Interner Bericht
DESY F35-72/3
June 1972

AN IMPROVED MODE OF OPERATION OF THE PROPOSED DESY e-p COLLIDING BEAM EXPERIMENT

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Introduction

In an earlier paper [1] the possibility of using DORIS as a e-p colliding beam facility by injecting protons into one of the rings was investigated. This was found to be possible with a maximum luminosity of about 10^{+31} cm^{-2} sec^{-1} at 3 GeV. In order to achieve such a luminosity the protons were bunched into two tightly held bunches occupying only a small fraction of the circumference of the accelerator. This lead, particularly in the longitudinal phase space, to a proton density much higher than those presently achieved in proton synchrotrons [2]. Also since the protons travel with a velocity different from c, collisions between the protons and the electrons can only take place at certain discrete energies.

In this note we propose to distribute the protons continuous around the circumference. The rf system is used only for acceleration and the rf voltage is given by the phase area occupied by the particles. Although the bunches are not compressed in this arrangement, we not only achieve the same luminosities as before, but collisions can now take place at all energies. A further advantage of this mode of operation is that the invariant proton phase densities are considerably reduced as compared to running in the bunched mode. The proton densities are now in fact of the same order as those presently achieved below the transition energy in proton synchrotrons.

Choise of Parameter and the Luminosity

As in our earlier paper we assume the protons are injected from a 3 MeV van de Graaff into a booster synchrotron. In the booster the protons are accelerated up to 1(GeV/c)and then, transferred to the storage ring.

For the purpose of this note we assume an injection emmitance of $\varepsilon_X = \varepsilon_Z = 10^{-5}$ rad m at 3 MeV so reducing the requirements of the van de Graaff injector. The total emmitance ($\varepsilon_X \cdot \varepsilon_Z$) is therefore a factor of 16 larger than assumed in our earlier paper. We further assume that the booster can deliver 5 x 10^{10} protons per pulse to the ring. This value is about a factor of two less than the lowest space charge limit which occurs at 3 MeV.

As the protons do not have speed of light the electric and magnetic fields produced by the beam do not cancel and give rise to a defocussing of the beam proportional to the space charge. This effect limits the total number of protons to $N_{max} \lesssim 4 \times 10^{13}$ protons in the storage ring. Another limit for the number of protons is given by the reduction of the stable buckets area defined by the rf system. The reduction in the stable bucket area due to this space charge effect is determined by a parameter $\Lambda\, A_{SP} = (4\pi \cdot h \cdot g_c \cdot m_p \cdot r_p \cdot N)/(R\ eV\ \gamma^2)$ [3].

In this formula:

- h = the harmonic number
- g_c = 1 + 2 ln (diameter of vacuum chamber/beam diameter)
- m_p = rest mass of the proton
- r_p = the classical proton radius = 1.53×10^{-18} m
- N = number of accelerated particles
- R = mean radius of the accelerator
- eV = peak r.f. voltage
- γ = total mass/rest mass

The peak r.f. voltage needed for a given reduction in the bucket area is thus proportional to h · N. Assuming the phase space can be reduced by a factor of two the numerical value is:

$$eV = (.6 \times 10^{-10}) \cdot h \cdot N(\text{eVolt})$$

The momentum acceptance of the reduced bucket at the injection is then given by:

$$\frac{\Delta p}{p} = \pm \frac{1}{\beta} \left(\frac{.3 \times 10^{-10} N}{\pi E \quad \eta} \right)^{1/2}$$

with E = total energy of the proton and $\eta = \gamma_{tr}^{-2} - \gamma^{-2}$. This momentum acceptance can be made at least $.7 \times 10^{-3}$, which is safe during injection.

The total number of protons N can now be written as $N = h \cdot k \cdot N_o$. Here k is the number of booster pulses injected into <u>one</u> r.f. bucket in the storage ring, i.e. $(k \cdot N_o)$ is the total charge in one bucket.

In order to transfer the protons from the booster into one of the buckets in the storage ring, the frequency of the r.f. in the storage ring must be an integral multiple of the r.f. frequency of the booster. Since the harmonic number of the booster is one and the radius is 1/12 of the radius of the storage ring, the value for h is limited to multiples of 12, i.e., h = 12, 24, 36 --.

