Superfield Supergravity and an Almost No-Scale Scenario

Björn S. Wonsak

May 10, 2004

Abstract

In this thesis we review the superfield description of four-dimensional supergravity in the compensator formalism. We derive the flat limit of the compensator formalism action and briefly discuss supersymmetry breaking. We investigate the 'Almost No-Scale' proposal of Luty and Okada (JHEP **0304** (2003) 050). Following their suggestion, we consider five-dimensional supergravity compactified on an orbifold with small perturbations around the no-scale potentials and a radion stabilised by Casimir energy. Gaugino mediation induced by a quasi-localised gauge multiplet provides an attractive mechanism for supersymmetry breaking in which the gravitino is much heavier than the gaugino. The required quasi-localisation can be realised if the five-dimensional gauge theory is near a non-trivial UV-fixed point.

Superfeld Supergravitation und ein 'Almost No-Scale' Scenario

Björn S. Wonsak

27. April 2004

Zusammenfassung

In dieser Arbeit geben wir einen Überblick über die Superfeldbeschreibung von vier-dimensionaler Supergravitation im Compensator-Formalismus. Wir leiten den flachen Grenzwert der Wirkung im Compensator-Formalismus her und wir diskutieren kurz Supersymmetriebrechung. Wir untersuchen den 'Almost No-Scale' Vorschlag von Luty und Okada (JHEP **0304** (2003) 050). Wie sie betrachten wir auf einer Orbifold kompaktifizierte, fünf-dimensionale Supergravitation mit kleinen Störungen um no-scale Potentiale und mit einem durch Casimir-Energie stabilisiertem Radion. Ein quasi-lokalisiertes Eichmultiplet stellt einen geeigneten Vermittler der Supersymmetriebrechung dar, wobei das Gravitiono eine sehr viel größere Masse als das Gaugino erhält. Die nötige Quasi-Localisierung kann realisiert werden, wenn die fünf-dimensionale Eichtheory nahe ihrem nicht-trivialen UV-Fixpunkt ist.

Contents

1 Introduction **4**

2 Supergeometry **8**
2.1 The Superspace and its Geometrical Structure 9
2.2 Prepotentials 17
2.3 The Chiral Representation 20
2.4 The Chiral Compensator Mechanism 21
2.5 The Gravitational Superfield and the Wess-Zumino Gauge . . 25

3 Supergravity Action **27**
3.1 The Supergravity Action and the Compensator Mechanism . . 27
3.2 Getting the Action in Components 31
3.3 The Flat Limit of the Action in Compensator Form 33
3.4 Short Review of the Results in Components 37

4 Supersymmetry Breaking **40**
4.1 Spontaneous Supersymmetry Breaking in the Flat Limit . . . 40
4.2 No-Scale Supergravity 43

5 Application to 'Almost No-Scale Supergravity' **46**
5.1 Basic Model and Derivation of the Scalar Potential 46
5.2 Cancelation of Cosmological Constant 49
5.3 Casimir Stabilisation of the Radion 51
5.4 Supersymmetry Breaking for a Quasi-localised Gauge Multiplet 55

6 Conclusions **60**

A Notations and Conventions **62**

Bibliography **64**

Chapter 1

Introduction

One aim of modern physics is to describe our whole world in one theory, which should be as simple and natural (and in this sense aesthetic) as possible. So far, some important steps to such an underlying theory which should unify the fundamental forces of nature have been made: First Maxwell's unification of magnetism and electrostatics, later the electroweak unification of Glashow, Salam and Weinberg [1–4] and after that the development of quantum chromodynamics (QCD), which describes the strong interaction as a gauge theory. These theories form the standard model (SM) with the gauge group $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. This model very successfully describes the interactions of the known elementary particles and thus motivated a lot of work on possible grand unified theories (GUTs) where the gauge group of the SM is embedded in a larger gauge group. Possible gauge groups are SU(5) and SO(10), but also more exotic groups like E_6. Thus there exists a large spectrum of GUT models, but so far none of these was sufficiently simple and predictive to replace the SM as the most fundamental theory of particle physics.

The discovery of a new symmetry called supersymmetry [5–7] in the early 70th has opened new perspectives on the way to a GUT (for an introduction see [8]). There are several reasons to be interested in this new symmetry. First of all, new symmetries have often led to progress in particle physics in the past. Furthermore supersymmetry has a property no other symmetry discussed so far has: it transforms bosons into fermions and vice versa. A particularly attractive feature of supersymmetry is that the simplest theory, N=1 supersymmetry, is essentially free of quadratic divergences (N is the number of supersymmetry generators). For theories with more supersymmetry generators this gets even better; for example the N=4 supersymmetric Yang-Mills theory is finite [15]. Furthermore, supersymmetry leads to exact unification of the gauge couplings, which is not the case in non-supersymmetric theories.

Another problem that can be addressed by supersymmetry is the hierarchy problem. Through radiative corrections the Higgs boson mass is quadratically divergent and thus its small value, inferred from precision experiments, is highly unnatural. In the minimal supersymmetric standard model (MSSM) one introduces superpartners which cancel the divergence order by order in perturbation theory. Unfortunately this solution gives rise to new problems, like flavor-changing neutral currents (FCNC's). Last but not least, the lightest superpartner is a possible candidate for dark matter, which is another reason to consider supersymmetry.

To get a complete 'theory of everything' we also have to unify gravity with the other interactions. This turned out to be difficult because gravity is described in the general relativity theory as a property of our space-time. However, is has been shown that a local version of supersymmetry called supergravity (SUGRA) automatically includes gravity [9, 10]. Unfortunately supergravity is not renormalisable, but there is still the hope that supergravity is the effective theory of an underlying fundamental theory, where the most likely candidate is string theory. (There exists a variety of useful books [11–13] and reviews [14–17] on supersymmetry and supergravity.)

Since supersymmetry is a symmetry between fermions and bosons it predicts that every particle we have in the SM should have a supersymmetric partner of the same mass, i.e. bosons give rise to bosinos and fermions to sfermions. Experiments do not confirm the existence of these additional particles with different spin but identical mass. Thus, if supersymmetry is realised in nature, it has to be broken.

How exactly supersymmetry breaking is realised is a widely discussed problem. One main idea in this context is the existence of two sectors which interact only weakly with each other. Supersymmetry breaking then takes place in one sector, called the hidden sector, and is communicated to the visible sector by a weak interaction. The visible sector contains for example the MSSM or another theory describing the observed particles. The exact form of the communication has a lot of influence on the properties of the resulting model. Two standard ways to communicate supersymmetry breaking are gravity mediation and gauge mediation.

Another ansatz which is widely used in the construction of possible GUT's is the proposal of higher dimensions. This goes back to Kaluza [18] and Klein [19] who proposed a compact extra dimension already in the 20th. This idea has been pursued in many ways in later years and, in particular, reappears in the work of Horava and Witten [20–22] . They compactified an eleven-dimensional supergravity on $S^1/\mathbb{Z}_2$, that is, on an interval of length l bounded by mirror planes. Then they found that, for consistency reasons, on each plane a ten-dimensional E_8 super-Yang-Mills theory has to be localised.

These two gauge theories are then coupled to supergravity in the interior of the interval. Thus it is sensible to pick up the idea of hidden sectors and locate one sector on each plane. This leads to a lot of interesting possibilities to realise supersymmetry breaking. Further toroidal compactifications lead to brane world scenarios in less dimensions. The simplest of them is a five-dimensional orbifold model.

Another interesting possibility is the Randall-Sundrum scenario [23] [24], where our world is a three brane embedded in a non-compactified but instead warped higher-dimensional space. Furthermore, higher dimensions lead to new possible realisations of supersymmetry breaking; supersymmetry can be broken in a geometrical way [25–27] or supersymmetry breaking can be mediated by extra dimension [28–30]. For a review see [31].

At ernergies below the compactification scale, the effective theory will always be four-dimensional SUGRA. Thus we begin by attempting to understand four-dimensional supergravity in a superfield formalism. This is much easier in conformal supergravity [32–34] than in Poincaré/Einstein supergravity. Therefore, we will first work out conformal supergravity and than use the famous chiral (or superconformal) compensator [11, 32–34] to arrive at Einstein supergravity. This is done in Chapter 2, where we follow the lines of Buchbinder and Kuzenko (B&K) [13]. Here we first define our superspace and give this space a geometrical structure. After imposing the appropriate constraints on the geometrical objects we then arrive at conformal supergravity. Then we discuss the prepotential formalism [11], where we put particular emphasis on the implementation of Frobenius' theorem in solving the representation preserving constraint. After the introduction of the chiral representation we analyse the gauge freedom of conformal supergravity in this formalism, which directly leads us to the chiral compensator mechanism. We use the chiral compensator to get Einstein supergravity from our conformal theory. At the end of this chapter we discuss the most useful gauges, which we will use from there on.

In Chapter 3 we use the formalism developed in the previous chapter to derive the supergravity action. Again we follow the lines of B&K. Since the component form of Einstein supergravity is well known, we use it as a check for the consistency of our calculations by comparing it with standard literature, namely the book by Wess and Bagger (W&B) [12].

Motivated by some recent papers of Luty and Sundrum [29], we then derive the flat limit of the compensator formalism action (see also [35] [36]). This gives us a very important tool, the bosonic part of the low energy effective action. Using the chiral compensator mechanism it can be given in a rigid supersymmetric superfield form. We pay particular attention to a full superfield derivation, including the contributions to the kinetic term which

arise from the auxiliary vector field (see also [30] [37]). As it turns out there exists a contribution to the kinetic term which, although an essential part of the flat bosonic action, does not follow from the rigid superfield formalism. In the last section of this chapter we present some of the results W&B derive on component level for further reference. Later we will use these results to compare with the outcome of our calculations.

We use Chapter 4 to briefly discuss spontaneous breaking of supersymmetry with the help of our flat action derived in Chapter 3. The results are consistent with W&B. Since we want to use a related ansatz in the next chapter, we briefly discuss no-scale models, which give a naturally vanishing cosmological constant [38].

Chapter 5 is devoted to applications of the tools we have discussed in the other chapters. Following the interesting proposal of 'Almost No-Scale Supergravity' by Luty and Okada [39] we consider five-dimensional supergravity compactified on an orbifold with small perturbations around the no-scale potentials and a radion stabilised by Casimir energy. We explicitly reproduce the calculations which lead to the nearly vanishing scalar potential given in [39] and the analysis of the Casimir energy (using the techniques of [40]), which shows that at least three bulk hypermultiplets are necessary for stabilisation. Furthermore, we show that, as proposed by Luty and Okada, it is possible to cancel the resulting vacuum energy with the help of an additional supersymmetry breaking field in the hidden sector. In addition, Luty and Okada claim that a gauge theory in the bulk with a large brane kinetic term will lead to a gravitino mass $m_{3/2}$ much larger than the scale of the other supersymmetric partners given by the gaugino mass $m_{1/2}$ (both masses are induced by a non-vanishing VEV $\langle F_T \rangle$ of the auxiliary field of the radion superfield). This is a desirable property since it would solve the cosmological gravitino problem. However, our careful analysis reveals that a sufficiently large brane kinetic term is, a priori, unnatural.

Therefore, we try to find a possible resolution of this problem. As discussed by Seiberg et al. [49,50] five-dimensional super Yang-Mills theory may have of non-trivial UV-fixed points. Then a cutoff Λ which is much larger than the five-dimensional gauge coupling g_5^{-2} at zero momentum should be possible $\Lambda \gg g_5^{-2}$. A gauge theory near this fixed point may have the desired properties and realise $m_{3/2} \gg m_{1/2}$ in a natural way. Our discussion is related to the 'GUT precursor' proposal of Dudas, Dienes and Gherghetta [51].

Chapter 2

Supergeometry

Global supersymmetry has some interesting properties, which we would like to have in a possible theory beyond the standard model. Since a lot of other very successful theories use gauge symmetries, it is sensible to look for gauged supersymmetry. If we do this, i.e., go from a global to a local supersymmetry, we see that we automatically get gravity into such a theory [9, 10]. Since gravity is an innate property of our space-time it makes sense to look for a space which is supersymmetric and matches ordinary spacetime (gravity) in its bosonic part. This space will be called superspace.

What do we need to describe such a superspace? First we need to know the properties of the coordinates and how many dimensions our space should have. Since we want to include ordinary spacetime, we need the usual four real spacetime coordinates which are bosonic (i.e. commuting). In the description of global supersymmetry we use another kind of variables, the anticommuting Grassmann variables $\theta, \bar{\theta}$. They are two component Weyl spinors and can be regarded as four linearly independent real variables. Therefore, the space we will work in is called $\mathbb{R}^{4|4}$. The first four indicates the number of commuting variables and the second four the number of anticommuting variables. Since ordinary (Einstein) space is included, we demand that superspace should be subject to general coordinate transformations. This will give us the gauge group. In order to do differential geometry we will have to define a vielbein, which will relate the coordinate basis to the basis of the tangent space. In addition we need a structure group to identify physically equivalent systems and a connection for our covariant derivatives. With these geometrical objects we could in principle describe our superspace, i.e. calculate torsion and curvature. Later we will put constraints on these geometrical objects to reduce our degrees of freedom and get the simplest theory sufficient to describe physics in a gravitational superspace.

2.1 The Superspace and its Geometrical Structure

Let us now begin with the description of our superspace. A space becomes a superspace, if we add the two-component anticommuting Weyl-spinors θ^μ and $\bar{\theta}_{\dot{\mu}}$ to the ordinary spacetime coordinates x. Therefore we consider an eight-dimensional superspace, $\mathbb{R}^{4|4}$, with coordinates $z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$ where $\theta^\mu = (\bar{\theta}^{\dot{\mu}})^*$. Now we need to focus on possible symmetry transformations and their representations on this space. They will give us the supergravity gauge group. We first consider the general coordinate transformations

$$z^M \to z'^M = e^{-K} z^M \qquad K = K^M \partial_M = \bar{K} \; , \tag{2.1}$$

with K beeing a real supervector field which commutes.

In addition, we want to give our space a certain geometry. We achieve this by introducing the supervierbein ${E_A}^M$. The supervierbein determines our standard basis in the tangent space $T_p(\mathbb{R}^{4|4})$ for any point $p \in \mathbb{R}^{4|4}$. This basis is given by the set $\{E_A|_p\}$ with

$$E_A = (E_a, E_\alpha, \bar{E}^{\dot{\alpha}}) = {E_A}^M(z)\partial_M \; . \tag{2.2}$$

For this to be a basis, the supermatrix ${E_A}^M(z)$ must be non-singular at each point of the superspace, that is

$$E \equiv \mathrm{sdet}({E_A}^M) \neq 0 \; , \tag{2.3}$$

where $E = E(z)$ is a real superfield, $\bar{E} = E$. For the definition of the superdeterminat sdet, which is sometimes also called Berezinian and denoted by Ber, see Appendix. Furthermore, since ${E_A}^M(z)$ is non-singular, it is invertible and we can define its inverse supermatrix ${E_M}^A(z)$ by

$${E_A}^M {E_M}^B = {\delta_A}^B \quad {E_M}^A {E_A}^N = {\delta_M}^N \; . \tag{2.4}$$

These two supermatrices can be used to convert structure group indices A into space-time (Einstein) indices M and vice versa.

After the introduction of the supervierbein we have to choose the appropriate structure group. A structure group determines physically equivalent bases (or frames) in $T_p(\mathbb{R}^{4|4})$ for each point $p \in \mathbb{R}^{4|4}$. In order to get ordinary gravity in the non-supersymmetric limit, we have to include the Lorentz group in our structure group. Therefore, we choose the group of superlocal Lorentz transformations which, like the usual Lorentz transformations, transform vectors into vectors. Furthermore, the superlocal Lorentz transformations transform spinors into spinors (see eq. (2.11)). In this sense,

we can speak of Lorentz vector superfields and Lorentz spinor superfields. Unfortunately, the supervierbein does not transform covariantly under these transformations. That means $[E_A V]' \neq E'_A V'$, where V is a supertensor field. Therefore, in order to get supercovariant derivatives $\mathscr{D}_A$, we need to introduce the superconnection

$$\Omega = \mathrm{d}z^M \Omega_M = \mathrm{d}z^M {E_M}^A \Omega_A = E^A \Omega_A \; , \tag{2.5}$$

where

$$\Omega_A = \frac{1}{2}\Omega_A{}^{bc}(z) M_{bc} = \Omega_A{}^{\beta\gamma}(z) M_{\beta\gamma} + \Omega_A{}^{\dot{\beta}\dot{\gamma}}(z)\bar{M}_{\dot{\beta}\dot{\gamma}} \tag{2.6}$$

and

$$\Omega_A{}^{bc} = (\Omega_a{}^{bc}, \Omega_\alpha{}^{bc}, \bar{\Omega}^{\dot{\alpha}bc}) \; . \tag{2.7}$$

Here M_{ab} are the usual Lorentz generators (cf. Appendix). In analogy to the covariant derivatives in general relativity

$$\mathscr{D}_m A_n = \partial_m A_n - \Gamma_{mn}{}^l A_l \tag{2.8}$$

we define our supercovariant derivatives as

$$\mathscr{D}_A \equiv E_A + \Omega_A \; . \tag{2.9}$$

It is obvious that this operator changes covariantly with respect to the superspace general coordinate transformations. But it should also change covariantly under superlocal Lorentz transformations:

$$\mathscr{D}'_A = e^{\frac{1}{2}K^{ab}(z)M_{ab}} \mathscr{D}_A e^{-\frac{1}{2}K^{ab}(z)M_{ab}} \; , \tag{2.10}$$

where

$${K_A}^B = \begin{pmatrix} {K_a}^b & 0 & 0 \\ 0 & {K_\alpha}^\beta & 0 \\ 0 & 0 & -\bar{K}^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix}$$

$$K^{ab}(z) = (\sigma^{ab})_{\alpha\beta} K^{\alpha\beta} - (\bar{\sigma}^{ab})_{\dot{\alpha}\dot{\beta}} K^{\dot{\alpha}\dot{\beta}} \tag{2.11}$$

$$K_{\alpha\beta}(z) = K_{\beta\alpha}(z) \; .$$

Therefore, the superconnection has to transform inhomogeneous under the superlocal Lorentz group:

$$\begin{aligned}\Omega' &= \mathrm{d}z^M \Omega'_M = -\mathrm{d}(\Lambda)\Lambda^{-1} + \Lambda\Omega\Lambda^{-1} \\ \Lambda &= \exp\left\{\frac{1}{2}K^{ab}(z)M_{ab}\right\}\end{aligned} \tag{2.12}$$

Furthermore, we should note at this point, that the supervierbein ${E_A}^M$ is the gauge field for the superspace general coordinate transformations supergroup while the superconnection ${\Omega_A}^{bc}$ is the gauge field for the superlocal Lorentz group [13].

These two superfields, the supervierbein ${E_A}^M$ and the Lorentz-superconnection ${\Omega_A}^{bc}$, completely determine our supergeometry. We call these two superfields the 'supergeometry potentials'. Of all the possible supergeometries which can be constructed by choosing arbitrary ${E_A}^M$ and ${\Omega_A}^{bc}$, two are equivalent if the corresponding supergeometry potentials are connected by some superspace general coordinate and superlocal Lorentz transformation. One of these geometries is special in the sense that we can always find a gauge choice, such that the covariant derivative $\mathscr{D}_A$ is identical to the flat covariant derivative D_A, i.e. ${\Omega_A}^{bc} = 0$ and

$${E_A}^M = \begin{pmatrix} {\delta_a}^m & 0 & 0 \\ i(\sigma^m)_{\alpha\dot{\nu}}\bar{\theta}^{\dot{\nu}} & {\delta_\alpha}^\mu & 0 \\ i(\bar{\sigma}^m)^{\dot{\alpha}\nu}\theta_\nu & 0 & {\delta^{\dot{\alpha}}}_\mu \end{pmatrix} . \tag{2.13}$$

Such a supergeometry is called 'flat superspace geometry'. If we analyse matter in the theory belonging to this flat supergeometry we arrive at our global, and in this sense 'flat', supersymmetry.

