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DESY - H 4

COMPARISON OF STORAGE RING MAGNET STRUCTURES

Summary

The properties of the magnet structures for e^--e^+-storage rings as suggested by the groups at Frascati, Cambridge and Stanford have been recalculated on the DESY analog computer. The obtained beam envelopes, phase angles, closed orbit functions and fractional radiation damping constants are presented.

In addition, the change of damping constants with varying equilibrium orbit positron is calculated for the DESY synchrotron. For a similar structure composed of "window frame quadrupoles", the damping behaviour is investigated for varying magnet excitations. It appears that such a structure, as compared to a separated function structure, is not especially attractive.

Introduction

Intended as a quick survey of different types of magnet structures, this work puts no emphasis on maximum accuracy. Subsequent numerical checks, however, have shown that, in most cases, the obtained accuracy is quite satisfactory.

Method of calculation:

For systems with sector shaped deflecting magnets (orthogonal entry and exit of principal trajectory), the analog computer solves the trajectory equations

$$\begin{aligned} &\text{vertical coordinate:} && z'' + kz = 0 \\ &\text{horizontal coordinate:} && x'' - \left(k - \frac{1}{\rho^2}\right)x = -\frac{1}{\rho}\frac{\Delta p}{p_o} \end{aligned} \qquad (1)$$

with

$$\frac{1}{\rho(s)} = \frac{e}{p_o} B_z(s) \Big|_{z=x=0}$$ representing the deflecting structure

and

$$k(s) = \frac{e}{p_o} \frac{\partial B_z}{\partial x} \Big|_{z=x=0}$$ representing the focusing structure.

For rectangular homogeneous field magnets (parallel entry and exit face) of deflecting angle $\phi = l/\rho$, the computer integrates the equations

$$\begin{aligned} z'' + \frac{1}{\rho^2} z &= 0 \\ x'' &= -\frac{1}{\cos^2 \phi/2} \frac{1}{\rho} \frac{\Delta p}{p_o} \end{aligned} \qquad (2)$$

between the limits $s = s_o$ and $s = s_o + \rho \sin\phi$. This yields a very good approximation in x and a fair approximation in z.

The closed orbit function P(s), i.e. the closed orbit normalized to a relative momentum deviation $\frac{\Delta p}{p_o} = 1$, is found by inserting $\frac{\Delta p}{p_o} = 1$ and varying the initial conditions of the trajectory until they agree with the exit conditions after one period.

The fractional damping constant α_{syn}, referred to one period, is determined from

$$\alpha_{syn} = -\frac{\varepsilon_o}{E_o} \left(1 + \frac{\int \frac{1}{\rho} (kP - \frac{1}{2} \frac{1}{\rho^2} P) ds}{\int \frac{1}{\rho^2} ds} \right) \quad (3)$$

where E_o is the particle energy and ε_o is the average radiation of the reference particle per period. The integrations in (3) extend over one period and are easily performed by a simple additional circuit since the voltage functions representing the three terms of the integrands are directly available in the analog computer. In the case of no coupling between the vertical and horizontal motion, the fractional damping constants for the betatron oscillations are immediately obtained from α_{syn} by the equations [1)]

$$\alpha_{vert} = -\frac{1}{2} \frac{\varepsilon_o}{E_o}$$

$$\alpha_{hor} = -\frac{3}{2} \frac{\varepsilon_o}{E_o} - \alpha_{syn} \quad (4)$$

The periodic betatron oscillation envelopes $E_z(s)$ and $E_x(s)$ for a beam emittance ε (measured in rad·m) are calculated from

$$E(s) = \sqrt{y_1^2(s) + y_2^2(s)} \qquad (5)$$

where $y_1(s)$ and $y_2(s)$ are "conjugated trajectories" satisfying the initial conditions

$$\begin{pmatrix} y_1 \\ y_1' \end{pmatrix}_o = \begin{pmatrix} E_o \\ E_o' \end{pmatrix} \quad ; \quad \begin{pmatrix} y_2 \\ y_2' \end{pmatrix}_o = \begin{pmatrix} 0 \\ \frac{\varepsilon}{E_o} \end{pmatrix} \qquad (6)$$

The initial conditions E_o and E'_o leading to the periodic solution E(s) are quickly found again by trial and error.

The betatron phase angles per period, Φ_z and Φ_x, are evaluated from

$$\Phi = \int \frac{\varepsilon}{E^2} ds \qquad (7)$$

An alternative, more accurate method would be to determine the trace T of the transformation matrix from the displacement and slope of the principal trajectories after one period and obtain Φ from

$$\cos \Phi = \frac{1}{2} T \qquad (8)$$

Results:

For the various magnet structures investigated, the closed orbit and envelope functions as drawn by the analog computer are shown on the following pages.

Frascati storage Ring Structure

Reference:

LNF-63/62, Frascati (1963)

Both the closed orbit function P and the envelope functions E have extraordinarily small average as well as maximum amplitudes.

