Supersymmetry Breaking in Five and Six Space-Time Dimensions

Kai Schmidt-Hoberg

January 27, 2005

Abstract

In this thesis we calculate supersymmetry breaking scalar masses in two different orbifold models in five and six space-time dimensions. We perform the full five-/six-dimensional calculation. In five dimensions the result is infrared and ultraviolet finite. In six dimensions a logarithmic ultraviolet divergence has to be cancelled by a brane counterterm. The results are compared with a four-dimensional renormalisation group analysis.

Supersymmetriebrechung in fünf und sechs Raumzeitdimensionen

Kai Schmidt-Hoberg

27. Januar 2005

Zusammenfassung

In dieser Arbeit berechenen wir die supersymmetriebrechenden Massenparameter in zwei verschiedenen Orbifoldmodellen in fünf und sechs Raumzeitdimensionen. Wir führen die vollständige fünf- bzw. sechsdimensionale Rechnung durch. In fünf Dimensionen erhalten wir ein infrarot- und ultraviolettendliches Ergebnis. In sechs Dimensionen benötigen wir einen "Brane Counterterm", um eine ultraviolette Divergenz zu absorbieren. Wir vergleichen die Ergebnisse mit einer vier-dimensionalen Renormierungsgruppenanalyse.

Contents

Chapter 1

Introduction

Is our world supersymmetric? In the simplest supersymmetric world, each particle has a *superpartner* which differs in spin by 1/2 and is related to the original particle by a supersymmetry transformation. Hence, supersymmetry links the fermionic to the bosonic sector of the theory. Although there is as yet no experimental evidence for it, theories with low energy supersymmetry have emerged as the strongest candidates for physics beyond the standard model. But why is it generally believed that, at some energy scale, there are deviations from the standard model at all? The standard model (SM) provides a correct description of all microscopic nongravitational phenomena that we know of. It is a gauge field theory of all known particles and their electroweak and strong interactions. The standard model gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is spontaneously broken to $SU(3)_c \times U(1)_{EM}$ by a nonvanishing vacuum expectation value (VEV) of a fundamental scalar field, the Higgs field. Although no experiment is in conflict with the standard model (except neutrino oscillations), there are some theoretical issues it fails to explain which we briefly recall in the following.

Certainly a new framework will be required at the Planck scale $M_P = (8\pi G_{\text{Newton}})^{-1/2}$ $= 2.4 \times 10^{18}$GeV, where quantum gravitational effects become important. Suppose the standard model to be an effective field theory defined below a cutoff scale Λ, beyond which new ultraviolet physics sets in such that the effective low energy description is no longer valid. But even as a low energy theory, the SM Higgs sector has two "naturalness" problems. One is the gauge hierarchy problem associated with explaining the origin of the electroweak scale when the natural cutoff is of the order unification or Planck scale ($\sim 10^{16}$ - 10^{18}GeV). The second is the technical hierarchy problem. In the standard model, the Higgs mass is subject to quadratically divergent radiative corrections of order $\delta m_H^2 \propto \Lambda^2$, where Λ is the ultraviolet cutoff of the theory. The "natural" value of the Higgs mass is therefore $\mathcal{O}(\Lambda)$ instead of $\mathcal{O}(100\,\text{GeV})$. Since the mass correction is also proportional to the corresponding Yukawa coupling, a weakly coupled scalar would not necessarily suffer from this problem. In contrast to the fermionic case, there is no chiral symmetry to protect the scalar sector from obtaining these large corrections.

Supersymmetry provides a solution to the technical hierarchy problem, since all quadratic divergences of the SM are cancelled by the corresponding diagrams with the superpartners running in the loops.

The unification of all fundamental forces of nature is one of the great aims (if not *the* aim) of high energy physics. The path to this unified scheme started with Maxwell's unification of electrostatics with magnetism and continued with the electroweak unification of Glashow, Salam and Weinberg [1]. Inspired by these very successful theories and the belief that the gauge couplings undergo renormalisation group evolution in such a way that they meet at a point at a high scale, much effort has been put to find a grand unified theory (GUT). A grand unified theory is a quantum field theory in which the standard model gauge group is embedded into a larger simple gauge group like $SU(5)$ [2], $SU(4) \times SU(2) \times SU(2)$ [3] or $SO(10)$ [4]. From this simple gauge group the standard model gauge group originates via spontaneous symmetry breaking. As pointed out in the early 1980s [5], the minimal supersymmetric standard model (MSSM) allows for the unification of the gauge couplings – in contrast to the standard model without supersymmetry. The extrapolation of the low energy values of the gauge couplings using renormalisation group running and the MSSM particle content shows that the gauge couplings unify at the scale $M_G \sim 3 \times 10^{16}$ GeV [6].

Even though these theories are called grand unified theories, they do not include gravity, which would of course be necessary to really obtain a unified picture of nature. The inclusion of gravity is a very difficult task, since a no-go theorem forbids any direct symmetry transformations between fields of different integer spin. This leaves supersymmetric theories as the only field theoretical models [7], which might achieve a unification of all forces. Since the supersymmetry algebra includes the space-time translation operator P^μ, it includes the general coordinate transformations when it is gauged. Therefore it is natural that a locally supersymmetric theory includes gravity (which then is called supergravity or SUGRA). Supersymmetry does not only provide a framework to unify all forces but also relates the exchange particles (bosons) to matter (fermions) via the supersymmetry transformations. Since supergravity is not renormalisable, it cannot be the fundamental theory of everything. But it can be regarded as the low energy effective theory of an underlying more fundamental theory. The only candidate for such an underlying theory is superstring theory.

Besides the solution of the hierarchy problem and gauge coupling unification, there are more hints pointing to supersymmetry. One of the most important successes of supersymmetry is that it can provide a natural mechanism for understanding electroweak symmetry breaking [8]. In the MSSM, the up-type Higgs soft mass-squared parameter is driven to negative values via renormalisation group running due to the large top quark Yukawa coupling. The minimum of the Higgs potential must break $SU(2)_L \times U(1)_Y$, which leads to a condition on the parameters of this potential. The condition $m_{H_u}^2 < 0$ is neither necessary nor sufficient for electroweak symmetry breaking, but it helps. Hence the assumption of the "mexican hat" potential is more natural in the MSSM than in the SM. Another important point is that supersymmetry can provide a natural candidate for cold dark matter in the form of the lightest superpartner (LSP) if it is stable. All these issues suggest that supersymmetry may indeed be a symmetry of nature and might soon be directly discovered by experiment.

In supersymmetric theories a new question arises. *A priori* particles and their superpartners are degenerate in mass which follows directly from the supersymmetry algebra. If supersymmetric gauge field theories are to find realistic application in high energy physics, both supersymmetry and gauge symmetry must be broken, because this mass degeneracy is not observed and unbroken supersymmetry is ruled out. Although we know that supersymmetry must be broken, the breaking mechanism is not yet understood and depends on the form of the unknown underlying theory. But it is desirable that the breaking is of a certain type known as soft supersymmetry breaking, where soft means that the quadratic divergences of the scalar self-energy still cancel. It can be shown that if a renormalisable supersymmetric theory is broken spontaneously, i.e. that the Lagrangian is supersymmetric but the vacuum state breaks supersymmetry, the corresponding terms in the low energy effective Lagrangian are soft. However, in general only a subset of all possible soft terms is realised in such a case. Hence the assumption of spontaneous breaking puts constraints on the possible soft terms. In the simplest scenarios such a spontaneous supersymmetry breaking can be achieved by the Fayet-Iliopoulos or the O'Raifeartaigh mechanism. However, these mechanisms are not able to reproduce the observed mass spectrum, since a generalised sum rule applies to such spontaneously broken theories such that the superpartner masses cannot be lifted to a phenomenologically acceptable range. One way to circumvent this sum rule and to obtain a mass spectrum that is in agreement with experiment is to assume that the theory can be split into at least two sectors with no direct renormalisable couplings between them: The hidden sector, in which supersymmetry is broken by a dynamical mechanism, and the visible sector, which contains the standard model fields and their superpartners. Within this framework, supersymmetry breaking is communicated from the hidden sector where it originates, to the observable sector via suppressed interactions involving a third set of fields, the messenger fields. A very natural way to achieve such a scenario is through the introduction of additional spatial dimensions. These higher dimensional models are motivated by string theories which require higher dimensional space-times, with the extra dimensions being compactified with a small radius of compactification in such a way as to make them consistent with the four-dimensional description we are familiar with. The different sectors of the theory are then naturally given by four dimensional branes embedded in the higher dimensional space-time. In the simplest case, the standard model fields are strictly confined to one brane, while supersymmetry is broken on another brane which is separated in the extra dimension. This brane world scenario is a natural low energy limit of string theory, if one of the extra dimensions is much larger than the others [9,10]. If this extra dimension is large compared to the higher dimensional Planck scale, the use of only field-theoretic, and not intrinsically string theoretic, degrees of freedom can be justified [11]. This field theory could be described as a five dimensional supergravity field theory, perhaps with some additional bulk supermultiplets.

In this thesis we study a toy model in which we replace supergravity by a super-Yang-Mills multiplet. This super-Yang-Mills multiplet in the bulk will serve as the messenger field mediating supersymmetry breaking from the brane where the breaking originates, to the brane to which the standard model fields are confined. The thesis is organised as follows: In Chapter 2 we review a few basic properties of supersymmetric models, effective field theory and orbifold compactifications. In Chapter 3 we calculate the supersymmetry breaking mass of a specific model in five dimensions and compare the result with [12, 13], where the same quantity was calculated via a different approach. In Chapter 4 we generalise the scenario of the five-dimensional case to six dimensions.

Chapter 2

Supersymmetry Breaking

2.1 Global Supersymmetry

The possible symmetries of the S-Matrix of a relativistic quantum field theory are strongly restricted by the Coleman-Mandula theorem [14]. It states that, given some very general assumptions, the only possible Lie algebra of symmetry generators is given by the generators of the Poincaré group and an internal symmetry group.

Supersymmetry avoids the restrictions of the Coleman-Mandula theorem by extending the structure of the Lie algebra to that of a graded Lie algebra. This graded Lie algebra involves commutators as well as anticommutators and successfully embeds the Poincaré group into its larger group structure without modifying the notions of local quantum field theory. Haag, Sohnius and Łopuszanski proved that the supersymmetry algebra is the only graded Lie algebra of symmetries of the S-Matrix consistent with local quantum field theory [15]. The supersymmetry algebra is given in Appendix B. The supersymmetry generators $\mathcal{Q}, \bar{\mathcal{Q}}$ transform as spinors in the $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$ representation of the Lorentz group and hence change the spin of the states they act on by $1/2$. Therefore, the irreducible representations of supersymmetry, the supermultiplets, contain both fermions and bosons. Given the supersymmetry algebra, the supermultiplets can be constructed systematically; this procedure is described e.g. in [16, 17]. These supermultiplets by definition contain an equal number of bosonic and fermionic degrees of freedom, both in the *on-shell* and *off-shell* case. The *off-shell* supersymmetry transformations represent the supersymmetry algebra on the components of the multiplet, unconditionally and independently of the dynamics, i.e. of a Lagrangian. Looking at the equations of motion for the fields of a multiplet, they can be separated into two different classes, algebraic and wave equations. The fields which are subject to algebraic equations are non-propagating degrees of freedom and hence auxiliary fields, which are needed for the closure of the algebra. These non-dynamical fields can be eliminated using their equations of motion which leads to the *on-shell* version of the supermultiplet. Nevertheless these auxiliary fields are important for supersymmetry breaking since they play the role of order parameters as we will see later.

In the construction of supersymmetric theories it is often convenient to work in the *superfield* formalism where a superfield is equivalent to a supermultiplet for our purposes. In this formalism, the supersymmetry algebra can be expressed entirely through commutators when anticommuting spinorial parameters θ_α are introduced [17]. *Superspace* generalises the notion of space-time and is the supersymmetry group supermanifold with coordinates $z = (x, \theta, \bar{\theta})$. Superfields are functions $F(z)$ of superspace. The component fields may always be recovered from superfields by power series expansion in θ- and $\bar{\theta}$-coordinates. Since these coordinates anticommute, any power higher than θ^2 vanishes and the power expansion terminates. The superfield formalism exists for $N = 1, d = 4$ supersymmetry, where $N = 1$ refers to simple supersymmetry and d is the number of space-time dimensions. Unfortunately there is no known way to generalise superfields to higher space-time dimensions in full generality. Nevertheless some progress has been made to formulate higher dimensional super Yang-Mills theories in this framework [18,19]. In this thesis we will work with component fields only.

The most important representations concerning this thesis are the *chiral* and the *vector* superfields. A chiral superfield $\Phi = (\phi, \psi, F)$ contains a complex scalar ϕ, one two-component chiral fermion ψ, and an auxiliary scalar field F. It is of phenomenological interest as it can be seen as a supersymmetric generalisation of fermions as well as of the Higgs bosons. The vector superfield $V = (V^a_\mu, \lambda^a, D^a)$ contains one gauge boson V^a_μ, a Majorana spinor λ^a (the gaugino) and a scalar auxiliary field D^a. Here a labels the gauge group generators. The vector superfield corresponds to the supersymmetric generalisation of gauge bosons.

The interactions of supersymmetric theories are encoded in three functions of the matter fields Φ_i: the superpotential W, the Kähler potential K and the gauge kinetic function f. The superpotential W contains all the couplings necessary to describe all interactions except gauge interactions, the Kähler potential determines the kinetic terms of the chiral multiplet and the gauge kinetic function the kinetic terms for the vector multiplet. Besides providing a non-canonical kinetic structure, the Kähler potential and the gauge kinetic function can generate nonrenormalisable interactions as well.

Supersymmetry constrains the parameters of the Lagrangian since different terms transform into each other under supersymmetry transformations. In addition to constraints from gauge invariance, W and f are further constrained to be holomorphic functions of the fields, whereas the Kähler potential can be any real function. If we confine our attention to renormalisable interactions only, the corresponding Lagrangian can be written as

$$\mathcal{L}_{int} = -\frac{1}{2} W_{ij}\psi_i\psi_j + W_i F_i + \text{c.c.} \tag{2.1}$$

where $W_i = \frac{\partial W}{\partial \phi_i}$, $W_{ij} = \frac{\partial^2 W}{\partial \phi_i \partial \phi_j}$ and the superpotential is generically given by

$$W = Y_{ijk}\phi_i\phi_j\phi_k + \mu_{ij}\phi_i\phi_j. \tag{2.2}$$

Y is dimensionless and μ has dimension of mass. Note that in superfield language we could write the superpotential with the bosonic fields replaced by chiral superfields, giving the same results for the couplings.

In globally supersymmetric theories the scalar potential is given by a sum of F-terms and D-terms

$$V(\phi, \phi^*) = |F_i|^2 + \frac{1}{2} D^a D^a \tag{2.3}$$

where

$$\begin{aligned} F_i^* &= \frac{\partial W}{\partial \phi_i} \ , \\ D^a &= -g\left(\phi_i^* T_{ij}^a \phi_j\right) . \end{aligned} \tag{2.4}$$

The scalar potential is completely determined by the other interactions of the theory. The F-terms are fixed by Yukawa couplings and fermion mass terms, and the D-terms are fixed by the gauge interactions. T^a is the generator of the considered gauge group. The scalar potential is crucial for supersymmetry to be broken or unbroken respectively. From the supersymmetry algebra it is easily derived that the energy can be written in terms of the supersymmetry generators as follows:

$$P_0 = \frac{1}{4}\left(\mathcal{Q}_1\bar{\mathcal{Q}}_{\dot{1}} + \bar{\mathcal{Q}}_{\dot{1}}\mathcal{Q}_1 + \mathcal{Q}_2\bar{\mathcal{Q}}_{\dot{2}} + \bar{\mathcal{Q}}_{\dot{2}}\mathcal{Q}_2\right) . \tag{2.5}$$

This operator is obviously positive semi-definite since $\langle\psi|\mathcal{Q}\bar{\mathcal{Q}}|\psi\rangle \geqslant 0$ for any state $|\psi\rangle$. Only states with a vanishing vacuum expectation value (VEV) are supersymmetric, since $\bar{\mathcal{Q}}|0\rangle = 0$ in this case. Therefore states with nonvanishing VEV (i.e. $\langle V\rangle \neq 0$) break supersymmetry spontaneously. This breakdown can be due to F- or D-term breaking (or both), depending on which auxiliary field acquires a vacuum expectation value.