Now we can define the 'supertorsion' ${T_{AB}}^C$ and the 'supercurvature' ${R_{AB}}^{cd}$ in analogy to general relativity by the graded commutators of the covariant derivatives

$$[\mathscr{D}_A, \mathscr{D}_B\} = {T_{AB}}^C \mathscr{D}_C + R_{AB} \tag{2.14}$$

$$R_{AB} = \frac{1}{2}{R_{AB}}^{cd}M_{cd} = {R_{AB}}^{\gamma\delta}M_{\gamma\delta} + {R_{AB}}^{\dot{\gamma}\dot{\delta}}M_{\dot{\gamma}\dot{\delta}} \tag{2.15}$$

$${T_{AB}}^C = -(-1)^{\epsilon_A\epsilon_B}{T_{BA}}^C \quad R_{AB} = -(-1)^{\epsilon_A\epsilon_B}R_{BA} . \tag{2.16}$$

Comparing equations (2.9) and (2.14), we see that the supertorsion and supercurvature can be expressed in terms of the supervierbein and the superconnection as follows:

$${T_{AB}}^C = {C_{AB}}^C + {\Omega_{AB}}^C - (-1)^{\epsilon_A\epsilon_B}{\Omega_{BA}}^C \tag{2.17}$$

$${R_{AB}}^{cd} = E_A{\Omega_B}^{cd} + {\Omega_A}^{cl}{\Omega_{Bl}}^d - (-1)^{\epsilon_A\epsilon_B}(A \leftrightarrow B) - {C_{AB}}^L{\Omega_L}^{cd} \tag{2.18}$$

where

$${\Omega_{AB}}^C = \begin{pmatrix} {\Omega_{Ab}}^c & 0 & 0 \\ 0 & {\Omega_{A\beta}}^\gamma & 0 \\ 0 & 0 & -{\Omega_A}^{\dot{\beta}}{}_{\dot{\gamma}} \end{pmatrix} . \tag{2.19}$$

The ${C_{AB}}^C$ are the 'anholonomy supercoefficients':

$$[E_A, E_B\} = {C_{AB}}^C E_C \tag{2.20}$$

$${C_{AB}}^C = (E_A{E_B}^M - (-1)^{\epsilon_A\epsilon_B}E_B{E_A}^M){E_M}^C . \tag{2.21}$$

If we use equation (2.19), we get the following relations from equation (2.17)

$$\begin{aligned} {T_{\alpha\beta}}^\gamma &= {C_{\alpha\beta}}^\gamma + {\Omega_{\alpha\beta}}^\gamma + {\Omega_{\beta\alpha}}^\gamma & {T_{\alpha\beta\dot{\gamma}}} &= {C_{\alpha\beta\dot{\gamma}}} \\ {T_\alpha}^{\dot{\beta}\gamma} &= {C_\alpha}^{\dot{\beta}\gamma} + \bar{\Omega}^{\dot{\beta}}{}_\alpha{}^\gamma & {T_{\alpha\beta}}^c &= {C_{\alpha\beta}}^c \\ {T_{\alpha b}}^\gamma &= {C_{\alpha b}}^\gamma - {\Omega_{b\alpha}}^\gamma & {T_\alpha}^{\dot{\beta}c} &= {C_\alpha}^{\dot{\beta}c} \\ {T_{\alpha b}}^c &= {C_{\alpha b}}^c + {\Omega_{\alpha b}}^c & {T_{\alpha\beta\dot{\gamma}}} &= {C_{\alpha\beta\dot{\gamma}}} \\ {T_{ab}}^c &= {C_{ab}}^c + {\Omega_{ab}}^c - {\Omega_{ba}}^c & {T_{ab}}^\gamma &= {C_{ab}}^\gamma . \end{aligned} \tag{2.22}$$

Analogously to the covariant derivatives, which satisfy the Jacobi identities, supercovariant derivatives must satisfy the graded Jacobi identities:

$$\begin{aligned}[\mathscr{D}_A, [\mathscr{D}_B, \mathscr{D}_C\}\} + (-1)^{\epsilon_A(\epsilon_B+\epsilon_C)}[\mathscr{D}_B, [\mathscr{D}_C, \mathscr{D}_A\}\} \\ + (-1)^{\epsilon_C(\epsilon_A+\epsilon_B)}[\mathscr{D}_C, [\mathscr{D}_A, \mathscr{D}_B\}\} = 0\end{aligned} \tag{2.23}$$

We can write this more compactly as

$$[\mathscr{D}_A, [\mathscr{D}_B, \mathscr{D}_C\}\} + \text{ graded cyclic sum} = 0 .$$

If we replace the graded commutators by (2.14) we get an expression of the form

$${S_{ABC}}^E\mathscr{D}_E + {\Psi_{ABC}}^{dl}M_{dl} = 0 , \tag{2.24}$$

so that both ${S_{ABC}}^E$ and ${\Psi_{ABC}}^{dl}$ must vanish

$$S_{ABC}{}^{E} = 0 \tag{2.25}$$
$$\Psi_{ABC}{}^{dl} = 0\ . \tag{2.26}$$

These two equations are called the 'Bianchi identities'. Explicitly they read

$$R_{ABC}{}^{E} - \{\mathscr{D}_A T_{BC}{}^{E} + T_{AB}{}^{F} T_{FC}{}^{D}\} + \text{ graded cyclic sum} = 0 \tag{2.25}$$

and

$$\{-\mathscr{D}_A R_{BC} + T_{AB}{}^{D} R_{DC}\} + \text{ graded cyclic sum} = 0\ . \tag{2.26}$$

With these equations we can get a direct correlation between the supercurvature $R_{ABC}{}^{D}$ and the supertorsion $T_{AB}{}^{C}$, i.e. the supercurvature can be expressed completely in terms of the supertorsion (Dragon theorem). This is in strong contrast to general relativity, where the curvature and the torsion are independent and consequently the torsion can be choosen to be zero. losing the curvature. Thus the supertorsion is the main object determining superspace geometry, i.e. we could use the Bianchi identities to derive an Algebra of our $\mathscr{D}$'s, which depends only on the supertorsion.

But as experience shows this theory has much more degrees of freedom than necessary. It is possible to find much simpler supergravity theories on our superspace by introducing a number of constraints. The following constraints are convenient:

- representation preserving constraints

$$T_{\dot\alpha\dot\beta}{}^{c} = T_{\dot\alpha\dot\beta}{}^{\gamma} = T_{\alpha\beta}{}^{c} = T_{\alpha\beta}{}^{\dot\gamma} = 0 \tag{2.27}$$

- conventional constraints (I)

$$T_{\alpha\dot\beta}{}^{\gamma} = T_{\alpha\dot\beta}{}^{\dot\gamma} = R_{\alpha\dot\beta}{}^{cd} = 0 \tag{2.28a}$$
$$T_{\alpha\dot\beta}{}^{c} = -2i(\sigma^c)_{\alpha\dot\beta} \tag{2.28b}$$
$$\Leftrightarrow \mathscr{D}_{\alpha\dot\alpha} = \frac{i}{2}\{\mathscr{D}_\alpha, \mathscr{D}_{\dot\alpha}\}$$

- conventional constraints (II)

$$T_{\alpha\beta}{}^{\gamma} = T_{\dot\alpha\dot\beta}{}^{\dot\gamma} = T_{\alpha,\beta(\dot\beta}{}^{\beta}{}_{,\ \dot\gamma)} = T_{\dot\alpha,(\beta\dot\beta,\gamma)}{}^{\dot\beta} = 0 \tag{2.29}$$

Looking at the equations (2.22), we know that $T_{\alpha\beta}{}^{c} = C_{\alpha\beta}{}^{c}$ and $T_{\alpha\beta\dot\gamma} = C_{\alpha\beta\dot\gamma}$. This allows us to rewrite the representation preserving constraint (2.27) in the equivalent form

$$\{E_\alpha, E_\beta\} = C_{\alpha\beta}{}^{\gamma} E_\gamma\ , \tag{2.30}$$

which will be crucial for the generation of prepotentials in the following section. Further we see that the E_α do not mix with the $\bar E_{\dot\alpha}$. That allows for the existence of 'covariantly chiral scalar superfields', defined by

$$\bar{\mathscr{D}}_{\dot\alpha}\Phi = 0\ . \tag{2.31}$$

This is desired because chiral superfields are, along with real superfields, the simplest fields in global supersymmetry. Chiral superfields are the generalisation of fermionic fields in the SM and therefore describe matter in global supersymmetry. Hence they are needed to form a complete supersymmetric action.

The conventional constraints (I) determine the vector covariant derivatives in terms of the spinor covariant derivatives. The conventional constraints (II) determine the spinor superconnections Ω_α and $\bar\Omega_{\dot\alpha}$ in terms of the spinor supervierbeins E_α and $\bar E_{\dot\alpha}$. The result is that the E_α and $\bar E_{\dot\alpha}$ are our basic degrees of freedom, which determine all the other geometrical quantities like the superconnection, the derivatives, the supertorsion and the supercurvature. However, the E_α and $\bar E_{\dot\alpha}$ are also constrained; by the representation preserving constraints. These three constraints are called conformal constraints and the corresponding geometry is called conformal supergravity. This name refers to additional symmetries of this theory which are related to the conformal group; the super-Weyl symmetry.

Now we have constraints on the supertorsion which restrict our covariant derivatives, as can be seen from equation (2.14). This makes the Bianchi identities (2.25), (2.26) non-trivial in the sense that they give us conditions for the non-vanishing components of the supertorsion and the supercurvature. These conditions allow us to reduce our set of irreducible Lorentz representations contained in $T_{AB}{}^{C}$ and R_{AB}. In the end only four such irreducible representations, $T_\alpha, R, G_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$, are sufficient to describe our structure. Each of them has certain properties, which arise from the solution of our constraints and they are defined in such a way that the remaining Bianchi-identities are as simple as possible:

$$\mathscr{D}_\alpha T_\beta + \mathscr{D}_\beta T_\alpha = 0 \quad W_{\alpha\beta\gamma} = W_{(\alpha\beta\gamma)} \tag{2.32}$$

$$\left(\bar{\mathscr{D}}_{\dot\alpha} - \frac{1}{2}\bar{T}_{\dot\alpha}\right) W_{\alpha\beta\gamma} = 0 \tag{2.33}$$

$$\bar{\mathscr{D}}_{\dot\alpha} R = 0 \quad \bar{G}_a = G_a \tag{2.34}$$

$$\mathscr{D}_\alpha R = \left(\bar{\mathscr{D}}^{\dot\alpha} - \frac{1}{2}\bar{T}_{\dot\alpha}\right)\left(G_{\alpha\dot\alpha} - \frac{1}{8}\mathscr{D}_\alpha \bar{T}_{\dot\alpha} - \frac{1}{8}\bar{\mathscr{D}}_{\dot\alpha} T_\alpha\right) \tag{2.35}$$

$$\begin{aligned}(\mathscr{D}^\gamma - T^\gamma) W_{\alpha\beta\gamma} = {} & \frac{i}{2}\left(\mathscr{D}_\alpha{}^{\dot\alpha} - \frac{i}{2}\mathscr{D}_\alpha \bar{T}^{\dot\alpha}\right)\left(G_{\beta\dot\alpha} + \frac{1}{8}\mathscr{D}_\beta \bar{T}_{\dot\alpha} + \frac{1}{8}\bar{\mathscr{D}}_{\dot\alpha} T_\beta\right) \\ & + \frac{i}{2}\left(\mathscr{D}_\beta{}^{\dot\alpha} - \frac{i}{2}\mathscr{D}_\beta \bar{T}^{\dot\alpha}\right)\left(G_{\alpha\dot\alpha} + \frac{1}{8}\mathscr{D}_\alpha \bar{T}_{\dot\alpha} + \frac{1}{8}\bar{\mathscr{D}}_{\dot\alpha} T_\alpha\right)\end{aligned} \tag{2.36}$$

with

$$\mathscr{D}_\alpha{}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}(\sigma^a)_{\alpha\dot\beta}\mathscr{D}_a. \tag{2.37}$$

In principle, our old irreducible representations of our supergeometry can be expressed in terms of our new irreducible representations, i.e., the non-vanishing components of our supercurvature R_{AB} and of the supertorsion $T_{AB}{}^C$ can be expressed in terms of the $T_\alpha, R, G_{\alpha\dot\alpha}$ and $W_{\alpha\beta\gamma}$. For example:

$$R_{\alpha\beta\gamma\delta} = -2\bar{R}(\epsilon_{\alpha\gamma}\epsilon_{\beta\delta} + \epsilon_{\alpha\delta}\epsilon_{\beta\gamma}) \tag{2.38}$$

$$\begin{aligned}T_{\alpha,\beta\dot\beta,\dot\gamma} &= i\epsilon_{\alpha\beta}\epsilon_{\dot\beta\dot\gamma}\left(\bar{R} - \frac{1}{16}T^\gamma T_\gamma + \frac{1}{8}\mathscr{D}^\gamma T_\gamma\right) \\ T_{\dot\alpha,\beta\dot\beta,\gamma} &= i\epsilon_{\dot\alpha\dot\beta}\epsilon_{\beta\gamma}\left(R - \frac{1}{16}\bar{T}^{\dot\gamma}\bar{T}_{\dot\gamma} + \frac{1}{8}\bar{\mathscr{D}}^{\dot\gamma}\bar{T}_{\dot\gamma}\right) .\end{aligned} \tag{2.39}$$

In this sense R is a scalar supercurvature while T_α is the torsion trace. Furthermore it is very important to note that the scalar superfield R, which is the analogue to the Ricci scalar in normal relativity, is chiral. For the sake of simplicity we combine these four fields under the name supertorsions.

Now we can derive the algebra (2.14) of our covariant derivatives in our constrained superspace. Important parts of the algebra are

$$\{\mathscr{D}_\alpha, \bar{\mathscr{D}}_{\dot\alpha}\} = -2i\mathscr{D}_{\alpha\dot\alpha} \tag{2.40a}$$

$$\{\mathscr{D}_\alpha, \mathscr{D}_\beta\} = -4\bar{R}M_{\alpha\beta}, \quad \{\bar{\mathscr{D}}_{\dot\alpha}, \bar{\mathscr{D}}_{\dot\beta}\} = 4R\bar{M}_{\dot\alpha\dot\beta} . \tag{2.40b}$$

Now that we are equipped with this simplified algebra, we can check wether there exist further transformations other than general coordinate and super-local Lorentz transformations under which the algebra is covariant.

An algebra is covariant under a particular transformation if the transformation of the commutator is equivalent to the commutator of the transformed operators. The constraints (2.27), (2.28) and (2.29) should be invariant under these transformations. In particular, we could ask what would happen in the case of an arbitrary variation of supervierbein E_A

$$E_A \to E_A + \delta E_A . \tag{2.41}$$

In general relativity we sometimes consider a special form of such transformations; the Weyl transformation:

$$e_a{}^m \to e_a{}^m + \sigma e_a{}^m \quad \Longrightarrow \quad g_{mn} \to g_{mn} - 2\sigma g_{mn} . \tag{2.42}$$

The Weyl transformation is a scale transformation and thus no symmetry of general relativity. If we try to do something similar in our superspace, we see that our constraints are invariant under such a superlocal scale transformation if we perform another transformation simultanously, namely a superlocal chiral or γ_5-transformation

$$\begin{aligned}E_\alpha \to E'_\alpha &= e^{(\Delta+i\kappa)/2}E_\alpha = LE_\alpha \\ \bar{E}_{\dot\alpha} \to \bar{E}'_{\dot\alpha} &= e^{(\Delta-i\kappa)/2}\bar{E}_{\dot\alpha} = \bar{L}\bar{E}_{\dot\alpha} \\ E_a \to E'_a &= e^{\Delta}E_a + \dots .\end{aligned} \tag{2.43}$$

Here $\Delta(z)$ and $\kappa(z)$ are real superfields which we can also express in terms of an arbitrary (non-vanishing) complex scalar superfield $L(z)$

$$L\bar{L} = e^{\Delta} \quad \text{and} \quad L\bar{L}^{-1} = e^{i\kappa} , \tag{2.44}$$

which in this sense could be used to characterise these transformations. The parameter Δ induces the superlocal scale (or superspace Weyl transformation), while κ induces the superlocal chiral transformation. Together they are called generalized super Weyl transformations induced by κ and Δ. The objects of our algebra transform as

$$\mathscr{D}'_\alpha = L\mathscr{D}_\alpha - 2(\mathscr{D}^\beta L)M_{\alpha\beta} \tag{2.45}$$

$$\bar{\mathscr{D}}'_{\dot\alpha} = \bar{L}\bar{\mathscr{D}}_{\dot\alpha} - 2(\bar{\mathscr{D}}^{\dot\beta}\bar{L})\bar{M}_{\dot\alpha\dot\beta} \tag{2.46}$$

and, for example,

$$T'_\alpha = LT_\alpha - \mathscr{D}'_\alpha \ln(L^4 \bar{L}^2) \tag{2.47}$$

$$R' = -\frac{1}{4}(\bar{\mathscr{D}}^2 - 4R)\bar{L}^2 \, . \tag{2.48}$$

2.2 Prepotentials

What do we have so far? We have the two superpotentials $E_A{}^M$ and $\Omega_A{}^{bc}$ which determine our supergeometry. After imposing the constraints (2.28) and (2.29) they depend only on the spinor vierbein superfields $E_\alpha{}^M$ and $\bar{E}_{\dot\alpha}{}^M$, which are thus our basic degrees of freedom. These superfields are, as we have seen earlier, again constrained by the representation preserving constraint (2.27) in such a way that they form a closed algebra (see equation (2.30))

$$\{E_\alpha, E_\beta\} = C_{\alpha\beta}{}^\gamma E_\gamma \, .$$

The result is that we have to work with constrained basic degrees of freedom, which is unconvenient; it is much easier to work with unconstrained fields. Let us see what we can do about that. In classical electrodynamics we have a similar problem. We have the field strength F_{mn} which completly describes our physics but is restricted by the constraint $\mathrm{d}F = 0$. This constraint is automatically met if we define F by $F = \mathrm{d}A$. This allows us to work with the unconstrained potential A instead of the constrained field strength F. Therefore our present aim is to find a way to rewrite our conformal supergravity in a such way that we are able to use unconstrained superfields and automatically fulfil the whole set of conformal constraints. Since we want to substitute some constrained potentials with unconstrained quantities, we will call these unconstrained fields 'prepotentials' (see also [11]).

To achieve this aim we need to understand the geometrical meaning of the statement that the algebra of the E_α closes. A mathematical tool that can help us with this task is Frobenius' theorem: A set of q linearly independent vector fields V_i on a p-dimensional manifold ($p > q$) defines a set of q-dimensional submanifolds if the algebra of the V_i closes.

The V_i are then the tangent vectors of these surfaces with the surface coordinates y^i. The submanifold can be seen as defined by a system of equations $f^\alpha(x) = 0$ with $\alpha = 1, .., p-q$, where the $f^\alpha(x)$ are independent scalar functions such that $df^\alpha(V_i) = 0$. Then we can choose the variables $\tilde{x}^m = (y^1, ..., y^q, f^1, ..., f^{p-q})$ as a local coordinate system on the manifold. These coordinates can be expressed in terms of the coordinates x^j of the

p-dimensional manifold with the help of a vector field U by $\tilde{x}^m = e^U x^m$. Altogether we obtain

$$V_i = A_i{}^j \frac{\partial}{\partial y^j} = A_i{}^j e^U \frac{\partial}{\partial x^j} e^{-U} \tag{2.49}$$

with

$$\det(A_i{}^j) \neq 0 \text{ and } U = U^m(x) \frac{\partial}{\partial x^m} \, . \tag{2.50}$$

This allows us to express the constrained (the constraint is that they form a closed algebra) fields V_i in terms of the unconstrained vector field U, which is responsible for the reparametrisation of our coordinates. Now we can work with the unconstrained U and our 'physical' fields V_i will automatically form a closed algebra. A proof of Frobenius' theorem can be found in any differential geometry textbook.

Unfortunately, we cannot apply Frobenius' theorem directly to our case, since the E_α come together with the $\bar{E}_{\dot\alpha}$, but the E_α and $\bar{E}_{\dot\alpha}$ do not form a closed algebra. A solution to this problem would be to find a way to consider just the E_α without the $\bar{E}_{\dot\alpha}$, so that we can use Frobenius' theorem on the E_α. That means we have to consider ∂_α (or θ^α) without $\bar{\partial}_{\dot\alpha}$ (or $\bar{\theta}^{\dot\alpha}$), i.e. our Grassmann variables and their complex conjugates have to be independent from each other. As we will see in the following we can do this if we go from $\mathbb{R}^{4|4}(x, \theta, \bar{\theta})$ to $\mathbb{C}^{4|4}(y, \theta, \bar{\rho})$ and regard $\mathbb{R}^{4|4}$ as a subspace defined by $y = \bar{y}$ and $\bar{\theta} = \bar{\rho}$, where $\bar{\rho}$ is just another set of Grassman variables.

