6D Supergravity: Warped Solution and Gravity Mediated Supersymmetry Breaking

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Abstract

We consider compactified six-dimensional gauged supergravity and find the general warped solution with four-dimensional maximal symmetry. Important features of the solution such as the number and position of singularities are determined by a free holomorphic function. Furthermore, in a particular torus compactification we derive the supergravity coupling of brane fields by the Noether procedure and investigate gravity-mediated supersymmetry breaking. The effective Kähler potential is not sequestered, yet tree level gravity mediation is absent as long as the superpotential is independent of the radius modulus.

Zusammenfassung

Ich bestimme die allgemeine gewarpte Lösung der kompaktifizierten sechsdimensionalen Supergravitation mit maximaler vierdimensionaler Symmetrie. Wesentliche Eigenschaften der Lösung wie Anzahl und Position von Singularitäten werden von einer frei wählbaren holomorphen Funktion bestimmt. Weiterhin betrachte ich eine spezielle $T^2/\mathbb{Z}_2$-Kompaktifizierung, bestimme die lokal supersymmetrische Kopplung der Branefelder mit Hilfe der Noether-Methode und untersuche die gravitative Vermittlung von Supersymmetriebrechung. Das effektive Kählerpotential zerfällt nicht in separate Anteile der verschiedenen Branes. Supersymmetriebrechung kann damit bereits auf Bornniveau durch gravitative Effekte übermittelt werden, jedoch nur, wenn das Superpotential vom Radionmodulus abhängt.

Contents

1. Introduction

> Wenn es möglich ist, daß es Ausdehnungen von andern Abmessungen gebe, so ist es auch sehr wahrscheinlich, daß sie Gott wirklich irgendwo angebracht hat. Denn seine Werke haben alle die Größe und Mannigfaltigkeit, die sie nur fassen können. Räume von dieser Art könnten nun unmöglich mit solchen in Verbindung stehen, die von ganz anderm Wesen sind; daher würden dergleichen Räume zu unserer Welt gar nicht gehören, sondern eigene Welten ausmachen müssen.
>
> *(Immanuel Kant, Gedanken von der wahren Schätzung der lebendigen Kräfte)*

High-energy physics is in a peculiar situation today: On the one hand, there are big problems which imply that we have to extend the standard model. These include the recent cosmological precision measurements which indicate that the universe as a whole is dominated by an unknown substance – dubbed dark energy for a lack of better words – which causes the expansion to accelerate, and that the matter part mainly consists of unknown matter which is called dark matter. In the standard model, there is no candidate particle for dark matter, and the problem of dark energy might well require us to go beyond quantum field theory: Quantum field theory does not describe gravity, which is phenomenologically well established. On the other hand, the standard model successfully describes all laboratory experiments conducted so far (strictly speaking, neutrino masses are already beyond the standard model, but they are easily included and almost standard by now).

A possible programme to attack these problems is to assume that the standard model results as a low-energy effective theory from a string theory compactification. This generically incorporates several popular extensions of the standard model, notably supersymmetry and extra dimensions, on the low-energy side while providing a theory of quantum gravity on the high-energy side.

A complete model along these lines would have various issues: In a string theory setup, there are branes and fluxes to choose, there might be intermediate field theories in lower dimensions and an $\mathcal{N} = 1$ supersymmetric theory with a gauge group containing the standard model group and the correct particle spectrum. In this effective theory, the remaining supersymmetry is then spontaneously broken and the standard model, including masses and mixings, emerges. Along the way the moduli have to be fixed, the cosmological constant must be small and positive, and there must be some suitable dark matter candidate.

This is a formidable task, and it can be tackled from many angles. In this thesis, we will consider two aspects of intermediate field theories: The result of any compactification is a compactified space, that is, a spacetime that is split in a four-dimensional large, i.e. noncompact, part for standard model particles and physicists to propagate in, and a compact internal space, and thus the metric is block-diagonal. For simplicity, one often considers maximal symmetry for the large dimensions (i.e. "our" world is Minkowski, de Sitter or anti-de Sitter space), and then this almost fixes the four-dimensional part of the metric. The only part that is not fixed is a conformal factor that depends on the internal dimensions, so that the metric becomes

$$\mathrm{d}s^2 = W^2(y)\,\mathrm{d}s_4^2 + g_{mn}(y)\,\mathrm{d}y^m\mathrm{d}y^n\,.$$

Such a geometry is known as warped geometry and has important implications for e.g. the hierarchy between scales in the theory.

