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Fringing Field Effect of the Insertion Quadrupoles in DORIS

by

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FRINGING FIELD EFFECT OF THE INSERTION QUADRUPOLES IN D O R I S

It was experimentally observed that a displacement of the closed orbit in the insertion part of the machine, where the large quadrupoles stay, led to important wave numbers shifts. More over it was seen that an horizontal displacement gave a quadratic effect while a vertical displacement around the design closed orbit (off centered vertically) appeared practically linear (Fig.1, 2). This is characteristic of an octupolar effect, and the main natural octupolar terms one can expect come from the fringing fields of the quadrupole.

Let's look first to the effect of such fringing fields according to the natural closed orbit distortion which exists vertically in the quadrupoles WQ_1 and WQ_2.

With regard to the formulas which are reminded in the ANNEX one can write:

$$\Delta\nu_x = -\frac{1}{4\pi}\int K' z_{co}\, z'_{co}\, \beta_x\, ds - \frac{1}{16\pi}\int K'' z^2_{co}\, \beta_x\, ds$$

$$\Delta\nu_z = \frac{1}{16\pi}\int K'' z^2_{co}\, \beta_z\, ds$$

Integration by parts gives:

$$\Delta\nu_x = -\frac{1}{4\pi}\int \left(\frac{1}{2} z_{co} z'_{co}\, \beta_x - \frac{1}{4} z^2_{co}\, \beta'_x\right) K'\, ds$$

$$\Delta\nu_z = -\frac{1}{4\pi}\int \left(\frac{1}{2} z_{co} z'_{co}\, \beta_z + \frac{1}{4} z^2_{co}\, \beta'_z\right) K'\, ds$$

The integrals must be taken only in the fringing fields of the quadrupoles, and a good approximation consists of taking constant values for the optic parameters. Moreover, the quadrupoles having mirror plates, a constant K' will be also a good estimation for the fringing fields.

Then one gets:

$$\Delta\nu_x = -\frac{1}{4\pi}\sum_{WQ1.2}\left(\frac{1}{2} z_{co} z'_{co}\beta_x - \frac{1}{4} z^2_{co}\beta'_x\right)K'L$$

$$\Delta\nu_z = -\frac{1}{4\pi}\sum_{WQ1.2}\left(\frac{1}{2} z_{co} z'_{co}\beta_x + \frac{1}{4} z^2_{co}\beta'_z\right)K'L$$

where L is the length of the fringing field, on both sides of a quadrupole.

These wave numbers shifts have to be added to the linear optic in order to have the real operating point.

But we are more interested here in the effect corresponding to an additional displacement of the closed orbit around the design one. Then one has:

$$d\,(\Delta\nu_x) = -\frac{1}{4\pi}\sum_{1.2} K'L\,\{\left(\frac{1}{2} z'_{co}\beta_x - \frac{1}{2} z_{co}\beta'_x\right)dz_{co} + \frac{1}{2} z_{co}\beta_x dz'_{co}\}$$

$$d\,(\Delta\nu_z) = -\frac{1}{4\pi}\sum_{1.2} K'L\,\{\left(\frac{1}{2} z'_{co}\beta_z + \frac{1}{2} z_{co}\beta'_z\right)dz_{co} + \frac{1}{2} z_{co}\beta_z dz'_{co}\}$$

These formulaes show that in the case of antisymmetric beam bump (Fig.3) with regard to the center of the long straight section, the effect of WQ 1 an WQ 2 in both quadrants will be added - for instance the experiments shown on Fig. 1 - 2 were done in such a case.

In the case of a symmetric beam bump (Fig.4) the total effect from the two quadrants is zero. This also has been experimentally observed (Fig.5).