Let us consider our basis vector fields E_α as defined on $\mathbb{C}^{4|4}$:

$$E_\alpha = E_\alpha{}^m(y, \theta, \bar{\rho}) \frac{\partial}{\partial y^m} + E_\alpha{}^\mu(y, \theta, \bar{\rho}) \frac{\partial}{\partial \theta^\mu} + E_{\alpha\dot\mu}(y, \theta, \bar{\rho}) \frac{\partial}{\partial \bar{\rho}_{\dot\mu}} \tag{2.51}$$

$$\bar{E}_{\dot\alpha} = \bar{E}_{\dot\alpha}{}^m(y, \bar{\rho}, \theta) \frac{\partial}{\partial \bar{y}^m} + \bar{E}_{\dot\alpha}{}^\mu(y, \bar{\rho}, \theta) \frac{\partial}{\partial \theta^\mu} + \bar{E}_{\dot\alpha\dot\mu}(y, \bar{\rho}, \theta) \frac{\partial}{\partial \bar{\rho}_{\dot\mu}} \tag{2.52}$$

where the bar over the functions $E_{\dot\alpha}{}^m, E_{\dot\alpha}{}^\mu$ and $E_{\dot\alpha\dot\mu}$ is defined by

$$\bar{E}_{\dot\alpha}{}^m(y, \bar{\rho}, \theta) \equiv \overline{E_\alpha{}^m(\bar{y}, \rho, \bar{\theta})} \tag{2.53}$$

$$\bar{E}_{\dot\alpha}{}^\mu(y, \bar{\rho}, \theta) \equiv \epsilon^{\mu\nu} \overline{E_{\alpha\dot\nu}(\bar{y}, \rho, \bar{\theta})} \tag{2.54}$$

$$\bar{E}_{\dot\alpha\dot\mu}(y, \bar{\rho}, \theta) \equiv \epsilon_{\dot\mu\nu} \overline{E_\alpha{}^\nu(\bar{y}, \rho, \bar{\theta})} \, , \tag{2.55}$$

i.e. $\bar{E}_{\dot{\alpha}}{}^{m}(\bar{y},\bar{\rho},\theta)$ follows from $E_{\alpha}{}^{m}(y,\theta,\bar{\rho})$ (considered as a power series) by $y\to\bar{y},\theta\to\bar{\rho},\bar{\rho}\to\theta$ and a complex conjugation of the coefficients of the power series (analogical for $\bar{E}_{\dot{\alpha}}{}^{\mu}(y,\bar{\rho},\theta)$ and $\bar{E}_{\dot{\alpha}\dot{\mu}}(y,\bar{\rho},\theta)$).

Now the E_α can be linearly independent from the $\bar{E}_{\dot{\alpha}}$, since they depend on independent coordinates θ and $\bar{\rho}$. Thus we can apply Frobenius' theorem to the vector fields E_α on $\mathbb{C}^{4|4}$, which still form a closed algebra. Frobenius' theorem gives:

$$E_\alpha = A_\alpha{}^{\mu}(z)\hat{E}_\mu \qquad \hat{E}_\mu = e^{W}\partial_\mu e^{-W} \tag{2.56}$$

$$\bar{E}_{\dot{\alpha}} = \bar{A}_{\dot{\alpha}}{}^{\dot{\mu}}(z)\hat{\bar{E}}_{\dot{\mu}} \qquad \hat{\bar{E}}_{\dot{\mu}} = -e^{\bar{W}}\bar{\partial}_{\dot{\mu}}e^{-\bar{W}} \tag{2.57}$$

where

$$W = W^{M}\partial_M \neq \bar{W} \tag{2.58}$$

and it follows that

$$\{\hat{E}_\mu,\hat{E}_\nu\} = 0 \qquad \{\hat{E}_{\dot{\mu}},\hat{E}_{\dot{\nu}}\} = 0 \tag{2.59}$$

with $\det(A_\alpha{}^{\mu})\neq 0$. Here the superfield W plays the role of the U above. It is the reparametrisation of the coordinates of the manifold of $\mathbb{C}^{4|4}$, which is in this case tangential to the E_α. In addition, it is convenient to factor $A_\alpha{}^{\mu}$ into a Lorentz scalar F and a unimodular matrix $N_\alpha{}^{\mu}$

$$E_\alpha = FN_\alpha{}^{\mu}\hat{E}_\mu \quad \bar{E}_{\dot{\alpha}} = \overline{FN}_{\dot{\alpha}}{}^{\dot{\mu}}\hat{\bar{E}}_{\dot{\mu}} \tag{2.60}$$

$$\det(N_\alpha{}^{\mu}) = 1\,. \tag{2.61}$$

Our final prepotentials, which we will call 'supergeometry prepotentials', are W and the Lorentz scalar F as well as the unimodular matrix $N_\alpha{}^{\nu}$. In addition, we will now work in $\mathbb{C}^{4|4}$ with the independent complex coordinates y,θ and $\bar{\rho}$. However, our physical space is still $\mathbb{R}^{4|4}$ which we get if we restrict our coordinates by $y=\bar{y}$ and $\bar{\theta}=\bar{\rho}$. Thus we will, for the sake of simplicity, continue to write $z^M=(x^m,\theta^\mu,\bar{\theta}_{\dot{\mu}})$, but we have to keep in mind that ρ is independent of θ before the restriction to the physical, real space. The supergeometry prepotentials allow us, after imposing the constraints (2.28) and (2.29), to express all geometrical objects, E, T_α, R, $G_{\alpha\dot{\alpha}}$ and $W_{\alpha\beta\gamma}$, in terms of these unconstrained superfields. To get these expressions, we have to choose a gauge on the superlocal Lorentz group. The simplest is

$$N_\alpha{}^{\mu} = \delta_\alpha{}^{\mu}\,, \tag{2.62}$$

which is called the Lorentz gauge. From now on will always work in this gauge unless otherwise stated. If we introduce a semi-covariant supervierbein $\hat{E}_A$

$$\hat{E}_A = (\hat{E}_A,\hat{E}_\alpha,\hat{\bar{E}}^{\dot{\alpha}}) = \hat{E}_A{}^{M}\partial_M \tag{2.63}$$

$$\hat{E}_a = -\frac{i}{4}\bar{\sigma}_a{}^{\dot{\alpha}\alpha}\{\hat{E}_\alpha,\hat{\bar{E}}_{\dot{\alpha}}\} \qquad \hat{E} = \mathrm{sdet}(\hat{E}_A{}^{M}) \tag{2.64}$$

we get the following equations for E,T_α and R:

$$E = F^2\bar{F}^2\hat{E} \tag{2.65}$$

$$\begin{aligned} T_\alpha &= E_\alpha T \qquad & T &= \ln\left[\bar{F}^2F^4\hat{E}(1\cdot e^{\overleftarrow{\bar{W}}})\right] \\ \bar{T}_{\dot{\alpha}} &= \bar{E}_{\dot{\alpha}}\bar{T} \qquad & \bar{T} &= \ln\left[F^2\bar{F}^4\hat{E}(1\cdot e^{\overleftarrow{\bar{W}}})\right] \end{aligned} \tag{2.66}$$

$$R = -\frac{1}{4}\hat{\bar{E}}_{\dot{\mu}}\hat{\bar{E}}^{\dot{\mu}}\bar{F}^2\,. \tag{2.67}$$

The $(1\cdot e^{\overleftarrow{\bar{W}}})$ can be seen as defined by the super-Jacobi-determinant of the transformation induced by W:

$$(1\cdot e^{\overleftarrow{\bar{W}}}) = \mathrm{sdet}\left(\frac{\partial z'^{M}}{\partial z^{N}}\right) \text{ with } z'^{M} = e^{W}z^{M} \tag{2.68}$$

2.3 The Chiral Representation

So far we have worked in a representation we call the real representation because in this representation the conjugate $\bar{V}$ is identical to the superfield V, if V is real. On the other hand chiral superfields

$$\bar{\mathscr{D}}_{\dot{\alpha}}\phi = 0 \tag{2.69}$$

are not very handy in this representation because they can still depend explicitly on the Grassmann variable $\bar{\theta}$. But if we want to use chiral superfields instead of real superfields in our action, it would be convenient to work in a representation in which chiral superfields have a simple form and sacrifice the simplicity of the reality condition. So the next important step on our way to physical applications, i.e. the construction of an invariant action, is the introduction of the chiral representation. To switch between the real and

the chiral representation, we have to perform the following picture changing transformation for every superfield V:

$$\tilde{V} \equiv e^{-\bar{W}} V$$

$$\tilde{\mathscr{D}}_A = e^{-\bar{W}} \mathscr{D}_A e^{\bar{W}} \tag{2.70}$$

$$\tilde{E}_A = e^{-\bar{W}} E_A e^{\bar{W}}$$

The advantage of this is that if V is covariantly chiral, $\tilde{V}$ is flat chiral, which means that it has no explicit $\bar{\theta}$-dependence. On the other hand real superfields $V = \bar{V}$ are much more complicated in this representation:

$$\tilde{\bar{V}} = e^{-W} e^{\bar{W}} \tilde{V} \tag{2.71}$$

In this section we have used the tilde to distinguish between the supervierbein in the chiral representation $\tilde{E}$ and the $\hat{E}$ we introduced with the prepotentials. However, for the rest of this thesis we will change this notation and will indicate superfields in the chiral representaion by a hat as long as this does not lead to confusions with our $\hat{E}$. For example we will write $\hat{\phi}$ for the chiral compensator, which we will introduce in the next section, if it is meant to be in the chiral representation.

2.4 The Chiral Compensator Mechanism

An essential part of the subsequent description of Einstein supergravity will be the chiral compensator field ϕ. Thus, it seems appropriate to comment on "compensating" fields which are also called compensators [11].

If we have a theory whose action is not invariant under a certain transformation which we would like to have as a symmetry of our theory, we can modify our action by terms which exactly compensate for the change of our action due to the transformation. This will change our theory, because we have to introduce new fields with specific transformation properties and we will gain a new gauge symmetry. These new fields are called compensators [41]. If we choose a gauge for these compensator fields, then only the original symmetries leave this gauge invariant, thus the remaining degree of freedom is the same as in the original theory. Thus, compensators are in some sense additions to the theory because they can always be gauged away. But instead of writing the theory without them, which is always possible, we often use compensators to simplify our theory as we have done in the case of

the prepotentials. So our prepotentials are compensator fields, but we have yet to recover which gauge symmetries they compensate for. Furthermore, we will use one of these prepotentials to construct another, more useful and well-known compensator field, the chiral compensator ϕ.

This compensator will help us to get rid of some additional symmetries our conformal supergravity has compared to the simplest form of supergravity, Einstein/Poincaré supergravity.

In order to write our conformal supergravity without the representation preserving constraint, we introduced the prepotentials, but had to use $\mathbb{C}^{4|4}$ instead of $\mathbb{R}^{4|4}$. At the same time, our theory gets some new gauge invariances. This results in an additional gauge group, the λ-supergroup. This is the group of reparametrisations of $\mathbb{C}^{4|4}$, which leave the $\hat{E}_\mu$ invariant. So the full gauge group of the prepotentials is the product of the λ-supergroup, the superlocal Lorentz group and the general coordinate transformation supergroup (the 'K-supergroup'), which reparametrises $\mathbb{R}^{4|4}$. If we let each of the three subgroups of our whole gauge group act on the prepotentials, we see that certain transformations act only on one prepotential. This prepotential is then the compensator field for this gauge symmetry.

- superlocal Lorentz group (original gauge invariance)

$$E'_\alpha = \Lambda_\alpha{}^\beta E_\beta = \Lambda_\alpha{}^\beta N_\beta{}^\mu F \hat{E}_\mu \tag{2.72}$$
$$\Rightarrow E'_\alpha = N'_\alpha{}^\mu F' \hat{E}'_\mu \tag{2.73}$$
$$\text{with } W' = W,\ F' = F \text{ and } N'_\alpha{}^\mu = \Lambda_\alpha{}^\beta N_\beta{}^\mu \tag{2.74}$$

- K-supergroup (reparametrisation of $\mathbb{R}^{4|4}$)

$$E'_\alpha = e^K E_\alpha e^{-K} \text{ with } K = K^M \partial_M + \bar{K} \tag{2.75a}$$
$$\Rightarrow F' = e^K F \quad N'_\alpha{}^\mu = e^K N_\alpha{}^\mu \quad e^{W'} = e^K e^W \tag{2.75b}$$

- λ-supergroup (reparametrisation of $\mathbb{C}^{4|4}$)

$$e^{W'} = e^W e^{-\bar{\lambda}} \quad e^{\bar{W}'} = E^{\bar{W}} e^{-\lambda} \quad \lambda = \lambda^M \partial_M \tag{2.76a}$$
$$\text{with } \hat{E}'_\mu = e^{W'} \partial_\mu e^{-W'} = \hat{E}_\mu \tag{2.76b}$$
$$\text{and thus } \partial_\mu \bar{\lambda}^n = \partial_\mu \bar{\lambda}_{\dot{\nu}} = 0 = \bar{\partial}_{\dot{\mu}} \lambda^n = \bar{\partial}_{\dot{\mu}} \lambda^\nu \tag{2.76c}$$
$$\text{(the } \bar{\lambda}^\mu \text{ are unrestricted)}$$

- generalized super Weyl group (see equation (2.43) and (2.44))

$$E'_\alpha = e^{(\Delta + i\kappa)/2} E_\alpha \tag{2.77}$$
$$\Rightarrow F' = LF \quad N'_\alpha{}^\mu = N_\alpha{}^\mu \quad W' = W \tag{2.78}$$

We can immediately see that the prepotential $N_\alpha{}^\mu$ is the compensating superfield for the superlocal Lorentz group and the prepotential F compensates for the generalized super Weyl group. For the remaining prepotential it is not so easy to see which transformation they compensate for. In other words, how much of W can be gauged away with the help of the remainig symmetry transformations, the λ- and K-transformations? To see the answer we ask the reverse: If we have $W = 0$ which components of W' can be created with the help of the K- and λ-supergroups

$$e^{W'} = e^K e^W e^{-\bar{\lambda}} \ ? \tag{2.79}$$

In the infinitesimal case we get

$$\mathbb{1}+W' = \mathbb{1}+K-\bar{\lambda} = \mathbb{1}+(K^m-\bar{\lambda}^m)\partial_m+(K^\mu-\bar{\lambda}^\mu)\partial_\mu+(\bar{K}_{\dot{\mu}}-\bar{\lambda}_{\dot{\mu}})\bar{\partial}^{\dot{\mu}} \ . \tag{2.80}$$

From the fields involved in this equation K^m, $\bar{K}_{\dot{\mu}}$ and $\bar{\lambda}^\mu$ are unconstrained as can be seen from the transformation laws above. Thus we are able to generate all parts of W^M except the imaginary part of W^m, since K^m is real, with unconstrained fields. The reverse statement is also true. We can use $K^m, \bar{K}_{\dot{\mu}}$ and $\bar{\lambda}^\mu$ to find for arbitrary W a gauge where only Im W' remains. If we do this step by step, we see that we can assign each of the above transformations to a certain parts of W. Re W is the compensating superfield for the K-supergroup and the superfield W^μ is the compensating superfield for the $\bar{\lambda}^\mu$-transformations. Thus, the only prepotential which is no compensator field is ImW^m. In this sense it is the only prepotential which has real physical meaning. Indeed it is possible to show that Im W^m plays the role of the gravitational superfield.

Now we want impose an additional constraint on our theory and then we want to see how we can use a compensator to write the resulting theory in an easy way. As we have seen, generalized super Weyl transformations are analogous to ordinary Weyl-transformations in ordinary conformal gravity. But our physical world has scales und thus we have to demand the existence of an invariant measure

$$\mathrm{d}^4x\sqrt{g} = \mathrm{d}^4x'\sqrt{g'} \ . \tag{2.81}$$

Since the Weyl transformation (2.42) is a scale transformation, it changes the measure and thus we lose this symmetry in our theory restricted by (2.81). This simplifies our theory considerably. The theory without the Weyl symmetry is ordinary Einstein gravity.

In the context of supergravity we have a similar situation. Our conformal supergravity is invariant under scale transformations, i.e. generalized super

Weyl transformations. But to get a physical theory, we have to restrict our symmetries by demanding

$$\mathrm{d}^8zE^{-1} = \mathrm{d}^8z'(E')^{-1} \tag{2.82}$$

and thus lose the generalised super Weyl symmetry. The resulting theory is called Einstein supergravity. In principle, we could now work with our conformal supergravity theory with this constraint, but as we have seen before it is more convenient to work with unconstrained fields.

Since F is the compensating superfield for the generalised super Weyl symmetry, constraint (2.82) is equivalent to choosing a gauge for F and simultaneously restricting the symmetry to the part which leaves this gauge unchanged. The simplest gauge choice is $F = 1$, but this gauge is not invariant under the λ-supergroup. Thus we would have to work with a constrained W if we want to preserve this gauge. A more useful gauge is the 'super Weyl gauge'

$$T_\alpha = \bar{T}_{\dot{\alpha}} = 0 \ . \tag{2.83}$$

This can be regarded as a gauge since the T_α transform under the generalised super Weyl group as

$$T'_\alpha = E'_\alpha T \quad T' = T + \ln(L^4\bar{L}^2) \tag{2.84}$$

(see equation (2.66)) and thus the allowed gerneralized super Weyl transformations are restricted to

$$L = e^{\sigma/2-\bar{\sigma}} \quad \text{with} \quad \bar{\mathscr{D}}_{\dot{\alpha}}\sigma = 0 \ . \tag{2.85}$$

However, it is more convenient to regard (2.83) just as an additional constraint imposed on the prepotentials F and W by equation (2.66). Now we want to use a compensator field to get rid of this constraint.

Therefore, we use the constraint to define another superfield, the chiral compensator, with the help of the prepotentials. From (2.66) we see that

$$\phi^3 = F^2\bar{F}^4\hat{E}(1\cdot e^{\overleftarrow{\bar{W}}})^{-1} \quad \text{with} \quad \bar{E}_{\dot{\alpha}}\phi = 0 \tag{2.86}$$

ensures that the the constraint (2.84) is fulfilled. The power of three is just the common convention. If we go to the chiral representation we see

$$\phi = e^{\bar{W}}\hat{\phi} \quad \bar{\partial}_{\dot{\mu}}\hat{\phi} = 0 \ , \tag{2.87}$$

so that the name chiral compensator is warranted. With (2.65) we get from (2.86) another useful form of ϕ^3:

$$\phi^3 = E^{-1}\bar{F}^{-2}(1 \cdot e^{\overleftarrow{\bar{W}}})^{-1} \,. \tag{2.88}$$

Now, if we calculate the transformation of $\hat{\phi}$ under the combined K-λ-transformations which preserve the gravitational superfield gauge (see below), we find precisely the infinitesimal version of

$$\hat{\phi}'(z', \theta') \longrightarrow \text{sdet}\, \left(\frac{\partial(y,\theta)}{\partial(y',\theta')}\right)^{1/3} \hat{\phi} \,, \tag{2.89}$$

which is sometimes seen as the definition of the chiral compensator.

We conclude that Einstein supergravity is given by the conformal constraints with the addition of the $T_\alpha = 0$ constraint and it can be characterised with $N_\alpha{}^\mu$, W^M and ϕ. The superfields $N_\alpha{}^\mu$, W^M and ϕ are called the minimal supergravity prepotentials. The superfield ϕ is the compensating field for the super Weyl transformation. It replaces the old prepotential F.

