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Periodic dispersion suppressors II

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Abstract

There exist a few dispersion suppressors of practical value that have a strictly periodic horizontal beta function. Given here as the result of an analytic survey, they provide criteria for the optimum design of dispersion suppressors.

Introduction

In large storage rings, the dispersion is usually matched to vanish in the straight section. This matching is usually done in the last section of the arc, called "dispersion suppressor", which is here assumed to consist of a periodic magnet geometry similar to the normal cell structure of the arc. However, in order to gradually decrease the dispersion, some of the cell parameters as bending angle, quadrupole strengths or cell length will differ from those in the arc.

Once the number n and length ℓ of dispersion suppressor half cells is chosen, it is the designer's art to individually fix the quadrupole strengths such that, with vanishing dispersion, the beam will stay confined to about the same size as in the arc. In this optimum case, the beam size assumes a quasi-periodic appeal throughout the dispersion suppressor. A good dispersion suppressor will thus be close to periodic, and this is the reason why a survey of strictly periodic dispersion suppressors might tell us in which vicinity to look for a good design.

Classification of periodic dispersion suppressors

In order to keep the analysis transparent, we assume equal focusing strengths in all quadrupoles of type F (horizontally focusing) and type D (horizontally defocusing) in the normal cell of the arc and in the dispersion suppressor cell, respectively. Then the dispersion suppressor cell is characterized by the following 3 parameters:

- The quadrupole focusing strength or, equivalently, the horizontal betatron phase advance $\underline{\varphi}$ per half cell,
- the length $\underline{\ell}$ of the half cell,
- the bending angle $\underline{\varepsilon_m}$ of the dipole magnet in the centre of the half cell.

Equivalently, we characterize the normal arc cell by $\underline{\Phi}$, $\underline{L}$, and $\underline{\varepsilon_M}$.

Let us assume that the normal arc cell is fully determined by general design considerations concerning beam emittance (for electrons), chromaticity correction, dynamic aperture and technical aspects, for example. Then for given Φ, L, and ε_M, the 3 dispersion suppressor parameters φ, ℓ, and ε_m can be chosen such that the dispersion D, its slope D' and the periodic horizontal beta function are matched at the joint to the normal arc cell. A simultaneous match of also the vertical beta function would require one more free parameter which could be an independent variation of the families of F- and D-quadrupoles, but this would complicate the analysis so much that we here prefer to ignore the vertical match. In numerical design tests it turns out anyway that, by slight varation of individual quadrupoles, the vertical beam envelope can be corrected without seriously affecting the dispersion match and the quasi-periodic appeal of the horizontal envelope.

In our model, then, everything is determined once we know in addition

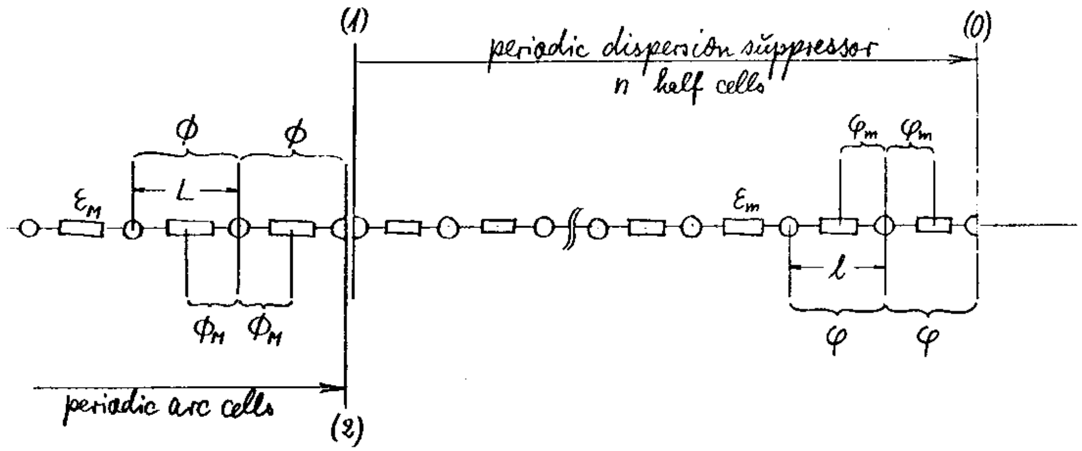
- the number $\underline{n}$ of half cells in the dispersion suppressor and
- the type of quadrupole (F or D) at the end of the dispersion suppressor after the last bending magnet.