Let's look now to the order of magnitude of the fringing fields effects of WQ 1 and WQ 2, considering the case of an antisymmetric closed orbit displacement. The following table gives the data concerning the injection optic IV; moreover an extra closed orbit displacement is considered in the vertical plane, having a maximum value of about 1 cm in WQ 2 .

		z_{co} (m)	z'_{co} (rd)	β_x	β_z	β'_x	β'_z	K'L	dz_{co} (m)	dz'_{co} (rd)
WQ 1	left side	$3.21\cdot10^{-2}$	$1.2\cdot10^{-2}$	29	8	22	5	-.685	$3.2\cdot10^{-3}$	$1.2\cdot10^{-3}$
	right side	$6.2\cdot10^{-2}$	$4.5\cdot10^{-2}$	27	29	-24	42	.685	$6.2\cdot10^{-3}$	$4.5\cdot10^{-3}$
WQ 2	left side	$7.7\cdot10^{-2}$	$4.5\cdot10^{-2}$	20	44	-20	50	.53	$7.7\cdot10^{-3}$	$4.5\cdot10^{-3}$
	right side	$9.84\cdot10^{-2}$	$-.9\cdot10^{-2}$	10	72	- 1	-13	-.53	$9.84\cdot10^{-3}$	$-.9\ 10^{-3}$

The total effect of the two quadrants is then:

$$d(\Delta\nu_x) = -\ 2.44\cdot10^{-3}$$

$$d(\Delta\nu_z) = -\ 5.30\cdot10^{-3}$$

while the experiment gives:

$$d(\Delta\nu_x) \simeq -\ 3.5\cdot10^{-3}$$

$$d(\Delta\nu_z) \simeq -\ 4.5\cdot10^{-3}$$

Conclusion.

It appears that the fringing field of the large quadrupoles leads to important wave numbers shifts which are in good agreement with the experimental values. If it was really necessary to compensate this effect one should remind that the fringing field looks like an octupole, even if it has not the right symmetry, and the sextupolar effect which is obtained in the vertical plane is only a consequence of the large initialclosed orbit distorsion. Then the best way of compensation, if we consider only the vertical plane will be to put additional octupolar coils in the fringing field of the quadrupoles, to avoid resonance effects.

Notice, however, that an operational use of the machine means that the closed orbit is fixed and then the preceeding effect doesn't appear any more.

I would like to thank K.Steffen for helpful discussions.

A N N E X

The analytic expression for the fringing field of a quadrupole is [1] [2]

$$B_x = G\,z - \frac{1}{12}\,\frac{d^2G}{ds^2}\,(3x^2 + z^2)z + \ldots .$$

$$B_s = G'xz - \frac{1}{12}\,\frac{d^3G}{ds^3}\,(x^2 + z^2)\,xz + \ldots .$$

$$B_z = G\,x - \frac{1}{12}\,\frac{d^2G}{ds^2}\,(x^2 + 3z^2)\,x + \ldots .$$

The wave number deviations versus betatron amplitudes and closed orbit corresponding to such a perturbation are[3]:

$$\Delta\nu_x = -\frac{1}{4\pi}\int\left[\left(\frac{1}{4}\,\frac{dK}{ds}\,\beta'_z\beta_x\,\frac{E_x}{\pi} + \frac{1}{4}\,\frac{d^2K}{ds^2}\,\left(\frac{\beta_x^2}{4}\,\frac{E_x}{\pi} + \frac{\beta_x\beta_z}{2}\,\frac{E_z}{\pi}\right)\right]ds$$

$$-\frac{1}{4\pi}\int\frac{dK}{ds}\,z_{co}z'_{co}\,\beta_x ds - \frac{1}{16\pi}\int\frac{d^2K}{ds^2}\,(x_{co}^2 + z_{co}^2)\beta_x\,ds$$

$$\Delta\nu_z = \frac{1}{4\pi}\int\left[\left(\frac{1}{4}\,\frac{dK}{ds}\,\beta'_x\beta_z\,\frac{E_x}{\pi} + \frac{1}{4}\,\frac{d^2K}{ds^2}\,\left(\frac{\beta_z^2}{4}\,\frac{E_z}{\pi} + \frac{\beta_x\beta_z}{2}\,\frac{E_x}{\pi}\right)\right]ds$$

$$+\frac{1}{4\pi}\int\frac{dK}{ds}\,x_{co}\,x'_{co}\,\beta_z\,ds + \frac{1}{16\pi}\int\frac{d^2K}{ds^2}\,(x_{co}^2 + z_{co}^2)\beta_z\,ds$$

with the following definitions:

