Supergravity in Four and Five Dimensions

Christoph Lüdeling

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Abstract

We review supergravity in the 'old minimal' approach and in the compensator formalism where we can express the gravitational multiplet in terms of unconstrained superfields (prepotentials). We find the same scalar potential in both cases. We comment on several mechanisms of supersymmetry and supergravity breaking with particular emphasis on the five-dimensional orbifold models.

Supergravitation in vier und fünf Raumzeitdimensionen

Christoph Lüdeling

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Zusammenfassung

Ich gebe einen Überblick über Supergravitation im "Old Minimal"-Ansatz sowie im Kompensatorformalismus, wobei die Felder des Gravitationsmultipletts durch Präpotentiale ohne Einschränkungen ausgedrückt werden. In beiden Fällen ergibt sich das gleiche skalare Potential. Ich gehe auf verschiedene Mechanismen der Supersymmetrie- und Supergravitationsbrechung ein, insbesondere auf fünfdimensionale Orbifold-Konstruktionen.

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Chapter 1

Introduction

The standard model of particle physics provides a very successful description af all particles and interactions that have been observed so far. However, there are theoretical and aesthetical reasons to believe that it is not the fundamental theory. These problems can be summarised under hierarchy, naturality, unification and gravity.

Let us first consider unification. The standard model is a gauge theory based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. At low energies, it is broken down to $SU(3)_C \times U(1)_{\mathrm{em}}$ by the Higgs mechanism, and as a result, the weak and electromagnetic interactions appear as separate forces of nature. It is therefore natural to inquire whether the standard model could be just a low-energy theory which originates from a grand unified theory based on a larger gauge group via a similar Higgs mechanism [1, 2]. Various models of such theories have been constructed. The most popular unified gauge groups have been $SU(5)$ [3] and $SO(10)$ [4, 5], which allow a convenient embedding of standard model particles in their multiplets[1], though there have been a range of other group choices.

The Higgs mechanism breaking these gauge symmetries operates at a high energy scale M_{GUT}. This scale can be estimated by considering the running of the standard model coupling constants with the energy. If a unified theory encorporating the standard model exists, the coupling constants should meet at a certain energy, thereby fixing the scale M_{GUT} [6].

Unified theories include new gauge bosons which, among other effects, mediate proton decay. This gives the most stringent lower bounds on the masses of these gauge fields, usually in the range of 10^{16} GeV. The simplest models of $SU(5)$ based unification are already ruled out experimentally.

The next puzzle concerns gravity. The theory of general relativity very elegantly explains gravitational interactions as the geometry of a curved spacetime manifold. However, it is a classical theory in the sense that it does not include quantum effects. So far, any attempt to formulate gravity as a gauge theory analogous to the other fundamental interactions has failed, because even it can be classically posed in a similar form, the theory is not renormalisable. There is, however, a theory that ventures to unify all interactions including gravity, string theory. String theory is not a gauge field theory as the standard model, but rather treats fundamental particles as one-dimensional objects (the 'strings') and automatically includes gravity. Unfortunately, string theory is mathematically and conceptually quite complex and is not yet able to make detailed testable predictions, but it does require eleven spacetime

[1]Since results from SuperKamiokaNDe, KamLAND and SNO give compelling evidence for the existence of right-handed neutrinos, $SO(10)$-models seem more natural since a complete family of the standard model can be incorporated into the 16-dimensional spinor representation.

dimensions for consistency. Most of these dimensions have not been observed so far, so they have to be compactified, curled up on very small distances into manifolds, orbifolds, conifolds, orientifolds or similar geometric objects.

Related to gravity is the hierarchy problem. The fundamental scale of gravity is generally thought to be the Planck scale, $M_{\mathrm{P}} = 2.4 \cdot 10^{18}$ GeV. Another fundamental scale might be the unification scale $M_{\mathrm{GUT}} \sim 10^{16}$ GeV. Both are more than ten orders of magnitude above the electroweak scale M_{Z}. The appearance of so vastly different scales in a theory has not been explained in a satisfactory manner.

High scales also lead to another problem known as naturality. The Higgs particle of the standard model is thought to have a mass below 1 TeV. Direct searches at LEP2 have excluded masses up to ≈ 114 GeV, but precision data favour a light Higgs with a mass not much above this limit. The mass of a scalar particle like the Higgs, however, receives quantum corrections quadratic in the cutoff. Since the only cutoff scales present are 14 or 16 orders of magnitude above the Higgs mass itself, the counterterms have to be fine-tuned to an extraordinary precision in every order of perturbation theory. This seems highly unnatural unless there is a mechanism which automatically generates these counterterms.

Such a mechanism is provided by supersymmetry. This is a symmetry relating fermions and bosons, which means that every fermionic particle has a bosonic partner. This immediately solves the naturality problem, since fermions and bosons contribute with opposite signs to the quadratic divergences which therefore cancel exactly without the need for any fine-tuning. But supersymmetry can also shed light on the other problems of the standard model. The coupling constants exactly unify in a supersymmetric grand unified theory, while they do not meet in non-supersymmetric models.

A locally supersymmetric theory necessarily includes gravity (so it is usually called supergravity). While it is still a non-renormalisable theory, string theory seems to require supersymmetry as well, so supergravity might serve as an effective theory below the energy scale associated with string theory (usually the Planck scale). However, there is no experimental evidence for supersymmetric partner particles yet, so supersymmetry must be broken somehow. There are models which achieve this breaking, some of which automatically explain electroweak symmetry breaking as well, accounting for the hierarchy between the weak and Planck scale. On the other hand, the actual breaking scheme is probably the dominant source of arbitrariness in the construction of realistic models.

A development which has attracted some attention in recent years are grand unified theories in higher dimensions [7]. The first models which tried to unify gravity and electromagnetism were put forward by Kaluza and Klein in the 1920's. They envisaged a five-dimensional spacetime with one dimension compactified on a circle S^1. The fields can be Fourier decomposed into sine and cosine modes in the fifth dimension. If the extra dimension is integrated out, these modes correspond to an infinite Kaluza-Klein tower of states with increasing equidistant masses proportional to the inverse radius of the circle. Recent models consider orbifolds rather than circles or tori as compact extra dimensions. Orbifold constructions offer many possibilities to break grand unified theories or supersymmetries while avoiding many problems of four-dimensional models. From the point of view of string theory, higher dimensions are present in any case, and orbifold compactifications first arose in this context [8].

For simple supersymmetry, there exists an elegant and powerful formalism, based on the so-called superspace. This formalism can alas not be generalised to higher dimensions in a straightforward manner, but there exist embeddings of higher-dimensional supersymmetric theories in superspace which conserves much of its usefulness [9–11].

This thesis is organised as follows. In Chapter 2 we introduce supersymmetry, the superspace formalism and give the Lagrangean for supersymmetric Yang-Mills theories. In Chapter 3 we turn to supergravity and present two formalisms: The old minimal approach in terms of geometric quantities and a compensator formalism which makes use of prepotentials, unconstrained fields in terms of which the supergravity fields are expressible. Methods of supersymmetry breaking are discussed in Chapter 4, including higher-dimensional theories and orbifolds. Finally, we give a summary and outline on perspectives for further work. In the appendix, we gather our conventions and the superfield embeddings mentioned above.

Chapter 2

Supersymmetry

The transformations which commute with the S matrix of a relativistic quantum field theory are strongly constrained by the Haag-Łopuszański-Sohnius theorem [12]. They can include the Poincaré group of Lorentz transformations and translations, an internal symmetry group commuting with the Poincaré group, and supersymmetry transformations. This theorem holds under very general assumptions. The algebra of supersymmetry transformations is a graded algebra, i.e. it involves commutators as well as anticommutators. It is presented in Section 2.1. For $N = 1$ supersymmetry, i.e. supersymmetry with one pair $\mathcal{Q}$, $\bar{\mathcal{Q}}$ of generators in four space-time dimensions, there exists a very elegant description in terms of differential operators on a higher dimensional so-called superspace. In superspace some of the coordinates commute, while others anticommute, reflecting the grading of the algebra. The functions of superspace, the superfields, provide a natural description of supersymmetry multiplets, i.e. fields that transform into each other under supersymmetry transformations. Superspace and superfields are introduced in Section 2.2 following ref. [13]. In Section 2.3 we introduce superspace integration and give the Lagrangean for supersymmetric Yang-Mills theories.

2.1 The Supersymmetry Algebra

The symmetry group of a relativistic quantum field theory is strongly restricted by the Coleman-Mandula theorem ([14], see also [15]). It states that under very broad assumptions the only Lie algebra of symmetry generators of the S matrix of such a theory is given by the generators of the Poincaré group and an internal symmetry group. The assumptions include an analytic and non-trivial S-matrix and a non-degenerate vacuum. The latter assumption can be relaxed to allow the conformal group instead of the Poincaré group. The theorem applies to the S matrix, the theory itself may have more (especially gauge) or less symmetries.

Haag, Łopuszański and Sohnius [12] extended the result of Coleman and Mandula to include graded algebras (also called superalgebras or Grassmann algebras) and found that the range of possible symmetry generators is extended, namely that there are additional generators $\mathcal{Q}$ and $\bar{\mathcal{Q}}$ which satisfy anticommutation relations and transform as spinors in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representations of the Lorentz group. These generators change the spin of states on which they act by $\frac{1}{2}$ and thus the statistics. They commute with the momentum operator, so there are mass-degenerate multiplets of states. This is obviously not observed experimentally, so this symmetry has to be (spontaneously) broken. We will analyze possible breaking schemes in later chapters. Now we will present the algebra.

The algebra consists of the generators of Lorentz transformations M_{mn}, the translation gen-

erator (momentum operator) P_m, the generators of some compact internal symmetry Lie group B_a and the supersymmetry generators $\mathcal{Q}^I_\alpha$ and $\bar{\mathcal{Q}}_{\dot\alpha I}$. The label I runs from 1 to some number N. If $N = 1$, one speaks of simple supersymmetry, otherwise of N-extended supersymmetry. These generators satisfy the following algebra (see e.g. [13, 16]):

$$[P_m, P_n] = 0 \tag{2.1a}$$
$$[M_{mn}, M_{rs}] = \mathrm{i}(\eta_{mr}M_{ns} - \eta_{nr}M_{ms} - \eta_{ms}M_{nr} + \eta_{ns}M_{mr}) \tag{2.1b}$$
$$[P_m, M_{rs}] = \mathrm{i}(\eta_{ms}P_r - \eta_{mr}P_s) \tag{2.1c}$$
$$[B_a, B_b] = \mathrm{i}f_{abc}B_c \tag{2.1d}$$
$$[B_a, P_m] = [B_a, M_{mn}] = 0 \tag{2.1e}$$
$$[\mathcal{Q}^I_\alpha, P_m] = [\bar{\mathcal{Q}}_{\dot\alpha I}, P_m] = 0 \tag{2.1f}$$
$$[\mathcal{Q}^I_\alpha, M_{mn}] = \tfrac{1}{2}(\sigma_{mn})_\alpha{}^\beta \mathcal{Q}^I_\beta \tag{2.1g}$$
$$[\bar{\mathcal{Q}}_{\dot\alpha I}, M_{mn}] = -\tfrac{1}{2}\bar{\mathcal{Q}}_{\dot\beta I}(\bar\sigma_{mn})^{\dot\beta}{}_{\dot\alpha} \tag{2.1h}$$
$$[\mathcal{Q}^I_\alpha, B_a] = (b_a)^I{}_J \mathcal{Q}^J_\alpha \tag{2.1i}$$
$$[\bar{\mathcal{Q}}_{\dot\alpha I}, B_a] = -\bar{\mathcal{Q}}_{\dot\alpha J}(b_a)^J{}_I \tag{2.1j}$$
$$\{\mathcal{Q}^I_\alpha, \bar{\mathcal{Q}}_{\dot\alpha J}\} = 2\delta^I_J \sigma^m_{\alpha\dot\alpha} P_m \tag{2.1k}$$
$$\{\mathcal{Q}^I_\alpha, \mathcal{Q}^J_\beta\} = 2\varepsilon_{\alpha\beta}Z^{IJ} \quad \text{with } Z^{IJ} = -Z^{JI} \tag{2.1l}$$
$$\{\bar{\mathcal{Q}}_{\dot\alpha I}, \bar{\mathcal{Q}}_{\dot\beta J}\} = 2\varepsilon_{\dot\alpha\dot\beta}Z^\dagger_{IJ} \tag{2.1m}$$

Our conventions regarding the metric and σ-matrices are given in Appendix A. Equations (2.1a) to (2.1c) give the algebra of the ordinary Poincaré group, equations (2.1d) and (2.1e) signify what is meant by internal symmetry. The f_{abc} are the structure constants and the coefficients b_a form some matrix representation of the symmetry group. From equation (2.1f) we immediately conclude $[Q, P^2] = 0$, so supersymmetry transformations do not change the mass of a state. The spinor properties of the $\mathcal{Q}$'s are shown in eqs. (2.1g) and (2.1h), where the σ_{mn} are the generators of the Lorentz group in the spinor representation. The remaining equations are given merely for completeness: since we will be dealing with $N = 1$ supersymmetry, the "central charges" Z^{IJ} vanish and the only internal symmetry which can act non-trivially (i.e. with nonvanishing matrix representation b_a) on the $\mathcal{Q}$'s is a $U(1)$ symmetry called R-symmetry. In many models, a remnant of this R-symmetry, R-parity is realised.

2.2 Superspace and Superfields

The supersymmetry algebra has been given in the last section as a graded algebra in terms of commutators and anticommutators. We can, however, formulate it entirely in terms of commutators if we introduce anticommuting spinorial parameters θ_α for the Grassmann-odd generators, because

$$[\theta\mathcal{Q}, \bar\theta\bar{\mathcal{Q}}] = \theta\{\mathcal{Q}, \bar{\mathcal{Q}}\}\bar\theta. \tag{2.2}$$

So we can express the supersymmetry algebra as a Lie algebra with anticommuting parameters and can thus define a group element by

$$G(x, \theta, \bar\theta) = e^{\mathrm{i}(-x^m P_m + \theta\mathcal{Q} + \bar\theta\bar{\mathcal{Q}})}. \tag{2.3}$$

This defines the supersymmetry group. It is the coset of the Super-Poincaré-Group with respect to the Lorentz group, i.e. it contains translations and supersymmetry transformations.

The product of two group elements is given by the Baker-Campbell-Hausdorff formula which involves only commutators and is now applicable. The left multiplication of two elements gives

$$G(0,\xi,\bar{\xi})G(x,\theta,\bar{\theta}) = G(x^m + \mathrm{i}\theta\sigma^m\bar{\xi} - \mathrm{i}\xi\sigma^m\bar{\theta}, \theta+\xi, \bar{\theta}+\bar{\xi}). \tag{2.4}$$

This motion in parameter space can (for infinitesimal ξ, $\bar{\xi}$) be generated by the differential operator $\xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}$, where the Q and $\bar{Q}$ form a representation of the algebra of the $\mathcal{Q}$'s and $\bar{\mathcal{Q}}$'s:

$$Q_\alpha = \partial_\alpha - \mathrm{i}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m \tag{2.5a}$$
$$\bar{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + \mathrm{i}\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\partial_m \tag{2.5b}$$

Here, $\partial_\alpha \equiv \frac{\partial}{\partial\theta^\alpha}$ and $\partial_{\dot{\alpha}} \equiv \frac{\partial}{\partial\theta^{\dot{\alpha}}}$. The properties of the partial spinor derivatives with respect to index positions and raising or lowering of indices are given in Appendix A.

In a similar manner, the right multiplication leads to differential operators D and $\bar{D}$:

$$D_\alpha = \partial_\alpha + \mathrm{i}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_m \tag{2.6a}$$
$$\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - \mathrm{i}\theta^\alpha\sigma^m_{\alpha\dot{\alpha}}\partial_m \tag{2.6b}$$

These operators by construction anticommute with the Q's and thus act as covariant derivatives. They will be used to impose covariant constraints on superfields.

Superspace is the supersymmetry group supermanifold parametrised by the coordinates $z = (x, \theta, \bar{\theta})$. Superfields are elements of the representation space of the supersymmetry group, that is, they are functions $F(z)$ of superspace. Since the θ- and $\bar{\theta}$-coordinates anticommute, any powers higher than θ^2 vanish and the superfield can be expanded in the Grassmann-odd coordinates:

$$\begin{aligned} F(x,\theta,\bar{\theta}) = f(x) + \theta\eta(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma^m\bar{\theta}v_m(x) \\ + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\chi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x) \end{aligned} \tag{2.7}$$

The x-dependent fields are called component fields and have Lorentz representation properties such as to make the superfield a Lorentz scalar, i.e. f, m, n and d are scalars, v_m is a vector and $\eta, \psi, \bar{\chi}$ and $\bar{\lambda}$ are Weyl spinors in the $(\frac{1}{2}, 0)$ or $(0, \frac{1}{2})$-representation.

The supersymmetry variation of a superfield $F(z)$ is given by

$$\delta F = (\xi Q + \bar{\xi}\bar{Q})F, \tag{2.8}$$

where Q and $\bar{Q}$ are the operators defined in (2.5). The commutator of two supersymmetry transformations is

$$[(\xi Q + \bar{\xi}\bar{Q}), (\eta Q + \bar{\eta}\bar{Q})]F = 2\mathrm{i}(\xi\sigma^m\bar{\eta} - \eta\sigma^m\bar{\xi})\partial_m F \tag{2.9}$$

A general superfield contains quite a lot of component fields and is not irreducible. However, we can impose constraints on superfields to obtain smaller multiplets. These constraints should be covariant, i.e. the constrained fields should still transform as superfields (eq. (2.8)). Two constrained fields which are of particular importance are chiral and vector superfields.