The use of extra dimensions in purely field-theoretical settings has also become rather popular in recent years, in particular the use of singular internal spaces, like orbifolds. They are smooth manifolds, but for isolated singular points. In the full space, these points correspond to special hyperplanes, called branes. Fields on such spaces can live in the full spacetime (the bulk), in which case there can be boundary conditions at the branes which lead e.g. to a breaking of gauge symmetries or to a reduction of the spectrum of light states. Alternatively, fields can be confined to branes, so that one can simply realise e.g. four-dimensional theories which are coupled to higher-dimensional gravity and gauge fields. Such models go by the name of brane-worlds and realise higher dimensions in a way Kant did not imagine.

Orbifolds provide a convenient method to construct such singular spaces, by dividing a smooth manifold by a non-freely acting discrete symmetry. However, the resulting space is what matters, and one can study such spaces without resorting to the orbifold procedure.

Considering singularities, six-dimensional compactifications are particularly interesting because four-dimensional branes then have two transverse dimensions. Codimension-two branes have an important property, namely, their curvature is independent of their tension, or energy density. The reason is that two-dimensional spaces admit a special kind of singularity, the conical singularity. As the name suggests, a conical singularity locally looks like the tip of a cone, and it is completely characterised by the deficit angle which gives the "pointiness" of the cone. The curvature close to the singularity is of the type (finite) $+\,\delta(x-x_0)$, and there is no $1/r$-divergence as e.g. for a Schwarzschild singularity which is of codimension three. A brane with a certain tension will just generate a corresponding deficit angle, but the curvature (i.e. the effective cosmological constant) on the brane vanishes. This is the idea of self-tuning.

We will study this idea in the context of six-dimensional gauged supergravity and a warped compactification. This supergravity theory comes with a positive definite potential which allows to circumvent the usual no-go theorems on warped compactifications, and it has a gauge field whose field strength can have nontrivial flux in the internal space. We find that, also for warped geometries, Minkowski space is the unique maximally symmetric four-dimensional space. Fine-tuning of the cosmological constant is thus unnecessary, the reason being that the internal space is compact. For the same reason, however, the gauge flux is quantised, and this leads to a different fine-tuning condition, between the brane tensions.

For six-dimensional supergravity, we find the general warped solution. Its most interesting feature is a free holomorphic function which determines the number of singularities in the internal space via its simple poles and zeroes. Hence, we can find solutions with any number of branes, and give examples of two- and many-brane solutions.

Many-brane solutions with matter on different branes are frequently employed for supersymmetry breaking. The spatial separation in the extra dimensions serves to hide the hidden sector, while the mediation of supersymmetry breaking is effected by bulk fields. A very generic candidate for the mediation is gravity, since gravitational fields always propagate in the whole of spacetime. Gravity mediation in this sense does not mean graviton exchange, but any interaction that is encoded in the Kähler potential, which generically includes non-universal couplings which imply strong flavour-changing neutral currents. If, however, the extra-dimensional separation leads to a particular structure in the Kähler potential, the sequestered form, schematically

$$\Omega = \Omega_{\text{bulk}} + \Omega_{\text{observable}} + \Omega_{\text{hidden}} \,,$$

such contact terms are absent. In five dimensional supergravity, the Kähler potential is indeed sequestered. We will study a six-dimensional setup in which this structure is not realised. This is due to a certain bulk field which has nontrivial couplings to the branes, and upon integrating out this field, we potentially generate contact terms which lead to direct mediation of supersymmetry breaking. However, the situation is actually more subtle: masses in the observable sector depend on the moduli dependence of the superpotential as well, and hence they are related to the issue of moduli stabilisation.

In the next chapter, we will introduce higher-dimensional supergravity and motivate why it is worthwhile to analyse such theories by placing it in the context of general beyond standard model physics. In Chapter 3, we will find the general warped solution of six-dimensional supergravity. For a particular compactification, we find the locally supersymmetric coupling of branes to the bulk supergravity in Chapter 4 and analyse gravity mediated supersymmetry breaking. Finally, we conclude in Chapter 5.

2. Supergravity in Higher Dimensions

We first present a brief overview of the standard model and give some reason why it should be extended. In the second and third section we then outline three possible directions of extension, higher dimensions, supersymmetry and grand unification. All are justified in two ways: From a top-down perspective, string theory (almost) requires both extra dimensions and supersymmetry, and it easily provides large gauge groups like E_8, depending on the type of string theory. On the other hand, these concepts are also quite fruitful in a bottom-up approach, that is, in (effective) field theories which arise as direct extensions of the standard model with additional ingredients.

