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SELECTED TOPICS ON Υ PHYSICS

by

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Abstract

We review some consequences of QCD for decays of the Υ-resonance.

Instroduction

The Υ familiy is well established now and its gross features are known. In particular, there is no doubt that the Υ-resonances are bound states of heavy b-quarks with electric charge -1/3. Furhter experiments are under way and one may hope for a continuous flow of experimental information.

It would be useful, therefore, to present a coincise summary of theoretical expectations to confront them with experimental data. In this review we partly undertake this task. The basic limitation of any review of such kind is the prejudiced evaluation by the authors of the importance of one or another particular problem. We are primarily interested in consequences of QCD and, moreover, such consequences which, we believe, follow from fundamental QCD with no model assumptions involved. These are reduced, to our mind, mostly to that is called Appelquist-Politzer recipe and QCD sum rules.

Indeed, fundamental QCD applies now direclty to short space-time intervals while large-distance dynamics is mostly beyond the reach of the theory. By virtue of the uncertainty principle

$$\Delta E \sim 1/\Delta t \,. \tag{1}$$

This implies that theory can predict outcome of an experiment with poor energy resolution, or properly averaged over a large energy interval.

The well known example of such predictions are

$$\sigma(e^+e^- \to hadrons) = \sigma(e^+e^- \to \bar{q}q) \tag{2}$$
$$\Gamma(\Upsilon \to hadrons) = \Gamma(\Upsilon \to 3g)$$

(q is quark and g is gluon) which hold provided that the total energy is large or that the Υ mass is large. The price for identifying the quark or gluon cross section with the physical one is summation over all the final states. Such predictions can be generically called Appelquist-Politzer recipe[1)]. They are expected to hold point by point in the experimental data.

Another way to ensure poor energy resolution is not to go immediately to high energies bu to smear, say, resonance cross section over large energy interval. Then QCD tells us again that the physical and quark-gluon cross sections are the same. For example

$$\int_{threshold}^{\infty} R^c_{phys}(s)\,\frac{ds}{s^n} = \int_{4m_c^2}^{\infty} R^c_{quark}(s)\,\frac{ds}{s^n} \tag{3}$$

where R^c is the chamred quark fraction of the total R in e^+e^- annihilation and n is some number, m_c is the quark mass; R^c_{phys} is directly measurable and consists of contributions of J/ψ, ψ', ψ'', $D\bar{D}$ ... while R^c_{quark} is directly computable in terms of simple quark graphs. The number n should not be too large ($n \lesssim 4$) to ensure that the smearing is smooth enough.

Equations like (2), (3) we call QCD sum rules and, in a rough approximation, one can think of them as of a refined quark-resonance duality. Although sum rules integrate over the initial energy and include contributions of various channels, in practical applications some of them are saturated by low-lying resonances, say, by J/ψ. The QCD sum rules again enjoy the status of consequences of fundamental QCD. (Moreover, corrections to asymptotic freedom are controllable in many cases.)

The sum rules were mostly developed by a group of theorists from ITEP (Moscow) and INF (Novosibirsk)[2]. Sum rules which exploit asymptotic freedom alone are summarized in the 'Physics Reports' volume [3], written by N. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin, and V. Zakharov. Further development was to understand the connection between the resonances and vacuum properties in QCD. From the practical point of view this allowed to derive even stronger constraints on the properties of a single resonance. In particular, the sume rules were powerful enough to rule out[5] identification of the X(2.83) state with η_c long before the experimental situation clarified. This development is described in original papers[4-6] and reference [4] contains most detailed exposition of the approach. Specifically the physics of b-quark bound states is studied in papers by Voloshin[6].

The only reason to give such a detailed list of references on the sume rule is that they might not be known so widely. Otherwise, we would not like to give a complete list of references since it would be too large. Therefore we widely use the review article[3] as a starting point. Let us also mention review articles[7,8] on the potential model whose approach is different from what we pursue here but can, of course, be very much instructive. As a reference to experimental data we mostly use the most recent review by Wolf[9].

Although we emphasize reliable predictions of QCD, to get a more complete picture we are forced in many cases to intorduce some model assumptions as well. However, we try in all the cases to indicate the limited character of additional assumptions.

To facilitate confrontation with experimental data, we discuss the physical processes one by one (while for a purely theorectical review it would be more natural to list consequences of one or another theoretical model separately).

The final remark is that we consistently suppress discussion of the jet physics. The only reason is that detailed original papers and very good review articles are around[8,10,11].

Υ - Meson

Most interesting quantities to measure here are leptonic width (check of the QCD sum rules) and Υ total hadronic width (measuring strong interactions running coupling constant). We discuss also inclusive radiative decays and inclusive charm production.Estimates of the decays into the lowest parabottonium level, η_b, are presented but they merely show that these decays will not be observed soon.

$\Upsilon \to \bar{e}e$

The leptonic width was used first to learn the b-quark electric charge. To differentiate between $(2/3)^2$ and $(-1/3)^2$ it suffices to have rough measurements so that the factor 4 is resolved. The only comment which should be added is that more precise measurements are also of great importance for the theory. Namely, QCD sum rules bound the leptonic width in following way[6]

$$\Gamma(\Upsilon \to e^+e^-)_{theory} = 1.15 \pm 0.20 \text{ KeV} \tag{4}$$

Current experimental data tend to a higher value, although the error bars are large (the DASP group result[12] is $1.35 \pm 0.11 \pm 0.22$ KeV).

Experimental number falling outside the range indicated above would necessiate revision of the theory in a direction which is difficult to foresee and premature to discuss at the moment.

Note also that experimental determination of the width with accuracy better than the present theoretical uncertainty could be immediately used to constrain further such fundamental parameters as the intensity of the vacuum gluon field (vacuum expectation value $\langle 0 | G^a_{\mu\nu} G^a_{\mu\nu} | 0 \rangle$ where $G^a_{\mu\nu}$ is the gluon field strength tensor) and to tighten the whole theoretical framework in this way.

$\Upsilon \to$ hadrons

There are two underlying mechanisms contributing to the decay. First, the decay via virtual photon:

$$\Upsilon \to \gamma^* \to \text{hadrons}$$

and, second, direct annihilation of $b\bar{b}$ into three gluons:

$$\Upsilon \to 3g \to hadrons$$

Respectively,

$$\frac{\Gamma(\Upsilon \to hadrons)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = R + \frac{\alpha_s^3}{\alpha^2}\frac{10}{9\pi}(\pi^2 - 9) \tag{5}$$

where R is the famous ratio $\sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ at energy just outside resonance ($\tau^+\tau^-$ is not included), $\alpha = 1/137$ and α_s is the strong interaction coupling constant.