2.5 The Gravitational Superfield and the Wess-Zumino Gauge

As we have seen, our $\mathbb{C}^{4|4}$ description has a lot of gauge degrees of freedom. To work in our prepotential description of supergravity, it is thus important to know which gauge choices are the easiest and which are the most useful. As already said in Section 2.2, we will always work in the Lorentz gauge. This fixes the prepotential $N_\alpha{}^\mu$ which leaves us with the superfields W^M and ϕ to get rid of the remaining gauge degrees of freedom. Let us first consider the W^M. The simplest possibility is

$$W = W^m \partial_m \text{ with } \text{Re}W = 0 \,. \tag{2.90}$$

Unfortunately, this gauge obscures some very important properties of our theory, which are easy to see in a more useful gauge, the gravitational superfield gauge. To get this gauge we take the above gauge and allow $\text{Re}W \neq 0$. This allows us to make the gauge choice

$$e^{\bar{W}^n \partial_n} x^m = x^m + i\mathscr{H}^m(x, \theta, \bar{\theta}) \,,\ \mathscr{H}^m = \bar{\mathscr{H}}^m \,,\ W_\alpha = W_{\dot{\alpha}} = 0 \,. \tag{2.91}$$

But our gauge fixing is not yet complete, since there are still some residual K- and λ-transformations which leave this gauge unchanged:

$$K^m = \frac{1}{2}\lambda^m(x + i\mathscr{H}, \theta) + \text{h.c.} \equiv \text{Re}\lambda^m(x + i\mathscr{H}, \theta) \tag{2.92}$$

$$K^\mu = \lambda^\mu(x + i\mathscr{H}, \theta), \quad \bar{K}^{\dot{\mu}} = \bar{\lambda}^{\dot{\mu}}(x + i\mathscr{H}, \bar{\theta}) \,. \tag{2.93}$$

Thus, the remaining gauge transformations allowed for $\mathscr{H}$ are

$$\delta\mathscr{H}^m = \frac{1}{2}(\lambda^m - \bar{\lambda}^m) + \left\{\frac{1}{2}(\lambda^n - \bar{\lambda}^n)\partial_n + \lambda^\nu \partial_\nu + \bar{\lambda}^{\dot{\nu}}\partial_{\dot{\nu}}\right\}\mathscr{H}^m \tag{2.94}$$

with

$$\lambda^m = \lambda^m(x + i\mathscr{H}, \theta) \,. \tag{2.95}$$

This expression allows for a very simple interpretation of $\mathscr{H}$ and its residual gauge freedom. The e^W has been introduced as a diffeomorphism of $\mathbb{C}^{4|4}$. Now we have specialised to $W = W^m\partial_m$. This means that we do not really need $\mathbb{C}^{4|4}$ but only $\mathbb{C}^{4|2}$ parametrised by (y^m, θ^α), since $W_\alpha = W_{\dot{\alpha}} = 0$ and thus the θ-space remains unchanged if we want to express our conformal supergravity with prepotentials in this gauge. Furthermore, every covariantly chiral scalar superfield has now the form

$$\Phi(x, \theta, \bar{\theta}) = e^{\bar{W}}\hat{\Phi}(x, \theta) = \hat{\Phi}(x^m + i\mathscr{H}^m, \theta^\mu) \,. \tag{2.96}$$

To complete our gauge of W, we can use the residual K-λ-transformations to gauge away the lowest components of any given $\mathscr{H}^m$. If we do this, we arrive at

$$\mathscr{H}^m(x, \theta, \bar{\theta}) = \theta\sigma^a\bar{\theta}e_a{}^m + i\bar{\theta}^2\theta^\alpha\psi^m{}_\alpha(x) - i\theta^2\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{m\dot{\alpha}}(x) + \theta^2\bar{\theta}^2\tilde{A}^m(x) \tag{2.97}$$

which is called the Wess-Zumino-gauge. If we work with the minimal supergravity prepotentials we still have some gauge freedom left at this point, the gauge choice for the chiral compensator ϕ. The most convenient choice is

$$\hat{\phi}^3 = e^{-1}\left(1 - 2i\theta\sigma_a\bar{\psi}^a + \theta^2\tilde{B}\right) \,. \tag{2.98}$$

Here A^m and B are some arbitrary auxiliary fields. However, it is non-trival that both superfields $\mathscr{H}^m$ and ϕ contain the same spinor field $\psi^m{}_\alpha(x)$ which is the gravitino. The vielbein $e_a{}^m$ appears in the Wess-Zumino gauge in such a way that the zero order component of E is the determinant of the normal vielbein e. Since ϕ^3 is antiproportional to E it has to be antiproportional to e on component level.

Chapter 3

Supergravity Action

3.1 The Supergravity Action and the Compensator Mechanism

Now we want to use the supergeometry, the prepotential approach and the compensator mechanism to derive a general supergravity action. In order to do this we need some rules about integrals in our superspace, namely the chiral integration rule and integration by parts. With the help of the algebra of Einstein supergravity we get a special form of the 'integration by parts':

$$\int \mathrm{d}^8 z E^{-1} \mathscr{D}^\alpha \psi_\alpha = 0 \; ; \qquad \int \mathrm{d}^8 z E^{-1} \mathscr{D}^a V_a = 0 \tag{3.1}$$

And we get the chiral integration rule for chiral $\mathcal{L}_c$

$$S_c = \int \mathrm{d}^6 z \hat{\phi}^3 \hat{\mathcal{L}}_c = \int \mathrm{d}^8 z (E^{-1}/R) \mathcal{L}_c \; . \tag{3.2}$$

This rule is not valid if the part of R which is independent of the Grassmann variables vanishes that means in particular in the flat superspace limit, where $\mathrm{R} \to 0$. This rule is therefore a particular property of curved space. It enables us to turn every integral over $\mathbb{R}^{4|4}$ into an integral over $\mathbb{R}^4_c \times \mathbb{C}^2_a$. It is important to note that $\hat{\mathcal{L}}_c$ is in the chiral representation, meaning

$$\mathcal{L}_c = e^{\bar{W}} \hat{\mathcal{L}}_c \; . \tag{3.3}$$

In conformal supergravity we can show with the help of the algebra (2.40) and its Bianchi identities (2.32)

$$\bar{\mathscr{D}}_{\dot{\alpha}} (\bar{\mathscr{D}}^2 - 4R) U_{\alpha_1 \ldots \alpha_n} = 0 \; , \tag{3.4}$$

for an arbitrary tensor superfield $U_{\alpha_1 \ldots \alpha_n}$. This means that $[(\bar{\mathscr{D}}^2 - 4R) U_{\alpha_1 \ldots \alpha_n}]$ is covariantly chiral. The operator $(\bar{\mathscr{D}}^2 - 4R)$ is called the chiral projector, since it gives us the chiral component of a given field. Note however, that it is not canonically normalised in the sense that two times the projector is exactly the same projector. The canonically normalised projector is $((-1/4R)\bar{\mathscr{D}}^2 + 1)$.

By using integration by parts and the chirality of R we can write:

$$\begin{aligned} \int \mathrm{d}^8 z E^{-1} \mathcal{L} &= -\frac{1}{4} \int \mathrm{d}^8 z \left(\frac{E^{-1}}{R} \right) \left(\bar{\mathscr{D}}^2 - 4R \right) \mathcal{L} \\ &= -\frac{1}{4} \int \mathrm{d}^6 z \hat{\phi}^3 e^{-\bar{W}} \left[\left(\bar{\mathscr{D}}^2 - 4R \right) \mathcal{L} \right] \\ &= -\frac{1}{4} \int \mathrm{d}^6 z \hat{\phi}^3 e^{-\bar{W}} \mathcal{L}_c \end{aligned} \tag{3.5}$$

From now on we will always use the subscript c to indicate that a superfield is chiral. Now after having the form of a general invariant action in Einstein supergravity, we want to know what the simplest Einstein supergravity action looks like. We first consider

$$\frac{1}{\kappa^2} \int \mathrm{d}^8 z \frac{E^{-1}}{R} \mathcal{L}_c (T_{BC}{}^D, \mathscr{D}_A T_{BC}^{\;\;D}, \ldots) \; , \tag{3.6}$$

where $\mathcal{L}_c$ is a covariantly chiral scalar depending polynomially on the supertorsion and its covariant derivatives up to a finite order, while κ is the gravitational coupling constant with $\kappa^2 = 8\pi G_N = 1/M_P^2$. This general ansatz gives us a lot of possibilities for the exact form of $\mathcal{L}_c$. In order to reduce them, we try to do some dimensional analysis. We know that an action has mass dimension zero. The other quantities in the action have the dimensions

$$\left[\kappa^{-2}\right] = 2 \quad \left[\mathrm{d}^8 z\right] = -2 \quad [E] = 0 \quad [R] = 1 \; ; \tag{3.7}$$

thus $\mathcal{L}_c$ must have the dimension 1. The Lagrangian can depend on the $T_{BC}{}^D$ and its covariant derivatives. As we have seen before, when we solved the Bianchi identities for conformal supergravity, there are only four irreducible representations left in $T_{BC}{}^D$, the supertorsions $T_\alpha, R, G_{\alpha\dot{\alpha}}$ and $W_{\alpha\beta\gamma}$. In addition we lose T_α if we go to Einstein supergravity. We have already given the mass dimension of the scalar curvature R. The mass dimensions of the other supertorsions can be seen from the algebra of Einstein supergravity:

$$[G_a] = 1 \quad [W_{\alpha\beta\gamma}] = 3/2 \; . \tag{3.8}$$

Therefore, the covariant derivatives of the $T_{BC}{}^{D}$ have mass dimensions equal or greater than 3/2. Now we could use every combination of these fields with a mass dimension equal or greater than 1. If the combination has a mass dimension greater than one we have to multiply it with appropriate powers of the gravitational coupling constant κ. Therefore, the contribution of these combinations to the action is suppressed by the order of the Planck scale. Thus we assume that $\mathcal{L}_c$ has no explicit dependence on κ. So we can only use combinations of the supertorsions and their derivatives with a combined mass dimension 1. Since the Lagrangian is a scalar and G_a is a vector, we find by this dimensional analysis that $\mathcal{L}_c$ is proportional to R. At component level we want to reproduce the ordinary Einstein-Hilbert action. If we normalise in order to produce the correct vierbein contribution for this, we get

$$S_{SG} = -\frac{3}{\kappa^2}\int \mathrm{d}^8 z E^{-1}\ . \tag{3.9}$$

That means the pure supergravity action is proportional to the supervolume of the curved superspace. Again we can use the chiral integration rule and get

$$S_{SG} = -\frac{3}{\kappa^2}\int \mathrm{d}^6 z \hat{\phi}^3 \hat{R}, \quad R = e^{\bar{W}}\hat{R}\ . \tag{3.10}$$

This action corresponds to the action in the formalism of W&B. This can most easily be seen if we go to the component level (see next section). Furthermore, at some point it is always necessary to go from the superfield description to the component level, if we want to give statements about our real physical fields. So let me show how we can arrive at our action in component fields if we start with a superfield action. First we have to choose a gauge. We take the usual gauge for ϕ (2.98) as well as the Wess-Zumino-gauge for $\mathscr{H}^m$ (2.97):

$$\mathscr{H}^m(x,\theta,\bar{\theta}) = \theta\sigma^a\bar{\theta}e_a{}^m + i\bar{\theta}^2\theta^\alpha\psi^m{}_\alpha(x) - i\theta^2\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{m\dot{\alpha}}(x) + \theta^2\bar{\theta}^2\tilde{A}^m(x)$$

$$\hat{\phi}^3 = e^{-1}\left(1 - 2i\theta\sigma_a\bar{\psi}^a + \theta^2\tilde{B}\right)$$

Now we can express all superfields in our theory with these two dynamical objects. In order to get the components of all the geometrical objects (the supervierbein and the supertorsions), we use the space projection. The space projection $V|$ of a superfield $V(x,\theta,\bar{\theta})$ is defined as

$$V| \equiv V(x,\theta=0,\bar{\theta}=0)\ . \tag{3.11}$$

So it gives us the zero-order term in the power series expansion of V in $\theta,\bar{\theta}$. Similarly we can define the space projection of differential operators. Therefore we can use the space projection of the covariant derivative of a superfield $\mathscr{D}_\alpha V|$ to get the higher-order terms of the power series expansion of this superfield. Equipped like that, we determine the components of our dynamical objects $\mathscr{H}^m$ and $\hat{\phi}^3$ or E and ϕ. After that we can determine R, G_a and $W_{\alpha\beta\gamma}$.

For example, the space projections of R and G_a are

$$\bar{R}| = \frac{1}{3}\bar{B} \tag{3.12}$$

$$G_a| = \frac{4}{3}A_a \tag{3.13}$$

where we have redefined our auxiliary fields B and A^a as

$$\bar{B} = \bar{\tilde{B}} + \frac{1}{2}\psi^a\sigma_a\bar{\sigma}_b\psi^b + \frac{1}{2}\psi^a\psi_a \tag{3.14}$$

$$A^a = \tilde{A}^a + \frac{1}{8}\epsilon^{abcd}\mathscr{C}_{bcd} - \frac{1}{4}\left(\psi_a\bar{\sigma}_b\bar{\psi}^b + \psi^b\bar{\sigma}_b\bar{\psi}_a\right) - \frac{1}{2}\psi^b\sigma_a\bar{\psi}_b + \frac{i}{8}\epsilon^{abcd}\psi^b\sigma_c\bar{\psi}_d\ . \tag{3.15}$$

Here, the $\mathscr{C}_{bcd}$ are the anholonomy coefficients of the bosonic vielbein

$$[e_a, e_b] = \mathscr{C}_{ab}{}^c e_c\ . \tag{3.16}$$

To eventually get the superfield action in component form, we use [13]

$$\begin{aligned} S_c &= \int \mathrm{d}^8 z (E^{-1}/R)\mathcal{L}_c \\ &= \int \mathrm{d}^4 x\, e^{-1}\left\{-\frac{1}{4}\mathscr{D}^2\mathcal{L}_c| - \frac{i}{2}(\bar{\psi}^a\bar{\sigma}_a)^\alpha\mathscr{D}_\alpha\mathcal{L}_c| \right. \\ &\qquad \left. + \left(B - \frac{1}{2}\bar{\psi}^a\bar{\sigma}_a\sigma_b\bar{\psi}^b - \frac{1}{2}\bar{\psi}^a\bar{\psi}_a\right)\mathcal{L}_c|\right\}\ . \end{aligned} \tag{3.17}$$

For our pure supergravity action we finally arrive at

$$\begin{aligned} S = \frac{1}{\kappa^2}\int \mathrm{d}^4 x e^{-1}\{\ &\frac{1}{2}\mathscr{R}(e,\psi) - \frac{1}{3}\bar{B}B + \frac{4}{3}A^d A_d \\ &+ \frac{1}{4}\epsilon^{abcd}\left[\bar{\psi}_a\bar{\sigma}\left(\tilde{\nabla}_c\psi_d - \tilde{\nabla}_d\psi_c - \tilde{\mathscr{T}}_{cd}{}^e\psi_e\right) + \text{h.c.}\right]\}\ . \end{aligned} \tag{3.18}$$

Here the covariant derivatives $\tilde{\nabla}_a$ are defined from the ordinary space-time derivatives ∇_a by

$$\nabla_a = e_a + \frac{1}{2}\omega_{abc}(e,\psi,A)M^{bc} \tag{3.19}$$

$$\tilde{\nabla}_a = e_a + \frac{1}{2}\tilde{\omega}_{abc}(e,\psi)M^{bc} \tag{3.20}$$

$$\omega_{abc}(e,\psi,A^d) \equiv \tilde{\omega}_{abc}(e,\psi) - \frac{2}{3}\epsilon_{abcd}A^d \ . \tag{3.21}$$

The $\tilde{\mathscr{T}}_{cd}{}^{e}$ is then the corresponding torsion (compare (2.14)). We have used this form of the derivatives because we want to explicitly see the complete A^d dependence in the action. (The A^d correspond to the b^a in the W&B apart from a factor of one half.)

3.2 Getting the Action in Components

Now we want to show that W&B [12] and B&K [13] have to perform exactly the same calculations to get the action in components out of the action in superfields.

The W&B action is

$$\begin{aligned}\mathcal{L} &= \int \mathrm{d}^2\Theta 2\mathcal{E}[-\frac{1}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\tilde{R})\Omega + P] + \text{h.c.} \\ &= \int \mathrm{d}^2\Theta 2\mathcal{E}\mathcal{L}_c + \text{h.c.}\end{aligned} \tag{3.22}$$

with the chiral density

$$\mathcal{E} = \frac{1}{2}\hat{\phi}^3 \tag{3.23}$$

as can be seen from (2.98), (3.14) and [12]

$$2\mathcal{E} = e\left\{1 + i\Theta\sigma^a\bar{\psi}_a - \Theta\Theta[M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b]\right\} \ . \tag{3.24}$$

The quantities Ω and P are called the Kähler potential and the superpotential. Since the superpotential P is a linear combination of chiral superfields it is chiral itself. Before we can go on with the calculation of the components of this action, we have to make some remarks on the definition the W&B uses. First we have to note that the scalar curvature R in the B&K differs

from the $\tilde{R}$ in the W&B by a factor of two. That is the reason for the eight in the chiral projector above. But the most important change are the new Θ_α variables.

The usual θ^μ variables carry Einstein indices and thus are changed by a general coordinate transformation. So it is not very practical to define the component fields of a superfield as the coefficient function of a power series expansion in θ^μ and $\bar{\theta}_{\dot{\mu}}$. Instead the W&B defines the component fields of for example a chiral superfield Φ with the help of the covariant derivative as

$$\begin{aligned}A &= \Phi|_{\theta=\bar{\theta}=0} \\ \chi_\alpha &= \frac{1}{\sqrt{2}}\mathscr{D}_\alpha\Phi|_{\theta=\bar{\theta}=0} \\ F &= -\frac{1}{4}\mathscr{D}^\alpha\mathscr{D}_\alpha\Phi|_{\theta=\bar{\theta}=0} \ .\end{aligned} \tag{3.25}$$

Apart from the factors before the derivative this is quite the same as the space projection defined by B&K. These components are then covariant [12](do not change their form under general coordinate transformations), since they carry Lorentz indices. Thus W&B defines the Θ_α in such a way that their expansion coefficients are exactly the covariant components (3.25). The Θ^2 integration in (3.22) can then be regarded as a projection on the Θ^2-component. Thus we have:

$$\mathcal{L} = 2\mathcal{E}\mathcal{L}_c|_{\Theta^2} \tag{3.26}$$

$$= e\left\{1 + i\Theta\sigma^a\bar{\psi}_a - \Theta\Theta[M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b]\right\}\cdot\left(\mathcal{L}_c| + \Theta\mathscr{D}\mathcal{L}_c| - \frac{1}{4}\Theta^2\mathscr{D}^2\mathcal{L}_c|\right)|_{\Theta^2} \tag{3.27}$$

$$= e\left\{-\frac{1}{4}\Theta^2\mathscr{D}^2\mathcal{L}_c| + i\Theta\sigma^a\bar{\psi}_a\Theta\mathscr{D}\mathcal{L}_c| - \Theta\Theta[M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b]\mathcal{L}_c|\right\}|_{\Theta^2} \tag{3.28}$$

$$= e\left\{-\frac{1}{4}\Theta^2\mathscr{D}^2\mathcal{L}_c| - \frac{i}{2}(\sigma^a\bar{\psi}_a)_\alpha\mathscr{D}^\alpha\mathcal{L}_c|\Theta\Theta - \Theta\Theta[M^* + \bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b]\mathcal{L}_c|\right\}|_{\Theta^2} \tag{3.29}$$

$$= e\left\{-\frac{1}{4}\mathscr{D}^2\mathcal{L}_c| + \frac{i}{2}(\bar{\psi}_a\bar{\sigma}^a)^\alpha\mathscr{D}_\alpha\mathcal{L}_c| - [M^* + \frac{1}{2}\bar{\psi}^a\bar{\sigma}_a\sigma_b\bar{\psi}^b + \frac{1}{2}\bar{\psi}^a\bar{\psi}_a]\mathcal{L}_c|\right\} \ . \tag{3.30}$$

One can see that the last equation is identical with equation (3.17) if one knows that $M^* = -B$ and the gravitino ψ has the opposite sign. And this equation is the one B&K uses to get the component form of the action. For the whole calculation above it is very important that the covariant derivatives $\mathscr{D}_\alpha$ of both books are identical.

3.3 The Flat Limit of the Action in Compensator Form

In this section we want to derive a very useful form of our action. The flat limit of the general supergravity action in the chiral compensator form, which allows us to work with a rather simple action in superfields as long as gravity is weak. This form of the action is known for a long time [36] [35], but we were not able to find a satisfactory derivation in the superfield formalism.