Due to $n = \rho^2 k = \frac{1}{2}$ in the deflecting magnet, the fractional damping constants are given by

$$\alpha_{syn} = - \frac{\varepsilon_o}{E_o}$$

$$\alpha_{vert} = \alpha_{hor} = - \frac{1}{2} \frac{\varepsilon_o}{E_o}$$

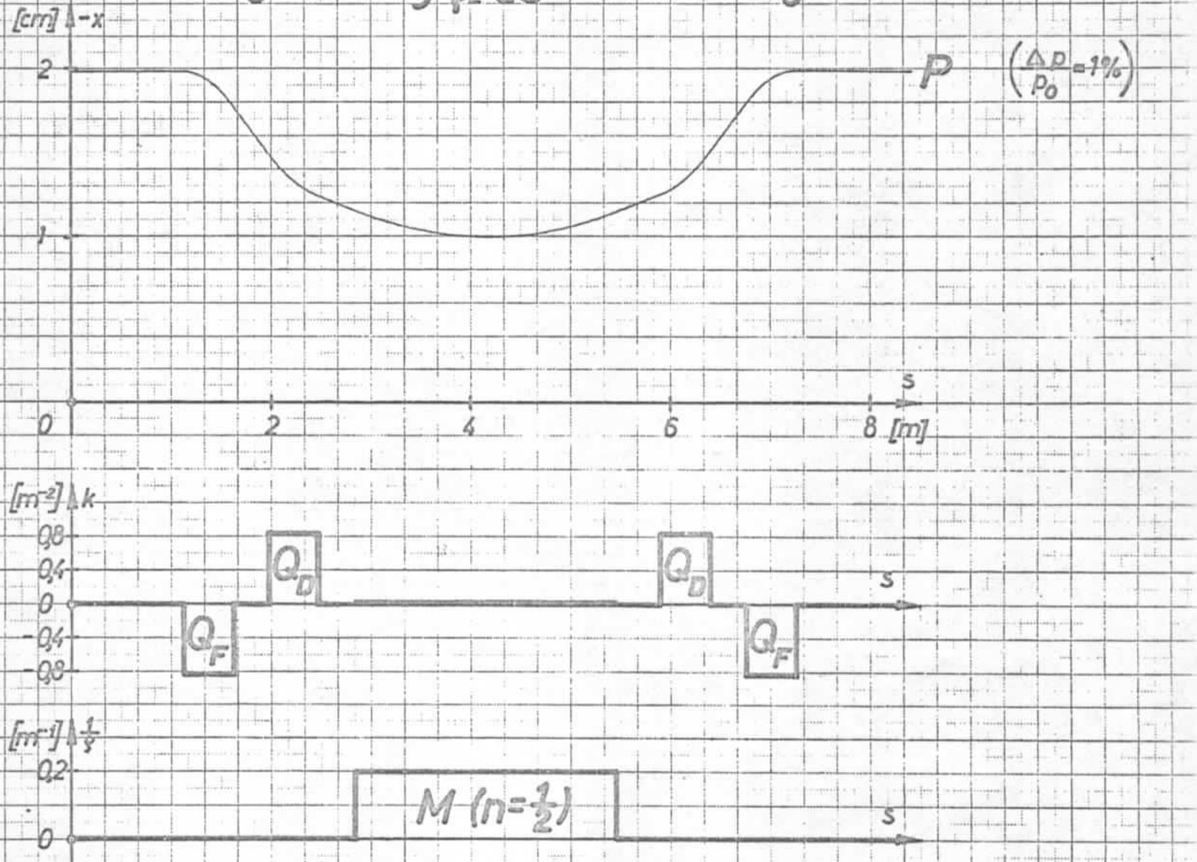
independent of quadrupole excitation.

Frascati Storage Ring Structure

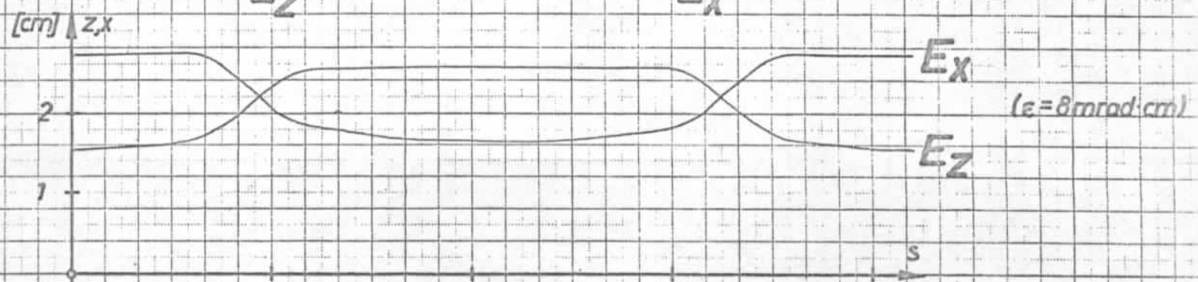
(one out of 12 periods)

M = sector magnet with $n = \rho^2 k = \frac{1}{2}$

$$\alpha_{syn} = -\frac{\varepsilon_0}{E_0}\left(1 + \frac{\int \frac{1}{\rho}\left(kP - \frac{1}{2}\frac{1}{\rho^2}P\right)ds}{\int \frac{1}{\rho^2}\,ds}\right) = -\frac{\varepsilon_0}{E_0}(1+0)$$



$$\bar{\Phi}_Z = \int \frac{\varepsilon}{E_Z^2}\,ds = 1{,}69 \qquad \bar{\Phi}_X = \int \frac{\varepsilon}{E_X^2}\,ds = 1{,}64$$

