

Phase Noise Measurements Of The New Master Oscillator For TTF2

Diplomarbeit im Fachbereich Elektrotechnik der

Fachhochschule Lübeck

vorgelegt von

Bastian Lorbeer

June 2004

Abstract

The timing and RF-Field control systems in the Tesla Test Facility 2 and X-Ray FEL in the future require ultra low phase noise and timing jitter performance. The short term timing jitter should not exceed 100fs and the long term stability 1ps respectively. In order to meet these requirements a new master oscillator is under construction. The task of verifying its quality in terms of phase noise is approached in this thesis. The complexity of building an oscillator at such a high demand is focused on and its related problems are tried to be solved.

Contents

1	Introduction	3
2	Theory on Phase Noise in Oscillators	4
2.1	Fluctuations of Signals	4
2.1.1	Time Domain	4
2.1.2	Frequency Domain	5
2.1.3	Time to Frequency Conversion	6
2.2	Stability	6
2.2.1	Long Term Stability	6
2.2.2	Short Term Stability	7
2.3	Phase Noise, Timing Jitter and Allan Variance	7
2.3.1	Phase Noise and Amplitude Noise	8
2.3.2	Timing Jitter	13
2.3.3	Allan Variance	13
2.4	Phase Noise Modeling Of An Oscillator	14
2.4.1	Basics Of Oscillators	14
2.4.2	Leeson's Phase Noise Model of Oscillators	16
3	Methods for Measuring Phase Noise	20
3.1	Direct Measurement	20
3.2	Delay Line Method	23
3.2.1	Procedure of measurement	24
3.3	Phase Locked Loop Method	27
3.3.1	The mixer working as a phase detector	28
3.3.2	Calibration with Beat Signal	29
3.3.3	Data correction	29
3.3.4	Advantages and disadvantages	30
4	The Master Oscillator for TTF2	31
4.1	Requirements of the Master Oscillator	31
4.1.1	Frequencies Provided By The M.O.	31
4.1.2	Stability Requirements	32
4.1.3	Phase Noise Requirements	32

4.2	Realization of the Master Oscillator	36
4.2.1	Building Blocks	36
4.2.2	The 27MHz Module	37
4.2.3	Locking 81MHz VCXO to 27MHz OCXO	39
4.2.4	The DRO locked to 81MHZ	45
5	The Measurement Setup and Results	48
5.1	Chosen method for measuring the phase noise	48
5.1.1	Mixer	48
5.1.2	Phase Detector	49
5.1.3	Loop Filter for locking the sources	50
5.1.4	The Low Noise Amplifier	51
5.2	Expected Values For The Measurement	52
5.3	Measurement	54
6	Conclusion and outlook	58
A	Phase Modulation	59
B	Transfer function of setup with delay line	61
C	Mixer operating as a Phase Detector	63
D	Converting Phase Noise to Allan Variance	65
E	Block Diagrams and schematics	66
E.1	Master Oscillator block diagram	66
E.2	Schematics of 27MHz module	66
E.3	Schematic of the lockboard	66
E.4	DRO schematic	66

Chapter 1

Introduction

To obtain the required performance of the timing system that should reach a short term stability of 100fs and a long term stability of 1ps and the RF-Field control system that should reach a phase stability of 0.02° and an amplitude stability of 10^{-4} a very high stable reference source is needed. This system is composed of a Master Oscillator and very complex reference frequency distribution system. This piece of work mostly focuses on the performance of the Master Oscillator and will not describe solutions of the frequency distribution.

One way to quantify the performance of oscillators is to measure its phase noise characteristics. This thesis gives an overview of different approaches to understand phase noise and describes a self implemented setup in order to measure phase noise.

A theoretical approach to phase noise and related parameters like timing jitter and the Allan Variance is given in Chapter 2. The historical but very practical Model of phase noise in oscillators by Leeson is established here as well [2].

In chapter 3 the methods commonly used for measuring phase noise are described and their advantages and disadvantages will be discussed.

The following Chapter 4 gives an overview of the requirements for the new M.O.. That means what short and longterm stability is tolerable, what frequencies are distributed, and of course what phase noise performance it should reach. Further more the new Master Oscillator and its different modules are presented and discussed.

The practical part is summarized in Chapter 5 where all parts of the measurement setup system are presented. A summary of the system components behavior is given too as far it is important for understanding the setup. A theoretical estimation of the expected values is made and finally the measurement results are presented.

Chapter 2

Theory on Phase Noise in Oscillators

At the beginning of this chapter an overview for describing noise signals in time and frequency domain is given. The characteristics of noise signals, the difference of short term and long term stability is shown and the terms of phase noise and its related terms are introduced.

In addition a focus on the oscillation condition in oscillators is made and Leeson's model for predicting phase noise in oscillators is brought up.

2.1 Fluctuations of Signals

2.1.1 Time Domain

The primary characteristic of noise signals is its randomness what means that you need statistical methods to describe the behavior of noise signals [1].

In time domain it is convenient to describe random fluctuations with a distribution function, in this case a *Gaussian Distribution*. Since noise can disturb different physical parameters - this function applies to voltage, current, frequency, phase and time where $x(t)$ is the disturbed parameter (Figure: 2.1). The expression for this reads:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\frac{-x^2}{2\sigma^2} \quad (2.1)$$

where σ is the standard deviation of $p(x)$. $p(x)$ is completely described by σ and can be calculated by means of statistical average or time average.

The *rms* value reads...

$$x(t)_{rms} = \sqrt{x^2(t)} \quad (2.2)$$

... and assuming that the noise is ergodic¹ σ reads:

$$\sigma = \sqrt{\overline{x^2(t)}} \quad (2.3)$$

Figure 2.1 pictures a random signal $x(t)$ and its distribution in amplitude:

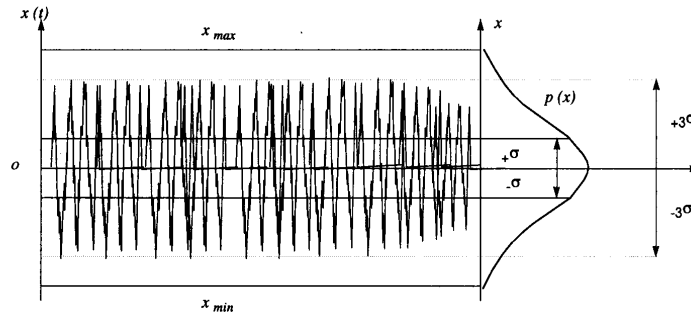


Figure 2.1: Random fluctuations in time domain [1]

In further discussions the quantities σ and $x(t)_{rms}$ appear again concerning the integrated phase noise power in rad_{rms} .

2.1.2 Frequency Domain

In frequency domain it is common to look at the power distribution of a given signal[1]. An undisturbed source is a single frequency and can be represented by a delta-function (see section 2.3.1). The real source is almost disturbed in amplitude, frequency and phase (see section 2.3.1). This is known as the spectral density of a signal source. Figure 2.2 therefor can describe all above mentioned kinds of noise:

$$\begin{aligned} \text{amplitude noise } S_x & \text{ in } \frac{\text{Volts}^2}{\text{Hz}} \\ \text{frequency noise } S_x & \text{ in } \frac{\text{Hz}^2}{\text{Hz}} \\ \text{phase noise } S_x & \text{ in } \frac{\text{radians}^2}{\text{Hz}} \end{aligned} \quad (2.4)$$

One obtains the total noise power whatever units you want by integrating the noise power in a defined bandwidth B over the frequency what is pictured in Figure 2.2 where $P_b = \sigma^2$.

¹ergodic means that the statistical average and time average are equal [8]

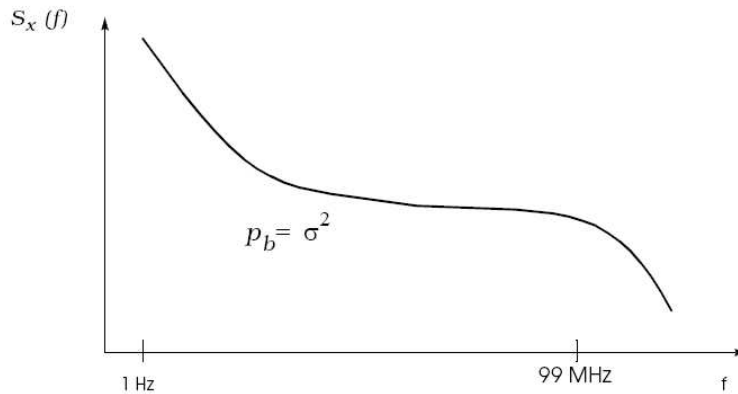


Figure 2.2: Spectral density in a single sideband [1]

2.1.3 Time to Frequency Conversion

One is able to convert the noise power from time into frequency domain and vice versa. P_b computes the overall noise power in a defined bandwidth b and reads:

$$P_b = \int_0^{\infty} S_x df \quad (2.5)$$

The relationship between the two domains is

$$P_b = \overline{x^2(t)} = \sigma^2 \quad (2.6)$$

2.2 Stability

For avoiding a misunderstanding of the terms in use when discussing about stability one has clearly to distinguish between long term stability and short term stability.

2.2.1 Long Term Stability

Long term stability describes all changes of the nominal frequency f_0 . It may also come up for real drifts in the nominal frequency. Later we will understand this as the close to the carrier phase noise. A typical plot of a drifting source is shown in figure 2.3.

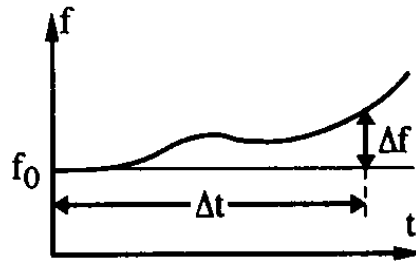


Figure 2.3: Long term stability [15]

2.2.2 Short Term Stability

This term accounts for all fast changes in a signal source and can be visualized with Figure 2.4. The short term stability comes up for the randomness based fluctuations in the source that are described by densities at higher offset frequencies from the carrier.



Figure 2.4: Short term stability [15]

Though one distinguishes between long term and short term stability there is no exact definition where the short term stability becomes long term stability. For the new Master Oscillator the long term stability is defined as the stability within minutes, hours and days [16].

2.3 Phase Noise, Timing Jitter and Allan Variance

This section gives an overview on how phase noise is understandable. Since a measurement setup may include amplitude noise and phase noise this chapter reviews an amplitude modulated and a phase modulated signal. There are several methods that are used to reduce the amount of amplitude noise

The terms single side band phase noise is introduced as well as the integrated timing jitter corresponding to phase noise and the Allan Variance as

a means to describe frequency stability of a frequency source in time domain.

2.3.1 Phase Noise and Amplitude Noise

When looking at an ideal undisturbed source in frequency domain one obtains just a single frequency that can be represented by a delta-function (Equation 2.8). The delta function in frequency domain (Figure 2.5) includes all energy of the source signal and corresponds to an undisturbed sinusoidal signal in time domain that reads:

$$u(t) = A \sin \omega_0 t \quad (2.7)$$

where A is the peak amplitude and f_0 is the resonance frequency. The corresponding delta function in frequency domain reads:

$$S_x(f) = \frac{A^2}{2\delta(f - f_0)} \quad (2.8)$$

where δ is the delta function and S_x is the total power in the sinusoidal signal from equation 2.7.

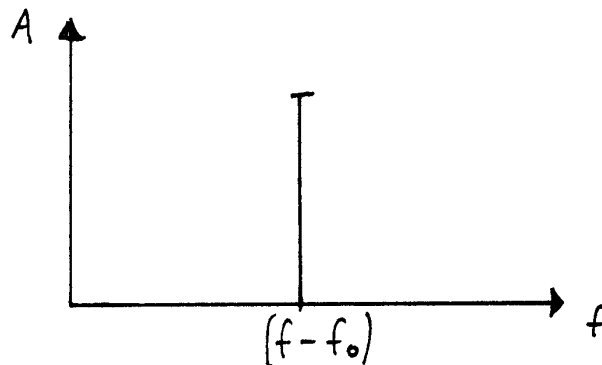


Figure 2.5: The ideal source

The real source is a disturbed source as introduced in section 2.1.2 and reads:

$$u_{dist}(t) = A [1 + \alpha(t)] \sin (\omega_0 t + \Delta\phi(t)) \quad (2.9)$$

where $\alpha(t)$ comes up for the noise contribution in amplitude and $\Delta\phi(t)$ for the fluctuations in phase. Figure 2.6 illustrates a signal only disturbed in amplitude and Figure 2.7 for a signal disturbed in phase.

Here ΔA are the rms deviation of the nominal amplitude.

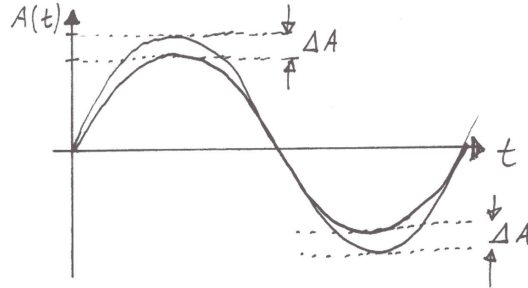


Figure 2.6: Amplitude Noise

Here $\Delta\phi$ are the rms deviation from the nominal phase.

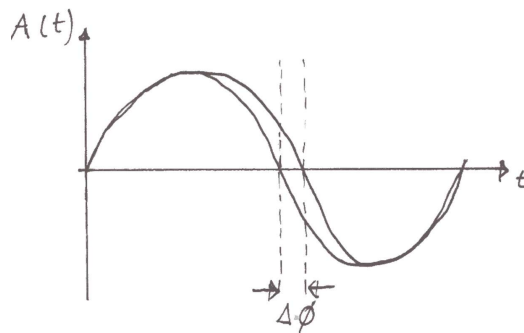


Figure 2.7: Phase Noise

A disturbed signal described by equation 2.9 gives a power distribution that is illustrated in Figure 2.8

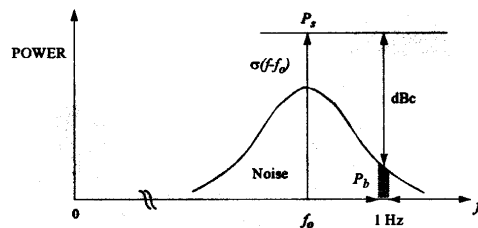


Figure 2.8: Power distribution of a real signal source [1]

The phase noise is given in units $\frac{dBc}{Hz}$ at an offset frequency f_m and quantifies the power P_b in a bandwidth of 1Hz.

Double Sideband Phase Noise (DSB):

Picture 2.8 shows the power distribution of a disturbed carrier in frequency domain. This is known as double sideband phase noise what means that the power in both sidebands of the carrier f_0 is meant. This distribution includes amplitude and phase noise. When neglecting any disturbances in amplitude the common term to describe the remaining disturbed parameter is the spectral density of phase fluctuations that read:

$$S_\phi(f_m) = \frac{\Delta\phi_{rms}^2(f_m)}{B} \left[\frac{rad^2}{Hz} \right] \quad (2.10)$$

what describes the phase noise power in rad^2 in a defined bandwidth B . When talking about phase noise the bandwidth is normalized to 1Hz (Figure 2.8).

Single Sideband Phase Noise (SSB):

The single sideband phase noise is namely just the noise power including one sideband of the carrier. When dealing with small deviation in phase ($\Delta\phi \ll 1rad$) the assumption that the SSB phase noise is half the DSB phase noise is valid.

The SSB phase noise reads:

$$\mathcal{L}_{\phi lin}(f_m) = \frac{1}{2} S_\phi(f_m) = \frac{1}{2} \frac{\Delta\phi_{rms}^2(f_m)}{B} \left[\frac{rad^2}{Hz} \right] \quad (2.11)$$

In a logarithmic scale:

$$\mathcal{L}_{dB_c}(f_m) = 10 \log L_{\phi lin}(f_m) \left[\frac{dB_c}{Hz} \right] \quad (2.12)$$

where the index c implies that this power is related to the carrier.

Frequency Fluctuations:

Also common for describing short term stability of signals is the spectral density of frequency fluctuations. This term describes the energy distribution as a continuous function, expressed in units of frequency variance per unit bandwidth [9].

$$S_{\Delta f}(f_m) = \frac{\Delta f_{rms}^2(f_m)}{\text{bandwidth used to measure } \Delta f_{rms}} \frac{Hz^2}{Hz} \quad (2.13)$$

Converting the different kinds of fluctuations:

Section 3.2 describes a measurement method where the frequency fluctuations will be measured. In this case the frequency fluctuations may be converted to SSB or DSB phase noise. The relations between frequency fluctuations, single sideband phase noise and double sideband phase noise are:

$$S_{\phi}(f_m) = \frac{S_{\Delta f}(f_m)}{f_m^2} \quad (2.14)$$

$$\mathcal{L}(f_m) = \frac{1}{2} \frac{S_{\Delta f}(f_m)}{f_m^2} \quad (2.15)$$

The relation between phase noise and frequency fluctuations is understandable from the following relation between *frequency* f and phase ϕ :

$$\Delta f(t) = 2\pi \frac{d\Delta\phi(t)}{dt} \quad (2.16)$$

Transforming equation 2.16 to frequency domain one obtains:

$$\Delta f(f_m) = f_m \Delta\phi(f_m) \quad (2.17)$$

... and the relation then reads:

$$S_{\Delta f}(f_m) = \Delta f_{rms}^2(f_m) = f_m^2 S_{\Delta\phi(f_m)} = 2f_m^2 \mathcal{L}(f_m) \quad (2.18)$$

Review of amplitude and phase modulation:

Regarding equation 2.9 one can also distinguish between a signal that is disturbed in amplitude and the signal that is disturbed in phase. Amplitude noise and phase noise is understandable from a modulation theoretical point of view.

