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PHOTOPRODUCTION OF PIONS IN FORWARD DIRECTION
AND REGGE POLES

von

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Photoproductions of single pions for high energies and small momentum transfer is considered. The production amplitude is approximated by exchange of ρ-, ω-, φ- and π-meson in the crossed channel $\gamma + \pi \dashrightarrow N + \bar{N}$. These mesons are treated either as "elementary" or as Regge poles. The angular distribution is calculated in the high energy limit and discussed in detail.

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Introduction

Photoproduction of pions on nucleons is a useful way to investigate strong interactions. At low energies the strong final state interaction between the emitted pion and the recoil nucleon dominates the behaviour of the photoproduction cross section. Theoretical analyses so far are mostly concerned with these low energy phenomena. They are usually based on the fixed momentum transfer dispersion approach of Chew, Goldberger, Low and Nambu [1] or modifications of it which start with the Mandelstam representation [2]. At higher energies (above one GeV) theoretical investigations are rather rare. In this energy range peripheral models with exchange of pion, ρ-meson, ω-meson and φ-meson can be applied. If these mesons are considered as dynamical bound or resonant states they may exhibit the typical behaviour found by Regge for nonrelativistic potential scattering [3]. In 1961 Chew and Frautschi [4] have suggested that all strongly interacting particles may show this Regge behaviour and several people [5] investigated the experimental consequences of this hypothesis with respect to particular processes as elastic $\pi - \pi$-, $\pi - N$-, and $N - N$- scattering together with the corresponding crossed reactions. For small momentum transfer and for high enough energy these reactions should be dominated by the Pomeranchuk Regge pole and should show equal diffraction scattering [4][5]. However, recent experimental studies of small-angle elastic scattering of protons on protons and $\pi^{\pm}$ on protons in the energy range 7 - 20 GeV clearly demonstrate that the diffraction pattern of π^+ - p and p - p scattering is different [6]. But this difference can be explained by assuming that besides the Pomeranchuk-pole also the ω-pole is present in this energy range [7]. Thus it cannot be excluded that even at rather high energies the vector meson like ω, φ and ρ play an important role. Therefore it seems to be worthwhile to consider reactions where these vector mesons contribute only and not the Pomeranchuk trajectory. Besides charge exchange pn - scattering where π and ρ are involved photoproduction of single pions on nucleons seems to be most suitable. Here small-angle scattering at high energies of the reaction $\gamma + p \dashrightarrow p + \pi^0$ should be determined by ω, φ and ρ^0 and of $\gamma + p \dashrightarrow n + \pi^+$ by π^+ and ρ^+. By measuring angular distributions of these two processes information about the Regge behaviour of ω, φ and ρ can be obtained.

In section I we discuss the relative contribution of various mesons to photoproduction on protons and neutrons which follow from isospin- and charge conjugation-invariance. The proper continuation of helicity amplitudes in the channel $\gamma + \pi \dashrightarrow N + \bar{N}$ into the complex angular momentum plane is introduced in section II. In section III we apply the Sommerfeld-Watson transformation to the expansion of the scattering amplitude into these helicity amplitudes and single out the contribution of special Regge poles. The implication of these pole contributions for small-angle photoproduction is dealt with in section IV *).

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*) After completion of this paper similar work on Regge poles and photoproduction appeared: R.W.Childers and W.G. Holladay: Phys.Rev.132, 1809 (1963) and G.Zweig: Institute of Technology, Pasadena, preprint.

§ 1. Application of Isospon- and G-Parity Invariance.

It is well known that the photoproduction amplitude considered in isospin space consists of three independent amplitudes with different isospin transformation properties.[1)] In this section we want to study the contribution of various mesons B characterized by their spin, parity, isospin and G-parity if they are exchanged between the γ, π - and N, $\bar{N}$ - pair (see fig. 1). The matrix element S_{fi} for photoproduction of pions

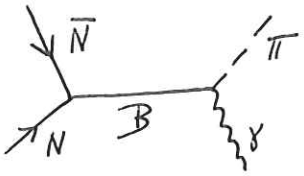

fig. 1

$$\gamma + N_1 \dashrightarrow N_2 + \pi$$

may be written

$$S_{fi} = (2\pi)^4 \, \delta(k+p_1-p_2-q) \, \varepsilon^P_\mu \langle \pi N_2 | j_\mu(0) | N_1 \rangle \qquad (1)$$

where ε^P is the polarization vector of the photon and $j_\mu(0)$ the electromagnetic current. We treat the coupling of the electromagnetic field in first order perturbation theory. Then the current can be decomposed in an isoscalar and an isovector part :

$$j_\mu = j_\mu^{(s)} + j_\mu^{(v)} \qquad (2)$$

The photoproduction amplitude T consists of the corresponding isoscalar and isovector component $T = T^{(s)} + T^{(v)}$. Since j_μ is odd under charge conjugation transformation C, $j_\mu^{(s)}$ and $j_\mu^{(v)}$ transform in the following way under G-parity operation ($G = C \, e^{-i\pi \, t_2}$, $\vec{t}$ isospin-operator) :

$$\begin{aligned} G \, j_\mu^{(s)} \, G^{-1} &= - \, j_\mu^{(s)} \\ G \, j_\mu^{(v)} \, G^{-1} &= \quad j_\mu^{(v)} \end{aligned} \qquad (3)$$

The pions have negative G-parity. Therefore we have for the right vertex in fig. 1

$$\langle \pi | j_\mu(0) | B \rangle = 0 \quad \begin{cases} \text{for } j_\mu = j_\mu^{(s)} \text{ and } G_B = -1 \\ \text{for } j_\mu = j_\mu^{(v)} \text{ and } G_B = +1 \end{cases} \tag{4}$$

We conclude that only such mesons B with positive (negative) G-parity contribute to $T^{(s)}$ ($T^{(v)}$). Furthermore $\langle \pi | j_\mu^{(s)} | B \rangle$ vanishes if B has isospin t = 0. Therefore from all known mesons [8]

$$\pi\,(0^-;1^-)\ ,\quad \eta\,(0^-;0^+)\ ,\quad f\,(2^+;0^+)\ ,\quad \rho\,(1^-;1^+)$$

$$\omega\,(1^-;0^-)\ ,\quad \varphi\,(1^-;0^-)$$

only the ρ contributes to $T^{(s)}$ and π, ω and φ contribute to $T^{(v)}$ (as usual the symbols in paranjheses stand for $(j^P;t^G)$ where j = spin, P = parity, G = G-parity, t = isospin).

The isospin dependence of T is usually removed by constructing all of the basid forms containing the isospin operators τ_1, τ_2 and τ_3 for the nucleon. If one denotes the isospin component of the outgoing pion by β there are three independent nucleon isospin combinations possible [1]:

$$\begin{aligned} g_\beta^+ &= \delta_{\beta 3} \\ g_\beta^- &= \tfrac{1}{2}\,[\tau_\beta, \tau_3] \\ g_\beta^0 &= \tau_\beta \end{aligned} \tag{5}$$

Then we have

$$\begin{aligned} T^{(s)} &= T^{(0)}\, g_\beta^0 \\ T^{(v)} &= T^{(+)}\, g_\beta^+ + T^{(-)}\, g_\beta^- \end{aligned} \tag{6}$$

with isospin independent amplitudes $T^{(0)}$, $T^{(+)}$ and $T^{(-)}$.

Since π has $t = 1$ it can only contribute to $T^{(-)}$ whereas the contributions of ω and φ are contained in $T^{(+)}$.

With nucleons the following four pionphotoproduction processes are possible :

$\gamma + p \rightarrow n + \pi^+$	with amplitude T_{π^+}
$\gamma + p \rightarrow p + \pi^0$	" " T_{π^0}
$\gamma + n \rightarrow p + \pi^-$	" " T_{π^-}
$\gamma + n \rightarrow n + \pi^0$	" " $T_{n\pi^0}$

The relations between these four amplitudes and $T^{(0)}$, $T^{(+)}$ and $T^{(-)}$ are well known. For completeness they are repeated here :[9)]

$$\begin{aligned} T_{\pi^+} &= \qquad\qquad -\sqrt{2}\,T^{(-)} - \sqrt{2}\,T^{(0)} \\ T_{\pi^0} &= T^{(+)} \qquad\qquad + T^{(0)} \\ T_{\pi^-} &= \qquad\qquad -\sqrt{2}\,T^{(-)} + \sqrt{2}\,T^{(0)} \\ T_{n\pi^0} &= T^{(+)} \qquad\qquad - T^{(0)} \end{aligned} \tag{7}$$

We denote the exchange contributions of the four mesons by $T(\pi)$, $T(\rho)$, $T(\omega)$ and $T(\varphi)$. They can be detected in the four reactions in proportions as following :

$$\begin{aligned} T_{\pi^+} &= -\sqrt{2}\,T(\pi) - \sqrt{2}\,T(\rho) \\ T_{\pi^0} &= T(\omega) + T(\varphi) + T(\rho) \\ T_{\pi^-} &= -\sqrt{2}\,T(\pi) + \sqrt{2}\,T(\rho) \\ T_{n\pi^0} &= T(\omega) + T(\varphi) - T(\rho) \end{aligned} \tag{8}$$

By experimental investigation of these four reactions it is therefore principally possible to obtain $T(\pi)$, $T(\rho)$ and $T(\omega) + T(\varphi)$ seperately.