The direct gluon decay is proportional to α_s^3 and therefore its measurement is one of the best ways to determine the value of α_s which is of fundamental importance in all the applications of QCD including such bizaar processes as the proton decay[13]. It is difficult to indicate at the moment any other measurement which could compete with determination of the Υ hadronic width in this respect.

Moreover, by comparing the Υ and J/ψ hadronic decay rates one could hope to test the QCD prediction for evolution of the coupling constant with the change of the characteristic mass scale from m_ψ to m_Υ:

$$\frac{\alpha_s(\Upsilon)}{\alpha_s(J/\psi)} = \left(1 + \frac{25}{12\pi}\alpha_s(J/\psi)\ln\frac{m_\Upsilon^2}{m_\psi^2}\right)^{-1} \tag{6}$$

This ratio is measurable via the relation

$$\frac{\Gamma(\Upsilon \to 3g)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = 4\,\frac{\Gamma(J/\psi \to 3g)}{\Gamma(J/\psi \to \mu^+\mu^-)}\left(\frac{\alpha_s(\Upsilon)}{\alpha_s(J/\psi)}\right)^3 \tag{7}$$

The numbers here are as follows. The branching ratio $B(J/\psi \to \mu^+\mu^-) = (7 \pm 1)\%$ corresponds to

$$\frac{\Gamma(J/\psi \to 3g)}{\Gamma(J/\psi \to \mu^+\mu^-)} = 9.1 \pm 1.8 \tag{8}$$

where we accounted for the fact that the total width of J/ψ is contributed by decays via γ^*, the 3g decay and by radiative decay into $2g + \gamma$, $\frac{\Gamma(J/\psi \to \gamma 2g)}{\Gamma(J/\psi \to 3g)} \simeq 0.12$ (see below). The number (8) corresponds to

$$\alpha_s(J/\psi) = 0.185 \pm 0.012 \tag{9}$$

Using eqs. (6) and (7) one finds

$$\frac{\Gamma(\Upsilon \to 3g)}{\Gamma(\Upsilon \to \mu^+\mu^-)} = 17.6 \pm 2.8 \tag{10}$$

which eventually results in the prediction

$$B(\Upsilon \to \mu^+\mu^-) = (4.0 \pm 0.5)\% \tag{11}$$

while the corresponding value of $\alpha_s(\Upsilon)$ is

$$\alpha_s(\Upsilon) = 0.145 \pm 0.010 \tag{12}$$

So far we assumed that the perturbative treatment of the hadronic decays is perfect both for J/ψ and Υ. This probably is not very good approximation for J/ψ where what is called higher twist or power-like corrections could be important. The corrections are proportional to $(m_0/m_\psi)^4$, where m_0 is mass parameter inherent to the gluon physics. Most probably, uncertainty from neglecting such corrections is of order of the error bars indicated above so that prediction for $B_\mu(\Upsilon)$ is quite reliable.

There exists, however, an alternative point of view[24] according to which the coupling constant $\alpha_s(\Upsilon)$ is large so that the very perturbative treatment of the J/ψ hadronic decay is irrelevant. If so, one should take $\alpha_s(\Upsilon)$ as given in terms of the QCD mass parameter Λ:

$$\alpha_s(\Upsilon) \simeq \frac{12\pi}{25\ln(m_\Upsilon^2/\Lambda^2)}$$

and fit Λ from the measured Υ hadronic width. If we use our favorite value $\Lambda \simeq 0.1$ GeV nothing is changed in the analysis outlined above. However, if one assumes that $\Lambda \simeq 0.5$ GeV as advocated by many, then

$$\alpha_s(\Upsilon) \simeq 0.25$$

This coupling constant is small enough to invoke perturbative calculation of the $\Upsilon \to 3g$ rate. A straightforward calculation with this value of $\alpha_s(\Upsilon)$ gives

$$B(\Upsilon \to \mu^+\mu^-) \lesssim 1\%$$

So, the difference between the two predictions amounts to the factor of 4 and it would be important to resolve the theoretical ambiguity as soon as possible (our bet is that eq. (11) does hold)).

Let us also mention a very interesting recent paper by Parisi and Petzonzio[25]. Convincint arguments are given here to the effect that large perturbative correction claimed in ref.24) vanishes as a result of a proper choice of the normalization point. Moreover, the authors of ref. 25) assume that gluons develop some kind of an effective mass which suppresses greatly the $J/\psi \to 3g$ decay (one can consider this mechanism as a phenomenological realization of large power corrections). As a result it is possible to keep the coupling constant much larger. To our mind, a crucial test of the hypothesis can be provided by measurement of $\Upsilon \to 3g$ decay. According to the scheme of ref. 25) it is given by

$$B_\mu(\Upsilon) \simeq 1.5\ \% .$$

$\Upsilon \to \gamma$ + hadrons

According to QCD the Υ decays first into two gluons plus photon and then the gluons transform into hadrons. Thus there arises a unique opportunity to probe the gluon masses as high as ~ 9 GeV. The hadronic mass is directly related to the photon energy:

$$m_h^2 = m_\Upsilon^2 (1-x)$$

where $x = 2E_\gamma/m_\Upsilon$.

Lowest order QCD predicts both the photon spectrum and the total radiative decay width[3]:

$$\frac{\Gamma(\Upsilon \to \gamma + \text{hadrons})}{\Gamma(\Upsilon \to 3g)} = \frac{4}{5}\,\frac{\alpha}{\alpha_s} \simeq 0.04 \,,$$

$$\frac{d\Gamma(\Upsilon \to \gamma + \text{hadrons})/dx}{\Gamma_{tot}(\Upsilon)} = \varphi(x)\,\frac{16}{9\pi}\,\frac{\alpha_s^2}{\alpha}\,\frac{\Gamma(\Upsilon \to e^+e^-)}{\Gamma_{tot}(\Upsilon)} \simeq$$

$$\simeq 0.06\,\varphi(x) \tag{13}$$

and $\varphi(x)$ reads as

$$\varphi(x) = \frac{x(1-x)}{(2-x)^2} + \frac{(2-x)}{x} + \left[\frac{(1-x)}{x^2} - \frac{(1-x)^2}{(2-x)^3}\right] \ln (1-x)^2 \tag{14}$$

These simple relations are modified by final state interaction, in particular by resonance formation (glueballs). Similarily, the bare quark cross section is strongly modified near the ρ meson. However, starting from $s \gtrsim 1.5$ GeV the asymptotic behaviour sets in for the $q\bar{q}$ channel and one can use the lowest order perturbation theory.

In the gluon channel the mass starting from which parton-like and physical cross sections are close to each other has not been determined experimentally yet. There are good reasons to expect that this mass scale is higher for gluons. Rough estimates give[14]:

$s_{asymptotic} \gtrsim 3$ GeV^2 (tensor two-gluon channel)

$s_{asymptotic} \gtrsim 6$ GeV^2 (scalar two-gluon channel)

Lower values of s are populated by resonances.