The matching will be different for having an even or odd number n and for ending with an F- or D-quad, respectively. Thus, the dispersion suppressors are classified by the distinctions

- no. of half cells n: even/odd
- quadrupole type at the end: F/D

and each of the 4 classes will, when matched, have different cell parameters.

Matching conditions



In the periodic cell we have in short lens approximation [1)]

$$\frac{\hat{\beta}}{\ell} = \frac{2(1+\sin\varphi)}{\sin 2\varphi} \; ; \qquad \frac{\check{\beta}}{\ell} = \frac{2(1-\sin\varphi)}{\sin 2\varphi}$$

$$\frac{\beta_m}{\ell} = \frac{1}{2\ell}(\hat{\beta} + \check{\beta}) = \frac{2}{\sin 2\varphi}$$

where the index m refers to the centre of the dipole magnet. The phase advance between dipole centre and quadrupole as defined in the sketch is

$$\varphi_m = \frac{\varphi}{2} \pm \frac{\ell}{12}\left(\frac{1}{\check{\beta}} - \frac{1}{\hat{\beta}}\right) = \frac{\varphi}{2} \pm \frac{1}{6}\sin\varphi\tan\varphi = \frac{\varphi}{2} \pm \Delta\varphi_m$$

where the sign of $\Delta\varphi_m$, and similarly of $\Delta\Phi_M$, depends on quadrupole polarity (D or F) and thus on the class of dispersion suppressor as follows:

	F-quad	D-quad
even n	class ① $\Delta\Phi_M > 0$, $\Delta\varphi_m > 0$	class ② $\Delta\Phi_M < 0$, $\Delta\varphi_m < 0$
odd n	class ④ $\Delta\Phi_M < 0$, $\Delta\varphi_m > 0$	class ③ $\Delta\Phi_M > 0$, $\Delta\varphi_m < 0$

Then, in order to identically vanish at the end (point 0), the values of the dispersion and its slope at the beginning of the dispersion suppressor (point 1) must be

$$D_1 = \sqrt{\beta_1}\sqrt{\beta_m}\,\varepsilon_m \frac{\cos\varphi_m}{\sin\varphi}(\cos n\varphi - 1) \qquad \text{f. even n (classes ①, ②)}$$ [2)]

$$D_1 = \sqrt{\beta_1}\sqrt{\beta_m}\,\varepsilon_m \frac{\cos\varphi_m}{\sin\varphi}\left(\cos n\varphi - \frac{\cos(\varphi - \varphi_m)}{\cos\varphi_m}\right) \qquad \text{f. odd n (classes ③, ④)}$$

$$D_1' = \frac{\sqrt{\beta_1}}{\sqrt{\beta_m}}\,\varepsilon_m \frac{\cos\varphi_m}{\sin\varphi}\sin n\varphi \qquad \text{①, ②, ③, ④}$$

On the other hand, the periodic dispersion in the arc cell is given by

$$D_2 = -\sqrt{\beta_2}\sqrt{\beta_M}\,\varepsilon_M \frac{\cos\Phi_M}{\sin\Phi}$$

$$D_2' = 0$$

The cell parameters φ, ℓ, ε_m of the matched periodic dispersion suppressor are then determined by the 3 equations

$$\beta_1 = \beta_2 \quad \text{with} \quad \begin{cases} \beta = \hat{\beta} & \text{in classes ①, ③} \\ \beta = \check{\beta} & \text{in classes ②, ④} \end{cases} \qquad (1)$$

$$D_1 = D_2 \qquad (2)$$

$$D_1' = D_2' = 0 \qquad (3)$$

Explicitly, we subsequently obtain

- from eq.(3) the <u>phase shift φ</u> per half period

$$n\varphi = k\pi \quad , \quad k \text{ odd integer} \qquad (k = 1 \text{ in practical cases}) \qquad (4)$$

and thus for $D_1' = 0$, i.e. with $\cos n\varphi = -1$

$$\begin{cases} D_1 = -\sqrt{\beta_1}\sqrt{\beta_m}\,\varepsilon_m \dfrac{2\cos\varphi_m}{\sin\varphi} & \text{f. even } n \\ D_1 = -\sqrt{\beta_1}\,\sqrt{\beta_m}\,\varepsilon_m \dfrac{\cos\varphi_m + \cos(\varphi - \varphi_m)}{\sin\varphi} & \text{f. odd } n \end{cases}$$