The value of k is given by the natural size of electron beam in the ring. That is, the width of the proton beam should be at least as wide as the natural width of the electron beam.

$$\text{i.e., } W_p \approx W_e \text{ (natural width)}$$

The width of the proton beam can be expressed by the amplitude function β_X and the emittance ε_{pX} as:

$$W_p = 2(\varepsilon_{pX} \cdot \beta_X)^{1/2}$$

The emittance at a momentum p is given by Liouvilles theorem as:

$$\varepsilon_{pX}(p) \approx \left(\frac{\varepsilon_{pX}(p_o) \cdot p_o}{p}\right) (k/\alpha),$$

Here $\varepsilon_{pX}(p_o)$ is the emittance at the injection into booster i.e., p_o = 75 MeV, $\varepsilon_{pX} = 10^{-5}$ rad m., and α is the stacking efficiency. The protons will be stacked into the transverse phase space as shown in our earlier paper [1] with an estimated stacking efficiency of 30%. At 3.5 GeV the natural size of the electron beam at the interaction region is about 1.5 mm corresponding to $\pm$ 1.5σ i.e., $W_p^o = W_e^o$ = 1.5 mm. We assume that at lower energies, where the natural size of the electron beam is smaller, the size of the electron beam can be increased to match the size of the proton beam. With a β_{OX} of 5 cm we find k = 16 corresponding to 8 x 10^{11} protons in each bucket at this energy.

The luminosity is given by: [4]

$$L = (f \cdot N_p \cdot N_e) \frac{1}{(\pi/4)\ W_p \left(h_0^2 + \delta^2 \ell^2\right)^{1/2}} \approx \frac{4f\ N_p \cdot N_e}{\pi\ W_p\ \delta\ell}$$

Here N_p is the total number of protons and N_e the total number of electrons in the ring, f the revolution frequency, 2δ the crossing angle and ℓ the circumference of the ring, 288 meters. The luminosity is computed for a crossing angle of 2δ = 6 mrad. A method to obtain such a small crossing angle without disturbing the present optic is shown below for the case of positron-proton collisions. Since the effective length λ of the interaction region due to finite beam height is large compared to β_{OX} we have to use in the luminosity formula the average value for $\frac{1}{W_p}$ i.e.,

$$\frac{1}{W_p} = \frac{1}{W_o}\frac{1}{\lambda}\int_{-\lambda/2}^{+\lambda/2}\frac{1}{(1 + s^2/\beta_{OX}^2)^{1/2}}dS$$

That is, the gain in going to a smaller crossing angle is partially offset by the larger value of W_p. For example, changing 2δ from 16 mrad to 6 mrad the luminosity increases only by a factor of about 1.5. The luminosity as a function of momentum are listed in table 1 for h = 36. Also listed is the peak r.f. voltage and the tune shifts due to the beam-beam interaction.

The luminosities listed is the sum of both interaction regions. The maximum total luminosity is about 1.7×10^{31}/sec cm^2 at 3 GeV and still about $.3 \times 10^{31}$/sec cm^2 at 4 GeV. Above 2.5 GeV where the electron beam is radiation limited these luminosities can in principle be increased by a factor of 2 by using all of the available r.f. power on one storage ring. However this will require substantial changes in the hardware of the r.f. system. An increase of the number of protons stored in the storage ring at a constant cross section of the beam is possible only by increasing the harmonic number. That is the luminosity will only increase with the square root of the r.f. voltage.

The tune shift ΔQ due to the beam-beam interaction (Amman-Ritson) are listed for protons and electrons. The tune shifts for electrons were computed from the expresseion:

$$\Delta Q^e = \left(\frac{r_e \cdot N_p}{\gamma_e}\right)\frac{4}{\pi}\frac{\beta}{(W_p \cdot \delta\ell)}$$

To find the tune shift for protons only the electron and proton indices must be reversed using the same formula. To make a conservative estimate the value for β at the end of the interaction region of length λ was used i.e., a β decreasing from 100 cm at 2 GeV to 48 cm at 4 GeV. Experimental values for ΔQ^e from existing e^+-e^- storage rings range between .018 and .05, or about a factor of 5 larger than the computed values. No experimental limits on the permitted value of ΔQ^p for protons are known, however the computed values for ΔQ^p at high energies are very small. It is also clear that by going to a crossing angle of 16 mrad, the tune shifts will be reduced by roughly an order of magnitude from the values given in the table.