2.2 Supersymmetry in Higher Dimensions

In recent years the idea of additional compact space-time dimensions has become a popular feature in quantum field theories. Motivated by string theory the possibility of gauge fields living in higher dimensions has been discussed to shed new light on long standing problems of particle physics such as the naturalness and hierarchy problem or the unification of gauge field theories with gravity. In order to be able to work with extra dimensions, we need to know how supersymmetric theories can be consistently built in higher dimensions. Higher dimensional Lorentz algebras lead to different spinor representations. Since supersymmetry by definition involves spinorial as well as tensorial quantities, we have to know what spinors look like in higher dimensions. A short review is given in Appendix C, in which we follow the lines of [16]. A more detailed discussion can be found in [20]. Important for our discussion here is the complex dimension of the spinor representation which is given by

$$D = 2^{d/2} \quad \text{for } d \text{ even} \ , \tag{2.6a}$$

$$D = 2^{(d-1)/2} \quad \text{for } d \text{ odd.} \tag{2.6b}$$

The increasing size of the spinor representations puts limits on the largest dimensions which can possibly sustain supersymmetry at all. This is because dimensionally reduced higher dimensional supersymmetric theories correspond to extended ($N > 1$) supersymmetric theories in four dimensions. There are maximally extended theories, namely $N = 4$ super-Yang-Mills theory and $N = 8$ supergravity. These limits stem from the fact that any multiplet of N-extended supersymmetry will contain particles with spin at least as large as $1/4\ N$. The mass dimension of fields which describe particles increase with spin. For spin $\geq 3/2$ their presence requires the introduction of coupling constants with negative mass dimension which renders flat-space quantum field theories non-renormalisable. In addition, gravity cannot be coupled consistently to particles with spin $\geq 5/2$ which leads to these well known limits. This translates to a limit on the total number of real spinorial charges, which must not exceed 16 or 32 respectively. It is therefore of interest to establish the smallest possible dimension of a spinor representation for a given space-time dimension. There are two possible ways to reduce the number of dimensions of the representation space. For even space-time dimensions we can impose *chirality* conditions, since we have a non trivial Γ_{d+1}-matrix to form a projection operator. In this case the number of dimensions can be halved. This is not possible for odd space-time dimensions since in this case the $\Gamma_{d+1} \propto \mathbb{1}$. The other possible way to reduce the dimension is to impose a reality or *Majorana* condition on the spinors. This can be done consistently only in dimension one to four and in eight to twelve dimensions. Both conditions can be simultaneously imposed only in dimension two and ten. Putting all the pieces together we see immediately that the maximal space-time dimension for a supersymmetric Yang-Mills theory is ten, for supergravity eleven.

In this thesis we concentrate on the case of one and two extra dimensions: In five-dimensional space-time, the smallest spinor is a four component Dirac spinor with eight real degrees of freedom. In the case of six dimensions, a chirality condition is possible and we also have eight real degrees of freedom. Therefore a $N = 1$ supersymmetric theory in $d = 5, 6$ corresponds to a $N = 2$ extended supersymmetry in four dimensions.

2.3 Effective Field Theory

In higher dimensional theories the standard model gauge fields can interact only through non-renormalisable interactions. This is because the gauge coupling in higher dimensions has negative mass dimension. Therefore we have to treat these higher dimensional theories as effective theories with a cutoff Λ, which may be seen as the fundamental scale of these theories. For the understanding of these higher dimensional theories it is therefore important to know what we can learn from effective field theory. If widely separated energy scales are involved, the low-energy dynamics of a theory can be studied independently of the high-energy interactions [21]. Effective field theories are the appropriate theoretical tool to describe low-energy physics (where low is defined with respect to the cutoff Λ). These theories explicitly only take into account states with $m < \Lambda$ while the heavier excitations with $M > \Lambda$ are integrated out from the action. This results in a string of nonrenormalisable interactions between the light states which can be organised in a power expansion [22]. At a given order in this expansion, the low energy theory is specified by a finite number of couplings, which allows for an order by order renormalisation.

An effective field theory is characterised by some effective Lagrangian,

$$\mathcal{L} = \sum_i c_i O_i, \tag{2.7}$$

where the O_i are operators constructed with the light fields. The information on any heavy degrees of freedom is hidden in the couplings c_i. The operators can be organised according to their dimension, d_i, which fixes the dimension of the coefficients c_i:

$$[O_i] = d_i \to c_i \sim \Lambda^{d-d_i} \quad , \tag{2.8}$$

with Λ the cutoff of the theory and d the space-time dimension. At energies below Λ, where the effective description is valid, the behaviour of the different operators is determined by their dimension. Three types of operators can be distinguished on dimensional grounds. Operators with $d_i < d$ are called *relevant*, operators with $d_i = d$ *marginal* and operators with $d_i > d$ *irrelevant*. This classification corresponds to the suppression by powers of $1/\Lambda$ as can be seen from Equation (2.8), e.g. the notion irrelevant means highly suppressed at small energies. Nevertheless the irrelevant operators can be quite important, since they contain the interesting information about the underlying dynamics at higher scales.

2.4 Soft Supersymmetry Breaking

If supersymmetry exists it must be a broken symmetry, because an exact symmetry would imply degenerate masses of particles and their superpartners which is ruled out by experiment. If supersymmetry is broken softly meaning that the quadratic divergences still cancel, the superpartner masses can be lifted to a phenomenologically acceptable range. One might wonder if there is any good reason why all of the superpartners of the standard model particles can be heavy enough to have avoided discovery so far. There is. All of the standard model particles would be massless in the absence of electroweak symmetry breaking. In particular the masses of all quarks and leptons and of the $W^{\pm}, Z^0$ bosons are given by a dimensionless coupling times the Higgs VEV, while the photon and gluons are required to be massless by gauge invariance. Conversely all superpartners can have mass terms in the absence of electroweak symmetry breaking.

Regardless if supersymmetry is explicitly or spontaneously broken in nature, there should be an effective Lagrangian at the electroweak scale parameterised by a set of soft supersymmetry breaking terms if the attractive features of supersymmetry are to be part of physics beyond the standard model. This means that the effective Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}. \tag{2.9}$$

For a review on the theory and experimental implications of the soft supersymmetrty breaking Lagarangian see [23]. From an effective field theory point of view the basic question is how to understand the explicit soft supersymmetry breaking encoded in the effective Lagrangian as a result of spontaneous supersymmetry breaking in a more fundamental theory. But the mechanism of supersymmetry breaking and how it might be implemented consistently within the (unknown) underlying theory is still not known.

The most straightforward approach is to look at spontaneous supersymmetry breaking via a F- or D-term VEV at the TeV scale in the MSSM. But when looking at the particle content of the MSSM it is easily seen that such a "visible sector" supersymmetry breaking leads to a pattern of bosonic and fermionic masses which is experimentally excluded. This can be inferred from the following sum rule for particles of spin J, the so called supertrace relation

$$\sum m^2_{J=0} - 2\sum m^2_{J=1/2} + 3\sum m^2_{J=1} = 0, \tag{2.10}$$

which is valid in the presence of tree level supersymmetry breaking. This sum rule holds separately for each sector of quantum numbers, since the conservation of electric charge, colour charge and global symmetry charges such as baryon and lepton number prevents mass mixing between these sectors. For example consider the right-handed down type quarks (charge -1/3 , colour e.g. red, baryon number 1/3 , lepton number 0) which contribute to the sum $2\,(m_d^2 + m_s^2 + m_b^2) \sim 2 \cdot (5\,\mathrm{GeV})^2$. Looking at the bosonic part of the sum rule this implies that none of the bosons should have a mass of more than about 7 GeV which is certainly ruled out by experiment.

The Hidden Sector Framework

Since all models with TeV scale supersymmetry breaking suffer from not generating sufficiently large superpartner masses, alternatives to this simplest scenario have to be considered. In the hidden sector framework, supersymmetry breaking is communicated from the hidden sector where it originates to the observable sector where the matter fields live via suppressed interactions involving another set of fields, the messenger fields. The results are effective SUSY breaking parameters in the observable sector. Because both, the fundamental scale of supersymmetry breaking M_S, and the scales associated with the messenger interactions are much larger than the electroweak scale, renormalisation group analysis is necessary in order to obtain the low energy values of the supersymmetry breaking parameters. Many different phenomenologically viable models within this hidden sector framework have been considered in the past few years. They can be divided into *gravity mediation* [24, 25], *gauge mediation* [26] and *bulk mediation* [27]. In gravity mediation the supersymmetry breaking is mediated via (non-renormalisable) Planck suppressed contact terms and as gravitational interactions are shared by all particles, gravity might be a good candidate for the messenger field. In gauge mediation, new messenger fields S_i with standard model quantum numbers are introduced. Supersymmetry is assumed to be broken dynamically such that non-zero F-term VEVs of the hidden sector fields are generated. The spontaneous breaking of global supersymmetry implies the existence of a massless Weyl fermion, the goldstino. The messenger fields couple to the goldstino of the hidden sector, which generates non-zero F_S terms. Supersymmetry breaking is then communicated to the observable sector through radiative corrections involving messenger field loops to the propagators of the observable fields. The feature which makes these models very attractive is that the masses of the squarks and sleptons depend only on their gauge quantum numbers, leading automatically to the degeneracy of squark and slepton masses needed for the suppression of FCNC effects [28].

In bulk mediation, the hidden and observable fields live on different branes separated in extra dimensions and supersymmetry breaking is mediated by fields living in the bulk and propagating between the branes. In this thesis we will mainly concentrate on this last case, where we have a super-Yang-Mills multiplet living in the bulk and supersymmetry breaking is communicated via gauginos belonging to this super-Yang-Mills multiplet. This is for obvious reasons often referred to as *gaugino mediation* [12, 13].

2.5 Orbifold Compactification

In all higher dimensional field theories, the extra dimensions have to be compactified in some way to reproduce the four-dimensional world we live in. The simplest kind of compactification in five dimensions would be to have the usual four-dimensional Minkowski space and the fifth dimension compactified on a circle. However, there is a serious drawback that makes such theories unsuitable as candidates for a physical theory. The problem lies within the spinor representation in five dimensions. Unlike the four-dimensional case, the Dirac spinor is an *irreducible* representation of the proper orthochronous Lorentz group and not a *reducible* one. This implies that we cannot have a chiral Lorentz invariant theory, which is unsuitable for the theory of weak interactions. One possible cure for this problem is the introduction of an additional discrete space-time symmetry. In the simplest case this results in the orbifold $S_1/\mathbb{Z}_2$, where the reflection symmetry $y \to -y$ is introduced.

Formulating this more abstractly, the field theoretic orbifolding procedure is based on a discrete symmetry group acting in physical space and in field space [29]. Consider a higher dimensional quantum field theory defined on a manifold $\mathbb{R}^4 \times C$ with both the manifold C and the QFT possessing a symmetry under a discrete group K. The action of K on the internal manifold C is geometrical,

$$K : (x, y) \to (x, k[y]), \tag{2.11}$$

where $k[y]$ is the image of the point y under the operation $k \in K$ with y the coordinates of C. The action of K in field space is given by

$$K : \Phi_i \to R(k)_{ij}\Phi_j \tag{2.12}$$

with Φ comprising all the fields of the theory and $R(k)$ a matrix representation of the symmetry group K. Declaring only field configurations to be physical which are invariant under these two actions, we orbifold (or mod out) the theory by K. Modding out by just the geometrical action results in a smaller physical space which is now C/K instead of C. An orbifold is a space where K acts non-freely, i.e. the action of K has fixed points ($k(y) = y$ for some $y \in C, k \neq 1$). By freely we mean that non-trivial elements of K move all points of C:

$$k[y] \neq y, \qquad \forall y \in C, \qquad \forall k \neq 1 \in K. \tag{2.13}$$

When K acts in such a way, the space C/K is smooth and is again a manifold. In the non-freely case the physical space C/K is not smooth having singularities at the fixed points. Coming back to the simplest case of an orbifold $S_1/\mathbb{Z}_2$, fields living on this orbifold have to be assigned a certain parity under the transformation of $\mathbb{Z}_2$. Fields with even parity will have zero modes in addition to their Kaluza-Klein-towers whereas the zero modes of fields with odd parity will be projected out. Therefore we can construct a theory, where the left/right symmetry of the fermions constrained to an orbifold fixed point can be lifted simply by assigning different parities to the left-/right-handed components of the Dirac spinor. Additionally, as will be shown in the next chapter, this results in the reduction of supersymmetry from $N = 2$ to $N = 1$ on the orbifold fixed points (remember that $N = 1$

supersymmetry in five and six dimensions corresponds to $N = 2$ supersymmetry in four dimensions).

Orbifold compactifications have also been used in the context of grand unified theories. Given the success of gauge coupling unification in supersymmetric extensions of the standard model, theories which include both, supersymmetry and larger gauge groups, are the most attractive candidates for effective theories below the Planck scale. The basic idea is that such a grand unified theory naturally lives in higher dimensions and the GUT gauge group is only broken down to the SM via symmetry violating boundary conditions on an orbifold compactification. Such an orbifold GUT was first suggested by Kawamura [30, 31]. He considered a $SU(5)$-GUT in five dimensions broken down to a $N = 1$ supersymmetric model with standard model gauge group by a compactification on $S_1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$. A more involved model of a supersymmetric orbifold GUT with gauge group $SO(10)$ was explored in [32, 33]. One of the advantages of grand unified theories with gauge group $SO(10)$ is that one generation of quarks and leptons, including the right handed neutrino, fits into a single irreducible representation. Whereas the breaking of $SU(5)$ down to the standard model gauge group can be achieved in a five-dimensional orbifold compactification, the breaking of $SO(10)$ is more involved and favours a six-dimensional space-time. The authors use the orbifold $T^2/(\mathbb{Z}_2 \times \mathbb{Z}_2^{GG} \times \mathbb{Z}_2^{PS})$ to achieve the breaking of the extended supersymmetry and of the GUT group $SO(10)$.

In this thesis, we follow the lines of these orbifold GUTs and evaluate the supersymmetry breaking mass term of matter fields living on one fixed point of an orbifold. In the five-dimensional case we use the orbifold $S_1/\mathbb{Z}_2$. We only mod out the circle with one $\mathbb{Z}_2$-transformation since we are not considering any specific gauge group here and therefore do not need to care about a mechanism to break the gauge group down to the standard model gauge group. We only want to break the unwanted extended supersymmetry which results from dimensional reduction. For the same reason we use the orbifold $T^2/\mathbb{Z}_2$ in the case of six space-time dimensions.

Chapter 3

Gaugino Mediated SUSY Breaking in a 5d Orbifold Model

In this chapter we want to calculate the supersymmetry breaking mass correction in a five-dimensional theory. We study a toy model in which supergravity is replaced by a super-Yang-Mills multiplet, living in a five-dimensional bulk. The setup of the theory is the same as in [12]. The physical space is chosen to be $\mathbb{R}^4 \times S_1/\mathbb{Z}_2$ with metric $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1,-1)$. The action of $\mathbb{Z}_2$ is given by the reflection symmetry $y \to 2L - y$. Such a space is known as an orbifold (see Section 2.5) and has two fixed points, at $y = 0$ and $y = L$. These fixed points correspond to four-dimensional branes embedded in the fifth dimension: the matter and the source brane, which correspond to the visible and the hidden sector of the theory. The matter fields, quarks, leptons, Higgs and their superpartners are localised on the matter brane whereas supersymmetry is broken on the spatially separated source brane. While matter is strictly confined to the orbifold fixed points the gauge bosons and gauginos can propagate through the bulk. The main idea of gaugino mediated supersymmetry breaking is that the gauginos couple to the source brane and become massive whereas the gauge bosons remain massless. In contrast to the fermions the matter superpartners receive masses via loop contributions through the bulk and hence supersymmetry breaking is mediated from the source brane to the matter brane via these gaugino loop contributions.

3.1 Super-Yang-Mills Theory on $\mathbb{R}^4 \times S_1/\mathbb{Z}_2$

In our theoretical setup, we have to couple the five-dimensional super-Yang-Mills multiplet to a four-dimensional boundary. A convenient strategy to see how this can be done is to work in the *off-shell* formulation, including auxiliary fields [11].

The field content of the super-Yang-Mills multiplet, when extended to an off-shell multiplet by adding a $SU(2)_R$-triplet X^a of auxiliary fields, is given by (A^M, Φ, ψ^i, X^a). Here A^M is a vector field, Φ a real scalar field and ψ^i a gaugino. The indices a, i are internal $SU(2)_R$ indices with $a = 1, 2, 3$ and $i = 1, 2$. Note that off-shell the gauge field has four and not five degrees of freedom, since one degree of freedom is removed by a gauge-fixing condition. The symplectic Majorana spinors ψ^i are Dirac fermions and satisfy the constraints

$$\psi^i = \varepsilon^{ij} C \bar{\psi}_j^T , \tag{3.1}$$

with C the charge conjugation matrix given in Appendix C. This condition can conveniently be written as a decomposition of the $5d$ symplectic Majorana spinor ψ^i into two $4d$ Weyl spinors λ_L, λ_R:

$$\psi^1 = \begin{pmatrix} (\lambda_L)_\alpha \\ (\bar{\lambda}_R)^{\dot{\alpha}} \end{pmatrix} , \quad \psi^2 = \begin{pmatrix} (\lambda_R)_\alpha \\ -(\bar{\lambda}_L)^{\dot{\alpha}} \end{pmatrix} , \quad \bar{\psi}_1 = \begin{pmatrix} (\lambda_R)^\alpha \\ (\bar{\lambda}_L)_{\dot{\alpha}} \end{pmatrix}^T , \quad \bar{\psi}_2 = \begin{pmatrix} -(\lambda_L)^\alpha \\ (\bar{\lambda}_R)_{\dot{\alpha}} \end{pmatrix}^T . \tag{3.2}$$

The supersymmetry transformation laws for the fields of the super-Yang-Mills multiplet are given by [11]

$$\begin{aligned} \delta_\xi A^M &= \mathrm{i}\bar{\xi}^i \gamma^M \psi^i , \\ \delta_\xi \Phi &= \mathrm{i}\bar{\xi}^i \psi^i , \\ \delta_\xi \lambda^i &= \left(\sigma^{MN} F_{MN} - \gamma^M D_M \Phi\right) \xi^i - \mathrm{i} \left(X^a \sigma^a\right)^{ij} \xi^j , \\ \delta_\xi X^a &= \bar{\xi}^i \left(\sigma^a\right)^{ij} \gamma^M D_M \psi^j - \mathrm{i} \left[\Phi, \bar{\xi}^i \left(\sigma^a\right)^{ij} \psi^j\right] , \end{aligned} \tag{3.3}$$

where the supersymmetry parameter ξ^i is a symplectic Majorana spinor. The members of the multiplet are written as matrices in the adjoint representation of the gauge group, $A^M = A^{MA} T^A$, etc. The covariant derivative is defined to be $D_M \equiv \partial_M - \mathrm{i}\,[A_M, \cdot\,]$ and $\sigma^{MN} \equiv 1/4 \left[\gamma^M, \gamma^N\right]$ with γ^M the five-dimensional Dirac matrices. The Lagrangian of the five-dimensional super-Yang-Mills multiplet left invariant under these transformations is given by

$$\mathcal{L}_5 = -\frac{1}{2} \operatorname{tr}[(F_{MN})^2] + \operatorname{tr}(D_M \Phi)^2 + \operatorname{tr}(\bar{\psi}_i \mathrm{i} \Gamma^M D_M \psi^i) + \operatorname{tr}(X^a)^2 - \operatorname{tr}(\bar{\psi}_i [\Phi, \psi^i]). \tag{3.4}$$

Capital indices M, N run over $\{0, 1, 2, 3, 5\}$.