So far inclusive radiative decays for onium states were observed only in the case of J/ψ[15]. The signal to background ratio becomes favourable starting from $x \simeq 0.5$ in this case so that the masses in the gluon channel are confined to

$$m_h^2 \lesssim 5\ \text{GeV}^2.$$

The data do not reproduce the lowest order QCD prediction and the study of the corresponding resonances is under way now.

Thus, it seems to be a priviledge of future experiments with Υ-mesons to observe the turnover to the smooth cross section in the gluon channel, as predicted by lowest order QCD. Search for genuine glueblls which could be quite heavy is also very promising (at present it is difficult to speculate on the decay modes of glueballs. Let us mention, however, an unexpected experimental indication that gluons 'like' K-mesons. Indeed, the so called E-meson in the decay $J/\psi \to \gamma$ + hadrons is detected through its $K\bar{K}\pi$ mode and this turns out to be large[16]. Moreover, there is a report[17] of a huge, ~ 28 %, branching ratio for the $KK\pi$ decay mode of the $\eta_c(2.98)$. Certainly, we would not recommend to take these first experimental data too seriously, but one should keep eye on further development).

If the mentioned above estimates of the characteristic mass scale for gluons channel are reasonable, then the turnover to a smooth cross section in inclusive radiative decays occurs at

$$x \sim 0.95$$

and we present a schematic view of the photon spectrum in Fig. 1.

It might worth noting that the decay width for exclusive channels, such as $\Upsilon \to \eta\gamma$ goes down with the onium mass very fast, faster than $(m_J/m_\Upsilon)^4$ and should be negligible in the case of Υ.

We conclude this section with emphasizing once more that measuring the photon spectrum in inclusive radiative decays of Υ would have the same impact on the gluon physics as the measurements of the famous ratio R (for energies up to 7 GeV) had on the quark physics. Thus, there is little doubt that inclusive radiative decays will be intensely studied in the years to come.

$\Upsilon \to \bar{c}c\, X$

Charm production in Υ decays is quite a rare process and not readily identifiable. So, it might suit only the next generation of experiments. Still, it has some nice features from a theoretical point of view: inclusive rate for charm production is calculable[18] in terms of the same coupling constant α_s providing with a new test of the whole framework; moreover, the $^1P_1$ state of charmonium could be observed in this way, and this state is difficult to reach otherwise.

If look for an analogy, then inclusive charm production is similar to the Dalitz pair decay of π^0. First, Υ goes into three gluons and then one of the gluons is converted into the $c\bar{c}$ pair. The corresponding Feynman graph is represented in Fig. 2. Since charmed quark is heavy the process is controlled by short distances and, therefore, is calculable in QCD.

The full calculation can be found in the literature[18]. Here we confine ourselves to a simple estimate. The point is that as far as $m_c \ll m_b$ one can readily find the rate in the leading log approximation. Actually, m_c is not small and we impose on the leading log formula phase space correction.

In this way we come to

$$\frac{\Gamma(\Upsilon \to c\bar{c}X)}{\Gamma(\Upsilon \to 3g)} \simeq \frac{2\alpha_s(J/\psi)}{4\pi}\left(\ln\frac{m_\Upsilon^2}{m_\psi^2}\right)\rho \simeq 0.07\rho \simeq 0.03, \qquad (15)$$

where ρ is the mentioned above factor due to the mass correction, evaluated at the J/ψ mass. Note also that we use $\alpha_s \approx 0.2$.

Thus, the inclusive rate turns out to be quite noticable. Exact calculation[18] gives around 2 % instead of 3 %.

It is much more difficult to predict how this total rate is distributed among particular channels. It is more or less safe only to predict that the share of the resonance production is sizable, if not dominating. The reason is that the resonances, like J/ψ, are dual to a large energy interval in the bare quark cross sections. Thus, we do not expect the resonance production to be lower than, say, 50 % of the total.

Share of each of the resonances is determined by the final state interaction. Charmed quarks are produced in a colored state and to materialize as hadrons they must exchange color with outgoing gluons. As past experience (with, say, D-meson decays) shows it is rather dangerous to speculate on the results of such an exchange.

Still, in the most naive picture, exchange of a single soft gluon in the final state would favor production of the P-wave C-even charmonium states since quarks are produced in S-state and the emitted gluon carries angular mometntum. It might worth emphasizing that once the data on resonance production start to accumulate experimentally many theorists will undoubtly step forward to propose their models of the final state interaction.

From a phenomenological point of view, observation of the C-odd $^1P_1$ state would be most exiting. The reason is that it is not produced in the radiative decays of ψ' because of its C-parity. However, the branching ratio for the decay

$$\Upsilon \to ({}^1P_1)_c + X$$

is, at best, a few units times 10^{-3}. As for the fate of the $^1P_1$ state, it decays primarily into η_c[3]:

$$({}^1P_1)_c \to \eta_c\gamma$$

(the corresponding branching ratio is no less than 50%).

$\Upsilon \to \eta_b \gamma$

The mass splitting between Υ and the parabottonium ground state, η_b, is expected to be small:

$$m(\Upsilon) - m(\eta_b) \simeq 30\ MeV.$$

As a result, the radiative M1 transition is strongly suppressed

$$\Gamma(\Upsilon \to \eta_b \gamma) = \frac{16}{27} \alpha \frac{\omega^3}{m_\Upsilon^2} \simeq 1.2\ eV \left(\frac{\omega}{30\ MeV}\right)^3,$$

or

$$B(\Upsilon \to \eta_b \gamma) \simeq 4 \cdot 10^{-5} \left(\frac{\omega}{30\ MeV}\right)^3,$$

where ω is the actual photon energy.

Thus, observation of η_b would be very difficult. Unfortunately, there is no more chance to reach η_b through Υ' radiative decay (see the corresponding section in the Υ' chapter).

Υ'

All the tests of QCD discussed for the Υ apply to the case of Υ' as well. Moreover, there are a few new interesting decays. First, as is well known from charmonium decays one can look for intermediate P-levels. QCD sum rules for P-levels are much more powerful in the case of bottonium than for charmonium and, therefore, these radiative decays are even of greater importance. Moreover, hadronic transitions between the bottonium levels, like $\Upsilon'' \to \Upsilon\, 2\pi$, can provide with a new insight into gluon physics[19,20]. It is quite a rare case when large distance dynamics can be reliably treated in QCD.