The Regge properties of ω plus φ alone could be studied also by pion photoproduction on nuclei with isotopic spin zero, like deuterons and C nuclei. Of course other residua than in photoproduction on nucleons are encountered this way.

It is clear that some of the mesons listed above are also present in η- photoproduction. Let us consider η production in isospin space. We decompose the amplitude in two parts

$$T = T^{(s)} + T^{(v)}\tau_3 \tag{9}$$

with two isospin independent amplitudes $T^{(s)}$ and $T^{(v)}$. The two possible reactions $\gamma + p \to p + \eta\,(T_p)$ and $\gamma + n \to n + \eta\,(T_n)$ are simply related to $T^{(s)}$ and $T^{(v)}$:

$$T_{p,n} = T^{(s)} \pm T^{(v)}$$

We consider exchange of a meson B as in fig. 1 with π replaced by η. It follows immediately that B with isospin $t = 0$ ($t = 1$) contributes to $T^{(s)}$ ($T^{(v)}$). $T^{(s)}$ and $T^{(v)}\tau_3$ are the isoscalar and isovector part of T respectively. Since η has positive G-parity B must have $G = -1$ to make a nonvanishing contribution to $T^{(s)}$. For $T^{(v)}$ it must have $G = +1$. Therefore $T^{(s)}$ is connected with B-exchange if $G_B = -1$ and $T = 0$ and $T^{(v)}$ if $G_B = +1$ and $t = 1$. The candidates for $T^{(s)}$ are ω and φ and for $T^{(v)}$ it is only the ρ.

In the following section we shall study the isospin independent amplitudes. We shall restrict ourselves to a discussion of pion production. But most of the results can be carried over to η- production quite easily since pion and η have the same spin and parity.

§ 2 Continuation of the Partial Wave Amplitudes for $\gamma + \pi \longrightarrow N + \bar{N}$ into the complex angular momentum plane.

We define the T - matrix for photoproduction of pions on nucleons $\gamma + N_1 \longrightarrow N_2 + \pi$ by :

$$S_{fi} = \frac{i}{(2\pi)^2} \frac{m}{(4 E_1 E_2 \omega k)^{1/2}} \bar{u}(p_2) T u(p_1) \delta(p_1 + k - p_2 - q) \tag{10}$$

denoting the momenta of the incoming nucleon and gamma by p_1 and k respectively and the momenta of the outgoing nucleon and pion by p_2 and q.

The most general form of the matrix T allowed by Lorentz invariance has been constructed by Ball [2) :

$$T = \sum_{i=1}^{8} B_i(s,t) \; N_i(p_1, p_2, k, \varepsilon, \gamma) \tag{11}$$

Here s and t are the Mandelstam variables $s = -(p_1 + k)^2$ $t = -(q-k)^2$ and $u = -(p_2-k)^2$ with $s + t + u = 2m^2 + \mu^2$ (m nucleon mass, μ pion mass) and the eight matrices N_i are all independent Lorentz-invariant matrices which can be formed with the four vectors p_1, p_2, k, ε and γ where ε is the polarization vector of the gamma and the γ are the Dirac gamma matrices. These N_i can be found in Ball's paper [2)]. The only property of the N_i needed for our discussion is the fact that the $B_i(s,t)$ as defined by equ (10) are free from kinematical singularities. This has been explicitly demonstrated in the appendix of ref. 2. It can be seen quite easily that the matrix T expanded in terms of the matrices N_i is not gauge invariant. This requirement yields the following relations between the B_i's [2)]:

$$B_2\, p \cdot k + B_3\, q \cdot k = 0$$
$$B_5 + B_6\, p \cdot k + B_8\, q \cdot k = 0 \tag{12}$$

where $p = \frac{1}{2}(p_1 + p_2)$.

It is convenient to express the gauge invariant T-matrix by the CGLN matrices M_i :

$$T = \sum_{i=1}^{4} A_i(s,t)\, M_i \tag{13}$$

where [1])

$$\begin{aligned} M_1 &= i\gamma_5\, \gamma\cdot\varepsilon\, \gamma\cdot k \\ M_2 &= 2i\gamma_5\, (p\cdot\varepsilon\; q\cdot k - p\cdot k\; q\cdot\varepsilon) \\ M_3 &= \gamma_5\, (\gamma\cdot\varepsilon\; q\cdot k - \gamma\cdot k\; q\cdot\varepsilon) \\ M_4 &= 2\gamma_5\, (\gamma\cdot\varepsilon\; p\cdot k - \gamma\cdot k\; p\cdot\varepsilon - i m\, \gamma\cdot\varepsilon\, \gamma\cdot k) \end{aligned} \tag{14}$$

The matrices M_i are explicitly gauge invariant. The amplitudes $A_i(s,t)$ can be expressed by the $B_i(s,t)$. The relations are:

$$\begin{aligned} A_1 &= B_1 - m\, B_6 \\ A_2 &= B_2 / q\cdot k = \frac{2B_2}{t-\mu^2} \\ A_3 &= -\, B_8 \\ A_4 &= -\frac{1}{2} B_6 \end{aligned} \tag{15}$$

B_4 and B_7 do not contribute because of $k\cdot\varepsilon=0$ for real photons. Now A_2 has a kinematical singularity. It seems to be generally true that gauge invariant amplitudes for reactions with an odd number of photons have kinematical singularities.[10]) But the new singularity of A_2 is only a pole in t. So for fixed t the amplitudes $A_i(s,t)$ have the same analytical properties in the variables s and u as the amplitudes $B_i(s,t)$.

In order to derive asymptotic properties of the photoproduction amplitude for high energies with the help of the Sommerfeld - Watson transformation the analytic continuation of the partial wave amplitudes to complex angular momentum in the t-channel ($\gamma + \pi \dashrightarrow N + \bar{N}$) must be found. For this purpose we start with the Legendre expansion of the invariant amplitudes $B_i(s,t)$:

$$B_i(s,t) = \sum_{\ell} (2\ell + 1)\, B_i(\ell,t)\, P_\ell\,(\cos\Theta_t) \tag{16}$$

We assume that the invariant amplitudes satisfy dispersion relations for fixed momentum transfer t :

$$B_i(s,t) = \int_{(m+\mu)^2}^{\infty} \frac{ds'}{\pi} \frac{\operatorname{Im} B_i^{(s)}(s',t)}{s'-s} + \int_{(m+\mu)^2}^{\infty} \frac{du'}{\pi} \frac{\operatorname{Im} B_i^{(u)}(u',t)}{u'-u} \tag{17}$$

Since $B_i(\ell,t) = \frac{1}{2}\int B_i(s,t) \quad P_\ell\,(\cos\Theta_t)\; d\cos\Theta_t$

we obtain for integer values of ℓ from representation equ (17) :

$$B_i(\ell,t) = \frac{1}{4pk'} \left\{ \int_{(m+\mu)^2}^{\infty} \frac{ds'}{\pi} \operatorname{Im} B_i^{(s)}(s',t)\, Q_\ell\left(\frac{s'-m^2+2Ek'}{2pk'}\right) - \int_{(m+\mu)^2}^{\infty} \frac{du'}{\pi} \operatorname{Im} B_i^{(u)}(u',t)\, Q_\ell\left(-\frac{u'-m^2+2Ek'}{2pk'}\right) \right. \tag{18}$$

where E and k' and cos Θ_t are given by :

$$E = \frac{\sqrt{t}}{2}, \quad k' = \frac{t-\mu^2}{2\sqrt{t}}, \quad s = m^2 - 2Ek' + 2pk'\cos\Theta_t \tag{19}$$