This section is not intended to be comprehensive. Rather, its aim is to motivate why it is ultimately worthwhile to investigate higher-dimensional supergravity in the light of general beyond the standard model physics.

2.1. The Standard Model

The standard model is a gauge theory based on the gauge group $\mathsf{SU}(3)_C \times \mathsf{SU}(2)_L \times \mathsf{U}(1)_Y$. Associated are the gauge bosons in the adjoint representations of the factors: The gluons G^a_μ, the $\mathsf{SU}(2)_L$ gauge bosons W^i_μ and the hypercharge gauge field B_μ, where $a = 1, \ldots, 8$ and $i = 1, 2, 3$ label the $\mathsf{SU}(3)_C$ and $\mathsf{SU}(2)_L$ generators, respectively.

The particle content is split into the matter fermions and the Higgs sector:

- The matter sector contains three families, i.e. three copies which follow the same pattern. All fermions are chiral, and they are divided between different representations as follows:
  - The quarks come in the fundamental or antifundamental of $\mathsf{SU}(3)_C$. The left handed up and down quarks form a $\mathsf{SU}(2)_L$-doublet, while their right-handed counterparts are singlets of weak isospin:

$$Q = \begin{pmatrix} u_\mathrm{L} \\ d_\mathrm{L} \end{pmatrix} \sim (\mathbf{3}, \mathbf{2})_{1/6} \;, \qquad u^\mathrm{c}_\mathrm{R} \sim \left(\overline{\mathbf{3}}, \mathbf{1}\right)_{-2/3} \;, \qquad d^\mathrm{c}_\mathrm{R} \sim \left(\overline{\mathbf{3}}, \mathbf{1}\right)_{1/3} \tag{2.1}$$

  - The leptons are similar, but for one exception: They are colourless. Strictly speaking, the standard model does not contain a right-handed neutrino (which is a complete singlet), but neutrino masses, which are by now firmly established, strongly suggest their existence:

$$L = \begin{pmatrix} \nu_\mathrm{L} \\ e_\mathrm{L} \end{pmatrix} \sim (\mathbf{1}, \mathbf{2})_{-1/2} \;, \qquad e^\mathrm{c}_\mathrm{R} \sim (\mathbf{1}, \mathbf{1})_1 \;, \qquad \left[\nu^\mathrm{c}_\mathrm{R} \sim (\mathbf{1}, \mathbf{1})_0\,\right] \tag{2.2}$$

- The Higgs field is a complex scalar doublet:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2})_{1/2} \tag{2.3}$$

Since left- and right-handed fermions have different gauge quantum numbers, any direct mass term is forbidden by gauge invariance, just as for the gauge fields. To obtain massive particles, one can employ the Higgs mechanism: The Higgs field has a potential

$$\mathscr{L}_V = \mu^2 \left(\Phi^\dagger \Phi\right) - \frac{\lambda}{4} \left(\Phi^\dagger \Phi\right)^2 , \tag{2.4}$$

which generates a vacuum expectation value $\langle \Phi \rangle \neq 0$. The potential fixes the modulus to be $\left\langle \Phi^\dagger \right\rangle \langle \Phi \rangle = 4\mu^2/\lambda = (246 \text{ GeV})^2$, but the phase of $\langle \Phi \rangle$ is not fixed. Since Φ is charged under $\mathsf{SU}(2)_L \times \mathsf{U}(1)_Y$, this group gets broken to the subgroup $\mathsf{U}(1)_{\text{em}}$ (whose relation to the original $\mathsf{SU}(2)_L \times \mathsf{U}(1)_Y$ is determined by the phase of the vacuum expectation value). The W^i_μ and B_μ fields mix in this process to give the $W^\pm_\mu$, Z_μ and A_μ fields. The first three of these get massive: A mass term comes from the covariant derivative acting on $\langle \Phi \rangle$, the required longitudinal components are provided by the Higgs field. Being a complex doublet, it had four real degrees of freedom to start with, so one remains: The Higgs boson ϕ, the only as yet undetected particle of the standard model.

The Higgs not only couples to the gauge field, but also has Yukawa couplings to the fermions $\sim \Phi\, \overline{\psi}_{\text{L}} \psi_{\text{R}}$. The vacuum expectation value $\langle \Phi \rangle$ thus turns the Yukawa coupling into a mass term for the fermions. The three generations can mix in the mass terms via coupling matrices, and the mass terms need not be (and are not) diagonal in the same basis as the kinetic terms. This leads into the realm of flavour physics: important examples are the B meson system and neutrino oscillations. The Yukawa matrices show a complicated structure, the origin of which is not understood at the moment. Nevertheless, in the quark sector, the mixings are small, and the absence of flavour-changing neutral currents (FCNCs) is an important constraint for theories beyond the standard model.