Phase Space Densities

In principle the invariant phase space density for a proton beam should be constant during the life time of the beam. However in an actual accelerator the invariant phase space density is reduced by the space charge, by passing through the transition energy and by various mishandling of the beam. It is therefore interesting to compare the invariant proton density required for the e-p colliging beam with those presently achieved. It is customary to define the normalized emmittances as:

$$\bar{\varepsilon}_L = \Delta(\beta\gamma) \cdot \ell$$

$$\bar{\varepsilon}_H = \pi(\beta\gamma) \cdot \varepsilon_H$$

$$\bar{\varepsilon}_V = \pi(\beta\gamma) \cdot \varepsilon_V$$

The invariant 6 dimensional phase space density σ is commonly written as:

$$\sigma = \frac{N}{\bar{\varepsilon}_L \, \bar{\varepsilon}_H \, \bar{\varepsilon}_V} \quad ,$$

where N is the number of protons contained in $\bar{\varepsilon}_L \cdot \bar{\varepsilon}_H \cdot \bar{\varepsilon}_V$.
The best value for σ obtained at the CERN P.S. at 19 GeV i.e., above the transition energy is $.8 \times 10^{21}/m^3$ [2]. Under the best conditions the phase space density gets diluted by about a factor of 6 passing through the transition energy. i.e., we assume a phase space density of $5 \times 10^{21}/m^3$ below the transition energy. Both the booster and the storage ring is operating well below the transition energy.

The values for the normalized emittances in the storage ring at p = 1 GeV/c are:

$$\bar{\varepsilon}_H = 4 \times \pi \times 10^{-5} \text{ mrad}$$
$$\bar{\varepsilon}_V = 8 \times \pi \times 10^{-6} \text{ mrad}$$
$$\bar{\varepsilon}_L = (2.2 \times 10^{-3}) \times 288 \text{ m} = .72 \text{ m}$$

The number of protons in the ring is at the most 4×10^{13}. This leads to $\sigma = 2 \times 10^{22}/m^3$, or a value about 4 times larger than the value presently achieved at the CERN P.S. We should note that CERN expects to improve the observed σ by a factor of 5 with the new booster.

Method to Adjust for Optimum Crossing Angle

Ideally we would like to be able to use a crossing angle 2δ smaller than the 16 mrad presently allowed. This can be done for positron-proton collisions without any major disturbances by the scheme outlined in Fig. 1. A pair of small bending magnets B1 and B2 are located on the axis as defined by the quadrupoles adjacent to the interaction region. The magnets are arranged symmetric to the interaction point and bend a particle through angles α and $-\alpha$ i.e., the total bend is 0. Since the protons and the positrons have the same charge but are travelling in opposite directions, they will be bend in opposite angles. With this arrangement crossing angles from 16 mrad to 0 can be achieved. The requirements on the bending magnets are modest. Since only a bend angle on the order of 20 mrad is required $B d\ell \approx 2.4$ k gauss · m at p = 3.5 GeV/c. The gap is in the order of 4 cm wide and 10 cm high.

Calculations showed that it is possible with the present magnet system in the storage ring, to separate the centers of magnets B1 and B2 up to 2m so giving a free space for experimenters of about 1.0 to 1.5 m.

Beam Lifetime due to Touschek Effects

Due to the larger mass the maximum momentum deviation Δp allowed within the stable phase area is much less for protons than for electrons. Since the Touschek lifetime is proportional to $(\Delta p)^2$ the lifetime for protons is reduced correspondingly. However in the non-bunched mode of operation this is more than offset by the large volume occupied by the particles. Computations leads to Touschek lifetimes of the order of 1,000 hours at injection i.e., losses or increases of phase space due to this effect will be negligible.

REFERENCES

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H.Gerke, H.Wiedemann, B.H.Wiik, G.Wolf,
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2) Barbalat MPS-DL-Note 71-16 - Internal CERN Report

3) K.H.Reich - BNL, AADD Techn. Note 13 (1965)

4) M.Sands - SLAC Report No. 121

TABLE 1

P(GeV/c)	2.0	2.5	3.0	3.5	4	4.2
$N_e(10^{13})$	3.3	1.3	.65	.25	.09	.05
λ(cm)	43	37	33	31	29.7	29.3
W_p(cm)	.30	.28	.26	.25	.25	.25
			$h = 24$			
$N_p(10^{13})$	1.1	1.3	1.6	1.9	2.2	2.3
$V_{r.f.}$(kV)	15.8	18.7	23.0	27.4	31.7	33.1
$L(10^{30}/cm^2 sec)$	35.7	17.8	11.5	5.6	2.3	1.4
			$h = 36$			
$N_p(10^{13})$	1.6	2.0	2.3	2.9	3.2	3.4
$V_{r.f.}$(kV)	34.6	43.2	49.7	62.6	69.1	13.9
$L(10^{30}/cm^2 sec)$	53.5	26.7	17.1	8.44	3.2	2.1
ΔQ^e	.0073	.0056	.0048	.0048	.0040	.0038
ΔQ^p	.0111	.0028	.0010	.0003	.00005	.00002