The analytic continuation of $B_i(\ell,t)$ to complex ℓ can be defined by equ (18). Then $B_i(\ell,t)$ is regular for Re$\ell > 0$. But the second term in equ (18) is not suitable for a Sommerfeld-Watson transformation because of the factor $(e^{\pm i\pi\ell})$ coming from the Legendre function of the second kind

$$\left(Q_\ell(-z) = -e^{\pm i\pi\ell}\, Q_\ell(z) \quad \text{for } \operatorname{Im} z \gtrless 0 \right)$$

Following Squires [11] we define therefore a direct and an exchange part of $B_i(\ell,t)$ (ℓ = integer) :

$$B_i(\ell,t) = B_i^{d}(\ell,t) + e^{\pm i\pi\ell} B_i^{e}(\ell,t) \tag{20}$$

where
$$B_i^{d}(\ell,t) = \frac{1}{4pk'} \int_{(m+\mu)^2}^{\infty} \frac{ds'}{\pi} \, \mathrm{Im}\, B_i^{(s)}(s',t)\, Q_\ell\left(\frac{s'-m^2+2Ek'}{2pk'}\right) \tag{21}$$

and

$$B_i^{e}(\ell,t) = \frac{1}{4pk'} \int_{(m+\mu)^2}^{\infty} \frac{du'}{\pi} \, \mathrm{Im}\, B_i^{(u)}(u',t)\, Q_\ell\left(\frac{u'-m^2+2Ek'}{2pk'}\right)$$

It can be shown that these two functions have the same asymptotic behaviour for $|\ell| \to \infty$ [11].

In general the dispersion relation equ (17) for $B_i(s,t)$ will not hold without subtractions. But if a subtracted dispersion relation is assumed we can conclude that the $B_i(\ell,t)$ are regular for $\mathrm{Re}\,\ell > N > 0$ (N integer). In the same way $B_i^{d}(\ell,t)$ and $B_i^{e}(\ell,t)$ are regular for $\mathrm{Re}\,\ell > N$. It is clear that the separation in a direct and in an exchange part is also possible if the dispersion relations are written in a subtracted form.

Regge proved that the partial wave amplitudes in potential scattering are meromorphic for $\mathrm{Re}\,\ell > -\frac{1}{2}$ [3]. It has been speculated that also in relativistic field theory partial wave amplitudes might be meromorphic as a function of ℓ for $\mathrm{Re}\,\ell$ positive. Following these suggestions we make the strong assumption that all $B_i(\ell,t)$ are meromorphic in ℓ for $-(\Lambda+1) \leq \mathrm{Re}\,\ell \leq N$ with $\Lambda > 0$.

Since for fixed t the amplitudes $A_i(s,t)$ have the same analytical properties in the variables s and u as the $B_i(s,t)$, we can conclude that the $A_i(\ell,t)$ defined by

$$A_i(s,t) = \sum_\ell (2\ell+1) A_i(\ell,t) P_\ell(\cos\theta_t) \tag{22}$$

have also analytic continuations in the variable ℓ which are regular for Re $\ell > N$ and are meromorphic in the domain $-(\Lambda+1) \leq \mathrm{Re}\,\ell \leq N$.

To establish a connection between the poles in the angular momentummplane and known particles or resonances present as intermediate particles in the t-channel the partial wave projections of the invariant amplitudes are not useful. For this purpose we consider the partial wave projections $T^{j,m}_{\lambda\bar\lambda}(t)$ of the helicity amplitude $T^m_{\lambda\bar\lambda}$ which are defined by [12),2)] :

$$T^m_{\lambda\bar\lambda} = \sum_j \left(j+\tfrac{1}{2}\right) T^{j,m}_{\lambda\bar\lambda}(t)\, d^j_{m,\lambda-\bar\lambda}(\cos\theta_t) \tag{23}$$

Here the $d^j_{m\mu}(x)$ are the usual rotation functions as found for instance in ref. 13. The helicity amplitudes are related to the invariant amplitudes by :

$$T^m_{\lambda\bar\lambda} = \bar{u}_\lambda(p_2) \sum_i A_i M_i(\vec{\varepsilon}_m)\, u^c_{-\bar\lambda}(p_1') \tag{24}$$

if the right side is evaluated in the cm - system of the reaction $\gamma + \pi \longrightarrow N + \bar{N}$. $m = \pm 1$ stands for the polarization of the photon and λ and $\bar\lambda$ are the helicities of the nucleon and the antinucleon respectively. If the vector $\vec{p}_2$ is parallel to the z - axis of our coordinate system the spins of nucleon and antinucleon are as follows :

$$\begin{aligned} \lambda &= \pm\tfrac{1}{2} \leftrightarrow m_3^N = \pm\tfrac{1}{2} \\ \bar\lambda &= \pm\tfrac{1}{2} \leftrightarrow m_3^{\bar N} = \mp\tfrac{1}{2} \end{aligned} \tag{25}$$

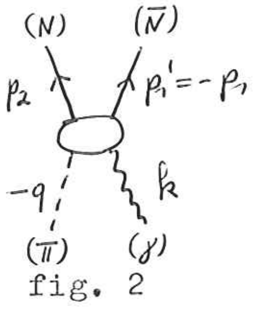

fig. 2

The notation of the momenta of nucleon, antinucleon, pion and photon can be seen in fig. 2, j is now the total angular momentum of the channel $\gamma + \pi$ or $N + \bar{N}$. Finally we shall apply the Sommerfeld-Watson tranformation to an expansion of the scattering matrix elements into total angular momentum states. An analytic continuation of the $T^{j,m}_{\lambda\bar{\lambda}}(t)$ to complex j can be performed with the help of the continuation of the partial wave amplitudes $A_i(\ell,t)$ to complex ℓ considered before. For real ℓ and j we have the following relation :

$$T^{j,m}_{\lambda\bar{\lambda}}(t) = \frac{2}{2j+1} \sum_{\ell,i} \tilde{c}_i^{\lambda\bar{\lambda}m}(t)\, A_i(\ell,t)\,(2\ell+1) \times C(\ell 1 j, 0 m)\, C(\ell 1 j, 0\, \lambda-\bar{\lambda}) \tag{26}$$

The relation equ (26) can be derived quite easily by means of functions $G_i(s,t)$ first introduced by Ball [2]. The $G_i(s,t)$ are rotational invariant functions in the c.m-system reducing the S-matrix element for $\gamma + \pi \longrightarrow N + \bar{N}$ to Pauli spinor form.

The helicity amplitudes are linear combinations of the four functions G_i with $\cos\Theta_t$-dependent coefficients :

$$T^{m}_{\lambda\bar{\lambda}} = \sum_{i=1}^{4} d^{1}_{m,\lambda-\bar{\lambda}}(\cos\Theta_t)\, c_i^{\lambda\bar{\lambda}m}\, G_i(s,t)$$

The $c_i^{\lambda\bar{\lambda}m}$ are independent of t and $\cos\Theta_t$ and have the property

$$c_i^{\lambda\bar{\lambda}m} = - c_i^{-\lambda,-\bar{\lambda},-m} \tag{28}$$

which follows from parity conservation :

$$T^{m}_{\lambda\bar{\lambda}} = - T^{-m}_{-\lambda,-\bar{\lambda}}\,(-1)^{m-\lambda+\bar{\lambda}} \tag{29}$$

On the other hand the $G_i(s,t)$ are linearly related to the four amplitudes $A_i(s,t)$ with t-dependent coefficients $b_{ii'}$:

$$G_i(s,t) = \sum_{i'} b_{ii'}(t)\, A_{i'}(s,t) \tag{30}$$

If we define

$$\tilde{c}_i^{\,\lambda\bar{\lambda}m} = \sum_{i'} c_{i'}^{\,\lambda\bar{\lambda}m}\, b_{i'i}(t) \tag{31}$$

and insert the Legendre expansion equ (22) of $A_i(s,t)$ into equ (30) and express the product of two d-functions encountered by means of the Clebsch-Gordan series [13]:

$$d^{j}_{\nu\mu}\, d^{j'}_{\nu'\mu'} = (-1)^{\nu'-\mu'} \sum_{J}{}' \, C(jj'J, \nu-\nu')\, C(jj'J, \mu-\mu')\, d^{J}_{\nu-\nu',\mu-\mu'} \tag{32}$$

equ (26) follows with the help of equ (23). Equ (26) now defines an analytic continuation of the amplitudes $T^{j,m}_{\lambda\bar{\lambda}}(t)$ to complex j if for $A_i(\ell,t)$ the analytic continuation to complex ℓ introduced above is used. Since the product of Clebsch-Gordan coefficients times $2\ell+1$ appearing in equ (26) has no singularities for Re $j > 0$ the $T^{j,m}_{\lambda\bar{\lambda}}(t)$ are meromorphic in j for Re $j > 0$. This follows from the analytical properties of the $A_i(\ell,t)$ as functions of ℓ ($A_i(\ell,t)$ meromorphic for Re $\ell \geq -(\Lambda+1)$, $\Lambda > 0$)

§ 3 The Sommerfeld-Watson transformation for the Invariant Amplitudes.