2.1.1. Problems of the Standard Model

While the standard model is very successful experimentally, there are a number of issues which suggest that there might be a more fundamental theory beyond it:

1. Neutrino masses and coupling unification: The right-handed neutrino, which is already present in (2.2), is actually beyond the standard model. It is (almost) required because we now know that neutrinos have masses. Neutrino masses, however, are extremely small (their size is unknown, but there are upper bounds of order 1 eV, and theoretical prejudice would prefer masses around 10^{-2} eV), and them being generated by the Higgs mechanism in the same way as for the other fermions seems highly unnatural. There is a more compelling solution to this problem which goes by the name of see-saw mechanism[1]. The key observation

is that right-handed neutrinos are complete standard model singlets, so they can be Majorana particles. As such they can have two types of mass terms, the usual Higgs coupling Y_ν and a Majorana mass term which involves only the right-handed fields,

$$\mathscr{L} \supset Y_\nu \epsilon_{ij} \left\langle \Phi^i \right\rangle \overline{\nu}_{\mathrm{R}} L^j + M \nu_{\mathrm{R}}^T C \nu_{\mathrm{R}} + \mathrm{H.c.} \tag{2.5}$$

Assuming that the Majorana mass M is much larger than the Dirac mass $m_{\mathrm{D}} = Y_\nu \left\langle \Phi^i \right\rangle$, one can find two mass eigenstates, the heavy one with a mass $\sim M$ and a light one with mass $\sim m_{\mathrm{D}}^2/M$. Assuming a Dirac mass of order of the other fermion masses, $\mathcal{O}\left(1\,\mathrm{GeV}\right)$, the Majorana mass scale would be around $\mathcal{O}\left(10^{11}\,\mathrm{GeV}\right)$. This model nicely fits into e.g. SO(10) GUTs, see Section 2.2.1.

2. The hierarchy problem: The highest scale in the standard model, the vacuum expectation value of the Higgs field, is $\langle \phi \rangle = 246$ GeV. This is 16 orders of magnitude below the Planck mass $M_{\mathrm{P}} = 2.4 \cdot 10^{18}$ GeV, the relevant scale of gravity (in other words, gravity is extremely weak compared with e.g. electromagnetic forces). This is regarded as a very unnatural hierarchy between gravity and the other interactions.

3. The preceding problems were not fundamental but rather aesthetic ones. There are, however, observational results which directly require physics beyond the standard model, to wit, dark matter and dark energy.

   Observations of the cosmic microwave background[2], the large scale structure of the universe[3] and of galactic rotation curves show that the matter content of the universe is split into about 15 % visible, i.e. baryonic (and leptonic) matter and 85% dark matter. This matter is not really dark, but transparent, that is, electrically neutral. It is detected by its gravitational effects: It clumps and thus is important for structure formation in the early universe. No standard model particle is a suitable dark matter candidate, and while there are some possible solutions which do not involve new fundamental particles (primordial black hole remnants, topological defects), these are the most natural. Extensions of the standard model usually come with plenty of those, but most of them do not have the right properties: They need to be electrically neutral, stable, and be in the right mass range to be nonrelativistic now (that is, cold dark matter). Supersymmetry offers a natural candidate in the form of the lightest neutralino, a mixture of the superpartners of the weak gauge bosons and the Higgses, or in form of the gravitino which is the superpartner of the graviton[4]. Alternatively, more complicated supersymmetric models contain various other particles which are neutral and stable and thus could constitute dark matter.

   Much more mysterious is another, and actually dominant, component of the universe: dark energy. The distance-redshift relation of type Ia supernovae[5], the age of globular clusters, and the microwave background data together with the Hubble parameter measurement[6] tell us that about 70% of the energy of the universe are not in matter, but in a smoothly distributed form with negative pressure, which manifests itself most prominently in an acceleration of the cosmic expansion. The nature of this energy is totally unclear at the moment. The simplest explanation would be a cosmological constant as a modification of gravity. Other mechanisms involve slowly evolving scalar fields or the vacuum energy of a quantum field theory.

4. The issue of dark energy, as well as the hierarchy problem, might well be inaccessible in a purely quantum field theoretical setting, since in QFT, gravity is not included, and it is not obvious how to treat e.g. zero-point energies of quantum fields.