- from eq.(1) the <u>length ratio λ</u>

$$\lambda = \frac{\ell}{L} = \frac{\dfrac{1 \pm \sin\Phi}{\sin 2\Phi}}{\dfrac{1 \pm \sin\varphi}{\sin 2\varphi}} \qquad \begin{matrix} \text{(upper sign for classes ①, ③)} \\ \text{(lower sign for classes ②, ④)} \end{matrix} \qquad (5)$$

- from eq.(2) the <u>bend ratio r</u>

$$r = \frac{\varepsilon_m}{\varepsilon_M} = \sqrt{\frac{1}{\lambda}\frac{\sin 2\varphi}{\sin 2\Phi}}\;\frac{\cos\Phi_M}{\sin\Phi}\;\frac{\sin\varphi}{2\cos\varphi_m} \qquad \text{for even } n \text{ (classes ①, ②)}$$

$$r = \frac{\varepsilon_m}{\varepsilon_M} = \sqrt{\frac{1}{\lambda}\frac{\sin 2\varphi}{\sin 2\Phi}}\;\frac{\cos\Phi_M}{\sin\Phi}\;\frac{\sin\varphi}{\cos\varphi_m + \cos(\varphi - \varphi_m)} \qquad \text{for odd } n \text{ (classes ③, ④)} \qquad (6)$$

<u>Numerical results and discussion</u>

The results are given in the tables at the end and are graphically displayed in Figs. 1 and 2 where the length ratio λ and the bend ratio r are shown as a function of the phase advance 2Φ (normal arc cell) for various numbers n of half cells in the dispersion suppressor.

As already apparent from eqs.(5) and (6), there is not a big difference between even and odd n-values, and the main class distinction is whether the joint between arc and dispersion suppressor occurs at maximum or minimum beta, i.e. in an F-type or D-type quadrupole. So the curves of classes ① and ③, when superimposed, fold smoothly into each other, and also the curves of classes ② and ④.

With a horizontally focusing quadrupole at the joint (classes ①, ③), the length ratio λ stays nearly constant and close to $\lambda = 1$ for large values of Φ and increases only somewhat at small Φ. The bend ratio r is generally smaller than λ, starting around $r = \frac{1}{2}$ at large Φ and rising more steeply toward smaller values of Φ. Only for n = 3, 4, and 5, the r-curve crosses the corresponding λ-curve at phase advances $2\Phi \approx 80°$, $45°$ and $25°$, respectively.

With a horizontally defocusing quadrupole at the joint (classes ②, ④), the bend ration r stays nearly constant and close to $r = \frac{1}{2}$ for large values of Φ and increases only somewhat at small Φ. It again is generally smaller than the length ratio λ which rises much more steeply toward smaller values of Φ. For n-values between 5 and 9, the λ-curves intersect their corresponding r-curve at large Φ-values in the vicinity of $2\Phi \approx 90°$.

For practical application, we are mostly interested in those dispersion suppressor configurations which use the ring length economically and thus have about the same bending per meter as the regular arc cell; i.e. we are looking for the configurations with $r \approx \lambda$. These configurations are emphasized in the tables and also in Figs. 1 and 2 where, in addition, the crossing points between r- and λ-curves are indicated. Good dispersion suppressor designs are expected to be in the neighbourhood of these crossing points. Numerical design tests reveal that satisfactory quasi-periodic solutions can even be found there when deviating from the ideal length ratio and making the dispersion suppressor half cells physically identical to the arc cells, which may be of great technical advantage.

For rather small phase advances around $2\Phi = 45°$ in the arc cell, good solutions can thus be expected in class ① with n = 4 half cells in the dispersion suppressor, and also in class ③ with n = 5. For large phase advances around $2\Phi = 90°$, good solutions appear in class ③ with n = 3 and in classes ② or ④ with 5-9 half cells. The solutions in classes ① and ③ (F-quad at the joint) require in the dispersion suppressor increased quadrupole strengths and therefore, possibly, longer quadrupoles. In this respect, the solutions (for large Φ) in classes ②, ④ (D-quad at the joint) are more favourable since they demand less quadrupole strength.