First we want to derive the expansion of the amplitudes $A_i(s,t)$ in terms of partial wave projections $T^{j,m}_{\lambda\bar{\lambda}}$ of the helicity amplitudes. The structure of this expansion which was proposed by Gell-Mann for the general case of two body reactions[14], is :

$$A_i(s,t) = \sum_{\varkappa, j} Z_j^{\varkappa, i}(t, \cos\theta_t)\, d_j^{\varkappa}(t) \tag{33}$$

where the $d_j^{\varkappa}(t)$ are linear combinations of the $T_{\lambda\bar{\lambda}}^{j,m}$ (see equ (36)). In this context it is sufficient to restrict m to m = 1 because the $T_{\lambda\bar{\lambda}}^{j,-1}$ are linearly related to the $T_{\lambda\bar{\lambda}}^{j,1}$ according to equ (29).

We derive the expansion equ (33) with the help of the functions $G_i(s,t)$ which are very simply related to the helicity amplitudes (see equ (27)). From an inversion of equ (30) the $A_i(s,t)$ can be expressed by the $G_i(s,t)$. These relations are :

$$
\begin{aligned}
A_1 &= \frac{4im}{(t-\mu^2)(t-4m^2)} \sqrt{t}\left(\sqrt{t}\, G_3 - (2m-\sqrt{t})\, G_4\right)\\
A_2 &= \frac{-4im}{(t-\mu^2)(t-4m^2)\sqrt{t}} \left(\sqrt{t}\, G_3 - (2m-\sqrt{t})\, G_4 + \sqrt{t-4m^2}\, G_1\right)\\
A_3 &= \frac{-4im}{(t-\mu^2)\sqrt{t-4m^2}}\, G_2\\
A_4 &= \frac{4im}{(t-\mu^2)(t-4m^2)} \left(2m\, G_3 + (2m-\sqrt{t})\, G_4\right)
\end{aligned}
\tag{34}
$$

Ball has also derived the formulas for the expansion of the $G_i(s,t)$ in total angular momentum states : These formulas for all four $G_i(s,t)$ are [2] :

$$
\begin{aligned}
G_1 &= -\sum_j \left(j+\tfrac{1}{2}\right) d_j^4\, P_j'(\cos\theta_t)\\
G_2 &= -\sum_j \left(j+\tfrac{1}{2}\right)\left\{-d_j^1 P_j'' + d_j^2 \frac{1}{2j+1}\left(j P_{j+1}'' + (j+1) P_{j-1}''\right)\right\}\\
G_3 + G_4 &= -\sum_j \left(j+\tfrac{1}{2}\right) d_j^3\, P_j'\\
G_4 &= -\sum_j \left(j+\tfrac{1}{2}\right)\left\{d_j^1 \frac{1}{2j+1}\left(j P_{j+1}'' + (j+1) P_{j-1}''\right) - d_j^2 P_j''\right\}
\end{aligned}
\tag{35}
$$

The $\alpha_j^{\varkappa}$ which are only functions of t are identical with the following linear combinations of the $T_{\lambda\bar{\lambda}}^{j,1}$:

$$\alpha_j^{1(2)} = \frac{1}{\sqrt{2}\, j(j+1)} \left\{ T^{j,1}_{1/2,-1/2} \underset{(-)}{+} T^{j,1}_{-1/2,1/2} \right\}$$

$$\alpha_j^{3(4)} = \frac{1}{\sqrt{2j(j+1)}} \left\{ T^{j,1}_{1/2,1/2} \underset{(-)}{+} T^{j,1}_{-1/2,-1/2} \right\}$$

The branch points at $j = 0, \ -1$ cancel with a corresponding factor $\sqrt{j(j+1)}$ contained in the Clebsch-Gordan coefficients in equ (26). The poles at $j = 0, - 1$ in $\alpha_j^{1(2)}$ cancel against a factor $j(j+1)$ contained in Z_j^{1i} and Z_j^{2i} respectively.

Now we combine equ (34) and equ (35), then the functions $Z_j^{\varkappa,i}(t,X_t)$ follow ($\cos\Theta_t \equiv X_t$) :

equ (37) see next page :

$$Z_j^{11} = \frac{4im^2\sqrt{t}}{(t-\mu^2)(t-4m^2)} \left(j\, P''_{j+1} + (j+1)\, P''_{j-1} \right)$$

$$Z_j^{21} = \frac{-8im^2\sqrt{t}}{(t-\mu^2)(t-4m^2)} (j+1/2)\; P''_j$$

$$Z_j^{31} = \frac{-4imt}{(t-\mu^2)(t-4m^2)} (j+1/2)\; P'_j \quad ; \quad Z_j^{41} = 0$$

$$Z_j^{12} = -\frac{1}{t} Z_j^{11} \; ; \quad Z_j^{22} = -\frac{1}{t} Z_j^{21}$$

$$Z_j^{32} = -\frac{1}{t} Z_j^{31} \; ; \quad Z_j^{42} = -\frac{1}{t} Z_j^{31} \tag{37}$$

$$Z_j^{13} = \frac{\sqrt{t-4m^2}}{2m\sqrt{t}} Z_j^{21} \; ; \quad Z_j^{23} = \frac{\sqrt{t-4m^2}}{2m\sqrt{t}} Z_j^{11}$$

$$Z_j^{33} = Z_j^{43} = 0$$

$$Z_j^{14} = \frac{1}{2m} Z_j^{11} \; ; \quad Z_j^{24} = \frac{1}{2m} Z_j^{21}$$

$$Z_j^{34} = \frac{2m}{t} Z_j^{31} \; ; \quad Z_j^{44} = 0$$

We notice that $Z_j^{\varkappa,i}(t,x_t)$ have a definite parity for $x_t \longrightarrow -x_t$ directly related to the parity of the Legendre functions ($P_j(x) = (-1)^j\, P_j(-x)$). Therefore the terms in the expansion equ (33) have definitely so called j-parity.

Directy connected with this fact is the occurrence of special combinations of the $T_{\lambda\bar{\lambda}}^{j,l}$ instead of these functions itself as coefficients of the $Z_j^{\varkappa i}$ in this equation.

In section 2 we mentioned that the $B_i(\ell,t)$ must be divided into two parts $B_i^{d}(\ell,t)$ and $B_i^{e}(\ell,t)$ to make the Sommerfeld-Watson transformation possible. The complete $B_i(\ell,t)$ is then given by :

$$
\begin{aligned}
B_i(\ell,t) &\equiv B_i^{(+)}(\ell,t) = B_i^{d}(\ell,t) + B_i^{e}(\ell,t) \qquad \text{for even } \ell \\
B_i(\ell,t) &\equiv B_i^{(-)}(\ell,t) = B_i^{d}(\ell,t) - B_i^{e}(\ell,t) \qquad \text{for odd } \ell
\end{aligned}
\tag{38}
$$

The $B_i^{d}(\ell,t)$ and $B_i^{e}(\ell,t)$ have been defined earlier in equ (21). Instead of the branches $B_i^{d}(\ell,t)$ and $B_i^{e}(\ell,t)$ we have introduced $B_i^{(+)}(\ell,t)$ and $B_i^{(-)}(\ell,t)$ which are connected with even and odd ℓ. Of course for these two branches we can also perform the Sommerfeld-Watson transformation if if we can apply it to the sum over the $B_i^{d}(\ell,t)$ and the sum over the $B_i^{e}(\ell,t)$ separately. But the $B_i^{(\pm)}(\ell,t)$ can be easily related to the even and odd part of $\alpha_j^{\varkappa}(t)$ for which we shall use to following notation :

$$
\begin{aligned}
\alpha_j^{\varkappa(+)}(t) &\equiv \alpha_j^{\varkappa}(t) \qquad \text{for even } j \\
\alpha_j^{\varkappa(-)}(t) &\equiv \alpha_j^{\varkappa}(t) \qquad \text{for odd } j
\end{aligned}
\tag{39}
$$

They are obtained from equ (26) if this equation is considered for even and odd j separately. Because $\ell = j \pm 1, j$ the two branches of $B_i(\ell,t)$ appear ($A_i(\ell,t)$ can be replaced by the $B_i(\ell,t)$ by means of equ (15)) in these two equations. Equ (36) gives the $\alpha_j^{\varkappa}$ for even or odd j in terms of the even and odd part of $T^{j,l}$. This way two distinct analytic continuations of $\alpha_j^{\varkappa}(t)$ are introduced, one which interpolates $\alpha_j^{\varkappa}(t)$ for even j, one for odd j. The two continuations are related to the two continuations of $B_i(\ell, t)$ for even and odd ℓ called $B_i^{(\pm)}(\ell, t)$ (equ (38)). Our next task is to rewrite the expansion (33) in such a form that the two branches $\alpha_j^{\varkappa(\pm)}(t)$ appear.