$\Upsilon' \to \bar{e}e$

The leptonic width is not so strongly bounded by the QCD sum rules as $\Gamma(\Upsilon \to \bar{e}e)$. However, there is a feeling based both on the potential model calculations[7,8] and QCD sum rules[21] that the width $\Gamma(\Upsilon' \to e^+e^-)$ should be no less than .4 keV which is slightly higher than the face value of the current data:

$$\Gamma(\Upsilon' \to e^+e^-) = (0.33 \pm 0.10)\ KeV$$

On the other hand, it is unlikely that this width exceeds 1/4 $\Gamma(\psi' \to e^+e^-)$ $\simeq$ 0.52 keV (the factor 1/4 is due to the difference in the quark charges). Therefore, we summarize theoretical expectations as

$$\Gamma(\Upsilon' \to e^+e^-)_{theor.} \simeq (0.46 \pm 0.06)\ KeV \tag{16}$$

and use this guess in all the estimates below.

The leptonic width determines also the rate of the decays via virtual photon:

$$\begin{aligned} &\Gamma(\Upsilon' \to e^+e^-) + \Gamma(\Upsilon' \to \mu^+\mu^-) + \Gamma(\Upsilon' \to \tau^+\tau^-) + \\ &+ \Gamma(\Upsilon' \to \gamma^* \to hadrons) = \Gamma(\Upsilon' \to e^+e^-)(3 + R) \simeq \\ &\simeq 6.5\ \Gamma(\Upsilon' \to e^+e^-) \simeq 3\ Kev\ (1 \pm 0.15). \end{aligned} \tag{17}$$

$\Upsilon' \to 3g \to$ hadrons

All the derivation here is similar to the case of Υ and we just quote the final result:

$$\begin{aligned} \Gamma(\Upsilon' \to 3g) &\simeq \frac{10}{9\pi}(\pi^2 - 9) \frac{\alpha_s^3(\Upsilon)}{\alpha^2} \Gamma(\Upsilon' \to e^+e^-) \simeq \\ &\simeq (17.6 \pm 2.8)\, \Gamma(\Upsilon' \to e^+e^-) = 8.1\ KeV\ (1 \pm 0.2), \end{aligned} \tag{18}$$

where we use our prejudiced analysis of the 3g decay of bottonium and charmonium. For practical reasons, however, Υ' might be less convinient than Υ, because hadronic decays are contributed by cascades $\Upsilon' \to 2\pi\, \Upsilon \to$ hadrons (see below), to say nothing of other possible setbacks.

$\Upsilon' \to P_B\, \gamma$

Looking for the radiative decays of Υ' should allow to uncover the 1P-states of bottonium. While the very existence of the P-levels is granted beyond any doubts, the radiative decay rates are not calculable directly in QCD. Indeed, the decay rate is controlled both by the mass differences between the P- and 2S-states and by overlap of the corresponding wave functions. The mass pattern is provided by QCD while the calculation of the matrix elements is a more delicate matter.

First of all, the fine splitting between the 1^3P_J-states is expected to be of order several MeV and, unlike the case of the charmonium, the P-levels will not be resolved, at least in the first generation of experiments. Therefore, we discuss here the total rate of the radiative decays. As for the mass difference between Υ' and the P-states the QCD sum rules give[6)]

$$m(\Upsilon') - m(1P) = (185 \pm 30)\ MeV , \tag{19}$$

while the potential model[7,8)] produces a somewhat smaller number

$$m(\Upsilon') - m(1P) \simeq (130 - 140)\ MeV . \tag{20}$$

Although the difference is not dramatic at first sight, its measuring could give preference to one of the theoretical approaches which in many other cases lead to close predictions.

Now, as for the rate of the radiative decays it is given by

$$\Gamma(\Upsilon' \to \sum_J 1^3P_J + \gamma) = \frac{4}{27}\ \alpha\omega^3\ |I|^2 \tag{21}$$

where ω is the photon energy and I is the radial part of the transition matrix element.

There exist reliable nonrelativistic sum rules which bound the dimensionless quantity $m_b\omega|I|^2$ summed over various transitions. In particular, the Thomas-Reiche-Kuhn sum rule reads

$$\sum_i m_b\,(E_i - E_n)\,|I_{in}|^2 = 3 \tag{22}$$

and for the 2S-1P transition we are interested in, these sum rules give the upper bound:

$$m\omega\,|I|^2 \le 2 . \tag{23}$$

From the measured 2^3S_1-1^3P_J transitions in charmonium we learn that in that case the relevant combination takes the value

$$m_c\,\omega\,|I|^2 \simeq (1 \pm 0.3) \tag{24}$$

(charmonium)

It is quite plausible that this quantity changes only slightly for the bottonium. Then we get

$$\Gamma(\Upsilon' \to \gamma + \sum_J 1^3P_J) \simeq 7\ KeV \left(\frac{\omega}{180\ MeV}\right)^2 (1 \pm 0.3) \tag{25}$$

Cascade decays $\Upsilon' \to P_B\,\gamma \to \Upsilon\,\gamma\,\gamma$

The ^3P_J states can undergo further radiative transitions into $\Upsilon\gamma$ so that photon cascade appears. The rate of the decay $^3P_J \to \Upsilon\gamma$ is governed by a similar equation

$$\Gamma(^3P_J \to \Upsilon\gamma) = \frac{4}{81}\cdot\ \alpha\omega^3\,|I|^2 \tag{26}$$

However, in this case the relevant combination of the $m\omega|I|^2$ is poorly known for charmonium and one should rely on potential model calculations and (or) on nonrelativistic sum rules. The latter give the bounds

$$1 \le m\omega\,|I|^2 \le 3 \tag{27}$$

and the estimate $m\omega|I|^2 \simeq 2$ looks plausible. In this way we come to

$$\Gamma(^3P_J \to \Upsilon\gamma) \simeq 20\ KeV \left(\frac{\omega}{370\ MeV}\right)^2 \tag{28}$$

(where 370 MeV is the QCD based expectation for the mass splitting between 1P and 1S levels).

The radiative decay of the intermediate levels compete with their hadronic decays which are given theoretically by the following expressions[3)]:

$$\begin{aligned} \Gamma(^3P_0 \to 2g \to hadrons) &= 96\,\alpha_s^2\,|R_P'(0)|^2/M_P^4 , \\ \Gamma(^3P_2 \to 2g \to hadrons) &= \frac{4}{15}\,\Gamma(^3P_0 \to 2g \to hadrons), \\ \Gamma(^3P_1 \to g + q\bar{q} \to hadrons) &\simeq \frac{16}{27}\,\frac{\alpha_s(m_b)}{\pi}\,\ln RM \cdot \Gamma(^3P_0 \to hadr.) \end{aligned} \tag{29}$$

Here $R'_P(0)$ is the derivative of the P-wave function at the origin, M_P is the mass of the state and R is its characteristic radius, $R \sim (1\ GeV)^{-1}$. The width $\Gamma(^3P_0 \to 2g)$ is estimated[6)] as

$$\Gamma(^3P_0 \to 2g \to hadrons) \simeq 360\ KeV \tag{30}$$

(similar estimates hold in potential models). Therefore, we get

$$\Gamma(^3P_2 \to hadrons) \simeq 100\ KeV \tag{31}$$

and

$$\Gamma(^3P_1 \to hadrons) \simeq 25\ KeV \tag{32}$$

Combined with the estimates of radiative decays of the P-levels these hadronic widths produce the following estimates of the two-photon cascade:

$$\Gamma(\Upsilon' \to \gamma\gamma\Upsilon) \simeq 0.25\,\Gamma(\Upsilon' \to \gamma + \sum_J {}^3P_J) \simeq 2\ \mathrm{KeV} \qquad (33)$$

As is clear from the presentation above, the final result is subject to various theoretical uncertainties so that the prediction obtained can serve for the purpose of orientation only. Still, we believe that it demonstrates that the photonic decays could be within the reach of experiments in the near future. If the experiment would bring nothing else but knowledge of the mass of the P-states, it would be still a very important piece of information.