   This is a general problem: It is already very hard to formulate QFT on curved spacetimes, and a quantum theory of gravity is still lacking. A straightforward quantisation of general relativity fails because the theory is (perturbatively) non-renormalisable. The – at the moment – most promising candidate for a theory unifying quantum theory and gravity is string theory. String theory generically lives in ten dimensions, but one might hope that the standard model in four dimensions emerges as a low-energy limit. More about this in Section 2.2.2.

2.2. Beyond the Standard Model

In the previous section, we have argued that the standard model, despite its phenomenological success, is not the ultimate theory of nature. When going beyond the standard model, there are generally two approaches:

- The bottom-up approach, where one starts form the standard model and enriches it with more ingredients like supersymmetry, unification or higher dimensions, to name the most popular ones.
- The top-down approach, which assumes a fundamental theory (or model) of everything, presumably at a very high energy scale, and proceeds to lower energies to try and find the standard model as a low-energy limit.

Both ways have their virtues and problems, and we will briefly discuss both of them in this section. The most obvious problem is that we do not know the fundamental model, and hence it is not clear whether we arrive at a consistent theory (bottom-up) or end at the standard model (top-down). However, we will for the purpose of this section restrict the top-down approach to string theory compactifications, and gear the bottom-up ingredients in accordance, that is, we mention extensions of the standard model which have some motivation in string theory.

2.2.1. Bottom-Up: Supergravity and Higher Dimensions

The most common addition certainly is supersymmetry in a more or less simple form. It has been around for about thirty years, and so has grand unification. In recent years, extra dimensions have also become very popular. All three have interesting benefits in their own right, and they are motivated by string theory as well. Thus, one might hope that there is a model starting from a consistent string theory, which by way of compactification, arrives at the standard model at low energies. En route, there might be stages where one deals with a higher-dimensional supergravity theory, and it seems advantageous to tackle such a quest from both sides, that is, by compactifying string theories and by extending the standard model with higher dimensions and supersymmetry.

All such theories are to be regarded as effective theories, that is, they are nonrenormalisable and only valid up to some energy cutoff. Beyond that cutoff, one has to assume a (more) fundamental theory, from which the effective theory arises as a low-energy limit.

Higher Dimensions

In the simplest higher-dimensional setup, spacetime is modelled as a d-dimensional manifold, where $d > 4$. Spacetime has a product structure where four of these dimensions are large, e.g. Minkowski space, while the additional $d-4$ dimensions form a small compact manifold.

The compact internal space imposes periodicity or boundary conditions on the fields, so that the dependence on the extra-dimensional coordinates can be expanded in a set of basis functions. For one extra dimension, a circle parametrised by $y \in [0, 2\pi R)$, a scalar field $\phi(x, y)$ can be Fourier expanded as

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{\mathrm{i}ny/R} \phi_n(x) \ . \tag{2.6}$$

The kinetic term in the Lagrangean then leads to mass terms for all modes (after integrating over the fifth dimension),

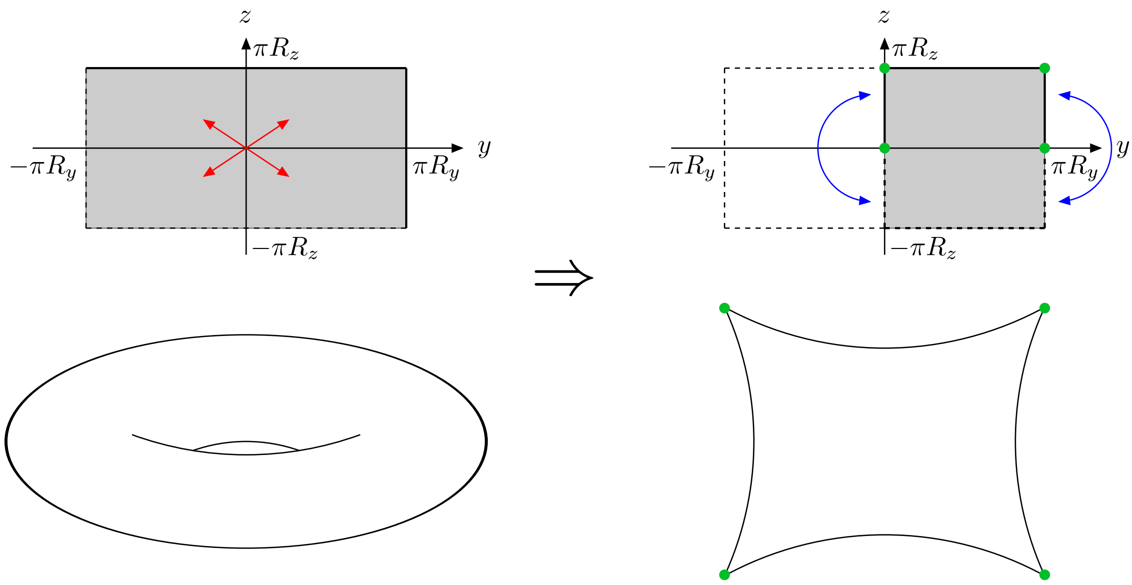
$$\int \mathrm{d}y \, \partial_5 \phi \, \partial^5 \phi = - \sum_{n=-\infty}^{\infty} \left(\frac{n}{R} \right)^2 \phi_n \phi_n^\dagger \, . \tag{2.7}$$