The minimal amount of supersymmetry in five dimensions corresponds to $N = 2$ extended supersymmetry in four dimensions. This complicates the coupling of higher dimensional supersymmetric fields to four-dimensional ones. In order to break the unwanted $N = 2$ supersymmetry of the bulk gauge field to $N = 1$ supersymmetry at the four-dimensional boundary, we compactify the extra dimension on the orbifold $S_1/\mathbb{Z}_2$. To promote the $\mathbb{Z}_2$ symmetry of the orbifold to a symmetry of our theory, we have to specify the $\mathbb{Z}_2$ parities of the component fields living in the bulk (leaving the bulk Lagrangian invariant). The P assignments of the bulk fields can be chosen as follows:

	P = +1	P = -1
A^M	A^m	A^5
Φ	-	Φ
ψ^i	λ_L	λ_R
X^a	X^3	$X^{1,2}$

The $\mathbb{Z}_2$ breaks half of the supersymmetry by distinguishing the components of the vector superfield. Only the fields with $P = +1$ are non-vanishing at the boundaries and hence couple to the fields on the brane. In order to write couplings between the bulk and brane fields we note that the $N = 2$ bulk fields with even parity with respect to the branes correspond to a $4d$ vector multiplet, with A^m, λ_L, and $(X^3 - \partial_5\Phi)$ as the vector, gaugino, and auxiliary D field [11]. Therefore we can couple them to the boundary fields in the same way as we would couple a four-dimensional $N = 1$ vector multiplet. The action including fields on the boundaries can be written as

$$S = \int \mathrm{d}^5x \Big[\mathcal{L}_5 + \delta(y)\mathcal{L}_m + \delta(y-L)\mathcal{L}_s\Big], \tag{3.5}$$

with $\mathcal{L}_5$ the bulk Lagrangian for the five-dimensional super-Yang-Mills multiplet, $\mathcal{L}_m$ the matter and $\mathcal{L}_s$ the source brane Lagrangian. Matter fields generalise to chiral superfields when working in a supersymmetric framework. This is the case since only chiral supermultiplets can contain fermions whose left-handed parts transform differently under the gauge group than their right-handed parts [28]. All of the standard model fermions have this property, so they must be members of chiral superfields. Hence the boundary Lagrangian of the matter brane should be the standard Lagrangian for a chiral model built from supermultiplets (ϕ, ψ, F) and coupled to a vector multiplet (A, λ, D):

$$\begin{aligned} \mathcal{L}_m = \ & D_m\phi^\dagger D^m\phi + \bar{\psi} i\bar{\sigma}^m D_m\psi + F^\dagger F \\ & - \sqrt{2}\mathrm{i}g_5(\phi^\dagger\lambda_L\psi - \bar{\psi}\bar{\lambda}_L\phi) + g_5\phi^\dagger(X^3 - \partial_5\Phi)\phi\,. \end{aligned} \tag{3.6}$$

Here the gauge fields (A, λ, D) were replaced by the boundary terms of the bulk fields $(A_m, \lambda_L, X^3 - \partial_5\Phi)$.

The source brane Lagrangian is in general very complicated since it involves all the fields necessary to break supersymmetry as well as couplings to the bulk gauge fields. In this thesis we assume the leading supersymmetry breaking VEV to be a F-term of a chiral superfield S living on the source brane at $y = L$. The leading term of the source Lagrangian, which couples the chiral superfield S to the gauge fields living in the bulk is of the form

$$\mathcal{L}_s = \frac{F_S}{M^2}\lambda_L^\alpha\lambda_{L\alpha} + \text{h.c.} + \dots \ . \tag{3.7}$$

When integrating out the extra dimension for the zero mode, which leads to the light four-dimensional gaugino, the resulting gaugino mass from the F-term of the chiral superfield is given by

$$m_\lambda = \frac{F_S}{2LM^2} \ . \tag{3.8}$$

The factor of $1/(2L)$ originates from normalising the mass term relative to the kinetic term making the kinetic term canonical again. The extra two in the denominator corresponds to our definition of the orbifold propagator, where we will use a trick to extend the theory to the whole circle again, though the physical space is given by the interval $(0, L)$.

The calculation of the resulting supersymmetry breaking mass term on the matter brane is more involved. Since supersymmetry is by definition a symmetry between fermions and bosons which results in degenerate masses, supersymmetry breaking can be parameterised by the mass difference between the two species. We will concentrate on the scalar self-energy first.

3.2 Self-Energy of the Scalar Field

The mass correction originating from higher order diagrams due to self-interactions of the scalar field is expressed in terms of the function $M^2(p^2)$, the sum of all one-particle irreducible diagrams with two external scalar lines. The full (interacting) propagator can be expressed as a sum which forms a geometric series in this function $M^2(p^2)$. This means the scalar propagator can be rewritten as

$$\begin{aligned} S_F(p^2) &= \frac{\mathrm{i}}{p^2 - m_0^2} + \frac{\mathrm{i}}{p^2 - m_0^2}(-\mathrm{i}M^2)\frac{\mathrm{i}}{p^2 - m_0^2} + ... \\ &= \frac{\mathrm{i}}{p^2 - m_0^2 - M^2(p^2)} \ . \end{aligned} \tag{3.9}$$

This propagator has two poles, which are shifted away from m_0 by the $M^2(p^2)$ term. The physical mass is given by the location of these poles. Therefore the physical mass m can be written as

$$m^2 = m_0^2 + \delta m^2 = m_0^2 + M^2(p^2 = m^2) \ . \tag{3.10}$$

Expanding $M^2(p^2)$ around $p^2 = m^2$ we get

$$\begin{aligned} M^2(p^2) &= \delta m^2(m, m_0, \Lambda) + (p^2 - m^2)\,\frac{\mathrm{d}}{\mathrm{d}p^2}M^2(p^2)|_{p^2=m^2} + \mathcal{O}(p^2 - m^2)^2 \\ &= \delta m^2 + (p^2 - m^2)\frac{Z-1}{Z} + \mathcal{O}(p^2 - m^2)^2 \end{aligned} \tag{3.11}$$

with Z the residue of the propagator known as field strength renormalisation. The last step can easily be seen when comparing the exact two-point function

$$\frac{\mathrm{i}Z}{p^2 - m^2} + (\text{terms regular at } \ p^2 = m^2) \tag{3.12}$$

to

$$\frac{\mathrm{i}}{p^2 - m_0^2 - M^2(p^2)} \ . \tag{3.13}$$

The field strength renormalisation Z is particularly important when calculating S-matrix elements. In general δm^2 and Z can be infinite and the theory has to be renormalised.

In this thesis we are mainly interested in the mass correction δm^2. To evaluate it we can do an analogous expansion as in Equation (3.11):

$$\delta m^2 = M^2(m^2) = M^2(m_0) + (m^2 - m_0^2)\frac{\mathrm{d}}{\mathrm{d}m^2}M^2(m^2)|_{m^2=m_0^2} + \mathcal{O}(m^2 - m_0^2)^2. \tag{3.14}$$

When calculating the mass correction this way, we have to be careful to consistently keep the right orders of the coupling constant g. To order $\alpha \propto g^2$, the mass shift is

$$\delta m^2 = M^2(m^2) = M^2(m_0). \tag{3.15}$$

What does this mass correction look like in our specific model? Writing out the boundary Lagrangian (3.6) we see that the scalar field ϕ couples to the auxiliary field X^3 through the term $g_5\, \phi^\dagger(X^3 - \partial_5\Phi)\phi$. Integrating out the auxiliary field results in [11]

$$\begin{aligned}\mathcal{L}_m = {} & D_m\phi^\dagger D^m\phi + \bar{\psi}_L \mathrm{i}\,\bar{\sigma}^m D_m\psi_L + F^\dagger F - \sqrt{2}\,\mathrm{i}g_5(\phi^\dagger\lambda_L\psi_L - \bar{\psi}_L\bar{\lambda}_L\phi) \\ & - g_5\phi^\dagger(\partial_5\Phi)\phi - \frac{1}{2}g_5(\phi^\dagger t^A\phi)^2\delta(0)\,.\end{aligned} \tag{3.16}$$

From this expression we can read off the couplings of the complex scalar field ϕ. The Feynman diagrams contributing to the self-energy of this field to one-loop order are given by

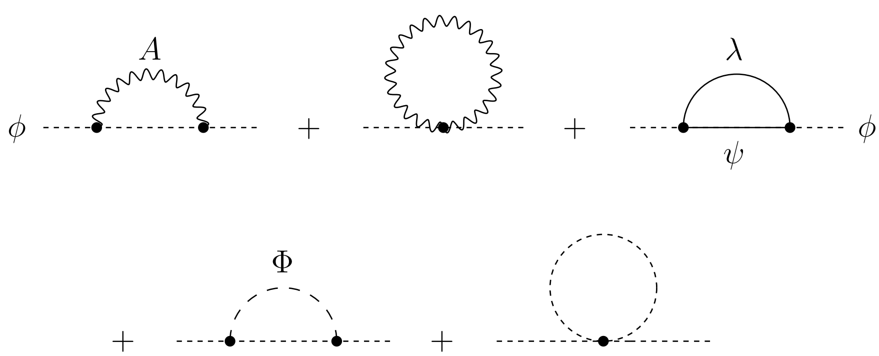


Figure 3.1: Feynman diagrams contributing to the ϕ self-energy at one-loop order.

As long as supersymmetry is unbroken, the ϕ cannot obtain a mass in perturbation theory. This means that all these diagrams sum up to zero. The corresponding calculation is explicitly done in [11], where the $\delta(0)$-term enters as a counterterm cancelling the singular behaviour of the Φ exchange diagram.

Only the gaugino obtains a supersymmetry breaking mass via the F-term VEV located on the source brane. To calculate the leading term of the scalar self-energy, $M_2^2(p^2)$, we therefore consider the gaugino loop diagram starting from the matter brane with gaugino mass insertions on the source brane (see Figure 3.2). The subscript 2 of the self-energy refers to two mass insertions. All other fields including the massless gaugino do not know about supersymmetry breaking and therefore still sum up to zero.

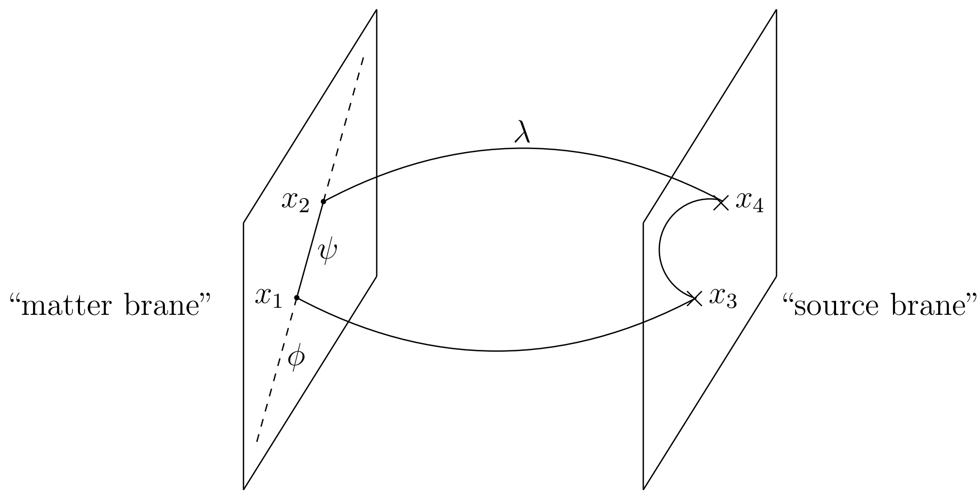


Figure 3.2: Loop diagram through the bulk.

We need an even number of mass insertions on the source brane, since otherwise we cannot combine the spinor structure of our propagators with the mass insertion vertices. In order to calculate this one-loop contribution we need the corresponding Feynman rules, which we have to derive from the Lagrangian. We start by considering the five-dimensional gaugino propagator.

3.2.1 The Feynman Rules

In our case it is convenient to use mixed propagators, where the coordinate of the compactified dimension is kept in configuration space whereas the coordinates of the other four dimensions are Fourier-transformed to momentum space. Since the interactions are restricted to the branes, the y-integration over the interaction point is trivial.

To derive a suitable propagator, we start with the kinetic term in our five-dimensional Lagrangian,

$$\mathrm{tr}[\bar{\psi}_i \mathrm{i}\gamma^M \partial_M \psi^i] \ . \tag{3.17}$$

Using the condition for symplectic Majorana spinors (3.1), we express this in terms of ψ^1 only. Taking into account the properties of the charge conjugation matrix (see Appendix C) and after an integration by parts, we finally obtain

$$2\,\mathrm{tr}[\bar{\psi}_1 \mathrm{i}\gamma^M \partial_M \psi^1] \ . \tag{3.18}$$

We define the trace over two group generators as usual, $\mathrm{tr}\,[t^A t^B] = 1/2\,\delta^{AB}$. The kinetic term is then canonical and we do not have to rescale the fields.

Given the fermionic term in the Lagrangian and choosing the following representation of the Dirac Algebra

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix}, \tag{3.19}$$

the Dirac equation is given by

$$\begin{pmatrix} \partial_5 & \mathrm{i}\sigma^\mu\partial_\mu \\ \mathrm{i}\bar{\sigma}^\mu\partial_\mu & -\partial_5 \end{pmatrix} \psi = 0 \ . \tag{3.20}$$

From here we can calculate the five-dimensional propagator defined by

$$\begin{pmatrix} \partial_5 & \mathrm{i}\sigma^\mu\partial_\mu \\ \mathrm{i}\bar{\sigma}^\mu\partial_\mu & -\partial_5 \end{pmatrix} S_F(x-x', y, y') = \mathrm{i}\delta^4(x-x')\delta(y-y')\,\mathbb{1} \ , \tag{3.21}$$

which is

$$\begin{aligned} S_F(x-x', y-y') &= \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \int \frac{\mathrm{d}p^5}{2\pi} \frac{\mathrm{i}\left(\not{p} + \mathrm{i}\gamma^5\partial_5\right)}{p^2 - p_5^2 + \mathrm{i}\varepsilon} e^{-\mathrm{i}p\cdot(x-x')} e^{\mathrm{i}p^5(y-y')} \\ &= \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\mathrm{i}\left(\not{p} + \mathrm{i}\gamma^5\partial_5\right)}{2\chi} e^{\mathrm{i}\chi|y-y'|} e^{-\mathrm{i}p\cdot(x-x')} \end{aligned} \tag{3.22}$$

where χ is simply $\sqrt{p^2 + \mathrm{i}\varepsilon}$. The integration over p^5 was done using the residue theorem.

Starting from this uncompactified propagator, we somehow have to incorporate the additional requirements of periodicity and reflection symmetry. Here we follow the approach by Puchwein and Kunszt [34]. A periodic propagator can be obtained by summing over all winding modes, where the propagator $S_F(q, y + 2Ln, y')$, which is now Fourier transformed in the four flat dimensions, winds n times around the circle:

$$
\begin{aligned}
S_F^c(q,y,y') &= \sum_{n=-\infty}^{+\infty} S_F(q, y+2Ln, y') \\
&= \frac{\mathrm{i}(\not{q} + \mathrm{i}\gamma^5 \partial_5)}{2\chi} \sum_{n=-\infty}^{+\infty} e^{\mathrm{i}\chi|y-y'+2Ln|} \\
&= (\not{q} + \mathrm{i}\gamma^5 \partial_5) \frac{\mathrm{i}\cos\chi(L - |y-y'|)}{2\chi \sin \chi L} \; .
\end{aligned}
\tag{3.23}
$$

This formula is valid for $y, y' \in [0, 2L)$ only. Therefore the periodic property of this propagator is hidden, but evaluating the sum with y replaced by $y + 2Ln$ we would get exactly the same result. The radiative corrections for the masses are dominated by high energy effects, which implies small distances. As can be seen from Equation (3.23), contributions with $n > 0$ are exponentially suppressed after Wick rotation. Therefore the small n contributions dominate by far over large n. This is an advantage over a Kaluza-Klein decomposition, where this is not the case [35]. Nevertheless, we can do the sum explicitly, so we do not have to think about possible approximations.