Let us look at our action, equation (3.5)

$$\begin{aligned}\int \mathrm{d}^8 z E^{-1}\mathcal{L} &= -\frac{1}{4}\int \mathrm{d}^8 z \left(\frac{E^{-1}}{R}\right)\left(\bar{\mathscr{D}}^2 - 4R\right)\mathcal{L} \\ &= -\frac{1}{4}\int \mathrm{d}^6 z \hat{\phi}^3 e^{-\bar{W}}\left[\left(\bar{\mathscr{D}}^2 - 4R\right)\mathcal{L}\right] \\ &= -\frac{1}{4}\int \mathrm{d}^6 z \hat{\phi}^3 e^{-\bar{W}}\mathcal{L}_c \,.\end{aligned}$$

The general ansatz for the Lagrangian in supergravity is

$$\mathcal{L} = K(Q, Q^*) + \frac{1}{R}P(Q)\,. \tag{3.31}$$

The superfield Q is chiral and we have

$$K = b + c_i Q_i + c_i^* Q_i^* + h_{ij} Q_i Q_j^* + ... \tag{3.32}$$

$$P = d + a_i Q_i + \frac{1}{2} m_{ij} Q_i Q_j + \frac{1}{3} g_{ijk} Q_i Q_j Q_k + ... \tag{3.33}$$

Therefore P is chiral as well. We call P the superpotential and K the Kähler potential. If we plug these superpotentials into our action we get

$$S = \int \mathrm{d}^8 z E^{-1}\mathcal{L} = \int \mathrm{d}^8 z E^{-1}(K + \frac{1}{R}P) \tag{3.34}$$

$$= \int \mathrm{d}^8 z E^{-1} K + \int \mathrm{d}^6 z \hat{\phi}^3 e^{-\bar{W}} P\,. \tag{3.35}$$

Further we can show with the help of (2.86) and (2.88)

$$E^{-1} = \phi\bar{\phi}(1 \cdot e^{\overleftarrow{W}})^{\frac{1}{3}}(1 \cdot e^{\overleftarrow{\bar{W}}})^{\frac{1}{3}}\hat{E}^{-\frac{1}{3}}\,, \tag{3.36}$$

which gives us the final result for the action (compare with [35] [36]):

$$S = \int \mathrm{d}^8 z \phi\bar{\phi}(1 \cdot e^{\overleftarrow{W}})^{\frac{1}{3}}(1 \cdot e^{\overleftarrow{\bar{W}}})^{\frac{1}{3}}\hat{E}^{-\frac{1}{3}}K + \int \mathrm{d}^6 z \hat{\phi}^3 \hat{P}\,. \tag{3.37}$$

It is important to recognise that $\hat{\phi}^3$ is naively chiral and likewise is $\hat{P} = e^{-\bar{W}}P$, so that the superpotential part of the action has the same simple structure as in global supersymmetry. The operator $e^{\bar{W}}$ acts on $\hat{\phi}^3$ and $\hat{P}$ like the transition from the x into the y basis in normal superfields. But in our case the coordinate transformation consists not only of the gravitational part $i\Theta\sigma^m\bar{\Theta}$ but also of a spinor term and a term containing the auxiliary field A (B&K) or b (W&B).

If we want to bring the first part of the action in the same form we have to write

$$\phi\bar{\phi} = \hat{\phi}\bar{\hat{\phi}}e^{\bar{W}}e^{W}\,. \tag{3.38}$$

In the end we have to calculate

$$e^{\bar{W}}e^{W}(1 \cdot e^{\overleftarrow{W}})^{\frac{1}{3}}(1 \cdot e^{\overleftarrow{\bar{W}}})^{\frac{1}{3}}\hat{E}^{-\frac{1}{3}}\,, \tag{3.39}$$

where the result in the lowest order should be $e^{-\frac{1}{3}}$.

In the Wess-Zumino gauge the gravitational superfield $\mathcal{H}^m$ of Einstein Supergravity is

$$\mathscr{H}^m = \theta\sigma^a\bar{\theta}e_a{}^m + i\bar{\theta}^2\theta^\alpha\psi^m{}_\alpha - i\theta^2\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{m\dot{\alpha}} + \theta^2\bar{\theta}^2 A^m\,. \tag{3.40}$$

Now we want to go to the flat limit. In this limit we can ignore the graviton $\psi^m{}_\alpha$ and $e_a{}^m$ is set to $\delta_a{}^m$.

For the derivatives of $\mathscr{H}^m$ we therefore get:

$$\frac{\partial\mathscr{H}^m}{\partial x^n} = \theta^2\bar{\theta}^2(\partial_n A^m) \tag{3.41}$$

$$\partial_\alpha\mathscr{H}^m = (\sigma^m\bar{\theta})_\alpha + 2\theta_\alpha\bar{\theta}^2 A^m \tag{3.42}$$

$$\bar{\partial}_{\dot{\alpha}}\mathscr{H}^m = -(\theta\sigma^m)_{\dot{\alpha}} - 2\theta^2\bar{\theta}_{\dot{\alpha}}A^m\,. \tag{3.43}$$

Using (2.56) and (2.91) results in

$$(1 + i\partial\mathscr{H})_n{}^m = \delta_n{}^m + i\theta^2\bar{\theta}^2(\partial_n A^m) \tag{3.44}$$
$$\hat{E}_\alpha = \partial_\alpha + i[(\sigma^m\bar{\theta})_\alpha + 2\theta_\alpha\bar{\theta}^2 A^m]\partial_m \tag{3.45}$$
$$\hat{\bar{E}}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i[(\theta\sigma^m)_{\dot{\alpha}} + 2\theta^2\bar{\theta}_{\dot{\alpha}} A^m]\partial_m \tag{3.46}$$

and therefore

$$\{\hat{E}_\alpha, \hat{\bar{E}}_{\dot{\alpha}}\} = -2i[(\sigma^m)_{\alpha\dot{\alpha}} + 4\theta_\alpha\bar{\theta}_{\dot{\alpha}} A^m]\partial_m \,. \tag{3.47}$$

This gives us together with equation (2.64)

$$\hat{E}_a{}^m\partial_m = -\frac{i}{4}(\bar{\sigma})^{\dot{\alpha}\alpha}\{\hat{E}_\alpha, \hat{\bar{E}}_{\dot{\alpha}}\} \tag{3.48}$$
$$= [\delta_a{}^m - 2(\theta\sigma_a\bar{\theta})A^m]\partial_m \,. \tag{3.49}$$

In principle we could now calculate

$$\hat{E} = \mathrm{sdet}(\hat{E}_A{}^M) = \det(\hat{E}_a{}^m) = \det(\mathbb{1} + \mathbb{X}) \tag{3.50}$$
$$\text{with } \mathbb{X}_a{}^m = -2(\theta\sigma_a\bar{\theta})A^m \,. \tag{3.51}$$

But instead we want to expand this in a power series in $\mathbb{X}_a{}^m$:

$$\hat{E}^{-1/3} = (\det\hat{E}_a{}^m)^{-1/3} = \exp\left[-\frac{1}{3}\log\det\hat{E}_a{}^m\right] \tag{3.52}$$
$$= \exp\left[-\frac{1}{3}\mathrm{tr}\,\log(\mathbb{1} + \mathbb{X})\right] = \exp\left[-\frac{1}{3}\mathrm{tr}(\mathbb{X} - \frac{1}{2}\mathbb{X}^2)\right] \tag{3.53}$$

with

$$\mathrm{tr}\mathbb{X} = -2(\theta\sigma_m\bar{\theta})A^m \equiv x \quad \mathrm{tr}\mathbb{X}^2 = x^2 \,. \tag{3.54}$$

This power series terminates after the second term and we get

$$\hat{E}^{-1/3} = 1 + \frac{2}{3}(\theta\sigma_m\bar{\theta})A^m - \frac{4}{9}\theta^2\bar{\theta}^2 A^2 \,. \tag{3.55}$$

Similarly we get

$$(1 \cdot e^{\overline{W}})^{1/3} = 1 - \frac{1}{3}i\theta^2\bar{\theta}^2(\partial A) \tag{3.56}$$

and thus

$$(1 \cdot e^{\overline{W}})^{1/3}(1 \cdot e^{\overline{W}})^{1/3} = (1 + \frac{1}{3}i\theta^2\bar{\theta}^2(\partial A))(1 - \frac{1}{3}i\theta^2\bar{\theta}^2(\partial A)) = 1 \,. \tag{3.57}$$

Now we have everything we need to calculate the flat limit of the supergravity action. For the chiral compensator we choose:

$$\hat{\phi}(x,\theta) = A_\phi(x) + \sqrt{2}\theta\psi_\phi(x) + \theta^2 F_\phi(x) \tag{3.58}$$
$$\hat{\phi}(x,\theta,\bar{\theta}) = A_\phi(x + i\mathscr{H}) + \sqrt{2}\theta\psi_\phi(x + i\mathscr{H}) + \theta^2 F_\phi(x + i\mathscr{H}) \tag{3.59}$$

with

$$\mathscr{H}^m = \theta\sigma^m\bar{\theta} + \theta^2\bar{\theta}^2 A^m = \mathscr{H}_0^m + \theta^2\bar{\theta}^2 A^m \,. \tag{3.60}$$

Then the result for the Kähler part of the action is

$$\int \mathrm{d}^4x \int \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\left(1 + \frac{2}{3}(\theta\sigma_m\bar{\theta})A^m - \frac{4}{9}\theta^2\bar{\theta}^2 A^2\right)$$
$$\cdot\,\phi(x + i\mathscr{H}, \theta)\overline{\phi(x + i\mathscr{H}, \theta)}K(Q(x + i\mathscr{H}, \theta)\overline{Q(x + i\mathscr{H}, \theta)}) \tag{3.61}$$
$$= \int \mathrm{d}^4x\mathcal{L}$$

with (discarding spinors)

$$\mathcal{L} = \mathcal{L}_{flat} + \frac{2}{3}(\theta\sigma_m\bar{\theta})A^m\left[A_\phi\bar{A}_\phi\frac{\partial K}{\partial Q}(\partial_n A_Q)i\theta\sigma^n\bar{\theta} + \bar{A}_\phi\partial_n A_\phi i\theta\sigma^n\bar{\theta}K + \mathrm{h.c.}\right]\Big|_{\theta^2\bar{\theta}^2}$$
$$-\frac{4}{9}A^2|A_\phi|^2K + \left[A_\phi\bar{A}_\phi\frac{\partial K}{\partial Q}(\partial_n A_Q)i\theta^2\bar{\theta}^2 A^n + \bar{A}_\phi\partial_n A_\phi i\theta^2\bar{\theta}^2 A^n K + \mathrm{h.c.}\right]\Big|_{\theta^2\bar{\theta}^2} \,. \tag{3.62}$$

Partially integrating the last term and using the equation [12]

$$\theta\sigma^m\bar{\theta}\theta\sigma^n\bar{\theta} = -\frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\eta^{mn} \tag{3.63}$$

we finally get

$$\mathcal{L} = \mathcal{L}_{flat} - \frac{4}{9}A^2|A_\phi|^2K + \frac{2}{3}i(A^n\partial_n A_Q|A_\phi|^2\frac{\partial K}{\partial Q} + A^n\partial_n A_\phi\bar{A}_\phi K - \mathrm{h.c.}) \tag{3.64}$$

where

$$\mathcal{L}_{flat} = \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \overline{\phi(x + i\mathscr{H}_0, \theta)} \phi(x + i\mathscr{H}_0, \theta) K\left(\overline{Q(x + i\mathscr{H}_0, \theta)}, Q(x + i\mathscr{H}_0, \theta)\right) . \tag{3.65}$$

At this point we should remark that this flat action is only flat with respect to the spacetime coordinates. The superspace part is still curved. If we want the superspace part to be flat as well, we have to gauge the chiral compensator to one. Thus a VEV of the auxiliary field indicates that supersymmetry is not flat, i.e. broken.

Integrating out the auxiliary fields the bosonic part of $\mathcal{L}_{flat}$ is

$$\begin{aligned} -|A_\phi|^2 (\partial_n A_Q) i\theta\sigma^n\bar{\theta} (\partial^m A_Q^*) i\theta\sigma_m\bar{\theta} \left.\frac{\partial^2 K}{\partial Q \partial Q^*}\right|_{\theta^2\bar{\theta}^2} + \text{ h.c.} \\ = -|A_\phi|^2 \partial^n A_Q^* \partial_n A_Q \frac{\partial^2 K}{\partial Q \partial Q^*} . \end{aligned} \tag{3.66}$$

This is the usual kinetic term in supergravity. Since $\mathscr{H} = \bar{\mathscr{H}}$ in the Wess-Zumino gauge, we see from equation (3.60) that $A_n = A_n^*$. Taking into account that $A_\phi = \text{const.}$ (see eq. (2.98)) the equation of motion for A_m is

$$A_n = i\frac{3}{4K}\left(\partial_n A_Q \frac{\partial K}{\partial Q} - \text{h.c.}\right) . \tag{3.67}$$

Plugging this back into (3.64) we get for the bosonic part of $\mathcal{L}$:

$$\begin{aligned} \mathcal{L} = &-|A_\phi|^2 \partial^n A_Q^* \partial_n A_Q \frac{\partial^2 K}{\partial Q \partial Q^*} \\ &- \frac{|A_\phi|^2}{4K}\left(\partial^n A_Q \frac{\partial K}{\partial Q} - \text{ h.c.}\right)\left(\partial_n A_Q \frac{\partial K}{\partial Q} - \text{ h.c.}\right) . \end{aligned} \tag{3.68}$$

Comparing this equation to the four-dimensional supergravity action Luty and Sundrum use [30] (equation (2.7)), we see that we reproduce their kinetic terms.

3.4 Short Review of the Results in Components

From the previous sections we know that the calculations in W&B are identical to our approach. Thus we want to give a few results of W&B for further

reference. Since we are interested in supersymmetry breaking and the cosmological constant problem, our main interest lies in the scalar potential V. In order to compare our results with those from W&B, we will first have a look at the nomenclature. Then we can bring the scalar potential in a form, which can be compared with our later results.

If we make the general ansatz for the Lagrangian as above and try to get the component fields we will get a very complicated expression. That does not change after we have integrated out the auxilliary fields. To simplify this, we have to redefine some of our fields. Furthermore, we have to normalise our physical fields in such a way that the gravitational part of the action, which depends on $\mathscr{R}$, becomes

$$-\frac{1}{2} e \mathscr{R} . \tag{3.69}$$

Therefore we have to perform a Weyl rescaling of the gravitational field:

$$e_n{}^a \to e_n{}^a e^\lambda \text{ with } e^{2\lambda} = -\frac{3}{K} \tag{3.70}$$

This will change the normalisation of the matter fields, which can be restored by a redefinition of the spinors and an additional shift of the gravitino. However this does not affect the scalar potential. For the scalar potential we get

$$V = e^\Omega [\Omega^{ij^*} (D_i P)(D_j P)^* - 3P^* P] . \tag{3.71}$$

Note at this point, that in our notation Ω and K have exactly the opposite definition than in W&B. Here we used the definitions

$$\Omega = -3 \log\left(-\frac{K}{3}\right) \quad \Omega^{ij^*} = \Omega_{ij^*}^{-1} \tag{3.72}$$

$$\text{and } D_i P = P_i + \Omega_i P , \tag{3.73}$$

where the subscripts on Ω and P denote derivatives with respect to the scalar field of the superfields Q and the superpotentials are the same as before, except that the superfields are replaced by their lowest components A_Q, i.e.

$$\Omega_{ij^*} = \frac{\partial^2 \Omega(A_Q, A_Q^*)}{\partial Q_i \, \partial Q_j^*} \quad K_{ii^*} = \frac{\partial^2 K(A_Q, A_Q^*)}{\partial Q_i \, \partial Q_j^*} \qquad P_i = \frac{\partial P(A)}{\partial Q_i} , \text{ etc.} \tag{3.74}$$

In terms of the old Kähler potential the scalar potential reads

$$V = \frac{9}{K^2(KK_{ij^*} - K_iK_{j^*})}[KP_iP_{j^*} - 3K_iPP_{j^*} - 3K_{j^*}P^*P_i + 9K_{ij^*}PP^*] \,. \tag{3.75}$$

In global supersymmetry a non-vanishing scalar potential is equivalent to supersymmetry breaking. This is not true for supergravity. Firstly, we can see from (3.71) that a negative scalar potential is possible, in contrast to global supersymmetry, where the scalar potential is the sum over the square of all auxiliary fields (F and D). Then we would have an Anti de Sitter space(AdS), which allows unbroken supersymmetry. Furthermore, since supergravity should give us Einstein's equations, we would expect the scalar potential und thus the cosmological constant to be zero in order to get a flat space solution. However, it is possible (as we will see in the next chapter) to get both a vanishing cosmological constant and an F-term VEV which will break supersymmetry.

In the next section we will use the flat limit compensator action, to derive the scalar potential. The result has to be identical with (3.75), if our flat action is correct.

Chapter 4

Supersymmetry Breaking

4.1 Spontaneous Supersymmetry Breaking in the Flat Limit

Now we want to analyse how we can establish supersymmetry breaking in our scenario. Further we would like our cosmological constant to vanish. To do so, we first have to know which of our component fields gets a VEV and how it depends on the superpotentials. We again consider the (flat) supergravity Lagrangian $L = \phi^+\phi K(Q, Q^*)|_{\Theta^4} + \phi^3 P(Q)|_{\Theta^2}$ with the chiral compensator ϕ. For the sake of simplicity we consider only one chiral superfield Q although more could in principle be allowed. Possible fields for a VEV we have to consider are A_ϕ, A_Q, F_ϕ, F_Q. The superpotential is of the form

$$P(Q) = p + p_iQ + \frac{1}{2}p_{ii}Q^2 + \frac{1}{6}p_{iii}Q^3 + ... \tag{4.1}$$

while $K(Q, Q^*)$ is the Kähler potential

$$K(Q, Q^*) = k + k_iQ + k_{i^*}Q^* + k_{ii^*}QQ^* + ... \tag{4.2}$$

with

$$p_i = P_i(A_Q)|_{A_Q=0} \quad k_i = K_i(A_Q, A_Q^*)|_{A_Q=A_Q^*=0} \quad \text{, etc.} \tag{4.3}$$

Spinors cannot get a VEV, since this would violate Lorentz invariance. Thus we can drop the corresponding parts. Likewise we can drop all derivatives. Therefore we get in components:

$$\begin{gathered} \phi = A_\phi + F_\phi\theta^2 \Rightarrow \phi^3 = A_\phi^3 + 3A_\phi^2F_\phi\theta^2 \\ Q = A_Q + F_Q\theta^2 \end{gathered} \tag{4.4}$$

(Higher terms in θ are zero because Grassman variables anticommutate.) This results in

$$\begin{aligned} L = {} & A_\phi A_\phi^* F_Q F_Q^* K_{ii^*} + A_\phi F_\phi^* F_Q K_i + A_\phi^* F_\phi F_Q^* K_{i^*} \\ & + F_\phi F_\phi^* K + A_\phi^3 F_Q P_i + 3 A_\phi^2 F_\phi P \,, \end{aligned} \tag{4.5}$$

where the K_{ii*} and P_i are defined as before in (3.74). After that we have to vary the Lagrangian with respect to the component fields if we want to find a consistent vacuum:

$$\delta_{A_\phi} L = \left[A_\phi^* F_Q F_Q^* K_{ii^*} + F_\phi^* F_Q K_i + 3 A_\phi^2 F_Q P_i + 6 A_\phi F_\phi P \right] \tag{4.6}$$

$$\delta_{F_\phi} L = \left[A_\phi^* F_Q^* K_{i^*} + F_\phi^* K + 3 A_\phi^2 P \right] \tag{4.7}$$

$$\delta_{F_Q} L = \left[A_\phi A_\phi^* F_Q^* K_{ii^*} + A_\phi F_\phi^* K_i + A_\phi^3 P_i \right] \tag{4.8}$$

$$\begin{aligned} \delta_{A_Q} L = {} & [\, A_\phi A_\phi^* F_Q F_Q^* K_{iii^*} + A_\phi F_\phi^* F_Q K_{ii} + A_\phi^* F_\phi F_Q^* K_{ii^*} \\ & + F_\phi F_\phi^* K_i + A_\phi^3 F_Q P_{ii} + 3 A_\phi^2 F_\phi P_i \,] \end{aligned} \tag{4.9}$$

These expressions have to vanish in the vacuum. If we consider the simplest example with $P = \text{const.} = P^*$, $A_\phi = 1$ and $K = k_i Q + k_{i^*} Q^*$ we get:

$$\delta_{A_\phi} L = \left[k_i F_\phi^* + 6 p F_\phi \right] \tag{4.10}$$

$$\delta_{F_\phi} L = \left[k_{i^*} F_Q^* + F_\phi^* K + 3 p \right] \tag{4.11}$$

$$\delta_{F_Q} L = \left[k_i F_\phi^* \right] \tag{4.12}$$

$$\delta_A L = \left[k_i F_\phi F_\phi^* \right] \tag{4.13}$$

From these equations we immediately get $\langle F_\phi^* \rangle = 0 = \langle F_\phi \rangle$. The second equation then leads to the important result

$$\langle F_Q^* \rangle = -\frac{3p}{k_{i^*}} \,. \tag{4.14}$$

This is an indicator for spontaneous supersymmetry breaking, in this case F-term breaking. Further we can immediately see from (3.75) that the cosmological constant vanishes for our choice of the potentials K and P.