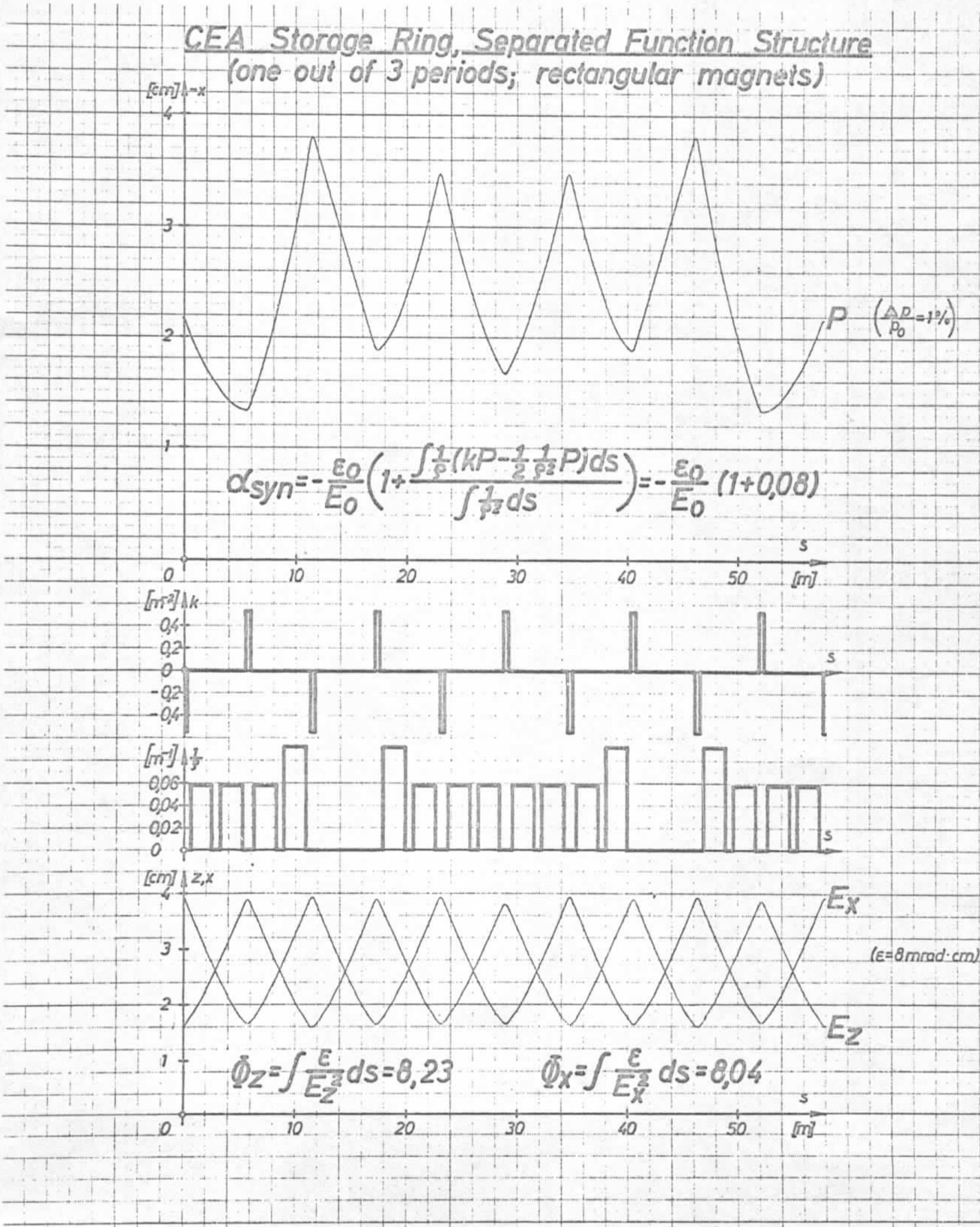


Proposed separated function structure for CEA storage ring

Reference:

K. W. Robinson, CEAL-TM-118, Cambridge (1963)

The amplitude of the closed orbit function in the center of the two long straight sections as well as the peak amplitude of closed orbit and envelope functions are relatively large.


CEA Storage Ring, Separated Function Structure
(one out of 3 periods; rectangular magnets)
[cm] −x
P $\left(\frac{\Delta p}{p_0} = 1\%\right)$
$\alpha_{syn} = -\frac{\varepsilon_0}{E_0}\left(1 + \frac{\int \frac{1}{\rho}(kP - \frac{1}{2}\frac{1}{\rho^2}P)ds}{\int \frac{1}{\rho^2} ds}\right) = -\frac{\varepsilon_0}{E_0}(1+0{,}08)$
s
[m]
0
10
20
30
40
50
[m⁻²] k
0,4
0,2
0
−0,2
−0,4
[m⁻¹] 1/ρ
0,06
0,04
0,02
[cm] z,x
E_x
E_z
(ε=8 mrad·cm)
$\Phi_z = \int \frac{\varepsilon}{E_z^2} ds = 8{,}23$
$\Phi_x = \int \frac{\varepsilon}{E_x^2} ds = 8{,}04$

Proposed synchrotron magnet structure for CEA storage ring

Reference:

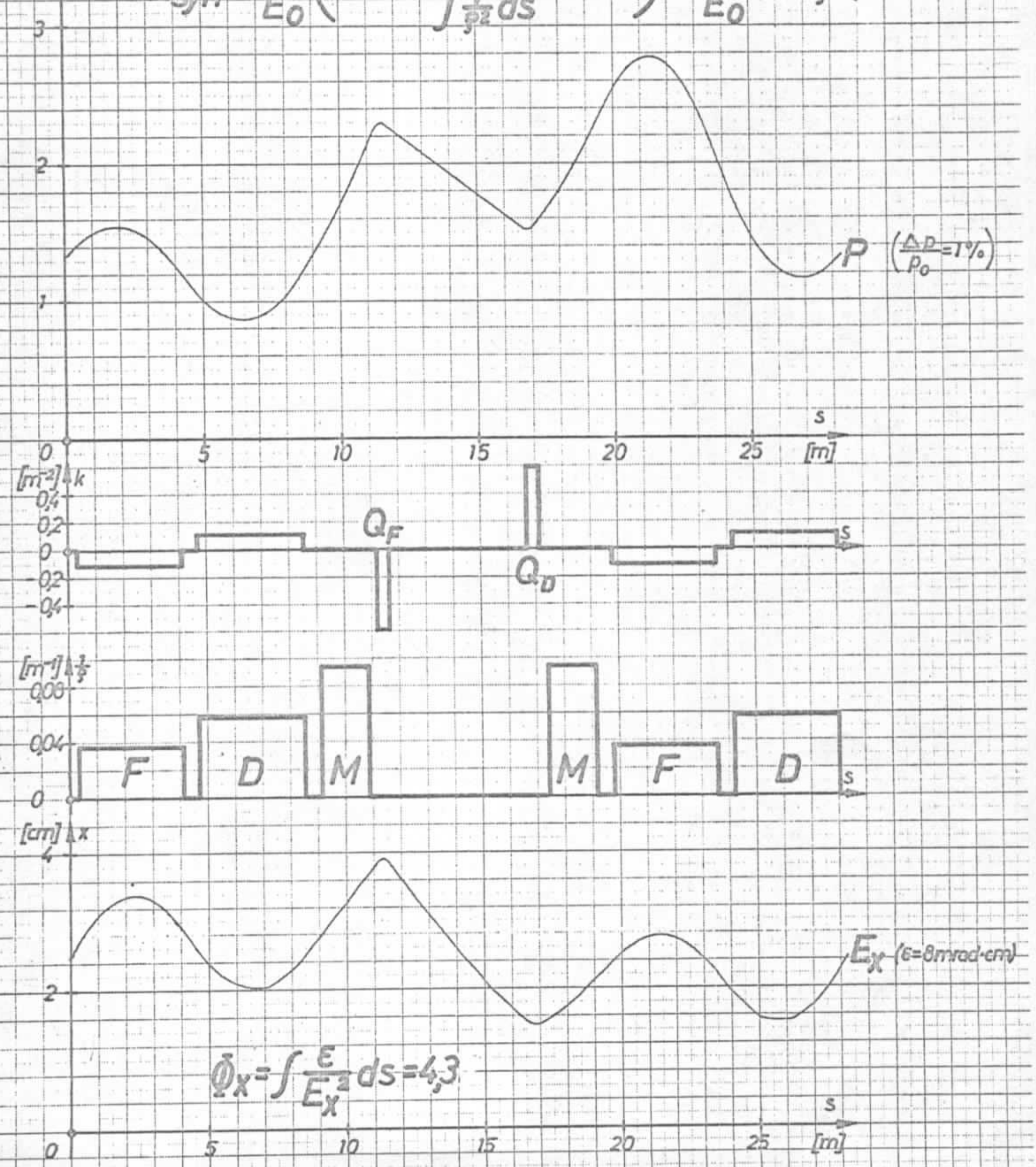
K. W. Ronbinson, CEAL-TM-118, Cambridge (1963)

The structure incorporates 4 different types of magnets.

CEA Storage Ring, Synchrotron Magnet Structure

(one out of 6 periods)

$$\alpha_{syn} = -\frac{\varepsilon_0}{E_0}\left(1+\frac{\int \frac{1}{\rho}\left(kP-\frac{1}{2}\frac{1}{\rho^2}P\right)ds}{\int \frac{1}{\rho^2}ds}\right) = -\frac{\varepsilon_0}{E_0}(1+0{,}15)$$



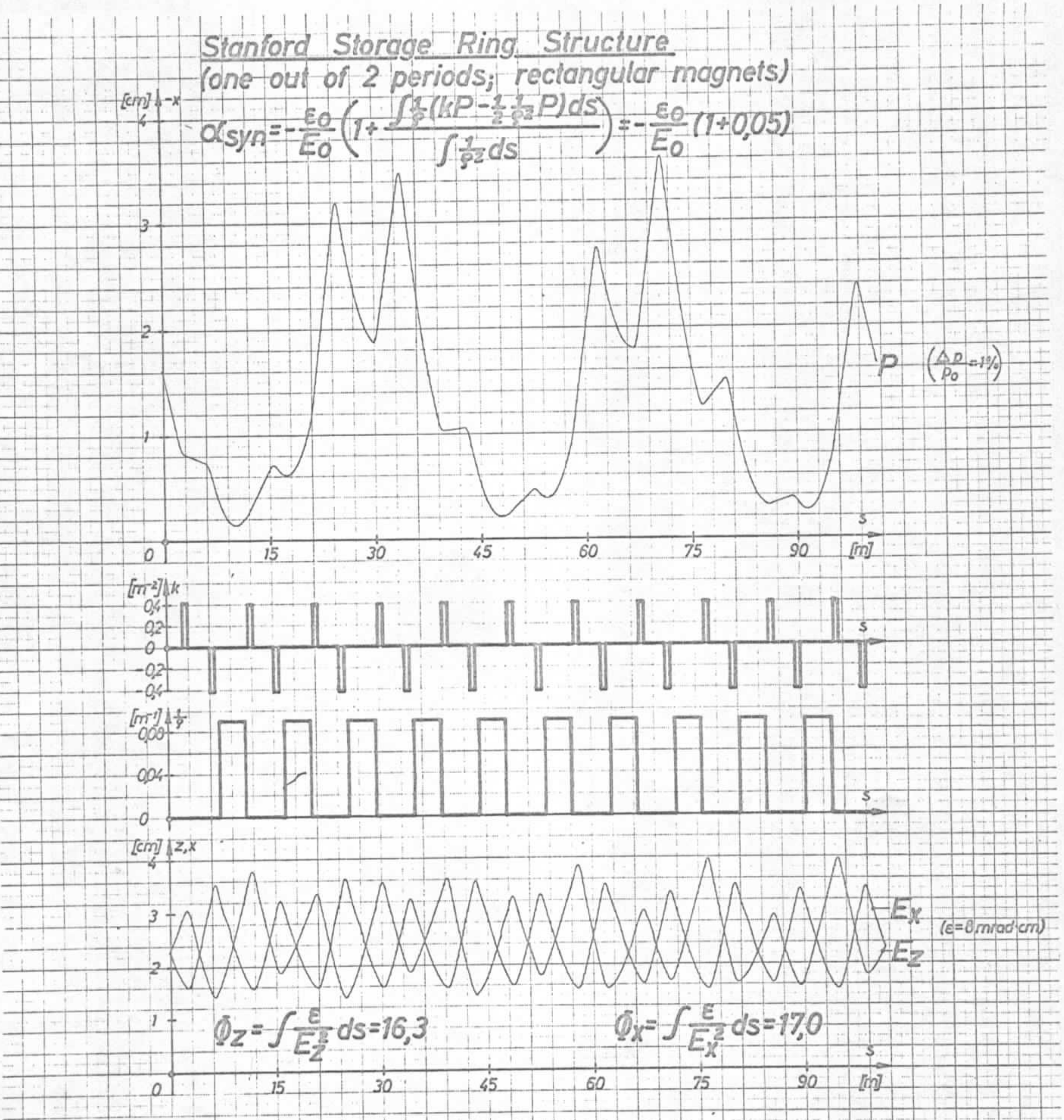
Proposed Stanford storage ring structure