In this way we have traded the dependence of fields on the extra dimension for an infinite tower of massive fields, the Kaluza-Klein tower. If the circle is very small, the Kaluza-Klein masses are large, and for the effective low-energy theory, we can keep only the massless mode, the zero mode.

For fermionic fields, there is a subtlety: In five dimensions, there are only Dirac spinors, so there can be no chiral fermions. The dimension of spinors grows quickly with spacetime dimension, so that e.g. the smallest five-dimensional spinor corresponds to a four-dimensional Dirac spinor, i.e. two chiral ones. This also implies that the simplest supersymmetry in five or six dimensions actually is $\mathcal{N} = 2$ extended supersymmetry in four dimensions. Details about spinors in six dimensions can be found in Appendix A.

Branes and Orbifolds

The simple higher-dimensional setup discussed in the previous section can be extended in an important way by the introduction of branes, that is, four-dimensional subspaces to which fields can be confined. Branes arise if the internal space is not a smooth manifold, but has singular points.



(a) *The torus as a rectangle in* $\mathbb{R}^2$ *with opposite sides identified. The* $\mathbb{Z}_2$ *action is indicated by the red arrows. The lower picture shows the torus as embedded in* $\mathbb{R}^3$*.*

(b) *The orbifold. The fundamental domain is halved, the green dots mark the fixed points of the* $\mathbb{Z}_2$ *action. The vertical edges are identified, with a fold in the middle.*

Figure 2.1: *The* $\mathbb{Z}_2$ *action takes the torus to the orbifold* $T^2/\mathbb{Z}_2$ *by identifying opposing points. The orbifold has four conical singularities, each with a deficit angle of* π*.*

A convenient way to construct such singular internal spaces is orbifolding. In this case, one starts with a regular manifold and mods out by a non-freely acting discrete symmetry. The fixed points of this action correspond to singular points on the resulting orbifold, and to branes in the full setup.

As an example, consider the two-dimensional torus T^2, with the fundamental domain $(y, z) \in (-\pi R_y, \pi R_y] \times (-\pi R_z, \pi R_z]$ (see Fig. 2.1(a)). Let the group $\mathbb{Z}_2$ act on the torus by

$$r : \ (y, z) \mapsto -(y, z) \tag{2.8}$$

and identify points which are mapped onto each other under this reflection. This action has four fixed points,

$$P_1 = (0, 0) \qquad P_2 = (\pi R_y, 0) \qquad P_3 = (0, \pi R_z) \qquad P_4 = (\pi R_y, \pi R_z) \ , \tag{2.9}$$

and the fundamental domain is halved. One can choose the fundamental domain to be the "right", i.e. $y \geq 0$ part of the original one as in Fig. 2.1(b). Then the left and right edges are identified with themselves, such that the resulting object embedded in three-dimensional space looks like a pillow with four corners, corresponding to the fixed points of the group action. However, the orbifold is flat everywhere (except for the fixed points), so the pillow picture might be misleading.

In orbifold field theories, the discrete group acts on the fields as well. In the case of a $\mathbb{Z}_2$ symmetry, the action is quite simple: Collecting all fields of the theory in a vector Φ, the reflection acts by a matrix R,

$$r : \Phi\left(x, y, z\right) \mapsto R\,\Phi\left(x, -y, -z\right)\,, \quad R^2 = \mathbb{1}\,. \tag{2.10}$$

The dependence on the four-dimensional coordinates x^μ is unaffected. We can diagonalise the matrix R to obtain a set of even and odd fields, that is, in the diagonal basis, the reflection acts as

$$r : \Phi_i\left(x, y, z\right) \mapsto \pm\Phi_i\left(x, -y, -z\right)\,. \tag{2.11}$$