Amplitude Modulation:

An amplitude modulated signal reads:

$$u_{AM} = A [1 + \alpha(t)] \sin(\omega_0 t) \quad (2.19)$$

where $\alpha(t)$ is the modulating amplitude and reads:

$$\alpha(t) = \hat{\alpha} \sin(\Delta\omega t) \quad (2.20)$$

When putting this into equation 2.19 one obtains with respect to the trigonometrical identity:

$$\sin a \sin b = \frac{1}{2} [\cos(a - b) + \cos(a + b)] \quad (2.21)$$

the common expression for an amplitude modulated signal, that reads:

$$u_{AM} = A \left[\underbrace{\sin(\omega_0 t)}_{Carrier} + \underbrace{\frac{\hat{\alpha}}{2} \cos(\omega_0 t - \Delta\omega t)}_{LowerSideband} - \underbrace{\frac{\hat{\alpha}}{2} \cos(\omega_0 t + \Delta\omega t)}_{UpperSideband} \right] \quad (2.22)$$

where A is the amplitude of the carrier with the angular frequency $\omega_0 = 2\pi f_0$ and the other two terms are the upper and lower sideband of the modulated carrier with a modulated signal at $\pm\Delta\omega$.

Phase Modulation:

A phase modulated signal may be written as [17]:

$$u_{PM} = A \sin (\omega_0 t + \phi(t)) \quad (2.23)$$

with $\phi(t)$ as the timedependent change in phase:

$$\phi(t) = \hat{\phi} \sin (\Delta\omega t) \quad (2.24)$$

where $\hat{\phi}$ is the peak angular deviation in radians. Putting equation 2.24 into equation 2.23 one yields an expression that reads:

$$u_{PM} = A \sin [\omega_0 t + \hat{\phi} \sin(\Delta\omega t)] \quad (2.25)$$

$$u_{PM} = A [\sin (\omega_0 t) \cos (\hat{\phi} \sin (\Delta\omega t)) + \cos (\omega_0 t) \sin (\hat{\phi} \sin (\Delta\omega t))] \quad (2.26)$$

Using the identities:

$$\cos (x \sin y) \equiv J_0(x) + 2 [J_2(x) \cos (2y) + J_4(x) \cos (4y) + \dots] \quad (2.27)$$

and

$$\sin (x \sin y) \equiv 2 [J_1(x) \sin y + J_3(x) \sin 3y + \dots] \quad (2.28)$$

where $J_0(x), J_1(x), \dots$ are Bessel Functions of the first kind of argument (x) and order $0, 1, \dots$ respectively. The expressions for u_{PM} are put into the appendix (see Appendix A). With the assumption that we are dealing with small angular phase modulation (derived in Appendix A) one gets an expression for u_{PM} that reads:

$$u_{PM} = A [\sin \omega t + \frac{\hat{\phi}}{2} \sin (\omega + \Delta\omega)t - \frac{\hat{\phi}}{2} \sin (\omega - \Delta\omega)t] \quad (2.29)$$

where A is the peak amplitude of the modulated carrier and $\frac{\hat{\phi}}{2}$ is the peak amplitude of the modulation signal at $\Delta\omega$.

Looking at equations 2.22 and 2.29 one may conclude that for small angular modulation the peak amplitudes for an amplitude and phase modulated signal are equal. That means if you look at the power distribution of a real source you are able to predict the phase deviation at this offset frequency in a bandwidth of $1Hz$. These predictions will be made in section 5.3.

2.3.2 Timing Jitter

The integrated timing jitter is the error in terms of time due to the nominal zero crossings of an oscillator. The equation to evaluate this reads:

$$\Delta T_{rms} = \frac{1}{2\pi f_0} \sqrt{\int_0^\infty S_\phi(f) df} \text{seconds}_{rms} \quad (2.30)$$

S_ϕ is the double sideband phase noise and f_0 the main resonance frequency. This is the integrated timing jitter for all frequencies from 0 to ∞ . In a practical sense it will be a defined bandwidth B .

2.3.3 Allan Variance

The Allan Variance is a widely accepted measure of frequency stability [17] in time domain. Phase Noise at small offset frequencies can not easily be measured in frequency domain. But this region is often of great interest for ultra high stable reference sources as is needed for the Tesla Test Facility 2. The reasonable method for describing stability close to the carrier is the Allan Variance [15] of the relative frequency deviations $y = \frac{f_k}{f_0}$ plotted over the interval T of the frequency measurements:

$$\sigma_y^2(T) = \frac{1}{2(M-1)} \sum_{k=1}^{M-1} \left[\frac{f_{k+1}}{f_0} - \frac{f_k}{f_0} \right]^2 \quad (2.31)$$

with f_k as the mean frequency in an time interval $[t + (k-1)T, t + kT]$ and M is the number of measurements. The advantage of the Allan Variance towards the ordinary variance is that merely the difference of two following measurements is used. The consequence is that even for large T (meaning long time measurements) do not have an influence on the Allan Variance [15]. Normally the root of $\sigma_y^2(T)$ is displayed.

The equation for converting phase noise to Allan Variance reads:

$$\sigma_y^2(T) = \frac{1}{(\pi f_0 T)^2} \int_0^B S_\phi(f_m) \sin^4(\pi f_m T) df_m \quad (2.32)$$

<i>with</i>	T	<i>the measurement time interval</i>
	B	<i>the bandwidth of interest</i>
	f_m	<i>the offset frequency from carrier</i>
	$S_\phi(f_m)$	<i>the DSB phase noise of the source</i>

(2.33)

The Allan - variance is mentioned here because it is an important term for describing frequency stability in time domain but will not be focused on further more in this thesis. Future studies may make use of this approach.

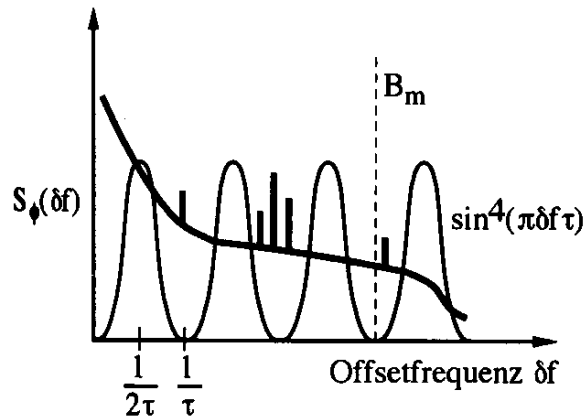


Figure 2.9: Calculating the Allan variance from DSB phase noise [15]

In appendix D you will find a conversion table to convert phase noise to Allan-Variance.

2.4 Phase Noise Modeling Of An Oscillator

2.4.1 Basics Of Oscillators

The basic idea of an oscillator is to convert dc power to a periodic, sinusoidal RF output signal. Though all oscillators need a nonlinear description of their behavior a linear technique approach is sufficient for its analysis and design. The block diagram 2.10 includes all necessary components of an oscillator [6].

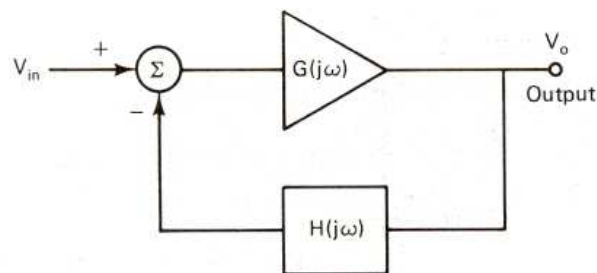


Figure 2.10: Oscillator block diagram [13]

This block diagram is composed of an amplifier with a frequency dependent gain $G(j\omega)$ and a frequency dependent feedback network $H(j\omega)$. The feedback system $H(j\omega)$ is the resonator circuit and $G(j\omega)$ is the frequency dependent amplification network. The resonating circuit has losses and can

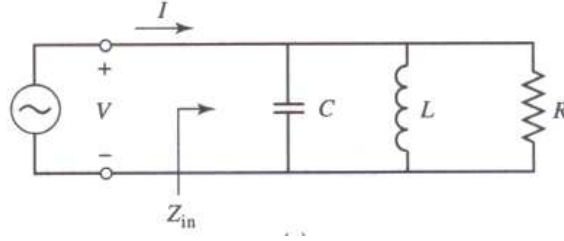


Figure 2.11: Parallel resonance circuit [10]

be modeled by a parallel resonance circuit (Figure 2.11). The inductivity L and capacitance C determine the resonance frequency ω_0 and the resistor R represents the losses in the circuit. The resistor R is the device to determine the Q (quality) of the resonator. The impedance of the circuit reads:

$$z_{in} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C \right)^{-1} \quad (2.34)$$

The resonance frequency can be established when the imaginary part of equation 2.34 is equal to zero. This means that the energy is oscillating between the inductor and capacitor. The oscillation condition then reads:

$$\frac{1}{\omega_0 C} = \omega_0 L \quad (2.35)$$

and the resonance frequency reads:

$$\omega_0 = \sqrt{\frac{1}{LC}} \quad (2.36)$$

The Q is defined as the (Figure 2.12) bandwidth of the resonance curve.

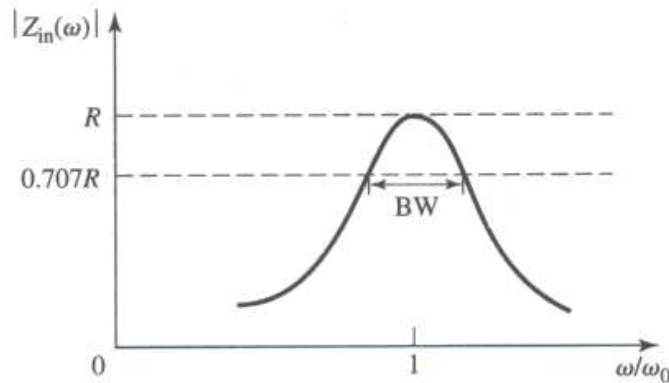


Figure 2.12: Normalized bandwidth of a resonator [10]

The Q for a resonator with losses represented by R reads:

$$Q = \frac{R}{\omega_0 L} = \omega_0 C R \quad (2.37)$$

Looking at figure 2.10 one has to derive the transfer function for the complete block.

$$\frac{v_o}{v_i} = \frac{V_o}{V_i} = \frac{G(j\omega)}{1 + G(j\omega) H(j\omega)} \quad (2.38)$$

For an oscillator V_o is nonzero when $V_i = 0$, what only is possible if the forward gain $G(j\omega)$ is infinite (what is impracticable) or the denominator

$$1 + G(j\omega) H(j\omega) = 0 \quad (2.39)$$

is equal to zero [11]. This is called the oscillation condition at the frequency ω_0

$$G(j\omega_0) H(j\omega_0) = -1 \quad (2.40)$$

This condition dates back to Nyquist and is named after him the Nyquist Criterion. Equation 2.40 implies that the magnitude of the open loop should be equal to one.

$$|G(j\omega_0) H(j\omega_0)| = 1 \quad (2.41)$$

and the phase shift should be:

$$\arg[G(j\omega_0) H(j\omega_0)] = 180^\circ \quad (2.42)$$

The magnitude (equation 2.41) and phase condition (equation 2.42) have to be fulfilled to get a stable oscillation at the output v_o of the oscillator.

2.4.2 Leeson's Phase Noise Model of Oscillators

In 1966 D.B.Leeson published a model for describing the output noise behavior of a feedback oscillator [2]. This model seems to be historic but still is in use for estimating the output spectral density of phase noise of an oscillator. This oscillator model is composed of a noiseless amplifier, a phase modulator and a resonator (Figure 2.13).

Assuming the resonator to be modeled by a parallel resonant circuit transfer function the single sideband phase noise is described with the following equation:

$$\mathcal{L}(\omega_m) = \frac{1}{1 + j\omega_m 2Q_{loaded}/\omega_0} \quad (2.43)$$

where ω_0 is the resonance frequency and ω_m the modulating frequency. An equivalent circuit is given with a phase locked loop circuit 2.14. And the phase fluctuations at the output of the oscillator can be described with:

$$\Delta\phi(\omega_m) = \left(1 + \frac{\omega_0}{j\omega_m 2Q_{loaded}}\right) \Delta\Phi\omega_m \quad (2.44)$$

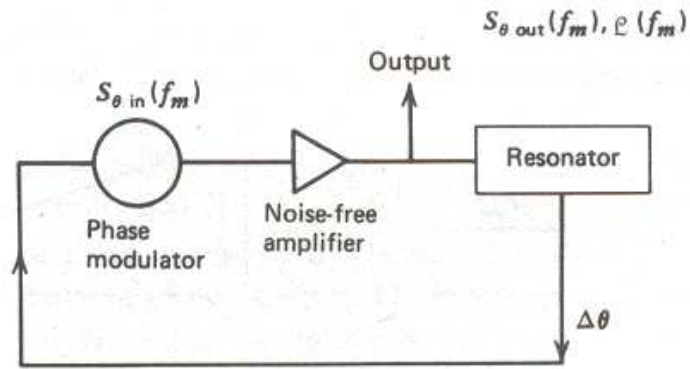


Figure 2.13: Phase Noise Model by Leeson [12]

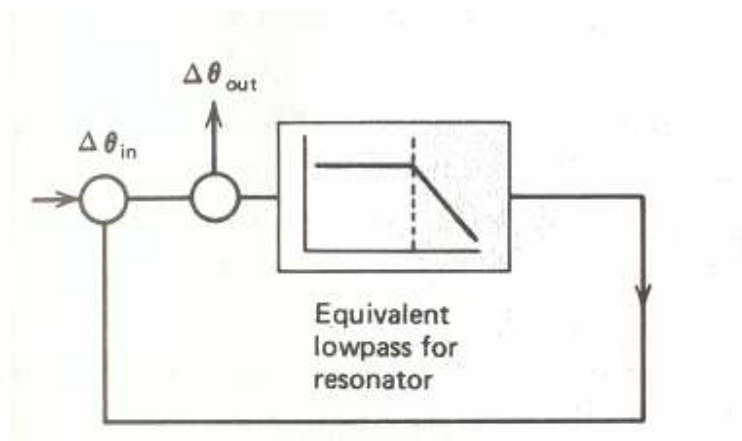


Figure 2.14: The equivalent phase locked loop [12]

The related power transfer function reads:

$$S_\phi(f_m) = \left[1 + \frac{1}{f_m^2} \left(\frac{f_0}{2Q_{loaded}} \right)^2 \right] S\Delta\Phi(f_m) \quad (2.45)$$

where the expression in the brackets is the squared phase transfer function 2.44 and $S\Delta\Phi(f_m)$ term are the phase disturbances.

The single sideband phase noise reads:

$$\mathcal{L}(f_m) = \frac{1}{2} \left[1 + \frac{1}{f_m^2} \left(\frac{f_0}{2Q_{loaded}} \right)^2 \right] S\Delta\Phi(f_m) \quad (2.46)$$

where $S\Delta\Phi(f_m)$ reads:

$$S\Delta\Phi(f_m) = \frac{FkT}{P_{s,av}} \left(1 + \frac{f_c}{f_m} \right) \quad (2.47)$$

where F is the noise figure of the amplifier, $kT = 4 * 10^{-21} Ws$ and f_c is the flicker corner frequency of the amplifier.

The phase noise in the half bandwidth:

For $f_m < \frac{f_0}{2Q_{loaded}}$ equation 2.46 reduces to:

$$\mathcal{L}(f_m) = \frac{1}{2} \frac{1}{f_m^2} \left(\frac{f_0}{2Q_{loaded}} \right)^2 \frac{FkT}{P_{s,av}} \left(1 + \frac{f_c}{f_m} \right) \quad (2.48)$$

Q_{loaded} may be understood as the ratio of reactive power over the total power dissipation:

$$Q_{loaded} = \frac{\omega_0 W_e}{P_{diss,tot}} \quad (2.49)$$

where W_e is the energy supplied in the resonator:

$$W_e = \frac{1}{2} C u^2 \quad (2.50)$$

The total power dissipation $P_{diss,tot}$ is the sum of the output signal power P_{sig} , the dissipated power in the resonator $P_{res} = \frac{\omega_0 W_e}{Q_0}$ (Q_0 is the quality of the resonator) and the input power P_{in} :

$$P_{diss,tot} = P_{sig} + P_{res} + P_{in} \quad (2.51)$$

This yields to a more detailed expression to characterize the SSB phase noise of an oscillator:

$$\mathcal{L}(\omega_m) = \frac{1}{8} \underbrace{\frac{FkT}{P_{s,av}}}_1 \frac{\omega_0^2}{\omega_m^2} \left(\underbrace{\frac{P_{in}}{\omega_0 W_e}}_2 + \underbrace{\frac{1}{Q_0}}_3 + \underbrace{\frac{P_{sig}}{\omega_0 W_e}}_4 \right)^2 \underbrace{\left(1 + \frac{\omega_c}{\omega_m} \right)}_5 \quad (2.52)$$

The numbering in the formula is made to explain the different contributions to the single sideband phase noise:

1. phase perturbations
2. ratio of input power to reactive power
3. the damping of the unloaded quality Q_0 of the resonator
4. ratio of output power to reactive power
5. the flicker effect

From equation 2.52 one can obtain some rules for designing of a low noise oscillator device. These are:

1. The unloaded quality Q_0 should be chosen very high.
2. The reactive power $\omega_0 W_e$ in the resonator should also be kept as high as possible.
3. The noise figure of the active device should be chosen very low.
4. The flicker corner frequency should be as low as possible to obtain a good phase noise characteristic close to the carrier.