$\Upsilon' \to \eta_b\ \gamma$

The decay $\Upsilon' \to \eta_b\ \gamma$ is a M1 transition which is forbidden in the nonrelativistic limit due to orthogonality of the 2S and 1S eigenfunctions. Therefore, the decay rate can be roughly rescaled from the observed value of $\psi' \to \eta_c$ (2.98) γ accounting for the fact that nonrelativistic picture works better for the heavier bottonium than for charmonium.

As a result, we come to the estimate:

$$\frac{\Gamma(\Upsilon' \to \eta_b\gamma)}{\Gamma(\psi' \to \eta_c\gamma)} \sim \frac{1}{4}\left(\frac{m_\psi}{m_\Upsilon}\right)^2 \left[\frac{(v^2/c^2)_{b\bar b}}{(v^2/c^2)_{c\bar c}}\right]^2$$

where the v^2/c^2 factor characterizes the relativistic corrections and can be estimated as

$$(v^2/c^2)_{c\bar c} \simeq \frac{m_{\psi'} - m_{J/\psi}}{m_{J/\psi}} \simeq 0.2\ ; \quad (v^2/c^2)_{b\bar b} \simeq \frac{m_{\Upsilon'} - m_\Upsilon}{m_\Upsilon} \simeq 0.06\ .$$

Finally, we get

$$\Gamma(\Upsilon' \to \eta_b\gamma) \sim 2.5\ \mathrm{eV} \qquad (34)$$

Extremely small expected value of this width as well as of that for the decay $\Upsilon \to \eta_b\ \gamma$ (see the corresponding section on Υ decays) makes the experimental search for η_b very questionable at least in near future.

$\Upsilon' \to \Upsilon\ \pi\pi$

Hadronic transitions between quarkonium levels can be visualized as a two-step process: first emission of soft gluons by quarkonium and then conversion of the gluons into light mesons[19]. The first stage is governed by the atom-like

mechanics of quarkonium and one can use multipole expansion for interaction of quarkonium with soft gluons. The gluon conversion can be described in terms of gluonic matrix elements over light mesons, such as

$$\langle \pi\pi \,|\, \pi\alpha_s\, G^a_{\mu\nu}\, G^a_{\mu\nu} \,|\, 0 \rangle \qquad (35)$$

which effectively measure gluonic field inside hadrons. On theoretical side[20), a few matrix elements of this kind are related to the so called triangle anomalies in the divergence of the axial current and in the trace of the energy-momentum tensor. These anomalies rooted deeply into the theory and measure color charge of quarks and gluons much in the same way as the famous $\pi^0 \to 2\gamma$ decay measures electric charges of quarks.

In particular, matrix element (35) is calculable via anomaly in trace of the energy momentum tensor:

$$\langle \pi\pi \,|\, \pi\alpha_s\, G^a_{\mu\nu} G^a_{\mu\nu} \,|\, 0 \rangle = -\frac{8\pi^2}{b}\left(q^2 - \lambda\mu_\pi^2\right)\frac{1}{2}\,\varphi_\pi^\alpha\,\varphi_\pi^\alpha \qquad (36)$$

where ϕ_π^α the pion isotopic amplitude, $\phi^\alpha\phi^\alpha = 2\,\phi^+\phi^- + \phi^0\phi^0$, μ_π is the pion mass and b is the coefficient in the QCD Gell-Mann-Low function (related to the color charges of the quarks and gluons): $b = 11 - 2/3\ n_{\text{light flavor}} = 9$, and finally, $q^2 \equiv m^2_{\pi\pi}$ is the two pion invariant mass squared. The dimensionless parameter λ is not fixed by the theory, but it can be determined from experimental analysis of the $\pi\pi$ invariant mass spectrum in the decay. The data on the $\psi' \to J/\psi\ \pi\pi$ decay give $\lambda \simeq 4$.

In terms of the matrix element (36) the amplitude of the $\Upsilon' \to \Upsilon\ \pi\pi$ decay is given by

$$A(\Upsilon' \to \Upsilon\pi\pi) = \frac{2\pi^2}{b}\left(m^2_{\pi\pi} - \lambda\mu_\pi^2\right)\left(\frac{1}{2}\varphi_\pi^\alpha\varphi_\pi^\alpha\right)\left(\vec e\,'\vec e\right) A'_{\bar b b} \qquad (37)$$

where A'_{BB} is the transition matrix element for gluon emission by quarkonium which is determined by wave functions of the $b\bar b$ states and $\vec e\,'$,$\vec e$ are the polarization amplitudes of the Υ and Υ', respectively.

The linear $m^2_{\pi\pi}$ dependence of the amplitude results in a specific form of the spectrum in the decay:

$$\frac{d\Gamma}{d m^2_{\pi\pi}} \sim \left(m^2_{\pi\pi} - \lambda\mu_\pi^2\right)^2 \sqrt{\Delta^2 - m^2_{\pi\pi}}\ \sqrt{1 - \frac{4\mu_\pi^2}{m^2_{\pi\pi}}} \qquad (38)$$

where $\Delta = m_{\Upsilon'} - m_{\Upsilon}$. The plot of the spectrum with $\lambda = 4$ is presented in Fig. 3. Such a form of the $\pi\pi$ invariant mass spectrum was first observed in the $\psi' \to \psi\,\pi\pi$ decay and then understood in terms of the current-algebra approach[22].

The total rate of the decay depends on the amplitude A' which cannot be calculated reliably at present. It is proportional to the squared radius of the quarkonium system and one can roughly rescale this amplitude from the charmonium case where the decay rate $\psi' \to \psi\,\pi\pi$ is measured. Roughly, $\langle r^2 \rangle$ scales as m_Q^{-1} [7,23]:

$$A'_{b\bar{b}} / A'_{c\bar{c}} \simeq m_\psi / m_\Upsilon \tag{39}$$

Such a rescaling finally fixes all the parameters and the decay width is predicted to be equal to

$$\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-) = 2\Gamma(\Upsilon' \to \pi^0\pi^0) \simeq 4\,KeV. \tag{40}$$

This, rather large decay width and quite a specific kinematics allows for a relatively easy detection of the decay. It would be very instructive to compare experimental results with the prediction (40).