G = gradient of the quadrupole

K = $G/(p/e)$

E = emittance $(\pi\cdot\frac{\hat{y}^2}{\beta_y})$

x_{co}, z_{co} = closed orbit

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(1) E.Regenstreif - Proceeding of the Los Alamos Linac Conference (1966)

(2) R.L.Gluckstern - Brookhaven National Laboratory AADD-122 (1966)

(3) G.Gendreau, J.Le Duff, G.Neyret - CERN SPRING STUDY June 1972

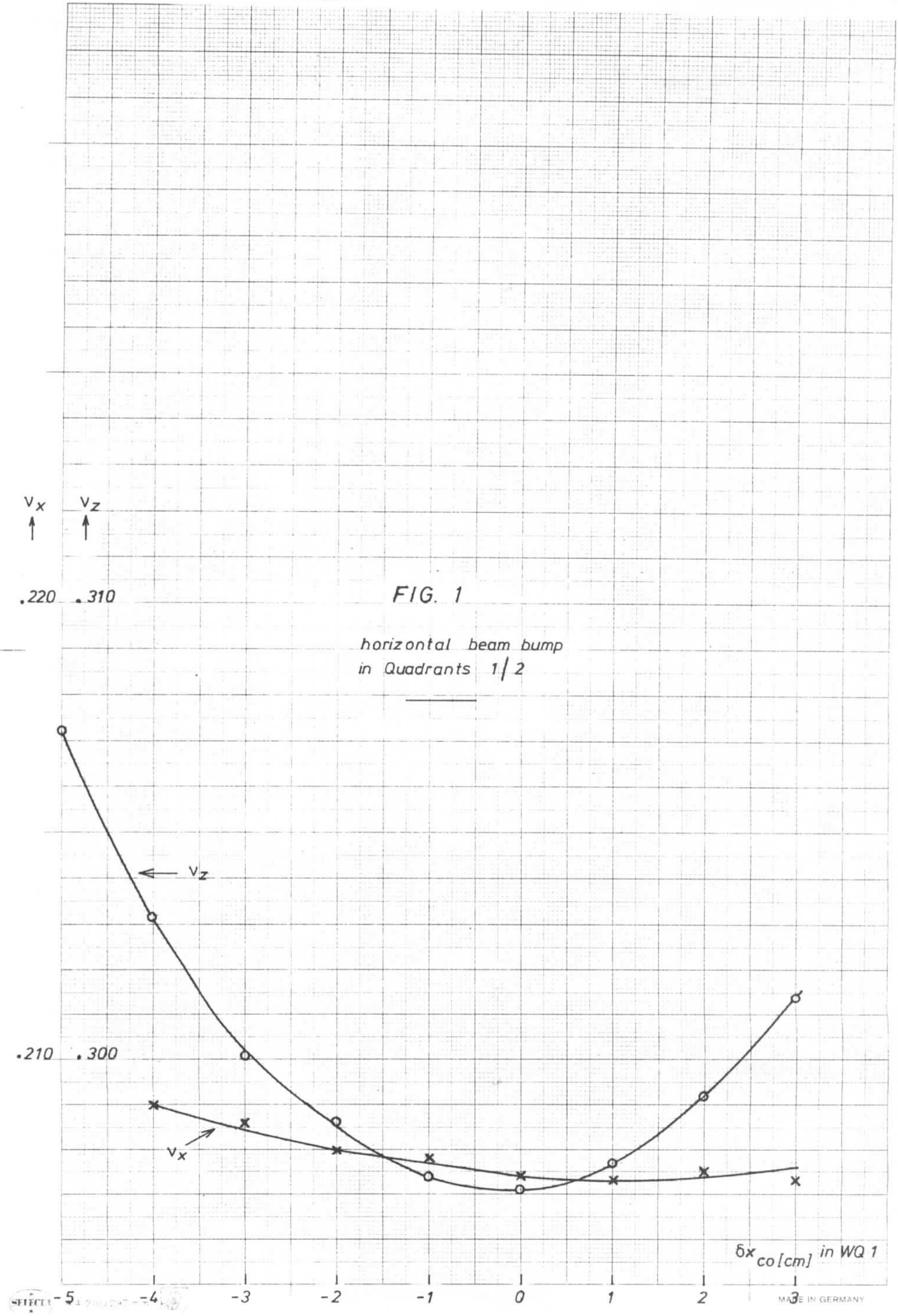

FIG. 1

horizontal beam bump
in Quadrants 1/2