Chiral superfields ϕ form an irreducible multiplet. They are defined by the constraint

$$\bar{D}_{\dot{\alpha}}\phi = 0 \tag{2.10}$$

Their θ-expansion is particularly simple in the basis defined by y and θ, where $y^m = x^m + \mathrm{i}\theta\sigma^m\bar{\theta}$:[1]

$$\begin{aligned}\phi &= A(y) + \sqrt{2}\theta\chi(y) + \theta\theta F(y) \\ &= A(x) + \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m A(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x) \\ &\quad + \sqrt{2}\theta\chi(x) - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\partial_m\chi(x)\sigma^m\bar{\theta} + \theta\theta F(x)\end{aligned} \tag{2.11}$$

The fields A and F are complex scalars and ψ is a two-component Weyl spinor. A and χ will later turn out to be dynamical fields, while F is an auxiliary field, i.e. its equations of motion are purely algebraic. This difference is already obvious in the $(x, \theta, \bar{\theta})$-expansion, since it contains derivatives of A and χ, but not of F. This was to be expected from the mass dimension: Since θ has mass dimension $-\frac{1}{2}$, and we want spinors to have canonical dimension $\frac{3}{2}$, F has mass dimension 2. So a term proportional to F^*F cannot appear in the Lagrangean together with a derivative.

Alternatively, the component fields can be defined in the following way:

$$A = \phi\big| \tag{2.12a}$$

$$\chi_\alpha = \tfrac{1}{\sqrt{2}} D_\alpha\phi\big| \tag{2.12b}$$

$$F = -\tfrac{1}{4} DD\phi\big|, \tag{2.12c}$$

where $D_\alpha\phi\big|$ denotes taking the $\theta{=}\bar{\theta}{=}0$-component of $D_\alpha\phi$.

Analogously, antichiral superfields $\bar{\phi}$ are defined by

$$D_\alpha\bar{\phi} = 0 \tag{2.13}$$

Their expansion is especially simple in terms of $y^{\dagger m} = x^m - \mathrm{i}\theta\sigma^m\bar{\theta}$ and $\bar{\theta}$:[2]

$$\begin{aligned}\bar{\phi} &= A^*(y^\dagger) + \sqrt{2}\bar{\theta}\bar{\chi}(y^\dagger) + \bar{\theta}\bar{\theta}F^*(y^\dagger) \\ &= A^*(x) - \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m A^*(x) + \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A^*(x) \\ &\quad + \sqrt{2}\bar{\theta}\bar{\chi}(x) + \frac{\mathrm{i}}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^m\partial_m\bar{\chi}(x) + \bar{\theta}\bar{\theta}F^*(x)\end{aligned} \tag{2.14}$$

Any superfield which is both chiral and antichiral is constant, and the Hermitian adjoint $\phi^\dagger$ of a chiral field ϕ is antichiral, so chiral fields can not be Hermitian unless they are constant. Products of (anti)chiral superfields are again (anti)chiral.

The other important superfield is the so called vector superfield V, which is defined by the reality condition

$$V = V^\dagger. \tag{2.15}$$

This condition is preserved by supersymmetry transformations. V has the component field expansion

$$\begin{aligned}V(x, \theta, \bar{\theta}) = C &+ \mathrm{i}\theta\chi - \mathrm{i}\bar{\chi}\bar{\theta} + \tfrac{1}{2}\mathrm{i}\theta\theta(M + \mathrm{i}N) - \tfrac{1}{2}\mathrm{i}\bar{\theta}\bar{\theta}(M - \mathrm{i}N) \\ &- \theta\sigma^m\bar{\theta}v_m - \mathrm{i}\bar{\theta}\bar{\theta}\theta(\lambda + \tfrac{1}{2}\mathrm{i}\sigma^m\partial_m\bar{\chi}) + \mathrm{i}\theta\theta\bar{\theta}(\bar{\lambda} + \tfrac{1}{2}\mathrm{i}\,\bar{\sigma}^m\partial_m\chi) \\ &+ \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D + \tfrac{1}{2}\Box C)\end{aligned} \tag{2.16}$$

[1] The y-basis corresponds to the so-called chiral parametrisation of the group element (2.3) as $G_c(x, \theta, \bar{\theta}) = e^{-\mathrm{i}x^m P_m} e^{\mathrm{i}\theta Q} e^{\mathrm{i}\bar{\theta}\bar{Q}}$

[2] This corresponds to the antichiral parametrisation $G_a(x, \theta, \bar{\theta}) = e^{-\mathrm{i}x^m P_m} e^{\mathrm{i}\bar{\theta}\bar{Q}} e^{\mathrm{i}\theta Q}$

The scalar fields M, N, D and C and the vector field v_m all have to be real. The component field v_m gives the name to the whole superfield, which is itself a superspace scalar. It is not irreducible, but not fully reducible into smaller superfields either.

To define a supersymmetric generalisation of ordinary gauge theories, we need to define gauge transformations of the vector superfield and of the matter fields which we place in chiral superfields. The we can couple the matter to the gauge field and find an appropriate field strength tensor. We first consider Abelian gauge groups.

To be supersymmetric, the gauge parameter cannot be just a single function of spacetime, since any supersymmetry-invariant quantity is constant (cf. eq. (2.9)). Rather, it has to be a full multiplet which we choose to be a chiral one. If we furthermore like to reproduce the standard gauge transformation of the vector field, $v_m \to v_m + \partial_m \beta$ and to preserve the reality condition of the vector superfield (2.15), we are led to the transformation

$$V \to V + \mathrm{i}(\Lambda - \Lambda^\dagger) \tag{2.17}$$

with Λ a chiral multiplet. Under this transformation the vector field indeed transforms as

$$v_m \to v_m + \partial_m (A + A^*). \tag{2.18}$$

The fields λ and D are gauge invariant. We can actually use all the remaining gauge freedom in Λ and set C, χ, M and N to zero and end up with the vector field v_m, the gaugino λ and the auxiliary field D. This is the so-called Wess-Zumino gauge which has the advantage that any powers of V higher than V^2 vanish, since the lowest component is the $\theta\bar{\theta}$-component. This is important since we will see that the Lagrangean is non-polynomial in V. On the other hand, this gauge breaks supersymmetry since the supersymmetry variations of M, N and χ contain terms proportional to λ and v_m, respectively. So a supersymmetry transformation has to be followed by a field-dependent gauge transformation to restore the Wess-Zumino gauge.

The supersymmetric analogues of the field strength are the gauge-invariant superfields

$$\begin{aligned} W_\alpha &= -\tfrac{1}{4}\bar{D}\bar{D}D_\alpha V \\ \bar{W}_{\dot{\alpha}} &= -\tfrac{1}{4}DD\bar{D}_{\dot{\alpha}} V. \end{aligned} \tag{2.19}$$

These superfields are (anti)chiral and contain the standard field strength v_{mn} in their $\bar{\theta}$- or $\bar{\theta}$-component.

The matter fields ϕ_k, $\bar{\phi}_k$ transform as

$$\phi_k \to e^{-\mathrm{i}g_k\Lambda}\phi_k \qquad \bar{D}_{\dot{\alpha}}\Lambda = 0 \tag{2.20}$$

$$\bar{\phi}_k \to e^{\mathrm{i}g_k\Lambda^\dagger}\bar{\phi}_k \qquad D_\alpha \Lambda^\dagger = 0. \tag{2.21}$$

The charge of the ϕ_k is denoted by g_k. We see that the chirality of Λ preserves the chirality of ϕ.

For non-Abelian gauge groups, the fields ϕ are members of some representation of that group and both the gauge parameter Λ and the gauge superfield V become matrix valued:

$$\Lambda^j{}_i = (T^a)^j{}_i \Lambda^a \qquad V^j_i = (T^a)^j{}_i V^a, \tag{2.22}$$

where T^a are the (Hermitean) generators of the group in the appropriate representation. We normalize the generators in the adjoint representation such that

$$\mathrm{tr}\, T^a T^b = k\delta^{ab}, \qquad k \text{ real} \tag{2.23}$$

and thus the structure constants f^{abc} are completely antisymmetric.

The matter fields then transform according to

$$\phi^i \to e^{-\mathrm{i}\Lambda^i{}_j}\phi^j \qquad\qquad \bar{\phi}_j \to \bar{\phi}_i e^{\mathrm{i}(\Lambda^\dagger)^i{}_j} \tag{2.24}$$

The gauge transformation for the vector superfield are changed to restore gauge invariance of the kinetic term:

$$e^{2gV} \to e^{-\mathrm{i}\Lambda^\dagger} e^{2gV} e^{\mathrm{i}\Lambda} \tag{2.25}$$

Here we have included the coupling constant g.

Evaluating this for an infinitesimal transformation gives the transformation of V as a lengthy expression containing the hyperbolic cotangent of commutators of V and Λ but reducing to eq. (2.17) for an Abelian gauge group.

The field strength has to be generalised as well to

$$W_\alpha = -\tfrac{1}{4}\bar{D}\bar{D}e^{-2gV}D_\alpha e^{2gV} \qquad\qquad \bar{W}_{\dot{\alpha}} = -\tfrac{1}{4}DDe^{-2gV}\bar{D}_{\dot{\alpha}}e^{2gV}, \tag{2.26}$$

which is no longer invariant but transforms as

$$W_\alpha \to e^{-\mathrm{i}\Lambda} W_\alpha e^{\mathrm{i}\Lambda} \qquad\qquad \bar{W}_{\dot{\alpha}} \to e^{\mathrm{i}\Lambda^\dagger}\bar{W}_{\dot{\alpha}} e^{-\mathrm{i}\Lambda^\dagger}. \tag{2.27}$$

2.3 Lagrangeans

To define a Lagrangean, we first introduce superspace integrals. The integral over an anticommuting variable η is known as Berezin integral [17] and is defined by

$$\int \mathrm{d}\eta\, \eta = 1\,, \quad \int \mathrm{d}\eta\, 1 = 0. \tag{2.28}$$

We see from the definition that the mass dimension $[\mathrm{d}\eta]$ is $-[\eta]$. Since any function $f(\eta)$ is expandable in a short power series, $f(\eta) = c_0 + c_1\eta$, this definition completely fixes the integration of functions of Grassmann variables. Note that if we define a δ-function by

$$\int \mathrm{d}\eta\, \delta(\eta) f(\eta) = f(0) = c_0, \tag{2.29}$$

we see that $\delta(\eta) = \eta$. For superspace integration over the two-component spinorial variables θ and $\bar{\theta}$, we define

$$\mathrm{d}^2\theta = -\tfrac{1}{4}\mathrm{d}\theta^\alpha \mathrm{d}\theta^\beta \varepsilon_{\alpha\beta}, \tag{2.30a}$$

$$\mathrm{d}^2\bar{\theta} = -\tfrac{1}{4}\mathrm{d}\bar{\theta}_{\dot{\alpha}} \mathrm{d}\bar{\theta}_{\dot{\beta}} \varepsilon^{\dot{\alpha}\dot{\beta}}, \tag{2.30b}$$

$$\mathrm{d}^4\theta = \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}, \tag{2.30c}$$

$$\mathrm{d}^8 z = \mathrm{d}^4\theta \mathrm{d}^4 x, \tag{2.30d}$$

and find

$$\int \mathrm{d}^2\theta\, \theta\theta = \int \mathrm{d}^2\bar{\theta}\, \bar{\theta}\bar{\theta} = \int \mathrm{d}^4\theta\, \theta\theta\bar{\theta}\bar{\theta} = 1. \tag{2.31}$$

From the definition we can see that for Grassmann variables, integration and differentiation give the same result, which for any superfield F extends to

$$\int \mathrm{d}^8 z\, D_\alpha F = \int \mathrm{d}^8 z\, \bar{D}_{\dot{\alpha}} F = 0 \tag{2.32a}$$

and

$$\int \mathrm{d}^8 z\, F = -\tfrac{1}{4} \int \mathrm{d}^4 z\, \mathrm{d}^2\theta\, \bar{D}^2 F = -\tfrac{1}{4} \int \mathrm{d}^4 x\, \mathrm{d}^2\bar{\theta}\, D^2 F \tag{2.32b}$$

since the simple and covariant spinor derivatives differ only by a surface term. Thus we have for chiral fields ϕ

$$\int \mathrm{d}^8 z\, \phi = 0 = \int \mathrm{d}^8 z\, \bar{\phi}, \tag{2.33}$$

which means that chiral fields have to appear in a product with non-chiral fields or a $\delta\big(\bar{\theta}\big)$-function under the $\mathrm{d}^4\theta$-integral.

To define a supersymmetric theory, we need an action invariant under supersymmetry transformations. The Lagrangean itself cannot be invariant without being constant, so it has to transform into a total derivative. Both the F-component of a chiral superfield and the D-component of a vector superfield do have this property, as can be seen from the explicit form of the Q's (eq. (2.5)). So if we consider a theory involving only chiral fields ϕ_i, the most general renormalisable (i.e. containing no coupling constants of negative mass dimension) Lagrangean is

$$\mathscr{L} = \int \mathrm{d}^4\theta \left\{ \bar{\phi}_i \phi_i + \big[\underbrace{\big(\tfrac{1}{2} m_{ij} \phi_i \phi_j + \tfrac{1}{3} \lambda_{ijk} \phi_i \phi_j \phi_k\big)}_{\text{superpotential}} \delta^2\left(\bar{\theta}\right) + \text{h.c.} \big] \right\} \tag{2.34}$$

The expression containing mass and trilinear coupling terms is called superpotential. Its coefficients m_{ij} and g_{ijk} are completely symmetric in their indices. A linear term $a_i \phi_i$ in the superpotential can be absorbed by a field redefinition.

If we additionally introduce (non-Abelian) gauge interactions and the corresponding gauge vector superfields V and field strengths W_α, the kinetic term is not invariant and has to be modified. The kinetic term for the gauge field is constructed in analogy to ordinary gauge theory, and the resulting Lagrangean is

$$\begin{aligned} \mathscr{L} = \int \mathrm{d}^8 z \, \Big\{ & \frac{1}{16 k g^2} \mathrm{Tr}\left(W^\alpha W_\alpha \delta^2\left(\bar{\theta}\right) + \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} \delta^2\left(\theta\right)\right) + 2\xi V \\ & + \bar{\phi} e^{2gV} \phi + \big[\big(\tfrac{1}{2} m_{ij} \phi_i \phi_j + \tfrac{1}{3} \lambda_{ijk} \phi_i \phi_j \phi_k\big)\, \delta^2\left(\bar{\theta}\right) + \text{h.c.} \big] \Big\} \end{aligned} \tag{2.35}$$

For an Abelian gauge group, each superfield ϕ_i can have a different charge g_i and the m_{ij} and λ_{ijk} have to be invariant tensors of the gauge group to ensure gauge invariance of the mass and Yukawa terms. The Fayet-Iliopoulos term $2\xi V$ is only possible for Abelian gauge groups (or Abelian factors of the group) since it is not gauge invariant otherwise. It leads to a spontaneous breaking of supersymmetry and possibly gauge symmetry, depending on the parameters of the superpotential.

Chapter 3

Supergravity

Given a symmetry, we can ask whether it can be made local, i.e. whether it can be gauged. For supersymmetry this is indeed the case, and the resulting theory is called supergravity. As the name suggests, gravity is included. The reason for this can be seen from eq. (2.9): The commutator of two supersymmetry transformations is a partial spacetime derivative, so if the supersymmetry parameters are local, so is the parameter of the derivative. Since we want the algebra to close, the commutator of two symmetry transformations has to be another symmetry transformation, and general coordinate transformations become symmetries of the theory.

We can also interpret eq. (2.4) as a coordinate transformation in superspace. Since rigid supersymmetry (i.e. with constant parameters) should be a limit of supergravity, it is rather suggestive to define supergravity transformations as coordinate transformation in a curved superspace. Some aspects of superspace geometry are discussed in Section 3.1 (following [13], see also [18] for a more detailed discussion). In Section 3.2 we derive a Lagrangean for supergravity based on geometric expressions. A somewhat different approach is pursued in the following sections. We will express the supergravity multiplet, which is subject to constraints, in terms of unconstrained superfields, the prepotentials. First we reconsider supersymmetric gauge theories as an analogy in Section 3.3 and then turn to supergravity in Section 3.4.

3.1 Differential Forms

As the setup for the geometrical description of supergravity, we consider superspace as an 8-dimensional (super-)manifold with coordinates $z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$. As before, the x-coordinates correspond to ordinary spacetime and hence commute while the θ-coordinates are spinorial Grassmann variables and anticommute. This is expressed by

$$z^M z^N = (-)^{mn} z^N z^M \tag{3.1}$$

where m and n are the grading of the coordinates z^M and z^N, i.e. $m = 0$ for vector and $m = 1$ for spinor indices. For the exterior product of differential forms, there is an additional minus sign:

$$\mathrm{d}z^M \mathrm{d}z^N = -(-)^{mn} \mathrm{d}z^N \mathrm{d}z^M \tag{3.2}$$

The product of differential forms $\mathrm{d}z^M \mathrm{d}z^N$ is understood to be the wedge product $\mathrm{d}z^M \wedge \mathrm{d}z^N$ throughout.