It might worth emphasizing that there is a room for a detailed check of the mechanism of the hadronic decay. Thus, the theory predicts strong suppression of the $\pi\pi$ D-wave in the decay. Namely, the D-wave is absent in the nonrelativistic approximation for bottonium and should be of order $v^2/c^2 \sim .06$ as compared to the amplitude of the dominant S-wave. Moreover, the theory predicts factorisation of the amplitudes of the gluon emission and conversion. If so, the numerical coefficient λ in the matrix element $(A \sim (m^2_{\pi\pi} - \lambda\,\mu^2_\pi))$ is universal and equal to 4 in the case of the bottonium decay as well as for charmonium.

$\Upsilon' \to \Upsilon\,\eta$

This transition can be treated along the same lines as the $\pi\pi$ emission. The difference is that the η meson is pseudoscalar and, therefore, interference of electric- and magnetic-type gluon fields is involved. This leads to nontrivial modifications of the formulas obtained in the previous subsection which can still be persued to the very end. In particular, coupling to magnetic field implies some suppression of the amplitude since heavy nonrelativistic quark interacts with magnetic field rather weakly. The corresponding factor looks like p_η/m_Q in the amplitude, where p_η is the meson 3-momentum and m_Q is the quark mass.

The conversion of the gluons into the η meson is governed by the matrix element which can be related to the triangle anomaly in the divergence of the axial current:

$$\langle \eta | \pi\alpha_s G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} | 0 \rangle = \frac{4\pi^2}{3}\sqrt{\frac{3}{2}}\, f_\pi\, m^2_\eta\,, \tag{41}$$

where $\tilde{G}^a_{\mu\nu} = 1/2\, \varepsilon_{\mu\nu\alpha\beta}\, G^a_{\alpha\beta}$ and f_π is the $\pi \to \mu\nu$ decay constant, $f_\pi = 130$ MeV.

The final expression for the $\Upsilon' \to \Upsilon\eta$ transition is

$$A(\Upsilon' \to \Upsilon\eta) = \frac{\pi^2}{9}\sqrt{\frac{3}{2}}\, f_\pi\, m^2_\eta\, m_Q^{-1} \left(\varepsilon_{ijk}\, e_i\, e'_j\, (p_\eta)_k\right) A'_{b\bar{b}} \tag{42}$$

where A'_{BB} is the same quarkonium matrix element which enters eq. (37). To eliminate it consider the ratio of the $\Upsilon' \to \Upsilon\eta$ and $\Upsilon' \to \Upsilon\pi\pi$ rates[20]:

$$\frac{\Gamma(\Upsilon' \to \Upsilon\eta)}{d\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-)/dm^2_{\pi\pi}} = 16\pi^2 f^2_\pi \left(\frac{p_\eta}{m_\Upsilon}\right)^2 \left(\frac{m^2_\eta}{m^2_{\pi\pi} - \lambda\mu^2_\pi}\right)^2 \frac{p_\eta}{\sqrt{\Delta^2 - m^2_{\pi\pi}}} \cdot \left(1 - 4\mu^2_\pi / m^2_{\pi\pi}\right)^{-1/2} \tag{43}$$

Numerically, $\Gamma(\Upsilon' \to \Upsilon\eta)$ depends rather crucially on the mass difference $\Delta = m_{\Upsilon'} - m_\Upsilon$ and we present our prediction in the form

$$\frac{\Gamma(\Upsilon' \to \Upsilon\eta)}{\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-)} \simeq 4 \cdot 10^{-3} \left(\frac{\Delta - m_\eta}{10\,MeV}\right)^{3/2} \tag{44}$$

So, the rate of the $\Upsilon' \to \Upsilon\eta$ decay is rather suppressed due to the small phase space. The perspectives for a study of the η emission by bottonium are much better in the Υ'' case.

Υ''

Properties of the Υ'' resonance are difficult to predict now in a more or less model-independent way. For example, the rates of radiative transitions to 1P and 2P levels are sensitive to the details of the Υ'' wave function. Such transitions could be a good place to test potential models.

The point we would like to emphasize here is that the hadronic transition $\Upsilon'' \to \Upsilon\pi\pi$ dominates the whole width (its branching ratio $\gtrsim 50$ %). Therefore, it seems to be the best place to study the physics of hadronic transitions in quarkonium as well as $\pi\pi$ interaction at invariant mass up to $M_{\Upsilon''} - M_{\Upsilon'} \simeq 890$ MeV.

The theoretical estimate of the $\Upsilon'' \to \Upsilon\pi\pi$ decay width is subject to a large uncertainty because of poor knowledge of the corresponding matrix element A''_{BB} (analog of the matrix element A'_{BB} entering the amplitudes of hadronic decays of Υ'). This matrix element is probably suppressed compared to A' because of the nodes in the 3S wave function of bottonium describing Υ''. An educated guess for the suppression factor is

$$A''_{b\bar{b}} / A'_{b\bar{b}} \simeq 1/4 - 1/2 , \tag{45}$$

which results in the estimate

$$\Gamma(\Upsilon'' \to \Upsilon\pi\pi) \simeq (25 - 100)\ KeV \tag{46}$$

while the other decays are unlikely to give more than ~ 25 keV in total.

Let us also mention that the $\pi\pi$ transitions from Υ'' to other bottonium states are greatly suppressed: the one to Υ' is suppressed by the phase space, while those to the 1P states are forbidden in the nonrelativistic limit.

Study of the decays $\Upsilon'' \to \Upsilon\pi\pi$ and $\Upsilon'' \to \eta\Upsilon$ can test our prediction (43) (with an obvious substitution of Υ'' instead of Υ') which relates the ratio of the decay rates to that of the anomalies in the divergence of the axial vector current and in the trace of the energy-momentum tensor. For the ratio of the total widths eq. (43) gives

$$\Gamma(\Upsilon'' \to \Upsilon\eta)/\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-) \simeq 0.020 \tag{47}$$

Let us emphasize once more that due to its connection to the triangle anomalies of QCD, experimental verification of this relation is of fundamental importance.