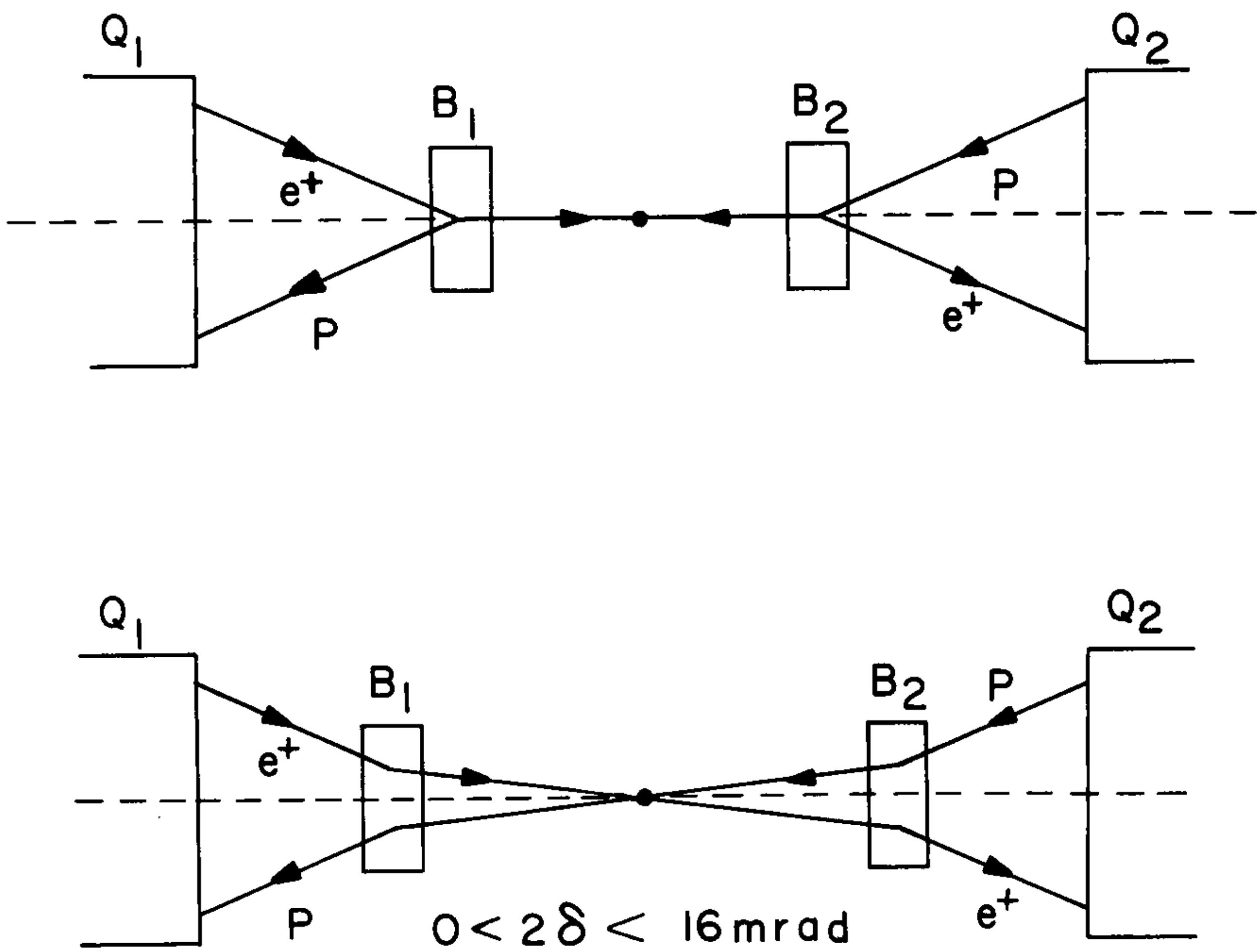

Fig. 1