We can now define p-forms and exterior derivatives just as in the usual case:

$$X = \mathrm{d}z^{M_1} \cdots \mathrm{d}z^{M_p} X_{M_p \cdots M_1} \tag{3.3}$$

and

$$\mathrm{d}X = \mathrm{d}z^{M_1} \cdots \mathrm{d}z^{M_p} \mathrm{d}z^M \partial_M X_{M_p \cdots M_1} \tag{3.4}$$

Note the relative order of the indices. The grading of the coefficient function $X_{M_p \cdots M_1}$ is given by $\sum_i m_i$ (modulo 2). The multiplication of differential forms is linear, graded commutative and associative:

$$\begin{aligned} (aX + bY)Z &= aXZ + bYZ \\ XY &= (-)^{pq} YX \quad \text{for } X \text{ a } p\text{-form and } Y \text{ a } q\text{-form} \\ X(YZ) &= (XY)Z \end{aligned} \tag{3.5}$$

The exterior derivative has the usual properties as well:

$$\begin{aligned} \mathrm{d}(X + Y) &= \mathrm{d}X + \mathrm{d}Y \\ \mathrm{d}(XY) &= X\mathrm{d}Y + (-)^p (\mathrm{d}X)Y \quad \text{for } Y \text{ a } p\text{-form} \\ \mathrm{dd} &= 0 \end{aligned} \tag{3.6}$$

Differential forms are automatically covariant under coordinate transformations. We can additionally require covariance under a tangent space group (e.g. a compact Lie group for gauge theories or the Lorentz group for supergravity), such that differential forms transform as tensors in a representation of that group like

$$X^i \to X^j g_j{}^i = X'^i. \tag{3.7}$$

As usual, this transformation rule is not preserved under exterior derivatives since

$$\mathrm{d}X' = X\mathrm{d}g + \mathrm{d}X g, \tag{3.8}$$

so we have to introduce a connection one-form

$$\Omega = \mathrm{d}z^M \Omega_M^r \, \mathrm{i}T^r \tag{3.9}$$

where the T^r are the generators of the group under consideration. The connection has to transform as

$$\Omega \to g^{-1}\Omega g - g^{-1}\mathrm{d}g, \tag{3.10}$$

so covariant derivatives of tensors X

$$\mathscr{D}X = \mathrm{d}X + X\Omega \tag{3.11}$$

again transform as tensors under the group:

$$(\mathscr{D}X)^i \to (\mathscr{D}X)^j g_j{}^i. \tag{3.12}$$

From the connection we can then construct the curvature (or field strength) tensor

$$R = \mathrm{d}\Omega + \Omega\Omega = \tfrac{1}{2} E^B E^A R_{AB} \tag{3.13}$$

which is a Lie algebra valued two-form satisfying the Bianchi identities

$$\mathscr{D}\mathscr{D}X = XR \tag{3.14}$$

and

$$\mathscr{D}R = 0. \tag{3.15}$$

The differential forms $\mathrm{d}z^M = (\mathrm{d}z^m, \mathrm{d}z^\mu, \mathrm{d}z_{\dot\mu})$ define a basis in the cotangent bundle of superspace. The derivatives $\partial_M = (\partial_m, \partial_\mu, \partial^{\dot\mu})$ form the dual basis in the tangent bundle. However, this basis might not be the most convenient, e.g. when dealing with the transformation of superfields. Generally, we might want to work in a different basis $E_A = (E_a, E_\alpha, E^{\dot\alpha})$. This must be accompanied by a change of basis in the cotangent bundle to a new frame given by the (super)vielbein forms

$$E^A = \mathrm{d}z^M E_M{}^A \tag{3.16}$$

This new basis provides us with a flat frame of reference at each point of superspace. In supergravity, the supervielbein will later be identified with the graviton and the gravitino whereas it is constant in flat superspace (e.g. in Yang-Mills theories where we can choose $E_A = D_A$, i.e. the vielbein derivatives as the supersymmetry-covariant derivatives). It is a straightforward extension of the standard vielbein formalism of general relativity. The upper index is taken to transform under the tangent space group. The vielbein $E_M{}^A$ and its inverse $E_A{}^M$ can be used to convert indices of one kind to another:

$$V_M = E_M{}^A V_A, \quad V_A = E_A{}^M V_M \tag{3.17}$$

with

$$E_M{}^A E_A{}^N = \delta_M{}^N \text{ and } E_A{}^M E_M{}^B = \delta_A{}^B. \tag{3.18}$$

Just as for the connection, we can consider the covariant derivative of the vielbein, the torsion:

$$T^A = \mathrm{d}E^A + E^B \Omega_B{}^A = \tfrac{1}{2} E^C E^D T_{DC}{}^A \tag{3.19}$$

As an example, let us consider flat superspace corresponding to rigid supersymmetry. If derivatives of superfields are to transform as superfields again, the basis $\mathrm{d}z^M$ and ∂_M is not appropriate, so it is convenient to choose $E_A = D_A = (\partial_a, D_\alpha, \bar D_{\dot\alpha})$ as a basis in the tangent bundle. In terms of the vielbein fields, this means that $D_A = E_A{}^M \partial_M$. The matrix $E_A{}^M$ is constant and can be read off directly from eq. (2.6) as

$$E_A{}^M = \begin{pmatrix} \delta_a{}^m & 0 & 0 \\ \mathrm{i}\sigma^m_{\alpha\dot\alpha}\bar\theta^{\dot\alpha} & \delta_\alpha{}^\mu & 0 \\ \mathrm{i}\theta^\alpha \sigma^m_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha} & 0 & \delta^{\dot\alpha}{}_{\dot\mu} \end{pmatrix}, \tag{3.20}$$

so its inverse $E_M{}^A$ can be found to be

$$E_M{}^A = \begin{pmatrix} \delta_m{}^a & 0 & 0 \\ -\mathrm{i}\sigma^a_{\mu\dot\mu}\bar\theta^{\dot\mu} & \delta_\mu{}^\alpha & 0 \\ -\mathrm{i}\theta^\nu \sigma^a_{\nu\dot\nu}\varepsilon^{\dot\nu\dot\mu} & 0 & \delta^{\dot\mu}{}_{\dot\alpha} \end{pmatrix}. \tag{3.21}$$

The basis differentials are given by $E^A = \mathrm{d}z^M E_M{}^A$. This basis has the disadvantage that the torsion is not zero:

$$T_{\alpha\dot\alpha}{}^a = T_{\dot\alpha\alpha}{}^a = 2\mathrm{i}\sigma^a_{\alpha\dot\alpha} \tag{3.22}$$

This geometrical setup is too general so far to give a minimal supergravity theory. In general relativity, the torsion could be constrained to vanish. However, it is non-zero even in flat superspace, so a torsion-free geometry would rule out flat supersymmetry as a solution. There are, however, constraints on the torsion which simplify the theory considerably without overly restricting it. These constraints will be discussed in Section 3.2.

3.2 The Supergravity Lagrangean

In supergravity we are dealing with local supersymmetry transformations, so the theory has to be covariant under general coordinate transformations. As structure group we will take the Lorentz group, such as to allow flat superspace as a limit of the theory.

Under an infinitesimal coordinate transformation

$$z^M \to z'^M = z^M - \xi^M(z) \tag{3.23}$$

the vielbein changes as

$$\delta E_M{}^A = -\xi^L \partial_L E_M{}^A - (\partial_M \xi^L) E_L{}^A. \tag{3.24}$$

The Lorentz transformations $L_B{}^A$ act on the upper index which is therefore called Lorentz (or flat space) index:

$$\delta E_M{}^A = E_M{}^B L_B{}^A \tag{3.25}$$

The lower index will be called world index or Einstein index. In general, world indices are taken form the middle of the alphabet and Lorentz indices from the beginning.

The Lorentz transformation $L_B{}^A$ will not intertwine spacetime and spinor nor dotted and undotted spinor coordinates. It is thus reducible into three blocks $L_b{}^a$, $L_\beta{}^\alpha$ and $L^{\dot\beta}{}_{\dot\alpha}$ which describe the same transformation in the appropriate representations:

$$\sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta} L_{ab} = -2\varepsilon_{\alpha\beta} L_{\dot\alpha\dot\beta} + 2\varepsilon_{\dot\alpha\dot\beta} L_{\alpha\beta} \tag{3.26}$$

The connection one-form associated with Lorentz transformations is called spin connection

$$\Omega_B{}^A = \mathrm{d}z^M \Omega_{MB}{}^A \tag{3.27}$$

We recall the definition of the torsion (eq. (3.19)):

$$T^A = \mathrm{d}E^A + E^B \Omega_B{}^A$$

and the curvature (eq. (3.13))

$$R = \mathrm{d}\Omega + \Omega\Omega.$$

We can now state the torsion constraints which will lead to a minimal formulation of supergravity. These constraints can be divided into four classes:

- Constraints which allow (anti)chiral superfields: In rigid supersymmetry, chiral fields ϕ with $\bar D_{\dot\alpha}\phi = 0$ were important fields containing the matter fields. Analogously, we would like to define covariantly chiral superfields Φ by a similar condition:

  $$\bar{\mathscr{D}}_{\dot\alpha}\Phi = 0 \tag{3.28}$$

  We will see that the torsion appears in the (anti)commutator of covariant derivatives, so nonvanishing torsion components $T_{\dot\alpha\dot\beta}{}^\gamma$ and $T_{\dot\alpha\dot\beta}{}^c$ would constrain chiral fields to be constant:

  $$0 = \{\bar{\mathscr{D}}_{\dot\alpha}, \bar{\mathscr{D}}_{\dot\beta}\} = -T_{\dot\alpha\dot\beta}{}^\gamma \mathscr{D}_\gamma \Phi - T_{\dot\alpha\dot\beta}{}^c \mathscr{D}_c \Phi \tag{3.29}$$

  The same argument applies to antichiral superfields satisfying $\mathscr{D}_\alpha \bar\Phi = 0$. Therefore we will impose the constraints

  $$T_{\dot\alpha\dot\beta}{}^\gamma = T_{\dot\alpha\dot\beta}{}^c = T_{\alpha\beta}{}^{\dot\gamma} = T_{\alpha\beta}{}^c = 0 \tag{3.30}$$

- We can require the anticommutator of covariant dotted and undotted spinor derivatives to be identical to flat superspace:

$$\{\mathscr{D}_\alpha, \bar{\mathscr{D}}_{\dot\alpha}\} = -2\mathrm{i}\sigma^a_{\alpha\dot\alpha}\mathscr{D}_a \tag{3.31}$$

  This corresponds to the torsion constraints

$$T_{\alpha\dot\alpha}{}^a = T_{\dot\alpha\alpha}{}^a = 2\mathrm{i}\sigma^a_{\alpha\dot\alpha} \tag{3.32a}$$

$$T_{\alpha\dot\alpha}{}^\beta = T_{\alpha\dot\alpha}{}^{\dot\beta} = 0 \tag{3.32b}$$

- We would like to express the spin connection in terms of the vielbein. This is enabled by the constraints

$$T_{\alpha\beta}{}^\gamma = T_{\dot\alpha\dot\beta}{}^{\dot\gamma} = T_{ab}{}^c = 0 \tag{3.33}$$

- The constraints so far correspond to conformal supergravity. Since we are interested in an extension of ordinary general relativity, we finally require

$$T_{Ab}{}^c = 0. \tag{3.34}$$

We can now solve the Bianchi identity (3.14) in the form

$$\mathscr{D}\mathscr{D}E^A = \mathscr{D}T^A = E^B R_B{}^A \tag{3.35}$$

with these constraints (see [13,19]) and find that the torsion and the curvature can be expressed in terms of three superfields R, G_a and $W_{\alpha\beta\gamma}$. R and $W_{\alpha\beta\gamma}$ are chiral, G_a is real. The second Bianchi identity (3.15) $\mathscr{D}R_A{}^B = 0$ is satisfied by these expressions as well.

Now that we have introduced connection, torsion and curvature, we can define a covariant derivative. Acting on a Lorentz tensor, the covariant derivative is

$$\mathscr{D}_M V^A = \partial_M V^A + (-)^{mb} V^B \Omega_{MB}{}^A. \tag{3.36}$$

In this expression, m and b are the gradings of the indices M and B. Since world and Lorentz indices can be transformed into each other by the vielbein, we can choose to work with Lorentz tensors if possible because their transformation properties are simple.

The (anti)commutator of two covariant derivatives is

$$[\mathscr{D}_A, \mathscr{D}_B]_\pm V^C = -T_{AB}{}^D \mathscr{D}_D V^C + (-)^{d(a+b)} V^D R_{ABD}{}^C, \tag{3.37}$$

where $[\mathscr{D}_A, \mathscr{D}_B]_\pm$ denotes the anticommutator for two spinor derivatives and the commutator if one of the derivatives carries a vector index.

Under a combined coordinate transformation $z^M \to z'^M = z^M - \xi^M$ and a Lorentz transformation $L_B{}^A$, a Lorentz tensor V^A transforms as

$$\delta V^A = -\xi^M \partial_M V^A + V^B L_B{}^A, \tag{3.38}$$

which is not Lorentz covariant. If we replace the partial with a covariant derivative, the transformation picks up an extra term:

$$\begin{aligned} \delta V^A &= -\xi^M \mathscr{D}_M V^A + V^B \xi^M \Omega_{MB}{}^A + V^B L_B{}^A \\ &= -\xi^B \mathscr{D}_B V^A + V^B (\xi^C \Omega_{CB}{}^A + L_B{}^A) \end{aligned} \tag{3.39}$$

Since the connection is Lie algebra valued, it satisfies a condition similar to eq. (3.26). We can thus choose a particular Lorentz transformation which depends on the parameter of the coordinate transformation such that the last bracket in eq. (3.39) vanishes:

$$L_B{}^A = -\xi^C \Omega_{CB}{}^A \tag{3.40}$$

so that the transformation of any tensor superfield becomes

$$\delta V^A = -\xi^B \mathscr{D}_C V^A \tag{3.41}$$

These combination of coordinate transformation and Lorentz transformation is called supergauge transformation.

The vielbein and the connection transform inhomogeneously under supergauge transformations:

$$\delta E_M{}^A = -\mathscr{D}_M \xi^A - \xi^B T_{BM}{}^A \tag{3.42}$$

$$\delta \Omega_{MA}{}^B = -\xi^C R_{CMA}{}^B \tag{3.43}$$

We can, however, perform general coordinate and Lorentz transformations on $E_M{}^A$ and $\Omega_{MA}{}^B$, which then transform as

$$\delta E_M{}^A = -\mathscr{D}_M \xi^A - \xi^B T_{BM}{}^A + E_M{}^B L_B{}^A \tag{3.44}$$

$$\delta \Omega_{MA}{}^B = -\xi^C R_{CMA}{}^B + \Omega_{MA}{}^C L_C{}^B - (-)^{m(a+c)} L_A{}^C \Omega_{MC}{}^B - \partial_M L_A{}^B \tag{3.45}$$

By these transformations, we can gauge away the higher components of $E_M{}^A$ and $\Omega_{MA}{}^B$ and restrict the $\theta = \bar{\theta} = 0$-components to

$$\begin{array}{lll} E_m{}^a\big| = e_m{}^a & E_m{}^\alpha\big| = \frac{1}{2}\psi_m{}^\alpha & E_{m\dot\alpha}\big| = \frac{1}{2}\bar\psi_{m\dot\alpha} \\ E_\mu{}^a\big| = 0 & E_\mu{}^\alpha\big| = \delta_\mu{}^\alpha & E_{\mu\dot\alpha}\big| = 0 \\ E^{\dot\mu a}\big| = 0 & E^{\dot\mu\alpha}\big| = 0 & E^{\dot\mu}{}_{\dot\alpha}\big| = \delta^{\dot\mu}{}_{\dot\alpha} \end{array} \tag{3.46}$$

and

$$\Omega_{mA}{}^B\big| = \omega_{mA}{}^B \qquad \Omega_{\mu A}{}^B\big| = 0 \qquad \Omega^{\dot\mu}{}_A{}^B\big| = 0. \tag{3.47}$$

The field $e_m{}^a$ corresponds to the graviton (or just vielbein as opposed to the "supervielbein" $E_M{}^A$), $\psi_m{}^\alpha$ and $\bar\psi_{m\dot\alpha}$ to its superpartner, the gravitino. Due to (3.33), the remaining part of the torsion, $\omega_{mA}{}^B$, can be expressed in terms of the graviton and gravitino as

$$\begin{aligned} \omega_{nml} &= E_m{}^B \omega_{nB}{}^A E_{Al} = e_m{}^b \omega_{nb}{}^a e_{al} \\ &= \tfrac{1}{2}\big[-\tfrac{\mathrm{i}}{2} e_{la}\left(\psi_m\sigma^a\bar\psi_n - \psi_n\sigma^a\bar\psi_m\right) - \tfrac{\mathrm{i}}{2} e_{ma}\left(\psi_n\sigma^a\bar\psi_l - \psi_l\sigma^a\bar\psi_n\right) \\ &\quad + \tfrac{\mathrm{i}}{2} e_{na}\left(\psi_l\sigma^a\bar\psi_m - \psi_m\sigma^a\bar\psi_l\right) - e_{la}\left(\partial_n e_m{}^a - \partial_m e_n{}^a\right) \\ &\quad - e_{ma}\left(\partial_l e_n{}^a - \partial_n e_l{}^a\right) + e_{na}\left(\partial_m e_l{}^a - \partial_l e_m{}^a\right)\big] \end{aligned} \tag{3.48}$$

Of the superfields R, G_a and $W_{\alpha\beta\gamma}$ used to describe the torsion and curvature tensors, all but the lowest components of R and G_a can either be gauged away or expressed in terms of the graviton, gravitino and the lowest components

$$-\tfrac{1}{6}M = R\big| \qquad\qquad -\tfrac{1}{3}b_a = G_a\big| \tag{3.49}$$

and their derivatives. These fields are auxiliary fields and can be eliminated from the theory by their equations of motion.

Now we have collected all fields forming the supergravity multiplet. They are the graviton ${e_m}^a$, the gravitino ${\psi_m}^\alpha$, $\bar\psi_{m\dot\alpha}$, the complex scalar field M and the real vector field b_a.