In electron rings, a large variation of Φ may be required in order to match the beam emittance to the ring acceptance at various energies. Then, the class ③ dispersion suppressor is the favourite because it permits periodic operation with $r = \lambda$ at higher Φ-values (with 3 half cells) as well as at lower Φ-values (with 5 half cells). However, an alternative configuration may also be quite good for variable-Φ operation. Observing that class ① and class ④ both end with an F-quad and, in an existing machine, can thus be transformed into each other by merely changing the strength (not the sign!) of some quadrupoles, we find here also periodic solutions with $r = \lambda$ at low and medium Φ (class ①, n = 4) as well as at high Φ (class ④, n = 5 or 7).

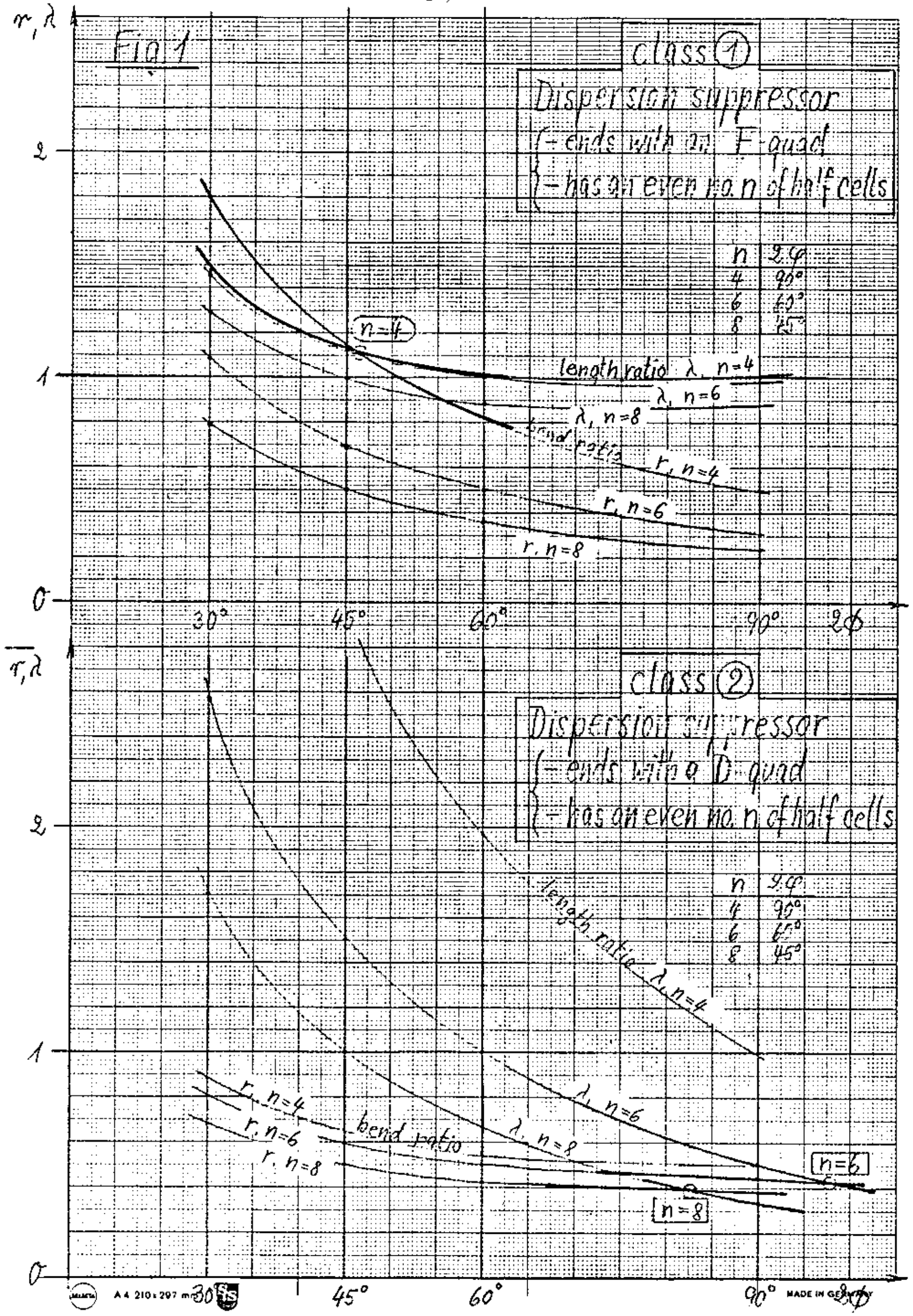
References

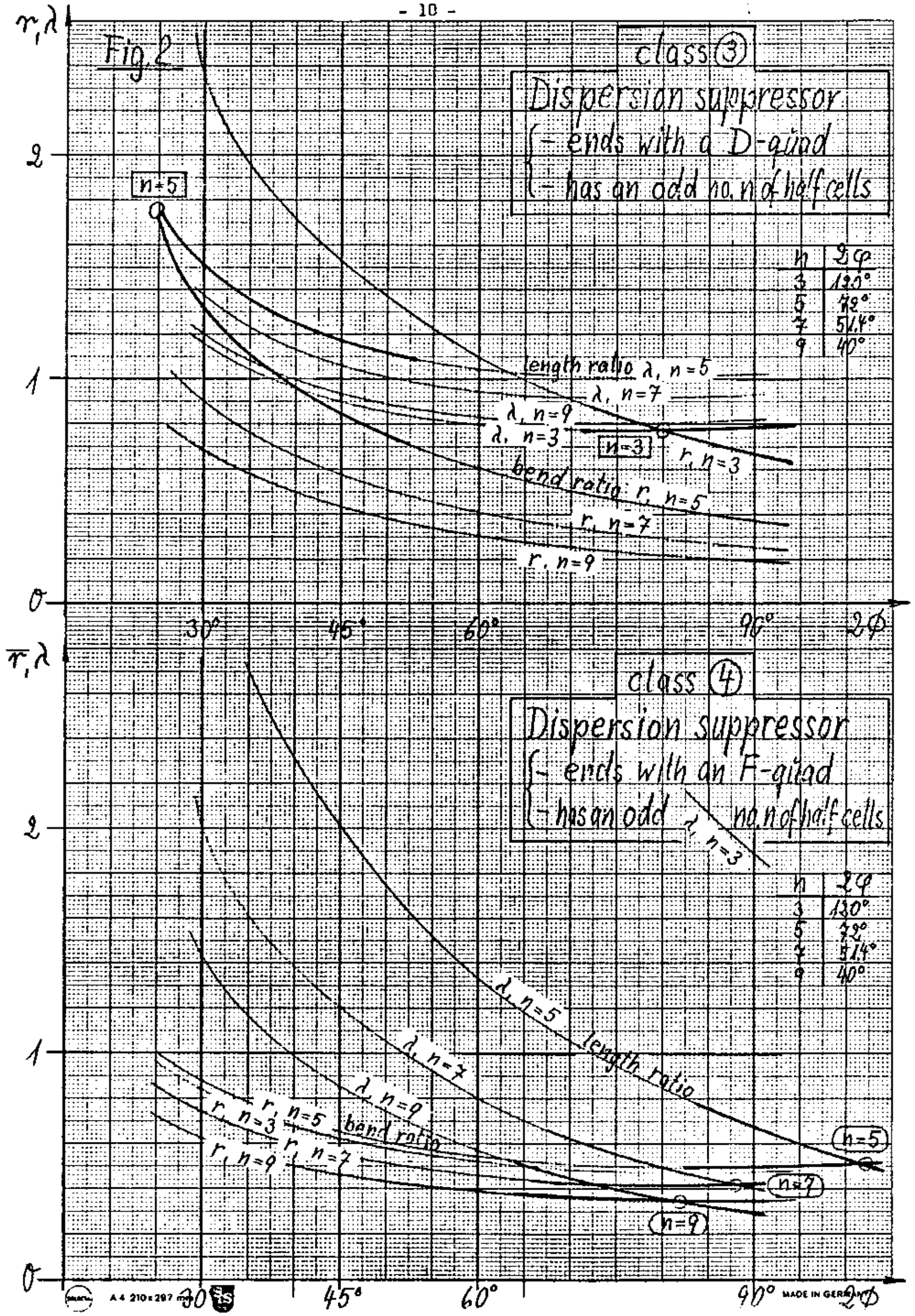
1) K. Steffen, DESY PET-79/03 (March 79)