Chapter 3

Methods for Measuring Phase Noise

This chapter describes three different methods for measuring phase noise. The aim is to show the complexity of the measurement setups and derive the expressions necessary to determine the single sideband phase noise.

3.1 Direct Measurement

The easiest way to measure the phase noise of an oscillator is to directly make use of a spectrum analyzer. As you can see in the measuring setup 3.1 (from [15]) it is easy to plug the output of an oscillator to a spectrum analyzer and look at the resonance curve of the oscillator.

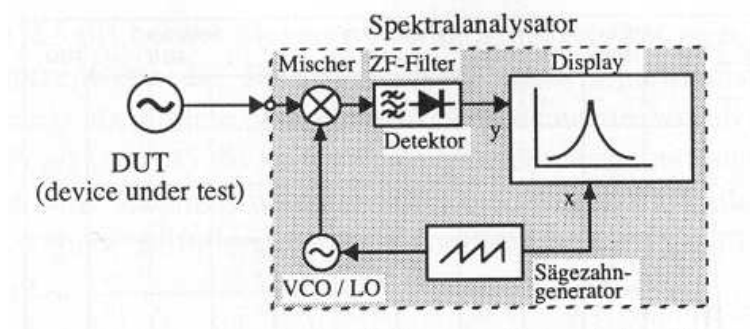


Figure 3.1: Measuring phase noise with a spectrum analyzer [15]

The D.U.T. in figure 3.1 is a device under test that is any kind of oscillator in the frequency range of the spectrum analyzer where it is plugged to. The local oscillator in the spectrum analyzer that mixes the oscillator down to DC. The spectrum analyzer displays the SSB phase noise of the oscillator. For understanding the mixing process inside the spectrum analyzer you may assume two sinusoidal ... signals being multiplied. A more detailed treatment of the mixing process is given in section 3.2. A typical display of an oscillator mixed down to DC is shown in figure 3.2.

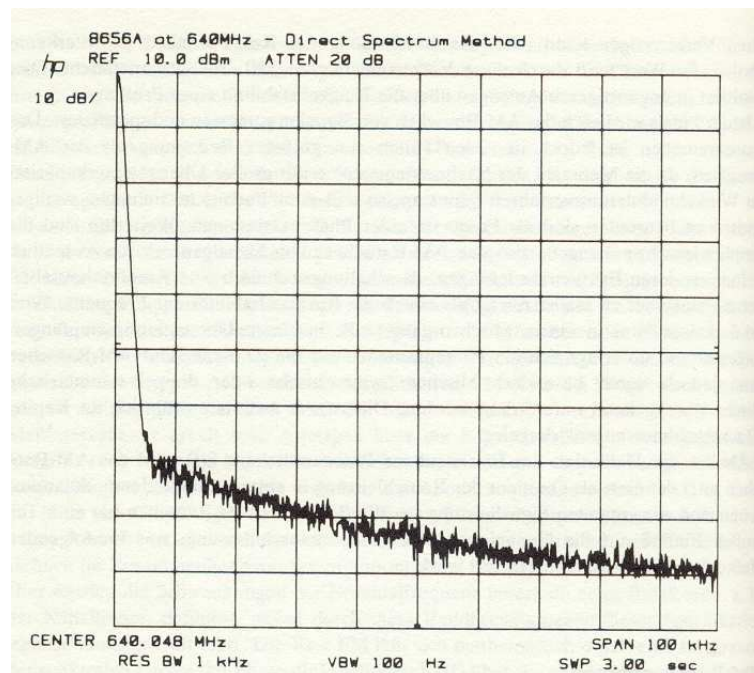


Figure 3.2: Example SSB Phase Noise with direct measurement method [5]

The main advantage of this method is its simple setup. Another advantage is that it is possible to measure phase noise at high offset frequencies from the carrier.

But the simplicity of measuring phase noise directly has several disadvantages:

- The spectrum analyzer does not make a difference between amplitude noise and phase noise when you do not have any idea about the noise power in amplitude and phase of your test object.
- The sensitivity is limited by the high noise floor of the spectrum analyzer.
- When measuring the phase noise at small offset frequencies the purity

of the internal local oscillator (LO) is limiting the sensitivity range of the setup.

- Since the phase noise power is normalized to a bandwidth of 1Hz of an ideal rectangular filter but the resolution bandwidth filter is nonideal, some correction factors for this error have to be taken into account.

Data correction:

The noise power has to be normalized to a bandwidth of 1Hz . This is done for a gaussian filter characteristic (the true characteristic of the resolution bandwidth filter):

$$B_{noise} = 1.2 B_{3dB} \quad (3.1)$$

where B_{3dB} is the 3dB corner frequency of the resolution bandwidth filter and 1.2 is the correction factor for the error between the ideal rectangular filter and the real gaussian filter (Figure 3.3). For describing the phase

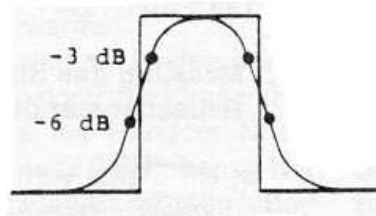


Figure 3.3: Error between the rectangular filter and the gaussian filter [15]

noise in $\frac{dBc}{Hz}$ one has to subtract the power in the carrier. Since all spectrum analyzers are calibrated for displaying sinusoidal signals a systematic error occurs when displaying noise power. The display of the spectrum analyzer reveals a level that is 2.5dB too small[15].

Example for direct measurement with bandwidth correction factors:

Regarding figure 3.2 one can establish the phase noise power at a given offsetfrequency as the following example will show:

Let the offsetfrequency be 50kHz with a power of -74dBm , the resolution bandwidth is 1kHz and the power in the carrier is 10dBm .

The systematic error is 2.5dB and the normalized bandwidth is:

$$10 * \log B_{noise} = 10 * \log 1.2 * 1\text{kHz} = 30.8\text{dB} \quad (3.2)$$

... and the resulting corrected noise power is:

$$-74\text{dBm} + 2.5\text{dB} - 30.8\text{dB} = -102.3\text{dBm} \quad (3.3)$$

Now the power in the carrier has to be added and one yields the phase noise in $\frac{dBc}{Hz}$:

$$\mathcal{L} = -102.3dBm + 10dBm = -112.3 \frac{dBc}{Hz} \quad (3.4)$$

Chapter 4.1.3 will show that we are not able to measure the phase noise of the Master Oscillator with our spectrum analyzer FSEM30 from ROHDE und SCHWARZ.

3.2 Delay Line Method

The delay line method is also called the frequency discriminator method. With this method one obtains the spectral density of frequency fluctuations. Finally these frequency fluctuations need to be converted to a spectral density in terms of phase noise due to equation 2.13. The idea is to split the signal from the device under test, send one of these signals through a delay line. The delay line output gives a phaseshift $\Delta\phi$ that is proportional to the frequency fluctuations Δf of the DUT. Keeping both signals in quadrature and finally mix them down to DC with the help of a double balanced mixer and get an output voltage ΔV that is proportional to the phase shift $\Delta\phi$. The output signal of the mixer is amplified with a low noise amplifier to get into the dynamic range of the spectrum analyzer.

Figure 3.4 shows the setup of this measurement [9].

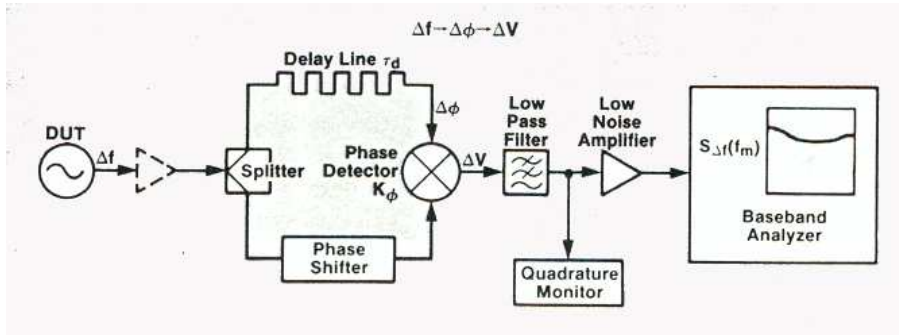


Figure 3.4: Measurement setup with delay line method [15]

The expression for $\Delta V(f_m)$ as function from the offsetfrequency f_m is developed in Appendix B and reads:

$$\Delta V(f_m) = K_\phi 2\pi\tau_d \Delta f(f_m) \frac{\sin(\pi f_m \tau_d)}{(\pi f_m \tau_d)} \quad (3.5)$$

where:

1. K_ϕ is the phase detector (mixer gain) constant in $\frac{V}{rad}$

2. τ_d is the delay time of the delay line in *seconds*
3. $\Delta f(f_m)$ are the frequency fluctuations (phase noise) as a function from the offsetfrequency f_m in $\frac{Hz^2}{Hz}$

3.2.1 Procedure of measurement

System Setup

The setup of this method is shown in figure 3.4. For best measurement results one needs to take care of several parameters in the setup. These are the power that is delivered by the source under test, the attenuation of the delay-line, the delay time τ_d the 1dB compression point of the mixer, the conversion gain K_ϕ of the mixer and the phase shifter to adjust a phaseshift of 90° between the two input signals at the mixer to guarantee a maximum phase sensitivity between the two input signals [9] (Developed in Appendix C)

System Calibration

To establish the right measurement values one has to calibrate the setup. That means to excite the system with a known input. For small offsetfrequencies f_m equation 3.5 the $\frac{\sin x}{x}$ term is equal to one and the response reads:

$$\Delta V(f_m) = K_\phi 2\pi\tau_d \Delta f(f_m) \quad (3.6)$$

For small offsetfrequencies K_ϕ and τ_d can be determined independently from each other. The τ_d can be found by varying the frequency at the input in a way that the output signal makes two zero crossings during this frequency change. The τ_d then reads:

$$\tau_d = \frac{1}{2\Delta f_0} \quad (3.7)$$

where Δf_0 is the change in frequency to obtain two zero crossings. The K_ϕ is the phase detector constant in $\left[\frac{V}{rad}\right]$ found with the help of a beat note like developed in appendix C.

The other way to calibrate the measurement is to put a known FM - modulation to the system like shown in figure 3.5. The calibration should be done with both detector input signals in quadrature since the true measurement will also be made in this operation mode. For small modulation index $\beta < 0.2rad$ the power in the higher order sidebands is neglectible.

The calibration signal K_d is calculated with the following relationship:

$$K_d[dB] = P_{cal}[dB] - (\Delta SB_{cal} \left[\frac{dB_c}{Hz}\right] + 20\log f_{m_{cal}}[dBHz] + 3[dB]) \quad (3.8)$$

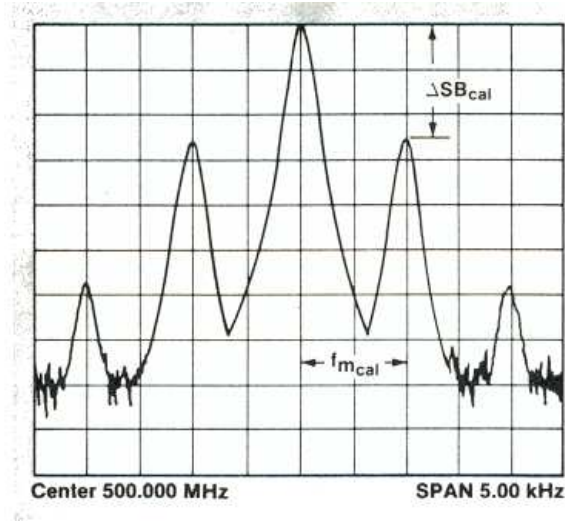


Figure 3.5: FM calibration signal [9]

where P_{cal} is the system response to a known FM signal and is measured with a spectrum analyzer.

Noise Measurement

Assume the detected output to be $S_v(f_m)$ it can be converted into the spectral density of frequency fluctuations of the source. By definition the spectral density of frequency fluctuations in a bandwidth of $1Hz$ reads:

$$S_{\Delta f}(f_m) = \Delta f_{rms}^2 \quad (3.9)$$

and the calibration signal K_d reads:

$$K_d^2 = \frac{\Delta V_{rms}^2}{\Delta f_{rms}^2} \quad (3.10)$$

So the frequency fluctuations read:

$$S_{\Delta f}(f_m) = \frac{\Delta V_{rms}^2}{K_d^2} \quad (3.11)$$

or in a logarithmic form:

$$S_{\Delta f}(f_m) \left[\frac{dB_{Hz}}{Hz} \right] = S_v(f_m) [dB_m] - K_d [dB_m] \quad (3.12)$$

The bandwidth correction for the resolution bandwidth reads:

$$RBW_{corr} [dB] = 10 \log \frac{1.2 RBW_{spectrumanalyzer}}{1Hz} \quad (3.13)$$

Then the spectral density of frequency fluctuations read:

$$S_{\Delta f}(f_m) \left[\frac{dB_{Hz}}{Hz} \right] = S_v(f_m) [dB_m] - K_d[dB_m] - RBW_{corr}[dB] \quad (3.14)$$

or the spectral density of phase fluctuations reads:

$$S_{\phi}(f_m) \left[\frac{dB_{rad}}{Hz} \right] = S_v(f_m) [dB_m] - K_d[dB_m] - RBW_{corr}[dB] - 20 \log f_m \quad (3.15)$$

Or the single sideband phase noise for phase deviations $\Delta\phi \ll 1rad$ reads:

$$\mathcal{L} \left[\frac{dB_c}{Hz} \right] = S_v(f_m) [dB_m] - K_d[dB_m] - RBW_{corr}[dB] - 20 \log f_m - 3dB \quad (3.16)$$

The advantages of this method are:

1. The frequency fluctuations $S_{\Delta f}$ of a DUT (device under test) can directly be measured.
2. A second source is not necessary in this setup.
3. Even strong drifting DUT can be measured.
4. The carrier is suppressed in this method and the AM-Noise is neglectible [9].

On the other hand this method has several disadvantages. These are:

1. For measuring $S_{\phi}(f_m)$ or $\mathcal{L}(f_m)$ the sensitivity is decreasing quadratically due to the term: $S_{\phi} = \frac{S_{\Delta f}}{f_m^2}$ for small offsetfrequencies f_m .
2. The sensitivity also decreases at higher offsetfrequencies f_m . The limit is $f_m = \frac{1}{\tau_d}$.
3. The sensitivity depends invers quadratically on K_D^2 what includes the delay time τ_d . Since in the microwave frequency range a long delaytime is connected with high losses in the line the delaytime cannot be chosen as long as one wishes.
4. A big effort as shown above has to be put in the calibration of this setup.

3.3 Phase Locked Loop Method

The PLL method is the most sensitive of all described methods. Two oscillators are sent to the two RF ports of a mixer. The IF signal is lowpassfiltered to keep out the sum frequencies and then send back in a small bandwidth to lock one oscillator to the other.

The setup of this method is shown in figure 3.6:

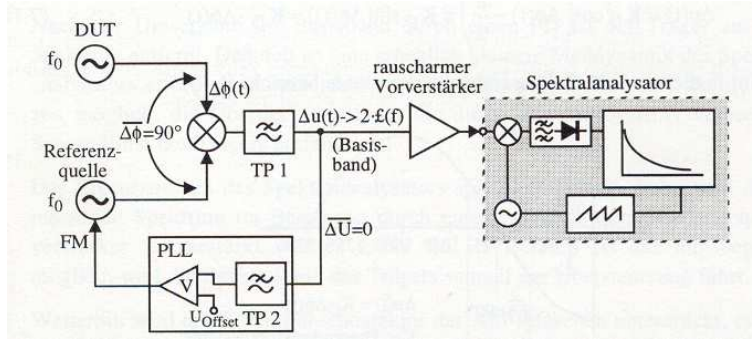


Figure 3.6: PLL locked method [15]

With the DUT and the reference source 90° out of phase the mixer outputs a voltage ΔV that is proportional to the phase difference $\Delta\phi$ between the two sources. In linear operation the phase detector equals the peak voltage of the sinusoidal beat signal. To avoid the two sources drifting out of quadrature a narrow bandwidth phase locked loop will track the two sources within the bandwidth of the PLL while outside the PLL bandwidth the phase fluctuations will be unaffected [14].

Both source contribute noise to the IF (intermediate frequency) - port of the mixer, which is the rms - sum of both sources. For making proper predictions on the phase noise source under test, one has to know the phase noise of at least one of the sources or the phase noise of one source is much smaller than the other. With three unknown sources it is possible to get enough data to calculate the phase noise of each source.