The transformation of these fields under local supersymmetry variations can be obtained from the general vielbein transformation law (3.44) by focusing on the lowest component. We restrict the lowest component of ξ^a and of ${L_A}^B$ to vanish, such that the transformation is parametrised by $\xi^\alpha\big| \equiv \zeta^\alpha$ and $\bar\xi_{\dot\alpha}\big| \equiv \bar\zeta_{\dot\alpha}$. We cannot, however, set all remaining components of ξ^A and ${L_A}^B$ to zero since that would destroy the gauge (3.46) and (3.47), so that we have to include higher components that depend on ζ^α, $\bar\zeta_{\dot\alpha}$ and the fields M, M^* and b_a. The transformation of M and b_a is obtained from the general tensor field transformation law (3.41). The resulting transformations are:

$$\delta {e_m}^a = \mathrm{i}\left(\psi_m\sigma^a\bar\zeta - \zeta\sigma^a\bar\psi_m\right) \tag{3.50a}$$

$$\delta {\psi_m}^\alpha = -2\mathscr{D}_m\zeta^\alpha + \mathrm{i}{e_m}^c\left[\tfrac{1}{3}M(\varepsilon\sigma_c\bar\zeta)^\alpha + b_c\zeta^\alpha + \tfrac{1}{3}b^d(\zeta\sigma_d\bar\sigma_c)^\alpha\right] \tag{3.50b}$$

$$\delta\bar\psi_{m\dot\alpha} = -2\mathscr{D}_m\bar\zeta_{\dot\alpha} - \mathrm{i}{e_m}^c\left[\tfrac{1}{3}M^*(\zeta\sigma_c)_{\dot\alpha} + b_c\bar\zeta_{\dot\alpha} + \tfrac{1}{3}b^d(\bar\zeta\bar\sigma_d\sigma_c)_{\dot\alpha}\right] \tag{3.50c}$$

$$\delta M = -\zeta\left(\sigma^a\bar\sigma^b\psi_{ab} + \mathrm{i}b^a\psi_a - \mathrm{i}\sigma^a\bar\psi_a M\right) \tag{3.50d}$$

$$\delta b_{\alpha\dot\alpha} = \zeta^\delta\Big[\tfrac{3}{4}\bar\psi_\alpha{}^{\dot\gamma}{}_{\delta\dot\gamma\dot\alpha} + \tfrac{1}{4}\bar\psi^{\gamma\dot\gamma}{}_{\gamma\dot\alpha\dot\gamma} - \tfrac{\mathrm{i}}{2}M^*\psi_{\alpha\dot\alpha\delta} \tag{3.50e}$$

$$+\tfrac{\mathrm{i}}{4}\left(\bar\psi_{\alpha\dot\rho}{}^{\dot\rho}b_{\delta\dot\alpha} + \bar\psi_{\delta\dot\rho}{}^{\dot\rho}b_{\alpha\dot\alpha} - \bar\psi_\delta{}^{\dot\rho}{}_{\dot\alpha}b_{\alpha\dot\rho}\right)\Big] + \text{ h.c.} \tag{3.50f}$$

Here we have used the notation

$$\begin{aligned}
{\psi_{ab}}^\alpha &= {e_a}^m{e_b}^n{\psi_{mn}}^\alpha \\
{\psi_{mn}}^\alpha &= \left(\partial_m{\psi_n}^\alpha + {\psi_n}^\beta{\omega_{m\beta}}^\alpha\right) - \left(\partial_n{\psi_m}^\alpha + {\psi_m}^\beta{\omega_{n\beta}}^\alpha\right) \\
{\psi_{\alpha\dot\alpha\beta\dot\beta}}^\gamma &= \sigma^a_{\alpha\dot\alpha}\sigma^b_{\beta\dot\beta}{\psi_{ab}}^\gamma
\end{aligned} \tag{3.51}$$

Just as in rigid supersymmetry, we would like to put matter in chiral superfields, and we have imposed the torsion constraints (3.30) explicitly because of this. A covariantly (anti)chiral field Φ ($\bar\Phi$) is defined in the obvious way by the condition

$$\bar{\mathscr{D}}_{\dot\alpha}\Phi = 0, \tag{3.52}$$

$$\mathscr{D}_\alpha\bar\Phi = 0. \tag{3.53}$$

Note, however, that due to the algebra of the covariant derivatives (3.37), covariant spinor derivatives do no longer anticommute, so the chiral and antichiral projectors are changed, so that for any scalar superfield U

$$\bar{\mathscr{D}}_{\dot\alpha}(\bar{\mathscr{D}}_{\dot\beta}\bar{\mathscr{D}}^{\dot\beta} - 8R)U = 0 \tag{3.54a}$$

$$\mathscr{D}_\alpha(\mathscr{D}^\beta\mathscr{D}_\beta - 8R^\dagger)U = 0. \tag{3.54b}$$

To be able to restrict ourselves to Lorentz tensors, we define the component fields analogous to (2.12):

$$A = \Phi\big| \tag{3.55a}$$

$$\chi_\alpha = \tfrac{1}{\sqrt{2}}\mathscr{D}_\alpha\Phi\big| \tag{3.55b}$$

$$F = -\tfrac{1}{4}\mathscr{D}\mathscr{D}\Phi\big| \tag{3.55c}$$

From (3.41), we can read off the transformation of the component fields under local supersymmetry transformations:

$$\delta A = -\sqrt{2}\zeta\chi \tag{3.56a}$$

$$\delta\chi_\alpha = -\sqrt{2}\zeta_\alpha F - \mathrm{i}\sqrt{2}\sigma^a_{\alpha\dot\alpha}\bar\zeta^{\dot\alpha}\left(\mathscr{D}_a A\big|\right) \tag{3.56b}$$

$$\delta F = -\tfrac{\sqrt{2}}{3}M^*\zeta\chi + \bar\zeta_{\dot\alpha}\left(\tfrac{\sqrt{2}}{6}b^{\dot\alpha\alpha}\chi_\alpha - \mathrm{i}\sqrt{2}\left(\mathscr{D}^{\dot\alpha\alpha}\chi_\alpha\big|\right)\right) \tag{3.56c}$$

Now all we need to write down invariant Lagrangeans is a volume element, the generalisation of $e = \det e_m{}^a$ form ordinary general relativity. We can construct such a field Δ which is called chiral density. It has to transforms as

$$\delta\Delta = -\partial_M[\xi^M\Delta(-)^m] \tag{3.57}$$

such that a product of this density and a scalar superfield Φ transforms again as a density:

$$\delta(\Delta\Phi) = -\partial_M[\xi^M(\Delta\Phi)(-)^m] \tag{3.58}$$

The chiral density we will consider here is a chiral field built from the determinant of the vielbein e [13]. Its components are

$$\Delta\big| = e \tag{3.59a}$$

$$\tfrac{1}{\sqrt{2}}\mathscr{D}_\alpha\Delta\big| = \tfrac{\mathrm{i}}{2}\sqrt{2}e\sigma^m\bar\psi_m \tag{3.59b}$$

$$-\tfrac{1}{4}\mathscr{D}\mathscr{D}\Delta\big| = -e\left(M^* + \bar\psi_m\bar\sigma^{mn}\bar\psi_n\right) \tag{3.59c}$$

Now we can consider the general setup for a supergravity theory involving only chiral matter fields Φ^i, $\bar\Phi^j$. The Lagrangean is determined by a Hermitean function $K(\Phi,\bar\Phi)$ – the Kähler potential – and the chiral superpotential $W(\Phi)$ depending only on the chiral fields. The Lagrangean is given by

$$\mathscr{L} = \frac{1}{\kappa}\int \mathrm{d}^2\theta\,\Delta\left\{\tfrac{3}{8}(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)\exp\left[-\frac{\kappa}{3}K(\Phi^i,\bar\Phi^j)\right] + \kappa W(\Phi^i)\right\} + \text{ h.c.} \tag{3.60}$$

where $\kappa = 8\pi G_N = M_{\mathrm{P}}^{-2}$ is the gravitational coupling constant with G_N being Newton's constant and M_{P} the Planck mass. For the remainder of this section,, we will choose mass units such that $M_{\mathrm{P}} = 1$. If we choose $K = 0$, we obtain the pure supergravity action

$$\mathscr{L}_{SG} = -3\int \mathrm{d}^2\theta\,\Delta R + \text{ h.c.} \tag{3.61}$$

We can evaluate this Lagrangean in terms of component fields and find a rather lengthy expression. The Lagrangean includes the usual terms for the matter fields, the vielbein and the Rarita-Schwinger gravitino, including the coupling of matter to supergravity, as well as four-fermion-interactions.

The name Kähler potential is not incidental: The chiral fields Φ^i, $\bar\Phi^j$ can be interpreted as coordinates on a complex Kähler manifold. This manifold is equipped with a metric g_{ij} and associated Christoffel symbols Γ^k_{ij} and Riemann curvature R_{ijkl}. These geometric objects are subject to certain conditions which mean that they can be obtained from the Kähler potential

K by differentiation. If we denote differentiation of K with respect to the coordinate Φ^i ($\bar{\Phi}^j$) by a subscript K_i (K_{j^*}), we have

$$g_{ij^*} \equiv K_{ij^*} \tag{3.62a}$$
$$g_{ij} = 0 \tag{3.62b}$$
$$g_{i^*j^*} = 0 \tag{3.62c}$$
$$g_{mj^*}\Gamma^m_{ik} = g_{ij^*,k} \tag{3.62d}$$
$$g_{im^*}\Gamma^{m^*}_{j^*k^*} = g_{ij^*,k^*} \tag{3.62e}$$
$$R_{ij^*kl^*} = g_{kl^*,ij^*} - g^{mn^*} g_{ml^*,j^*} g_{kn^*,i}. \tag{3.62f}$$

However, the gravitational action is not yet properly normalised, so we have to perform a rescaling of the vielbein and compensating redefinitions of the spinors to restore the normalisation of their kinetic terms:

$$e_m{}^a \to \exp\{\tfrac{1}{6}K\} e_m{}^a \tag{3.63a}$$
$$\chi_i \to \exp\{-\tfrac{1}{12}K\}\chi_i \tag{3.63b}$$
$$\psi_m \to \exp\{\tfrac{1}{12}K\}\psi_m + \tfrac{\sqrt{2}}{6}\mathrm{i}\sigma_m\bar{\chi}K_{i^*} \tag{3.63c}$$

Here K denotes the same Kähler potential as above, but as a function of the scalar components of the chiral superfields.

Using the language of Kähler geometry, the complete Lagrangian is given by

$$\begin{aligned} e^{-1}\mathscr{L} = &-\tfrac{1}{2}\mathscr{R} - g_{ij^*}\partial_m A^i \partial^m A^{*j} - \mathrm{i} g_{ij^*}\bar{\chi}^j\bar{\sigma}^m\mathscr{D}_m\chi^i \\ &+ \varepsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\tilde{\mathscr{D}}_m\psi_n - \tfrac{1}{\sqrt{2}}g_{ij^*}\partial_n A^{*j}\chi^i\sigma^m\bar{\sigma}^n\psi_m \\ &- \tfrac{1}{\sqrt{2}}g_{ij^*}\partial_n A^i\bar{\chi}^j\bar{\sigma}^m\sigma^n\bar{\psi}_m + \tfrac{1}{4}g_{ij^*}\left(\mathrm{i}\varepsilon^{klmn}\psi_k\sigma_l\bar{\psi}_m + \psi_m\sigma^n\bar{\psi}^m\right)\chi^i\sigma_n\bar{\chi}^j \\ &- \tfrac{1}{8}(g_{ij^*}g_{kl^*} - 2R_{ij^*kl^*})\,\chi^i\chi^k\bar{\chi}^j\bar{\chi}^l \\ &- \exp\{K/2\}\Big[W^*\psi_m\sigma^{mn}\psi_n + W\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n + \tfrac{\mathrm{i}}{\sqrt{2}}D_iW\chi^i\sigma^m\bar{\psi}_m \\ &+ \tfrac{\mathrm{i}}{\sqrt{2}}D_{i^*}W^*\bar{\chi}^i\bar{\sigma}^M\psi_m + \tfrac{1}{2}\mathscr{D}_iD_jW\chi^i\chi^j + \tfrac{1}{2}\mathscr{D}_{i^*}D_{j^*}W^*\bar{\chi}^i\bar{\chi}^j\Big] \\ &- e\exp\{K\}\left[g^{ij^*}(D_iW)(D_jW)^* - 3W^*W\right] \end{aligned} \tag{3.64}$$

This expression is rather compact since we have included the following covariant derivatives:

$$\mathscr{D}_m\chi^i = \partial_m\chi^i + \chi^i\omega_m + \Gamma^i_{jk}\partial_m A^j\chi^k - \tfrac{1}{4}(K_j\partial_m A^j - K_{j^*}\partial_m A^{*j})\chi^i \tag{3.65a}$$
$$\tilde{\mathscr{D}}_m\psi_n = \partial_m\psi_n + \psi_n\omega_m + \tfrac{1}{4}(K_j\partial_m A^j - K_{j^*}\partial_m A^{*j})\psi_n \tag{3.65b}$$
$$D_iW = W_i + K_iP \tag{3.65c}$$
$$\mathscr{D}_iD_jW = W_{ij} + K_{ij} + K_iD_jW + K_jD_iW - K_iK_jW - \Gamma^k_{ij}D_kW \tag{3.65d}$$

$\mathscr{R}$ is the usual Ricci scalar constructed from the lowest component ω of the spin connection Ω in the usual manner as

$$\mathscr{R} = e_a{}^n e_b{}^m\left(\partial_n\omega_m{}^{ab} - \partial_m\omega_n{}^{ab} + \omega_m{}^{ac}\omega_{nc}{}^b - \omega_n{}^{ac}\omega_{mc}{}^b\right) \tag{3.66}$$

The last line is the scalar potential which controls the spontaneous breaking of supergravity. Its vacuum expectation value determines the cosmological constant.

From equation (3.62), we see that a Kähler transformation $K(\Phi,\bar\Phi)\to K(\Phi,\bar\Phi)+F(\Phi)+F^\dagger(\bar\Phi)$ does not change the metric and the connection. The last terms in (3.65a) and (3.65b) transform as a connection under these transformations. However, the Lagrangean (3.60) is not invariant under these transformations. They have to be accompanied by super-Weyl-transformations of the vielbein and matter fields. These are rescalings of the superfields parametrised by a chiral field Σ and its conjugate $\bar\Sigma$. The superpotential must be rescaled as well. These combined transformations generate a symmetry and can be used to transform the Lagrangean to a slightly different form (if the expectation value $\langle W\rangle$ of the superpotential is nonzero) by changing the Kähler potential to $G=K+\ln|W|^2$ and the superpotential to unity:

$$\begin{aligned}
e^{-1}\mathscr{L} = &-\tfrac{1}{2}\mathscr{R} - g_{ij^*}\partial_m A^i \partial^m A^{*j} - \mathrm{i} g_{ij^*}\bar\chi^j\bar\sigma^m\mathscr{D}_m\chi^i \\
&+\varepsilon^{klmn}\bar\psi_k\bar\sigma_l\tilde{\mathscr{D}}_m\psi_n - \tfrac{1}{\sqrt{2}}g_{ij^*}\partial_n A^{*j}\chi^i\sigma^m\bar\sigma^n\psi_m \\
&-\tfrac{1}{\sqrt{2}}g_{ij^*}\partial_n A^i\bar\chi^j\bar\sigma^m\sigma^n\bar\psi_m + \tfrac{1}{4}g_{ij^*}\left(\mathrm{i}\varepsilon^{klmn}\psi_k\sigma_l\bar\psi_m + \psi_m\sigma^n\bar\psi^m\right)\chi^i\sigma_n\bar\chi^j \\
&-\tfrac{1}{8}(g_{ij^*}g_{kl^*}-2R_{ij^*kl^*})\,\chi^i\chi^k\bar\chi^j\bar\chi^l - \exp(G/2)\,[\psi_m\sigma^{mn}\psi_n \\
&+\bar\psi_m\bar\sigma^{mn}\bar\psi_n + \tfrac{\mathrm{i}}{\sqrt{2}}G_i\chi^i\sigma^m\bar\psi_m + \tfrac{\mathrm{i}}{\sqrt{2}}G_{i^*}\bar\chi^i\bar\sigma^m\psi_m \\
&+\tfrac{1}{2}\left(G_{ij}+G_iG_j-\Gamma^k_{ij}G_k\right)\chi^i\chi^j \\
&+\tfrac{1}{2}\left(G_{i^*j^*}+G_{i^*}G_{j^*}-\Gamma^{k^*}_{i^*j^*}G_{k^*}\right)\bar\chi^i\bar\chi^j\big] \\
&-\exp(G)\left[g^{ij^*}G_iG_{j^*}-3\right]
\end{aligned} \tag{3.67}$$

In this form we can read off the gravitino mass directly:

$$m_\psi = \langle e^{G/2}\rangle \tag{3.68}$$

3.3 Analogy: Supersymmetric Gauge Theories

We can use the geometric formalism of the previous chapter to analyse gauge theories. It will turn out that while there are constraints on the torsion and the field strength, we can express these quantities in terms of a single superfield without constraints. In the next section we will present a similar formalism for supergravity. First we will turn to gauge theories as an analogy and will find the same results as in Section 2.3 by a different route.

We take the structure group to be the (compact semisimple) gauge group $\mathcal{G}$ and superfields $\Psi,\bar\Psi$ in some representation of that group. The Hermitean generators are denoted by T^r with commutation relations

$$[T^r,T^s] = \mathrm{i}f^{rst}T^t, \tag{3.69}$$

the structure constants f^{rst} are real and completely antisymmetric. The connection is the gauge potential one-form superfield

$$A = \mathrm{d}z^M A_M = \mathrm{d}z^M E_M{}^A A_A = E^A A_A, \quad A_M = A^r_M\,\mathrm{i}T^r. \tag{3.70}$$

Under gauge transformations, the gauge potential and the fields transform as

$$A \to g^{-1}Ag - g^{-1}\mathrm{d}g \tag{3.71a}$$
$$\Psi \to g^{-1}\Psi \tag{3.71b}$$
$$\bar\Psi \to \bar\Psi g. \tag{3.71c}$$

The covariant derivatives of superfields

$$\mathscr{D}\Psi = \mathrm{d}\Psi - A\Psi \tag{3.72a}$$
$$\mathscr{D}\bar{\Psi} = \mathrm{d}\bar{\Psi} + \bar{\Psi}A \tag{3.72b}$$

then transform as the superfields themselves.