The action of the $\mathbb{Z}_2$-symmetry on the fermions is implemented via

$$
P_5\, \psi(x,y) = \mathrm{i}\gamma^5\, \psi(x, 2L-y) = \psi(x,y). \tag{3.24}
$$

Here we replaced $-y$ by $2L - y$ as to make the theory well defined again since the periodic propagator we found was valid only for $y, y' \in [0, 2L)$. One could now use the same trick as in Equation (3.23) to get the propagator of the orbifolded space. In addition to applying the group of translations by $2L$ to the propagator and summing over it, one would also sum the contributions obtained by acting with $\mathbb{Z}_2$. This procedure would result in the physical interval $y \in [0, L]$. However, calculations can be simplified when we extend the theory to the full circle. Both ways of defining the theory are equivalent, it is just a matter of convenience when doing calculations. An easy way to find a propagator satisfying the boundary condition (3.24) is to note that we can write our fermion field ψ in terms of an unconstrained field χ as

$$
\psi(x,y) = \frac{1}{2}\big(\chi(x,y) + \mathrm{i}\gamma^5\, \chi(x, 2L-y)\big) \tag{3.25}
$$

automatically satisfying (3.24). Note that since both y and $-y$ appear in this equation, the propagator depends on both, the difference $y - y'$ and the sum $y + y'$.

Putting everything together we see that both requirements are achieved by the following propagator for a Dirac fermion ψ:

$$\begin{aligned} S_F^{orb}(q,y,y') &= \frac{1}{2}\big(S_F^c(q,y,y') + \mathrm{i}\gamma^5\, S_F^c(q,2L-y,y')\big) \ , \\ &= (\not{q} + \mathrm{i}\gamma^5\partial_5)\frac{\mathrm{i}\Big[\cos\chi(L-|y-y'|) - \mathrm{i}\gamma^5\,\cos\chi(L-(y+y'))\Big]}{4\chi\sin\chi L} \ , \end{aligned} \tag{3.26}$$

where $S_F^c(q,y,y')$ is the Dirac propagator calculated above satisfying just the condition of periodicity.

Since the gaugino loop we want to calculate starts and ends on a brane, we only need the following special cases of the gaugino propagator:

$$S_F^{orb}(q,0,L) = S_F^{orb}(q,L,0) = \frac{\mathrm{i}P_L\not{q}}{2\chi\sin\chi L} \ , \tag{3.27a}$$

$$S_F^{orb}(q,L,L) = \mathrm{i}P_L\left(\frac{\not{q}\cos\chi L}{\chi\sin\chi L} + 1\right) . \tag{3.27b}$$

Now we turn to the vertices necessary in order to calculate the mass corrections. To consistently read off the Feynman rules, we have to express the mass interaction term (3.7) in terms of the Dirac fermion ψ^1 instead of the Weyl spinor λ_L. This is easily done via the condition for symplectic Majorana spinors (3.1). Note that we need both terms in the Lagrangian to consistently write down the expression for our loop integral. Replacing Weyl by Dirac spinors we obtain:

$$(\lambda_L)^\alpha\,(\lambda_L)_\alpha \;=\; -\bar{\psi}_2 P_L\psi^1 = \left(\psi^1\right)^T C^{-1}P_L\psi^1, \tag{3.28a}$$

$$\left(\bar{\lambda}_L\right)_{\dot\alpha}\left(\bar{\lambda}_L\right)^{\dot\alpha} \;=\; -\bar{\psi}_1 P_R\psi^2 = \bar{\psi}_1 C P_R\left(\bar{\psi}_1\right)^T . \tag{3.28b}$$

The projection operators $P_{L/R}$ and the charge conjugation matrix C commute, cf. Appendix C. The correct expression for the mass insertion for a Dirac spinor is given by

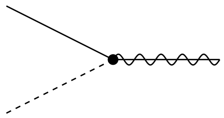
$$= \begin{cases} \mathrm{i}F_S/M^2\, C^{-1}\, P_L\,\delta(y-L) & (\psi)\ , \\ \mathrm{i}F_S/M^2\, C\, P_R\,\delta(y-L) & (\bar{\psi})\ . \end{cases}$$

Here the vertex is proportional to P_R or to P_L, depending on between which fields we insert this mass vertex.

The vertex between λ_L, ψ and ϕ comes from the following term in our boundary Lagrangian (3.6), where we replace the Weyl spinor λ_L by the Dirac spinor ψ^1 again:

$$\begin{aligned} \mathcal{L}_m \supset &\ -\sqrt{2}\mathrm{i}g_5\left(\phi^\dagger\,(\lambda_L)^\alpha\,(\psi_L)_\alpha - (\bar{\psi}_L)_{\dot\alpha}\left(\bar{\lambda}_L\right)^{\dot\alpha}\phi\right) \\ = &\ -\sqrt{2}\,\mathrm{i}g_5\left(\phi^\dagger\psi^T P_L C\psi^1 + \bar{\psi}_1 P_R C\bar{\psi}^T\phi\right) . \end{aligned} \tag{3.29}$$

From here we can now easily read off the Feynman expression for the vertex:



$$= \begin{cases} \sqrt{2} g_5 P_L C \delta(y) & (\psi) \ , \\ \sqrt{2} g_5 P_R C \delta(y) & (\bar{\psi}) \ . \end{cases}$$

3.2.2 The Loop Integral

In coordinate space we have to integrate over all possible space-time points for each vertex. Since every vertex comes with a delta distribution $\delta(y - y')$, the y-integration collapses and we only have to integrate over the uncompactified dimensions. A first approximation for the massive gaugino propagator would be to use the minimal number of mass insertions, in our case two. The self-energy in coordinate space with two mass insertions, with ψ the superpartner of ϕ and $P_\psi(x, x')$ the corresponding propagator is given by (cf. Figure 3.2):

$$\begin{aligned} -\mathrm{i}M_2^2(x_1 - x_2) =& 2g_5^2 \left(\frac{F_S}{M^2}\right)^2 \int \mathrm{d}^4x_3 \mathrm{d}^4x_4 \ \mathrm{tr}\left[P_R C \left(P_\psi(x_1, x_2)\right)^T P_L C S_F^{orb}(x_1, x_3, 0, L)\right. \\ & \left. \times\ C P_R \left[S_F^{orb}(x_3, x_4, L, L)\right]^T C^{-1} P_L S_F^{orb}(x_4, x_2, L, 0)\right] \\ =& -2g_5^2 \left(\frac{F_S}{M^2}\right)^2 \int \mathrm{d}^4x_3 \int \mathrm{d}^4x_4 \int \frac{\mathrm{d}^4 l}{(2\pi)^4}\ \mathrm{tr}\left[\frac{\mathrm{i}}{\not{l} + \mathrm{i}\varepsilon}\, e^{-\mathrm{i}l\cdot(x_1 - x_2)} \int \frac{\mathrm{d}^4 q}{(2\pi)^4}\right. \\ & \times \frac{\mathrm{i}P_L \not{q}}{2\chi_q \sin\chi_q L}\, e^{-\mathrm{i}q\cdot(x_1 - x_3)} \int \frac{\mathrm{d}^4 m}{(2\pi)^4} \frac{\mathrm{i}P_R \not{m} \cos\chi_m L}{2\chi_m \sin\chi_m L} \\ & \left. \times\ e^{-\mathrm{i}m\cdot(x_3 - x_4)} \int \frac{\mathrm{d}^4 n}{(2\pi)^4} \frac{\mathrm{i}P_L \not{n}}{2\chi_n \sin\chi_n L}\, e^{-\mathrm{i}n\cdot(x_4 - x_2)}\right] \\ =& -2g_5^2 \left(\frac{F_S}{M^2}\right)^2 \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathrm{i}}{l^2 + \mathrm{i}\varepsilon} \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{\mathrm{i}}{2\chi_q \sin\chi_q L} \frac{\mathrm{i}\cos\chi_q L}{2\chi_q \sin\chi_q L} \\ & \times\ \frac{\mathrm{i}}{2\chi_q \sin\chi_q L}\, e^{-\mathrm{i}(l+q)\cdot(x_1 - x_2)}\ \mathrm{tr}\left[\not{l} P_L \not{q}\not{q}\not{q}\right] \ . \end{aligned} \tag{3.30}$$

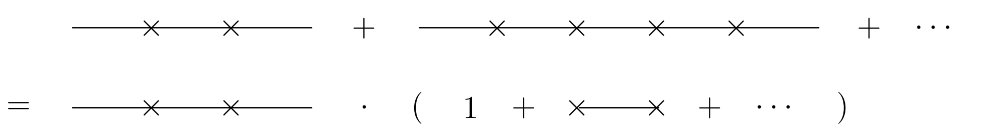
Here $\chi_p = \sqrt{p^2 + \mathrm{i}\varepsilon}$ with p the corresponding momentum. Note that a closed fermion loop contributes a factor of -1 since fermionic fields anticommute. We used the properties of the charge conjugation matrix. Fourier transforming to momentum space,

$$M^2(p^2) = \int_{-\infty}^{\infty} \mathrm{d}^4x\ e^{\mathrm{i}p\cdot x} M^2(x) \ , \tag{3.31}$$

we get a delta distribution $\delta(p-l-q)$, thus the l-integral collapses and we are left with

$$-\mathrm{i}M_2^2(p^2) = -\,2g_5^2\left(\frac{F_S}{M^2}\right)^2 \int \frac{\mathrm{d}^4q}{(2\pi)^4} \frac{\cos\chi_q L}{8\chi_q^3\,\sin^3(\chi_q L)[(p-q)^2+\mathrm{i}\varepsilon]}\;\mathrm{tr}\left[P_R(\not{p}-\not{q})\not{q}\not{q}\not{q}\right]. \tag{3.32}$$

Unfortunately this expression has an infrared divergence, but we will see that this divergence only results from approximating the massive gaugino propagator with two mass insertions. We obtain a finite result when we consider the correct one-loop diagram which is the sum of all diagrams with all different numbers of mass insertions. The corresponding gaugino propagator can be written diagrammatically as



We can evaluate the sum over all mass insertions explicitly as a formal geometric series, the only propagator involved is $S_F^{orb}(q,L,L)$. Note that the constant part of the propagator does not contribute, because it is always sandwiched between P_R and P_L. The sum over all additional terms is given by

$$\sum_{n=0}^{\infty}\left[\left(\frac{F_S}{M^2}\right)^2\left(\frac{q^2\cos^2(\chi_q L)}{4\chi_q^2\sin^2(\chi_q L)}\right)\right]^n = \left[1-\left(\frac{F_S}{M^2}\right)^2\left(\frac{q^2\cos^2(\chi_q L)}{4\chi_q^2\sin^2(\chi_q L)}\right)\right]^{-1}. \tag{3.33}$$

Performing the trace (cf. Appendix A)

$$\mathrm{tr}\left[P_R(\not{p}-\not{q})\not{q}\not{q}\not{q}\right] = 2q^2 q\cdot(p-q) \tag{3.34}$$

and then Wick rotating our expression in order to be able to use spherical coordinates we obtain

$$\begin{aligned} M^2(p^2) &= 2g_5^2\left(\frac{F_S}{M^2}\right)^2\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{q\cdot(p-q)\cosh qL}{4q\,\sinh^3(qL)(p-q)^2}\left[1+\left(\frac{F_S}{M^2}\right)^2\left(\frac{\cosh^2(qL)}{4\,\sinh^2(qL)}\right)\right]^{-1} \\ &= 2g_5^2\left(\frac{F_S}{M^2}\right)^2\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{q\cdot(p-q)\,\coth(qL)}{4q(p-q)^2}\left[\sinh^2(qL)+\frac{1}{4}\cosh^2(qL)\left(\frac{F_S}{M^2}\right)^2\right]^{-1}. \end{aligned} \tag{3.35}$$

This expression is now well defined even for q and p equal to zero. Now we can calculate the mass correction to order α with the help of Equation (3.15):

$$\delta m^2 = M^2(m_0) = M^2(0)\,. \tag{3.36}$$

We find

$$\delta m^2 = \frac{g_5^2}{16\pi^2}\left(\frac{F_S}{M^2}\right)^2 \int\limits_0^\infty \mathrm{d}q\; q^2 \coth(qL)\left[\sinh^2(qL) + \frac{1}{4}\cosh^2(qL)\left(\frac{F_S}{M^2}\right)^2\right]^{-1}. \tag{3.37}$$

Because we cannot perform the integration in a closed form, we have to make a suitable expansion. This will be done in the following section.

3.2.3 Perturbative Evaluation of the Loop Integral

To evaluate this mass correction, we first look at the denominator of the loop integral,

$$\int\limits_0^\infty \mathrm{d}q \frac{q^2\coth(qL)}{\sinh^2(qL) + a^2\cosh^2(qL)} = \frac{1}{L^3}\int\limits_0^\infty \mathrm{d}t \frac{t^2\coth(t)}{\sinh^2(t) + a^2\cosh^2(t)} \tag{3.38}$$

with $a^2 = F_S^2/4M^4$. Depending on the loop momentum, one term will dominate the other. Therefore we will split up the integral into two parts and write a perturbative expansion.

$$I(a) = I_\varepsilon(a) + I_\infty(a) = \int\limits_0^\varepsilon \mathrm{d}t \frac{t^2\coth(t)}{\sinh^2(t) + a^2\cosh^2(t)} + \int\limits_\varepsilon^\infty \mathrm{d}t \frac{t^2\coth(t)}{\sinh^2(t) + a^2\cosh^2(t)} \tag{3.39}$$

with $\varepsilon = \operatorname{arccoth} a$. This choice ensures that in the first integral the a^2-term is always bigger whereas in the second integral it is always smaller than the other term. Consider first the integral I_∞. Here we can expand the denominator as follows:

$$\frac{1}{\sinh^2(t) + a^2\ \cosh^2(t)} = \frac{1}{\sinh^2(t)}\sum_{n=0}^\infty \frac{(-1)^n a^{2n}\cosh^{2n}(t)}{\sinh^{2n}(t)} \tag{3.40}$$

Now we can integrate term by term. Starting with the $n = 0$-term, we obtain the following expression for $I_\infty(a)$

$$\begin{aligned} I_\infty^0(a) &= \left[-t\coth(t) - \frac{t^2}{2\sinh^2(t)} + \ln[\sinh(t)]\right]_\varepsilon^\infty \\ &= \left(\ln(1/2) + \frac{\sqrt{1+a^2}}{a}\operatorname{arcsinh}(a) + \frac{1}{2a^2}\operatorname{arcsinh}^2(a) - \ln(a)\right). \end{aligned} \tag{3.41}$$

For small a, $\operatorname{arccoth}(a) \sim a$ and hence this expression simplifies to

$$I_\infty^0(a) \sim \left(\ln(1/2) + \frac{3}{2} - \ln(a)\right). \tag{3.42}$$

The next order calculation yields for small a

$$I^1_\infty = -\frac{1}{2} + \mathcal{O}(a). \tag{3.43}$$

All other orders give a contribution

$$I^n_\infty = \frac{1}{2}(-1)^n/n + \mathcal{O}(a)\,. \tag{3.44}$$

We can sum up all $n > 0$-contributions yielding

$$\sum_{n=1}^{\infty} I^n_\infty = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n} = \frac{1}{2}\ln(1/2)\,. \tag{3.45}$$

Therefore the full result up to $\mathcal{O}(a)$ is given by

$$I_\infty(a) = -\ln(a) + \frac{3}{2}\big(1+\ln(1/2)\big)\,. \tag{3.46}$$

Now we turn to the integral I_ε, where we do an analogous expansion as in the case just considered. The result is given by

$$I_\varepsilon = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n} = \frac{1}{2}\ln(2) \tag{3.47}$$

plus higher order terms in a. Hence the mass correction up to terms constant in a is given by

$$\begin{aligned}\delta m^2 &= \frac{g_5^2}{16\pi^2}\left(\frac{F_S}{M^2}\right)^2\frac{1}{L^3}\left(-\ln\left(\frac{F_S}{2M^2}\right)+\frac{3}{2}\big(1+\ln(1/2)\big)-\frac{1}{2}\ln(1/2)\right)\\ &= \frac{g_4^2}{2\pi^2}\,m_\lambda^2\left(-\ln\left(2m_\lambda L\right)+\frac{3}{2}\right)\end{aligned} \tag{3.48}$$

where we used (3.8) and the following relation between the coupling constants in four and five dimensions:

$$\frac{1}{g_4^2} = \frac{2L}{g_5^2}\quad . \tag{3.49}$$

The leading term for small value of $F_S/M^2 = 2\,m_\lambda L$ is clearly given by the logarithmic term.

3.2.4 The High Momentum Approximation and Four-Dimensional Renormalisation Group Running

A different approach to evaluate the scalar mass correction is pursued in [12]. The authors notice that the theory behaves differently depending on the energy in question:

The theory is five-dimensional for scales larger than the compactification scale $1/L$ and effectively four-dimensional for smaller scales. The effective four-dimensional theory is obtained by integrating out the extra dimension at the compactification scale and contains the zero modes of the bulk fields. Therefore the approach in [12] is, to perform the integral given above (3.37) with the compactification scale as an infrared cutoff. This gives the scalar mass correction at the compactification scale. Of course the infrared cutoff renders the expression infrared insensitive. Therefore no infrared divergence shows up in [12] and the gaugino propagator does not have to be resummed. Nevertheless we start from the full expression (3.37). To turn this mass correction at the compactification scale into a physically meaningful quantity describing physics at the electroweak scale, the authors employ renormalisation group equations to run this mass correction down. In this section we will follow their approach and compare the result with the result we obtained by evaluating the full five-dimensional integral over all momenta.