Reference:

SLAC storage ring proposal, Stanford (1964)

The beginning and end of the period as shown refer to the centers of the two long straight sections designed for beam interaction.

The peak amplitudes of closed orbit and envelope functions are relatively large.

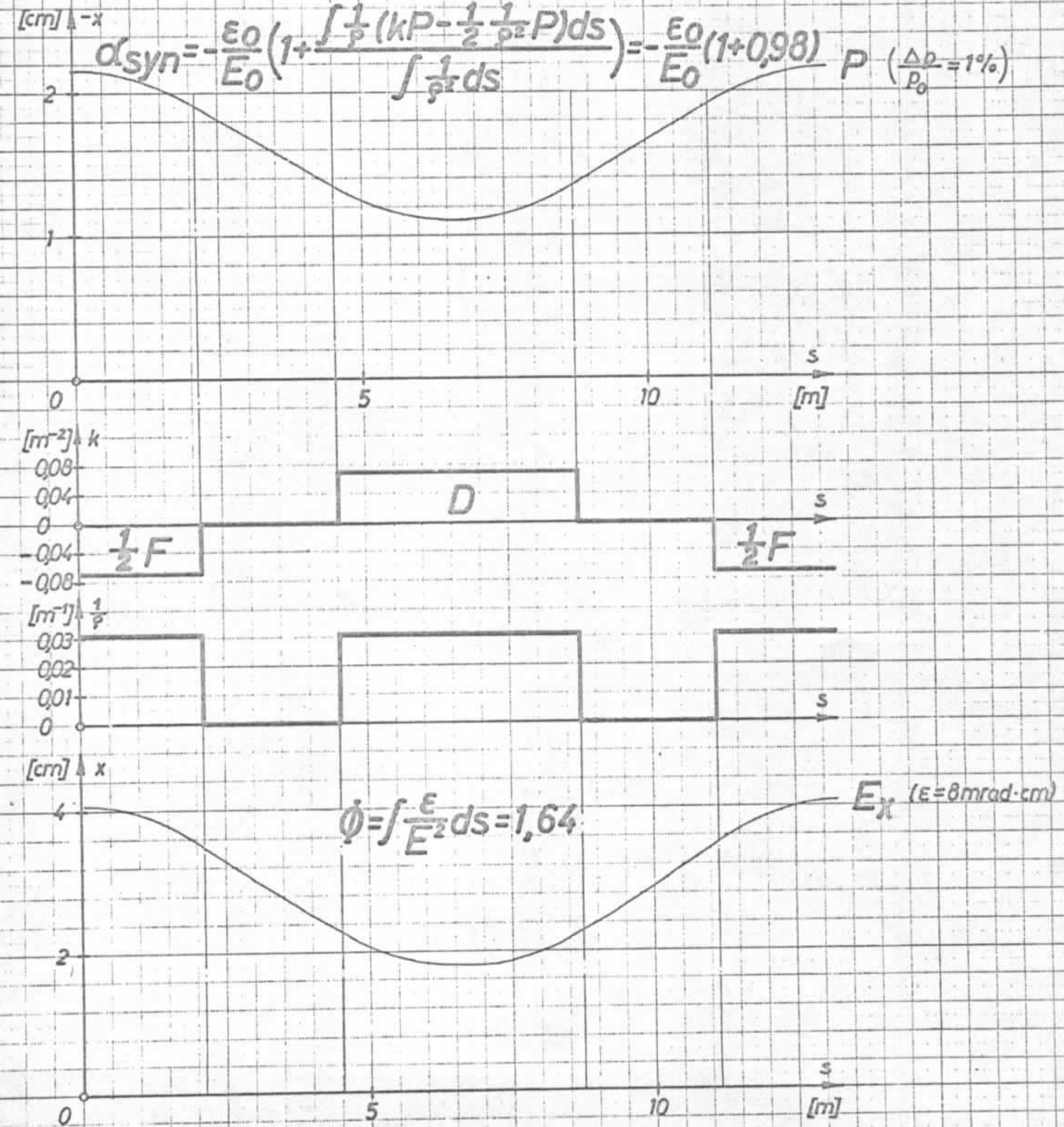

Stanford Storage Ring Structure
(one out of 2 periods; rectangular magnets)
$\alpha_{syn} = -\frac{\varepsilon_0}{E_0}\left(1 + \frac{\int \frac{1}{\rho}(kP - \frac{1}{2}\frac{1}{\rho^2}P)ds}{\int \frac{1}{\rho^2}ds}\right) = -\frac{\varepsilon_0}{E_0}(1+0,05)$
[cm]
-x
P
$\left(\frac{\Delta p}{p_0} = 1\%\right)$
[m]
s
[m^-2]
k
[m^-1]
$\frac{1}{\rho}$
[cm]
z,x
E_x
E_z
$(\varepsilon = 8\,mrad\,cm)$
$\Phi_z = \int \frac{\varepsilon}{E_z^2} ds = 16,3$
$\Phi_x = \int \frac{\varepsilon}{E_x^2} ds = 17,0$

DESY synchrotron structure

Reference:

DESY-Bericht A1.6, Hamburg (1959)

DESY Synchrotron Structure
(one out of 24 periods)



Variation of fractional damping constant α_{syn} in the DESY synchrotron structure as a function of equilibrium orbit displacement.