SUMMARY OF THEORETICAL EXPECTATIONS

Quantity	Expected value	Comments
	Υ	
$\Gamma(\Upsilon \to e^+e^-)$	1.15 ±0.20 keV	Check of the QCD sum rules. Better accuracy would further constrain QCD parameters
$B(\Upsilon \to \mu^+\mu^-)$	(4.0 ±0.5) %	Measures the value of the QCD coupling constant; tests perturbative picture of quarkonium annihilation into three gluons.
$B(\Upsilon \to c\bar{c} + \text{hadrons})$	~ 2 %	Measures $\alpha_s(m_c)$
$B(\Upsilon \to ({}^1P_1)_c + X)$	~10^{-3} (very rough estimate)	Allows to uncover the C-odd charmonium P-level.
$\dfrac{\Gamma(\Upsilon \to \gamma + \text{hadrons})}{\Gamma(\Upsilon \to 3g \to \text{hadrons})}$	4 %	Measures $\alpha_S(m_B)$. Determination of the photon spectrum allows to study gluon-hadron duality at energies up to $M_h^2 < 50$ GeV².
$\Gamma(\Upsilon \to \eta_b \gamma)$	1.2 eV $(\omega/30 \text{ MeV})^3$	η_B is very difficult to reach.
	Υ'	
$\Gamma(\Upsilon' \to e^+e^-)$	0.4 - 0.5 keV	Potential model, QCD sum rules.
$\Gamma(\Upsilon' \to \gamma^* \to X)$	3 keV (1 ±0.15)	Trivially related to $\Gamma(\Upsilon' \to e^+e^-)$
$\Gamma(\Upsilon' \to 3g \to \text{hadrons})$	8.1 keV (1 ±0.2)	Measures α_S.
$\Gamma(\Upsilon' \to \gamma P_b)$	7 keV · $(\omega/180 \text{ MeV})^2$ · (1 ±0.3)	Uncovers the bottonium P-levels. Determination of the position of the levels helps to choose between potential model and QCD sum rules.
$\Gamma(\Upsilon' \to \gamma\gamma\Upsilon)$	~ 2 keV (rough estimate)	Measures wave functions of the bottonium levels and, indirectly, total width of the P-states.
$\Gamma(\Upsilon' \to \eta_b \gamma)$	2.5 eV (rough estimate)	Very small

Quantity	Expected value Υ'	Comments
$\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-)$	4 keV for $\pi^+\pi^-$	Measuring the spectrum tests PCAC. Tests validity of the multipole expansion for interaction of quarkonium with soft gluons.
$\dfrac{\Gamma(\Upsilon' \to \Upsilon\eta)}{\Gamma(\Upsilon' \to \Upsilon\pi^+\pi^-)}$	$\leq 4.5 \times 10^{-3}$ (sensitive to $\Upsilon' - \Upsilon$ mass difference)	Measures QCD anomalies associated with quark and gluon color charges.
	Expected value Υ''	
$\Gamma(\Upsilon'' \to \Upsilon\pi\pi)$	Probably dominates the total width ($B \geq 50$ %)	Spectrum is sensitive to $\pi\pi$ interaction.
$\dfrac{\Gamma(\Upsilon'' \to \Upsilon\eta)}{\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-)}$	0.020	Measures QCD anomalies.

Figure Captions

Fig. 1 Inclusive photon spectrum in the decay of Υ. At the upper end the turnover from the perturbation theory prediction (dashed line) to the resonance region (solid line) is indicated.

Fig. 2 A diagram contributing to the inclusive charm production in the Υ decay.

Fig. 3 The expected two pion invariant mass distribution in the decay $\Upsilon' \to \Upsilon\pi\pi$ (solid line), the dashed line is the phase space.

References

1) T. Appelquist, H. Politzer, Phys. Rev. D12 (1975) 1404

2) V.A. Novikov et al., Phys. Rev. Lett. (1977) 626,
Phys. Lett. 67B (1977) 409

3) V.A. Novikov et al., Phys. Repts. 41C (1978) 1

4) M. Shifman, A. Vainshtein, V. Zakharov, Phys. Rev. Lett. 42 (1979) 277
Pisma ZhETF 27 (1978) 60,
Nucl. Phys. B147 (1979) 385, 448

5) M. Shifman et al., Phys. Lett. 77B (1978) 80

6) M. Voloshin, Nucl. Phys. B154 (1979) 365
prepring ITEP (Moscow)-21 (1980)
Yadernaya Fizika 29 (1979) 1368 (in Russian)

7) K. Gottfried, in Proceedings of the Intern. Symp. on Lepton and Photon Interactions, Hamburg, 1977, edited by F. Gurbrod (DESY, Hamburg, 1977)
E. Eichten et al., Phys. Rev. D17 (1980) 3090, D21 (1980) 203

8) M. Krammer, H. Krasemann, Acts. Phys. Austriaca Suppl., XXI (1979) 259

9) G. Wolf, preprint DESY 80/13 (1980)

10) K. Koller, T.F. Walsh, Phys. Lett. 72B (1977) 227
Nucl. Phys. B140 (1978) 449
T.A. DeGrand, Y.J. Ng, S.H.H. Tye, Phys. Rev. D16 (1977) 3251

11) M.K. Gaillard, LAPP-TH-13 (1980)
T.F. Walsh, in preparation

12) DASP Group, preprint DESY 80/30 (1980)

13) T. Goldman, D. Ross, Phys. Lett. 84B (1979) 208

14) V. Novikov et al., ITEP preprint (1980)

15) G.S. Abrams, Phys. Rev. Lett. 44 (1980) 114

16) H. Feldman, Talk at DESY Seminar, March 1980

17) T. Himmel, SLAC-Report No. 223 (1979)

18) R. Barbieri, M. Caffo, E. Remiddi, Phys. Lett. 83B (1979) 345

19) K. Gottfried, Phys. Rev. Lett. 40 (1978) 538

20) M. Voloshin, V. Zakharov, preprint DESY 80/28 (1980)

21) M. Voloshin, prepring ITEP-54 (1979)

22) M. Voloshin, JETP Letters 21 (1975) 347
L.S. Brown, R.N. Kahn, Phys. Rev. Lett. 35 (1975) 1

23) A. Billoiee et al., Nucl. Phys. B135 (1979) 493

24) K. Barbieri et al., Nucl. Phys. B154 (1979) 535

25) G. Parisi, R. Petronzio, preprint CERN TH-2804 (1980)