The double sideband for this setup reads:

$$S_{\Delta\phi}(f_m) = \frac{\Delta\phi_{rms}^2(f_m)}{B} = \frac{\Delta V_{rms}^2}{V_{Beat\ rms}^2 B} \quad (3.17)$$

where B is the bandwidth. Assuming that $\Delta\phi \ll 1rad$ the single sideband phase noise reads:

$$\mathcal{L}(f_m) = \frac{1}{2} \frac{S_{\Delta\phi}(f_m)}{B} = \frac{1}{4} \frac{\Delta V_{rms}^2}{V_{Beat\ rms}^2 B} \quad (3.18)$$

3.3.1 The mixer working as a phase detector

The most sensitive region of the mixer working as a phase detector is in the linear region of the output sinusoidal signal (like developed in the delay line method). The output voltage fluctuations ΔV then are proportional to the phase fluctuations $\Delta\phi$ at the input of the mixer. The gain of the phase detector is also in the linear operating range. The equation for the output voltage fluctuations reads:

$$\Delta V(t) = K_\phi \cos(\Delta\phi(t) - \overline{\Delta\phi}) \quad (3.19)$$

For the quadrature condition $\overline{\Delta\phi}$ is 90° and for small phase fluctuations $\Delta\phi$ the output voltage fluctuations depend linear on the phase fluctuations:

$$\Delta V(t) = K_\phi \Delta\phi(t) \quad (3.20)$$

This can be visualized with figure 3.7:

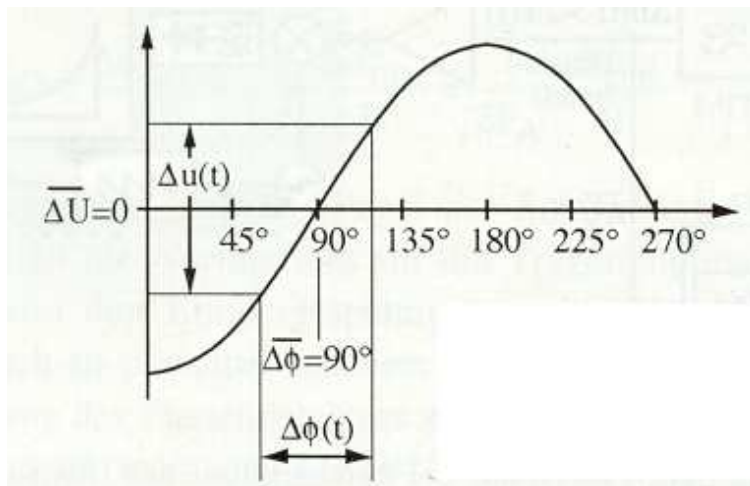


Figure 3.7: Determining K_ϕ [15]

The output signal at the IF port of the mixer includes all information of amplitude and phase [15] of the input signals. The carrier at f_0 is mixed to 0Hz. It depends on the phase difference $\overline{\phi}$ between the input signals what sidebands remain in the baseband signal at the IF port. For $\overline{\phi} = 0 \text{ degree}$ the mixer is working as an amplitude demodulator and for $\overline{\phi} = 90^\circ$ the amplitude modulated AM signals are suppressed and the phase modulated PM signals remain.

For measuring phase fluctuations it is necessary to keep the condition $\overline{\phi} = 90^\circ$. This is done with a phase locked loop control as introduced with the setup at the beginning of this chapter (Figure 3.6).

3.3.2 Calibration with Beat Signal

In frequency domain one has a response of the system that reads:

$$\Delta V(f_m) = K_\phi \Delta \phi(f_m) \quad (3.21)$$

Figure 3.8 shows the response of the measurement setup.

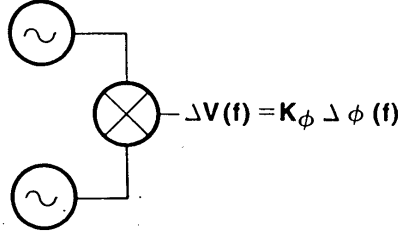


Figure 3.8: Response of PLL locked method [14]

The mixers conversion gain K_ϕ is the peak value $V_{B,peak}$ of the mixer output when the sources are in quadrature (90° out of phase). The *rms* - value is $\sqrt{2}$ times the peak value of the beat note:

$$K_\phi = V_{B,peak} = \sqrt{2} V_{B,rms} \quad (3.22)$$

With the knowledge of K_ϕ one can establish the *rms* phase fluctuations by rearranging equation 3.21. This leads to the following expression:

$$\Delta \phi_{rms}(f_m) = \frac{1}{k_\phi} \Delta V_{rms}(f_m) = \frac{1}{\sqrt{2} V_{B,rms}} \Delta V_{rms}(f_m) \quad (3.23)$$

By squaring the *rms* phase fluctuations one can determine the double sideband power spectral density. That reads:

$$S_{\Delta \phi}(f_m) = \frac{\Delta \phi_{rms}^2(f_m)}{1Hz} = \frac{1}{2} \frac{\Delta V_{rms}^2(f_m)}{V_{B,rms}^2 1Hz} \quad (3.24)$$

In the case of small angular modulation $\Delta \phi \ll 1rad$ the single sideband phase noise reads:

$$\mathcal{L}(f_m) = \frac{1}{2} S_{\Delta \phi}(f_m) = \frac{1}{4} \frac{\Delta V_{rms}^2(f_m)}{V_{B,rms}^2 1Hz} \quad (3.25)$$

3.3.3 Data correction

Following correction factors hav to be taken into account:

- Noise bandwidth normalization to a bandwidth of 1Hz with

$$B_{noise} = 1.2 B_{RBW} \quad (3.26)$$

- Normalization to the reference power.
- Subtracting of 2.5dB to account for the systematic error in the setup

These known correction factors that have already been introduced in the direct measurement setup three new data corrections have to be made in this setup:

- 2 times \mathcal{L} is measured. To obtain \mathcal{L} one has to subtract 3dB.
- The reference power can not be read out directly but is doubled power of a beat note when the DUT and the reference source have a small frequency offset
- Inside the bandwidth of the phase locked loop additional correction factors that will not be listed here have to be taken into account

An example from [15] has the following parameters:

measured power at $f_m = 20Hz$	$-40dB_m/Hz$
systematic error	2.5dB
bandwidth normalization	$-10\log 1.2Hz = -0.8dB$
corrected noise power	$-28.3dB_m$
-(beat note power + 3dB + att.)	$-(10dB_m + 3dB + 30dB) = -43dB$
from DSB to SSB	-3dB
SSB phase noise at $f_m = 20Hz$	$-74.3dB_c/Hz$

3.3.4 Advantages and disadvantages

- After converting the RF signal to the baseband a smaller dynamic range of the spectrum analyzer is necessary.
- The internal noise of the spectrum analyzer is not the limiting section in this setup. The low noise preamplifier is used to amplify the baseband into the range of the spectrum analyzer.
- The mixer operating as a phase detector is suppressing the amplitude noise due to the quadrature condition in this setup. Good mixers achieve an AM suppression from 30 - 40dB [15].
- The reference source is a high stable oscillator that is the limiting element of the setup.
- For the case of two identical source DUT and reference have the same characteristic the measurement result is 3dB higher.
- The only disadvantage is the need of two sources in this setup.

Chapter 4

The Master Oscillator for TTF2

4.1 Requirements of the Master Oscillator

This chapter gives an overview of what frequencies should be provided by the new Master Oscillator. At first it shows a tabular overview of the required frequencies from the Master Oscillator and what enduser finally will use them.

Then a list of the stability requirements is shown. In section 4.1.3 the phase noise requirements for the three main M.O. modules are revealed.

4.1.1 Frequencies Provided By The M.O.

The Master Oscillator has to provide several frequencies that are needed in the facility. The tabular gives an overview of the frequencies and the subsystems they are needed for.

Frequency	System / User
50Hz	The event timing of the system 1Hz, 2Hz, 5Hz or 10Hz has to be related to the zero crossings of the mains power supply
1MHz	master reference frequency divided by 9
9MHz	master reference frequency with TTL compatible output
13.5MHz	old Laser
27MHz	new Laser
81MHz	distribution frequency
108MHz	Streak camera near RF-Gun
1300MHz	reference frequency for linear accelerator
2856MHz	transverse deflecting cavity for bunch measurements

For further information refer to the requirements of the new Master Oscillator[16].

4.1.2 Stability Requirements

The stability requirements are derived from the correct timing stability in the operating accelerator. The stability is given in terms of timing jitter. For the main frequency 1.3GHz that is used in the accelerator the integrated timing jitter (bandwidth of $B = 1\text{MHz}$) should be $T_{rms} = 1\text{ps}$ what corresponds to a phase error of 0.5deg. The "slow" drifts should not exceed 1ps. During the pulses of 800us the jitter should be equal or better than 0.1ps and from pulse to pulse it should not exceed 0.3ps. Within minutes (long term) the jitter should be less than 1ps, within hours equal or better than 2ps and within days 10ps.

4.1.3 Phase Noise Requirements

The stability requirements of the M.O. in terms of phase noise are described in this section. It is necessary to modularize the setup of the M.O. to achieve the required phase noise goals. The proposed oscillator modules are at frequencies 9MHz, 1.3GHz and 2.856GHz. The other frequencies listed in the table above should be generated with the help of multipliers, dividers and phase locked loops.

Due to the requirements the phase noise values are given at the below listed offsetfrequencies.

Requirements for 9MHz:

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
1	-115
10	-140
100	-150
1k	-155
10k	-162
100k	-164
1M	-164

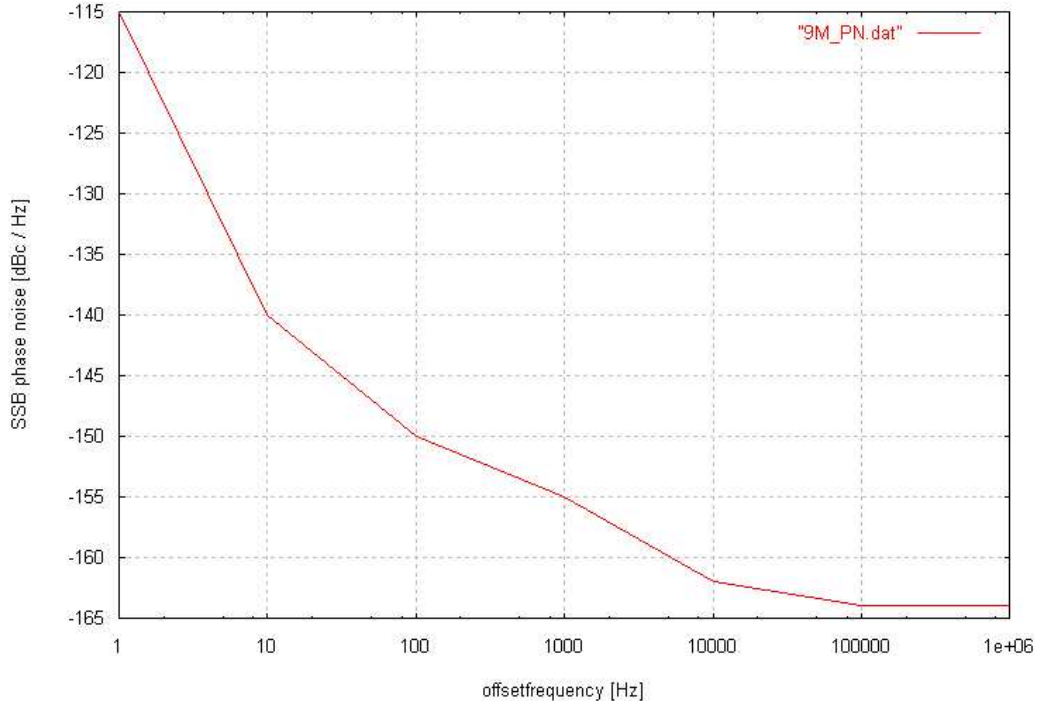


Figure 4.1: Phase Noise Requirements 9MHz module

The estimated integrated phase noise power ϕ in terms of rad_{rms} due to formula 2.5 and $P_b = \sigma^2$ is the squareroot of σ . For the different modules one can determine the integrated phase noise power in the different specified frequency ranges. For the 9MHz module one can give the different contributions to phase noise in the different frequency ranges as follows:

$$\begin{aligned}
 1Hz - 10Hz & : \phi = 5.33\mu rad_{rms} \\
 10Hz - 100Hz & : \phi = 0.95\mu rad_{rms} \\
 100Hz - 1kHz & : \phi = 0.95\mu rad_{rms} \\
 1kHz - 10kHz & : \phi = 1.7\mu rad_{rms} \\
 10kHz - 100kHz & : \phi = 2.4\mu rad_{rms} \\
 100kHz - 1MHz & : \phi = 6\mu rad_{rms}
 \end{aligned}
 \tag{4.1}$$

Summing up all noise contributions one yields the integrated phase noise power in the single sideband in a bandwidth from 1Hz to 1MHz. The value for this reads:

$$\Delta\phi_{rms,SSB} = 17.3\mu rad_{rms} \tag{4.2}$$

With the assumption for small phase deviations $\phi \ll 1rad$ the double sideband reads two times the value of the single sideband integrated phase

noise power:

$$\Delta\phi_{rms,DSB} = 2\phi_{SSB} = 2 \cdot 17.3\mu rad_{rms} = 34.6\mu rad_{rms} \quad (4.3)$$

With respect to formula 2.30 for determining the integrated timing jitter the value for it reads:

$$\Delta T_{rms} = \frac{\phi_{rms}}{2\pi f_0} = \frac{34.6\mu rad_{rms}}{2\pi \cdot 9MHz} = 611fs \quad (4.4)$$

Requirements for 1.3GHz:

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dBc}{Hz}$]
1	NA
10	-80
100	-105
1000	-135
10000	-145
100000	-155
1000000	-150

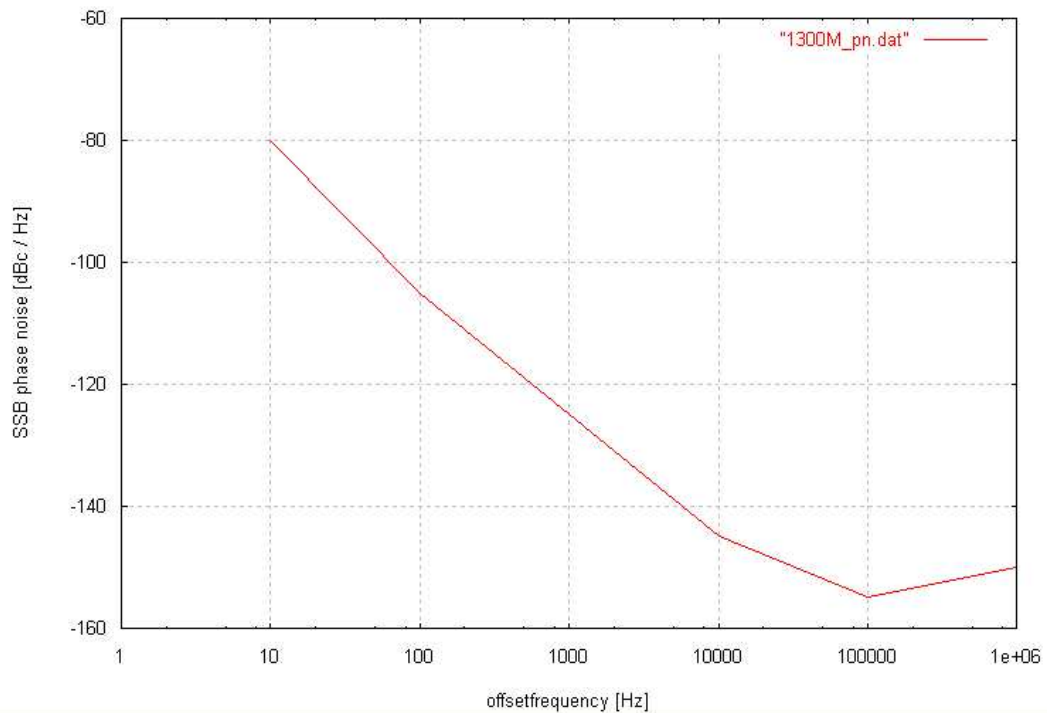


Figure 4.2: Phase Noise Requirements 1.3GHz module

For the 1300MHz module one can angeben the different contributions to phase noise in the different frequency ranges as well:

$$\begin{aligned}
10Hz - 100Hz & : \phi = 984\mu rad_{rms} \\
100Hz - 1kHz & : \phi = 168\mu rad_{rms} \\
1kHz - 10kHz & : \phi = 0.18\mu rad_{rms} \\
10kHz - 100kHz & : \phi = 16.9\mu rad_{rms} \\
100kHz - 1MHz & : \phi = 16.9\mu rad_{rms}
\end{aligned} \tag{4.5}$$

Summing up all noise contributions one yields the integrated phase noise power of the single sideband in a bandwidth from 10Hz to 1MHz. The value for this reads:

$$\Delta\phi_{rms,SSB} = 1.15mrad_{rms} \tag{4.6}$$

With the assumption for small phase deviations $\phi \ll 1rad$ the double sideband reads two times the value of the single sideband integrated phase noise power:

$$\Delta\phi_{rms,DSB} = 2\phi_{SSB} = 2 \cdot 1.15mrad_{rms} = 2.3mrad_{rms} \tag{4.7}$$

With respect to formula 2.30 for determining the integrated timing jitter the value for it reads:

$$\Delta T_{rms} = \frac{\phi_{rms}}{2\pi f_0} = \frac{2.3mrad_{rms}}{2\pi \cdot 1.3GHz} = 281fs \tag{4.8}$$

Requirements for 2.856GHz:

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
1	NA
10	-88
100	-97
1000	-117
10000	-137
100000	-145
1000000	-142

For the 2856MHz module one can angeben the different contributions to phase noise in the different frequency ranges as well:

$$\begin{aligned}
10Hz - 100Hz & : \phi = 377\mu rad_{rms} \\
100Hz - 1kHz & : \phi = 423\mu rad_{rms} \\
1kHz - 10kHz & : \phi = 134\mu rad_{rms} \\
10kHz - 100kHz & : \phi = 42\mu rad_{rms} \\
100kHz - 1MHz & : \phi = 53\mu rad_{rms}
\end{aligned} \tag{4.9}$$

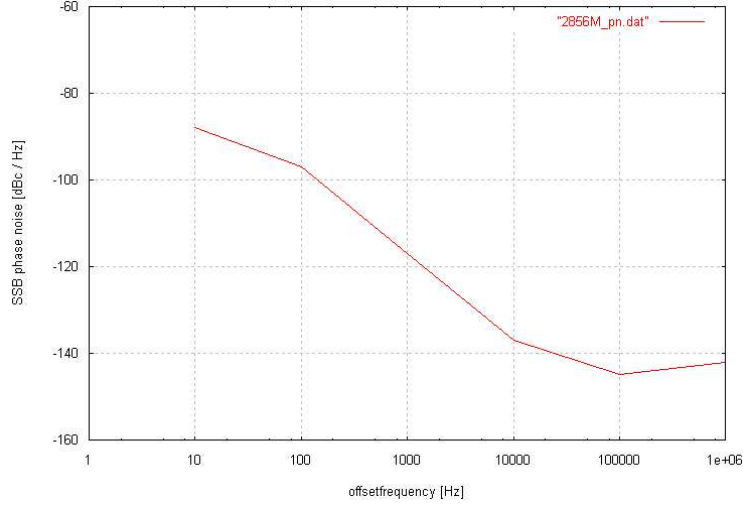


Figure 4.3: Phase Noise Requirements 2.856GHz module

Summing up all noise contributions one yields the integrated phase noise power for the single sideband in a bandwidth from 10Hz to 1MHz. The value for this reads:

$$\Delta\phi_{rms,SSB} = 1.03\text{mrad}_{rms} \quad (4.10)$$

With the assumption for small phase deviations $\phi \ll 1\text{rad}$ the double sideband reads two times the value of the single sideband integrated phase noise power:

$$\Delta\phi_{rms,DSB} = 2\phi_{SSB} = 2 \cdot 1.03\mu\text{rad}_{rms} = 2.06\text{mrad}_{rms} \quad (4.11)$$

With respect to formula 2.30 for determining the integrated timing jitter the value for it reads:

$$\Delta T_{rms} = \frac{\phi_{rms}}{2\pi f_0} = \frac{2.06\text{mrad}_{rms}}{2\pi \cdot 2.856\text{GHz}} = 114\text{fs} \quad (4.12)$$

4.2 Realization of the Master Oscillator

This section focuses on the realization of the M.O. It gives an overview of the building blocks of the M.O. and then describes the three main sources that have been chosen to develop the Master Oscillator in the way it meets the phase noise requirement goals.

