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On the Damping of the Head-Tail Instability by Octupoles

by

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The Head-Tail mechanism [1)2)3)4)] is able to produce beaminstabilities by shortrange interaction of particles within a single bunch. Several authors [2)3)] gave the frequencyshift Re $\Delta\omega$ and the rate of rise Im $\Delta\omega$ for the different possible oscillation modes assuming the following simple bunch model: all particles within one bunch have the same amplitude of synchrotron oscillation A and the wake function S being responsible for driving the tail, is constant all over the bunch and drops to zero before the next bunch passes by.

$$\Delta\omega_o = - \frac{N \cdot S}{4\omega_o} + i \frac{2NS\xi A}{\pi^2\alpha}$$

$$\Delta\omega_\mu = - i \frac{2NS\xi A}{\pi^2\alpha(4\mu^2-1)}$$

with

- N : number of particles within one bunch
- S : wake coupling strength
- ω_o: unperturbed betatron (angular)frequency
- A : Amplitude of the synchrotron phase oscillations
- α : momentum compaction
- ξ : chromaticity

The instability is damped by radiation damping and Landaudamping. In the following calculations the effect of radiation damping is neglected. The Landaudamping is assumed to be proportional to the octupole strength in the machine.

A variation of chromaticity is especially qualified for testing this theory, because the results of a variation of the other free variable, the bunch length, is strongly dependent on the assumption of a constant wakefunction [5)]. Fig. 1 shows measurements of the threshold currents I_{th} as a function of the provided octupol strength for different values of chromaticity ξ [5)]: The threshold is always proportional to the octupolcurrent; the slopes on the lefthand side of the minimum are constant for more than one order of magnitude in ξ and go only to infinity for extrem small chromaticities ($\xi < 0.05$).

At the right hand side of the minimum the slopes go continously to infinity for $\xi \to o$.

To explain the different slopes on the right and left side of the minimum the Landau damping has been calculated always assuming a ν-spectrum of the form [2)6)]

$$f(\nu) \sim e^{-\frac{\nu-\nu_o}{2\nu_o \langle a^2 \rangle D}}$$

with

ν_o = unperturbed betatron frequency

$\langle a^2 \rangle$ = average square value of the betatron oscillation amplitude

D = const. $\cdot I_{octupole}$

In this case the behaviour of the threshold of the o-mode-oscillation is described by the following two equations:[2)]

$$-\frac{U}{V}\pi = \pm \text{Ei}(y)$$

$$\frac{4\nu_o^2 \langle a^2 \rangle D}{N} \frac{U}{U^2+V^2} = e^{-y} \text{Ei}(y)$$

with

$U \sim \text{Re}\, \Delta\omega$

$V \sim \text{Im}\, \Delta\omega \sim \xi$

$$y = \frac{\bar{\nu} - \nu_o}{2\nu_o \langle a^2 \rangle D}$$

$\bar{\nu}$ = betatron frequency of the center of charge of the bunch

The + sign applies for the case of equal signs of U and D. It follows from those equations, that the threshold goes continously to infinity for $\xi \to o$ in the case of equal signs of D and U, but that the thresholds go to zero with decreasing chromaticity in the case of different signs of D and U. This behaviour is in disagreement with the measurements (Fig.2).

The reason for it is the strongly simplified form of the ν-spectrum, especially the jump in density at $\nu = \nu_o$. A more realistic spectrum has to take into account, that there are no jumps in density because of the always present $\Delta\nu$-spread due to the finite energy spread and that the frequency of the maximum of density changes with octupole strength.

A simple spectrum having those features is the following:

$$f(\nu) \sim \frac{\nu-\nu_o}{2\nu_o <a^2> D} \cdot e^{-\frac{\nu-\nu_o}{2\nu_o <a^2> D}}$$

Calculation of Landaudamping with this spectrum result in the following two equations:

$$-\frac{U}{V}\pi = \pm \frac{1-ye^{-y}\ Ei\ (y)}{ye^{-y}}$$

$$\frac{4\ \nu_o^2 <a^2> D}{N} \quad \frac{U}{U^2+V^2} = 1 - y\ e^{-y}\ Ei\ (y)$$

Fig. 3 shows the behaviour of the thresholds for different chromaticities: having equal sign of D and U (right hand side) there is a continously increase of the threshold for $\xi \to 0$. If U and D have opposite signs the threshold is constant for several orders of magnitude in ξ. For $\xi \to -\infty$ this threshold goes to zero. That the threshold doesn't go to infinity for $\xi \to o$ is due to the fact, that the radiation damping was not taken into account: For $\xi \to o$ one has $\bar{\nu} \to \nu_o$; at $\nu = \nu_o$ the spectrum density and therefore, the Landaudamping is exact zero while with consideration of radiation damping there remains a finite amount of damping.