To study the consequences of the $\mathbb{Z}_2$ action on the fields, we can start from the Kaluza-Klein decomposition of a field Φ similar to Eq. (2.6). Consider a scalar field for simplicity, extension to other spins is straightforward. It is convenient to rewrite the exponentials in terms of sines and cosines to display the symmetry properties:

$$\begin{aligned}\phi\left(x, y, z\right) = \frac{1}{\sqrt{4\pi^2 R_y R_z}} \sum_{m,n=-\infty}^{\infty} \phi_{(m,n)} \Bigg[&\cos\left(m\frac{y}{R_y} + n\frac{z}{R_z}\right)\\ &+ \mathrm{i}\sin\left(m\frac{y}{R_y} + n\frac{z}{R_z}\right)\Bigg]\end{aligned} \tag{2.12}$$

After integrating over the extra dimensions, the mode $\phi_{(m,n)}$ acquires a Kaluza-Klein mass $m^2_{(m,n)} = \left(m/R_y\right)^2 + \left(n/R_z\right)^2$. This means that the only massless mode is the $(m, n) = (0, 0)$ cosine mode.

Imposing the $\mathbb{Z}_2$ action, we immediately see that for even fields, the sine modes are forbidden, while odd fields lose their cosine modes. In particular, only the even fields retain a zero mode, while the lightest mode of odd fields already has a mass of the order of the inverse compactification radius. This is the main feature of orbifold theories: By choosing appropriate boundary conditions, one can eliminate unwanted fields from the low-energy theory. This includes gauge fields, so that gauge symmetries can be broken at orbifold fixed points, and a huge number of models of this type exists.

The construction of orbifolds in the way just described is quite intuitive. However, we end up with a theory defined on a manifold with singularities, and its origin as a smooth space with a discrete symmetry is not relevant. We could just as well have started with the fundamental domain of the orbifold and imposed the right boundary condition to end up with the same theory. Indeed, there are many two-dimensional manifolds with singularities that cannot be obtained as a manifold modded out by a discrete symmetry.

While much of the orbifold construction can be carried over to other dimensions, the severity of the resulting singularities depends on the dimension. In one-dimensional orbifolds, the singularities are simply end-of-the-world points on a closed interval. This is a reflection of the fact that, in one dimension, there is no curvature. In two dimensions, there is a special kind of singularity, the conical one. It can be thought of as a tip of cone obtained from a plane by cutting out a wedge of deficit angle δ and gluing the edges (see Fig. 2.2). The metric close to such a singularity can be brought into the following form (in polar coordinates (ρ, θ), where the singularity is at $\rho = 0$, and $\theta \in [0, 2\pi)$):

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 + \beta^2 \rho^2 \mathrm{d}\theta^2 \tag{2.13}$$

A circle of radius r around the singularity has circumference $2\pi r \beta$, which means the deficit angle is given by $\delta = 2\pi (1 - \beta)$.

Conical branes have two important properties: The curvature close to the singularity does not diverge, but is smooth up to a δ-function at the position of the singularity. This is in contrast to singularities in higher dimensions, like the Schwarzschild black hole, where curvature invariants diverge as powers of $1/r$. Thus, singularities of two-dimensional manifolds are weaker. The second feature concerns the use of a two-dimensional manifold as internal space of a higher-dimensional spacetime: The curvature of a four-dimensional brane located at a conical singularity is independent of the brane tension (i.e. the energy density): The bulk develops a conical singularity with deficit angle of the same value as the localised brane energy density. This effect is known as self-tuning.

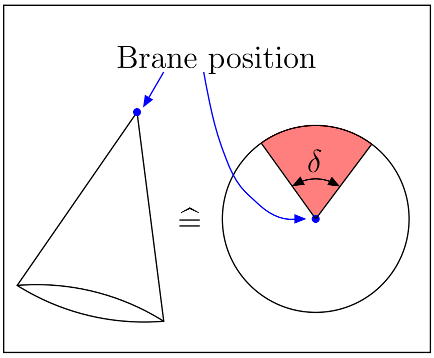


Figure 2.2: *A conical brane is obtained by cutting a wedge from a circle. The curvature does not diverge on approaching the tip.*