Introducing the infrared cutoff $1/L$ is similar to approximating the integral for momenta $q > 1/L$. Doing this approximation we obtain

$$\begin{aligned} \delta m^2 &= g_5^2 \left(\frac{F_S}{M^2}\right)^2 \frac{1}{4\pi^2} \int \mathrm{d}q\, q^2 \;\; \frac{1}{e^{2qL}\left(1 + \left(\frac{F_S}{M^2}\right)^2\right)} \\ &\approx g_5^2 \left(\frac{F_S}{M^2}\right)^2 \frac{1}{4\pi^2} \int \mathrm{d}q q^2 e^{-2qL} \end{aligned} \tag{3.50}$$

using $(\frac{F_S}{M^2})^2 \ll 1$. Hence the mass correction is given by

$$\delta m^2 = g_5^2 \left(\frac{F_S}{M^2}\right)^2 \frac{1}{16\pi^2 L^3} = \frac{g_4^2}{2\pi^2}\, m_\lambda^2\,. \tag{3.51}$$

We see, that the scalar mass correction at the compactification scale is suppressed by the third power of the brane separation, which can be absorbed in the four-dimensional coupling and gaugino masses.

In the approximation of pure gauge interaction, the one-loop renormalisation group equations can be written as [28]

$$16\pi^2 \frac{\mathrm{d}}{\mathrm{d}\ln(pL)}\, m_\phi^2 = -8\, g_4^2\, |M|^2 \tag{3.52}$$

for each scalar ϕ where we omitted group theoretical factors. The renormalisation group equations get additional contributions, if the Yukawa couplings cannot be neglected, like in the case of the third family squarks and sleptons. M is the corresponding running gaugino mass parameter given by

$$M(q) = \frac{g_4^2(q)}{g_4^2(q_0)}\, m_\lambda\,, \tag{3.53}$$

where q_0 is the input scale. An important feature of Equation (3.52) is that the right-hand side is strictly negative. Therefore the scalar mass grows as it is evolved from the compactification scale down to the weak scale. Using these equations to run down the scalar mass correction to the gaugino mass scale we obtain to order α

$$\delta m^2 = -\frac{g_4^2}{2\pi^2}\, m_\lambda^2 \big(\ln(m_\lambda L) - 1\big). \tag{3.54}$$

Clearly without supersymmetry breaking this expression is zero since $m_\lambda = 0$ in this case. Comparing the mass correction at the compactification scale with the mass correction obtained from four-dimensional renormalisation group running, we see that we can neglect the contribution from the high scale since the compactification scale is typically a factor 100 below the Planck scale [12]. Therefore the largest contribution comes from the logarithmic term. This means that the extra dimension only sets the scale from which the four-dimensional renormalisation group running starts.

Comparing this result (3.54) with the result obtained earlier (3.48), we see that the factor multiplying the logarithmic term is identical and the only small difference is in the constant terms. However, since we used different approximations small deviations are quite natural and the main result is that both methods yield the same leading term.

Instead of using the renormalisation group equations we could also calculate the four-dimensional part of the mass correction. The zero mode of the five-dimensional propagator reduces to the ordinary four-dimensional Dirac propagator after integrating out the extra dimension. The resulting integral seems to be both, infrared and ultraviolet divergent. The infrared divergence can be cured as before when summing over all mass insertions. The ultraviolet divergence has to be regularised and renormalised. This is in contrast to the five-dimensional theory where everything is finite. Of course we know that the four-dimensional theory is valid only up to the compactification scale, so we have a physical cutoff in our scenario. When integrating up to the compactification scale we get

$$\begin{aligned}\delta m^2 &= \frac{g_4^2}{16\pi^4}\, 2\pi^2 m_\lambda^2\, 4 \int\limits_0^{1/L} \mathrm{d}q \frac{q}{q^2 + m_\lambda^2} \\ &= -\frac{g_4^2}{2\pi^2}\, m_\lambda^2 \ln(m_\lambda L),\end{aligned} \tag{3.55}$$

which is the same result for the infrared part as before (3.54).

3.3 The Fermion Self-Energy

To be able to calculate the supersymmetry breaking mass, we have to subtract the contribution of the fermion self-energy from the scalar self-energy calculated above. The corresponding gaugino loop differs only in the scalar field ϕ running in the loop instead of the fermionic field ψ:

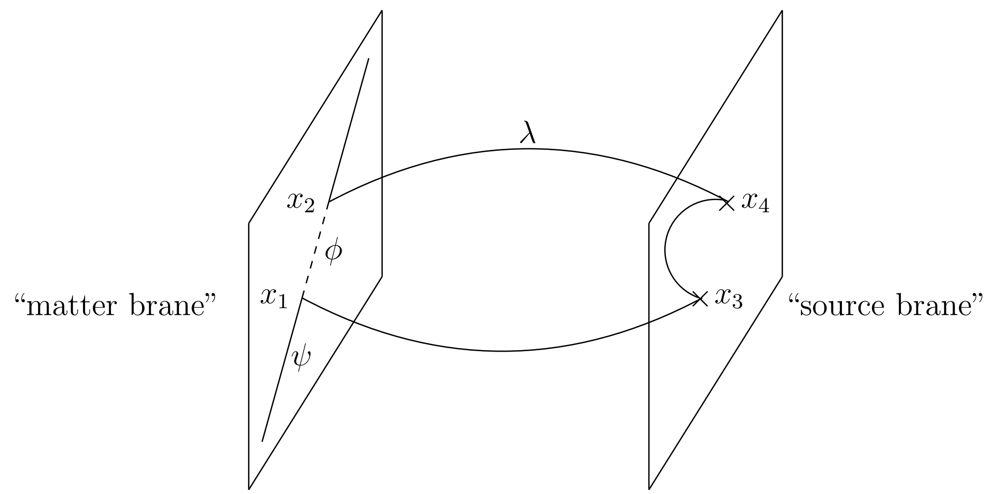


Figure 3.3: Fermionic self-energy with two mass insertions.

This diagram results in the following expression when summing over all mass insertions:

$$\begin{aligned} M(p) &= 2g_5^2 \left(\frac{F_S}{M^2}\right)^2 \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{\mathrm{tr}[\not{q}] \cosh(qL)}{8q \sinh^3(qL)[(p-q)^2)]} \left[1 + \left(\frac{F_S}{M^2}\right)^2 \left(\frac{\cosh^2(qL)}{4\sinh^2(qL)}\right)\right]^{-1} \\ &= 0 \end{aligned} \tag{3.56}$$

since the trace over an odd number of γ-matrices is zero (cf. Appendix A) and all terms which are not proportional to $\mathrm{tr}[\not{q}]$ are sandwiched between P_L and P_R (we have not written them explicitly here). Therefore, the parameter for supersymmetry breaking, which is given by the difference of the bosonic and fermionic mass correction, is equal to the bosonic mass correction (3.48).

Chapter 4

Supersymmetry Breaking in Six Dimensions

Orbifold compactifications have recently been applied to grand unified field theories, in which the standard model gauge group is embedded into larger gauge groups like $SU(5)$ or $SO(10)$ [30–33]. Whereas the breaking of $SU(5)$ down to the standard model gauge group can be achieved in a five-dimensional orbifold compactification, the breaking of $SO(10)$ is more involved and favours a six-dimensional space-time [32]. With the increasing experimental evidence for neutrino oscillations, which implies neutrino masses and mixings, this gauge group becomes particularly attractive. This is the case since neutrino masses imply the existence of an additional particle, the right handed neutrino, which can be unified with the corresponding quarks and leptons in a single representation of $SO(10)$. $SO(10)$ contains $SU(5)$ as well as the Pati-Salam group $SU(4)\times SU(2)\times SU(2)$ [3] and flipped $SU(5)$ [36] as subgroups. The intersection of all these subgroups yields the standard model gauge group with an additional $U(1)$-factor. Since $SO(10)$ plays a very important part in the discussion of grand unified theories, we will concentrate in this chapter on how supersymmetry breaking may be communicated in the six-dimensional case.

Similar to the five-dimensional case we will consider a super-Yang-Mills multiplet living in the six-dimensional bulk of a $M^4 \times T^2/\mathbb{Z}_2$ orbifold. This orbifold has four fixed points, corresponding to four-dimensional branes embedded in the bulk. We will assume supersymmetry to be broken spontaneously at one of the fixed points and matter to be confined to the three other branes. This setup is motivated by the $SO(10)$-GUT model described in [33], where three sequential quark-lepton families are localised at three different orbifold fixed points. The physical quarks and leptons are mixtures of these brane states and additional bulk zero modes. We only need one $\mathbb{Z}_2$-symmetry to break the extended $N = 2$ down to ordinary $N = 1$ supersymmetry at the orbifold fixed points. In [33] additional $\mathbb{Z}_2$-symmetries are introduced to break the gauge group

$SO(10)$ down to the different subgroups stated above. Though very important for the GUT-model, supersymmetry breaking is not affected by these additional $\mathbb{Z}_2$-symmetries.

4.1 Supersymmetric Yang-Mills Theory in Six Dimensions

The six-dimensional $N = 1$ supersymmetric Lagrangian is given by

$$\mathcal{L}_{6\mathrm{d}}^{\mathrm{YM}} = \mathrm{tr}\left(-\frac{1}{2}F_{MN}F^{MN} + \mathrm{i}\,\bar{\Lambda}_i\Gamma^M D_M\Lambda^i\right) \tag{4.1}$$

with $F_{MN} = F_{MN}^A T^A$ and $\Lambda = T^A\Lambda^A$, where T^A are the generators of the considered gauge group in the adjoint representation. The capital indices M, N run over $\{0, 1, 2, 3, 5, 6\}$. Furthermore, $D_M\Lambda = \partial_M\Lambda - \mathrm{i}g_6[V_M, \Lambda]$, $F_{MN} = [D_M, D_N]/(\mathrm{i}g_6)$ and Γ^M are the six-dimensional Dirac matrices. Here Λ is a left handed symplectic Majorana Weyl spinor, i.e.

$$\Lambda = \frac{1}{2}(1 - \Gamma_7)\Lambda \tag{4.2}$$

and

$$\Lambda^i = \varepsilon^{ij}C^{(6)}\bar{\Lambda}_j^T \tag{4.3}$$

where $C^{(6)}$ is the six-dimensional charge conjugation matrix given by

$$C^{(6)} = \mathrm{i}\sigma^2 \otimes C \tag{4.4}$$

with C the charge conjugation matrix encountered before. Γ_7 is the projection operator given in Appendix C.

The action, including fields on the boundaries, is given by a similar expression as in the five-dimensional case:

$$\begin{aligned} S = \int \mathrm{d}^6x \Big[&\mathcal{L}_{6\mathrm{d}}^{\mathrm{YM}} + \delta(y - \pi R_y)\delta(z)\mathcal{L}_m^1 + \delta(y)\delta(z - \pi R_z)\mathcal{L}_m^2 \\ &+ \delta(y - \pi R_y)\delta(z - \pi R_z)\mathcal{L}_m^3 + \delta(y)\delta(z)\mathcal{L}_s\Big]. \end{aligned} \tag{4.5}$$

We choose the source brane to be the fixed point $(0, 0)$, since we want to have a gaugino mediation scenario and in the corresponding GUT model the three sequential quark-lepton families reside on the other orbifold fixed points. The Lagrangians for the matter fields $\mathcal{L}_m^i$ are again the standard chiral Lagrangians used before (3.6), this time of course with the boundary terms of the six-dimensional gauge fields.

We will also assume a source brane Lagrangian analogous to the one in the five-dimensional case. The leading term of the source Lagrangian, which couples the chiral superfield S (cf. Section 3.1) to the gauge fields living in the bulk is of the form

$$\mathcal{L}_s = \frac{F_S}{M^3} \lambda_L^\alpha \lambda_{L\alpha} + \text{h.c.} + \ldots \ . \tag{4.6}$$

Observe that in the six-dimensional case the gaugino λ_α has mass dimension 5/2 and therefore we have to divide by M^3 to get a dimensionless action. The four-dimensional gaugino mass resulting from the F-term of the chiral superfield is, when normalised to give a canonical kinetic term,

$$m_\lambda = \frac{F_S}{4\pi^2 R_y R_z M^3} \ , \tag{4.7}$$

with R_y, R_z the radii of the torus. We extended the integration in the action to the full torus which will be compensated by the definition of the orbifold propagator. In contrast to the gaugino the gauge boson stays massless as before, hence supersymmetry is broken. This breaking is again mediated via gaugino loops through the bulk.

4.2 The Feynman Rules

The Γ-matrices are most conveniently chosen to be (cf. Appendix A)

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \gamma_\mu & 0 \end{pmatrix} , \qquad \Gamma_5 = \begin{pmatrix} 0 & \mathrm{i}\gamma_5 \\ \mathrm{i}\gamma_5 & 0 \end{pmatrix} , \qquad \Gamma_6 = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \tag{4.8}$$

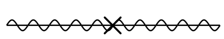
and

$$\Gamma_7 = \Gamma_0 \Gamma_1 \cdots \Gamma_6 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} , \tag{4.9}$$

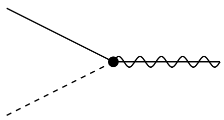
with $\gamma_5^2 = \mathbb{1}$ and $\{\Gamma_M, \Gamma_N\} = 2\,\eta_{MN} = 2\,\mathrm{diag}(1,-1,-1,-1,-1,-1)$. Note that the complex dimension of the Dirac matrices is $n = 2^{d/2}$ for even dimension d, which means that in our six-dimensional space-time we are dealing with eight-component spinors (cf. Appendix C). With this representation of the Dirac matrices, the chiral spinor $\Lambda = -\Gamma_7\,\Lambda$ can be written as $(\psi, 0)^T$, where ψ is now a four component spinor. The six-dimensional symplectic Majorana condition reduces to the one obtained in the five-dimensional case, with 4×4 C-matrix and four component spinors,

$$\psi^i = \varepsilon^{ij} C \bar{\psi}_j^T . \tag{4.10}$$

Hence the couplings of the gaugino are analogous to the ones we already know:



$$= \begin{cases} \mathrm{i}F_S/M^2\, C^{-1}\, P_L\, \delta(y)\delta(z) & (\psi) \\ \mathrm{i}F_S/M^2\, C\, P_R\, \delta(y)\delta(z) & (\bar{\psi}) \end{cases}$$



$$= \begin{cases} \sqrt{2}g_5 P_L C\delta(y-y_i)\delta(z-z_i) & (\psi) \\ \sqrt{2}g_5 P_R C\delta(y-y_i)\delta(z-z_i) & (\bar{\psi}) \end{cases} .$$

For the $\phi\psi\psi^1$-vertex the arguments of the delta distributions have to be adjusted to the fixed point we consider. With the choice of Dirac matrices given above the equation of motion simplifies to

$$\mathrm{i}(\not{\partial} + \mathrm{i}\gamma^5\partial_5 - \mathbb{1}\partial_6)\psi = 0. \tag{4.11}$$

Note that $\gamma^i = -\gamma_i$ for spatial index i. The defining relation for the two-point function is as before given by

$$\mathrm{i}(\not{\partial} + \mathrm{i}\gamma^5\partial_5 - \mathbb{1}\partial_6)\psi = \mathrm{i}\delta^4(x-x')\delta(y-y')\delta(z-z') \tag{4.12}$$

which leads to the following six-dimensional Dirac propagator in flat space:

$$S_F(x-x', y-y', z-z') = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\mathrm{d}p^5}{2\pi}\frac{\mathrm{d}p^6}{2\pi}\frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6\mathbb{1})}{p^2 - p_5^2 - p_6^2 + \mathrm{i}\varepsilon} e^{-\mathrm{i}p\cdot(x-x')} e^{\mathrm{i}p^5(y-y')} e^{\mathrm{i}p^6(z-z')}. \tag{4.13}$$

We will work with propagators in the mixed representation again. This means that we have to integrate over p_5 and p_6. The corresponding integral, Fourier transformed in four dimensions, is most conveniently written in spherical coordinates:

$$\begin{aligned} S_F(p, y-y', z-z') &= \frac{1}{4\pi^2}\int_0^\infty \mathrm{d}r\, r \int_0^{2\pi} \mathrm{d}\theta\, \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6\mathbb{1})}{p^2 - r^2 + \mathrm{i}\varepsilon}\, e^{\mathrm{i}rw\cos\theta} \\ &= \frac{1}{2\pi}\int_0^\infty \mathrm{d}r\, r\, \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6\mathbb{1})}{p^2 - r^2 + \mathrm{i}\varepsilon}\, J_0(rw) \\ &= -\frac{\mathrm{i}}{2\pi}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6\mathbb{1})\, K_0(-\mathrm{i}\chi_p w) \\ &= -\frac{\mathrm{i}\not{p}}{2\pi}\, K_0(-\mathrm{i}\chi_p w) - \frac{\gamma^5}{2\pi}\frac{\chi_p(y-y')}{w}\, K_1(-\mathrm{i}\chi_p w) \\ &\quad + \frac{\mathbb{1}}{2\pi}\frac{\chi_p(z-z')}{w}\, K_1(-\mathrm{i}\chi_p w) \end{aligned} \tag{4.14}$$

where J_0 is the Bessel function of the first kind, the K_i are modified Bessel functions, $\chi_p = \sqrt{p^2 + \mathrm{i}\varepsilon}$ and $w = \sqrt{(y-y')^2 + (z-z')^2}$. Basic properties of and integrals over Bessel functions are reviewed in Appendix D.