As in the previous chapter, the field strength tensor

$$F = \mathrm{d}A + AA = \tfrac{1}{2}\mathrm{d}z^M \mathrm{d}z^N F_{NM} = \tfrac{1}{2}E^A E^B F_{BA} \tag{3.73}$$

satisfies the Bianchi identities

$$\mathscr{D}F = \mathrm{d}F - AF + FA = 0 \tag{3.74}$$

and

$$\mathscr{D}\mathscr{D}\Psi = -F\Psi. \tag{3.75}$$

However, the field strength tensor contains far too many component fields. The number of these component fields can be greatly reduced by imposing appropriate constraints which preserve Lorentz invariance and supersymmetry (see e.g. [13]):

$$F_{\beta\alpha} = D_\beta A_\alpha + D_\alpha A_\beta - \{A_\beta, A_\alpha\} = 0 \tag{3.76a}$$
$$F^{\dot\beta\dot\alpha} = \bar{D}^{\dot\beta} A^{\dot\alpha} + \bar{D}^{\dot\alpha} A^{\dot\beta} - \{A^{\dot\beta}, A^{\dot\alpha}\} = 0 \tag{3.76b}$$
$$F_\beta{}^{\dot\alpha} = D_\beta A^{\dot\alpha} + \bar{D}^{\dot\alpha} A_\beta - \{A_\beta, A^{\dot\alpha}\} + 2\mathrm{i}\sigma^a_{\beta\dot\alpha} A_a = 0 \tag{3.76c}$$

We will now express the gauge potential A_A and the field strength F_{BA} in terms of real unconstrained superfields V, which are of course identified with the vector superfields of Section 2.3. We first define covariantly chiral superfields Φ and $\bar\Phi$ by the condition

$$\bar{\mathscr{D}}^{\dot\alpha}\Phi = 0, \quad \mathscr{D}_\alpha\bar\Phi = 0. \tag{3.77}$$

These covariantly chiral fields can be obtained from chiral fields ϕ, $\phi^\dagger$ satisfying $D_\alpha\phi^\dagger = 0 = \bar{D}^{\dot\alpha}\phi$ by introducing a field $V = V^r T^r$, where $V = V^\dagger$ and the T^r are the generators of the gauge group in the following way:

$$\Phi = e^V \phi \qquad \bar\Phi = \phi^\dagger e^V \tag{3.78a}$$
$$\mathscr{D}_\alpha = e^{-V} D_\alpha e^V \qquad \bar{\mathscr{D}}^{\dot\alpha} = e^V \bar{D}^{\dot\alpha} e^{-V} \tag{3.78b}$$

Obviously the conditions (3.77) are satisfied:

$$\bar{\mathscr{D}}^{\dot\alpha}\Phi = 0 \tag{3.79a}$$
$$\bar\Phi \overleftarrow{\mathscr{D}}_\alpha = 0 \tag{3.79b}$$

The spinorial components of the gauge connection can be easily read off:

$$\begin{aligned} \mathscr{D}_\alpha\Phi &= (D_\alpha - A_\alpha)\Phi \\ &= e^{-V} D_\alpha e^V \Phi \\ &= (D_\alpha + e^{-V}(D_\alpha e^V))\Phi \end{aligned} \tag{3.80}$$

$$\begin{aligned}\bar{\Phi}\overleftarrow{\bar{\mathscr{D}}}^{\dot\alpha} &= \bar{\Phi}(\overleftarrow{\bar{D}}^{\dot\alpha} + A^{\dot\alpha}) \\ &= \bar{\Phi}e^V\overleftarrow{\bar{D}}^{\dot\alpha}e^{-V} \\ &= \bar{\Phi}(\overleftarrow{\bar{D}}^{\dot\alpha} + (e^V\overleftarrow{\bar{D}}^{\dot\alpha})e^{-v}) \\ &= \bar{\Phi}(\overleftarrow{\bar{D}}^{\dot\alpha} - e^V(\bar{D}^{\dot\alpha}e^{-V}))\end{aligned} \tag{3.81}$$

They are thus given by

$$A_\alpha = -e^{-V}D_\alpha e^V \tag{3.82a}$$
$$A^{\dot\alpha} = -e^V\bar{D}^{\dot\alpha}e^{-V} \tag{3.82b}$$

The first two constraints of eq. (3.76) are automatically satisfied:

$$\begin{aligned}F^{\dot\beta\dot\alpha} &= \bar{D}^{\dot\beta}A^{\dot\alpha} + \bar{D}^{\dot\alpha}A^{\dot\beta} - \{A^{\dot\beta}, A^{\dot\alpha}\} \\ &= -(\bar{D}^{\dot\beta}e^V)\bar{D}^{\dot\alpha}e^{-V} - e^V\bar{D}^{\dot\beta}\bar{D}^{\dot\alpha}e^{-V} \\ &\quad - (\bar{D}^{\dot\alpha}e^V)\bar{D}^{\dot\beta}e^{-V} - e^V\bar{D}^{\dot\alpha}\bar{D}^{\dot\beta}e^{-V} \\ &\quad - e^V\underbrace{(\bar{D}^{\dot\beta}e^{-V})e^V}_{=-e^{-V}\bar{D}^{\dot\beta}e^V}\bar{D}^{\dot\alpha}e^{-V} - e^V\underbrace{(\bar{D}^{\dot\alpha}e^{-V})e^V}_{=-e^{-V}\bar{D}^{\dot\alpha}e^V}\bar{D}^{\dot\beta}e^{-V} \\ &= 0\end{aligned} \tag{3.83}$$

The calculation for $F_{\beta\alpha}$ is analogous. The constraint (3.76c), however, allows us to express the spacetime components A_a in terms of V:

$$A_a = -\tfrac{\mathrm{i}}{4}\bar{\sigma}_a^{\dot\alpha\beta}\left(D_\beta A_{\dot\alpha} + \bar{D}_{\dot\alpha}A_\beta - \{A_\beta, A_{\dot\alpha}\}\right) \tag{3.84}$$

For the simplest case of an $U(1)$ gauge group, we have just one real superfield V, and the spinorial parts of A are given by

$$A_\alpha = D_\alpha V, \quad A^{\dot\alpha} = -\bar{D}^{\dot\alpha}V \tag{3.85}$$

while the lowest component of the spacetime part is

$$A_a(x) = -\tfrac{\mathrm{i}}{4}\varepsilon_{\dot\alpha\dot\beta}(\bar{\sigma}_a)^{\dot\beta\beta}[D_\beta, \bar{D}^{\dot\alpha}]V\big| = \mathrm{i}v_a. \tag{3.86}$$

In this expression, $[D_\beta, \bar{D}^{\dot\alpha}]V|$ denotes evaluation of $[D_\beta, \bar{D}^{\dot\alpha}]V$ at $\theta = \bar\theta = 0$ and v_a is the vector field component of the real superfield (see eq. (2.16)).

Together with the constraints (3.76), the Bianchi identity $\mathscr{D}F = 0$ implies that we can express the field strength components F_{ab}, $F_{\beta a}$ and $F^{\dot\beta}{}_a$ in terms of superfields W_α, $\bar{W}^{\dot\alpha}$ which are themselves subject to some constraints:

$$F_{\beta a} = D_\beta A_a - \partial_a A_\beta = \mathrm{i}(\sigma_a)_{\beta\dot\beta}\bar{W}^{\dot\beta} \tag{3.87a}$$
$$F^{\dot\beta}{}_a = \bar{D}^{\dot\beta}A_a - \partial_a A^{\dot\beta} = \mathrm{i}\bar{\sigma}_a^{\dot\beta\beta}W_\beta \tag{3.87b}$$
$$F_{ab} = \partial_a A_b - \partial_b A_a = \tfrac{1}{2}(\bar{\mathscr{D}}_{\dot\alpha}(\bar{\sigma}_{ab})^{\dot\alpha}{}_{\dot\beta}\bar{W}^{\dot\beta} - \mathscr{D}^\alpha(\sigma_{ab})_\alpha{}^\beta W_\beta) \tag{3.87c}$$

where W_β and $\bar{W}^{\dot\beta}$ satisfy

$$\mathscr{D}_\alpha\bar{W}^{\dot\beta} = 0 \tag{3.88a}$$
$$\bar{\mathscr{D}}^{\dot\alpha}W^\beta = 0 \tag{3.88b}$$
$$\mathscr{D}^\alpha W_\alpha = \bar{\mathscr{D}}_{\dot\alpha}\bar{W}^{\dot\alpha} \tag{3.88c}$$

The fields W_α and $\bar{W}^{\dot{\alpha}}$ can be expressed in terms of V as well:

$$W^{\dot{\alpha}} = -\tfrac{1}{4} D^2 \bar{D}^{\dot{\alpha}} V \tag{3.89a}$$

$$W_\alpha = -\tfrac{1}{4} \bar{D}^2 D_\alpha V \tag{3.89b}$$

So we have succeeded in expressing the constrained fields F_{AB}, W_α and $\bar{W}^{\dot{\alpha}}$ as well as the connection components A_A by a single real unconstrained superfield V which is sometimes called prepotential. The same can be done for a non-Abelian gauge group and will lead to the known expressions from Section 2.3 which we now have rederived using differential geometry.

3.4 Supergravity in Compensator Formalism

We will now present a formalism which will express supergravity in terms of a real superfield H_m and the chiral compensator Ψ. These fields are not subject to constraints, but do contain some gauge degrees of freedom. We will largely follow ref. [18] in this section.

We start by describing the geometry in terms of the vielbein differential operators $E_A = (E_a, E_\alpha, \bar{E}^{\dot{\alpha}}) = E_A{}^M \partial_M$, where $\partial_M = (\partial_m, \partial_\mu, \partial^{\dot{\mu}})$. We will impose the same torsion constraints as in the previous approach (eqs. (3.30) to (3.34)). These imply that the spinorial vielbein derivatives form a closed Lie algebra

$$\{E_\alpha, E_\beta\} = C_{\alpha\beta}{}^\gamma E_\gamma. \tag{3.90}$$

The structure constants $C_{\alpha\beta}{}^\gamma$ are the spinorial anholonomy coefficients. Following Frobenius' theorem, they can be expressed as

$$E_\alpha = F N_\alpha{}^\mu \hat{E}_\mu, \qquad \hat{E}_\mu = e^{\mathscr{H}} \partial_\mu e^{-\mathscr{H}} \tag{3.91a}$$

$$\bar{E}_{\dot{\alpha}} = \bar{F} \bar{N}_{\dot{\alpha}}{}^{\dot{\mu}} \hat{\bar{E}}_{\dot{\mu}} \qquad \hat{\bar{E}}_{\dot{\mu}} = -e^{\bar{\mathscr{H}}} \partial_{\dot{\mu}} e^{-\bar{\mathscr{H}}}. \tag{3.91b}$$

In this expression, F is a scalar function, $N_\alpha{}^\mu$ is a matrix with unit determinant and $\mathscr{H} = \mathscr{H}^M \partial_M$ is a unconstrained complex vector superfield, $\mathscr{H} \neq \bar{\mathscr{H}}$. Under local Lorentz transformations, F and $\mathscr{H}$ are scalars, but the lower index of $N_\alpha{}^\mu$ is a Lorentz index, so that we can choose a gauge such that

$$N_\alpha{}^\mu = \delta_\alpha{}^\mu. \tag{3.92}$$

and consequently

$$E_\alpha = F \hat{E}_\alpha, \qquad \hat{E}_\alpha = e^{\mathscr{H}} \partial_\alpha e^{-\mathscr{H}} \tag{3.93a}$$

$$\bar{E}_{\dot{\alpha}} = \bar{F} \hat{\bar{E}}_{\dot{\alpha}} \qquad \hat{\bar{E}}_{\dot{\alpha}} = -e^{\bar{\mathscr{H}}} \partial_{\dot{\alpha}} e^{-\bar{\mathscr{H}}} \tag{3.93b}$$

$$\hat{E}_a = -\tfrac{\mathrm{i}}{4} \bar{\sigma}_a^{\dot{\alpha}\alpha} \{\hat{E}_\alpha, \hat{\bar{E}}_{\dot{\alpha}}\} \tag{3.93c}$$

We will later need the determinant of the vielbein to construct a Lagrangean for supergravity, so we define

$$\det\left(E_A{}^M\right) \equiv E^{-1} \tag{3.94a}$$

$$\det\left(\hat{E}_A{}^M\right) \equiv \hat{E}^{-1} \tag{3.94b}$$

$$\Rightarrow E^{-1} = F^2 \bar{F}^2 \hat{E}^{-1} \tag{3.94c}$$

To find $\hat{E}$ and F, we consider the torsion. The torsion constraints imply that the quantity

$$T_\alpha = (-)^b T_{\alpha B}{}^B \tag{3.95}$$

vanishes, $T_\alpha = 0$ (and also $T_{\dot\alpha} = 0$). We can, on the other hand, express T_α in terms of $\hat{E}_A$, F and $\mathscr{H}$ via the anholonomy coefficients and the connection:

$$\begin{aligned} T_\alpha &= (-)^b C_{\alpha B}{}^B - \Omega_{\beta\alpha}{}^\beta \\ (-)^b C_{\alpha B}{}^B &= E_\alpha \ln E - (1 \cdot \overleftarrow{E_\alpha}) && \text{(3.96a)} \\ \Omega_{\beta\alpha}{}^\beta &= -3E_\alpha \ln F \\ \Rightarrow T_\alpha &= F\hat{E}_\alpha \ln\left(F^4\bar{F}^2\hat{E}(1 \cdot e^{\overleftarrow{\mathscr{H}}})\right) && \text{(3.96b)} \\ T_{\dot\alpha} &= \bar{F}\hat{\bar{E}}_{\dot\alpha} \ln\left(\bar{F}^4F^2\hat{E}(1 \cdot e^{\overleftarrow{\bar{\mathscr{H}}}})\right) && \text{(3.96c)} \end{aligned}$$

Since $\ln(\bar{F}^4F^2\hat{E}(1 \cdot e^{\overleftarrow{\mathscr{H}}}))$ is a scalar, $E_{\dot\alpha}$ coincides with the covariant derivative $\mathscr{D}_{\dot\alpha}$, so $\bar{F}^4F^2\hat{E}(1 \cdot e^{\overleftarrow{\mathscr{H}}})$ is a covariantly chiral field which we define as

$$\begin{aligned} \bar{F}^4F^2\hat{E}(1 \cdot e^{\overleftarrow{\mathscr{H}}}) &\equiv \Psi^{-3} && E_{\dot\alpha}\Psi = 0 && \text{(3.97a)} \\ F^4\bar{F}^2\hat{E}(1 \cdot e^{\overleftarrow{\bar{\mathscr{H}}}}) &\equiv \bar{\Psi}^{-3} && E_\alpha\bar{\Psi} = 0 && \text{(3.97b)} \end{aligned}$$

We note that, just as in the previous section, a covariantly chiral field Ψ can be obtained from a flat chiral field ψ,

$$\begin{aligned} \Psi &= e^{\bar{\mathscr{H}}}\psi && \partial_{\dot\mu}\psi = 0 && \text{(3.98a)} \\ \bar{\Psi} &= e^{\mathscr{H}}\bar{\psi} && \partial_\mu\psi = 0, && \text{(3.98b)} \end{aligned}$$

since $E_\alpha = FN_\alpha{}^\mu e^{\mathscr{H}}\partial_\mu e^{-\mathscr{H}}$.

The field Ψ is called the chiral compensator. From eq. (3.97) we find

$$\begin{aligned} F &= \Psi^{1/2}\bar{\Psi}^{-1}(1 \cdot e^{\overleftarrow{\mathscr{H}}})^{-1/3}(1 \cdot e^{\overleftarrow{\bar{\mathscr{H}}}})^{1/6}\hat{E}^{1/6} && \text{(3.99a)} \\ \bar{F} &= \Psi^{-1}\bar{\Psi}^{1/2}(1 \cdot e^{\overleftarrow{\mathscr{H}}})^{1/6}(1 \cdot e^{\overleftarrow{\bar{\mathscr{H}}}})^{-1/3}\hat{E}^{1/6}. && \text{(3.99b)} \end{aligned}$$

So far we have expressed the vielbein derivatives in terms of unconstrained fields $\mathscr{H}^M$ and Ψ. However, we can perform some transformations on $\mathscr{H}$ that leave E_α and $\bar{E}_{\dot\alpha}$ and thus the algebra of covariant derivatives invariant. By these transformations, we can bring $\mathscr{H}$ into the form

$$\mathscr{H} = -\mathrm{i}H' \quad \text{with } H' = H'^m\partial_m = \bar{H}'. \tag{3.100}$$

Additionally, it is convenient to split H' into $H' = H + H_0$, where $H_0 = \theta\sigma^a\bar{\theta}\partial_a$ is the prepotential in flat superspace. This amounts to changing the partial derivatives ∂_α, $\partial_{\dot\alpha}$ appearing in $\hat{E}_\alpha$ and $\hat{\bar{E}}_{\dot\alpha}$ into supersymmetry-covariant ones (see eq. (2.6)):

$$\begin{aligned} \hat{E}_\alpha &= e^{-\mathrm{i}H}D_\alpha e^{\mathrm{i}H} && \text{(3.101a)} \\ \hat{\bar{E}}_{\dot\alpha} &= e^{\mathrm{i}H}\bar{D}_{\dot\alpha}e^{-\mathrm{i}H} && \text{(3.101b)} \end{aligned}$$

We are now ready to define the supergravity action. It should be invariant under coordinate transformations and Lorentz transformations. Furthermore, we would like to recover general relativity if we set all field except for the spacetime part of the vielbein to zero, i.e. then the action should reduce to the Einstein action.

By virtue of the identity

$$\int \mathrm{d}^8z\, E\, \mathscr{L} = -\tfrac{1}{4}\int \mathrm{d}^8z\, \frac{E}{R}(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)\mathscr{L} \tag{3.102}$$

where $(\bar{\mathscr{D}}\bar{\mathscr{D}} - 8R)$ is the chiral projector, we can restrict our attention to chiral integrals of the type

$$S_{SG} = \frac{1}{\kappa}\int \mathrm{d}^8z\, \frac{E}{R}\mathscr{L}_c + \text{ c.c.} \tag{3.103}$$

with a chiral Lagrangean $\mathscr{L}_c$. Considering the mass dimensions, we must have $[\mathscr{L}_c] = 1$, and for obtaining the correct general relativity limit, we require $\mathscr{L} \sim R$, and thus

$$S_{SG} = -\frac{3}{\kappa}\int \mathrm{d}^8z\, E \tag{3.104}$$

Using eqs. (3.94c) and (3.99), we can express the action in terms of the prepotential $\mathscr{H}$, the chiral compensator Ψ and $\hat{E}$:

$$S_{SG} = -\frac{3}{\kappa^2}\int \mathrm{d}^8z\, \bar{\Psi}\Psi \left\{(1\cdot e^{\overleftarrow{\mathscr{H}}})(1\cdot e^{\overleftarrow{\bar{\mathscr{H}}}})\hat{E}\right\}^{1/3} \tag{3.105}$$

This formalism contains the same gravitational multiplet as the previous one, albeit differently embedded in the superfields. To determine the components, we first express the covariantly chiral field Ψ by

$$\Psi = e^{\mathscr{H}}\psi = e^{-iH}\psi \tag{3.106}$$

and project out the components of the flat chiral field ψ as

$$\psi = \psi\big| \tag{3.107a}$$
$$\zeta_\alpha = \tfrac{1}{\sqrt{2}}D_\alpha\psi\big| \tag{3.107b}$$
$$M = -\tfrac{1}{4}DD\psi\big|. \tag{3.107c}$$

The spinorial component can be gauged away. The lowest component, which, following a widespread abuse of notation, we have denoted by the same ssymbolas the superfield itself, is a gauge degree of freedom as well. We will later fix this gauge to obtain the correct scalar potential. The auxiliary field component, on the other hand, cannot be gauged away and has to be eliminated by its equations of motion.