The expansion (33) can be written :

$$A_i(s,t) = \sum_{\varkappa} \Big\{ \sum_{j\,\text{even}} d_j^{\varkappa(+)} Z_j^{\varkappa i}(t, X_t) + \sum_{j\,\text{odd}} d_j^{\varkappa(-)} Z_j^{\varkappa i}(t, X_t) \Big\} \tag{40}$$

Since

$$(-1)^j \, \varepsilon_{\varkappa i} \; Z_j^{\varkappa i}(t, -X_t) = Z_j^{\varkappa i}(t, X_t)$$

where $\varepsilon_{\varkappa i} = \pm 1$, depending on the indices $\varkappa$ and i the form, equ(40) is equivalent to the following expression :

$$\begin{aligned} A_i(s,t) = \sum_{\varkappa, j} \Big\{ & d_j^{\varkappa(+)} (-1)^j \tfrac{1}{2} \left[\varepsilon_{\varkappa i} Z_j^{\varkappa i}(t, -X_t) + Z_j^{\varkappa i}(t, X_t) \right] \\ & + d_j^{\varkappa(-)} (-1)^j \tfrac{1}{2} \left[\varepsilon_{\varkappa i} Z_j^{\varkappa i}(t, -X_t) - Z_j^{\varkappa i}(t, X_t) \right] \Big\} \end{aligned} \tag{41}$$

It is well known that this sum is identical with a contour integral, the integration path surrounding the real axis in clock wise directions :

$$\begin{aligned} A_i(s,t) = -\frac{1}{2i} \sum_{\varkappa} \int_C \frac{d\lambda}{\sin \pi\lambda} \Big\{ & d_\lambda^{\varkappa(+)} \tfrac{1}{2} \left[\varepsilon_{\varkappa i} Z_\lambda^{\varkappa i}(t, -X_t) \right. \\ & \left. + Z_\lambda^{\varkappa i}(t, X_t) \right] + d_\lambda^{\varkappa(-)} \tfrac{1}{2} \left[\varepsilon_{\varkappa i} Z_\lambda^{\varkappa i}(t, -X_t) - Z_\lambda^{\varkappa i}(t, X_t) \right] \Big\} \end{aligned} \tag{42}$$

Here the functions $\alpha_\lambda^{\varkappa(\pm)}$ are analytical continuations of $\alpha_j^{\varkappa(+)}$ to complex $j \equiv \lambda$, defined by the continuation of equ (36), (26), (38) and (21) considered for even and odd j separately as explained above.

Following Regge we transform the integral in equ (42) by moving the integration path to a line parallel to the imaginary axis with $\operatorname{Re}\lambda = -\Lambda \; (\Lambda > 0)$. Then the $A_i(s,t)$ are equal to integrals from $-\Lambda - i\infty$ to $-\Lambda + i\infty$ in addition to the residues of the poles encircled by moving the integration path :

$$A_i(s,t) = \frac{i}{2}\sum_{\varkappa}\int_{-\Lambda-i\infty}^{-\Lambda+i\infty}\frac{d\lambda}{\sin\pi\lambda}\Big\{\alpha_\lambda^{\varkappa(+)}\frac{1}{2}\left[\varepsilon_{\varkappa i}\,Z_\lambda^{\varkappa i}(t,-X_t)+Z_\lambda^{\varkappa i}(t,X_t)\right]$$

$$+\ \alpha_\lambda^{\varkappa(-)}\frac{1}{2}\left[\varepsilon_{\varkappa i}\,Z_\lambda^{\varkappa i}(t,-X_t)-Z_\lambda^{\varkappa i}(t,X_t)\right]\Big\} \tag{43}$$

$$-\frac{\pi}{2}\sum_{\alpha_\pm}\frac{1}{\sin\pi\alpha_\pm}\sum_{\varkappa}\beta_\pm^{\varkappa}\left[\varepsilon_{\varkappa i}\,Z_{\alpha_\pm}^{\varkappa i}(t,-X_t)\pm Z_{\alpha_\pm}^{\varkappa i}(t,X_t)\right]$$

In equ (43) we have written $\beta_\pm^{\varkappa}$ for the residues of the amplitudes $\alpha_\lambda^{\varkappa(\pm)}(t)$ at the poles $\lambda = \alpha_\pm$, that is :

$$\lim_{\lambda\to\alpha_\pm}(\lambda-\alpha_\pm)\,\alpha_\lambda^{\varkappa(\pm)} = \beta_\pm^{\varkappa} \tag{44}$$

The residues $\beta_\pm^{\varkappa}$ and the poles $\alpha_\pm$ are functions of t.

As has been done for other processes [5] we investigate the asymptotic behaviour of the $A_i(s,t)$ for large s but finite t. X_t is related to s and t by :

$$X_t = \frac{s - m^2 + \frac{1}{2}(t-\mu^2)}{(t-\mu^2)\sqrt{\dfrac{t-4m^2}{4t}}} \tag{45}$$

Thus for $s \gg (m^2, t)$ the variable X_t increases as :

$$X_t \longrightarrow \frac{s}{(t-\mu^2)}\sqrt{\frac{4t}{t-4m^2}} \tag{46}$$

Because of

$$P_\alpha(X) \underset{X\to\infty}{\longrightarrow} \frac{1}{\sqrt{\pi}}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\,(2X)^\alpha \tag{47}$$

the pole term dominates over the integral in the limit s ---> as long as $\alpha(t) > -\Lambda$. In the following we shall consider only the dominating pole trajectories which meet this condition. It follows from equ (43),(46),(47) together with equ (37) that the $A_i(s,t)$ behave for $s \dashrightarrow \infty$ as

$$A_i(s,t) \sim s^{\alpha(t)-1} \quad \text{for all i.} \tag{48}$$

(here and in the following considerations the indices $\pm$ are omitted). In equ (37) only the terms proportional P_α' and $P_{\alpha+1}''$ are important for $s \to \infty$.

In the following we shall evaluate the contributions of one single Regge pole to $A_i(s,t)$.

We have from equ (43) :

$$A_i(s,t) = -\frac{\pi}{2\sin\pi\alpha}\sum_\varkappa \beta^\varkappa\left[\varepsilon_{\varkappa i} Z_\alpha^{\varkappa i}(-X_t) \pm Z_\alpha^{\varkappa i}(X_t)\right] \quad (49)$$

The asymptotic expressions of $Z_\alpha^{\varkappa i}$ are derived starting from equ (37) and equ (48). They are :

$$Z_\alpha^{11}(t,-X_t) = \frac{4im^2\sqrt{t}}{(t-\mu^2)(t-4m^2)}\,\alpha\, P_{\alpha+1}''(-X_t)$$

$$= \gamma^{11}(t)\,(-t)^{\frac{\alpha}{2}}\, s^{\alpha-1} \quad (50)$$

Here we have introduced the coefficient :

$$\gamma^{11}(t) \equiv (-1)^{\frac{\alpha}{2}-1}\,\frac{4im^2}{\sqrt{\pi}}\,2^{2\alpha-1}\,\frac{\Gamma(\alpha+\frac{3}{2})}{\Gamma(\alpha+2)}\,\alpha^2(\alpha+1)(t-\mu^2)^{-\alpha}(t-4m^2)^{-\frac{\alpha}{2}-\frac{1}{2}} \quad (51)$$

In the same way we obtain

$$Z_\alpha^{21} = 0\,(\,s^{\alpha-2}) \quad (52)$$

$$Z_\alpha^{31}(t,-X_t) = \gamma^{31}(t)\,(-t)^{\frac{\alpha}{2}+\frac{1}{2}}\,s^{\alpha-1} \quad (53)$$

where

$$\gamma^{31}(t) \equiv (-1)^{\frac{\alpha}{2}+\frac{1}{2}}\,\frac{2im}{\sqrt{\pi}}\,2^{2\alpha-1}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\,\alpha(2\alpha+1)(t-\mu^2)^{-\alpha}(t-4m^2)^{-\frac{\alpha}{2}-\frac{1}{2}} \quad (54)$$