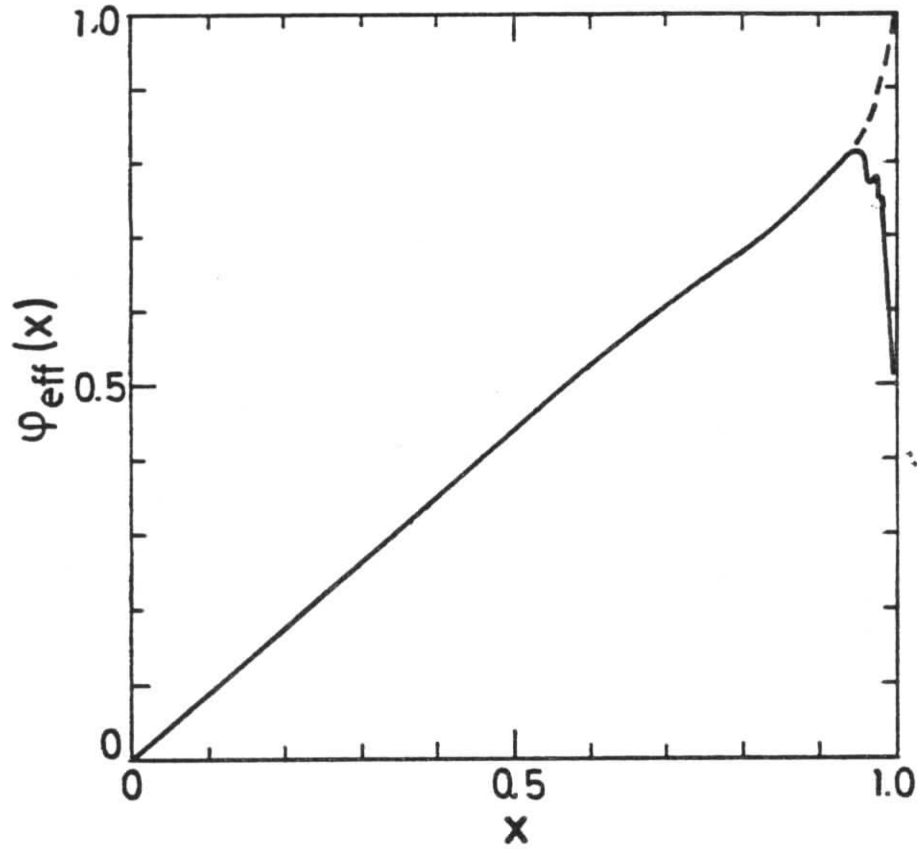

Fig.1

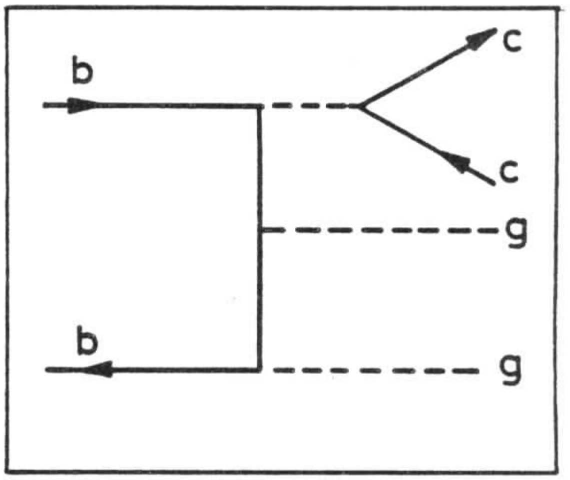

Fig.2

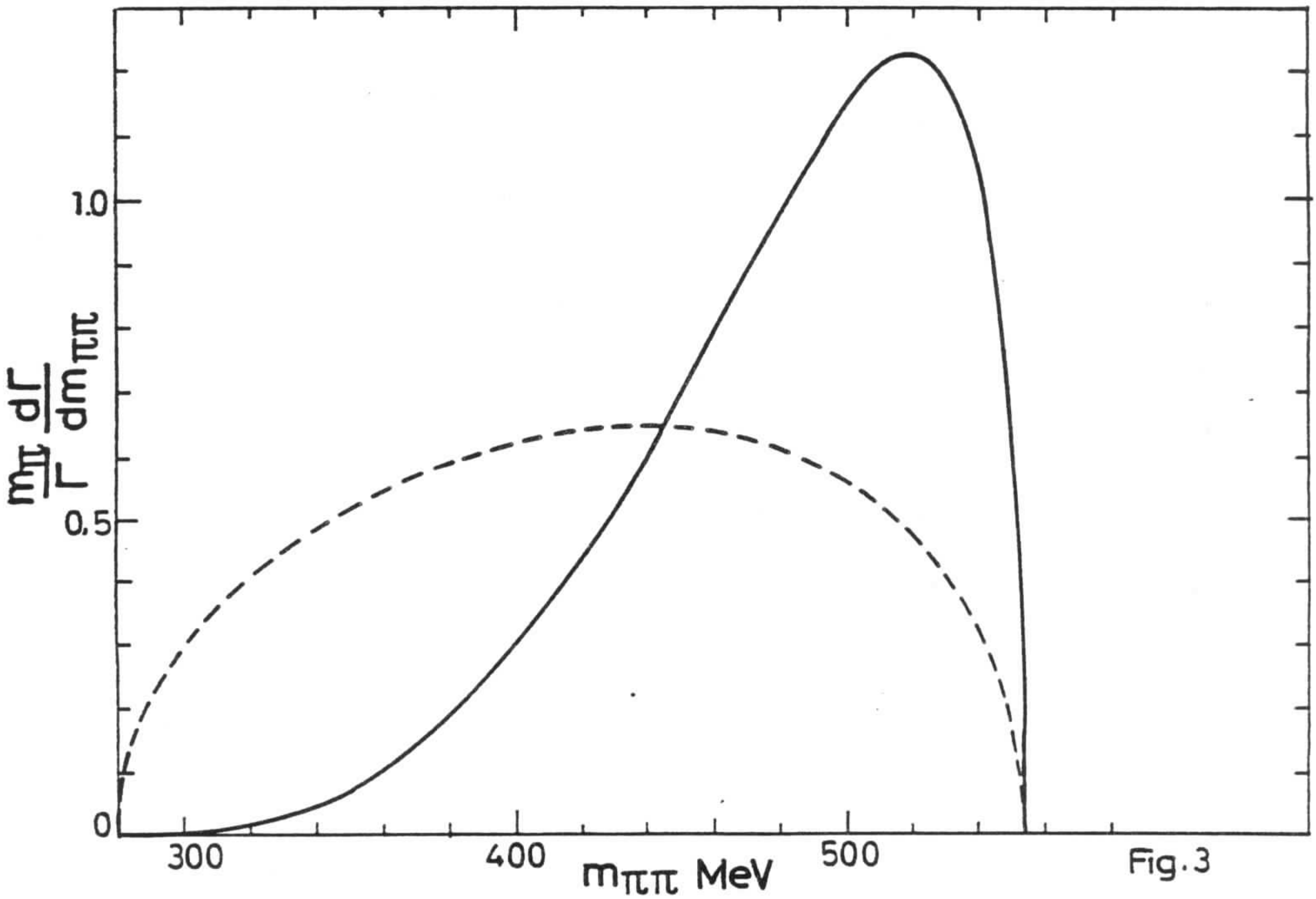

Fig.3