In the gaugino propagator only the term proportional to $\not{p}$ will survive in the loop integral, since the other terms will be multiplied by $P_L P_R$. Hence we will drop these non contributing terms from here on. To obtain the periodic propagator, we have to sum over all winding modes. Wick rotating to Euclidean space we obtain:

$$S_F^c(p, y-y', z-z') = \frac{-\mathrm{i}\not{p}}{2\pi} \sum_{n,m=-\infty}^{\infty} K_0\left(p\sqrt{(y-y'+2\pi R_y m)^2 + (z-z'+2\pi R_z n)^2}\right). \tag{4.15}$$

We leave $\not{p}$ in Minkowski space for now. After having done the trace over all Dirac matrices we will Wick rotate the result before performing the loop integral. This means we are effectively Wick rotating under the integral as usual. The gaugino propagator is divergent when starting and ending at the point $(0,0)$ in the extra dimensions. This divergence arises because we have taken the limit in which our brane is represented by a delta distribution, i.e. it is infinitely thin. Nevertheless the brane might have internal structure at short distance, and the divergence we encounter reflects the fact that the field theory we consider is not a valid description of the physics at these scales. To regularise the propagator we use a cutoff Λ. Since the divergence is logarithmic, it leads to running couplings and renormalisation group flow even at the classical level [37]. For small argument x, the Bessel functions can be approximated as follows:

$$K_0(x) = -\ln(x) - \gamma_E + \ln(2) + \mathcal{O}(x). \tag{4.16}$$

Regularising the divergent propagator with a momentum cutoff results in

$$S_F^c(p, 0, 0) = \frac{\mathrm{i}\not{p}}{2\pi}\left(\ln\left(\frac{p}{\Lambda}\right) - \sum_{n,m\neq(0,0)} K_0\left(p\sqrt{(2\pi R_y m)^2 + (2\pi R_z n)^2}\right)\right). \tag{4.17}$$

To renormalise this propagator we have to note that the source Lagrangian consists of many terms among which we can have a term like (see [12]):

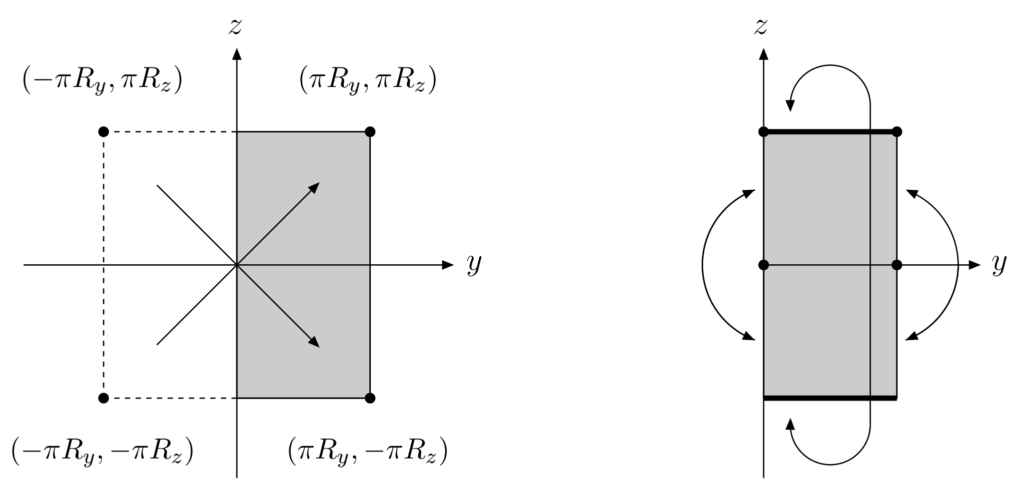
$$\mathcal{L}_S \supset \alpha \frac{F_S F_S^\dagger}{M^6} \bar{\lambda}_L \not{\partial} \lambda_L\,. \tag{4.18}$$

This term is a localised brane kinetic term for the gaugino and leads to an additional vertex at the orbifold fixed point $(0,0)$. When picking up the mass insertions at the source brane we have to include this brane kinetic term as well, acting as a counterterm. This means that the propagator with two mass insertions including the counterterm diagrammatically looks like

$$\propto \quad \left(\tfrac{F_S}{M^3}\right)^2 \not{p} \left(\ln\left(\tfrac{p}{\Lambda}\right) + c\right) \quad + \quad \alpha\left(\tfrac{F_S}{M^3}\right)^2 \not{p} \quad .$$

We choose the coefficient α such that the divergent piece is cancelled and we obtain $\ln(p/\mu)$ instead of $\ln(p/\Lambda)$ in the formerly divergent propagator. Here μ is an arbitrary renormalisation scale.

As with all higher dimensional field theories, the extra dimensions have to be compactified in some way to reproduce the four-dimensional world we live in. In this thesis we concentrate on the compactification on the orbifold $M^4 \times T^2/\mathbb{Z}_2$ similar to the space of the orbifold GUTs constructed in [32, 33]. The $\mathbb{Z}_2$-symmetry acting as $(y, z) \to (-y, -z)$ is introduced to break the unwanted $N = 2$ supersymmetry of the four-dimensional theory which is introduced by dimensional reduction from the six-dimensional case. The size of the physical space is halved when acting with the $\mathbb{Z}_2$ transformation: The physical region



(a) Implementing the $\mathbb{Z}_2$-symmetry (b) Identification of the edges

Figure 4.1: The $\mathbb{Z}_2$-Transformation acting on the Torus.

of this orbifolded space is obtained by identifying the edges of the rectangle as sketched in Figure 4.1. This results in a "pillow" with four fixed points: $(0, 0), (\pi R_y, 0), (0, \pi R_z)$ and $(\pi R_y, \pi R_z)$. The fixed points can be regarded as four-dimensional branes, similar to the five-dimensional case.

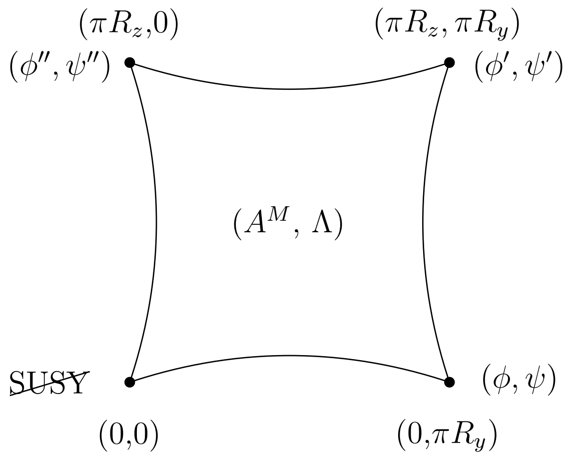


Figure 4.2: The $T^2/\mathbb{Z}_2$ orbifold with four fixed points.

Again we have to specify the $\mathbb{Z}_2$-parity of our fields to promote the $\mathbb{Z}_2$-symmetry to a symmetry of our theory:

$$\begin{aligned} V_\mu(x,-y,-z) &= +V_\mu(x,y,z) \quad , \\ V_{5,6}(x,-y,-z) &= -V_{5,6}(x,y,z) \quad . \end{aligned} \tag{4.19}$$

Under the corresponding reflection $(y,z) \to (-y,-z)$ the two Weyl spinors of the gaugino must have opposite parities to ensure the invariance of the Lagrangian:

$$\begin{aligned} \lambda_1(x,-y,-z) &= +\lambda_1(x,y,z) \quad , \\ \lambda_2(x,-y,-z) &= -\lambda_2(x,y,z) \quad , \end{aligned} \tag{4.20}$$

which can be rewritten to

$$\psi(x,-y,-z) = -\gamma^5 \psi(x,y,z) \tag{4.21}$$

with $\psi = (\lambda_1, \lambda_2)^T$. Hence we can write ψ in terms of an unconstrained field χ as follows:

$$\psi(x,y,z) = \frac{1}{2}\big(\chi(x,y,z) - \gamma^5 \chi(x,-y,-z)\big) \tag{4.22}$$

which causes the six-dimensional orbifold gaugino propagator to be of the same structure:

$$S_F^{orb}(p,y,y',z,z') = \frac{1}{2}\big(S_F^c(p,y,y',z,z') - \gamma^5 S_F^c(p,-y,y',-z,z')\big) \ . \tag{4.23}$$

4.3 The Self-Energy of the Scalar Field

We now want to calculate the scalar mass correction. For that, we have to evaluate the loop diagram from the fixed point to which our scalar field is confined, to the source brane $(0,0)$ where we have to pick up the supersymmetry breaking mass insertions. Again, we have to resum the gaugino propagator in order to obtain an infrared finite result. Written in a general form, the six-dimensional gaugino propagator, when resummed over all mass insertions, is given by

$$\begin{aligned}
\widetilde{S}_F^{orb}(y,y',z,z') &= \left(\frac{F_S}{M^3}\right)^2 S_F^{orb}(y,0,z,0)\,\mathrm{i}CP_R\left(S_F^{orb}(0,0,0,0)\right)^T \mathrm{i}C^{-1}P_L\,S_F^{orb}(0,y',0,z') \\
&\quad\times\left(1-\left(\frac{F_S}{M^3}\right)^2 S_F^{orb}(0,0,0,0)\,CP_R\left(S_F^{orb}(0,0,0,0)\right)^T C^{-1}P_L+\cdots\right) \\
&= \left(\frac{F_S}{M^3}\right)^2 \frac{S_F^{orb}(y,0,z,0)\,\mathrm{i}CP_R\left(S_F^{orb}(0,0,0,0)\right)^T \mathrm{i}C^{-1}P_L\,S_F^{orb}(0,y',0,z')}{1+\left(\frac{F_S}{M^3}\right)^2 S_F^{orb}(0,0,0,0)\,C\left(S_F^{orb}(0,0,0,0)\right)^T C^{-1}}\,.
\end{aligned}\tag{4.24}$$

We suppressed the four-momentum in this notation. Note that propagators on orbifolds generally depend on both, y and y', rather than on just the difference $y-y'$. Since we cannot evaluate the sum over all winding modes explicitly, we will divide the loop integral into an infrared and an ultraviolet part. For both regions we will perform suitable approximations.

4.3.1 The Infrared Part of the Mass Correction

For small momenta we can approximate the sum (4.15) with an integral, as long as the condition $2\pi R p \ll 1$ is valid. Note that for the propagator starting and ending at the same point in the extra dimensions we have to add an additional term (cf. (4.17)). Approximating the sum we obtain

$$\begin{aligned}
S_F^c(p,y-y',z-z') &= \frac{-\mathrm{i}\not{p}}{2\pi}\int\limits_{-\infty}^{\infty}\mathrm{d}m\;\mathrm{d}n\,K_0\left(p\sqrt{(y-y'+2\pi R_y m)^2+(z-z'+2\pi R_z n)^2}\right) \\
&= \frac{-\mathrm{i}\not{p}}{2\pi}\int\limits_{-\infty}^{\infty}\frac{\mathrm{d}u\;\mathrm{d}w}{(2\pi R_y p)(2\pi R_z p)}\,K_0\left(\sqrt{(u+p(y-y'))^2+(w+p(z-z'))^2}\right).
\end{aligned}\tag{4.25}$$

We can now shift variables and replace by spherical coordinates:

$$\begin{aligned} S_F^c(p, y-y', z-z') &= -\mathrm{i}\not{p} \int_0^\infty \frac{\mathrm{d}r\ r}{(2\pi R_y p)(2\pi R_z p)}\, K_0(r) \\ &= \frac{-\mathrm{i}\not{p}}{4\pi^2 R_y R_z p^2}\ . \end{aligned} \tag{4.26}$$

The orbifold boundary condition is implemented as before. This results in the following massless propagators in this approximation:

$$S_F^{orb}(p, y, y', z, z') = \frac{-\mathrm{i}P_L\not{p}}{4\pi^2 R_y R_z p^2}\ , \tag{4.27a}$$

$$S_F^{orb}(p, 0, 0, 0, 0) = \frac{\mathrm{i}P_L\not{p}}{2\pi}\left(\ln\left(\frac{p}{\mu}\right) - \frac{1}{2\pi R_y R_z p^2}\right). \tag{4.27b}$$

The first expression is not valid in the case that the propagator starts and ends at the same point in the extra dimensions and has to be replaced by the second one. These propagators are infrared divergent and we have to resum them over all mass insertions in order to obtain a finite expression. Plugging these massless propagators into the resummed propagator (4.24), we obtain

$$\begin{aligned} \widetilde{S}_F^{orb}(p, y, y', z, z') &= \left(\frac{F_S}{M^3}\right)^2 \frac{-\mathrm{i}P_L\not{p}}{4\pi^2 R_y R_z p^2}\, \mathrm{i}P_R C\, \frac{\mathrm{i}P_R\not{p}^T}{2\pi}\left(\ln\left(\frac{p}{\mu}\right) - \frac{1}{2\pi R_y R_z p^2}\right) \\ &\quad \times \mathrm{i}C^{-1}P_L \frac{-\mathrm{i}P_L\not{p}}{4\pi^2 R_y R_z p^2}\left[1 + \left(\frac{F_S}{M^3}\right)^2 \frac{p^2}{4\pi^2}\left(\ln\left(\frac{p}{\mu}\right) - \frac{1}{2\pi R_y R_z p^2}\right)^2\right]^{-1} \\ &= -\left(\frac{F_S}{M^3}\right)^2 \frac{\mathrm{i}P_L\not{p}}{32\pi^5 R_y^2 R_z^2 p^2}\left(\ln\left(\frac{p}{\mu}\right) - \frac{1}{2\pi R_y R_z p^2}\right) \\ &\quad \times \left[1 + \left(\frac{F_S}{M^3}\right)^2 \frac{p^2}{4\pi^2}\left(\ln\left(\frac{p}{\mu}\right) - \frac{1}{2\pi R_y R_z p^2}\right)^2\right]^{-1}. \end{aligned} \tag{4.28}$$

In this infrared approximation we can neglect the logarithmic term simplifying the expression for the propagator,

$$\widetilde{S}_F^{orb}(p, y, y', z, z') = -m_\lambda^2 \frac{\mathrm{i}P_L\not{p}}{2\pi^2 R_y R_z p^4}\left[1 + \frac{m_\lambda^2}{p^2}\right]^{-1}, \tag{4.29}$$

where we used (4.7). The scalar self-energy can be written analogously to the first line of (3.30). By analogous we mean that the structure is exactly the same, only the vertices and the propagators change and we already include the sum over all mass insertions.

In order to obtain the mass correction, we set the external momentum on-shell, Fourier transform in four dimensions and Wick rotate to Euclidean space. This leads to the following expression for the infrared part of the scalar mass correction with a the cutoff up to which the infrared approximation is valid:

$$
\begin{aligned}
\delta m^2 &= -2g_6^2 \frac{2\pi^2}{(2\pi)^4} m_\lambda^2 \int_0^a \mathrm{d}q\, q^3 \,\mathrm{tr}\left[\frac{\not{q}}{q^2}\frac{P_L \not{q}}{2\pi^2 R_y R_z q^4}\left[1+\frac{m_\lambda^2}{q^2}\right]^{-1}\right] \\
&= \frac{g_4^2 m_\lambda^2}{2\pi^2}\int_0^a \mathrm{d}q\, \frac{q}{q^2+m_\lambda^2} \\
&= \frac{g_4^2 m_\lambda^2}{4\pi^2}\ln\left(\frac{a^2+m_\lambda^2}{m_\lambda^2}\right) \sim \frac{g_4^2 m_\lambda^2}{2\pi^2}\ln\left(\frac{a}{m_\lambda}\right)
\end{aligned}
\tag{4.30}
$$

where we used a relation between the coupling constants in six and four dimensions similar to (3.49),

$$
g_4^2 = \frac{g_6^2}{4\pi^2 R_y R_z}\,. \tag{4.31}
$$

The last step in (4.30) uses that $a \sim 1/\max(R_y, R_z) \gg m_\lambda$. This means that a corresponds to the compactification scale with larger radius and hence to the point at which the theory starts to look four-dimensional again. Comparing this with the result we obtained from four-dimensional renormalisation group running (3.54) in the previous chapter, we see that the coefficient of the logarithmic term is again the same. Therefore, the infrared part of the mass correction obtained by calculating the six-dimensional loop is as before equivalent to four-dimensional RG running. The ultraviolet part of the mass correction will be calculated in the next section.

4.3.2 The Ultraviolet Part of the Mass Correction in the Asymmetric Case ($R_y \gg R_z$)

Now we turn to the ultraviolet part of the loop integral. To find a suitable approximation it is helpful to come back to the evaluation of the periodic gaugino propagator. It can be advantageous not to perform the Fourier integrals explicitly as we did in Equation (4.14), but to make use of the Poisson resummation formula. This formula relates a sum over a function with the sum over the Fourier transform of this function:

$$
\sum_{n=-\infty}^{\infty}\int \frac{\mathrm{d}k}{2\pi}\, e^{-\mathrm{i}k2\pi Rn} F(k) = \frac{1}{2\pi R}\sum_{n=-\infty}^{\infty} F(n/R). \tag{4.32}
$$

The periodic propagator can be generally written in terms of the propagator in flat space as:

$$\begin{aligned}
S_F^c(p,y,y',z,z') &= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} S_F(p, y+2\pi R_y m - y', z + 2\pi R_z n - z') \\
&= \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \int \frac{\mathrm{d}p^5}{(2\pi)} \int \frac{\mathrm{d}p^6}{(2\pi)} \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6 \mathbb{1})}{p^2 - p_5^2 - p_6^2 + \mathrm{i}\varepsilon} \\
&\qquad \times e^{-\mathrm{i}p^5(y+2\pi R_y m - y')}\, e^{-\mathrm{i}p^6(z+2\pi R_z n - z')} \qquad (4.33) \\
&= \frac{1}{4\pi^2 R_y R_z} \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6 \mathbb{1})}{p^2 - (m/R_y)^2 - (n/R_z)^2 + \mathrm{i}\varepsilon} \\
&\qquad \times e^{-\mathrm{i}m(y-y')/R_y}\, e^{-\mathrm{i}n(z-z')/R_z},
\end{aligned}$$

where we used the Poisson resummation formula to obtain the last line.