For $Z_\alpha^{\varkappa i}(t,-X_t)$ (i = 2,3,4) the asymptotic expressions are easily obtained using the relations equ (37) between the $Z_\alpha^{\varkappa i}$:

$$Z_\alpha^{12}(t,-X_t) = -\frac{1}{t} Z_\alpha^{11} = \gamma^{11}(t)\,(-t)^{\frac{\alpha}{2}-1}\, s^{\alpha-1} \tag{55}$$

$$Z_\alpha^{22} = O(s^{\alpha-2}) \tag{56}$$

$$Z_\alpha^{32}(t,-X_t) = Z_\alpha^{42}(t,-X_t) = -\frac{1}{t} Z_\alpha^{31}(t,-X_t) = \gamma^{31}(t)(-t)^{\frac{\alpha}{2}-\frac{1}{2}} s^{\alpha-1} \tag{57}$$

$$Z_\alpha^{13} = O(s^{\alpha-2}) \tag{58}$$

$$Z_\alpha^{23}(t,-X_t) = \frac{1}{2m}\sqrt{1-\frac{4m^2}{t}}\, Z_\alpha^{11}(t,-X_t) = \frac{i}{2m}\sqrt{t-4m^2}\,\gamma^{11}(t)(-t)^{\frac{\alpha}{2}-\frac{1}{2}} s^{\alpha-1} \tag{59}$$

$$Z_\alpha^{14}(t,-X_t) = \frac{1}{2m} Z_\alpha^{11}(t,-X_t) = \frac{1}{2m}\gamma^{11}(t)(-t)^{\frac{\alpha}{2}} s^{\alpha-1} \tag{60}$$

$$Z_\alpha^{24} = O(s^{\alpha-2})$$

$$Z_\alpha^{34}(t,-X_t) = \frac{2m}{t} Z_\alpha^{31}(t,-X_t) = -2m\,\gamma^{31}(t)(-t)^{\frac{\alpha}{2}-\frac{1}{2}} s^{\alpha-1} \tag{61}$$

We notice that in the limit of large s all amplitudes $A_i(s,t)$ are proportional to $t^{\gamma(t)}$ where $\gamma(t)$ depends on the trajectory $\alpha(t)$.

§ 4. Cross Section for Single Regge Poles.

The differential cross section for photoproduction in the barycentric system is usually expressed as :

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}\sum_{\vec{\varepsilon}} \frac{1}{4}\, Sp\,(\mathcal{F}\mathcal{F}^{+}) \tag{62}$$

$\mathcal{F}$ is given by :

$$\mathcal{F} = i\,(\vec{\sigma}.\vec{\varepsilon})\,\mathcal{F}_1 + \frac{1}{qk}(\vec{\sigma}.\vec{q})\,\vec{\sigma}.(\vec{k}\times\vec{\varepsilon})\,\mathcal{F}_2 + \frac{i}{qk}(\vec{\sigma}.\vec{k})(\vec{q}.\vec{\varepsilon})\,\mathcal{F}_3 + \frac{i}{q^2}(\vec{\sigma}.\vec{q})(\vec{q}.\vec{\varepsilon})\,\mathcal{F}_4 \tag{63}$$

Equ (62) stands for the cross section of unpolarized photons and unpolarized nucleons in the initial and final state. $\vec{\varepsilon}$ is the polarization vector of the photon. We substitute equ(63) into equ (62), the result is :

$$\frac{k}{q}\frac{d\sigma}{d\Omega} = |\mathcal{F}_1 - \mathcal{F}_2|^2 + (1-\cos\Theta)\, 2\, \mathrm{Re}(\mathcal{F}_1 \mathcal{F}_2^*) \qquad (64)$$
$$+\sin^2\Theta\left[\tfrac{1}{2}|\mathcal{F}_3+\mathcal{F}_4|^2 + \mathrm{Re}(\mathcal{F}_1\mathcal{F}_4^* + \mathcal{F}_2\mathcal{F}_3^*) - (1-\cos\Theta)\mathrm{Re}(\mathcal{F}_3\mathcal{F}_4^*)\right]$$

The F_i are functions of A_1, A_2, A_3 and A_4. The relations are derived in ref. 1 and 2. In the limit of large s they are :

$$\mathcal{F}^{(+)} \equiv \mathcal{F}_1 + \mathcal{F}_2 = \frac{S}{8\pi} A_4 \qquad (65)$$

$$\mathcal{F}^{(-)} \equiv \mathcal{F}_1 - \mathcal{F}_2 = \sqrt{\frac{S}{8\pi}}\; A_1$$

$$G^{(+)} \equiv \mathcal{F}_3 + \mathcal{F}_4 = \frac{S}{16\pi}\,(A_3 - A_4)$$

$$G^{(-)} \equiv \mathcal{F}_3 - \mathcal{F}_4 = \frac{S^{3/2}}{16\pi}\, A_2$$

In the limit $s \to \infty$ we can write for the differential cross section :

$$\frac{d\sigma}{d\Omega} = |\mathcal{F}^{(-)}|^2 + (1-\cos\Theta)\tfrac{1}{2}|\mathcal{F}^{(+)}|^2 \qquad (66)$$
$$+\sin^2\Theta\left[\tfrac{1}{2}|G^{(+)}|^2 + \tfrac{1}{2}\mathrm{Re}(\mathcal{F}^{(+)}G^{(+)*} - \mathcal{F}^{(-)}G^{(-)*}) + \frac{(1-\cos\Theta)}{4}|G^{(-)}|^2\right]$$

Since t is considered finite we have to express $d\Omega$ and $\cos\Theta$ by s and t. For $s \to \infty$ we have :

$$\cos\Theta = 1 + \frac{2t}{s} \qquad (67)$$

and therefore for equ (66) :

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}\Big\{|A_1|^2 - t\,|A_4|^2 \qquad (68)$$
$$- t\left[\tfrac{1}{2}|A_3 - A_4|^2 + \mathrm{Re}\left(A_4(A_3^* - A_4^*) - A_1 A_2^*\right) - \tfrac{t}{2}|A_2|^2\right\}$$

We see that the cross section for large s behaves like $s^{2(\alpha(t)-1)}$ since all $A_i \sim s^{\alpha(t)-1}$ according to equ (48). This factor gives rise to a diffraction like dependence which is modified by extra factors t^{γ} contained in the amplitudes A_i and in the cross section explicitly.

Formula equ (68) will be applied now to the exchange of the prominent candidates, a resonance with spin $j = 1^-$ which shall represent the ρ, the ω- or the φ-meson and a particle with spin $j = 0^-$ which will stand for the pion.

It must be asked whether all helicity amplitudes really appear in A_1, A_2, A_3 and A_4. We notice that α_j^2 was eliminated because it is multiplied with expansion functions $Z_\alpha{}^{2i}$ which vanish stronger for $s \to \infty$ than the contributions multiplied by $\alpha_j{}^{\varkappa}$ ($\varkappa = 1,3$ and 4). The $\alpha_j{}^{\varkappa}$ are related to the helicity amplitudes $T_{\lambda\bar{\lambda}}^{j,1}$ of the reaction $\gamma + \pi \to N + \bar{N}$ by equ (36). It can be seen that they describe transitions to triplet or singlet states of the $N\bar{N}$-system with definite parity π_f. α_j^1, α_j^2 and $\alpha_j{}^3$ lead to triplet states with parity $(-1)^j$, $(-1)^{j+1}$ and $(-1)^j$ respectively. $\alpha_j{}^4$ describes singlet transitions with $\pi_f = (1)^{j+1}$. These statements follow immediately from the definition of $\alpha_j{}^{\varkappa}$ in terms of helicity amplitudes and the property of $N\bar{N}$ - helicity states against the parity operation P [12]:

$$P\,|\,j,m,\lambda,\bar{\lambda},\rangle = (-1)^j\,|\,j,m,-\lambda,-\bar{\lambda}\rangle \qquad (69)$$

Therefore transitions to triplet $N\bar{N}$-states with negative (positive) parity and odd (even) total angular momentum j are described by $\alpha_j{}^1$ and $\alpha_j{}^3$ whereas $\alpha_j{}^2$ contributes to triplet transitions with positive (negative) parity and odd (even) j. It is clear that the exchange of particles with $j = 1^-$ can only contribute to $\alpha_j{}^1$ and $\alpha_j{}^3$. Pseudoscalar particles like the pion should contribute only to α_j^4 if a term with $j = 0$ is present in the expansion $A_i(s,t)$ into helicity states (equ (33)).

This is not the case because of the transversality of the photon. The sum in equ (33) starts with $j = 1$. Nevertheless the statement that pion exchange is determined by α_j^4 alone remains true as will be shown later.