It has been pointed out by Hereward[2] that, in an AG synchrotron, the distribution of radiation damping may be varied by displacing the synchronous equilibrium orbit, as may be done by shifting the frequency of the accelerating voltage.

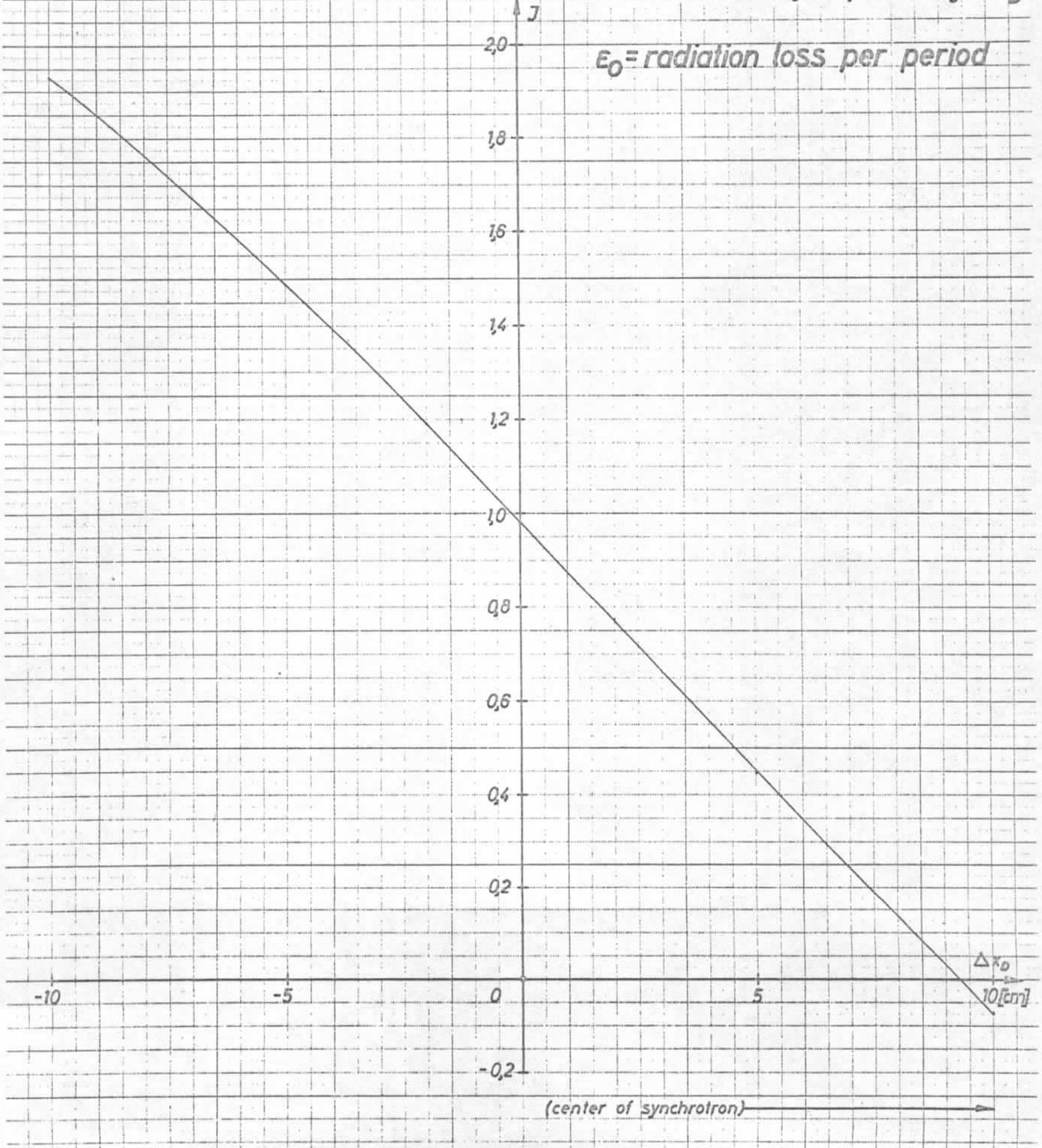
For the DESY synchrotron structure, this effect has been computed and is shown in the graph. It appears that, in the center of the horizontally defocusing magnet, the equilibrium orbit must be displaced by 4.5 cm towards the center of the ring in order to remove the antidamping of horizontal betatron oscillation. If the magnet structure were to be used for an e-p storage ring, about twice this displacement would be desirable to have about equal damping for vertical and horizontal betatron oscillations.