The real superfield H^m has the following components (see eq. (2.16)):

$$c^m = H^m\big| \tag{3.108a}$$
$$\chi^m_\alpha = D_\alpha H^m\big| \tag{3.108b}$$
$$a^m = -\tfrac{1}{4}D^2H^m\big| \tag{3.108c}$$
$$h_{\alpha\dot{\alpha}}{}^m = -\tfrac{1}{2}[D_\alpha, D_{\dot{\alpha}}]H^m\big| \tag{3.108d}$$
$$\psi^m_\alpha = \tfrac{\mathrm{i}}{4}D_\alpha\bar{D}^2H^m\big| \tag{3.108e}$$
$$b^m = \tfrac{1}{32}\{D^2, \bar{D}^2\}H^m\big| \tag{3.108f}$$

The c^m, χ_α^m and a_m components can be gauged away, so that H^m is in 'Wess-Zumino' gauge. The remaining fields can be identified with the graviton, the gravitino and an auxiliary vector field.

The field content of the theory is thus the same as in the previous formalism, namely the graviton ${e_{\alpha\dot\alpha}}^m$ and the gravitino ψ_α^m (not to be confused with the chiral compensator ψ) as dynamical degrees of freedom as well as two auxiliary fields, the complex scalar M and the real vector field b^m.

We will now proceed to evaluate (3.105) up to second order in the superfields. To this end we impose the gauge (3.100) and calculate the superdeterminant $\hat{E}$ using (3.101a). We further express the chiral compensator ψ by

$$\psi = e^\sigma \tag{3.109}$$

with a flat chiral field σ. The vielbein derivatives are given by

$$\hat{E}_\alpha = e^{-iH} D_\alpha e^{iH} = D_\alpha + i(D_\alpha H) + \tfrac{1}{2} H(D_\alpha H) - \tfrac{1}{2}(D_\alpha H)H + \mathcal{O}(H^3) \tag{3.110a}$$

$$\hat{\bar{E}}_{\dot\alpha} = e^{iH} \bar{D}_{\dot\alpha} e^{-iH} = \bar{D}_{\dot\alpha} - i(\bar{D}_{\dot\alpha} H) + \tfrac{1}{2} H(\bar{D}_{\dot\alpha} H) - \tfrac{1}{2}(\bar{D}_{\dot\alpha} H)H + \mathcal{O}(H^3) \tag{3.110b}$$

$$\hat{E}_a = -\tfrac{\mathrm{i}}{4} \bar\sigma_a^{\dot\alpha\alpha} \left\{ \hat{E}_\alpha, \hat{\bar{E}}_{\dot\alpha} \right\} = \left({\delta_a}^b + {U_a}^b \right) \partial_b + \mathcal{O}(H^3), \tag{3.110c}$$

with a matrix

$$\begin{aligned} {U_a}^b = -\tfrac{\mathrm{i}}{4} \bar\sigma_a^{\dot\alpha\alpha} \big(& -\mathrm{i}[D_\alpha, \bar{D}_{\dot\alpha}] H^b \\ & + 2(D_\alpha H^c)(\partial_c \bar{D}_{\dot\alpha} H^b) + 2(\bar{D}_{\dot\alpha} H^c)(\partial_c D_\alpha H^b) \\ & - \tfrac{1}{2} \left((\partial_a H^c)(\partial_c H^b) - H^c \partial_c \partial_a H^b \right) \big) \end{aligned} \tag{3.111}$$

Since $\hat{E}_a$ contains no spinor derivatives, the superdeterminant $\hat{E}$ is given just by the determinant of $(1+U)^{-1}$, so

$$\begin{aligned} \hat{E}^{1/3} &= \exp\{-\tfrac{1}{3} \operatorname{tr} \ln(1+U)\} \\ &= 1 + \tfrac{1}{12} [D_\alpha, \bar{D}_{\dot\alpha}] H^{\dot\alpha\alpha} + \tfrac{1}{6} \left((\partial_a H^c)(\partial_c H^a) - H^c \partial_c \partial_a H^a \right) \\ &\quad + \tfrac{\mathrm{i}}{6} \left((D_\alpha H^c)(\partial_c \bar{D}_{\dot\alpha} H^{\dot\alpha\alpha}) + (\bar{D}_{\dot\alpha} H^c)(\partial_c D_\alpha H^{\dot\alpha\alpha}) \right) \\ &\quad + \tfrac{1}{96} \left([D_\alpha, \bar{D}_{\dot\alpha}] H^{\dot\beta\beta} \right) \left([D_\beta, \bar{D}_{\dot\beta}] H^{\dot\alpha\alpha} \right) + \tfrac{1}{288} \left([D_\alpha, \bar{D}_{\dot\alpha}] H^{\dot\alpha\alpha} \right)^2 + \mathcal{O}(H^3). \end{aligned} \tag{3.112}$$

With

$$(1 \cdot e^{i\overleftarrow{H}})^{1/3} (1 \cdot e^{-i\overleftarrow{H}})^{1/3} = 1 - \tfrac{1}{3} H^a \partial_a \partial_b H^b + \mathcal{O}(H^3) \tag{3.113}$$

and numerous identities and partial integrations, we arrive at the linearised supergravity action

$$\begin{aligned} S_{SG}^{\mathrm{lin}} = \frac{1}{\kappa^2} \int \mathrm{d}^8 z \Big\{ & \tfrac{1}{8} H^a D^\alpha \bar{D}^2 D_\alpha H_a + \tfrac{1}{48} \left([D_\alpha, \bar{D}_{\dot\alpha}] H^{\dot\alpha\alpha} \right)^2 \\ & - (\partial_a H^a)^2 + 2\mathrm{i}(\sigma - \bar\sigma) \partial_a H^a - 3\sigma\bar\sigma \Big\} \end{aligned} \tag{3.114}$$

Now we can consider the coupling of matter to supergravity. We restrict ourselves to covariantly chiral matter fields Φ_k, $\bar\Phi_l$ related to flat chiral fields by $\Phi_k = e^{\mathrm{i}H} \phi_k$ and $\bar\Phi_l = e^{-\mathrm{i}H} \bar\phi_l$. The action is given by

$$S_M = -\frac{3}{\kappa} \int \mathrm{d}^8 z\, E \exp\{-\tfrac{\kappa}{3} K(\Phi_k, \bar\Phi_l)\} + \left\{ \int \mathrm{d}^8 z\, \frac{E}{R} W(\Phi_k) + \mathrm{h.c.} \right\} \tag{3.115}$$

with the superpotential $W(\Phi_k)$. Due to the chiral integration rule

$$\int \mathrm{d}^6 z\, \psi^3 \phi = \int \mathrm{d}^8 z \frac{E}{R} \Phi \tag{3.116}$$

for a (covariantly) chiral field $\Phi = e^{\mathrm{i}H}\phi$, we can rewrite the second term in eq. (3.115) to

$$S_{M,W} = \int \mathrm{d}^6 z\, \psi^3 W(\phi_k) + h.c. \tag{3.117}$$

For the purpose of supergravity breaking, one needs to know the scalar potential which can be computed from eq. (3.115). In a flat space background, the action reduces to

$$S_M = -\frac{3}{\kappa} \int \mathrm{d}^8 z\, \bar{\Psi}\Psi \exp\{-\tfrac{\kappa}{3} K(\Phi_k, \bar{\Phi}_l)\} + \left\{ \int \mathrm{d}^6 z\, \psi^3 W(\phi_k) + \mathrm{h.c.} \right\}. \tag{3.118}$$

For the scalar potential, we can further neglect spinor and vector fields and restrict our attention to constant superfields ψ, ϕ_k containing scalar and auxiliary components (ψ, M) and (A_k, F_k), respectively:

$$\psi = \psi + \theta\theta M \tag{3.119a}$$

$$\phi_k = A_k + \theta\theta F_k \tag{3.119b}$$

We can now calculate the component action from (3.118), eliminate the auxiliary fields and arrive at

$$\exp\{\tfrac{\kappa}{3} K\} (\psi^*\psi)^2 \left(|W_i + K_i W|^2 (K_{ij^*})^{-1} - 3W^* W \right), \tag{3.120}$$

where $K = K(A_i, A_j^*)$ is the same function as above, but the arguments are just the scalar components of the respective superfields, and $K_i = \frac{\partial}{\partial A_i} K$.

The scalar field ψ, however, is a gauge degree of freedom corresponding to the determinant of the vielbein. Analogous to the derivation of the Lagrangean in Section 3.2, we can rescale the vielbein by choosing

$$\psi^*\psi = \exp\{\tfrac{\kappa}{3} K\}, \tag{3.121}$$

thus arriving at exactly the same scalar potential as in the last line of eq. (3.64).

Chapter 4

Supersymmetry Breaking Scenarios

Since we do not observe superpartners of the Standard Model particles with the same mass, supersymmetry must be broken in nature (if it is present at all). To construct a realistic model, one has to find a breaking scheme which gives masses to the superpartners of ordinary standard model particles which are high enough so they are consistent with present experimental limits. In the first section we will discuss methods to break rigid supersymmetry spontaneously in some extension of the standard model (e.g. the Minimal Supersymmetric Standard Model MSSM which is briefly explained in the next section) and learn about the problems associated with this approach, namely that this approach will not give a phenomenologically acceptable mass spectrum. To circumvent this deficiency, we are led to a class of models where supersymmetry is broken by some fields in a "hidden sector" which is separated from the MSSM by a high mass and a weak interaction, e.g. gravity or gauge interactions on the loop level. These models are covered in Section 4.4 and 4.5. Both have their own problems, some of which can be avoided in a higher dimensional setup where one or more extra dimensions are compactified on an orbifold. These are manifolds from which a group acting non-freely is divided out. They were first considered in string theory to compactify the extra dimensions, but have become popular in recent years for the construction of grand unified theories, in particular the simplest example of an $S_1/\mathbb{Z}_2$ orbifold. In Section 4.6 we give a brief overview of orbifolds and their relevance for grand unified theories and supersymmetry breaking.

4.1 Soft Explicit Breaking: The MSSM

The straightforward supersymmetrisation of the standard model leads to the so-called minimal supersymmetric standard model MSSM [20, 21]. The procedure is simple: We replace every fermion by a chiral superfield and every gauge boson by a vector superfield. A slight subtlety occurs in the Higgs sector: Since the Higgs boson will receive a fermionic partner, cancellation of the triangle anomaly requires two Higgs doublets wwithopposite hypercharge. There are five physical Higgs bosons, a pair of charged bosons $H^{\pm}$, a CP-even neutral pair H^0, h^0 and a CP-odd boson denoted A^0. The particle spectrum is summarised in Table 4.1. The charged and neutral gauginos and Higgsinos can mix into charginos and neutralinos.

An additional discrete symmetry, R parity, is introduced. It is defined as

$$R = (-1)^{3(B-L)+2S} \tag{4.1}$$

where B, L and S denote baryon number, lepton number and spin of the particle, respectively. It amounts to assigning $R = +1$ to all standard model particles and $R = -1$ to all superpartners.

If this parity is conserved, it means that superpartners can only be produced in pairs, and that the lightest supersymmetric particle (LSP) is stable and might be a candidate for dark matter.

The mechanism of supersymmetry breaking is not fixed in the MSSM. The only requirement is that the breaking is soft, i.e. it does not introduce quadratic divergences. There are three possible types of soft breaking [22]:

- Scalar masses: Different masses for scalars and pseudoscalars are possible.
- Gaugino masses
- Trilinear couplings of chiral fields

In superspace language, these breakings are parametrised by constant 'spurion' superfields only containing F- and D-components.

In the MSSM, these possibilities induce 104 breaking parameters, most of them mass matrices for squarks and sleptons and Higgs-squark-squark and Higgs-slepton-slepton couplings, since these are 3×3-matrices in generation space. There are furthermore three gaugino masses, one Higgs mass, the ratio of the vacuum expectation values of the two Higgs bosons usually denoted $\tan\beta$ and a Higgs mixing parameter μ which is determined up to a sign.

However, for phenomenological studies, one has to reduce the parameters for practical purposes. To suppress flavour-changing neutral currents, mass and coupling matrices aareoften assumed to be diagonal or even universal. If grand unification occurs, all gaugino masses can be described by a single parameter, the Higgs parameters are constrained by electroweak symmetry breaking, and in the 'most minimal' case, one ends up with just five parameters: a universal scalar mass, a universal trilinear coupling, a universal gaugino mass, $\tan\beta$ and the sign of μ. This model has been widely investigated.

Superfield	Bosons	Fermions	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Vector superfields					
G	Gluons g	Gluinos $\tilde{g}$	**8**	**1**	0
V^a	W^a Bosons	Winos $\widetilde{W}^a$	**1**	**3**	0
V'	B Boson	Bino $\tilde{B}$	**1**	**1**	0
Chiral superfields					
Q_L	(S)quarks	$(\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	**3**	**2**	$\frac{1}{6}$
$U_R^{\rm C}$	(S)quarks	$\tilde{u}_R^*$	$u_R^{\rm C}$	$\bar{\mathbf{3}}$	**1**	$-\frac{2}{3}$
$D_R^{\rm C}$	(S)quarks	$\tilde{d}_R^*$	$d_R^{\rm C}$	$\bar{\mathbf{3}}$	**1**	$\frac{1}{3}$
L_L	(S)leptons	$(\tilde{\nu}_L, \tilde{e}_L)$	(ν_L, e_L)	**1**	**2**	$-\frac{1}{2}$
$E_R^{\rm C}$	(S)leptons	$\tilde{e}_R^*$	$e_R^{\rm C}$	**1**	**1**	1
H_d	Higgs(inos)	$(\tilde{h}_d^0, \tilde{h}^-)$	(h_d^0, h^-)	**1**	**2**	$-\frac{1}{2}$
H_u	Higgs(inos)	$(\tilde{h}^+, \tilde{h}_u^0)$	(h^+, h_u^0)	**1**	**2**	$\frac{1}{2}$

Table 4.1: The MSSM particle spectrum. The subscripts L and R on the sfermions denote their fermionic partner and their $SU(2) \times U(1)$ transformation, not the chirality.

4.2 Spontaneous Supersymmetry Breaking

The signal for the breaking of rigid supersymmetry is a positive vacuum expectation value of the scalar potential. This can be seen from the anticommutator (2.1k):

$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot{\alpha}}\} = 2\sigma^m_{\alpha\dot{\alpha}} P_m$$

which implies that

$$P_m = -\tfrac{1}{4}\bar{\sigma}_m^{\dot{\alpha}\alpha}\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot{\alpha}}\}, \tag{4.2}$$

thus the energy is

$$P_0 = \tfrac{1}{4}\left(\mathcal{Q}_1\bar{\mathcal{Q}}_{\dot{1}} + \bar{\mathcal{Q}}_{\dot{1}}\mathcal{Q}_1 + \mathcal{Q}_2\bar{\mathcal{Q}}_{\dot{2}} + \bar{\mathcal{Q}}_{\dot{2}}\mathcal{Q}_2\right) \tag{4.3}$$

which is obviously a positively semidefinite operator, since for any state ϕ

$$\langle\phi|\mathcal{Q}\bar{\mathcal{Q}}|\phi\rangle = \|\bar{\mathcal{Q}}|\phi\rangle\|^2 \tag{4.4}$$

If the vacuum is supersymmetric, it is annihilated by the $\mathcal{Q}$'s and the energy is zero. If, on the other hand, the energy, i.e. the vacuum expectation value of the scalar potential, is positive, supersymmetry is spontaneously broken in the vacuum.

This, however, is not true for supergravity. From eq. (3.64) we see that the scalar potential is not positively semidefinite anymore. It acts, on the other hand, as a source for the Einstein equations, so one has to require that the vacuum expectation value of the scalar potential vanishes to obtain a flat space solution.

The scalar potential for a theory containing chiral matter fields ϕ_i and vector superfields V_a is given by the F- and D-components,

$$V = \sum_i |F_i|^2 + \tfrac{1}{2}\sum_a D_a^2. \tag{4.5}$$

where F_i and D_a are determined by their algebraic equations of motion. Both contributions are non-negative, so a positive contribution of either one signifies a breakdown of supersymmetry. This breakdown can be classified in F- or D-term breaking, depending on which fields develop a vacuum expectation value.

The simplest model exhibiting F-term breaking is the O'Raifeartaigh model [23], which contains just three chiral fields ϕ_i, $i = 1, 2, 3$ and a superpotential

$$W_{OR} = \lambda\phi_1 + \mu\phi_2\phi_3 + \kappa\phi_1\phi_2^2. \tag{4.6}$$

The superpotential parameters are chosen such that there is no solution to the equations of motions of the auxiliary fields where all of them vanish, so supersymmetry is broken. In particular, the degeneracy in the masses in a multiplet is lifted. The mass eigenstates are linear combinations of the chiral superfield components, and scalars and spinors do not have the same masses anymore. However, the sum over the masses of all fermionic states is equal to that of all bosonic states. This property can be expressed as a sum over states with spin $j = 0, \frac{1}{2}$ where complex scalar fields are considered as two real ones:

$$\sum_j (-1)^j (2j+1) M_j^2 = 0 \tag{4.7}$$

The Fayet-Iliopoulos term in (2.35) corresponds to D-term breaking. Such a term is only possible for Abelian gauge groups, since a term $\propto V$ is not gauge invariant for non-Abelian

groups. Depending on the size of the coefficient ξ and the parameters in the superpotential, this term breaks supersymmetry only or gauge symmetry as well. In the latter case, D becomes non-zero. However, a sum rule similar to the previous case holds, the spins now ranging from $j = 0$ to $j = 1$:

$$\sum_j (-1)^j (2j+1) M_j^2 = -2g \operatorname{tr}(T^a) D^a \tag{4.8}$$

with an additional trace over the group generators. This trace vanishes for non-Abelian groups, whereas it gives the sum of all $U(1)$ charges for Abelian factors of the group $\sum g_i$, i.e. it is proportional to a trace anomaly (which vanishes in the MSSM).