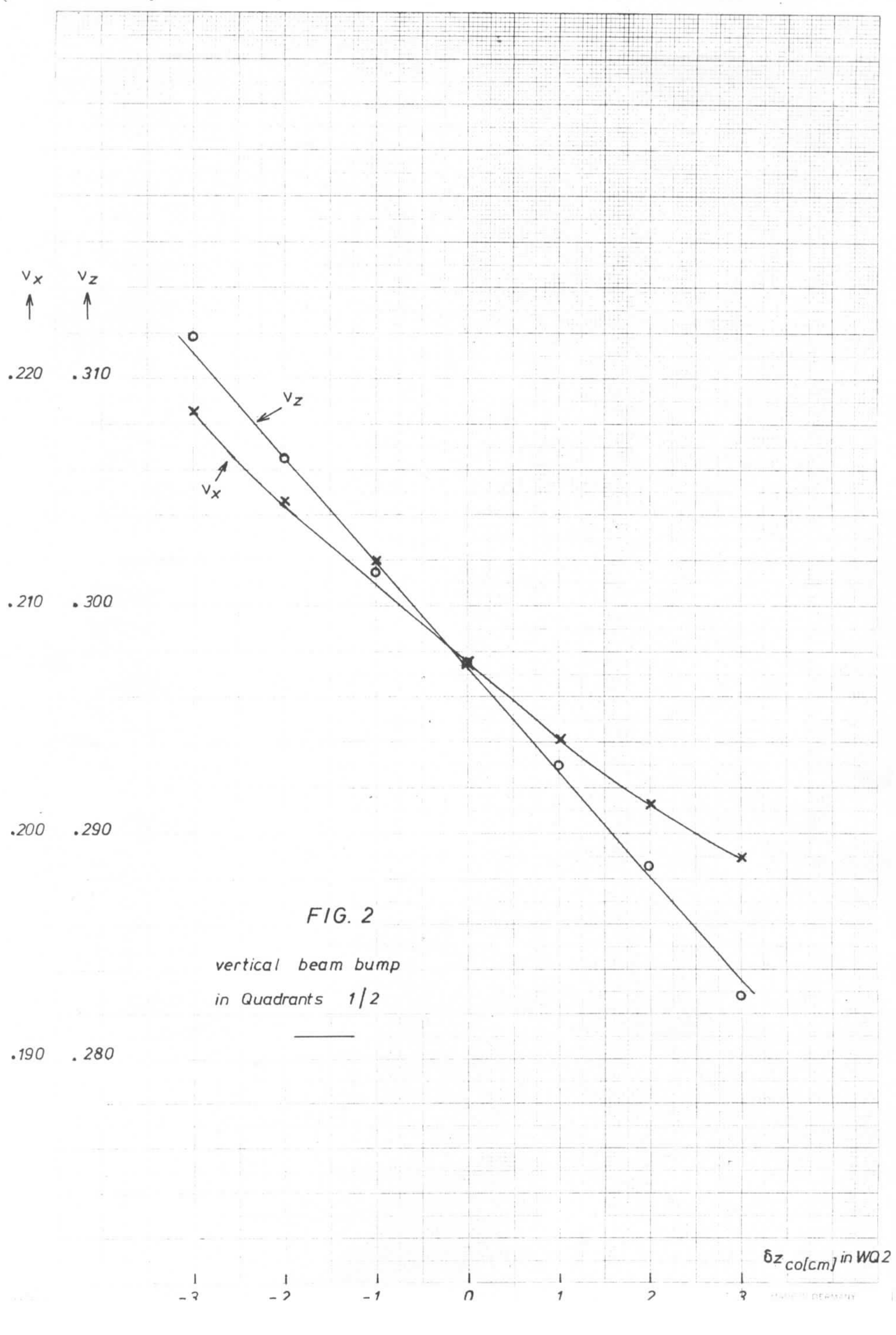

FIG. 2

vertical beam bump
in Quadrants 1/2

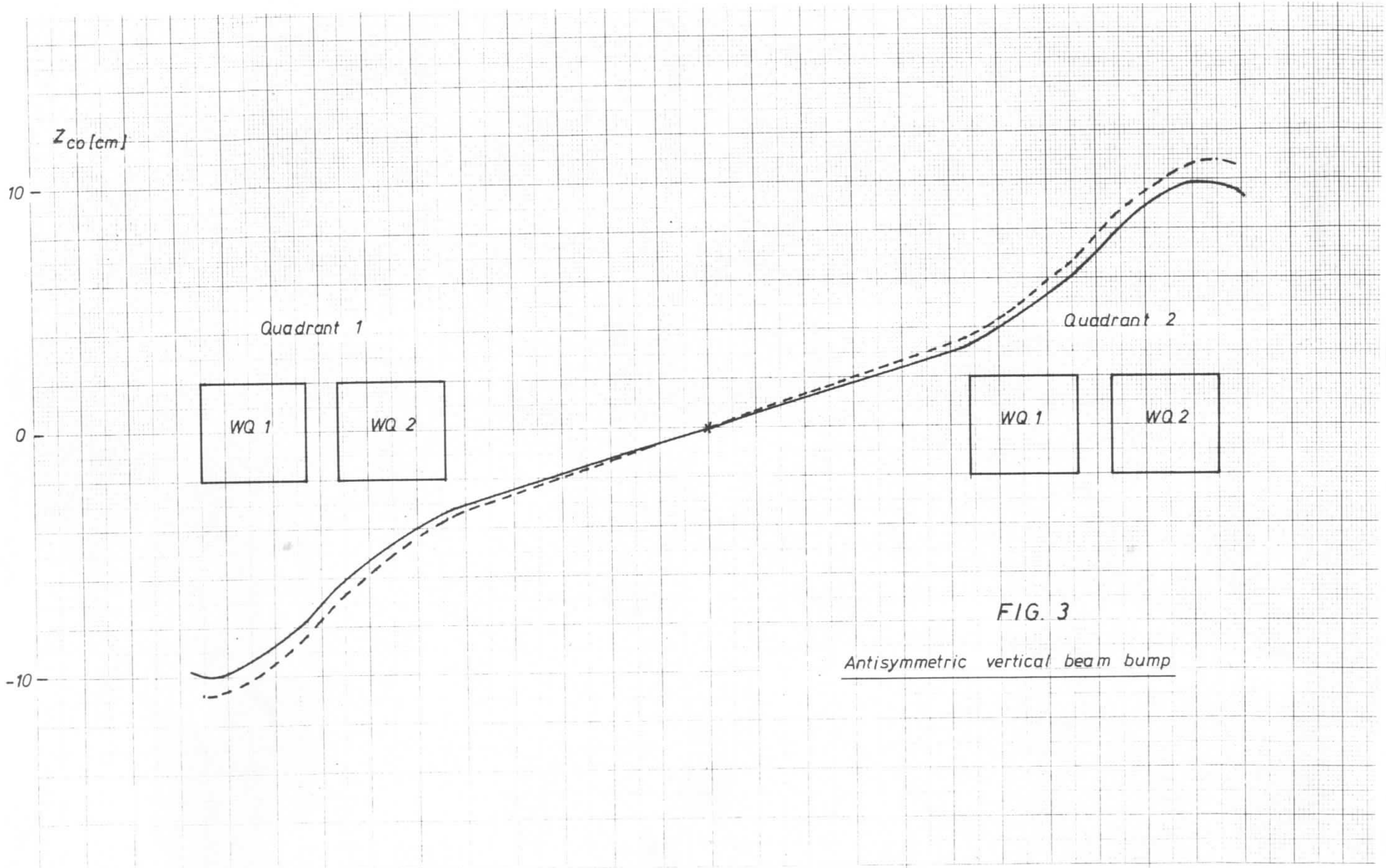

FIG. 3

Antisymmetric vertical beam bump