4.2.1 Building Blocks

The main blocks are a low power part and a high power part. The low power part is generating all frequencies (except 1.3GHz, 1.517GHz and 2.856GHz)

that are listed in the table at the beginning of this chapter. The reference frequency of the whole Master Oscillator is a 27MHz OCXO (ovenized crystal oscillator) with a very high long term stability. This signal is multiplied by three and then phase locked in a bandwidth of approximately 400Hz to a voltage controlled crystal oscillator (VCXO) at 81MHz. All other frequencies in the low power part of the Master Oscillator are derived from these two sources. This is done with the help of multipliers, dividers, a mixer and some direct digital synthesizers (DDS). In addition some amplifiers, couplers, attenuators and filters are found in the low power part.

A block diagram of the complete low power power part is put into the appendix E.1. The 1.3 GHz will be generated with a dielectric resonator oscillator (DRO) that will be locked to the 81MHz output signal of the low power part of the Master Oscillator (Will be described in more detail in section 4.2.4).

4.2.2 The 27MHz Module

The 27MHz reference source is a SC-cutted quartz that has a very high long term stability what means that the close in phase noise is very low. This reference oven at 27MHz is first amplified with 12.8dB then attenuated by 2dB for good matching to 50Ohm, then multiplied by three, afterwards filtered by a bandpass with the center frequency at 81MHz, then attenuated by 2dB for good matching reasons and not overdriving the following amplifier, then send to an amplifier with a gain of 20.5dB, then filtered by a lowpass with the corner frequency at 88MHz and finally attenuated again. The following block diagram shows what just has been described:

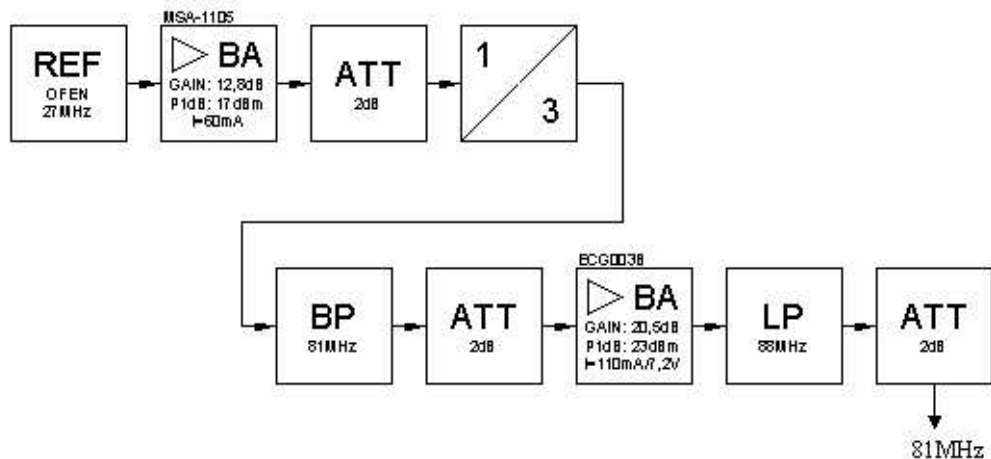


Figure 4.4: 27MHz module block diagram

For a view under the mask of the single blocks in the diagram look into appendix 4.11.

The output phase noise of the 27MHz reference has been measured with the PN9000 from Aeroflex and is shown in figure 4.5.

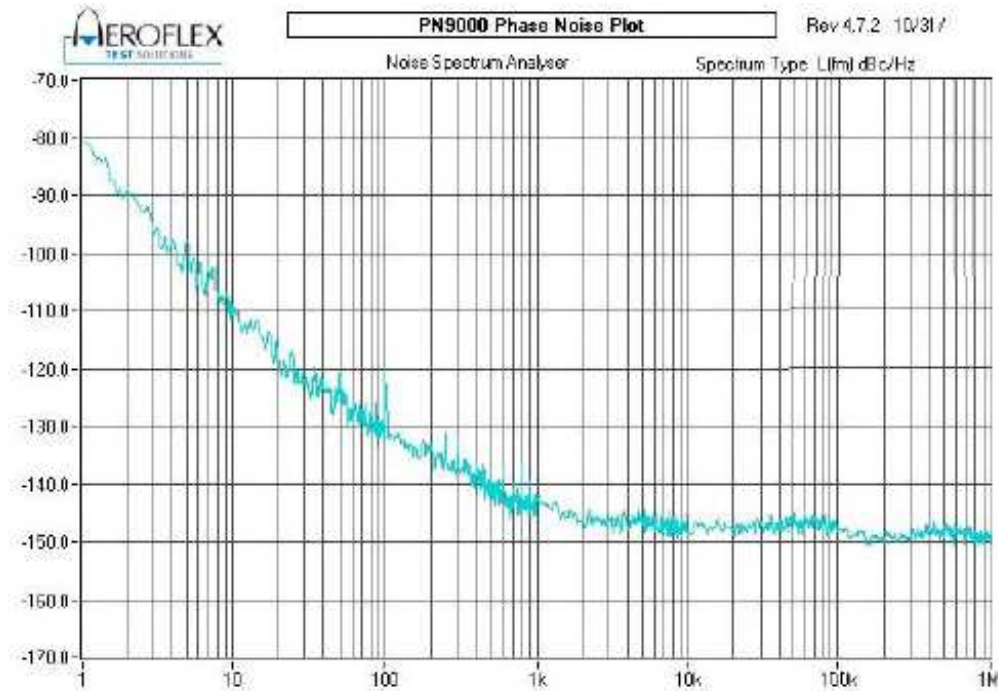


Figure 4.5: Phase Noise from 27MHz reference oven

The horizontal scale is the offset frequency f_m and the vertical scale is the SSB phase noise in $\frac{dB_c}{Hz}$. Some typical values for this are listed in the following table:

Offset frequency f_m [Hz]	SSB phase noise $[\frac{dB_c}{Hz}]$
10	-110
100	-130
1000	-142
10000	-148
100000	-148
1000000	-148

From the figure and tabular one can establish a phase noise floor at around $-148 \frac{dB_c}{Hz}$. This floor is not due to the requirements. This floor will be improved with the VCXO module at 81MHz.

After multiplying with N one gets a SSB phase noise output that is $20 \log N$ higher as shown in figure 4.5. So in this case after multiplication with 3 one

yields a phase noise output spectrum that is 9.5dB worse than the one shown in 4.5. The values for the output spectrum after multiplying with three assuming the multiplication to be ideal are listed in the following tabular:

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
10	-100.5
100	-120.5
1000	-132.5
10000	-138.5
100000	-138.5
1000000	-138.5

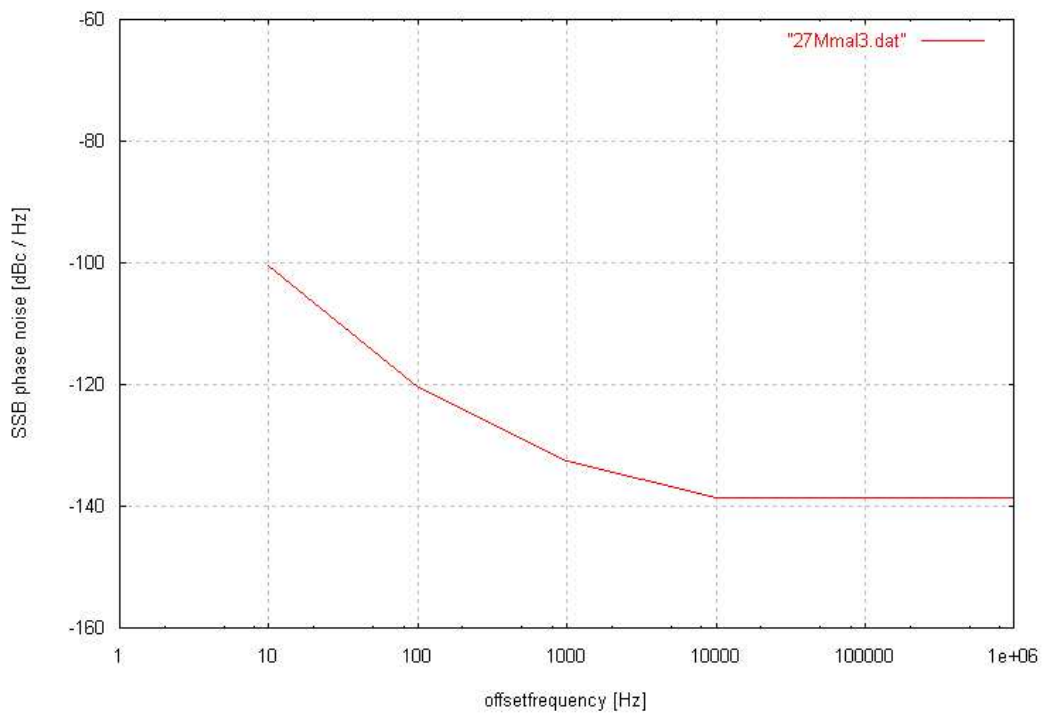


Figure 4.6: Reference source phase noise + 9.5dB

4.2.3 Locking 81MHz VCXO to 27MHz OCXO

The following blockdiagram 4.7 shows how the output signal of the 27MHz module multiplied by three and the 81MHz VCXO are locked to each other.

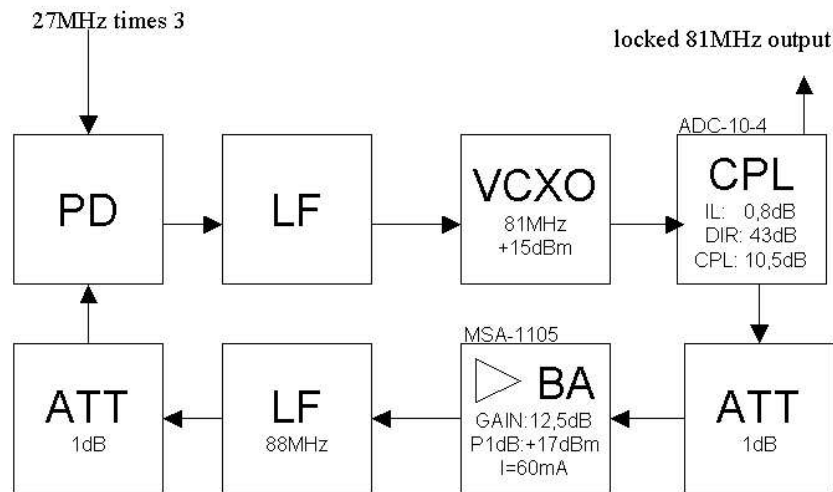


Figure 4.7: The PLL to lock the reference to the VCXO

The PD is a phase detector that detects a phase difference between the two input signals and has a voltage proportional to the input phase difference. The VCXO at 81MHz is a AT-cutted quartz with good stability in the medium frequency range (from 500Hz approximately up to 10kHz). It has a varactor diode to tune the quartz in a range of 6kHz with a DC voltage from 0 to 10V. So the gain of the VCXO is approximately 600Hz/V. The error voltage of the phase detector tunes the VCXO in a way so the resulting phase error is minimized. The LF is a lowpass filter that determines the bandwidth of the phase locked loop and will be set to approximately 400Hz. This is done to get a good overall phase noise performance at the output of the coupler CPL. Inside the loop bandwidth of 400Hz the 81MHz VCXO is locked to the 27MHz reference and one obtains a phase noise characteristic due to figure 4.6 up to an offsetfrequency of circa 400Hz. Outside this bandwidth one obtains the phase noise characteristic of the free running 81MHz VCXO. This phase noise of the free running VCXO at 81MHz has been measured and is shown in figure 4.8.

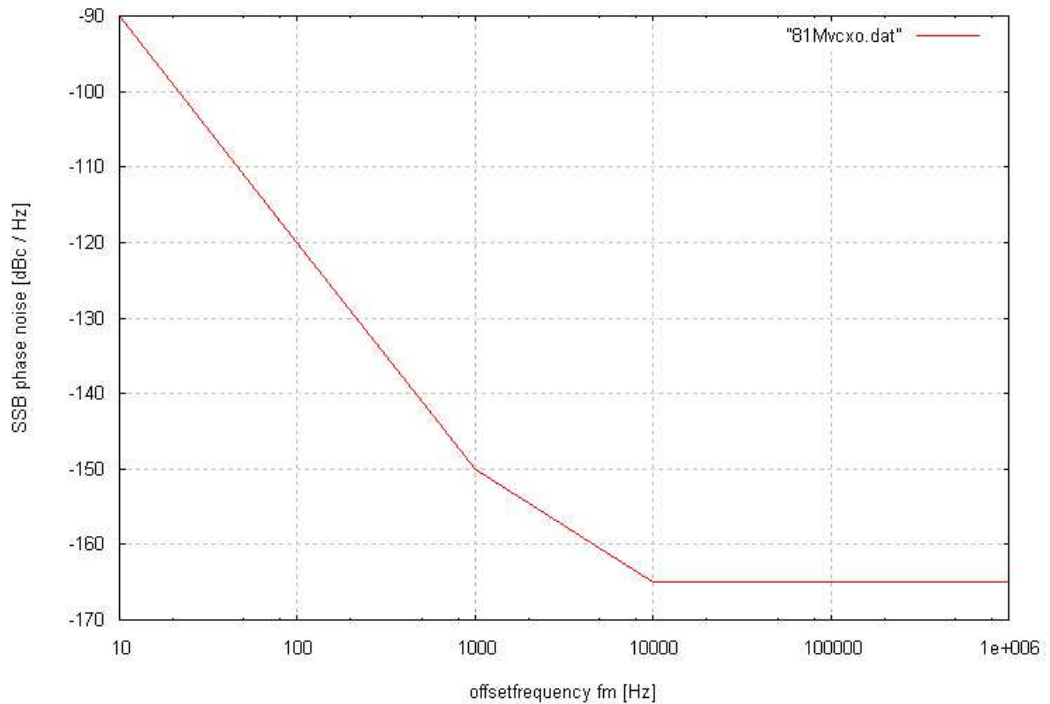


Figure 4.8: Phase Noise of free running VCXO

After locking the reference oven multiplied by three to the VCXO one obtains an improved overall output phasenoise. The expected values for this are listed in the table below and shown in figure 4.9.