Literature

1) C.Pellegrini , Int. Note LNF 69/49

2) C.Pellegrini , Corso 46 of Physics of Storage Rings (edited by B.Touschek, Acad. Press.1971)

3) M. Sands , Slac-TN-69/21

4) A. Piwinski , DESY H-70/21

5) A.Hofmann, G.Mülhaupt, 8. Int.Conf.High Energ. Acc. Genf 1971)

6) M.P.Level, J.Le Duff , Rapp.Technique Orsay 7-69

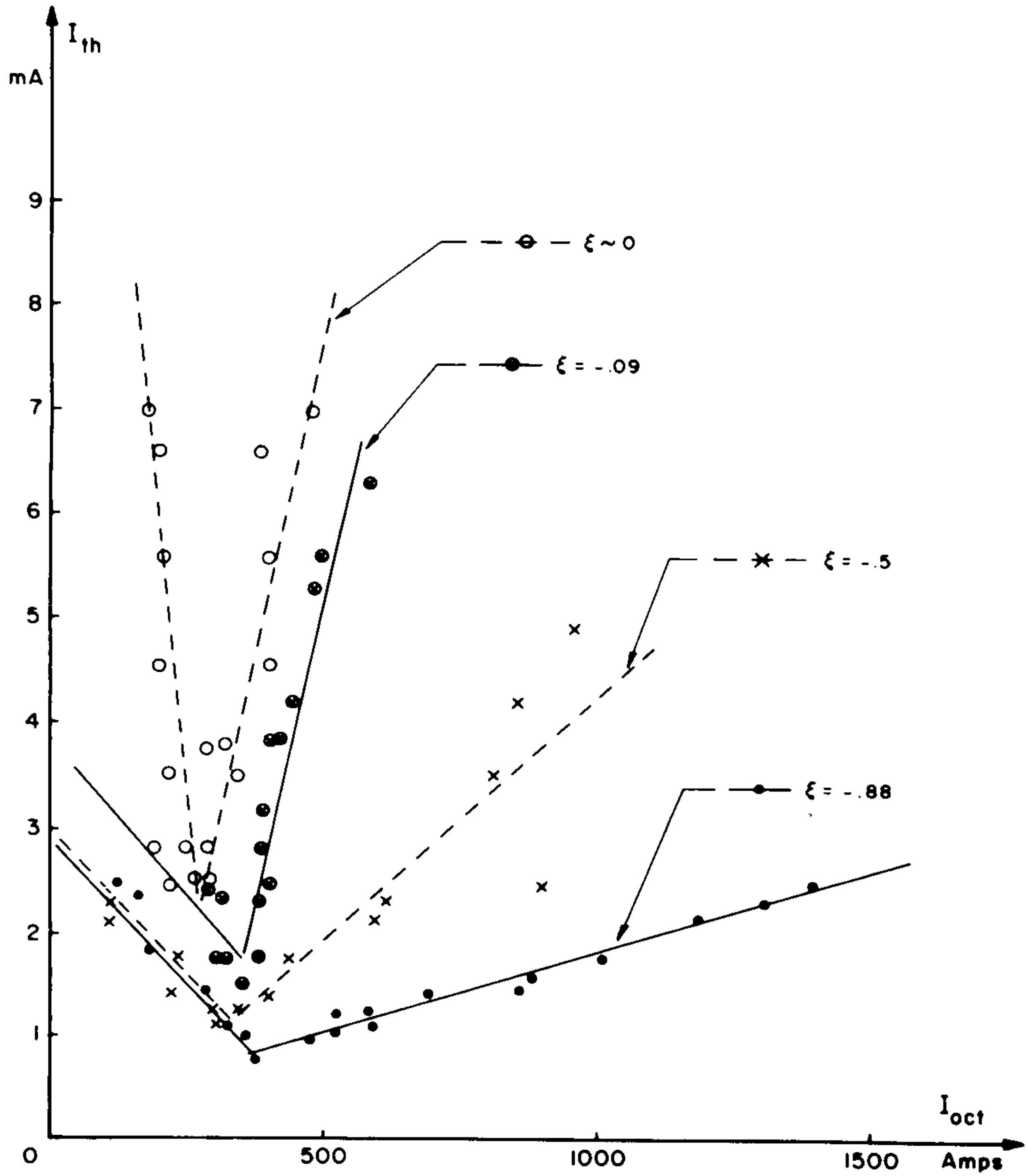

Fig. 1 THRESHOLD vs. OCTUPOLE CURRENT

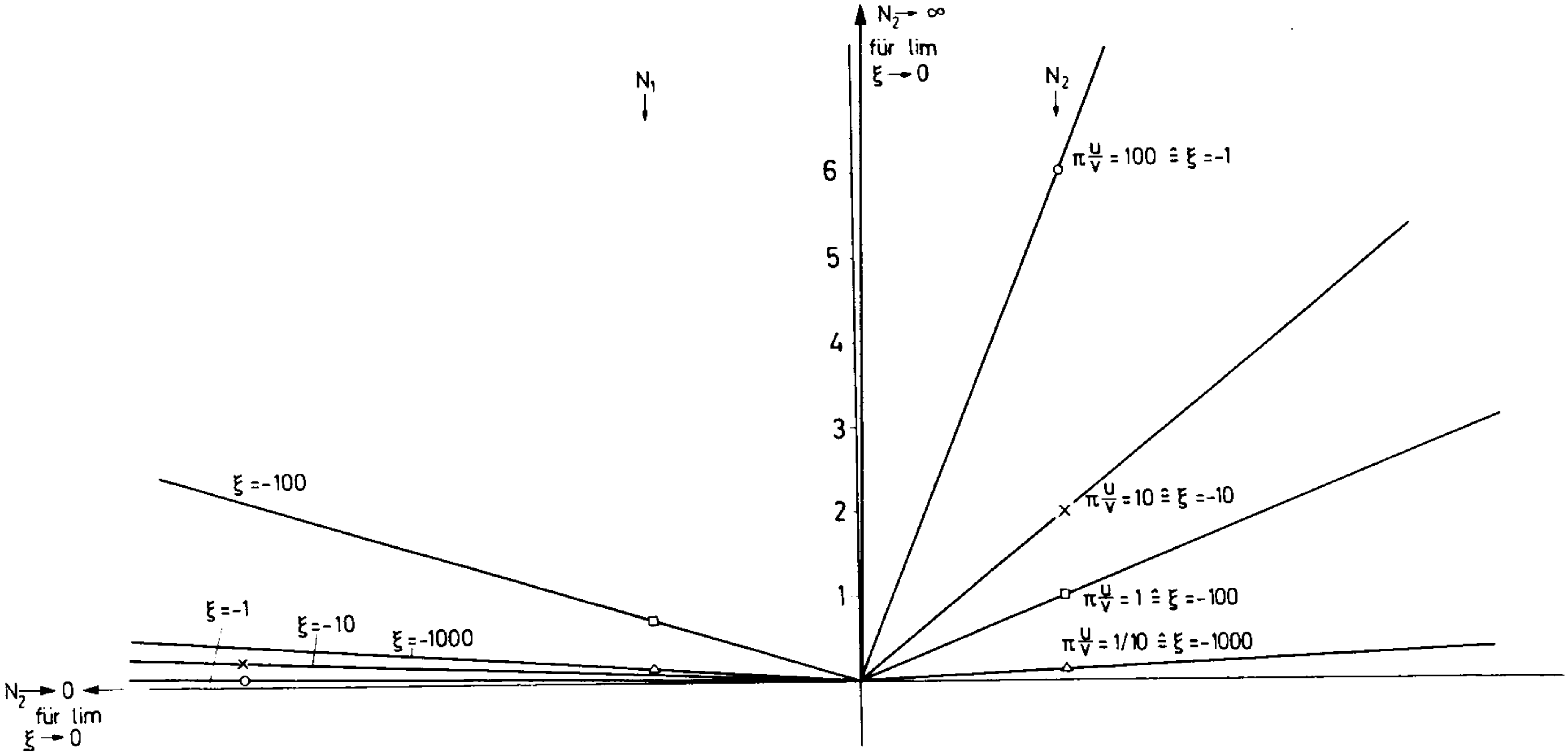

Fig. 2 Theor. threshold vs. octupole current $\left(f(v) \sim e^{-\frac{v-v_0}{2 v_0 \langle a^2 \rangle D}} \right)$