Fractional damping constant α_{syn} for one period of the DESY synchrotron structure as a function of the equilibrium orbit displacement Δx_D in the center of the D-magnet.

$$\alpha_{syn} = -\frac{\varepsilon_0}{E_0}(1+J) \qquad \text{with} \qquad J = \frac{\int \frac{1}{\rho}\left(kP - \frac{1}{2}\frac{1}{\rho^2}P\right)ds}{\int \frac{1}{\rho^2}\,ds}$$

P = closed orbit, displaced by Δx_D

ε_0 = radiation loss per period



DESY-type structure with window frame quadrupoles

The magnet cross section [3] is shown at the bottom. All magnets are of the same type, but have alternating polarity. The pole contours are concentric hyperbolas, and the magnetic field is a linear superposition of a homogeneous field, deflecting horizontally, and a quadrupole field which has its axis at the principal orbit and is rotated about this axis by 45^{o} with respect to the usual orientation. The heavy dashed lines at right indicate the ideal boundaries of the coil cross section.

The trajectory equations solved by the analog computer are modified according to the coupling between x- and z-coordinates. The upper graph a) shows the closed orbit function for the most symmetric magnet arrangement. The closed orbit is shifted horizontally by an almost constant amount and oscillates vertically.

The distribution of radiation damping is unfavourable for e-p storage rings since the two betatron oscillation modes are practically neither damped nor anti-damped.

The attempt of alternately varying the deflecting strength while keeping the focussing strength unchanged results in a vertical shift of the closed orbit, but leaves the distribution of radiation damping unaltered, as apparent from the lower graph b).

DESY – Type Structure with Window Frame Quadrupoles

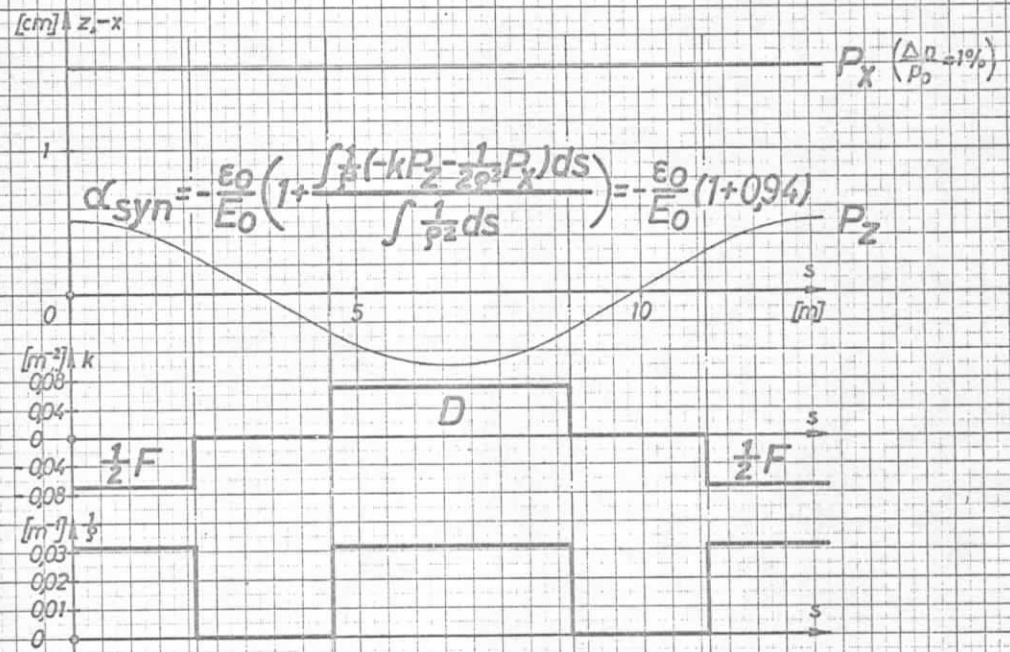
(Focusing fields twisted by 45°)

Differential equation:

$$z'' + kx = 0$$

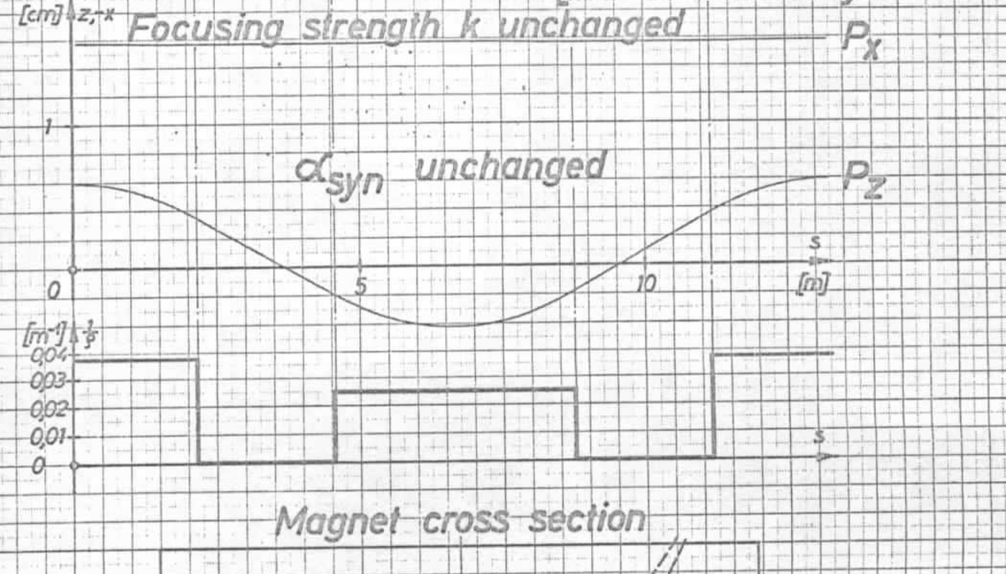
$$x'' + kz + \frac{1}{\rho^2}x = -\frac{1}{\rho}\frac{\Delta p}{p_0}$$

a) Equal deflecting strength $\frac{1}{\rho}$ in F and D Magnets

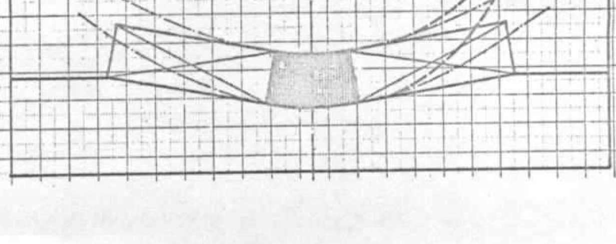


b) Deflecting strength $\frac{1}{\rho}$ $\begin{cases}\text{in F increased by 20\%}\\ \text{in D decreased by 20\%}\end{cases}$