This mass sum rule poses a serious problem, since it means that the scalar fields cannot be consistently heavier than their spinorial counterparts, so some of them should have been observed already. This sum rule, however, holds only on tree level. If, on the other hand, the mass splittings inside the multiplets are induced by radiative effects, this rule is not valid. Therefore, neither F- nor D-term breaking are usually considered for model building.

The sum rule does not hold for supergravity. However, scalar vacuum expectation values in supergravity should be of the order of the Planck mass to avoid a new fine tuning problem which was a motivation for ssupersymmetryin the first place, so one uusuallydoes not consider tree-level breaking of supergravity for model building either. One rather assumes the existence of a sector that is 'hidden' in the sense that in has only very weak interactions with the observable fields. In this hidden sector, supersymmetry is broken somehow, and the effects are mediated to the observable sector by some mechanism.

4.3 The Polonyi Model

The simplest model for the breaking of supergravity is the Polonyi model [24]. In contrast to rigid supersymmetry, one chiral field Φ is sufficient. We denote its scalar component by $\Phi\big| = A$.

The Kähler potential is the "canonical" one

$$K = \bar{\Phi}\Phi \tag{4.9}$$

leading to a flat Kähler metric with vanishing connection and curvature:

$$g_{ij^*} \equiv g_{AA^*} = 1 \tag{4.10}$$

The superpotential is given by

$$W(\Phi) = \mu^2 (\Phi + m) \tag{4.11}$$

with real parameters μ^2 and m.

From the last line of eq. (3.64) or from alternatively from eq. (3.121) we can read off the scalar potential

$$\begin{aligned} V &= e^K \left[g^{ij^*} (D_i W)(D_j W)^* - 3 W^* W \right] \\ &= e^{A^* A} \mu^4 \left[(A^* A)^2 + m A^* A (A^* + A) \right. \\ &\quad \left. + (m^2 - 1) A^* A - 2m(A^* + A) - 3m^2 + 1 \right] \end{aligned} \tag{4.12}$$

Since we have used mass units such that $M_{\mathrm{P}} = 1$, the terms in the scalar potential have to be multiplied with the appropriate powers of M_{P} to obtain the correct mass dimension 4. We

seek a vacuum expectation value $\langle A\rangle$ which minimises the potential. The parameter m will be adjusted such that $V(\langle A\rangle) = 0$, i.e. the cosmological constant vanishes. These conditions are fulfilled by the values (again in units of $M_{\rm P}$)

$$m = 2 - \sqrt{3} \quad \text{and } \langle A\rangle = \sqrt{3} - 1. \tag{4.13}$$

The complex field A is split into two real fields $A_1 = \mathrm{Re}\, A$ and $A_2 = \mathrm{Im}\, A$ with different masses $m_1^2 = 2\sqrt{3}m_{3/2}$ and $m_2 = 2(2-\sqrt{3})m_{3/2}$, where $m_{3/2}$ is the gravitino mass given by eq. (3.68),

$$m_{3/2} = \mu^4 e^{2-\sqrt{3}} \tag{4.14}$$

So, obviously supersymmetry is broken, since members of supermultiplets do not have the same mass anymore. If a local symmetry is spontaneously broken, the corresponding gauge field should become massive by "eating" a massless field, the goldstone field. In the case of local supersymmetry, the gauge field is the gravitino which indeed is massive. To identify the goldstone field, which has to be a fermion, the goldstino, we note that in eq. (3.64), there are terms mixing the gravitino ψ_m and the spinor from the chiral field,

$$\mathscr{L}_{\rm mix} = \tfrac{\mathrm{i}}{\sqrt{2}} G_i \chi^i \sigma^m \bar{\psi}_m + \tfrac{\mathrm{i}}{\sqrt{2}} G_{i^*} \bar{\chi}^i \bar{\sigma}^m \psi_m. \tag{4.15}$$

Under a supergravity transformation parametrised by ξ, $G_i\chi^i$ transforms inhomogeneously,

$$\delta_\xi G_i \chi^i = -3\sqrt{2} m_{3/2} \xi + \text{ homogeneous terms}, \tag{4.16}$$

so it can be shifted away, eaten by the gravitino. This is the super-Higgs effect.

4.4 Gravity Mediation

The main idea in either gravity or gauge mediated supersymmetry breaking is the existence of two (or more) sectors sharing only a weak interaction. In the hidden sector the actual breaking of supersymmetry takes place (via a vacuum expectation value of some field(s), e.g. a Polonyi type model, or fermion condensation). This effect is transferred to the observable sector (containing e.g. the MSSM or a grand unified model) by some mechanism. In supergravity, it is natural to consider gravity as mediating interaction. The theory then contains the Planck mass $M_{\rm P}$ as a relevant scale as well as a possible unification scale $M_{\rm GUT}$. However, it has been shown [25,26] that neither scale is present in the low-energy effective theory. Rather, if all interactions which are suppressed by powers of $M_{\rm P}$ are neglected, and the hidden sector fields are replaced by their vacuum expectation values, the theory reduces to a globally supersymmetric theory with soft breaking terms. In addition, the effective superpotential can also induce spontaneous breaking of gauge symmetries [15].

We will denote the chiral superfields in the hidden and visible sectors by η_a and ϕ_i, respectively. The condition that the sectors are separated then means that the superpotential is split into two parts:

$$W(\eta_a, \phi_i) = h(\eta_a) + g(\phi_i) \tag{4.17}$$

We take the Kähler potential to be canonical,

$$K = \sum_a \bar{\eta}_a \eta_a + \sum_i \bar{\phi}_i \phi_i \tag{4.18}$$

so the sectors have no common gauge or Yukawa interactions. Denoting the scalar components of η_a and ϕ_i by z_a and a_i and derivatives with respect to these fields by a subscript $_a$ and $_i$, respectively, the scalar potential is (cf. (3.67))

$$V = \exp\left\{\frac{|h_a|^2 + |a_i|^2}{M_{\rm P}}\right\}\left[\left|h_a + \frac{z_a^* W}{M_{\rm P}^2}\right|^2 + \left|g_i + \frac{a_i^* W}{M_{\rm P}^2}\right|^2 - 3\frac{|W|^2}{M_{\rm P}^2}\right] + \tfrac{1}{2}D^2, \tag{4.19}$$

with D-terms for the gauge multiplets. If we now assume that the potential has a minimum where the hidden sector fields get vacuum expectation values of the order of the Planck mass (or at least of a very high scale), parametrised by

$$\langle z_a\rangle = \kappa_a M_{\rm P} \qquad \langle h_a\rangle = \lambda_a m M_p \qquad \langle h\rangle = m M_{\rm P}^2, \tag{4.20}$$

where m is a parameter of mass dimension 1, but $m \ll M_p$. The visible sector fields are assumed to have small or zero vacuum expectation values. The cosmological constant vanishes, $\langle V(z_a, a_i)\rangle = 0$, if

$$\sum_a |\kappa_a + \lambda_a|^2 = 3. \tag{4.21}$$

If we now assume that in the low energy effective action the hidden sector fields are at their vacuum expectation values, and discard all terms which are suppressed by powers of $M_{\rm P}$, we can express the resulting scalar potential in terms of a rescaled superpotential

$$\tilde{g}(a_i) = e^{\frac{1}{2}|\kappa_i|^2} g(a) \tag{4.22}$$

as

$$V_{LE} = |\tilde{g}_i|^2 + m_{3/2}^2 |a_i|^2 + m_{3/2}\left[g_i a_i + (\kappa_a^*(\kappa_a + \lambda_a) - 3)\tilde{g} + \text{c.c.}\right]. \tag{4.23}$$

The fermionic part of this theory is the same as in a globally supersymmetric theory with soft breaking [27, 28].

Since low-energy masses and couplings depend on the nature of the actual breaking, the mass spectrum is highly model dependent. General features, however include strong flavour-changing neutral currents and CP violation. These effects have to be suppressed by explicit fine tuning, such as the assumption of universal scalar masses.

The spontaneous breaking of gauge symmetries can also be arranged for if the superpotential is chosen accordingly, i.e. if it is not positive definite. If appropriate Higgs fields are included and the parameters are in the right range, these fields can get vacuum expectation values, breaking the desired gauge symmetries. This is of course highly model dependent as well.

Another version of gravity mediation uses the moduli fields which arise in compactifications of string theory [29, 30]. These moduli are flat directions of the potential, i.e. their value is not fixed in the fundamental theory. The hidden sector fields themselves do not break supersymmetry, but induce a potential for the moduli fields which as a result get vacuum expectation values, breaking supersymmetry. Gravity then mediates this effect to the observable sector.

A general problem of gravity mediated supersymmetry breaking are the squark and slepton masses. Without further assumptions, there is no reason for these mass matrices to be diagonal in generation space, so flavour changing neutral currents might occur via the exchange of scalars. To avoid this so-called supersymmetric flavour problem, one has to assume (near) universality of the scalar mass matrices. This of course is an ad hoc assumption and represents a huge amount of fine tuning (cf. the remarks at the end of Section 4.1 concerning this problem in the MSSM).

4.5 Gauge Mediation

As stated above, gravity mediation suffers from the supersymmetric flavour problem, i.e. there are generically too strong flavour changing neutral currents without explicit fine tuning. A way to overcome this problem is a scenario where the mediation of supersymmetry breaking is done by gauge interactions (for a review, see [31]). These models usually contain three sectors, again a hidden sector where the breaking happens and a visible sector with the MSSM or some unified model, but additionally a messenger sector which transfers the supersymmetry breaking to the visible fields. In the simplest case with one field per sector, the messenger field T interacts with the hidden sector field X via a term $\propto \lambda X T \bar{T}$. If X acquires a vacuum expectation value $\langle X \rangle = M + \theta\theta F$, the messenger field scalar component t forms mass eigenstates $\frac{1}{\sqrt{2}}(t \pm t^*)$ with squared masses $(\lambda M)^2 \pm \lambda F$ and the spinor components gets a mass λM.

The main point is now that the messenger fields are charged under the standard model gauge group (or some unification group), so they interact with the visible sector. Since gauge interactions are flavour-blind, sfermion mass universality is automatically guaranteed in these models. Gauginos and sfermions are not protected by gauge invariance, so they can get masses which are consistently higher than their standard model counterparts. For gauginos, these masses of order $\frac{F}{M}$ (plus corrections $\mathcal{O}(\frac{F^3}{M^3})$) arise as a one-loop effect, while for sfermions, squared masses of order $\frac{F^2}{M^2}$ are induced at two-loop. This means that gaugino masses are of the same order as scalar masses, while in gravity mediation gaugino masses tend to be too low. Trilinear couplings are also induced at two-loop.

A phenomenologically very attractive feature is the predictivity. The mass spectrum of the visible sector is determined by known gauge interactions, so the only parameters are the mass scales M and F, $\tan\beta$ and the so-called messenger index, a number of order unity which is calculated from the Dynkin index of the gauge group representation of the messenger fields. It has been analysed in detail in refs. [32, 33]. In general, flavour-changing neutral currents are suppressed and squarks are heavier that sleptons. The lightest supersymmetric particle is the gravitino (which can be as light as a few keV) or, in rigid supersymmetry, a neutral gaugino or a right-handed slepton, usually the stau.

A problem is the μ parameter and the Higgs mixing mass. They are not generated by gauge interactions. Within the framework of gauge mediation, they are free parameters and have to be put in by hand. However, from electroweak symmetry breaking, they can be expressed in terms of $\tan\beta$ and the Z mass.

4.6 Orbifolds and Symmetry Breaking

To summarise, both gravity and gauge mediated supersymmetry have successes and problems:

- Both can account for a satisfying spectrum for scalar masses. Gravity mediation has more problems to generate large enough gravitino masses, while in some gauge mediation squared masses turn out negative, but both problems can be circumvented.

- Gravity mediation naturally can produce a μ parameter [34] and Higgs mixed masses of the right size, whereas this is not possible in gauge mediation without the theory becoming extremely complicated.

- Gauge mediation elegantly solves the flavour problem, while this needs major fine tuning in gravity mediation.

- Gauge mediation is more predictive, i.e. there is much less model dependence than in gravity mediation.

- All models involving a hidden sector have to forbid direct interaction terms between the hidden and observable sector in the effective theory (or suppress them very strongly). In supergravity, there are naturally Planck-suppressed higher dimensional operators directly coupling the hidden and observable sectors. These coupling generally do not respect flavour symmetries. Especially, there is no symmetry to protect the separation in eq. (4.18) from radiative corrections.

There is, however, a setup that can potentially solve the problems of these approaches. The idea is that we consider the world to be five dimensional, where the fifth dimension is compactified on an orbifold. The corresponding energy scale, the inverse of the compactification length, is assumed to be a few orders of magnitude below the Planck scale, $\sim 10^{10-16}$ GeV.

Orbifolds are manifolds from which a non-freely acting transformation group is divided out. The fixed points of this group action become singularities of the resulting object, which is therefore very nearly a manifold but for isolated singularities. The simplest example is given by the orbifold $S^1/\mathbb{Z}_2$. Let the manifold be parametrised by the coordinates $(-\pi R, \pi R]$, where $-\pi R$ and πR are identified. The group $\mathbb{Z}_2$ consists of the reflection and the identity, the reflection acts by $\mathcal{P} : y \to -y$. This action is not free, since 0 and πR are fixed points. The orbifold is now the space of all orbits of the group, that is, of all pairs $(x, -x)$ and, due to the fixed points, is isomorphic to the closed interval $[0, \pi R]$ without any identification, so it is very nearly a manifold, but not quite: The boundary points $y = 0$ and $y = \pi R$ are singularities.

If we now consider a five-dimensional spacetime where the fifth coordinate is compactified on such an orbifold, all spacetime is divided into three parts corresponding to the structure of the orbifold: The two boundary points become four-dimensional branes while the interior becomes the five-dimensional bulk. This offers very interesting possibilities for model building, since fields can be constrained to live in the bulk or on one of the branes.

We must specify how the reflection acts on fields living in the bulk. If we assemble all bulk fields in a vector Φ, the action of the reflection is given by

$$\mathcal{P} : \Phi(x, y) \mapsto P\Phi(x, -y) \tag{4.24}$$

where x are the four-dimensional coordinates and y is the coordinate along the orbifold. The matrix P forms a representation of the reflection with eigenvalues ± 1. If we choose a basis where P is diagonal, the fields have positive or negative parity. Those with positive parity have Kaluza-Klein expansions involving $\cos\left(\frac{ny}{R}\right)$-modes, while negative parity fields have expansions in terms of $\sin\left(\frac{ny}{R}\right)$-modes. Integrating out the fifth dimension, the expanded modes acquire masses $\propto \frac{n}{R}$. Thus, only the positive parity fields have massless zero modes.

This mechanism can be employed for symmetry breaking [35]. We can e.g. consider a $SU(5)$ grand unified theory with gauge bosons in the adjoint representation. If we simply assign negative parity to the X and Y bosons and positive parity to the standard model gauge fields, only these have zero modes while the additional ones get masses of the order of the GUT scale if the orbifold size is chosen small enough. This setup was first considered in [36], and now there have been a lot of models considering $SU(5)$ in five dimensions [37–40] or $SO(10)$ in six dimensions [41, 42]. These models also include Higgs fields on the brane(s) or in the bulk and more sophisticated orbifold constructions such as $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$, where the two branes correspond to different $\mathbb{Z}_2$-parities.

Orbifolds can also be used to break supersymmetry [43]. We immediately encounter a problem here: Supersymmetry generators live in a spinor representation of the Lorentz group, which in five dimensions is twice as large as in four. That means that simple five-dimensional supersymmetry corresponds to N=2 supersymmetry in four dimensions. So if the bulk theory is supersymmetric, at least half of the supersymmetry has to be broken on each brane to allow chiral spinors[1]. Another problem of supersymmetry in higher dimensions is that there exists no superspace formalism similar to the one in four dimensions for N=1. However, there is a way to embed the higher dimensional multiplets in terms of N=1 superfields which preserves much of its usefulness. This was done in ref. [10, 44] for the 5d gauge and matter multiplets[2] and in ref. [11] for linearised supergravity (cf. Appendix B).

The orbifold setup for supersymmetry breaking is now as follows. One brane contains our world, that is, the MSSM or some extension thereof. The other brane contains the hidden sector where the remaining supersymmetry is broken. This effect is transferred to our brane via some bulk fields. These might be gauge fields (gaugino mediation, see e.g. [45–47]), supergravity [11, 48–50], the effect of the super-Weyl anomaly[3] (see [43]) or the chiral field containing the radius of the extra dimension (radion mediation, see [51]). This setup provides solutions to the problem stated above: Scalar masses can be generated naturally to obtain a mass spectrum which is phenomenologically acceptable, interactions between the hidden and observable sectors are forbidden since they live on different branes, and it is possible to induce a μ term of the right order of magnitude.

[1] This is also obvious from the fact that the full five-dimensional supersymmetry generates translations in the fifth direction, which is not a symmetry anymore after the orbifolding. The branes act as symmetry-breaking defects in this respect.

[2] See also ref. [9] for an embedding of a ten-dimensional theory.

[3] This is called anomaly mediation. It is present in any hidden sector model, but it is usually not the dominant contribution. A problem is that is gives negative squared masses to sleptons, so there must be additional contributions to the slepton mass.