ξ in rel. units

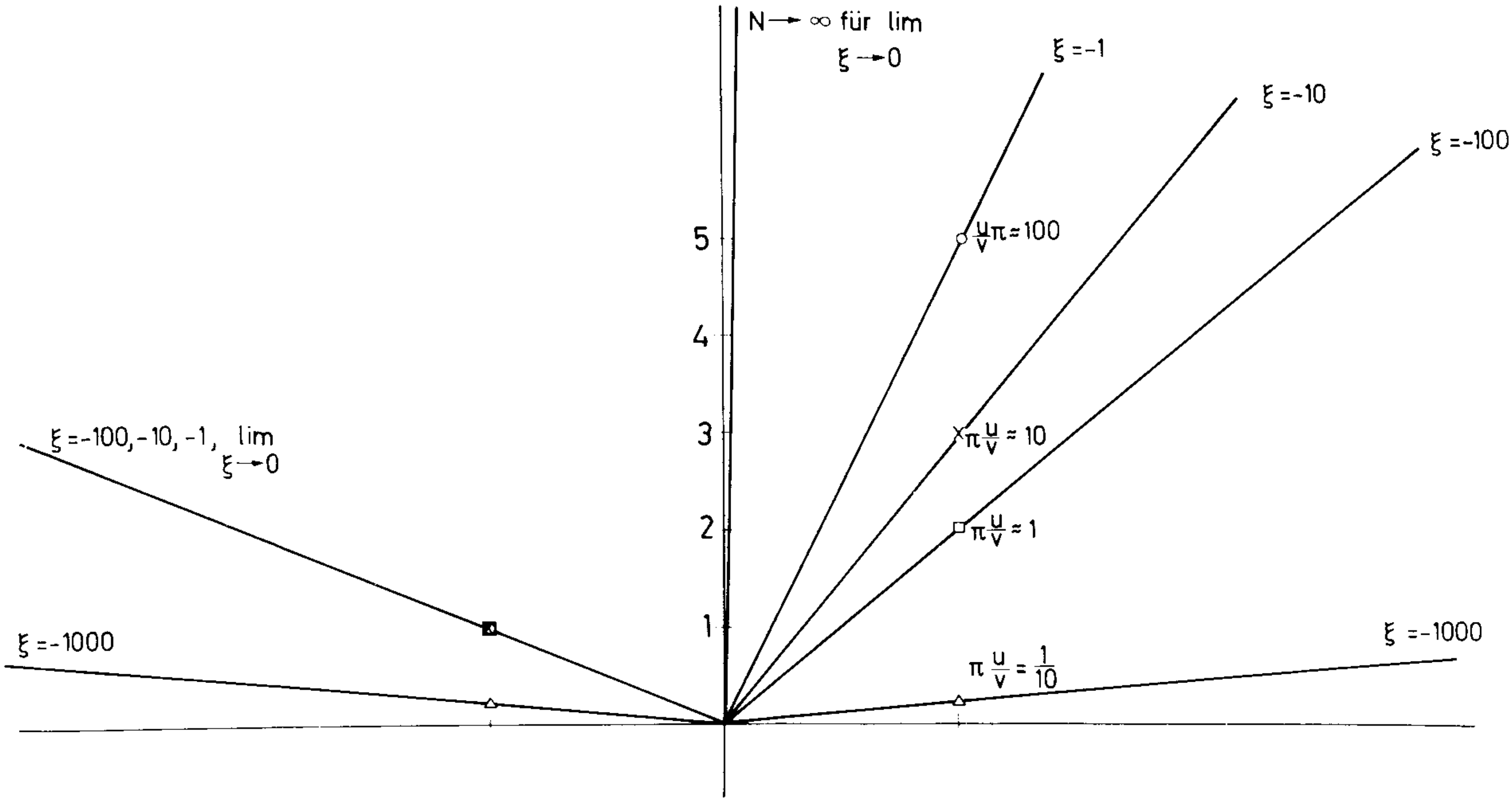

Fig. 3 Theor. theshold vs. octupole current ($f(v) \sim \frac{v - v_0}{2v_0 < a^2 > D} e^{-\frac{v - v_0}{2v_0 < a^2 > D}}$)
ξ in relative units