wigner-Bargman) equations.

V. Current algebra.

We have seen, that the Lagrangian formulation of broken symmetries may fulfil all our requirements with the exception of the symmetry requirement itself. But nevertheless we have certain relations between unrenormalized coupling constants.

Therefore the question arises: Is there any nonperturbation theoretic formulation of broken symmetries, which contains the mentioned ratios between unrenormalized coupling constants in a certain approximation. Indeed, such a scheme exists, namely the approach of equal time current commutators (for the details I remind you of the lectures given by Prof. Moffat). It turns out, that one obtains the same ratios between coupling constants as in first order perturbation theory, if one restricts oneself to the one-particle contributions within the dispersion theoretical treatment of current commutation relations according to Fubini , Furlan and Rossetti. The many-particle contributions,if they are physical interpretable, are a numerical measure of the renormalization effects (as an example I refer to the Adler-Weisberger relation). In the cases, where the many-particle contributions are not physical interpretable two different approximation schemes may be discussed:

1. The approximation of the two (resp. three) - particle contributions by the relevant resonances +).

+) I am grateful to the audience for mentioning this possibility.

2. The extension of the dispersion-theoretic one-particle contribution to a local one-particle contribution, whereby the latter contains the full contribution of the one-particle intermediate state in the commutator. In such an approach the full vertex-structure of the one-particle matrix elements of currents and not only the coupling constants are taken into account [++].

Finally, I only want to mention, that all problems connected with the dependence of certain results on current commutators on the frame of reference may be solved by considering only local approximations resp. local decompositions of matrix elements of current commutators [3].

VI. Miscellaneous remarks.

There are some boxes at the diagram I have not yet discussed.

Dispersion relations: In this context the usual N/D job is meant, whereby as an input certain one-particle exchange diagrams are taken as driving forces. The ratios of coupling constants at the vertices of the OPE - diagram may be taken from the first order perturbation results of broken symmetries.

'Bootstrap': If one does not like perturbation theory for the mentioned coupling constants, one may try to determine them, if possible, self-consistently. So, the bootstrap we are speaking about in this context is the bootstrap in a certain technical sense and not the general bootstrap-philosophy (which seems to be empty from the point of view of general local field theory!)

++) Some work along this line is in progress.

References:

1. Proceedings of the seminar on high-energy physics and elementary particle physics, Trieste, 3 May - 30 June 1965, and all the literature quoted there.

2. Seminar on high-energy physics and elementary particle physics, Trieste, 3 May - 30 June 1965, paper SMR 2/72 (presented by B.Schroer).

3. B.Schroer and P.Stichel: 'Current commutation relations in the framework of general quantum field theory' (Hamburg-preprint).