Chapter 5

Summary and Outlook

Supersymmetry is a very intersting concept which provides realistic models of particles and interactions. However, the breaking mechanism remains unknown so far. Supersymmetric theories on orbifolds offer promising breaking schemes which seem to avoid many problems of four-dimensional models. On the other hand, string theory is the only theory trying to give a truly unified picture of our world, including gravity. Extra dimensions and supersymmetry are automatically required by string theory, so supersymmetry and especially supergravity might well be appropriate effective theories below the string scale. In this thesis we have reviewed models of supersymmetry breaking, including orbifold constructions where supersymmetry breaking is transmitted from on brane to the other by bulk fields. We have also presented a embedding of higher-dimensional supermultiplets which enables us to employ much of the superspace formalism which has proven so useful in four dimensions. Furthermore, via a formulation in terms of prepotentials calculations in supergravity will be simplified considerably. This is meant to be the starting point of an analysis of realistic higher-dimensional models which we will pursue in future work.

Appendix A

Conventions and Algebra

I use the spinor conventions of [13]. The spinors are two-component Weyl spinors χ, $\bar{\psi}$ which can be combined to form a Dirac spinor Ψ:

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}. \tag{A.1}$$

Spinors carry undotted or dotted Greek indices, vectors carry Latin ones. In general, indices taken from the beginning of the alphabet like $A = (a, \alpha, \dot{\alpha})$ denote flat superspace, indices taken from the middle of the alphabet like $M = (m, \mu, \dot{\mu})$ are used in curved superspace.

- ε-symbols, metric:

$$\varepsilon^{12} = \varepsilon_{21} = 1, \quad \varepsilon^{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1 \tag{A.2}$$

$$\varepsilon^{\alpha\beta}\varepsilon_{\gamma\alpha} = \delta^\beta_\gamma, \quad \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\alpha}} = \delta^{\dot{\beta}}_{\dot{\gamma}} \tag{A.3}$$

$$\varepsilon_{0123} = -1 \tag{A.4}$$

$$\text{Minkowski metric: } \eta_{mn} = \text{diag}(-1, 1, 1, 1) \tag{A.5}$$

- σ-matrices:

$$\sigma^m = (-\mathbb{1}, \sigma^i), \quad \bar{\sigma}^m = (-\mathbb{1}, -\sigma^i) \tag{A.6}$$

- Raising and lowering of indices:

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \Rightarrow \psi^1 = \psi_2,\ \psi^2 = -\psi_1 \tag{A.7}$$

$$\bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\psi_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dot{\beta}} \Rightarrow \bar{\psi}^{\dot{1}} = \bar{\psi}_2,\ \bar{\psi}^{\dot{2}} = -\bar{\psi}_{\dot{1}} \tag{A.8}$$

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^m_{\beta\dot{\beta}} \tag{A.9}$$

$$\sigma^m_{\alpha\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}\bar{\sigma}^{m\dot{\beta}\beta} \tag{A.10}$$

$$\text{Bispinors: } X_{\alpha\dot{\alpha}} = \sigma^m_{\alpha\dot{\alpha}}X_m, \quad X^{\dot{\alpha}\alpha} = \bar{\sigma}^{\dot{\alpha}\alpha}_m X^m \tag{A.11}$$

$$X_m = -\tfrac{1}{2}X_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\alpha}\alpha}_m,\ X^m = -\tfrac{1}{2}X^{\dot{\alpha}\alpha}\sigma^m_{\alpha\dot{\alpha}},\ X^mY_m = -\tfrac{1}{2}X^{\dot{\alpha}\alpha}Y_{\alpha\dot{\alpha}} \tag{A.12}$$

- γ-matrices:

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}, \quad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix} \tag{A.13}$$

- Product of spinors: undotted indices are contracted top-down, dotted indices are contracted bottom up

$$\psi\chi = \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \tag{A.14}$$
$$\bar\psi\bar\chi = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = -\bar\psi^{\dot\alpha}\bar\chi_{\dot\alpha} = \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = \bar\chi\bar\psi \tag{A.15}$$
$$(\chi\psi)^\dagger = (\chi^\alpha\psi_\alpha)^\dagger = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = \bar\psi\bar\chi = \bar\chi\bar\psi \tag{A.16}$$

$$\theta^\alpha\theta^\beta = -\tfrac{1}{2}\varepsilon^{\alpha\beta}\theta\theta, \qquad \theta_\alpha\theta_\beta = \tfrac{1}{2}\varepsilon_{\alpha\beta}\theta\theta \tag{A.17}$$
$$\bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac{1}{2}\varepsilon^{\dot\alpha\dot\beta}\bar\theta\bar\theta, \qquad \bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\bar\theta\bar\theta \tag{A.18}$$

$$\theta_\alpha\theta^\beta = -\tfrac{1}{2}\theta\theta\delta_\alpha^\beta, \qquad \theta^\alpha\theta_\beta = \tfrac{1}{2}\theta\theta\delta_\beta^\alpha \tag{A.19}$$
$$\bar\theta_{\dot\alpha}\bar\theta^{\dot\beta} = \tfrac{1}{2}\bar\theta\bar\theta\delta_{\dot\alpha}^{\dot\beta}, \qquad \bar\theta^{\dot\alpha}\bar\theta_{\dot\beta} = -\tfrac{1}{2}\bar\theta\bar\theta\delta_{\dot\beta}^{\dot\alpha} \tag{A.20}$$
$$\tag{A.21}$$

- Algebra of σ-matrices:

$$\sigma^m_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta} = -\eta^{mn}\delta_\alpha^\beta + 2(\sigma^{mn})_\alpha{}^\beta \tag{A.22}$$

$$(\sigma^{mn})_\alpha{}^\beta = \tfrac{1}{4}(\sigma^m_{\alpha\dot\alpha}\bar\sigma^{n\dot\alpha\beta} - \sigma^n_{\alpha\dot\alpha}\bar\sigma^{m\dot\alpha\beta}) \tag{A.23}$$
$$(\sigma^{0i})_\alpha{}^\beta = \tfrac{1}{2}(\sigma^i)_\alpha{}^\beta, \quad (\sigma^{ij})_\alpha{}^\beta = -\tfrac{1}{2}\mathrm{i}\varepsilon^{ijk}(\sigma^k)_\alpha{}^\beta \tag{A.24}$$

$$\bar\sigma^{m\dot\alpha\alpha}\sigma^n_{\alpha\dot\beta} = -\eta^{mn}\delta_{\dot\beta}^{\dot\alpha} + 2(\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\beta} \tag{A.25}$$

$$(\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\beta} = \tfrac{1}{4}(\bar\sigma^{m\dot\alpha\alpha}\sigma^n_{\alpha\dot\beta} - \bar\sigma^{n\dot\alpha\alpha}\sigma^m_{\alpha\dot\beta}) \tag{A.26}$$
$$(\bar\sigma^{0i})^{\dot\alpha}{}_{\dot\beta} = \tfrac{1}{2}(\bar\sigma^i)^{\dot\alpha}{}_{\dot\beta}, \quad (\bar\sigma^{ij})^{\dot\alpha}{}_{\dot\beta} = \tfrac{1}{2}\mathrm{i}\varepsilon^{ijk}(\bar\sigma^k)^{\dot\alpha}{}_{\dot\beta} \tag{A.27}$$

$$(\sigma^{mn})_\alpha{}^\alpha = (\bar\sigma^{mn})^{\dot\alpha}{}_{\dot\alpha} = 0 \tag{A.28}$$
$$\mathrm{tr}(\sigma^m\bar\sigma^n) = -2\eta^{mn} \tag{A.29}$$
$$\sigma^m_{\alpha\dot\alpha}\bar\sigma_m^{\dot\beta\beta} = -2\delta_\alpha^\beta\delta_{\dot\alpha}^{\dot\beta} \tag{A.30}$$
$$(\sigma^m)_{\alpha\dot\alpha}(\sigma_m)_{\beta\dot\beta} = -2\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} \tag{A.31}$$

$$\sigma^a\bar\sigma^b\sigma^c = \eta^{ac}\sigma^b - \eta^{bc}\sigma^a - \eta^{ab}\sigma^c + \mathrm{i}\varepsilon^{abcd}\sigma_d \tag{A.32}$$
$$\bar\sigma^a\sigma^b\bar\sigma^c = \eta^{ac}\bar\sigma^b - \eta^{bc}\bar\sigma^a - \eta^{ab}\bar\sigma^c - \mathrm{i}\varepsilon^{abcd}\bar\sigma_d \tag{A.33}$$

$$(\sigma^{mn})_\alpha{}^\beta\varepsilon_{\beta\gamma} = (\sigma^{mn})_\gamma{}^\beta\varepsilon_{\beta\alpha} \tag{A.34}$$
$$\varepsilon^{mnkl}\sigma_{kl} = -2\mathrm{i}\sigma^{mn}, \quad \varepsilon^{mnkl}\bar\sigma_{kl} = 2\mathrm{i}\bar\sigma^{mn} \tag{A.35}$$

Appendix B

Spinors in Higher Dimensions

We give a short overview over spinors in higher (especially five) dimensions. Spinors form the representation space of the covering group of the d-dimensional Lorentz group $SO(1, d-1)$, *Spin*(d). The representations of *Spin(d)* can be obtained from representations of the d-dimensional Dirac algebra defined by

$$\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}\mathbb{1} \quad \text{with } a, b = 0, 1, \ldots, d-1 \tag{B.1}$$

with η_{ab} the d-dimensional Minkowski metric, $\eta_{ab} = \text{diag}(-, +, \ldots, +)$. A representation of the group is formed by the matrices

$$\tfrac{1}{2}\Sigma_{ab} = \mathrm{i}[\Gamma_a, \Gamma_b]. \tag{B.2}$$

The complex dimension of this representation is

$$D = 2^{d/2} \quad \text{for } d \text{ even} \tag{B.3a}$$

$$D = 2^{(d-1)/2} \quad \text{for } d \text{ odd}. \tag{B.3b}$$

There are, however, conditions that can reduce the dimension of the spinor representations, reality and chirality conditions.

In even dimensions the matrix

$$\Gamma_{d+1} = \Gamma_0\Gamma_1 \cdots \Gamma_{d-1} \tag{B.4}$$

can be used to define projection operators

$$P_{L,R} = \tfrac{1}{2}(\mathbb{1} \pm \beta\Gamma_{d+1}), \tag{B.5}$$

where $\beta = 1$ for $d = (2 \mod 4)$ and $\beta = \mathrm{i}$ for $d = (4 \mod 4)$. These operators project on the left- and right-handed spinors, which form subspaces half the size of the original representation. This is possible because Γ_{d-1} and the Γ_a anticommute. In odd dimensions, on the other hand, they commute and hence $\Gamma_{d-1} \propto \mathbb{1}$, so there are no chiral spinors in odd dimensions.

The other possibility is a reality (Majorana) condition. It is usually stated as

$$\psi^c \equiv C\bar{\psi}^T = \psi \tag{B.6}$$

where $\bar{\psi} = \psi^\dagger\Gamma_0$ and C is the charge conjugation matrix satisfying

$$C^{-1}\Gamma_a C = -\Gamma_a^T \quad \text{and} \quad C = \pm C^T, \tag{B.7}$$

where the sign in the last equation depends on the dimension. This condition can only be imposed in one to four dimensions and in eight to twelve dimensions. A Majorana condition halves the representation. However, it is not necessarily compatible with a chirality condition. Chiral Majorana spinors are possible only in $d = (2 \mod 8)$.

In four dimensions we can have either chiral (Weyl) or Majorana spinors, so the minimal real dimension is 4 (since (B.3) lists the complex dimension and we can eliminate half of the dimensions). In five dimensions, although the spinor dimension (B.3) is the same, we can have neither chiral nor Majorana spinors, and so the minimal dimension of the spinor representation is 8.

There is, however, a possibility to impose a symplectic Majorana condition involving two spinors $\psi^{1,2}$ in five dimensions. The condition reads

$$\psi^i = \varepsilon^{ij} C \bar{\psi}_j^T \tag{B.8}$$

and we can decompose these spinors into two Weyl spinors $\psi_{L,R}$ as

$$\psi_1 = \begin{pmatrix} (\psi_L)_\alpha \\ (\bar{\psi}_R)^{\dot{\alpha}} \end{pmatrix} \qquad \psi_2 = \begin{pmatrix} (\psi_R)_\alpha \\ -(\bar{\psi}_L)^{\dot{\alpha}} \end{pmatrix} \tag{B.9a}$$

$$\bar{\psi}_1 = \left((\psi_R)^\alpha, (\bar{\psi}_L)_{\dot{\alpha}}\right) \qquad \bar{\psi}_2 = \left(-(\psi_L)^\alpha, (\bar{\psi}_R)_{\dot{\alpha}}\right). \tag{B.9b}$$

We can now embed the Weyl spinors in a N=1 chiral ssuperfield This is done in Appendix C.

An important consequence of eq. (B.3) is that supersymmetry can live in at most eleven dimensions, since the minimal real dimension of a spinor representation in twelve spacetime dimensions is 64. A theory involving 64 real supercharges, however, inevitably contains states with spin $\frac{5}{2}$ which cannot be consistently coupled to matter. In rigid supersymmetry, where renormalisability is an issue, even spin-$\frac{3}{2}$-states are not allowed, so the spacetime dimension is restricted to $d = 10$.

Appendix C

5d Multiplets and Embeddings

C.1 Rigid Supersymmetry

We will consider the five-dimensional chiral and vector multiplets from a four-dimensional point of view, where they act as N=2 multiplets. Unfortunately, there is no superfield formalism for N=2 supersymmetry, so we have to consider the multiplets themselves.

The vector multiplet contains a real vector field v^M ($M = 0, 1, 2, 3, 5$), a real scalar field Σ, a $SU(2)_R$-triplet of auxiliary scalars X^a and a $SU(2)_R$-doublet of gaugino fields λ^i. They transform under two supersymmetries parametrised by symplectic Majorana spinors ξ_i (as defined in the last chapter). Since we are interested in a five-dimensional theory on an orbifold, one of the supersymmetries has to be broken in any case, so we can decompose the parameters ξ_i according to (B.9) and assume that the remaining supersymmetry is the one corresponding to ξ_L. Then the fields of the vector multiplet can be arranged in two N=1 superfields, a vector superfield V in Wess-Zumino gauge and a chiral superfield Φ in the following way (where Φ is taken in the y-parametrisation as given in (2.11)) [52]:

$$V = -\theta\sigma^m\bar{\theta}v_m + \mathrm{i}\theta\theta\bar{\theta}\bar{\lambda}_L - \mathrm{i}\bar{\theta}\bar{\theta}\theta\lambda_L + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(X^3 - D_5\Sigma) \tag{C.1}$$

$$\Phi = (\mathrm{i}\Sigma + \mathrm{i}v_5) + \sqrt{2}\theta(-\mathrm{i}\sqrt{2}\lambda_R) + \theta\theta(X^1 + \mathrm{i}X^2), \tag{C.2}$$

where $D_5 = \partial_5 + \mathrm{i}v_5$ appears in the D-component of the vector superfield for five-dimensional Lorentz covariance.

The matter multiplet ("hypermultiplet") consists of two complex scalar fields A_i, two complex auxiliary scalars F_i, both of which form a $SU(2)_R$-doublet, and a Dirac spinor ψ. It can be expressed as two chiral superfields

$$H_1 = A_1 + \sqrt{2}\theta\psi_L + \theta\theta(F_1 + D_5H_2 - \Sigma H_2) \tag{C.3}$$

$$H_2 = A_2^\dagger + \sqrt{2}\theta\psi_R + \theta\theta(-F_2^\dagger - D_5H_1^\dagger - H_1^\dagger\Sigma), \tag{C.4}$$

where again $\psi_{L,R}$ are related to ψ via the decomposition (B.9).

C.2 Supergravity

The on-shell supergravity multiplet in five dimensions can be embedded in N=1 superfields as well [11]. The multiplet contains the metric g_{MN} (or alternatively the vielbein) and the

gravitino ψ_M as dynamical fields and a auxiliary vector field B_M [53]. In the linearised theory, we express the metric as $g_{MN} = \eta_{MN} + h_{MN}$ transforming as

$$\delta h_{MN} = \partial_M \xi_N + L_{MN}. \tag{C.5}$$

The L_{55} component of the Lorentz transformation vanishes. ξ_M is a vector contained in the linearised transformation parameter superfield L_α. B_M transforms as

$$\delta B_M = \partial_M \alpha \tag{C.6}$$

The four-dimensional part h_{mn} is embedded in the supergravity fields H^m and σ (see (3.114)). The zero modes of the h_{55}-component and B_5 can be arranged as $h_{55} + \mathrm{i}B_5$ as the scalar component of a chiral multiplet T (the "radion multiplet") with suitably chosen transformation law

$$\delta T = \partial_5 \Omega \tag{C.7}$$

with a chiral field Ω containing the transformation parameters $\xi_5 + \mathrm{i}\alpha_5$ as their scalar components.

The h_{m5} component can be embedded in a vector superfield K in Wess-Zumino gauge as

$$K \sim \theta\sigma^m\bar{\theta}h_{m5} + \cdots \tag{C.8}$$

transforming as a gauge particle (see eq. (2.17)) with additional Lorentz part,

$$\delta K = \mathrm{i}(\Omega + \Omega^\dagger) - N. \tag{C.9}$$

Here N is a field containing the Lorentz transformation,

$$N \sim \theta\sigma^m\bar{\theta}L_{5m}. \tag{C.10}$$

The remaining component h_{5m} is embedded in a spinorial superfield ζ_α together with B_m as

$$\zeta_\alpha \sim \bar{\theta}^{\dot{\alpha}}(B_{\alpha\dot{\alpha}} + \mathrm{i}h_{5,\alpha\dot{\alpha}}) + \cdots \tag{C.11}$$

The fields ζ_α and K both transform under N. However, the N-dependence can be eliminated if the fields are combined to

$$\hat{\zeta}_\alpha = \zeta_\alpha + \tfrac{\mathrm{i}}{4}D_\alpha K. \tag{C.12}$$

The higher components of $\hat{\zeta}$ and T are given by the decomposed gravitino fields $\psi_m^\pm$ and $\psi_5^\pm$. The superscript $\pm$ denotes the $\mathbb{Z}_2$-parity.

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