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
10	-100.5
100	-120.5
1000	-150
10000	-165
100000	-165
1000000	-165

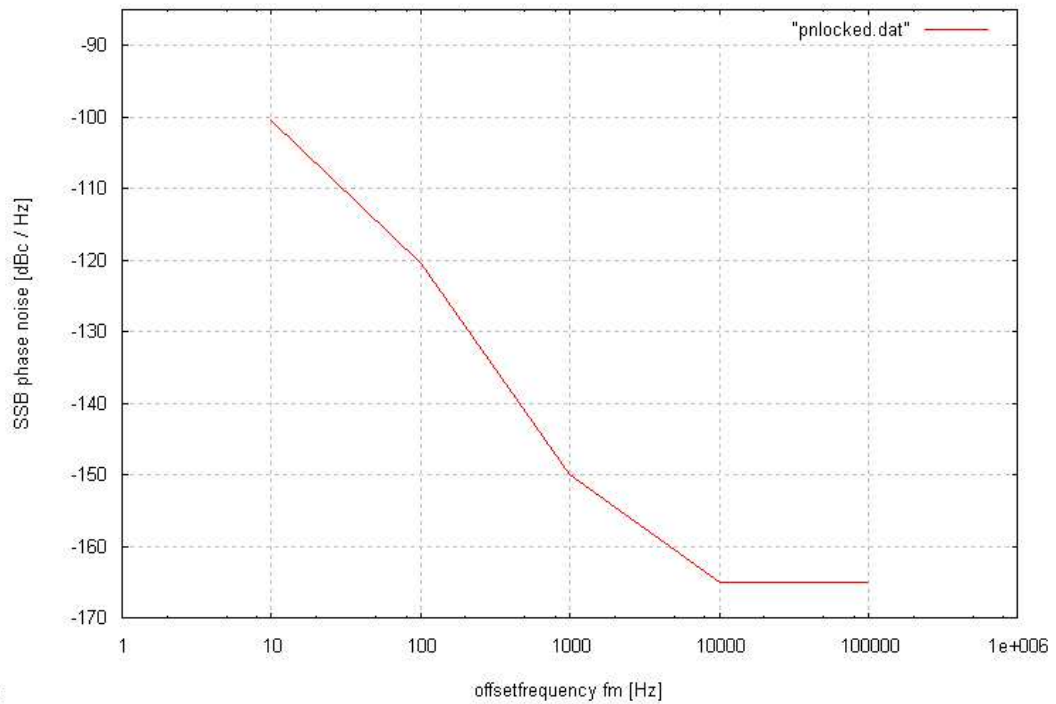


Figure 4.9: Phase Noise in locked condition with output frequency 81MHz

The 27MHz output of the Master Oscillator obtains a phase noise performance that is 3 times or 9.5dB better than the output at 81MHz. The following table shows values for the locked condition and the 81MHz divided by three. The corresponding plot is shown in figure 4.10.

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
10	-110
100	-130
1000	-159.5
10000	-174.5
100000	-174.5
1000000	-174.5

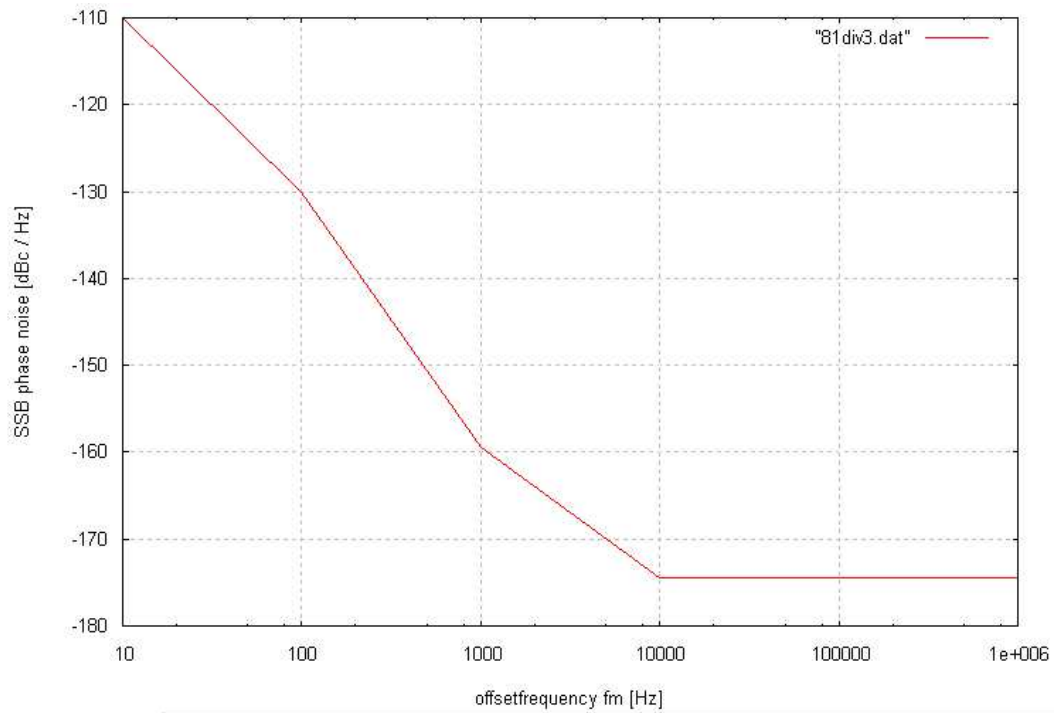


Figure 4.10: Locked condition with output frequency 27MHz

For the output phase noise at 9MHz one yields values that are 9 times or in logarithmic terms $20 \log 9 = 19db$ better than the locked oscillators with the output frequency of 81MHz.

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
10	-119.5
100	-139.5
1000	-169
10000	-184
100000	-184
1000000	-184

The single sideband of the 9MHz module output phase noise looks as follows:

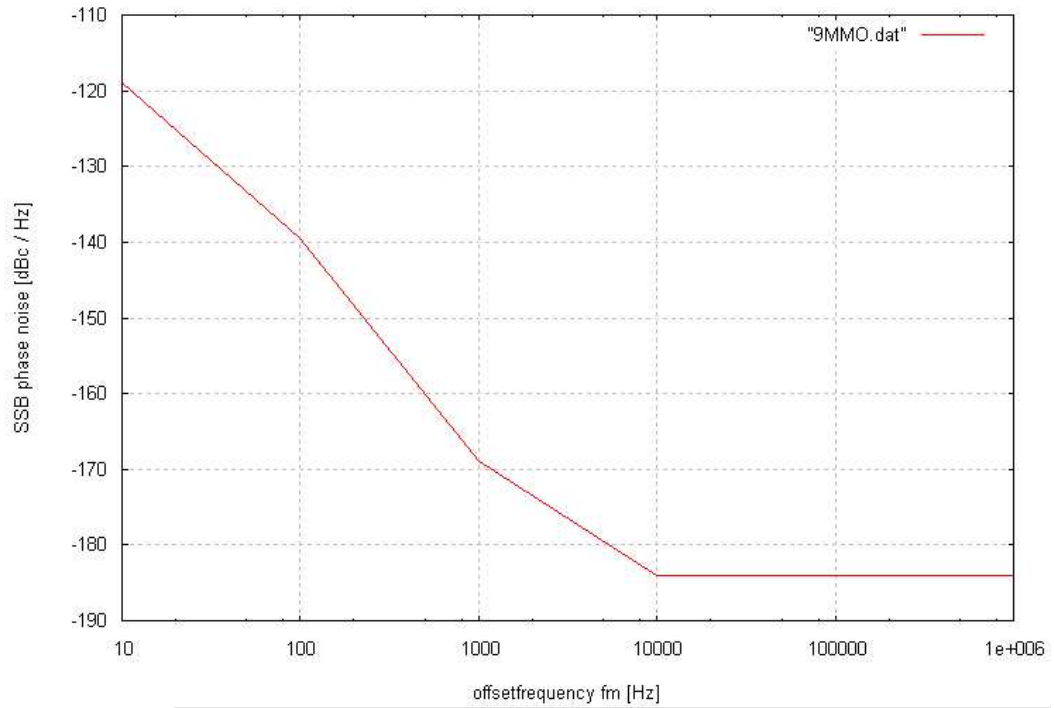


Figure 4.11: Phase Noise of Master Oscillator at 9MHz

The above mentioned phase noise plots are derived on ideal assumptions of the devices and with this exclude all losses in the dividing process. The real output phase noise of the Master Oscillator with his different modules can only be established in a real measurement setup that could not be done til now since the Master Oscillator is not assembled completely. Some additional estimations for the output phase noise at the frequency 1.3GHz will be made in chapter 4.2.4.

4.2.4 The DRO locked to 81MHz

The 81MHz output signal is locked to an oscillator that resonates at 1.3GHz. This oscillator is realized as a dielectric resonator oscillator (DRO). A small amount of the output power of the DRO is coupled out with a directional coupler and then divided by 16 with a divider chip. The 81MHz signal described in the previous section and the 1.3GHz/16 = 81MHz signal are compared with the phase detector HMC439. The output signal is the error voltage that is proportional to the input phase difference between the locked 81MHz VCXO signal and the 1.3GHz signal divided by 16. The error voltage tunes the 1.3GHz DRO to lock the DRO in the bandwidth that is determined by the LF loop filter. Inside the bandwidth of the PLL the phase noise at 1.3 GHz is then determined by the phase noise of the locked 81MHz signal while outside the bandwidth of the PLL the phase noise of the free running DRO determines the phase noise at the output. The blockdiagram 4.12 shows how the DRO should be locked to the 81MHz source.

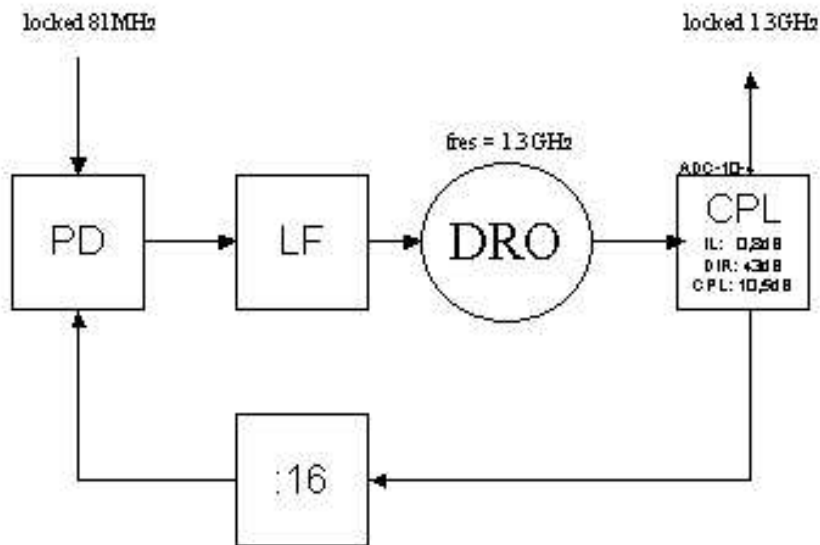


Figure 4.12: DRO locked to 81MHz signal

To achieve the phase noise goal that is due to the requirements at 1.3GHz one can estimate the phase noise inside the PLL to be the phase noise of the locked VCXO at 81MHz multiplied by 16 (81MHz x 16 = 1.3GHz) and outside the bandwidth of the PLL should reach a floor at an offset frequency $f_m = 40kHz$ of approximately $-150 \frac{dBc}{Hz}$. The phase noise excluding a PLL is $20 \log 16 = 24dB$ worse than at 81MHz. The values are calculated and plotted in figure 4.13.

Offsetfrequency f_m [Hz]	SSB phase noise [$\frac{dB_c}{Hz}$]
10	-76.5
100	-96.5
1000	-126
10000	-141
100000	-141
1000000	-141

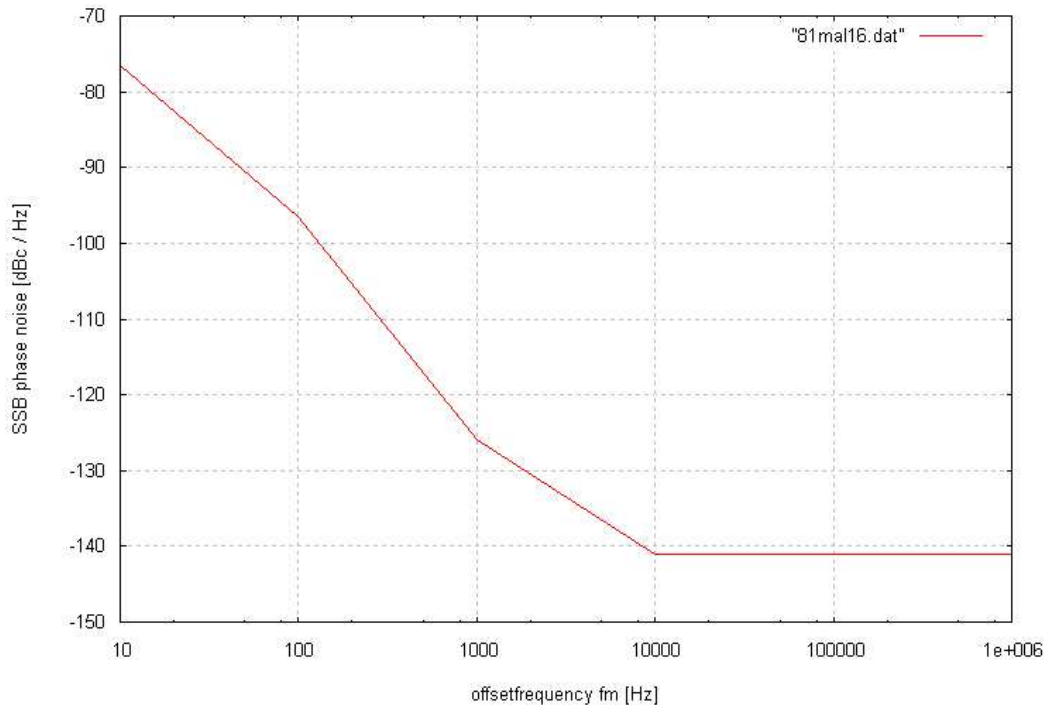


Figure 4.13: Phase Noise of locked 81MHz multiplied by 16

To improve the phase noise floor of $-141 \frac{dB_c}{Hz}$ starting at an offsetfrequency $f_m = 10kHz$ one has to lock the 81MHz signal with this bandwidth to the DRO assuming the DRO to have a better phase noise floor at this offsetfrequency. To get a phase noise floor at this offsetfrequency that is due to the requirements with a value of $-145dB_c/Hz$ at an offsetfrequency of 10kHz and a phase noise floor of $-150dB_c/Hz$ at 1MHz offsetfrequency the DRO should be designed in the proper way to achieve this goal. Some effort has been made to achieve this goal but could not be reached with the prototype.

The DRO

A dielectric resonator oscillator is almost a cylindrical pill composed of high permittivity material like bariumoxid - titanoxid with ϵ_r values in the range from 5 up to 150. The unloaded quality Q_0 is in the range of 7000 up to 10000 and is almost ten times smaller than those of crystal quartz resonators.

DRO are narrow band tunable resonators, are highly resistant for mechanical disturbances and also may be used for low phase noise applications. A loose coupling to the external load promises load independent frequency stability. With respect to figure 4.11 the resonator can be modeled by a parallel resonant circuit but with half the inductance on both sides of the circuit (figure 4.11). It is common to couple the excited field in the resonator to microstrips that are placed close to the pill. The distance between pill and microstrip effects the amount of coupling between the pill and the microstrip.

A prototype DRO has been assembled and its schematic is shown in appendix E.4. The output resonance is at 1.3GHz and its harmonics 2.6GHz, 3.9GHz etc. but the phase noise characteristic unfortunately is not due to the requirements.

Chapter 5

The Measurement Setup and Results

5.1 Chosen method for measuring the phase noise

The most sensitive method for measuring phase noise is the phase locked loop method. So this method has been chosen to be realized. The sensitivity for the phase noise measurement is limited by the first device in the setup (Figure: 5.1) which is a double balanced mixer working as a phase detector. A setup with a phase detector and a double balanced mixer has been realized and is presented in this chapter.

5.1.1 Mixer

The double balanced mixer is chosen from the company Minicircuits. It is the high level mixer TUF-5H. The mixer is a three port device with an RF (radio frequency) input, an LO (local oscillator) input and an IF (intermediate frequency) output. The sources to be tested will be put to the RF and LO port and the signal at the IF port is - after filtering out the sum-frequencies that are produced by the mixing process - represents the phase fluctuations. The double balanced mixer is composed of four diodes (Figure: 5.1) and the mixing process can be understood as a switching process of the diodes in the frequency of the LO signal. During the positive half period of the LO signal the diodes CR1 and CR3 are conducting and in the negative half period the diodes CR2 and CR4 are conducting. The output signal at the IF (intermediate frequency port) is a multiplication of the alternating positive and negative LO (local oscillator) signal and the signal at the RF (Radio Frequency) port.

For proper operation ("lossless switching") of the mixer one has to set high input levels at the LO and RF port. For the TUF5-H these levels are for the LO input signal: 20dBm. The operating frequency range of the TUF-5H is

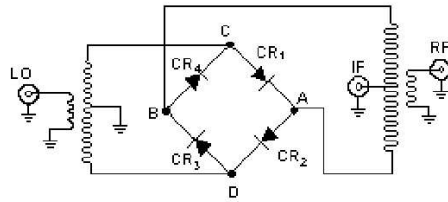


Figure 5.1: Double Balanced Mixer [7]

between 20MHz ; f_{RF} ; 1.5GHz. Some important notes of the mixer at the three main source frequencies are:

- @27MHz:
 - conversion loss \approx 6dB
 - isolation LO - RF Port: 75dB
 - isolation LO - IF Port: 50dB
 - maximum phase detection RF - LO: $U = -830\text{mV}$
- @81MHz:
 - conversion loss \approx 5.5dB
 - isolation LO - RF Port: 60dB
 - isolation LO - IF Port: 44dB
 - maximum phase detection RF - LO: $U = -900\text{mV}$
- @1.3GHz:
 - conversion loss \approx 7dB
 - isolation LO - RF Port: 35dB
 - isolation LO - IF Port: 20dB
 - maximum phase detection RF - LO: $U = -700\text{mV}$

For additional information application notes from MINICIRCUITS [7].

5.1.2 Phase Detector

The phase detector is a device that outputs a voltage at the output that is proportional to the phase difference of two RF signals. One source with a tuning input is be tuned by this error signal and the two sources will be in quadrature.

For measuring phase noise the phase detector HMC439 from HITTITE seems to be promising. It has a phase noise floor of $-153 \frac{\text{dBc}}{\text{Hz}}$ (Figure: 5.2).