Now we try to perform the same calculations for a more general superpotential

$$P(Q) = p + p_i Q + \frac{1}{2} p_{ii} Q^2 + \frac{1}{6} p_{iii} Q^3 \tag{4.15}$$

and the Kähler potential

$$K(Q, Q^*) = k + k_i Q + k_{i^*} Q^* + k_{ii^*} Q Q^* \,. \tag{4.16}$$

Then we get from (4.7)

$$\langle F_\phi^* \rangle = -\frac{\left(K \langle F_Q^* \rangle + 3P \right)}{K_{i^*}} \,, \tag{4.17}$$

where we have chosen $A_\phi = 1$. Plugging this back into equation (4.8) gives us

$$\langle F_\phi^* \rangle = \frac{3 P K_{ii^*} - K_{i^*} P_i}{K_i K_{i^*} - K K_{ii^*}} \,, \tag{4.18}$$

which together with the previous equation leads to

$$\langle F_Q^* \rangle = \frac{K P_i - 3 P K_i}{K_i K_{i^*} - K K_{ii^*}} \,. \tag{4.19}$$

At this point, it is important to note that we see from these equations that both auxiliary fields F get a VEV only, if we have a non vanishing superpotential $P(Q)$. From (4.19) we can see that the condition which indicates supersymmetry breaking is $\langle K P_i - 3 P K_i \rangle \neq 0$. This is equivalent to $\langle D_i P \rangle \neq 0$, the condition the W&B gives.

Using these equations of motion (4.18) and (4.19) in our Lagrangian (eq. (4.5)) results in the scalar potential

$$V = \frac{1}{K_i K_{i^*} - K K_{ii^*}} \left[K P_i P_i^* - 3 K_i P P_i^* - 3 K_{i^*} P^* P_i + 9 K_{ii^*} P P^* \right] \,. \tag{4.20}$$

We see that this is identical with equation (3.75) if we take into account that we have done a Weyl rescaling before we arrived at (3.75). In front of the scalar potential we have a factor e in the Lagrangian [12]. Together with the Weyl rescaling (3.70) this gives rise to a factor $\exp(4\lambda) = 9/K^2$, thus (4.20) is equivalent to (3.75) before the Weyl rescaling.

If we demand $P_i \neq 0$ we see from equations (4.6) and (4.8)

$$\langle F_Q \rangle = -\frac{3P}{P_i} \langle F_\phi \rangle \,. \tag{4.21}$$

This indicates that a VEV of F_ϕ directly leads to a VEV of F_Q and in this sense implies F-term supersymmetry breaking.

Using (4.18) and (4.19) in the equation above gives us

$$KP_iP_{i^*} - 3K_{i^*}P^*P_i = -9PP^*K_{ii^*} + 3PP_i^*K_i \,. \tag{4.22}$$

Since we used (4.6) to get this result, we see that the cosmoloical constant always vanishes if we demand $\delta_{A_\phi} L = 0$.

Assuming $P_{ii} \neq 0$ in addition to $P_i \neq 0$, equations (4.8) and (4.9) lead to

$$\langle F_Q \rangle = -\frac{2P_i}{P_{ii}} \langle F_\phi \rangle \,. \tag{4.23}$$

Together with (4.21) we get the condition

$$\frac{3P}{P_i} = \frac{2P_i}{P_{ii}} \,. \tag{4.24}$$

This is equivalent to the equation we get if we require that the scalar potential is extremised, i.e. $\langle \partial V / \partial A_Q^i \rangle = 0$.

4.2 No-Scale Supergravity

Looking at our scalar potential (3.71) we see that one way to get a vanishing cosmological constant is to set $\langle e^\Omega \rangle$ to zero which means that $|\langle K \rangle|$ would go to infinity (see eq. (3.72)). This, however, leads to the phenomenologically unacceptable case of unbroken supergravity, since $\langle F_\phi \rangle$ and thus $\langle F_Q \rangle$ would go to zero (see equations (4.18), (4.19) and (4.21)). So, if we want supergravity to be broken, we have to arrange our potentials K and P in such a way that the square bracket in (3.75) vanishes for the VEVs of our fields. This requires a lot of finetuning, which is not desirable. Instead we could demand the square bracket to vanish identically, i.e., for any value of the fields. If we could find fields satisfying this demand we would get a naturally vanishing (flat) scalar potential, without losing the possibility of broken supergravity. It is easy to see from (3.75), that the easiest of such solutions is a constant superpotential P together with a Kähler potential of the form

$$K = Q + Q^* \,. \tag{4.25}$$

That is exactly what we have done in our simple example (4.10) above. Note however, that we can use a Kähler-Weyl transformation to write (4.25) in a more familiar way. It is possible to show that the supergravity Lagrangian is invariant under the Kähler-Weyl transformation

$$\Omega(Q, Q^+) \to \Omega(Q, Q^+) + 6\Sigma(Q) + 6\Sigma^+(Q^+) \tag{4.26}$$

$$P \to e^{-6\Sigma} P \,. \tag{4.27}$$

This can be used to rescale the superpotential P to one (as long as $\langle P \rangle \neq 0$). In order to achieve this, we have to choose $6\Sigma = \log P$. Then the new Kähler potential, which we will call G, becomes

$$G \equiv \Omega + \log P + \log P^* \,. \tag{4.28}$$

Then G is in some sense the only potential. This changes our scalar potential to

$$V = e^G \left[g^{ij^*} G_i G_{j^*} - 3 \right] \,, \tag{4.29}$$

where g^{ij^*} is the inverse of the Kähler metric

$$g_{ij^*} = \frac{\partial^2 G(A_Q, A_Q^*)}{\partial A_Q^i \, \partial A_Q^{*j}} \,. \tag{4.30}$$

We can now set this new scalar potential to zero (vanishing cosmological constant) and regard the identity

$$\left[g^{ij^*} G_i G_{j^*} - 3 \right] = 0 \tag{4.31}$$

to be satisfied by G. Then the solution has the famous form [42]

$$G = -3 \log(Q + Q^*) \,. \tag{4.32}$$

With equation (3.72) we get

$$G = \log \left(- \left(\frac{K}{3} \right)^{-3} PP^* \right) \tag{4.33}$$

which together with (4.32) results in the identity

$$K = 3(Q + Q^*) \, \sqrt[3]{PP^*} \,. \tag{4.34}$$

As we have argued above, we can always use the Kähler-Weyl invariance to choose P to be one. So we have recovered the solution (4.25). The factor of three is just the usual convention.

Supergravity breaking is tantamount to a non-vanishing gravitino mass $m_{3/2}$. Looking into the Lagrangian in components (which we have not given here) [12] we could see that the gravitino mass is given by $m_{3/2} = e^{\langle G/2 \rangle}$. That means if we demand

$$\left\langle g^{ij^*} G_i G_{j^*} \right\rangle = 3 \tag{4.35}$$

instead of (4.31) we get a condition for the VEVs of the fields contained in G. Therefore, we have to finetune our fields if we want to get a certain $m_{3/2}$, which is the scale of supersymmetry breaking. However, that is not the case for our approach here, where we get:

$$m_{3/2} = \frac{1}{(\langle Q + Q^* \rangle^{3/2})} \, . \tag{4.36}$$

Since only the form of our potential is given, but not the VEVs, this expression is non-zero and undetermined. In this sense the supergravity breaking scale is undetermined and thus we can rightfully speak of 'No-Scale supergravity'.

Chapter 5

Application to 'Almost No-Scale Supergravity'

Since No-Scale supergravity has some very convenient properties, it seems natural to ask what will happen if we perturb around this solution. Therefore we want to have a look at a set of superpotentials K and P which have a small deviation from the no-scale form. We will mainly follow a paper of Luty and Okada [39], where the consequences for the radion stabilisation in an orbifold model are analysed.

5.1 Basic Model and Derivation of the Scalar Potential

As in the Luty-Okada paper [39] we consider 5-dimensional supergravity compactified on an $S^1/\mathbb{Z}^2$ orbifold. Furthermore, we will assume that there are constant superpotentials c_1 and c_0 localised on both orbifold fixed points. For simplicity we set $c_1 = 0$ and $c_0 = c \neq 0$. As shown in [44], it is possible to see from the Kaluza-Klein spectrum that c is responsible for the mass difference between $m_{3/2}$ and the graviton. Following Luty and Okada [39] we therefore demand $c/M_5^3 \ll 1$ in order to get a nearly supersymmetric spectrum. Furthermore we want to assume that we have an approximately flat bulk metric. To obtain a 4-dimensional effective theory we have to integrate out the Kaluza-Klein modes of the 5-dimensional SUGRA multiplet at the scale r. Then the 4-dimensional effective Lagrangian is [30]

$$\begin{aligned}\mathscr{L}_4 = &\int \mathrm{d}^4\theta\phi^+\phi\left[-3M_5^3(T+T^+)+\Delta K(T^+,T)\right]\\ &+\left(\int \mathrm{d}^2\theta\phi^3\left[c+\Delta P(T)\right]+\text{h.c.}\right)\,,\end{aligned} \tag{5.1}$$

where Δ indicates the small deviation from the no-scale superpotentials and $T=\pi r+...$ is the radion chiral superfield. For the chiral compensator we have $\phi = 1+\theta^2 F_\phi$.

Let us first consider the exact No-Scale Lagrangian. From equation (4.14) we see

$$\langle F_T\rangle = \frac{c^*}{M_5^3} \text{ and } \langle F_\phi\rangle = 0\,, \tag{5.2}$$

which means that SUSY is broken spontaneously by the F-term of the radion. In the 4-dimensional effective theory the condition $c/M_5^3 \ll 1$ becomes $\langle F_T\rangle \ll 1$.

Now we want to calculate what happens to the scalar potential and the VEVs of our auxiliary fields if we include the deviations from the No-Scale potentials. Then the Lagrangian of the auxiliary fields is

$$\begin{aligned}\mathscr{L}_{\text{Aux}} = &-3M_5^3(F_\phi F_\phi^+[A_T+A_T^+]+F_\phi^+F_T+F_\phi F_T^+)+3F_\phi c+3F_\phi^+c^*\\ &+\Delta K_{TT^+}F_TF_T^+ +\Delta K_{T^+}F_\phi F_T^+ +\Delta K_T F_\phi^+F_T+\Delta K(T,T^+)F_\phi F_\phi^+\\ &+\Delta P_T F_T+\Delta P_T^+F_T^+ +3\Delta P(T)F_\phi+3\Delta P^+(T)F_\phi^+\,.\end{aligned} \tag{5.3}$$

To get the scalar potential we have to integrate out the auxiliary fields like we did in Section 4.1. Therefore, we first have to vary the Lagrangian with respect to the auxiliary fields and their Hermitean conjugates:

$$\delta_{F_T}L = -3M_5^3F_\phi^+ +\Delta K_{TT^+}F_T^+ +\Delta K_TF_\phi^+ +\Delta P_T = 0 \tag{5.4}$$

$$\delta_{F_T^+}L = -3M_5^3F_\phi +\Delta K_{TT^+}F_T +\Delta K_{T^+}F_\phi +\Delta P_{T^+} = 0 \tag{5.5}$$

$$\begin{aligned}\delta_{F_\phi}L = &-3M_5^3(F_\phi^+[A_T+A_T^+]+F_T^+)+3c+\Delta K_{T^+}F_T^+\\ &+\Delta K(T,T^+)F_\phi^+ +3\Delta P(T)=0\end{aligned} \tag{5.6}$$

$$\begin{aligned}\delta_{F_\phi^+}L = &-3M_5^3(F_\phi[A_T+A_T^+]+F_T)+3c^*+\Delta K_TF_T\\ &+\Delta K(T,T^+)F_\phi+3\Delta P^+(T)=0\end{aligned} \tag{5.7}$$

From the first equation we get

$$F_T^+ = \frac{(3M_5^3-\Delta K_T)F_\phi^+ -\Delta P_T}{\Delta K_{TT^+}}\,, \tag{5.8}$$

while the third equation gives us

$$F_T^+ = \frac{\left(3M_5^3[A_T+A_T^+]-\Delta K(T,T^+)\right)F_\phi^+ -3c-3\Delta P(T)}{\Delta K_{T^+}-3M_5^3}\,. \tag{5.9}$$

Equating them results in

$$F_\phi^+ = \frac{1}{B}\left(3M_5^3\Delta P_T-\Delta K_{T^+}\Delta P_T+3\Delta K_{TT^+}c+3\Delta P(T)\Delta K_{TT^+}\right) \tag{5.10}$$

with the denominator

$$\begin{aligned}B = &9M_5^6-3M_5^3\Delta K_T-3M_5^3\Delta K_{T^+}-\Delta K_T\Delta K_{T^+}\\ &+3M_5^3[A_T+A_T^+]\Delta K_{TT^+}-\Delta K_{TT^+}\Delta K(T,T^+)\,.\end{aligned} \tag{5.11}$$

Plugging this back into (5.8) leads to

$$\begin{aligned}F_T^+ = \frac{1}{B}\big(&9M_5^3c+9M_5^3\Delta P(T)-3M_5^3[A_T+A_T^+]\Delta P_T\\ &-3c\Delta K_T-3\Delta P(T)\Delta K_T+\Delta P_T\Delta K(T,T^+)\big)\,.\end{aligned} \tag{5.12}$$

Neglecting higher order pertubative terms, i.e. terms with two or more Δ, and taking into account that $\Delta K(T,T^+)\ll c/M_5 \ll M_5^2$ and $\Delta P(T)\ll c\ll M_5^3$, we get the following solution

$$F_\phi^+ = \frac{1}{3M_5^6}\left(\Delta K_{TT^+}c+M_5^3\Delta P_T\right) \tag{5.13}$$

$$\begin{aligned}F_T^+ = \frac{1}{3M_5^6}(&3M_5^3c+3M_5^3\Delta P(T)+M_5^3\Delta P_T[A_T+A_T^+]\\ &-c[A_T+A_T^+]\Delta K_{TT^+}+c\Delta K_{T^+})\,.\end{aligned} \tag{5.14}$$

We see that if we ignore the perturbative terms, i.e. the terms with Δ, we have exactly the same VEVs as in the No-Scale case.

Now we want to calculate the scalar potential up to linear order in perturbation theoty. The exact No-Scale potential has a minimum at $V=0$.

Therefore, linear pertubations of the fields will lead to quadratic pertubations of the first line of (5.3). Thus we have to use only the unperturbed parts of F_ϕ and F_T in this line, so that the first line of (5.3) does not contribute to the pertubation up to linear order. The other parts of $\mathscr{L}_{\text{Aux}}$ already contain a linear pertubation, so that we again have to use the unperturbed VEVs for F_ϕ and F_T. Thus, since we have $\langle F_\phi \rangle = 0$ in the No-Scale case only terms without F_ϕ can contribute to the scalar potential. Therefore, plugging the exact No-Scale VEVs of F_T (5.2) back into the Lagrangian gives us the scalar potential

$$V = -\frac{|c|^2 \Delta K_{TT^+}}{M_5^6} - \left(\frac{c^* \Delta P_T}{M_5^3} + \text{h.c.} \right) . \tag{5.15}$$

This scalar potential ought to have a stable minimum. There is no reason for the Kähler and the superpotential contribution to the potential to be of the same size, so we expect one of them to dominate. If the contribution of the superpotential ΔP dominates it is not possible to get a stable minimum, since $c^* \Delta P_T$ is a holomorphic function of T and thus we can use the Chauchy-Riemann differential equations to show that the second order derivatives of its real part with respect to the imaginary and the real part of T can not have the same sign. Therefore, we will from now on assume the Kähler contribution to dominate. Then there exists a stable minimum of the scalar potential if ΔK_{TT^+} has a local maximum. In order to cancel the cosmological constant we get from this, we have to add an additional source of supersymmetry breaking. Since we want to avoid contact terms between this additional field and our visible sector, we assume the additional field to be on the hidden brane.

5.2 Cancelation of the Cosmological Constant by Additional Supersymmetry Breaking Sector

If we assume that our additional susy breaking field S is localised on the brane and therefore has no contact terms to the radion T we get the effective Lagrangian

$$\begin{aligned} \mathscr{L}_4 = &\int \mathrm{d}^4\phi^+\phi \left[-3M_5^3 (T + T^+) + \Delta K(T^+, T) + \Delta K(S, S^+) \right] \\ &+ \left(\int \mathrm{d}^2\theta \phi^3 \left[c + \Delta P(T) + \Delta P(S) \right] + \text{h.c.} \right) . \end{aligned} \tag{5.16}$$

If we want to vary the Lagrangian of the auxiliary fields with respect to F_S, we get equations similar to the equations for F_T in our previous calculation, with the difference that for F_S the term proportional to $-3M_5^3$ is missing. This results in

$$F_\phi = -\frac{\Delta P_S^+ + \Delta K_{SS^+} F_S}{\Delta K_{S^+}} \tag{5.17}$$

$$F_S = -\frac{\Delta P_S^+ + \Delta K_{S^+} F_\phi}{\Delta K_{SS^+}} . \tag{5.18}$$

Furthermore, the equation for the variation with respect to F_ϕ gets additional terms $\Delta K_{S^+} F_S^+$, $3\Delta P(S)$ and $\Delta K(S, S^+)$. If we plug the result for F_S (5.18) into this equation, we get instead of (5.9)

$$\begin{aligned} F_T^+ = \Bigg[&\left(3M_5^3 [A + A^+] + \frac{\Delta K_{S^+} \Delta K_S}{\Delta K_{SS^+}} - \Delta K(T, T^+) - \Delta K(S, S^+) \right) F_\phi^+ \\ &-3c - 3\Delta P(T) - 3\Delta P(S) + \frac{\Delta P_S \Delta K_{S^+}}{\Delta K_{SS^+}} \Bigg] \frac{1}{\Delta K_{T^+} - 3M_5^3} . \end{aligned} \tag{5.19}$$

Equating this and equation (5.8) gives us

$$\begin{aligned} F_\phi^+ = \frac{1}{B'} \Big(&3M_5^3 \Delta P_T - \Delta K_{T^+} \Delta P_T + 3\Delta K_{TT^+} c + 3\Delta P(T) \Delta K_{TT^+} \\ &+ 3\Delta P(S) \Delta K_{TT^+} - \frac{\Delta P_S \Delta K_{S^+}}{\Delta K_{SS^+}} \Delta K_{TT^+} \Big) \end{aligned} \tag{5.20}$$

with the denominator

$$B' = B + \frac{\Delta K_{S^+} \Delta K_S}{\Delta K_{SS^+}} \Delta K_{TT^+} - \Delta K(S, S^+) \Delta K_{TT^+} . \tag{5.21}$$

Dropping higher order terms and terms suppressed by c/M_5^3 we have the same F_ϕ as before (equation (5.13)). We then get for F_T

$$\begin{aligned} F_T^+ = \frac{1}{3M_5^6} \big(&3M_5^3 (c + \Delta P(T) + \Delta P(S)) - M_5^3 [A + A^+] \Delta P_T \\ &- M_5^3 \frac{\Delta P_S \Delta K_{S^+}}{\Delta K_{SS^+}} + c \Delta K_{T^+} - c[A + A^+] \Delta K_{TT^+} \big) \end{aligned} \tag{5.22}$$

and for F_S

$$F_S^+ = -\frac{\Delta P_S}{\Delta K_{SS^+}} \,. \tag{5.23}$$

If we plug this into the Lagrangian of the auxiliary fields, we see that, up to linear order in perturbation, only the parts which depend on S change the scalar potential. The new scalar potential reads

$$V = -\frac{|c|^2 \Delta K_{TT^+}}{M_5^6} - \left(\frac{c^* \Delta P_T}{M_5^3} + \text{h.c.}\right) + \frac{\Delta P_S \Delta P_S^+}{\Delta K_{SS^+}} \,. \tag{5.24}$$

If we assume ΔK_{TT^+} to dominate over ΔP_T, we see that F_{S^+} has to be very small in order to cancel the cosmological constant. The condition for that is

$$\frac{|c|^2 \Delta K_{TT^+}}{M_5^6} = \frac{\Delta P_S \Delta P_S^+}{\Delta K_{SS^+}} \,. \tag{5.25}$$

However, that would mean nothing, if S were not stabilised, i.e. the part of the scalar potential involving S must have a stable minimum. This is indeed possible. If we assume

$$\Delta P(S) = S \,, \tag{5.26}$$

then ΔP_S is constant. Thus ΔK_{SS^+} must have a stable maximum if we want to have a stable minimum for the potential in S. This is the case for

$$\Delta K(S, S^+) = SS^+ - \text{const.}(SS^+)^2 \,. \tag{5.27}$$

5.3 Casimir Stabilisation of the Radion

So far we have realised supersymmetry breaking and we have shown a method to finetune the cosmological constant to zero. The next problem involved in an orbifold theory is the problem of the radius stabilisation.