2) K. Steffen, "Periodic dispersion suppressors", DESY HERA 81/19 (Dec. 81)

see also

F. Willeke, DESY PET-81/28 (Nov. 81)





Class ① Dispersion suppressor - ends with an F-quad - has an even no. n of half cells		2Φ = 30° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.2618\\ .1425\end{pmatrix}$	2Φ = 45° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.3927\\ .2228\end{pmatrix}$	2Φ = 60° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.5236\\ .3099\end{pmatrix}$	2Φ = 90° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.7854\\ .5106\end{pmatrix}$
2φ = 45° n = 8	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.3927\\ .2228\end{pmatrix}$	λ = 1.2875 r = .7864	1 0.5	.8858 .3588	.8730 .2179
2φ = 60° n = 6	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.5236\\ .3099\end{pmatrix}$	λ = 1.4536 r = 1.0959	1.1290 .6968	1 0.5	.9856 .3036
2φ = 90° n = 4	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.7854\\ .5106\end{pmatrix}$	λ = 1.4748 r = 1.8048	1.1455 1.1475	1.0146 .8234	1 0.5

Class ② Dispersion suppressor - ends with a D-quad - has an even no. n of half cells		2Φ = 30° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.2618\\ .1193\end{pmatrix}$	2Φ = 45° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.3927\\ .1699\end{pmatrix}$	2Φ = 60° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.5236\\ .2137\end{pmatrix}$	2Φ = 90° $\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix}=\begin{pmatrix}.7854\\ .2749\end{pmatrix}$
2φ = 45° n = 8	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.3927\\ .1699\end{pmatrix}$	λ = 1.6980 r = .6797	1 0.5	.6613 .4216	.3355 .3836
2φ = 60° n = 6	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.5236\\ .2137\end{pmatrix}$	λ = 2.5675 r = .8060	1.5121 .5930	1 0.5	.5073 .4550
2φ = 90° n = 4	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix}=\begin{pmatrix}.7854\\ .2749\end{pmatrix}$	λ = 5.0611 r = .8859	2.9807 .6517	1.9712 .5495	1 0.5

Class ③ Dispersion suppressor - ends with a D-quad - has an odd no. n of half cells		$2\Phi = 30°$	$2\Phi = 45°$	$2\Phi = 60°$	$2\Phi = 90°$
		$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.2618\\ .1425\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.3927\\ .2228\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.5236\\ .3099\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.7854\\ .5106\end{pmatrix}$
$2\varphi = 40°$, $n = 9$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.3491\\ .1538\end{pmatrix}$	$\lambda = 1.2059$ $r = .6859$	.9366 .4361	.8296 .3129	.8177 .1900
$2\varphi = 5.14°$, $n = 7$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.4488\\ .1896\end{pmatrix}$	$\lambda = 1.3728$ $r = .9089$	1.0662 .5779	.9444 .4147	.9308 .2518
$2\varphi = 72°$, $n = 5$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.6283\\ .2430\end{pmatrix}$	$\lambda = 1.5080$ $r = 1.3307$	1.1713 .8461	1.0375 .6071	1.0225 .3687
$2\varphi = 120°$, $n = 3$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}1.0472\\ .2736\end{pmatrix}$	$\lambda = 1.1684$ $r = 2.4030$	.9075 1.5278	.8038 1.0963	.7923 .6657

Class ④ Dispersion suppressor - ends with an F-quad - has an odd no. n of half cells		$2\Phi = 30°$	$2\Phi = 45°$	$2\Phi = 60°$	$2\Phi = 90°$
		$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.2618\\ .1193\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.3927\\ .1699\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.5236\\ .2137\end{pmatrix}$	$\begin{pmatrix}\Phi\\ \Phi_M\end{pmatrix} = \begin{pmatrix}.7854\\ .2749\end{pmatrix}$
$2\varphi = 40°$, $n = 9$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.3491\\ .1953\end{pmatrix}$	$\lambda = 1.4481$ $r = .6278$	.8529 .4618	.5640 .3894	.2861 .3543
$2\varphi = 51.4°$, $n = 7$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.4488\\ .2592\end{pmatrix}$	$\lambda = 2.0472$ $r = .7465$	1.2057 .5492	.7973 .4631	.4050 .4211
$2\varphi = 72°$, $n = 3$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}.6283\\ .3853\end{pmatrix}$	$\lambda = 3.4201$ $r = .8863$	2.0142 .6520	1.3321 .5498	.6758 .5003
$2\varphi = 120°$, $n = 3$	$\begin{pmatrix}\varphi\\ \varphi_m\end{pmatrix} = \begin{pmatrix}1.0472\\ .7736\end{pmatrix}$	$\lambda = 9.5822$ $r = .8417$	5.6433 .6192	3.7321 .5221	1.8933 .4751