Some other important characteristics for this chip are listed above:

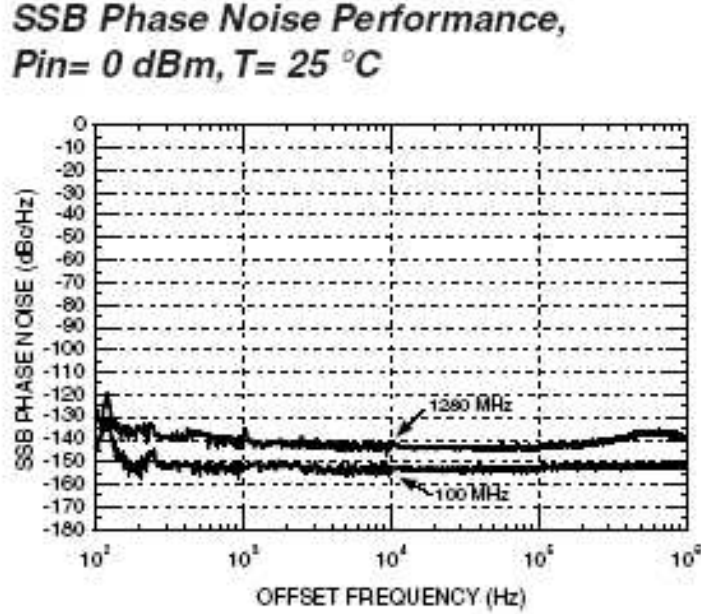


Figure 5.2: Phase Noise Floor HMC439 [4]

- input power ranges from $-10dBm < P_{in} < +10dBm$
- frequency range from $10MHz < f_{input} < 1.3GHz$

5.1.3 Loop Filter for locking the sources

For locking the two sources and for measuring the phase noise close to the carrier what means at offsetfrequencies f_m 100Hz a loop filter with a bandwidth in this range is necessary. The loop filter is realized with the operation amplifier LT1028 with a big time constant τ inversely proportional to the offsetfrequency f_m :

$$Offsetfrequency f_m = Loopbandwidth B = \frac{1}{Timeconstant \tau} \quad (5.1)$$

For the experimental setup it is common to start with locking the sources in a bigger bandwidth because it is more difficult to lock the sources with higher time constants and then step by step increase the time constant of the loop filter to decrease the bandwidth B so the oscillators are locked to each other in more and mor decreasing bandwidths. Outside the loop bandwidth it is possible to measure the phase noise of the oscillators.

The optimal configuration for the resistors and the capacitors will be found out experimentally. After a long experimenting time with the gain and the time constant of the loop filter it was possible to lock the VCXO at 81MHz

to the signal generator. The DC signal of the locked oscillators is shown in figure 5.3.

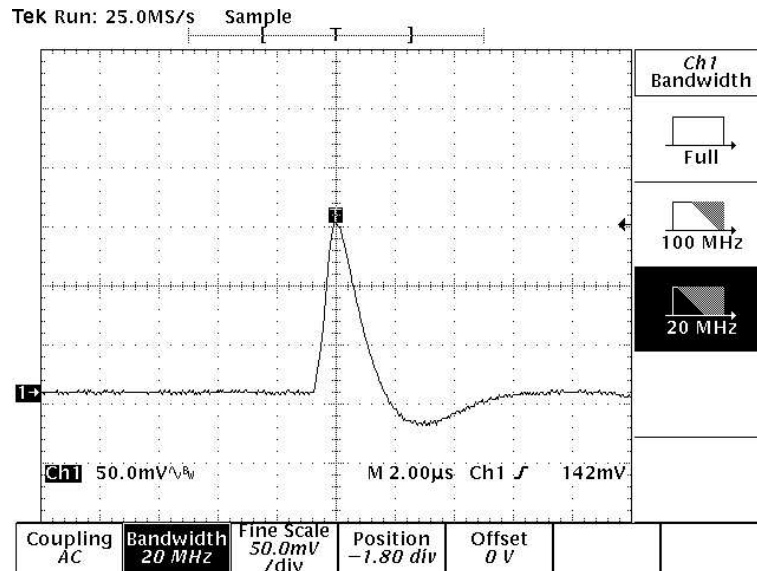


Figure 5.3: The signal at the output from the Loop Filter

5.1.4 The Low Noise Amplifier

Since the output signal of the mixer is in the nanovolt range (estimations follow in section 5.3) and the ordinary FFT - Analyzer or spectrum analyzer are not sensitive enough to detect this signal one has to amplify it.

The amplifier needs to have a lower noise level than the noise output signal of the mixer. For this purpose an amplifier has been built that is promisingly in the voltage noise density region of $\frac{1nV}{\sqrt{Hz}}$. The features of this amplifier are:

- Power Gain of 40dB and higher
- Voltage noise floor of $\frac{1nV}{\sqrt{Hz}}$ at a low offset frequency
- Has a bandwidth from DC to 1MHz

The noise spectral density can be seen in figure 5.4

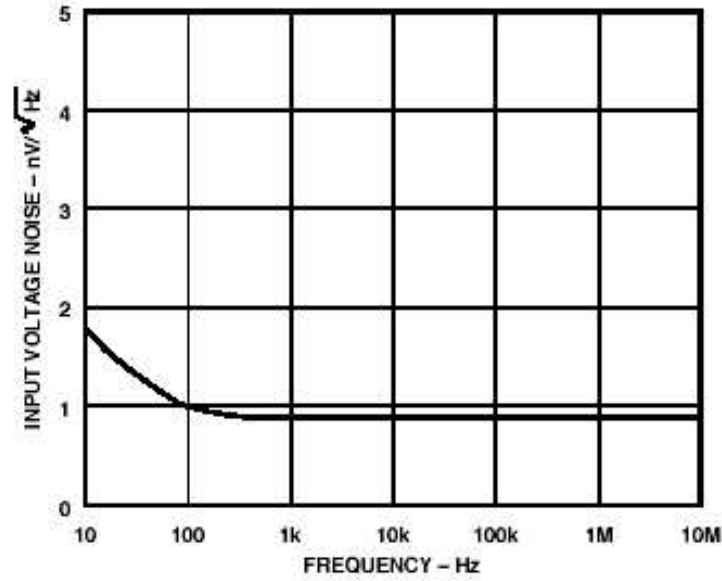


Figure 5.4: Voltage noise spectral density[3]

5.2 Expected Values For The Measurement

For the measurement setup the VCXO with $f_0 = 81.2524MHz$ and the signal generator SML03 from "Rohde und Schwarz" with the same frequency have been chosen. The setup is implemented with the double balanced mixer TUF-5H [7]. The setup is realized like the setup in chapter 3.3. Due to formula 3.25 and the assumption of a constant K_ϕ one obtains some values that will be expected from the measurement setup. These values can be calculated by rearranging formula 3.25 to:

$$\Delta V_{rms}^2(f_m) = 4\mathcal{L}(f_m) V_{B,rms}^2 1Hz \quad (5.2)$$

The sensitivity K_ϕ can be established by getting a beat note out of the mixer. You observe a beat note at the output of the mixer when you plug a level of $16dB_m$ from the signal generator "SML03" to the LO Port of the mixer and an RF Level of $13dB_m$ from the 81MHz VCXO to the RF Port and a small frequency shift $\Delta f = 1kHz$ is set on the signal generator. This frequency shift is the frequency that you observe at the IF Port of the mixer. This beat note is shown in figure 5.5.

The slope of this signal is the sensitivity of the phase detection that is highly dependent on the input signal power at the LO and RF port. In this case the beatnote K_ϕ is:

$$K_\phi = \frac{2.5V}{180^\circ} \frac{360^\circ}{2\pi} = 800 \frac{mV}{rad} \quad (5.3)$$

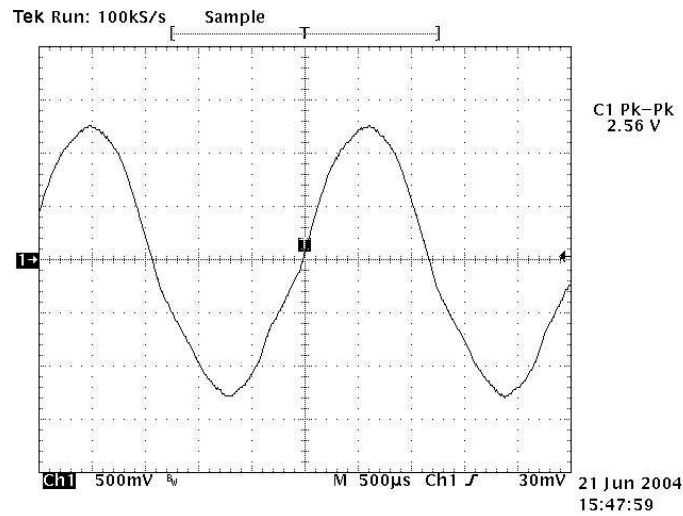


Figure 5.5: Beat note from double balanced mixer TUF-5H

With this beat-note one can estimate the powerlevels in dB_m that you may read out on the spectrum analyzer. The expected values are listed in the following tabular.

Offsetfrequency [Hz]	SSB_{lin}	$\sqrt{\Delta V}$	ΔV^2 at 50 Ω
10	10^{-9}	$50.6 \frac{\mu V}{\sqrt{Hz}}$	$-73 \frac{dB_m}{Hz}$
100	10^{-12}	$1.6 \frac{\mu V}{\sqrt{Hz}}$	$-103 \frac{dB_m}{Hz}$
1000	10^{-15}	$50.6 \frac{nV}{\sqrt{Hz}}$	$-132 \frac{dB_m}{Hz}$
10000	$10^{-16.5}$	$9 \frac{nV}{\sqrt{Hz}}$	$-147 \frac{dB_m}{Hz}$
100000	$10^{-16.5}$	$9 \frac{nV}{\sqrt{Hz}}$	$-147 \frac{dB_m}{Hz}$
1000000	$10^{-16.5}$	$9 \frac{nV}{\sqrt{Hz}}$	$-147 \frac{dB_m}{Hz}$

When measuring with a resolution bandwidth of 100Hz from the spectrum analyzer these values are $10 \log 1.2 * 100 Hz = 20 dB$ higher. So the smallest value at an offsetfrequency of 1MHz is $-127 dB_m$ and the highest value is at an offsetfrequency of 10Hz with $-53 dB_m$.

5.3 Measurement

The noise floor of the spectrum analyzer without any input signals from 0Hz to 1MHz with a resolution bandwidth of 100Hz is shown in figure 5.6.

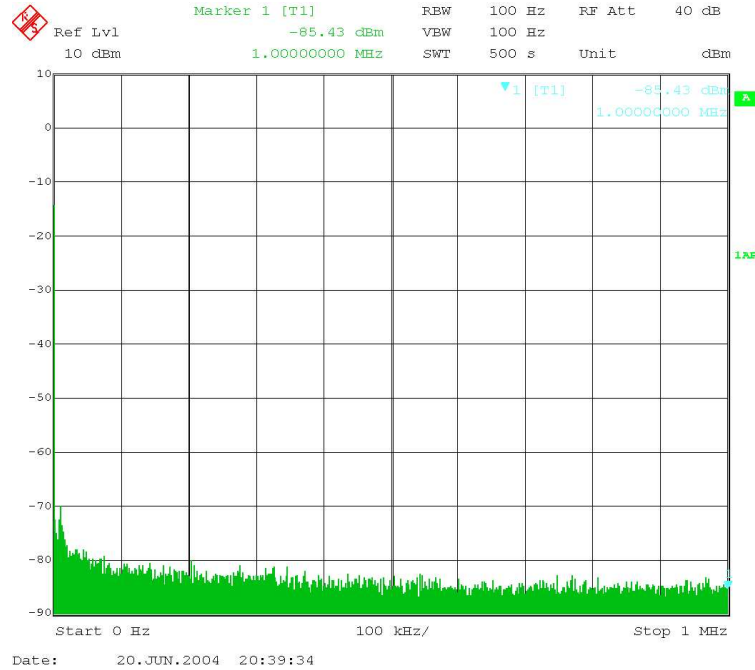


Figure 5.6: Noise Floor from Spectrum Analyzer

The noise floor is approximately at a power level of $-105dB_m$ when taking into account the noise bandwidth decrease of 20dB (Resolution bandwidth normalized to 1Hz). The expected power levels up to an offset frequency of 1kHz are directly measurable with the spectrum analyzer. Including an additional low noise amplifier as introduced in section 5.1.4 one is supposed to measure values that are below the dynamic range of the spectrum analyzer.

The self assembled amplifier realized as two cascaded operation amplifiers with a power gain of 40dB, matched to 50Ohm at the input is connected to the spectrum analyzer what is shown in figure 5.7.

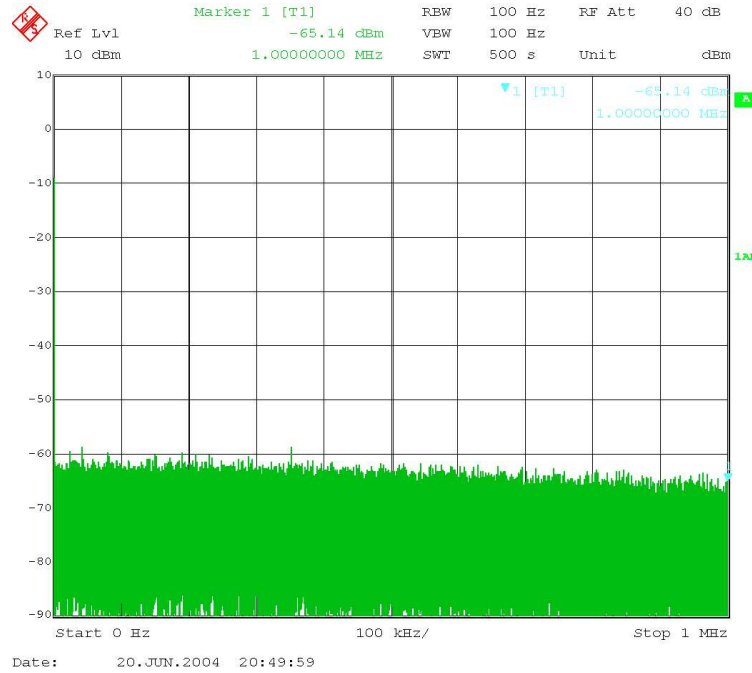


Figure 5.7: Noise Floor with amplifier connected to spectrum analyzer

The noise floor of the amplifier is approximately at $-65dB_m - 40dB(Gain) - 20dB(Resolution\ Bandwidth\ Correction) = -127dB_m$. This floor at $-127dB_m$ is the measuring limit of this setup.

The next step was to couple the output signal at the IF Port of the mixer to the input of the amplifier and look at the output spectrum that is shown in figure 5.8

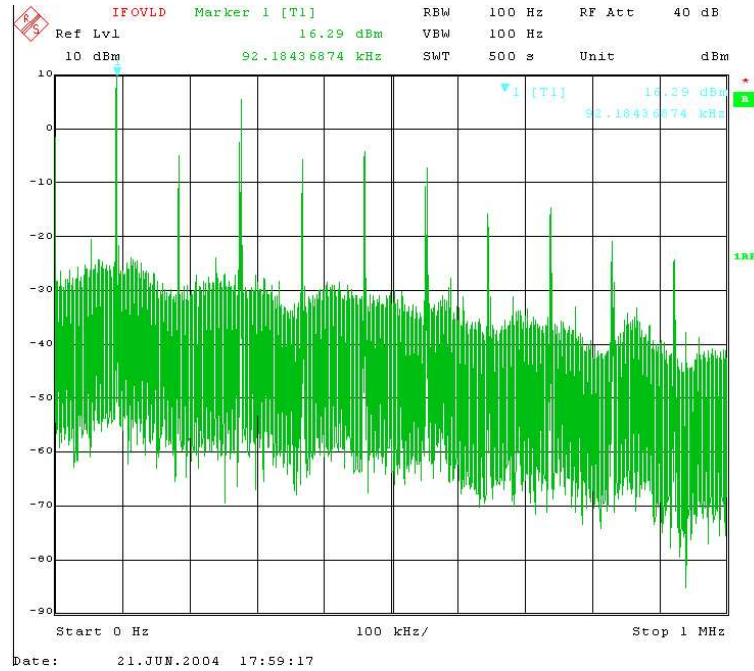


Figure 5.8: Output spectrum of amplified mixer IF-Port

The expected values after amplifying with 40dB and a resolution bandwidth correction of 20dB for the characteristic offsetfrequencies in the spectrum are listed in the following table. The measurement in figure 5.8 shows no relation to the expected values.

Offsetfrequency [Hz]	$P[\frac{dBm}{Hz}]$	$P[\frac{dBm}{Hz}] + 40dB + 20dB$
10	-73	-13
100	-103	-43
1000	-132	-72
10000	-147	-87
100000	-147	-87
1000000	-147	-87

These theoretical values cannot be obtained since the noise floor of the amplifier is at $-127dB_m$. At smaller offsetfrequencies than 1kHz the noise floor of the amplifier is smaller than the value one needs to amplify. But at smaller offsetfrequencies the two oscillators are locked and you will not measure the phase noise of the free running VCXO.

Chapter 6

Conclusion and outlook

Unfortunately the expected values can not be measured with this setup. The output spectrum shown in figure 5.8 is not comparable to the predicted values. Comparing the two amplifier stages output spectrum shown in figure 5.7 and the noise floor of the spectrum analyzer in figure 5.6 shows that the amplifier assuming a power gain of 40dB is able to amplify signals that are 20dB below the spectrum analyzers noise floor. The 50 Ohm input noise floor of the amplifiers are approximately -127dBm. That is not sufficiently low for measuring the predicted noise floor of the VCXO at -147dBm/Hz. After coupling the noise signal from the Mixer to the amplifiers the spectral analysis after the amplifiers does not include any correlation to the expectations.