Focusing strength k unchanged



Magnet cross section



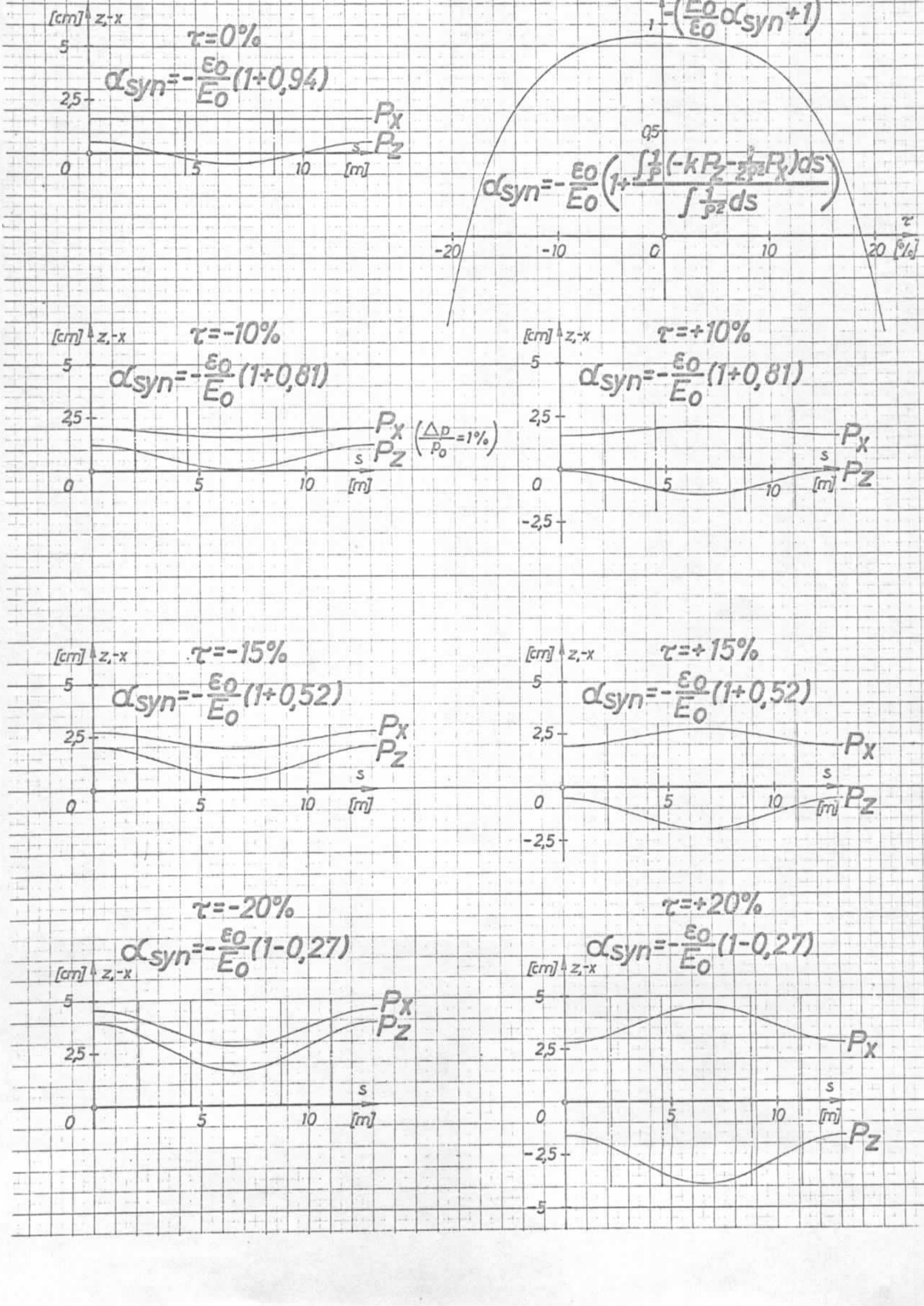
DESY structure with widow frame quadrupoles (continued)

The graphs show the closed orbit functions resulting from an alternating, proportional variation of deflecting and focusing strength by $\pm\tau$ per cent. As shown in the upper graph at right, this changes the fractional damping constant α_{syn} by an amount which is proportional to τ^2. At $\tau \approx 19$ %, one has $\alpha_{syn} = -\frac{\varepsilon_o}{E_o}$ as in a separated function structure.

It may be concluded from the aperture requirements and damping behaviour of this type of magnet structure that it is not very attractive for a storage ring design.

DESY-Type Structure with Window Frame Quadrupoles
(Focusing fields twisted by 45°)

Deflecting strength $\frac{1}{\rho}$ and focusing strength k { in F increased by τ percent; in D decreased by τ percent



Acknowledgement

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References:

1) K. W. Ronbinson, Phys. Rev. 111/373 (1958)

2) H. G. Hereward, 1961 International Conference on High Energy Accelerators, p. 222, Brookhaven (1961)

3. K. G. Steffen, DESY-Notiz A2.82, Hamburg (1961)