There is no physical reason which of the summations over Kaluza-Klein modes should come first. The two sums are absolutely symmetric and we can perform one of them explicitly. The second sum cannot be evaluated in a closed form and we have to approximate it in one way or the other. In the asymmetric case $R_y \gg R_z$ one could ask the question whether one sum is more important than the other. This is indeed the case since after Wick rotation the summands of the sum over m decrease much slower than the summands of the other sum. Therefore we should sum over m in this case and approximate the sum over n. Evaluating the sum over m we get

$$\begin{aligned}
&\frac{1}{4\pi^2 R_y R_z} \sum_{n=-\infty}^{\infty} \left(\sum_{m=1}^{\infty} \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6 \mathbb{1})}{p^2 - (m/R_y)^2 - (n/R_z)^2 + \mathrm{i}\varepsilon} 2\cos(m(y-y'/R_y)) \right. \\
&\qquad \left. + \frac{1}{p^2 - n^2/R_z^2 + \mathrm{i}\varepsilon} \right) e^{-\mathrm{i}n(z-z')/R_z} \\
= \;& \frac{\mathrm{i}(\not{p} - \gamma^5\partial_5 + \mathrm{i}\partial_6 \mathbb{1})}{4\pi R_z} \\
&\qquad \times \sum_{n=-\infty}^{\infty} \left(\frac{\cos\left(\sqrt{p^2 - n^2/R_z^2 + \mathrm{i}\varepsilon}\,(\pi R_y - |y-y'|)\right)}{\sqrt{p^2 - n^2/R_z^2 + \mathrm{i}\varepsilon}\,\sin(\pi R_y \sqrt{p^2 - n^2/R_z^2 + \mathrm{i}\varepsilon})} \right) e^{-\mathrm{i}n(z-z')/R_z}
\end{aligned} \tag{4.34}$$

where we used

$$\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2 - a^2} = \frac{1}{2a^2} - \frac{\pi \cos a((2m+1)\pi - x)}{2a\sin(\pi a)} \tag{4.35}$$

for $x \in [2\pi m, 2\pi(m+1)]$ and $a \notin \mathbb{Z}$.

Since we want to calculate the loop contribution to the self energies of fields living on the orbifold fixed points, we are mainly interested in the gaugino propagators going from one fixed point to another. It is worth noting that the structure of the loop integral is such that only terms proportional to $\not{q}$ will contribute. Therefore the derivative terms drop out and we will neglect them from here on. Putting in the values for y and z, we obtain the following propagator after a Wick rotation, with the high momentum limit given in the second line:

$$\begin{aligned} S^c(p, \pi R_y, 0, 0, 0) &= \frac{\mathrm{i}\not{p}}{4\pi R_z} \sum_{n=-\infty}^{\infty} \frac{1}{\sqrt{p^2 + n^2/R_z^2}\,\sinh(\pi R_y \sqrt{p^2 + n^2/R_z^2}\,)} \\ &\approx \frac{\mathrm{i}\not{p}}{2\pi R_z} \sum_{n=-\infty}^{\infty} \frac{e^{-\pi R_y \sqrt{p^2+n^2/R_z^2}}}{\sqrt{p^2 + n^2/R_z^2}} \ . \end{aligned} \tag{4.36}$$

Here we have to write the orbifold condition in a slightly different way, because the values of y, y' are restricted to the interval $[0, \pi R_y]$ after the summation:

$$S_F^{orb}(p, y, y', z, z') = \frac{1}{2} \left(S_F^c(p, y, y', z, z') - \gamma^5 S_F^c(p, 2\pi R_y - y, y', 2\pi R_z - z, z') \right) \ . \tag{4.37}$$

Only the propagator $S_F^c(p, \pi R_y, 0, 0, 0)$ is really periodic, since we just performed the sum over m, not over n. This means that only in this case we can consistently apply the orbifold condition. We see that for $n > 0$ the summands are exponentially damped. Therefore we can try to approximate the sum with the lowest order term.

The mass correction for the scalar field confined to the fixed point $(\pi R_y, 0)$ can now be written in the approximation of two mass insertions as

$$\begin{aligned} \delta m^2 &= \frac{2g_6^2}{16\pi^4} \left(\frac{F_S}{M^3} \right)^2 2\pi^2 \int \mathrm{d}q\, q^3\, \mathrm{tr} \left[\frac{\not{q}}{q^2} P_L \frac{1}{8\pi^3 R_z^2} \not{q}\, \ln(q/\mu)\, e^{-2\pi R_y q} \right] \\ &= -\frac{3}{2\pi^3}\, g_4^2 m_\lambda^2\, \frac{R_z}{R_y}\, \ln(\mu R_y) \, . \end{aligned} \tag{4.38}$$

Since we considered the case $R_y \gg R_z$, only the limits $R_y \to \infty$ and $R_z \to 0$ make sense and are well defined. We see that the ultraviolet part of the scalar masses is suppressed and the dominant part of these scalar masses comes from the infrared part or equivalently from four-dimensional renormalisation group running. As in the five-dimensional case the mass correction for the fermionic fields on the orbifold fixed points is identically zero and hence the bosonic mass correction is a supersymmetry breaking parameter by itself. In the asymmetric case we considered here, the mass correction including infrared and ultraviolet effects is given by

$$\delta m^2 = -\frac{g_4^2 m_\lambda^2}{2\pi^2} \left(\ln(m_\lambda R_y) + \frac{3R_z}{\pi R_y}\, \ln(\mu R_y) \right) \ . \tag{4.39}$$

When taking the limit $R_z \to 0$ we reproduce the result obtained in five dimensions. In contrast to the five-dimensional case this result is logarithmically scale dependent. Of course a physical parameter should be independent of this scale, i.e. when summing over all orders in the coupling g_4, μ should drop out from the scalar mass correction. Nevertheless this is a fixed order expression, which can be renormalisation scale dependent. The quest for the in some sense "optimal" scale is vital for meaningful applications but has so far no generally accepted solution. Nevertheless this scale dependence is an indication of the direction in which the perturbation theory is going, since one knows that the higher-order terms must conspire to cancel it exactly.

Chapter 5

Summary and Outlook

Theories on orbifolded space-times offer promising breaking schemes for supersymmetry as well as gauge symmetries. These orbifold constructions in higher dimensional theories lead quite naturally to hidden sector models possessing a nice geometrical picture of sequestering. Hidden sector models in turn can lead to a viable superpartner mass spectrum and hence are very attractive phenomenologically for mediating spontaneous supersymmetry breaking to the MSSM particles.

In this thesis we calculated supersymmetry breaking mass parameters in an orbifold model in five and six space-time dimensions. We obtained the supersymmetry breaking masses using two different methods. The first method being the full five-/six-dimensional calculation can be seen as the main part of this work. Starting with the five-dimensional case, we derived the Feynman rules of our model and performed the loop calculation to extend existing results in the literature. Here we find that the five-dimensional theory is infrared and ultraviolet finite after resumming the gaugino propagator. The main contribution to the mass correction stems from the infrared part of the loop integral. As a second approach we turned to an approximation based on [12]. The authors realise that the five-dimensional theory can be split into two regimes: For energies above the compactification scale the theory is five-dimensional, while for energies below the compactification scale the theory is effectively four-dimensional. The supersymmetry breaking parameters calculated in the high energy part of the higher dimensional theories give initial conditions for the four-dimensional renormalisation group. In this second approach we obtained small supersymmetry breaking parameters for the sfermions at the compactification scale. Evolving these parameters down to the electroweak scale, the soft scalar masses receive large positive flavour diagonal contributions from renormalisation group running. Comparing the two different approaches, we find that both results are dominated by infrared effects. Both yield the same leading terms, confirming the consistency of our calculations.

Furthermore, we extended the theory to six space-time dimensions. In this case the scalar mass correction has only been roughly estimated [13], without specifying any details. Performing the corresponding calculation, we encountered a divergence arising from short distance singularities which is not mentioned in [13]. We notice that the theory can be renormalised with the help of a brane kinetic counter term, resulting in the dependence of the gaugino propagator on an arbitrary renormalisation scale μ. This μ-dependence carries through such that the mass correction in six dimensions depends logarithmically on this scale parameter. The ultraviolet part is again suppressed. The infrared part comes out analogously to the five-dimensional case, i.e. the six-dimensional calculation yields the same result as the corresponding four-dimensional renormalisation group running. Again, this infrared contribution dominates the supersymmetry breaking mass terms.

A next step would be the inclusion of Higgs fields in the bulk, possibly leading to additional contributions to the scalar mass parameters. Furthermore, one should include supergravity effects as well, especially when considering higher energies.

Appendix A

Notations and Conventions

Throughout the thesis we have used a time-like space-time metric

$$\eta_{mn} = \mathrm{diag}(1, -1, ..., -1). \tag{A.1}$$

The four-dimensional Dirac matrices are chosen in the Weyl representation

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix} \tag{A.2}$$

satisfying $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$. Here $\sigma^m = (\mathbb{1}, \sigma^i)$, $\bar{\sigma}^m = (\mathbb{1}, -\sigma^i)$ and the σ^i are the Pauli matrices. Though we mainly work in terms of Γ-matrices rather than σ-matrices, we state their algebra for completeness:

$$\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\beta} = -\eta^{mn}\delta^{\beta}_{\alpha} + 2(\sigma^{mn})_{\alpha}{}^{\beta} \tag{A.3a}$$

$$(\sigma^{mn})_{\alpha}{}^{\beta} = \tfrac{1}{4}(\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{n\dot{\alpha}\beta} - \sigma^n_{\alpha\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\beta}) \tag{A.3b}$$

$$(\sigma^{0i})_{\alpha}{}^{\beta} = \tfrac{1}{2}(\sigma^i)_{\alpha}{}^{\beta}, \quad (\sigma^{ij})_{\alpha}{}^{\beta} = -\tfrac{1}{2}\mathrm{i}\varepsilon^{ijk}(\sigma^k)_{\alpha}{}^{\beta} \tag{A.3c}$$

$$\bar{\sigma}^{m\dot{\alpha}\alpha}\sigma^n_{\alpha\dot{\beta}} = -\eta^{mn}\delta^{\dot{\alpha}}_{\dot{\beta}} + 2(\bar{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}} \tag{A.3d}$$

$$(\bar{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\beta}} = \tfrac{1}{4}(\bar{\sigma}^{m\dot{\alpha}\alpha}\sigma^n_{\alpha\dot{\beta}} - \bar{\sigma}^{n\dot{\alpha}\alpha}\sigma^m_{\alpha\dot{\beta}}) \tag{A.3e}$$

$$(\bar{\sigma}^{0i})^{\dot{\alpha}}{}_{\dot{\beta}} = \tfrac{1}{2}(\bar{\sigma}^i)^{\dot{\alpha}}{}_{\dot{\beta}}, \quad (\bar{\sigma}^{ij})^{\dot{\alpha}}{}_{\dot{\beta}} = \tfrac{1}{2}\mathrm{i}\varepsilon^{ijk}(\bar{\sigma}^k)^{\dot{\alpha}}{}_{\dot{\beta}} \tag{A.3f}$$

$$(\sigma^{mn})_{\alpha}{}^{\alpha} = (\bar{\sigma}^{mn})^{\dot{\alpha}}{}_{\dot{\alpha}} = 0 \tag{A.3g}$$

$$\mathrm{tr}(\sigma^m\bar{\sigma}^n) = -2\eta^{mn} \tag{A.3h}$$

$$\sigma^m_{\alpha\dot{\alpha}}\bar{\sigma}^{\dot{\beta}\beta}_m = -2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}} \tag{A.3i}$$

$$(\sigma^m)_{\alpha\dot{\alpha}}(\sigma_m)_{\beta\dot{\beta}} = -2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} \tag{A.3j}$$

$$\sigma^a\bar{\sigma}^b\sigma^c = \eta^{ac}\sigma^b - \eta^{bc}\sigma^a - \eta^{ab}\sigma^c + \mathrm{i}\varepsilon^{abcd}\sigma_d \tag{A.3k}$$
$$\bar{\sigma}^a\sigma^b\bar{\sigma}^c = \eta^{ac}\bar{\sigma}^b - \eta^{bc}\bar{\sigma}^a - \eta^{ab}\bar{\sigma}^c - \mathrm{i}\varepsilon^{abcd}\bar{\sigma}_d \tag{A.3l}$$

$$(\sigma^{mn})_\alpha{}^\beta\varepsilon_{\beta\gamma} = (\sigma^{mn})_\gamma{}^\beta\varepsilon_{\beta\alpha} \tag{A.3m}$$

$$\varepsilon^{mnkl}\sigma_{kl} = -2\mathrm{i}\sigma^{mn}, \quad \varepsilon^{mnkl}\bar{\sigma}_{kl} = 2\mathrm{i}\bar{\sigma}^{mn} \tag{A.3n}$$

The Greek indices $(\alpha, \dot{\alpha}, \beta, \dot{\beta}...)$ run from 1 to 2 and denote two-component Weyl spinors. Raising and lowering of the indices can be done with the help of the ε-tensor. Choosing

$$\varepsilon^{12} = \varepsilon_{21} = 1, \quad \varepsilon^{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1 \tag{A.4a}$$

$$\varepsilon^{\alpha\beta}\varepsilon_{\gamma\alpha} = \delta^\beta_\gamma, \quad \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\alpha}} = \delta^{\dot{\beta}}_{\dot{\gamma}} \tag{A.4b}$$

one can define raising and lowering via

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta \Rightarrow \psi^1 = \psi_2,\ \psi^2 = -\psi_1 \tag{A.5a}$$

$$\bar{\psi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\psi_{\dot{\beta}}, \quad \bar{\psi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dot{\beta}} \Rightarrow \bar{\psi}^{\dot{1}} = \bar{\psi}_{\dot{2}},\ \bar{\psi}^{\dot{2}} = -\bar{\psi}_{\dot{1}} \tag{A.5b}$$

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \varepsilon^{\dot{\alpha}\dot{\beta}}\varepsilon^{\alpha\beta}\sigma^m_{\beta\dot{\beta}} \tag{A.5c}$$

$$\sigma^m_{\alpha\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}\bar{\sigma}^{m\dot{\beta}\beta}. \tag{A.5d}$$

With these definitions one finds

$$\psi\chi = \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \tag{A.6a}$$

$$\bar{\psi}\bar{\chi} = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = -\bar{\psi}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} = \bar{\chi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \bar{\chi}\bar{\psi} \tag{A.6b}$$

$$(\chi\psi)^\dagger = (\chi^\alpha\psi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi}. \tag{A.6c}$$

Here we used the fact that spinors anticommute.

Coming back to the framework of γ-matrices, we note that Dirac spinors Ψ_D contain two Weyl spinors

$$\Psi_D = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}. \tag{A.7}$$

In five dimensions we use the following convention for γ-matrices

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}, \quad \gamma^5 = -\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & \mathrm{i} \end{pmatrix}. \tag{A.8}$$

Note that in this convention the γ^5-matrix is different from the one used in the projection operators $P_{L/R}$.