The discussion above shows that for j fixed the particular $\alpha_j^{\varkappa}$ are connected with definite parity and spin of the $N\bar{N}$-system. If now the total angular momentum j is continued to complex values these relations between $\varkappa$ on one side and parity and spin on the other side remain fixed.

From other reactions it is known that the $j = 1$ resonances have negative signature. This is in agreement with the construction following equ (40) of section 3.

Besides the trajectory function $\alpha_-(t) \equiv \alpha_\rho(t)$ we encounter two unknown functions $\gamma_1(t)$ and $\gamma_2(t)$ to be defined later. This reflects the fact that two independent coupling constants appear in the coupling of a vector meson to the $N\bar{N}$-system, for instance the so called "charge" and the "anomalous moment" coupling constant. In terms of these coupling constants (f_1 and f_2) the amplitudes for exchange of an "elementary" vector ρ-meson with mass m_ρ^2 and width Γ_ρ in lowest order perturbation theory are :

$$A_1 = -t \, \frac{f_2}{2m} \, \frac{\lambda}{m_\rho^2 - i\,\Gamma_\rho m_\rho - t} \quad ; \quad A_2 = -\frac{1}{t} A_1 \qquad (70)$$

$$A_3 = 0 \quad ; \quad A_4 = f_1 \, \frac{\lambda}{m_\rho^2 - i\,\Gamma_\rho m_\rho - t}$$

Here λ is the $\rho - \pi - \gamma$ coupling constant. By comparison with equ (49) up to equ (61) the constants λf_1 and λf_2 can be related to the residues $\beta^{\varkappa}$ at $t = m_\rho^2$. Furthermore $\mathrm{Re}\,\alpha_\rho(m_\rho^2)=1$ and Γ_ρ is related to $\mathrm{Im}\,\alpha_\rho(m_\rho^2)$ and $\mathrm{Re}\,\alpha'_\rho(m_\rho^2)$ as has been written down in ref. 5 for instance.

The differential cross section for "elementary" ρ - exchange in the limit $s \rightarrow \infty$ is :

$$\frac{d\sigma}{dt} = \frac{1}{32\pi} \left| \frac{\lambda}{m_\rho^2 - i\Gamma_\rho m_\rho - t} \right|^2 (-t) \left(|f_1|^2 - t \left| \frac{f_2}{2m} \right|^2 \right) \tag{71}$$

This formula is easily derived by substituting the $A_i(s,t)$ (equ (70)) into the cross section formula equ (68).

Now we want to derive the cross section for the exchange of a 1^- particle called ρ meson in the following as a Regge pole. We mentioned already that in this case only the amplitudes α_j^1 and α_j^3 contribute. Therefore as residues only β^1 and β^3 appear. Starting from equ (49) and substituting the asymptotic expressions for $Z_\alpha^{\varkappa i}$, given in equ (50) - (59), the complete amplitudes for ρ exchange in the limit $s \rightarrow \infty$ are :

$$A_1 = \frac{\pi}{\sin\pi\alpha_\rho} \left(1 - e^{-i\pi\alpha_\rho}\right) s^{\alpha_\rho - 1} (-t)^{\frac{1}{2}(\alpha_\rho + 1)} \gamma_2(t) \tag{72}$$

where $$\gamma_2(t) \equiv (-t)^{-\frac{1}{2}} \gamma^{11}(t)\, \beta^1(t) + \gamma^{31}(t)\, \beta^3(t) \tag{73}$$

$$A_2 = -\frac{1}{t} A_1 \quad , \quad A_3 = 0 \tag{74}$$

$$A_4 = \frac{\pi}{\sin\pi\alpha_\rho} \left(1 - e^{-i\pi\alpha_\rho}\right) s^{\alpha_\rho - 1} (-t)^{\frac{1}{2}(\alpha_\rho - 1)} \gamma_1(t) \tag{75}$$

with $$\gamma_1(t) \equiv \frac{(-t)^{\frac{1}{2}}}{2m} \gamma^{11}(t)\, \beta^1(t) - 2m\, \gamma^{31}(t)\, \beta^3(t) \tag{76}$$

The residue functions $\gamma_1(t)$ and $\gamma_2(t)$ are defined in a way that the amplitudes A_1, A_2 and A_4 behave near $t = m_\rho^{\,2}$ as the amplitudes of the "elementary" ρ written down in equ (70). Therefore we expect $\gamma_1(t)$ and $\gamma_2(t)$ to be relatively constant near the pole. Whether this is still the case in the physical region $t < 0$, we are interested in, is an open question. In particular because of the $\frac{1}{\sqrt{-t}}$ appearing in $\gamma_2(t)$ one might think that $\gamma_2(t)$ changes rapidly with t. This $(-t)^{-1/2}$ can as well be of kinematical origin (see equation for Z_α^{1i} in equ (37)) and then might be compensated by a factor $\sqrt{t}$ in $\beta^1(t)$.

We remark that the relations $A_3 = 0$ and $A_2 = -\frac{1}{t} A_1$ are valid for a ρ-meson "elementary" as well as a Regge pole.

Now we calculate the cross section with the amplitudes in equ (72) - (76). The result is :

$$\frac{d\sigma}{dt} = \frac{\pi}{32} \left| \frac{1 - e^{-i\pi\alpha_\rho}}{\sin \pi\alpha_\rho} \right|^2 s^{2(\alpha_\rho - 1)} (-t)^{\alpha_\rho} \times \left\{ |\gamma_1(t)|^2 - t\, |\gamma_2(t)|^2 \right\} \tag{77}$$

We see that the formula for Regge pole exchange (equ (77)) is distinguished from the "elementary" ρ-exchange by the following properties :

1) The pole term which changes strongly with t is replaced by $\left| \frac{1 - e^{-i\pi\alpha_\rho(t)}}{\sin \pi\alpha_\rho(t)} \right|^2$ which is less t dependent if $\alpha_\rho(t)$ differs from 1.

2) the factor (-t) which causes a minimum of $\frac{d\sigma}{dt}$ in the forward direction is reduced to $(-t)^{\alpha_\rho}$ with a t-dependent exponent $\alpha_\rho(t) < 1$.

3) the cross section decreases with energy as $s^{2(\alpha_\rho(t)-1)}$ the decrease depends on t. In $\pi-\pi$, $\pi-N$ or $N-N$ scattering the equivalent factor is responsible for shrinking of the diffraction peak.

If we write with an arbitrary constant S_0 :

$$F_\rho(t) = \frac{\pi}{32}\, S_0^{\,2(\alpha_\rho-1)} \left| \frac{1-e^{-i\pi\alpha_\rho}}{\sin\pi\alpha_\rho} \right|^2 \left(|\gamma_1(t)|^2 - t|\gamma_2(t)|^2 \right) \tag{78}$$

and approximate $\alpha_\rho(t)$ for small t by :

$$\alpha_\rho(t) = \alpha_\rho(0) + t\alpha_\rho'(0)$$

the cross section for large s and very small t becomes :

$$\frac{d\sigma}{dt} = \left(\frac{s}{s_0}\right)^{-2(1-\alpha_\rho(0))} F_\rho(t)\,(-t)^{\alpha_\rho(0)}\, e^{2t\alpha_\rho'(0)\ln\frac{s}{s_0}} \tag{79}$$

If $F_\rho(t)$ is less t dependent than the rest the characteristic angular dependence is given by a diffraction pattern modified by the factor $(-t)^{\alpha_\rho(0)}$.