Casimir energy could be a solution for this problem. In an orbifold model we have a compact extra dimension and thus we get a tower of Kaluza-Klein states for our fields. If we had an infinitely large dimension we would have a continuous spectrum of such modes and no IR-cutoff. Thus the compact extra dimension results in a non-zero vacuum energy, called Casimir energy. To calculate this Casimir energy we have to add up all possible energy states. From Ref. [44] we get for the gravitational part of the Casimir energy

$$V_{\text{Grav}} = -\frac{3}{32\pi^6 r^4}\left[\zeta(5) - \text{Li}_5(e^{2i\pi a}) + \text{h.c.}\right] \,. \tag{5.28}$$

Here $L = 2\pi r$ is the size of the extra dimension, $\zeta(n)$ is the zeta function and Li_n is a polylogarithmic function defined by

$$\text{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n} \text{ and thus } \text{Li}_n(1) = \zeta(n) \,. \tag{5.29}$$

The difference between the gravitino and the graviton mass is given by $m_{3/2} - m_{\text{Grav}} = a/r$. The graviton is massless, while the gravitino mass is related to $\langle F_T \rangle$. In the exact no-scale case we have (5.2) and for $c/M_5 \ll 1$ we get [39]

$$m_{3/2} = \frac{c/2\pi M_5^3}{r} = \frac{F_T/2\pi}{r} \,. \tag{5.30}$$

Therefore, we can identify

$$a = \frac{F_T}{2\pi} \ll 1 \,. \tag{5.31}$$

Thus it makes sense to expand (5.28) as a power series in a and ignore all but the first order of this expansion. At first we expand $e^{2i\pi a}$, which gives us

$$V_{\text{Grav}} = -\frac{3}{32\pi^6 r^4}\left[\zeta(5) - \text{Li}_5(1 + i2\pi a - 2\pi^2 a^2) + \text{h.c.}\right] \,. \tag{5.32}$$

If we then expand the function Li we get

$$\begin{aligned} V_{\text{Grav}} &= -\frac{3}{32\pi^6 r^4}\left[\zeta(5) - \left(\text{Li}_5(1) + (2\pi i a - 2\pi^2 a^2)\text{Li}_5'(1) - 2\pi^2 a^2 \text{Li}_5''(1)\right) + \text{h.c.}\right] \\ &= -\frac{3}{32\pi^6 r^4}\left[4\pi^2 a^2 \text{Li}_5'(1) + 4\pi^2 a^2 \text{Li}_5''(1)\right] \,. \end{aligned} \tag{5.33}$$

From (5.29) we derive

$$\text{Li}_n'(z) = \frac{1}{z}\text{Li}_{n-1} \tag{5.34}$$

and

$$\text{Li}_n''(z) = \frac{z\text{Li}_{n-1}'(z) - \text{Li}_{n-1}(z)}{z^2} = \frac{\text{Li}_{n-2}(z) - \text{Li}_{n-1}(z)}{z^2} \tag{5.35}$$

which leads us to the result

$$V_{\text{Grav}} = -\frac{3}{8\pi^4 r^4}\zeta(3)a^2 + \mathcal{O}(a^4) \,. \tag{5.36}$$

Now we want to do the same calculations for massive hypermultiplets in this theory. We begin with the KK spectrum of such multiplets:

$$m_0^{2(n)} = \left(\frac{\pm n + F_T/2\pi}{r}\right)^2 + m^2 \quad m_{1/2}^{2(n)} = \left(\frac{n}{r}\right)^2 + m^2 \;\; n = 0, 1, ... \tag{5.37}$$

Here each of these KK levels has two states of each orbifold parity (± 1). To calculate the resulting Casimir energy we have to solve

$$\begin{aligned} V_{\text{Hyper}} = &\frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\log\left(k^2 + \left(\frac{n+F_T/2\pi}{r}\right)^2 + m^2\right) \\ &-\frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\log\left(k^2 + \left(\frac{n}{r}\right)^2 + m^2\right) . \end{aligned} \tag{5.38}$$

Like in Ref. [40] we make the ansatz

$$V_{\text{Hyper}} = -\frac{\mathrm{d}}{\mathrm{d}s}\left(\zeta_0(s) - \zeta_{1/2}(s)\right)|_{s=0} \tag{5.39}$$

with

$$\zeta_0(s) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\left(k^2 + \left(\frac{n+F_T/2\pi}{r}\right)^2 + m^2\right)^{-s} \tag{5.40}$$

and an analogous definition of $\zeta_{1/2}(s)$. This integral can be evaluated, resulting in

$$\zeta_0(s) = \frac{1}{32\pi^2}\frac{1}{(2-s)(1-s)}\frac{\pi^{4-2s}}{(r^2\pi^2)^{2-s}}F(s-2; F_T/2\pi, mr) , \tag{5.41}$$

where $F(u; a, c)$ is defined in [40]

$$F(u; a, c) = \sum_{n=-\infty}^{\infty}\frac{1}{[(n+a)^2 + c^2]^u} . \tag{5.42}$$

In order to calculate V_{Hyper} we need the derivative of this function $F(u)$ (see the appendix of Ref. [40])

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}u}F(u; a, c)|_{s=-2} = &-\frac{16\pi}{15}|c|^5 + \frac{1}{2\pi^4}\left(\, 4\pi^2|c|^2\mathrm{Li}_3(q)\right. \\ &\left. + 6\pi|c|\mathrm{Li}_4(q) + 3\mathrm{Li}_5(q) + \text{h.c.}\, \right) , \end{aligned} \tag{5.43}$$

where we have defined $q = e^{2\pi i(a+i|c|)}$. Plugging this, together with (5.40), into (5.39) gives us

$$\begin{aligned} V_{\text{Hyper}} = \frac{1}{32\pi^2 r^4}\frac{1}{2\pi^4}\,[\, &4\pi^2(mr)^2\left(\mathrm{Li}_3(e^{iF_T - 2\pi/mr}) - \mathrm{Li}_3(e^{-2\pi/mr})\right) \\ &+ 6\pi(mr)\left(\mathrm{Li}_4(e^{iF_T-2\pi/mr}) - \mathrm{Li}_4(e^{-2\pi/mr})\right) \\ &+ 3\left(\mathrm{Li}_5(e^{iF_T-2\pi/mr}) - \mathrm{Li}_5(e^{-2\pi/mr})\right)\,] . \end{aligned} \tag{5.44}$$

Since F_T is small in this scenario, we want to write the Casimir energy as a power series in F_T. For the $\mathrm{Li}_n(e^{iF_T-2\pi/mr})$ we get

$$\begin{aligned} \mathrm{Li}_n(e^{iF_T-2\pi/mr}) = &\,\mathrm{Li}_n(e^{-2\pi/mr}) + (e^{iF_T} - 1)\mathrm{Li}_{n-1}(e^{-2\pi/mr}) \\ &+ (e^{iF_T} - 1)^2\frac{1}{2}\left(\mathrm{Li}_{n-2}(e^{-2\pi/mr}) - \mathrm{Li}_{n-1}(e^{-2\pi/mr})\right) \\ &+ \text{higher terms} . \end{aligned} \tag{5.45}$$

If we expand e^{iF_T} in a power series as well, this becomes

$$\begin{aligned} \mathrm{Li}_n(e^{iF_T-2\pi/mr}) = &\,\mathrm{Li}_n(e^{-2\pi/mr}) + iF_T\mathrm{Li}_{n-1}(e^{-2\pi/mr}) + \frac{1}{2}F_T^2\mathrm{Li}_{n-2}(e^{-2\pi/mr}) \\ &+ \text{higher terms} . \end{aligned} \tag{5.46}$$

All imaginary parts, which are the parts linear in F_T, drop out in the calculation of $F(s-2; F_T/2\pi, mr)$. If we use $L = 2\pi r$ and keep in mind that we have two states of each orbifold parity, the final result for the Casimir energy of a hypermultiplet is

$$\begin{aligned} V_{\text{Hyper}} = &(+2)\frac{3}{8\pi^2L^4}\left[\frac{1}{3}(mL)^2\mathrm{Li}_1(e^{-mL})\right. \\ &\left. + (mL)\mathrm{Li}_2(e^{-mL}) + \mathrm{Li}_3(e^{-mL})\;\right]|F_T|^2 + \mathcal{O}(F_T^4) . \end{aligned} \tag{5.47}$$

We can see imediately that the contribution of the hypermultiplet to the Casimir energy has a different sign than the gravitational contribution. To get a better feeling for these potentials, so that we can compare the gravitational Casimir energy with the contribution of a hypermultiplet, we have to look at the asymptotic behaviour. For $L \ll \frac{1}{m}$ we have to expand Li_n around 1 which gives us $\zeta(n)$ at zeroth order. Ignoring terms of higher order in L we get

$$V_{\text{Hyper}} = (+2)\frac{3\zeta(3)}{8\pi^2 L^4}|F_T|^2 + \mathcal{O}(F_T^4) \ . \tag{5.48}$$

For $L \gg \frac{1}{m}$ we have to expand the Li_n around e^{-Lm} so that the zeroth order term is e^{-Lm}. This results in

$$V_{\text{Hyper}} = (+2)\frac{m^2 e^{-mL}}{8\pi^2 L^2}|F_T|^2 + \mathcal{O}(F_T^4) \text{ for } L \gg \frac{1}{m} \ . \tag{5.49}$$

Now we see that for $L \ll \frac{1}{m}$ the hypermultiplet contribution and the gravitational part of the Casimir energy have the same behaviour in F_T but different signs and the gravitational contribution is larger by a factor of two. Thus the repulsive hypermultiplet contribution dominates if there are three or more hypermultiplets. For $L \gg \frac{1}{m}$ the Casmir energy, which arises from the hypermultiplet, is exponentially suppressed, so that the attractive gravitational part dominates. This gives us a minimum at $L \sim \frac{1}{m}$. Thus the radion, i.e. the size of the extra dimension, can be stabilized by the Casimir energy. The Casimir energy is negative at the minimum and of order $F_T^2 m^4/\pi^2$. We can cancel this contribution to the cosmological constant as before by an additional source of supersymmetry breaking localised on the hidden brane.

5.4 Supersymmetry Breaking for a Quasi-localised Gauge Multiplet

So far we only have supersymmetry breaking in the gravitational and in the hidden sector. But in our physical world we do not have a degenerated mass spectrum, i.e. SUSY is broken. Therefore, we have to discuss how SUSY is broken in the visible sector. One possibility is to localise the SUSY breaking sector on the visible brane, but this would give rise to flavor-violating contact terms between the SUSY breaking fields and our physical fields. To avoid this we localise our SUSY breaking sector on the hidden brane. SUSY breaking then has to be communicated to the visible brane by other means like gaugino mediation or anomaly mediation [29]. However, the bulk hypermultiplets also give rise to flavor violating contact terms. This contribution to supersymmetry breaking is of order 10^4 times larger than a possible anomaly mediation contribution [39].

Therefore, we consider gaugino mediation. For that we need a Super-Yang Mills sector in the bulk. A Super-Yang-Mills sector on a brane gives rise to the Lagrangian

$$\frac{1}{g_{\text{brane}}^2}\int d\theta^2 W^\alpha W_\alpha \ , \tag{5.50}$$

while a Super-Yang-Mills field in the bulk gives [48]

$$\frac{1}{g_5^2}\int d\theta^2 T W^\alpha W_\alpha \ . \tag{5.51}$$

If we assume that the brane field is induced by the bulk gauge field, we get for the field strength part of the Lagrangian

$$\frac{1}{g_{\text{brane}}^2}F^{MN}F_{MN} + \frac{L}{g_5^2}F^{MN}F_{MN} \ . \tag{5.52}$$

From that we can directly read off the four-dimensional effective gauge coupling

$$\frac{1}{g_4^2} = \frac{1}{g_{\text{brane}}^2} + \frac{L}{g_5^2} \ . \tag{5.53}$$

Now we want to estimate the size of the gauge couplings on the brane and in the bulk. Therefore, we assume that there is a scale Λ where one-loop corrections become important, i.e. the coupling becomes strong. Following [47], we then use naive dimensional analysis to get a relation between Λ and the gauge couplings. A typical loop integral in a D-dimensional theory can be written as

$$\int \frac{\mathrm{d}^D P}{(2\pi)^D} f(P^2) = \frac{\Omega_D}{(2\pi)^D}\int \mathrm{d}p p^{D-1} f(p^2) \ , \tag{5.54}$$

where $\Omega_D = \pi^{D/2}/\Gamma(D/2)$ is the surface area of a D-dimensional sphere of radius one. We use this to define the D-dimensional loop factor

$$\frac{1}{l_D} = \frac{\Omega_D}{(2\pi)^D} = \frac{1}{2^D \pi^{D/2}\Gamma(D/2)} \ , \tag{5.55}$$

which indicates the kinematical suppression of a loop. Then the naive loop integral for gravitons looks like

$$\left(\frac{1}{M_D^{(D-2)/2}}\right)^2 \frac{1}{l_D}\int_{1/L}^{\Lambda} \mathrm{d}p p^{D-1}\left(\frac{1}{p^2}\right)^2 p^2 = \frac{1}{l_D M_D^{D-2}}\int_{1/L}^{\Lambda}\mathrm{d}p p^{D-3} \sim \frac{\Lambda^{D-2}}{l_D M_D^{D-2}} \ . \tag{5.56}$$

Here the first factor is the strength of the gravitational coupling squared, the factor $(1/p^2)$ comes from the two propagators in a loop and the p^2 at the end

arises from the gravitational vertex. All loops should become unsuppressed at the scale Λ so that our last equation should be of order 1. This gives us the estimate

$$\Lambda^3 \sim l_5 M_5^3 \,. \tag{5.57}$$

For D-dimensional gauge fields we get the following loop integral

$$\frac{g_D^2}{l_D}\int_{1/L}^{\Lambda} \mathrm{d}p p^{D-5} \sim \frac{\Lambda^{D-4} g_D^2}{l_D} \,. \tag{5.58}$$

This should again be of order one, resulting in

$$g_5^2 \sim \frac{l_5}{\Lambda} \quad \text{and} \quad g_{\text{brane}}^2 \sim \frac{l_4}{\log(L\Lambda)} \,. \tag{5.59}$$

We want to use these results to get an estimate for the relative size of the gravitino mass and the gaugino mass. In the No-Scale case the gravitino mass is given by

$$m_{3/2} = \frac{\langle F_T \rangle}{L} \,. \tag{5.60}$$

From equation (5.50) and equation (5.51) we get for the gaugino mass

$$m_{1/2} \sim \frac{\langle F_T \rangle}{g_5^2} \,. \tag{5.61}$$

That means that a non-vanishing VEV of F_T breaks supersymmetry and gives the gauginos a mass. In this sense we can speak of radion mediated supersymmetry breaking [45]. If we normalise the corresponding kinetic term canonically (5.52), we have

$$m_{1/2} = \frac{g_4^2 \langle F_T \rangle}{g_5^2} \,. \tag{5.62}$$

Together with our estimate for the couplings this results in

$$\frac{m_{3/2}}{m_{1/2}} = \frac{g_5^2}{g_4^2 L} \sim \frac{1}{l_4 L\Lambda}(l_5 \log(L\Lambda) + l_4 L\Lambda) \,. \tag{5.63}$$

Now we could ask what happens if the bulk gauge field dominates, i.e. $L/g_5^2 \gg 1/g_{\text{brane}}^2$. Then we would get $m_{3/2}/m_{1/2} \sim 1$. This would lead to the cosmological gravitino problem. To avoid this, Luty and Okada make the opposite assumption that the standard-model gauge multiplet is not completely localised on the brane. Instead it should be quasi-localised [46] in the sense that the brane kinetic terms dominate over the bulk kinetic terms.

From equation (5.53) we see that this requires small radius. Furthermore, we want to assume that the gauge coupling g_4^2 is of order one. Then we get

$$\frac{m_{3/2}}{m_{1/2}} \sim \frac{g_5^2}{L} \sim \frac{l_5}{L\Lambda} \,. \tag{5.64}$$

However, we want strictly divided branes, i.e. we want to have sequestering [29]. But this requires large radius, which seems to be in contradiction to the assumption that the brane localised gauge field dominates. But maybe both requirements are compatible in a small region of intermediate radius r. In sequestered sector scenarios coordinate space propagators for a bulk state of mass m will be suppressed by $e^{-mr_c} \sim e^{-m/\mu_c}$, where $\mu_c \ll M_P l$ is the compactification scale. To avoid direct interactions between the two branes we want this propagator to be small. Therefore, Luty and Okada [39] assume that sequestering requires $e^{-\Lambda L} \lesssim 10^{-3}$ or in other words $\Lambda L \gtrsim 7$. With (5.59) we see that this results in 4D effective gauge couplings, g_4^2 and g_5^2/L, which are of the same size ($l_4 = 16\pi^2$ and $l_5 = 24\pi^3$). Thus sequestering and quasi-localisation of the gauge multiplet are not compatible. In the case where we assume both, sequestering and quasi-localisation of the gauge multiplet, we get from equation (5.63)

$$\frac{m_{3/2}}{m_{1/2}} \gtrsim 2 \,. \tag{5.65}$$

We see that the gravitino mass is only slightly larger than the weak scale, which does not solves the gravitino problem.

However, there might be still a solution of this problem. In five-dimensional supersymmetric gauge theories exists the possibility for non-trivial UV-fixed points [49–51]. In a gauge theory near such a fixed point we would expect the bulk contribution to the effective gauge coupling to be independent of the cutoff. So that we can hope that quasi-localisation, in the sense that $g_5^{-2} L \ll g_4^{-2}$, is consistent with sequestering.