In the case of six space-time dimensions, the Γ-matrices are 8×8 matrices which we choose to be

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \gamma_\mu & 0 \end{pmatrix}, \qquad \Gamma_5 = \begin{pmatrix} 0 & \mathrm{i}\gamma_5 \\ \mathrm{i}\gamma_5 & 0 \end{pmatrix}, \qquad \Gamma_6 = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \tag{A.9}$$

with the $4 \times 4\, \gamma^\mu$-matrices as given above and $(\gamma^5)^2 = 1$. Note that we changed the convention of the γ^5-matrix from the five-dimensional to the six-dimensional case. Finally

$$\Gamma_7 = \Gamma_0 \Gamma_1 \cdots \Gamma_6 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}. \tag{A.10}$$

Traces over Dirac Matrices

When considering diagrams with fermions running in a loop, we have to take the trace over all corresponding γ matrices. These can be evaluated as follows:

$$\mathrm{tr}(\mathbb{1}) = 4 \tag{A.11a}$$

$$\mathrm{tr}(\text{any odd \# of } \gamma\text{'s}) = 0 \tag{A.11b}$$

$$\mathrm{tr}(\gamma^\mu \gamma^\nu) = 4 g^{\mu\nu} \tag{A.11c}$$

$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) \tag{A.11d}$$

$$\mathrm{tr}(\gamma^5) = 0 \tag{A.11e}$$

$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^5) = 0 \tag{A.11f}$$

$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5) = -4\mathrm{i}\epsilon^{\mu\nu\rho\sigma} \tag{A.11g}$$

Appendix B

The Supersymmetry Algebra

The supersymmetry algebra is a graded Lie algebra, i.e. it is given in terms of commutators as well as anticommutators. The algebra consists of the translation generator P_m, the generators of the Lorentz group M_{mn}, the supersymmetry generators $\mathcal{Q}^I_\alpha$ and $\bar{\mathcal{Q}}_{\dot{\alpha}I}$ and possibly the generators of some internal symmetry Lie group B_a. The Greek indices $(\alpha, \dot{\alpha}, \beta, \dot{\beta}...)$ run from 1 to 2 and denote two-component Weyl spinors, whereas the Latin indices $(m, n...)$ run from 1 to 4 and refer to Lorentz four vectors. The label I runs from 1 to some number N. If $N > 1$ one speaks of N-extended supersymmetry in contrast to simple supersymmetry for $N = 1$. As discussed earlier, there are restrictions on the maximal value of N which lead to maximally extended supersymmetric theories. The generators satisfy the following algebra which can be found e.g. in [17]:

$$[P_m, P_n] = 0 \tag{B.1a}$$

$$[M_{mn}, M_{rs}] = \mathrm{i}(\eta_{mr}M_{ns} - \eta_{nr}M_{ms} - \eta_{ms}M_{nr} + \eta_{ns}M_{mr}) \tag{B.1b}$$

$$[P_m, M_{rs}] = \mathrm{i}(\eta_{ms}P_r - \eta_{mr}P_s) \tag{B.1c}$$

$$[B_a, B_b] = \mathrm{i}f_{abc}B_c \tag{B.1d}$$

$$[B_a, P_m] = [B_a, M_{mn}] = 0 \tag{B.1e}$$

$$[\mathcal{Q}^I_\alpha, P_m] = [\bar{\mathcal{Q}}_{\dot{\alpha}I}, P_m] = 0 \tag{B.1f}$$

$$[\mathcal{Q}^I_\alpha, M_{mn}] = \tfrac{1}{2}(\sigma_{mn})_\alpha{}^\beta \mathcal{Q}^I_\beta \tag{B.1g}$$

$$[\bar{\mathcal{Q}}_{\dot{\alpha}I}, M_{mn}] = -\tfrac{1}{2}\bar{\mathcal{Q}}_{\dot{\beta}I}(\bar{\sigma}_{mn})^{\dot{\beta}}{}_{\dot{\alpha}} \tag{B.1h}$$

$$[\mathcal{Q}^I_\alpha, B_a] = (b_a)^I{}_J \mathcal{Q}^J_\alpha \tag{B.1i}$$

$$[\bar{\mathcal{Q}}_{\dot{\alpha}I}, B_a] = -\bar{\mathcal{Q}}_{\dot{\alpha}J}(b_a)^J{}_I \tag{B.1j}$$

$$\{\mathcal{Q}^I_\alpha, \bar{\mathcal{Q}}_{\dot{\alpha}J}\} = 2\delta^I_J \sigma^m_{\alpha\dot{\alpha}} P_m \tag{B.1k}$$

$$\{\mathcal{Q}^I_\alpha, \mathcal{Q}^J_\beta\} = 2\varepsilon_{\alpha\beta} Z^{IJ} \quad \text{with } Z^{IJ} = -Z^{JI} \tag{B.1l}$$

$$\{\bar{\mathcal{Q}}_{\dot{\alpha}I}, \bar{\mathcal{Q}}_{\dot{\beta}J}\} = 2\varepsilon_{\dot{\alpha}\dot{\beta}} Z^\dagger_{IJ} \tag{B.1m}$$

Equations (B.1a) to (B.1c) give the algebra of the ordinary Poincaré group, equations (B.1d) and (B.1e) the internal symmetry algebra. The f_{abc} are the structure constants and the coefficients b_a form some matrix representation of this internal symmetry group. The spinor properties of the $\mathcal{Q}$'s are shown in eqs. (B.1g) and (B.1h), where the σ_{mn} are the generators of the Lorentz group in the spinor representation. Note that the appearance of P^m on the right-hand side of (B.1k) is not surprising, since it transforms under Lorentz boosts and rotations as a spin 1 object while $\mathcal{Q}$, $\bar{\mathcal{Q}}$ each transform as spin 1/2 objects. The mass operator P^2 commutes with the operators $\mathcal{Q}, \mathcal{Q}^\dagger$ and with all space-time operators, so it follows immediately that particles inhabiting the same irreducible supermultiplet must have equal eigenvalues of P^2 and hence are degenerate in mass. The remaining equations are non-trivial only in the case of extended supersymmetry. The "central charges" Z^{IJ} vanish for simple supersymmetry and the only internal symmetry which can act non-trivially (i.e. with nonvanishing matrix representation b_a) on the $\mathcal{Q}$'s is a $U(1)$ symmetry called R-symmetry.

Appendix C

Spinors in Higher Dimensions

When higher dimensional supersymmetric theories are considered, the first thing which needs attention is how spinors look like in higher dimensions. In this Appendix we give a short overview. Spinors form the representation space of the covering group of the d-dimensional Lorentz group $SO(1,d-1)$, *Spin*(d). The representations of *Spin(d)* can be obtained from representations of the d-dimensional Dirac algebra defined by

$$\{\Gamma_a, \Gamma_b\} = 2\eta_{ab}\mathbb{1} \quad \text{with } a,b = 0,1,\ldots,d-1 \tag{C.1}$$

with η_{ab} the d-dimensional Minkowski metric, $\eta_{ab} = \mathrm{diag}(+,-,\ldots,-)$. A representation of the group is formed by the matrices

$$\tfrac{1}{2}\Sigma_{ab} = \frac{i}{2}\Gamma_{[a}\Gamma_{b]}. \tag{C.2}$$

The complex dimension of this representation is

$$D = 2^{d/2} \quad \text{for } d \text{ even} \tag{C.3a}$$

$$D = 2^{(d-1)/2} \quad \text{for } d \text{ odd.} \tag{C.3b}$$

which grows exponentially with d. There are, however, conditions that can reduce the dimension of the spinor representations as discussed already in Chapter 2.2: *reality* and *chirality* conditions. In even dimensions the matrix

$$\Gamma_{d+1} = \Gamma_0\Gamma_1\cdots\Gamma_{d-1} \tag{C.4}$$

is non-trivial and can be used to define projection operators

$$P_{L,R} = \tfrac{1}{2}(\mathbb{1} \pm \beta\Gamma_{d+1}), \tag{C.5}$$

where $\beta = 1$ for $d = (2 \mod 4)$ and $\beta = \mathrm{i}$ for $d = (4 \mod 4)$. These operators project on the left- and right-handed spinors, which form subspaces half the size of the original representation. This is possible because Γ_{d-1} and the Γ_a anticommute. In odd dimensions, on the other hand, they commute and hence $\Gamma_{d-1} \propto \mathbb{1}$, so there are no chiral spinors in odd dimensions.

The other possibility is a reality (Majorana) condition. It is usually stated as

$$\psi^c \equiv C\bar{\psi}^T = \psi \tag{C.6}$$

where $\bar{\psi} = \psi^\dagger \Gamma_0$ and C is the charge conjugation matrix satisfying

$$C^{-1}\Gamma_a C = -(-1)^d \Gamma_a^T \quad \text{and} \quad C^T = C^{-1} = C^\dagger = \pm C, \tag{C.7}$$

where the sign in the last equation depends non-trivially on the dimension. This condition can only be imposed in one to four dimensions and in eight to twelve dimensions. A Majorana condition halves the representation. However, it is not necessarily compatible with a chirality condition. Chiral Majorana spinors are possible only in $d = (2 \mod 8)$.

In four dimensions we can have either chiral (Weyl) or Majorana spinors, so the minimal real dimension is 4 (since (C.3) lists the complex dimension and we can eliminate half of the dimensions). In five dimensions, although the spinor dimension (C.3) is the same, we can have neither chiral nor Majorana spinors, and so the minimal real dimension of the spinor representation is 8.

There is, however, a possibility to impose a symplectic Majorana condition involving two spinors $\psi^{1,2}$ in five dimensions. This is the condition we used frequently in Chapter 3 and it reads

$$\psi^i = \varepsilon^{ij} C \bar{\psi}_j^T. \tag{C.8}$$

The five-dimensional charge conjugation matrix is in our case explicitly given by $C = -\mathrm{i}\sigma^2 \otimes \mathbb{1}_2$. As shown before we can decompose these spinors into two Weyl spinors $\psi_{L,R}$ as

$$\psi_1 = \begin{pmatrix} (\psi_L)_\alpha \\ (\bar{\psi}_R)^{\dot{\alpha}} \end{pmatrix} \qquad \psi_2 = \begin{pmatrix} (\psi_R)_\alpha \\ -(\bar{\psi}_L)^{\dot{\alpha}} \end{pmatrix} \tag{C.9a}$$

$$\bar{\psi}_1 = \left((\psi_R)^\alpha, (\bar{\psi}_L)_{\dot{\alpha}}\right) \qquad \bar{\psi}_2 = \left(-(\psi_L)^\alpha, (\bar{\psi}_R)_{\dot{\alpha}}\right). \tag{C.9b}$$

This gives an easy handle on the relation between theories formulated in terms of two and four component spinors in five dimensions.

In six dimensions the real dimension is 16, but can be halved by imposing a chirality condition. Furthermore the spinors can be subject to a symplectic Majorana condition like in the five-dimensional case. These two conditions can be written as

$$\Gamma^7 \Lambda = -\Lambda \tag{C.10a}$$

$$\Lambda^i = \varepsilon^{ij} C^{(6)} \Lambda_j^T. \tag{C.10b}$$

where the six-dimensional charge conjugation matrix is given in terms of the five-dimensional one as $C^{(6)} = i\sigma^2 \otimes C$. When using an appropriate representation of the Γ-matrices, the spinor Λ can be written as a four component spinor due to the chirality condition.

Appendix D

Bessel Functions

Bessel functions are encountered in the evaluation of six-dimensional propagators when Fourier transforming the two extra dimensions in order to obtain a mixed propagator. In this Appendix we will give a very short and basic overview over the definition and a few properties of Bessel functions. For a more detailed discussion we refer the reader to e.g. [38].

Bessel functions are special cases of cylinder functions. The Bessel functions $J_n(z)$ and $Y_n(z)$ are linearly independent solutions to the differential equation

$$z^2 y'' + z y' + (z^2 - n^2) y = 0. \tag{D.1}$$

For integer n, the $J_n(z)$ are regular at $z = 0$, while the $Y_n(z)$ have a logarithmic divergence at $z = 0$. Bessel functions arise in solving differential equations for systems with cylindrical symmetry. $J_n(z)$ is often called the Bessel function of the first kind, or simply the Bessel function. $Y_n(z)$ is referred to as the Bessel function of the second kind or the Neumann function (denoted $N_n(z)$).

The modified Bessel functions $I_n(z)$ and $K_n(z)$ are solutions to the differential equation

$$z^2 y'' + z y' - (z^2 + n^2) y = 0. \tag{D.2}$$

For integer n , $I_n(z)$ is regular at $z = 0$ whereas $K_n(z)$ always has a logarithmic divergence. Therefore we had to regularise the gaugino propagator when starting and ending at the same point in the two co-dimensions.

The Bessel function of the first kind can be written as

$$J_\nu(z) = \frac{z^\nu}{2^\nu} \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{2^{2k} k! \Gamma(\nu + k + 1)} \ , \quad [|\arg z| < \pi]. \tag{D.3}$$

A very important integral representation we used is given by

$$J_0(z) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}t\, e^{iz\cos(t)} \tag{D.4}$$

and by the relation

$$\int_0^\infty dq \frac{q}{p^2 - q^2} J_0(qr) = -K_0(-ipr)\,, \quad [\arg p^2 \neq 0,\, r > 0]\,. \tag{D.5}$$

We also used the following integrals involving modified Bessel functions:

$$\int_0^\infty dq q^5 K_0^2(aq) = \frac{16}{15a^6}\,, \; \mathrm{Re}(a) > 0\,, \tag{D.6a}$$

$$\int_0^\infty dq q^5 \ln(q) K_0^2(aq) = \frac{c - 16\ln(a)}{15a^6}\,, \; \mathrm{Re}(a) > 0\,, \tag{D.6b}$$

$$\int_0^\infty dq q^5 K_1^2(aq) = \frac{8}{5a^6}\,, \; \mathrm{Re}(a) > 0\,, \tag{D.6c}$$

$$\int_0^\infty dq q^5 \ln(q) K_1^2(aq) = \frac{c' - 8\ln(a)}{5a^6}\,, \; \mathrm{Re}(a) > 0\,, \tag{D.6d}$$

$$\int_0^\infty dq q K_0(q) = 1\,. \tag{D.6e}$$

Finally the derivative of the modified Bessel function $K_n(z)$ is given by

$$\partial_z K_n(z) = -\frac{1}{2}\left(K_{n-1}(z) + K_{n+1}(z)\right). \tag{D.7}$$

Bibliography

[1] A. Salam, J.C. Ward, Nuovo Cim. **11**,568 (1959) ; S.L. Glashow, Nucl. Phys. **22**, 569 (1961) ; J. Goldstone, A. Salam, S. Weinberg, Phys. Rev. **127**, 965 (1962) ; A. Salam, J.C. Ward, Phys. Lett. **13**, 168 (1964)

[2] H. Georgi, S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974)

[3] J.C. Pati, A. Salam, Phys. Rev. **D 10**, 275 (1974)

[4] H. Fritzsch, P. Minkowski, Ann. Phys. **93**, 193 (1975)

[5] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Rev. **D 24**, 1681 (1981) ; S. Dimopoulos, H. Georgi, Nucl. Phys. **B 193**, 150 (1981)

[6] C. Giunti, C.W. Kim, U.W. Lee, Mod. Phys. Lett. **A 6**, 1745 (1991) ; U. Amaldi, W.de Boer, H. Furstenau, Phys. Lett. **B 260**,447 (1991)

[7] J. Wess, B. Zumino, Nucl. Phys. **B 70**, 39 (1974)

[8] L.E. Ibanez, G.G. Ross, Phys. Lett. **B 110**, 215 (1982) ; L. Alvarez-Gaume, M. Claudson, M. Wise, Nucl. Phys. **B 207**, 96 (1982)

[9] T. Kobayashi, S. Raby, R.-J. Zhang, Phys. Lett. **B 593**, 262 (2004)

[10] S. Förste, H.P. Nilles, P. Vaudrevange, A. Wingerter, Phys. Rev. **D 70**, 106008 (2004)

[11] E. A. Mirabelli and M. E. Peskin, Phys. Rev. **D 58**, 065002 (1998)

[12] D.E. Kaplan, G.D. Kribs, M. Schmaltz, Phys. Rev. D **62** (2000) 035010

[13] Z. Chacko, Markus A. Luty, Ann E. Nelson, Eduardo Ponton, JHEP 0001:003,2000.

[14] S.R. Coleman, J. Mandula, Phys. Rev. **159**,1251-1256 (1967)

[15] R. Haag, J.T. Lopuszanski, M. Sohnius, Nucl. Phys. **B 88**, 257 (1975)

[16] M.F. Sohnius, Phys. Rept. **128**, 39-204 (1985)

[17] J. Wess, J. Bagger, *Supersymmetry and Supergravity*, Second ed., Princeton Series, Princeton University Press, Princeton, New Jersey (1992) Front. Phys. **58**(1983)

[18] N. Arkani-Hamed, T. Gregoire, J. Wacker, JHEP **03**, 055 (2002)

[19] A. Hebecker, Nucl. Phys. **B 632**, 101-113 (2002)

[20] J. Strathdee, Int. J. Mod. Phys. **A2**, 273-300 (1987)

[21] S. Weinberg, *The Quantum Theory of Fields II*, Cambridge University Press (1996)

[22] A. Pich, arXiv:hep-ph/9806303

[23] D.J.H. Chung *et. al*, arXiv:hep-ph/0312378

[24] H.P. Nilles, Phys. Rep. **110**, 1 (1984)

[25] V. Kaplunovsky, J. Louis, Phys. Lett. **B 306**, 269 (1993)

[26] M. Dine, A.E. Nelson, Phys. Rev. **D 48**, 1277 (1993); M. Dine, A.E. Nelson, Y. Shirman, Phys. Rev. **D 51**, 1362 (1995); M. Dine, A.E. Nelson, Y. Shirman, Phys. Rev. **D 53**, 2658 (1996)

[27] L. Randall, R. Sundrum, Nucl. Phys. **B 557**, 79-118 (1999)

[28] S.P. Martin, arXiv:hep-ph/9709356 (1999)

[29] A. Hebecker, J.M. Russell, Nucl.Phys. **B 625**, 128-150 (2002)

[30] Y. Kawamura, Prog. Theor. Phys. **103**, 613, (2000)

[31] Y. Kawamura, Prog. Theor. Phys. **105**, 999-1006, (2001)

[32] T. Asaka, W. Buchmüller, L. Covi, Phys. Lett. **B 523**, 199 (2001) ; L. Hall, Y. Nomura, T. Okui, D. Smith, Phys. Rev. **D 65**, 035008 (2002)

[33] T. Asaka, W. Buchmüller, L. Covi, Phys. Lett. **B 563**, 209 (2003)

[34] M. Puchwein, Z. Kunszt, Ann. Phys. **311**, 288-313 (2004)

[35] L. Rold, Phys. Rev. **D 69**, 105015 (2004)

[36] S.M. Barr, Phys. Lett. **B 112**, 219 (1982)

[37] W.D. Goldberger, M.B. Wise, Phys. Rev. Lett. **D 65** 020511 (2001)

[38] I.S. Gradstein, I.M. Ryshik, Tables of Series, Products and Integrals, Verlag Harri Deutsch, Thun · Frankfurt/M (1981)

Acknowledgements

I am very grateful to Wilfried Buchmüller for the supervision of my Diploma thesis, his foresight in choosing this interesting subject and his advice throughout the year. I also would like to thank Jan Louis for his co-supervision on behalf of the Universität Hamburg. Furthermore, I have to thank my office mates Jan and Dirk as well as all the members of the DESY theory group, especially Adam and Toffi for helpful discussions and comments. Finally I would like to thank my parents who always supported me in every possible way, all of my friends and especially Marcia for her love, understanding and support.