In the future more effort will be put on the low noise amplifiers that should reach a lower noise floor. The mixer output coupling to the low noise amplifier will be focused on to obtain some reasonable output spectrum that is comparable to the predicted values for this setup.

The treatment of this topic gave me a good insight to characterize the quality of oscillators as well as the practical problems that occur during the work on this subject. It also introduced me to closely related problems like the distribution of signals required in an accelerator like the Tesla Test Facility and their stability far away from the point where they have been generated. Understanding phase noise and building a setup to measure phase noise are two different subjects - I will go on studying both in the future.

Appendix A

Phase Modulation

The phase modulated signal

$$u_{PM} = A [\sin(\omega_0 t) \cos(\hat{\Phi} \sin(\Delta\omega t)) + \cos(\omega_0 t) \sin(\hat{\Phi} \sin(\Delta\omega t))] \quad (\text{A.1})$$

using the identities:

$$\cos(x \sin y) \equiv J_0(x) + 2 [J_2(x) \cos(2y) + J_4(x) \cos(4y) + \dots] \quad (\text{A.2})$$

and

$$\sin(x \sin y) \equiv 2 [J_1(x) \sin y + J_3(x) \sin 3y + \dots] \quad (\text{A.3})$$

where $J_0(x), J_1(x), \dots$ are Bessel Functions of the first kind of argument (x) and order $0, 1, \dots$ respectively. reads:

$$\begin{aligned} u_{PM} = A \sin \omega t [& J_0(\hat{\phi}) + 2J_2(\hat{\phi}) \cos 2\Delta\omega t \\ & + 2J_4(\hat{\phi}) \cos 4\Delta\omega t + \dots] \\ & + \cos \omega t [2J_1(\hat{\phi}) \sin \Delta\omega t \\ & + 2J_3(\hat{\phi}) \sin 3\Delta\omega t + \dots] \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} u_{PM} = A [& J_0(\hat{\phi}) \sin \omega t + 2J_2(\hat{\phi}) \cos 2\Delta\omega t \sin \omega t \\ & + 2J_4(\hat{\phi}) \cos 4\Delta\omega t \sin \omega t + \dots] \\ & + A [2J_1(\hat{\phi}) \cos \omega t \sin \Delta\omega t \\ & + 2J_3(\hat{\phi}) \cos \omega t \sin 3\Delta\omega t + \dots] \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} u_{PM} = A [& J_0(\hat{\phi}) \sin \omega t + J_2(\hat{\phi}) \sin(\omega + 2\Delta\omega)t \\ & + J_2(\hat{\phi}) \sin(\omega - 2\Delta\omega)t \\ & + J_4(\hat{\phi}) \sin(\omega + 4\Delta\omega)t \\ & + J_4(\hat{\phi}) \sin(\omega - 4\Delta\omega)t + \dots] \\ & + A [J_1(\hat{\phi}) \sin(\omega + \Delta\omega)t \end{aligned}$$

$$\begin{aligned}
& - J_1(\hat{\phi}) \sin(\omega - \Delta\omega)t \\
& + J_3(\hat{\phi}) \sin(\omega + 3\Delta\omega)t \\
& + J_3(\hat{\phi}) \sin(\omega - 3\Delta\omega)t + \dots] \quad (\text{A.6})
\end{aligned}$$

$$\begin{aligned}
u_{PM} = A [& \underbrace{J_0(\hat{\phi}) \sin \omega t}_{\text{carrier}} + \underbrace{J_1(\hat{\phi}) \sin(\omega + \Delta\omega)t}_{\text{first upper sideband}} \\
& - \underbrace{J_1(\hat{\phi}) \sin(\omega - \Delta\omega)t}_{\text{first lower sideband}} \\
& + \underbrace{J_2(\hat{\phi}) \sin(\omega + 2\Delta\omega)t}_{\text{second upper sideband}} \\
& - \underbrace{J_2(\hat{\phi}) \sin(\omega - 2\Delta\omega)t}_{\text{second lower sideband}} \\
& + \dots] \quad (\text{A.7})
\end{aligned}$$

For small values of $\hat{\phi}$ meaning that we have small angular phase modulation such as occur in random noise modulation one yields approximations as follows [17]:

$$J_0(\hat{\phi}) \approx 1 \quad (\text{A.8})$$

$$J_1(\hat{\phi}) \approx \frac{\hat{\phi}}{2} \quad (\text{A.9})$$

$$J_2(\hat{\phi}) \approx J_3(\hat{\phi}) \approx J_4(\hat{\phi}) \approx 0 \quad (\text{A.10})$$

For this approximation formula A.7 reduces to:

$$\begin{aligned}
u_{PM} = A [& \sin \omega t + \frac{\hat{\phi}}{2} \sin(\omega + \Delta\omega)t \\
& - \frac{\hat{\phi}}{2} \sin(\omega - \Delta\omega)t] \quad (\text{A.11})
\end{aligned}$$

Appendix B

Transfer function of setup with delay line

Consider a measurement setup as in Figure 3.4 and let the source signal be:

$$v_{source} = \hat{v} \cos [\omega_0 t + \frac{\Delta\omega}{\omega_m} \cos(\omega_m t)] \quad (\text{B.1})$$

After splitting the signal and sending one signal through the delay-line one yields the two following signals at the input port from the mixer:

$$\begin{aligned} \underline{\text{Port 1}} & : \\ v_{delay} & = \frac{\hat{v}}{2} \cos [\omega_0(t - \tau_d) + \frac{\Delta\omega}{\omega_m} \cos(\omega_m(t - \tau_d))] \quad (\text{B.2}) \end{aligned}$$

$$\begin{aligned} \underline{\text{Port 2}} & : \\ v_{\overline{delay}} & = \frac{\hat{v}}{2} \cos [\omega_0 t + \frac{\Delta\omega}{\omega_m} \cos(\omega_m t)] \quad (\text{B.3}) \end{aligned}$$

Assuming the mixing process to be linear the output port of the mixer delivers the product of the delayed (B.2) and non-delayed (B.3) signal that reads:

$$\begin{aligned} v_{Intermediate} = K_\phi \cos [\omega_0(t - \tau_d) & + \frac{\Delta\omega}{\omega_m} \cos(\omega_m(t - \tau_d)) \\ & - \omega_0 t - \frac{\Delta\omega}{\omega_m} \cos \omega_m t] \\ & + \cos[\omega_0(t - \tau_d) + \frac{\Delta\omega}{\omega_m} \cos \omega_m(t - \tau_d) \\ & + \omega_0 t + \frac{\Delta\omega}{\omega_m} \cos \omega_m t] \quad (\text{B.4}) \end{aligned}$$

Now assuming the sum frequency to be low pass filtered and rearranging the equation one yields:

$$v_{filtered} = K_\phi \cos[-2\pi\omega_0\tau_d + 2 \frac{\Delta\omega}{\omega_m} \sin(\pi \frac{\omega_m}{2} \tau_d) \sin \omega_m (t - \frac{\tau_d}{2})] \quad (\text{B.5})$$

APPENDIX B. TRANSFER FUNCTION OF SETUP WITH DELAY LINE 62

With both signals in quadrature at Port 1 and Port 2 meaning following assumption:

$$\begin{aligned}\omega_0\tau_d &= (2K + 1)\frac{\pi}{2} \\ \text{with } K &= 0, 1, 2, 3, 4, \dots\end{aligned}\tag{B.6}$$

equation B.5 reads:

$$v_{quadrature} = K_\phi \underbrace{\sin\left[2\frac{\Delta\omega}{\omega_m} \sin\left(\pi\frac{\omega_m}{2}\tau_d\right) \sin\omega_m\left(t - \frac{\tau_d}{2}\right)\right]}_{\text{for } \frac{\Delta\omega}{\omega_m} < 0.2 \text{ rad} \approx 2\frac{\Delta\omega}{\omega_m} \sin\left(\pi\frac{\omega_m}{2}\tau_d\right) \sin\omega_m\left(t - \frac{\tau_d}{2}\right)}\tag{B.7}$$

The Transfer Function now reads:

$$\begin{aligned}\Delta V &= K_\phi 2\frac{\Delta\omega}{\omega_m} \sin(\pi\omega_m\tau_d) \\ &= K_\phi 2\pi\tau_d\Delta\omega \underbrace{\frac{\sin(\pi\omega_m\tau_d)}{(\pi\omega_m\tau_d)}}_{\approx 1 \text{ for } \omega_m < \frac{1}{2\pi\tau_d}}\end{aligned}\tag{B.8}$$

The simplified transfer function for the setup in Figure 3.4 then reads:

$$\Delta V = K_\phi 2\pi\tau_d\Delta\omega\tag{B.9}$$

Appendix C

Mixer operating as a Phase Detector

A mixer operating as a phase detector outputs a voltage $v(t)$ that is proportional to the fluctuating phase difference between the input signal at the L-Port and the R-Port. Assuming two input like given in figure C.1:

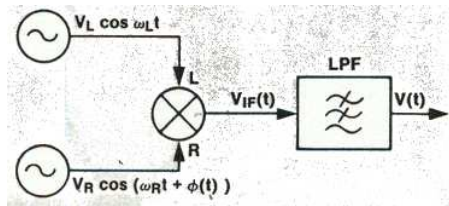


Figure C.1: Mixer operating as phase detector

The output signal of the mixer after lowpass-filtering the sum frequencies reads:

$$u(t) = K_L V_R \cos[(\omega_R - \omega_L)t + \phi(t)] \quad (\text{C.1})$$

Where K_L is the conversion loss of the mixer. The signal looks like figure C.2

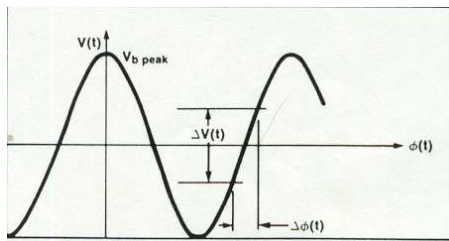


Figure C.2: Filtered mixer output

The peak amplitude is:

$$u_{beat} = K_L V_R \quad (C.2)$$

where K_L is the mixer efficiency or conversion loss in the mixer, then equation C.1 becomes:

$$u(t) = u_{beat} \cos[(\omega_R - \omega_L)t + \phi(t)] \quad (C.3)$$

With $\omega_L = \omega_R$ and $\phi(t) = 90deg + \Delta\phi(t)$ we come to:

$$\Delta u(t) = u_{beat} \sin \Delta\phi(t) \quad (C.4)$$

For $\Delta\phi_{peak} \ll 1radian$ leads to $\sin\Delta\phi(t) \approx \Delta\phi(t)$ and equation C.4 shows a linear relationship between the voltage and phase fluctuations:

$$\Delta u(t) = u_{beat} \Delta\phi(t) \quad (C.5)$$

If the beat signal u_{beat} is K_ϕ the output voltage fluctuations read:

$$\Delta u = K_\phi \Delta\phi \quad (C.6)$$

The phase detector constant K_ϕ is the measured rms value times $\sqrt{2}$! When looking at the voltage as a function of the offsetfrequency f_m the function reads:

$$\Delta u(f_m) = K_\phi \Delta\phi(f_m) \quad (C.7)$$

That means that the voltage displayed on the spectrum analyzer are a means to describe $\Delta\phi_{rms}(f_m)$:

$$\Delta\phi_{rms}(f_m) = \frac{1}{K_\phi} \Delta u_{rms}(f_m) \quad (C.8)$$

Appendix D

Converting Phase Noise to Allan Variance

To convert phase noise to Allan Variance [15] make use of the following tabular. It separates the different slope regions of phase noise with the factor α .

$S_\phi(f_m) \approx f_m^{-\alpha}$	$\sigma_y^2(\tau) = K S_\phi(f_m)$
$\alpha = 0$	$K = \frac{3B_m}{4\pi^2 f_0^2} \frac{1}{\tau^2}$
$\alpha = 1$	$K = \frac{f_m}{4\pi^2 f_0^2} \frac{(1.038 + 3\ln(2\pi B_m \tau))}{\tau^2}$
$\alpha = 2$	$K = \frac{f_m^2}{2f_0^2} \frac{1}{\tau}$
$\alpha = 3$	$K = \frac{f_m^3}{f_0^2} 2 \ln(2)$
$\alpha = 4$	$K = \frac{f_m^4}{3f_0^2} 2\pi^2 \tau$

Appendix E

Block Diagrams and schematics

- E.1 Master Oscillator block diagram
- E.2 Schematics of 27MHz module
- E.3 Schematic of the lockboard
- E.4 DRO schematic

blockdiagramm

27MHz modul

lockboard1

lockboard2

DROSCHEMATIC

List of Figures

2.1	Random fluctuations in time domain [1]	5
2.2	Spectral density in a single sideband [1]	6
2.3	Long term stability [15]	7
2.4	Short term stability [15]	7
2.5	The ideal source	8
2.6	Amplitude Noise	9
2.7	Phase Noise	9
2.8	Power distribution of a real signal source [1]	9
2.9	Calculating the Allan variance from DSB phase noise [15]	14
2.10	Oscillator block diagram [13]	14
2.11	Parallel resonance circuit [10]	15
2.12	Normalized bandwidth of a resonator [10]	15
2.13	Phase Noise Model by Leeson [12]	17
2.14	The equivalent phase locked loop [12]	17
3.1	Measuring phase noise with a spectrum analyzer [15]	20
3.2	Example SSB Phase Noise with direct measurement method [5]	21
3.3	Error between the rectangular filter and the gaussian filter [15]	22
3.4	Measurement setup with delay line method [15]	23
3.5	FM calibration signal [9]	25
3.6	PLL locked method [15]	27
3.7	Determining $K\phi$ [15]	28
3.8	Response of PLL locked method [14]	29
4.1	Phase Noise Requirements 9MHz module	33
4.2	Phase Noise Requirements 1.3GHz module	34
4.3	Phase Noise Requirements 2.856GHz module	36
4.4	27MHz module block diagram	37
4.5	Phase Noise from 27MHz reference oven	38
4.6	Reference source phase noise + 9.5dB	39
4.7	The PLL to lock the reference to the VCXO	40
4.8	Phase Noise of free running VCXO	41
4.9	Phase Noise in locked condition with output frequency 81MHz	42

4.10	Locked condition with output frequency 27MHz	43
4.11	Phase Noise of Master Oscillator at 9MHz	44
4.12	DRO locked to 81MHz signal	45
4.13	Phase Noise of locked 81MHz multiplied by 16	46
5.1	Double Balanced Mixer [7]	49
5.2	Phase Noise Floor HMC439 [4]	50
5.3	The signal at the output from the Loop Filter	51
5.4	Voltage noise spectral density[3]	52
5.5	Beat note from double balanced mixer TUF-5H	53
5.6	Noise Floor from Spectrum Analyzer	54
5.7	Noise Floor with amplifier connected to spectrum analyzer . .	55
5.8	Output spectrum of amplified mixer IF-Port	56
C.1	Mixer operating as phase detector	63
C.2	Filtered mixer output	63

Bibliography

- [1] Application Note Aeroflex/Comstron. Phase noise theory and measurement. Internet, www.aeroflex.com.
- [2] D.B.Leeson, editor. *A simple model of feedback oscillator noise spectrum*. IEEE Proceedings, vol.54, February 1966.
- [3] ANALOG DEVICES. *Datasheet from AD797*. www.analog.com.
- [4] HITTITE. *Datasheet from phase detector HMC439*. www.hittite.com.
- [5] K.
- [6] Darko Kajfez and Pierre Guillon. *Dielectric Resonators*. Noble Publishing Corp., 1998.
- [7] MINICIRCUITS. *FAQ's on double balanced mixers*. www.minicircuits.com.
- [8] R. Mueller. *Halbleiterelektronik, Band 15 , Rauschen*. Springer Verlag, 1990.
- [9] Application Note/Hewlett Packard. Phase noise characterization of microwave oscillators. Product Note: 11729C-2. Frequency discriminator method.
- [10] David M. Pozar. *Microwave Engineering*. John Wiley and Sons, Inc., 1998.
- [11] Ulrich Rohde. *Digital PLL Frequency Synthesizers / Theory and Design*. Prentice Hall, Inc., Eaglewood Cliffs, N.J., 07632, 1983.
- [12] Vendelin / Pavo / Rohde. *Microwave Circuit Design Using...* John Wiley and Sons, Inc., 1990.
- [13] Dieter Scherer. *Entwicklungsprinzipien und pruefmethoden fuer hf - und mikrowellengeneratoren mit niedrigem phasenrauschen*, 1981. *nachrichten elektronik* 35.

- [14] Dieter Scherer, editor. *The Art of Phase Noise Measurement*. Hewlett Packard, May 1983.
- [15] Thumm/Wiesbeck/Kern. *Hochfrequenzmesstechnik, Verfahren und Messysteme*. B.G.Teubner Stuttgart Leipzig, 1998.
- [16] Henning Weddig. Requirements for the new m.o. for ttf2. DESY paper.
- [17] W.P.Robins. *Phase Noise In Signal Sources*. Peter Peregrinus Ltd., London, UK., 2.edition edition, First Published 1982.

Acknowledgements

I would like to thank Daniel Kotthaus, Henning Weddig, K.Czuba, Frank Eints, Matthias Felber, Matthias Hoffmann, Wojtek, Günther Möller, Richard Wagner and Mr. Bienkowski for inspiration, discussion, support and a nice time in the last year.

I would also like to thank my supervisor Prof.Dr.Bartels from the Fachhochschule Lübeck who introduced me to the interesting field of Radio Frequencies and Microwaves .

I would like to thank Dr.Stefan Simrock who gives me the opportunity to work in his group and supported me in all circumstances!