The discussion of pion exchange, elementary or as Regge pole, becomes complicated by the fact that the exchange graph fig. 1 for B replaced by the pion is not gauge invariant. Therefore in the sum equ (33) a term with j = 0 which could be related to the pion is not present. To get some insight into the problem we shall start the discussion with the charge part of the amplitude in lowest order perturbation theory (also with respect to the strong interactions). Then the amplitude T is given by the sum of the three Feynman-graphs presented in fig. 3

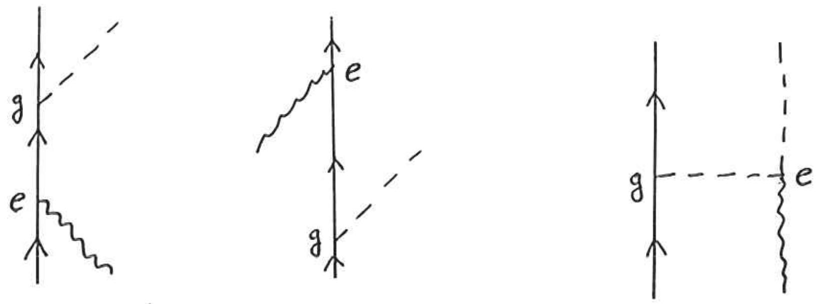

fig. 3

with coupling constants e for the charge and g for π-N-coupling. First we write down the isospin decomposition of these graphs. If the isospin independent matrixelements of the graphs in fig. 3 are D_1, D_2 and D_3 respectively, their sum is identical to:

$$T^{e}_{Born} = (D_1 + D_2)\,\frac{1}{2}\,(g_\beta^{+} + g_\beta^{o}) + \frac{1}{2}\,g_\beta^{-}\,(D_1 - D_2 + 2D_3) \qquad (80)$$

with isospon matrices $g_\beta^{\pm}$, g_β^{o} defined in section 1. It is well known that $D_1 + D_2$ and $D_1 - D_2 + D_3$ are gauge invariant. One pion exchanged characterized by a pole term $\frac{1}{t - \mu^2}$ is only contained in $D_1 - D_2 + 2D_3$. Therefore "elementary" π-exchange contributes only to the isospin invariant amplitude $T^{(-)}$ but not to $T^{(+)}$ and $T^{(o)}$ as was already affirmed in section 1.

The next step is to find out which invariant functions $A_i(s,t)$ are present in $\frac{1}{2}\,(D_1 - D_2 + 2D_3)\,g_\beta^{-}$. The only amplitude with a pole in t at $t = \mu^2$ is A_2. It is equal to :

$$A_2 = \frac{2s + t - 2m^2 - \mu^2}{(s - m^2)\,(u - m^2)(t - \mu^2)}\,eg \qquad (81)$$

That only A_2 contains a pole is already clear since D_3 is proportional to B_3 which is related to the gauge invariant amplitude A_2 only (see equ (12) and (15)). $A_2 \sim s^{-1}$ as expected for spin 0 "elementary" particles (equ (38)).

Earlier it was mentioned that π-exchange contributes only to α_j^4. We expand the expression equ (81) into functions $Z_j^{\varkappa 2}(t, X_t)$ defined in equ (37). The result is :

$$\alpha_j^1 = \alpha_j^2 = \alpha_j^3 = 0 \qquad \text{for all } j,$$

$$\alpha_j^4 = \frac{i}{2m} \frac{t}{t - \mu^2} \frac{1}{2j+1} \left(Q_{j-1}(b) - Q_{j+1}(b) \right) \tag{82}$$

where $b = \left(\frac{t}{t - 4m^2} \right)^{1/2}$ and $j = 2, 4, 6, \text{-------}$

This proves the statement made earlier that for pion exchange $\alpha_j^1 = \alpha_j^2 = \alpha_j^3 = 0$ and $\alpha_j^4 = 0$. Furthermore $\alpha_j^4 = 0$ for j odd consistent with positive signature of the pion.

It is interesting to calculate the cross section for "elementary" π-exchange in the limit s $\dashrightarrow \infty$. We substitute the asymptotic A_2 into equ (68), the result is :

$$\frac{d\sigma}{dt} = \frac{1}{8\pi} e^2 g^2 \frac{t^2}{s^2 (t - \mu^2)^2} \tag{83}$$

Now we derive the formula of the angular distribution for Regge exchange of the pion. Since $\alpha_\pi = 0$ for $T = \mu^2$ we expect $\alpha_\pi(t) < 0$ in the physical region of $\gamma + N_1 \dashrightarrow \pi + N_2$. Therefore from equ (48) it follows that the pion contribution decreases stronger with energy than the cross section originating from the vector mesons. Since physical amplitudes for photoproduction on protons or neutrons are linear combinations of pion and vector meson amplitudes, comparison with experiment must start with the appropriate combinations defined by equ (8). To compare with elementary pion exchange discussed earlier we give the formula for pion Regge exchange alone.

First the amplitude for this case is :

$$A_2 = \frac{\pi}{\sin \pi \alpha_\pi} \left(1 + e^{-i\pi\alpha_\pi}\right) (-t)^{\alpha_\pi/2} \, s^{\alpha_\pi - 1} \, g(t) \tag{84}$$

where $g(t) = (-t)^{-1/2} \gamma^{31}(t) \beta^4(t)$ (85)

Concerning the factor $(-t)^{-1/2}$ in the residue function g(t) we refer to the similar situation in ρ-exchange. g(t) is defined such that $A_2(s,t)$ reduces to A_2 in equ (81) if $s \dashrightarrow \infty$ and $t \dashrightarrow \mu^2$. Then the cross section for π-Regge exchange is :

$$\frac{d\sigma}{dt} = \frac{\pi}{2} \left| \frac{1 + e^{-i\pi\alpha_\pi}}{\sin \pi \alpha_\pi} \right|^2 s^{2(\alpha_\pi - 1)} (-t)^{\alpha_\pi + 2} g^2(t) \tag{86}$$

which reduces near t = 0 to :

$$\frac{d\sigma}{dt} = \left(\frac{s}{s_0}\right)^{-2(1-\alpha_\pi(0))} (-t)^{\alpha_\pi(0)+2} \, e^{2t\alpha_\pi'(0) \ln s/s_0} F_\pi(t) \tag{87}$$

with

$$\alpha_\pi(t) = \alpha_\pi(0) + t\,\alpha_\pi'(0) \tag{88}$$

and

$$F_\pi(t) \equiv \frac{\pi}{32} \left| \frac{1 + e^{-i\pi\alpha_\pi}}{\sin \pi \alpha_\pi} \right|^2 g^2(t) \, s_0^{2(1-\alpha_\pi(t))} \tag{89}$$

Conclusions

The cross section formulas we have derived for ρ-meson and pion Regge exchange make some prediction about the behaviour of the angular distribution as a function of s and t in the forward direction. The study of the s-dependence tells us something about the Regge trajectories near t = 0. We tried also to obtain a result about the t-dependence of the cross section. This is given by two factors, first the rather definite factor $e^{2t\alpha' \ln s/s_0}$ which depends on the derivative α' of the trajectory, second a factor $(-t)^{\gamma(t)}$ F(t) where $\gamma(t)$ depends on the trajectory function $\alpha(t)$. $(-t)^{\gamma(t)}$ was factored out to make F (t) $\approx$ constant near the pole. Whether this behaviour of F(t) survives in the physical region $t < 0$ is not known. From this point of view the factor $(-t)^{\gamma(t)}$ appears somewhat arbitrary. It is an interesting problem to look whether the cross section goes to zero for t ---> 0 as proposed here and to determine the exponent $\gamma(t)$.

We think that the pion is not important for very high energies as explained in the last section. Then the hypothesis of dominance of ρ, ω- and φ-meson exchange correlates the cross sections for the four reactions $\gamma + N_1 \dashrightarrow N_2 + \pi^{\pm,0}$ as given by equ (8) of section 1. In particular the situation is very simple if only one Regge pole dominates. For instance if only the ρ is present we have

$$\left(\frac{d\sigma}{dt}\right)_{\pi^+} : \left(\frac{d\sigma}{dt}\right)_{\pi^0} : \left(\frac{d\sigma}{dt}\right)_{\pi^-} : \left(\frac{d\sigma}{dt}\right)_{n\pi^0} = 2:1:2:1$$

On the other side if $\omega + \varphi$ dominate we have

$$\left(\frac{d\sigma}{dt}\right)_{\pi^0} = \left(\frac{d\sigma}{dt}\right)_{n\pi^0} \quad \text{and} \quad \left(\frac{d\sigma}{dt}\right)_{\pi^\pm} \ll \left(\frac{d\sigma}{dt}\right)_{\pi^0}$$

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Addendum to:

PHOTOPRODUCTION OF PIONS IN FORWARD DIRECTION AND REGGE POLES

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In formula equ (20) and equ (21) the kinematic singularities with respect to t should be separated [16) in the following form:

$$B_i^{d,e}(\ell,t) = (2\,pk')^{\ell}\,\tilde{B}_i^{d,e}(\ell,t)$$

If now the residues $\beta_{\pm}^{x}(t)$ in equ (44) are defined with respect to $\tilde{B}^{d,e}(\ell,t)$ instead of $B_i^{d,e}(\ell,t)$ the factor $(-t)^{\alpha_\rho}$ in equ (77) and equ (79) respectively and the factor $(-t)^{\alpha_\pi}$ in equ (86) and equ (87) respectively do not appear. Of course, the t-dependence of the new functions $\tilde{\gamma}_1(t)$, $\tilde{\gamma}_2(t)$ and $\tilde{g}(t)$ is still undetermined.

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