In five-dimensional super-Yang-Mills theory the gauge coupling at zero momentum is given by

$$g_5^{-2}(0) = g_5^{-2}(\Lambda) - c\Lambda \,. \tag{5.66}$$

The first part on the right side of this equation is the tree level contribution while the second part is the one-loop correction. It can be shown that no higher order contributions exist [49] [50]. Furthermore, we expect $g_5^{-2}(0)$ to give us the fundamental infrared scale of the theory $g_5^{-2}(0) = M$. Then, as shown above, the maximal possible cutoff in our model is given by equation

(5.59). Seiberg et al. claim that there may exist a non-trivial UV-fixed point, i.e.

$$\frac{g_5^{-2}(\Lambda)}{\Lambda} = \text{const.} \tag{5.67}$$

From equation (5.66) then follows that $g_5^{-2}(0) = 0$ at the fixed point. Now we want assume that we are near such a fixed point which means $g_5^{-2}(0) \ll g_5^{-2}(\Lambda)$. This gives us

$$g_5^{-2}(\Lambda) = M + c\Lambda \quad \text{with} \quad M \ll c\Lambda \,. \tag{5.68}$$

The phenemenological relevant scale in our model is given by the size of the extra dimension L. Thus we have

$$g_5^{-2}(1/L) = M + \frac{c}{L} \,, \tag{5.69}$$

and we consider the situation $M \ll c/L$. To get the four-dimensional effective coupling we have to multiply g_5^{-2} with L so that we arrive at

$$g_{5,\,\text{eff}}^{-2}(1/L) = LM + c \,. \tag{5.70}$$

Thus we get for the relevant part of the action

$$M \int \mathrm{d}^2\theta T W^\alpha W_\alpha + \frac{1}{g_4^2(1/L)} \int \mathrm{d}^2\theta W^\alpha W_\alpha \tag{5.71}$$

with

$$g_4^{-2}(1/L) = g_{\text{brane}}^{-2}(1/L) + c \,. \tag{5.72}$$

If we demand a canonical kinetic term for the gauginos, we get a gaugino mass of

$$m_{1/2} = \langle F_T \rangle M \left(LM + c + g_{\text{brane}}^{-2} \right)^{-1} \,. \tag{5.73}$$

Together with equation (5.60) this results in

$$\frac{m_{3/2}}{m_{1/2}} = 1 + \frac{c + g_{\text{brane}}^{-2}}{LM} \,. \tag{5.74}$$

Now it is possible to realise both, sequestering and a quasi-localisation of the gauge multiplet. As before we have to demand $L\Lambda > 7$ for sequestering. The requirement for quasi localisation is $LM \ll c + g_{\text{brane}}^{-2}$. And we get the phenomenological acceptable case of large gravitino mass, since the ratio (5.74) wich can be made much greater than 2.

Chapter 6

Conclusions

Clearly supersymmetry and the concept of higher dimensions lead to many interesting possibilities for the description of nature. In particular local supersymmetry, i.e. supergravity, seems to be very attractive, since it automatically includes gravity. However, in supergravity the Lagrangians tend to be very complicated if we work them out in component fields. Calculations become lengthy and quite unclear. Therefore, it makes sense to work in a superfield formalism like in global supersymmetry.

Motivated by this we study supergeometry. In this thesis we work out the superfield description of conformal supergravity and Poincaré supergravity following the book of Buchbinder und Kuzenko [13]. Emphasis is placed on the prepotential and the compensator formalism, which play an essential part in the construction of supergravity in superfields. We derive the supergravity action in this formalism and then compare with the well known results of calculations at the component level as a consistency check. We then give a full superfield derivation of the flat limit of the compensator formalism action. From that we obtain the bosonic part of the low energy effective action. It includes an extra contribution to the kinetic term which, although an essential part of the flat bosonic action, does not follow from the rigid superfield formalism. We give a brief analysis of supersymmetry breaking in this flat limit. The results of these computations serve as an additional consistency check. After a brief discussion of No-Scale supersymmetry breaking we finally come to some applications.

Here we are interested in a recent proposal of 'Almost No-Scale Supergravity' by Luty and Okada. Following their paper [39], we consider five-dimensional supergravity compactified on an orbifold with small perturbations around the No-Scale superpotential and a radion stabilised by Casimir energy. We calculate the resulting scalar potential and show that, as claimed by Luty and Okada, it can be canceled by an additional supersymmetry

breaking field in the hidden sector. After that we compute the gravitational contribution to the Casimir energy in this model and the Casimir energy which arises from massive hypermultiplets. Our calculations reproduce the results of [39], which indicates that at least three bulk hypermultiplets are necessary for stabilisation. At last we analyse possibilities for supersymmetry breaking in this model. Luty and Okada propose gaugino mediated supersymmetry breaking with a quasi-localised gauge field. However, our careful analysis reveals that a sufficiently large brane kinetic term is, a priori, unnatural, as naive dimensional analysis reveals.

We point out that the existence of a non-trivial UV-fixed point in five-dimensional Yang-Mills theory [49] [50] may solve this problem. This fixed-point allows for a cutoff Λ which is much larger than the five-dimensional gauge coupling g_5^{-2} at zero momentum $\Lambda \gg g_5^{-2}$. This leads to a four-dimensional effective gauge coupling which is dominated by the brane contribution. Therefore, quasi-localisation of the gauge field and the sequestering of the supersymmetry breaking sector are compatible in this scenario. Furthermore, we show that a gauge theory near this fixed point realises $m_{3/2} \gg m_{1/2}$ in a natural way. This is a desirable property since it avoids the cosmological gravitino problem. However, we have not analysed how the gauge multiplet used for supersymmetry breaking affects the Casimir energy of our model and thus radion stabilisation.

For further work it might be interesting to consider warped scenarios [23]. Here we have a similar form of the four-dimensional effective action [37], we just have to work with a modified Kähler-Potential. If the radius of the curvature $1/k$ is large compared to the radius of compactification r, we are again in an 'Almost No-Scale' situation. However there might be differences in the perturbation. A possible mechanism for radius stabilisation which could be interesting in such a case is given by [53].

Appendix A

Notations and Conventions

My aim in this work is to describe the supergeometry as it is defined in B&K and to show how one can get a SUGRA-action out of this. After that I want to connect these results to the appropriate action in components in W&B.

To achieve this goal it is first of all important to be sure which conventions are to be used. Therfore, I will now introduce the most important conventions and definitions. Further conventional aspects are discussed later if they are of importance.

At first I have to mention some basic notations, mainly concerning the indices. SL(2,$\mathbb{C}$) indices(spinor indices) are denoted by small Greek letters. SO(1,3) ('Space') indices are denoted by small Latin letters. If they are from the begining of the alphabet they are flat-space-time indices or Lorentz indices. Chosen from the middle of the alphabet they indicate curved-space-time and are called Einstein indices. Capital Latin letters denote the whole superspace, e.g. $A = (a, \alpha, \dot{\alpha})$.

An important group of objects are invariant tensors of the Lorentz group. They are usful for lowering, raising or covariant contractions of indices.

One notable set of such tensors are the spinor metrics and their inverse. They are antisymmetric and both books use them to lower or rise dotted and undotted spinor indices

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha} \quad \epsilon_{12} = -1 \tag{A.1}$$

$$\epsilon^{\alpha\beta} = -\epsilon^{\beta\alpha} \quad \epsilon^{12} = 1 \tag{A.2}$$

$$\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^{\alpha}{}_{\gamma} \tag{A.3}$$

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \quad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \tag{A.4}$$

$$\epsilon_{\dot\alpha\dot\beta} = -\epsilon_{\dot\beta\dot\alpha} \quad \epsilon_{\dot1\dot2} = -1 \tag{A.5}$$

$$\epsilon^{\dot\alpha\dot\beta} = -\epsilon^{\dot\beta\dot\alpha} \quad \epsilon^{\dot1\dot2} = 1 \tag{A.6}$$

$$\psi^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\psi_{\dot\beta} \quad \psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\psi^{\dot\beta} \; . \tag{A.7}$$

Another important set of invariant Lorentz tensors are the σ-matrices. They are usually used to convert spacetime indices into spinor ones and vice versa.

$$(\tilde\sigma_m)^{\dot\alpha\alpha} \equiv \epsilon^{\dot\alpha\dot\beta}\epsilon^{\alpha\beta}(\sigma_m)_{\beta\dot\beta} \tag{A.8}$$

$$\tilde\sigma_m = (+1, -\overrightarrow{\sigma}) \; . \tag{A.9}$$

One vector index is equivalent to a pair of a dotted and an undotted spinor index

$$V_{\alpha\dot\alpha} = (\sigma^a)_{\alpha\dot\alpha}V_a \quad V_a = -\frac{1}{2}(\tilde\sigma_a)^{\dot\alpha\alpha}V_{\alpha\dot\alpha} \; . \tag{A.10}$$

This kind of conversion often leads to some technical advantages. For example one can use the σ-matrices to decompose a second-rank tensor X_{ab} into irreducible components. If we introduce the matrices

$$(\sigma_{ab})_\alpha{}^\beta = -\frac{1}{4}(\sigma_a\tilde\sigma_b - \sigma_b\tilde\sigma_a)_\alpha{}^\beta \tag{A.11}$$

$$(\tilde\sigma_{ab})^{\dot\alpha}{}_{\dot\beta} = -\frac{1}{4}(\tilde\sigma_a\sigma_b - \tilde\sigma_b\sigma_a)^{\dot\alpha}{}_{\dot\beta} \tag{A.12}$$

$$(\sigma_{ab})_{\alpha\beta} \equiv \epsilon_{\beta\gamma}(\sigma_{ab})_\alpha{}^\gamma = (\sigma_{ab})_{\beta\alpha} \tag{A.13}$$

$$(\bar\sigma_{ab})_{\dot\alpha\dot\beta} \equiv \epsilon_{\dot\alpha\dot\gamma}(\bar\sigma_{ab})^{\dot\gamma}{}_{\dot\beta} = (\bar\sigma_{ab})_{\dot\beta\dot\alpha} \tag{A.14}$$

we can see, that an arbitrary antisymmetric tensor X_{ab} is equivalent to a pair of symmetric bi-spinors

$$X_{ab} = (\sigma_{ab})_{\alpha\beta}X^{\alpha\beta} - (\bar\sigma_{ab})_{\dot\alpha\dot\beta}X^{\dot\alpha\dot\beta} \tag{A.15}$$

$$X_{\alpha\beta} = \frac{1}{2}(\sigma^{ab})_{\alpha\beta}X_{ab} \quad X_{\dot\alpha\dot\beta} = -\frac{1}{2}(\bar\sigma^{ab})_{\dot\alpha\dot\beta}X_{ab} \; . \tag{A.16}$$

While B&K uses a tilde above the σ, W&B uses a $\bar\sigma$ and there are no brackets around the σ. I will always use the $\bar\sigma$, because the bar indicates correctly what happens if we go from σ to $\bar\sigma$.

To arrange complicated calculations more clearly we will suppress contracted indices according to the following conventions:

$$\psi\chi \equiv \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha \quad \psi^2 \equiv \psi\psi \tag{A.17}$$

$$\bar\psi\bar\chi \equiv \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = -\bar\psi^{\dot\alpha}\bar\chi_{\dot\alpha} \quad \bar\psi^2 \equiv \bar\psi\bar\psi \tag{A.18}$$

$$\psi\sigma_a\bar\chi \equiv (\sigma_a)_{\alpha\dot\alpha}\psi^\alpha\bar\chi^{\dot\alpha} \quad \bar\chi\bar\sigma_a\psi \equiv \bar\chi_{\dot\alpha}(\bar\sigma_a)^{\dot\alpha\alpha}\psi_\alpha \tag{A.19}$$

Furthermore, we will need the superspace equivalent of a determinant, the superdeterminant called Berezinian. If F is a c-type non-singular supermatrix

$$F^M{}_N = \begin{pmatrix} A^m{}_n & B^m{}_\nu \\ C^\mu{}_n & D^\mu{}_\nu \end{pmatrix} \tag{A.20}$$

then our superdeterminant is defined by

$$\text{sdet } F \equiv \text{Ber } F = \det\,(A - BD^{-1}C)\,\det{}^{-1}D \tag{A.21}$$

$$= \det\,A\,\det{}^{-1}(D - CA^{-1}B). \tag{A.22}$$

Bibliography

[1] A. Salam and J. C. Ward, "Weak And Electromagnetic Interactions," Nuovo Cim. **11** (1959) 568.

[2] S. L. Glashow, "Partial Symmetries Of Weak Interactions," Nucl. Phys. **22** (1961) 579.

[3] J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. **127** (1962) 965.

[4] A. Salam and J. C. Ward, Phys. Lett. **13** (1964) 168.

[5] Y. A. Golfand and E. P. Likhtman, "Extension Of The Algebra Of Poincare Group Generators And Violation Of P Invariance," JETP Lett. **13** (1971) 323 [Pisma Zh. Eksp. Teor. Fiz. **13** (1971) 452].

[6] D. V. Volkov and V. P. Akulov, "Possible Universal Neutrino Interaction," JETP Lett. **16** (1972) 438.

[7] J. Wess and B. Zumino, "Supergauge Transformations In Four-Dimensions," Nucl. Phys. B **70** (1974) 39.

[8] M. F. Sohnius, "Introducing Supersymmetry," Phys. Rept. **128** (1985) 39.

[9] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, "Progress Toward A Theory Of Supergravity," Phys. Rev. D **13** (1976) 3214.

[10] S. Deser and B. Zumino, "Consistent Supergravity," Phys. Lett. B **62** (1976) 335.

[11] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, "Superspace, Or One Thousand And One Lessons In Supersymmetry," Front. Phys. **58** (1983) 1 [arXiv:hep-th/0108200].

[12] J. Wess and J. Bagger, "Supersymmetry And Supergravity,"

[13] I. L. Buchbinder and S. M. Kuzenko, "Ideas And Methods Of Supersymmetry And Supergravity: Or A Walk Through Superspace,"

[14] P. Van Nieuwenhuizen, "Supergravity," Phys. Rept. **68** (1981) 189.

[15] H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110** (1984) 1.

[16] N. Dragon, U. Ellwanger and M. G. Schmidt, "Supersymmetry And Supergravity," Prog. Part. Nucl. Phys. **18** (1987) 1.

[17] P. Binetruy, G. Girardi and R. Grimm, "Supergravity couplings: A geometric formulation," Phys. Rept. **343** (2001) 255 [arXiv:hep-th/0005225].

[18] T. Kaluza, "On The Problem Of Unity In Physics," Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **1921** (1921) 966.

[19] O. Klein, "Quantum Theory And Five-Dimensional Theory Of Relativity," Z. Phys. **37** (1926) 895 [Surveys High Energ. Phys. **5** (1986) 241].

[20] P. Horava and E. Witten, "Heterotic and type I string dynamics from eleven dimensions," Nucl. Phys. B **460** (1996) 506 [arXiv:hep-th/9510209].

[21] P. Horava and E. Witten, "Eleven-Dimensional Supergravity on a Manifold with Boundary," Nucl. Phys. B **475** (1996) 94 [arXiv:hep-th/9603142].

[22] E. Witten, "Strong Coupling Expansion Of Calabi-Yau Compactification," Nucl. Phys. B **471** (1996) 135 [arXiv:hep-th/9602070].

[23] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett. **83** (1999) 3370 [arXiv:hep-ph/9905221].

[24] L. Randall and R. Sundrum, "An alternative to compactification," Phys. Rev. Lett. **83** (1999) 4690 [arXiv:hep-th/9906064].

[25] J. Scherk and J. H. Schwarz, "Spontaneous Breaking Of Supersymmetry Through Dimensional Reduction," Phys. Lett. B **82** (1979) 60.

[26] A. Pomarol and M. Quiros, "The standard model from extra dimensions," Phys. Lett. B **438** (1998) 255 [arXiv:hep-ph/9806263].

[27] R. Barbieri, L. J. Hall and Y. Nomura, "Softly broken supersymmetric desert from orbifold compactification," Phys. Rev. D **66** (2002) 045025 [arXiv:hep-ph/0106190].

[28] E. A. Mirabelli and M. E. Peskin, "Transmission of supersymmetry breaking from a 4-dimensional boundary," Phys. Rev. D **58** (1998) 065002 [arXiv:hep-th/9712214].

[29] L. Randall and R. Sundrum, "Out of this world supersymmetry breaking," Nucl. Phys. B **557** (1999) 79 [arXiv:hep-th/9810155].

[30] M. A. Luty and R. Sundrum, "Radius stabilization and anomaly-mediated supersymmetry breaking," Phys. Rev. D **62** (2000) 035008 [arXiv:hep-th/9910202].

[31] M. Quiros, "New ideas in symmetry breaking," arXiv:hep-ph/0302189.

[32] S. Ferrara and P. van Nieuwenhuizen, "Tensor Calculus For Supergravity," Phys. Lett. B **76** (1978) 404.

[33] K. S. Stelle and P. C. West, "Relation Between Vector And Scalar Multiplets And Gauge Invariance In Supergravity," Nucl. Phys. B **145** (1978) 175.

[34] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, "Yang-Mills Theories With Local Supersymmetry: Lagrangian, Transformation Laws And Superhiggs Effect," Nucl. Phys. B **212** (1983) 413.

[35] T. Kugo and S. Uehara, "Conformal And Poincare Tensor Calculi In N=1 Supergravity," Nucl. Phys. B **226** (1983) 49.

[36] T. Kugo and S. Uehara, "Improved Superconformal Gauge Conditions In The N=1 Supergravity Yang-Mills Matter System," Nucl

[37] M. A. Luty and R. Sundrum, "Hierarchy stabilization in warped supersymmetry," Phys. Rev. D **64** (2001) 065012 [arXiv:hep-th/0012158].

[38] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, "Naturally Vanishing Cosmological Constant In N=1 Supergravity," Phys. Lett. B **133** (1983) 61.

[39] M. A. Luty and N. Okada, "Almost no-scale supergravity," JHEP **0304** (2003) 050 [arXiv:hep-th/0209178].

[40] E. Ponton and E. Poppitz, JHEP **0106** (2001) 019 [arXiv:hep-ph/0105021].

[41] V. Kaplunovsky and J. Louis, "Field dependent gauge couplings in locally supersymmetric effective quantum field theories," Nucl. Phys. B **422** (1994) 57 [arXiv:hep-th/9402005].

[42] D. V. Nanopoulos, "The March towards no scale supergravity," arXiv:hep-ph/9411281.

[43] J. Bagger, F. Feruglio and F. Zwirner, "Brane induced supersymmetry breaking," JHEP **0202** (2002) 010 [arXiv:hep-th/0108010].

[44] J. Bagger, F. Feruglio and F. Zwirner, "Brane induced supersymmetry breaking," JHEP **0202** (2002) 010 [arXiv:hep-th/0108010].

[45] Z. Chacko and M. A. Luty, "Radion mediated supersymmetry breaking," JHEP **0105** (2001) 067 [arXiv:hep-ph/0008103].

[46] G. R. Dvali, G. Gabadadze and M. A. Shifman, "(Quasi)localized gauge field on a brane: Dissipating cosmic radiation to extra dimensions?," Phys. Lett. B **497** (2001) 271 [arXiv:hep-th/0010071].

[47] Z. Chacko, M. A. Luty and E. Ponton, "Massive higher-dimensional gauge fields as messengers of supersymmetry breaking," JHEP **0007** (2000) 036 [arXiv:hep-ph/9909248].

[48] D. Marti and A. Pomarol, "Supersymmetric theories with compact extra dimensions in N = 1 superfields," Phys. Rev. D **64** (2001) 105025 [arXiv:hep-th/0106256].

[49] N. Seiberg, "Five dimensional SUSY field theories, non-trivial fixed points and string dynamics," Phys. Lett. B **388** (1996) 753 [arXiv:hep-th/9608111].

[50] K. A. Intriligator, D. R. Morrison and N. Seiberg, "Five-dimensional supersymmetric gauge theories and degenerations of Calabi-Yau spaces," Nucl. Phys. B **497** (1997) 56 [arXiv:hep-th/9702198].

[51] K. R. Dienes, E. Dudas and T. Gherghetta, "GUT precursors and non-trivial fixed points in higher-dimensional gauge theories," Phys. Rev. Lett. **91** (2003) 061601 [arXiv:hep-th/0210294].

[52] K. R. Dienes, E. Dudas and T. Gherghetta, "Extra spacetime dimensions and unification," Phys. Lett. B **436** (1998) 55 [arXiv:hep-ph/9803466].

[53] N. Maru and N. Okada, arXiv:hep-th/0312148.

Acknowledgements

Special thanks go to my supervisor Arthur Hebecker for proposing this interesting subject, his constant advice and the time he spent with me on this subject. Further I would like to thank Christoph (Toffi) Lüdeling for helpful discussions and advice as well as my officemates Kai Schmidt-Hoberg, Jan Hamann (H-man) and Benjamin Janutta (Benny) for a comfortable atmosphere, cookies and tea. I also like to thank Jan Louis for his co-supervision on behalf of the Universität Hamburg.

Last but not least I have to thank my parents for their continuing support and encouragement as well as my friends for all the fun we had and my brother for being my twin.