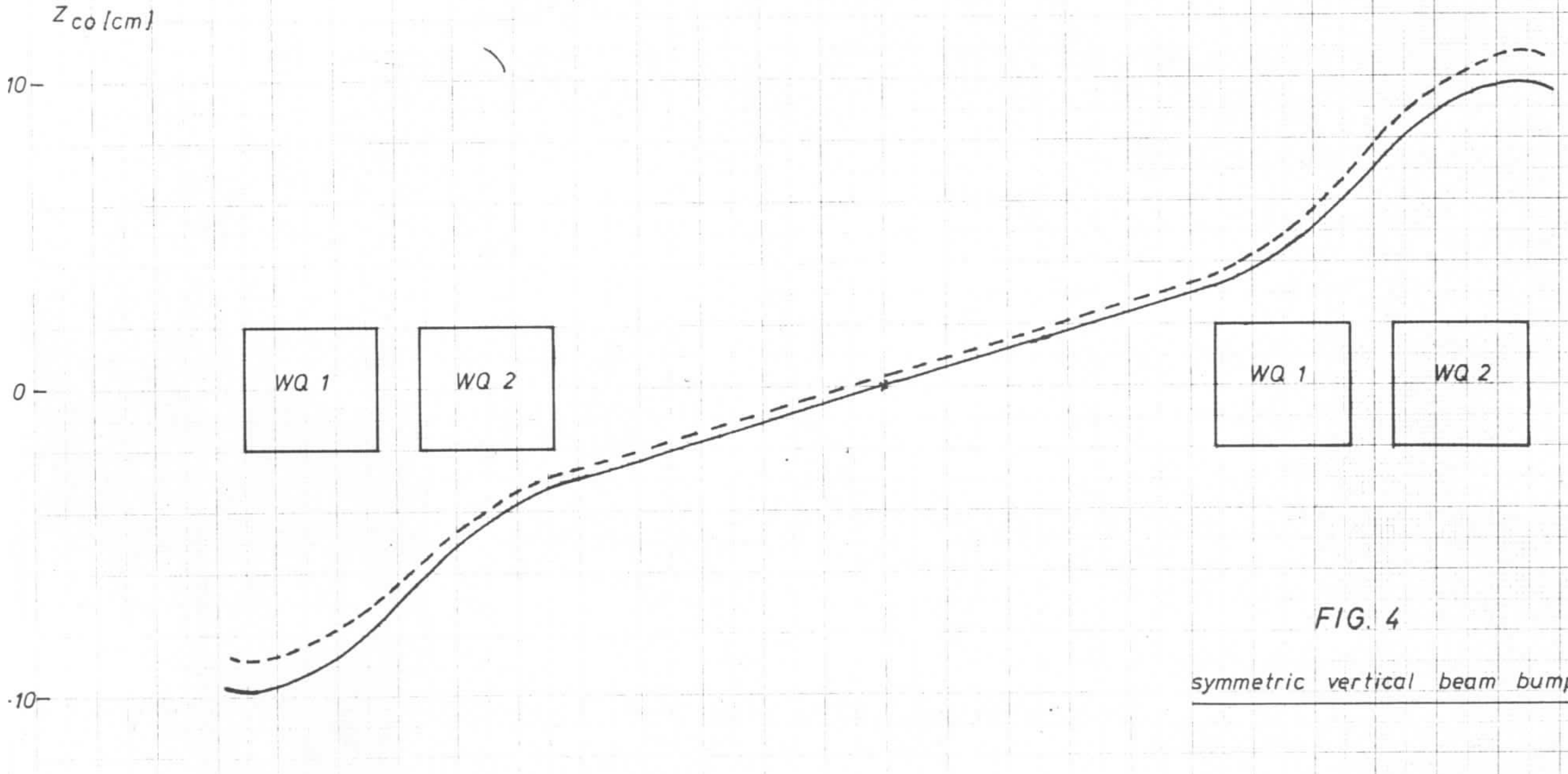

FIG. 4

symmetric vertical beam bump

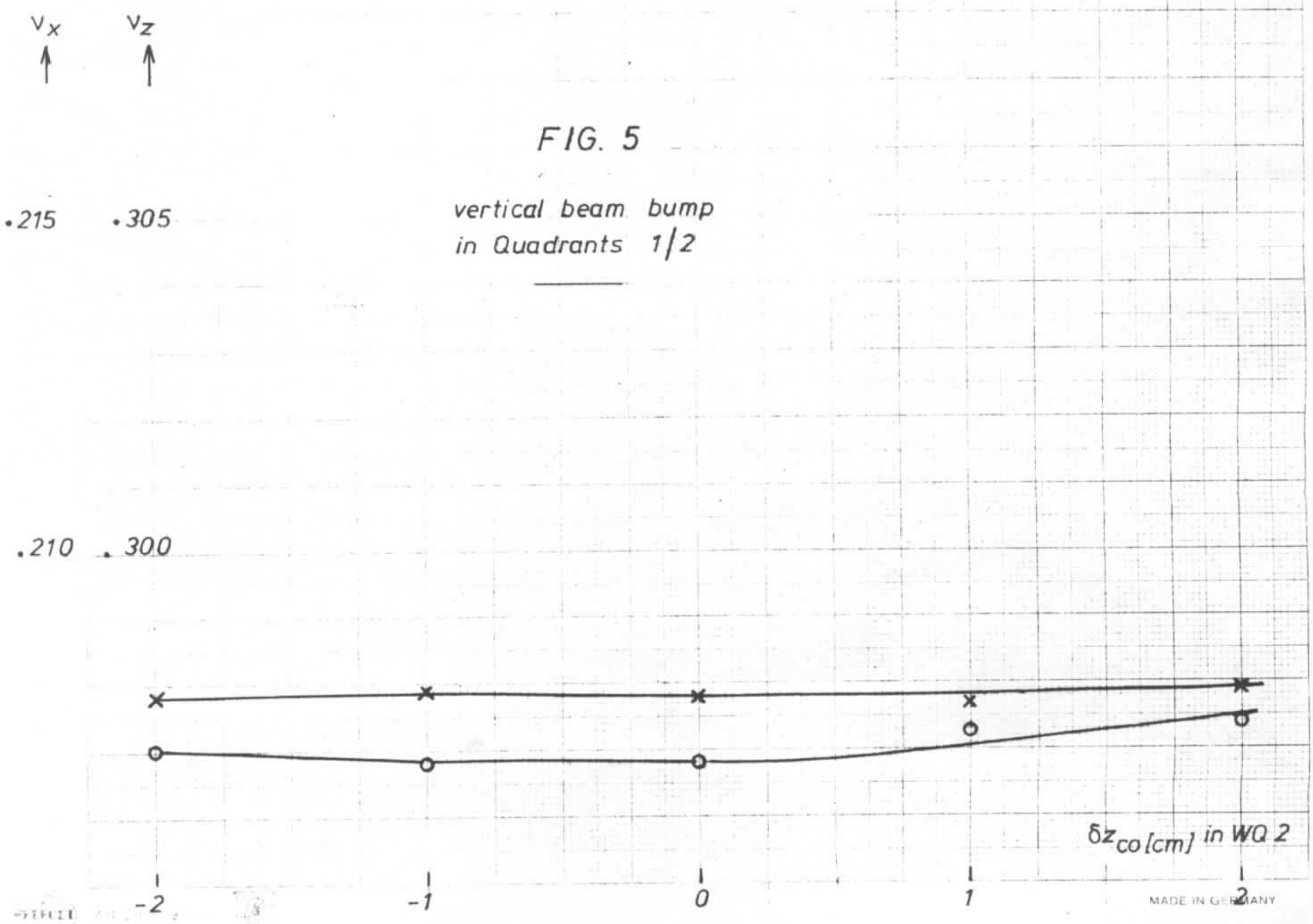

FIG. 5

vertical beam bump
in Quadrants 1/2