Warped Geometries

Now we turn to the metric of higher-dimensional theories. A product spacetime $\mathcal{S}_{4+d} = \mathcal{S}_4 \times \mathcal{S}_d$ suggests a split metric, $G_{MN} = g_{\mu\nu}(x) \oplus g_{mn}(y)$, where the four-dimensional metric has certain symmetries (e.g. Minkowski). An interesting generalisation of this, however, is a so-called warped geometry, where the line element is

$$\mathrm{d}s_d^2 = W^2(y)\,\mathrm{d}s_4^2 + g_{mn}(y)\,\mathrm{d}y^m \mathrm{d}y^n\,, \tag{2.14}$$

that is, the 4d metric gets multiplied by a warp factor depending on the extra coordinates. Its main significance is that the warp factor can generate large hierarchies between scales. In particular, if there is a brane at position y_0, with a mass parameter m, this corresponds to a physical mass $W(y_0) \cdot m$ [7]. Thus, a large variation of the warp factor can generate a large hierarchy between initially comparable mass scales.

Supersymmetry and Supergravity

Certainly supersymmetry is the most popular route beyond the standard model. The idea of unifying particles with different spins was first pursued in the early 60's, but in 1967 Coleman and Mandula[8] proved that there was no way to have a symmetry between fermions and bosons in QFT. In the beginning of the 70's, Golfand and Likhtman[9] and Volkov and Akulov[10] proposed a way to circumvent this theorem, and in 1975, Haag, Łopuszanski and Sohnius[11] were able to show that this way is unique: The extension of the usual symmetry algebra by fermionic generators, that is, generators in the spinor representation of the Lorentz group which obey anticommutation relations. The first field theory model exhibiting supersymmetry was found in 1974 by Wess and Zumino[12], and before the end of the decade, starting from Refs. [13, 14], gauged supersymmetry – supergravity – was widely studied. By now, supersymmetry is included in the majority of models beyond the standard model, from MSSM parameter scans to KKLT flux compactifications. There is a large number of excellent books and reviews available, some of which are collected in Reference [15]. Here I will just collect a few tidbits of information to argue why supersymmetry is an interesting approach and what issues are particularly relevant for the later discussion.

Supersymmetry has several attractive features:

- From the theoretical point of view, it is the only possible extension of the Poincaré group as a symmetry group of a quantum field theory. The extensions proceed via a graded symmetry algebra to avoid the Coleman-Mandula no-go theorem, and it is essentially fixed. The generators of supersymmetry transformations Q_i, $i = 1, \ldots, \mathcal{N}$ carry spin $1/2$ and fulfill anticommutation relations among themselves, in particular,

$$\left\{Q_i, \overline{Q}_j\right\} = 2\delta_{ij}\gamma^\mu P_\mu\,. \tag{2.15}$$

  For notations and conventions, see Appendix A. If $\mathcal{N} = 1$, one speaks of simple, otherwise of $\mathcal{N}$-extended supersymmetry. The more generators, the larger the multiplets, and for $\mathcal{N} = 4$ all multiplets contain a vector field, that is, all fields are gauge fields. For $\mathcal{N} = 8$, there always is a spin-2 particle, so one is lead to supergravity. $\mathcal{N} > 8$ would imply spins greater than 2, and there is common belief that such a theory cannot be consistently formulated (unless it is free). Here the aforementioned spinor growth in higher dimensions comes in: Simple supersymmetry in five or six dimensions corresponds to $\mathcal{N} = 2$ supersymmetry in four dimensions, simple supersymmetry in seven to ten dimensions to $\mathcal{N} = 4$. In eleven dimensions, we find $\mathcal{N} = 8$ and for more than eleven dimensions, $\mathcal{N} > 8$.

- Since fermions and bosons come in pairs and contribute with opposite signs to loop diagrams, many divergences cancel. In particular, there are no quadratic divergences, including the notorious Higgs self-energy, of which only a logarithmic divergence remains. In general, supersymmetry protects against large radiative corrections. There are powerful non-renormalisation theorems which state that the superpotential, i.e. the part of the Lagrangean containing masses and Yukawa couplings, does not get renormalised at all (in perturbation theory). This even holds for spontaneously broken supersymmetry.

- Supersymmetry suggests a new global symmetry, called R-symmetry, which ensures that some particles can only be produced in pairs, which in turn means that the lightest of these particles is stable and might be a candidate for dark matter. R-symmetries are symmetries that do not commute with supersymmetry, so particles in a supersymmetry multiplet have different R-charges. This also means that extended supersymmetry can have larger R-symmetry groups, while simple supersymmetry at most has a $\mathsf{U}(1)_R$-symmetry.

- When supersymmetry is gauged, the theory contains gravity as well, hence the name supergravity. The gauge field is now a spin-$3/2$ fermion, the gravitino.

Supersymmetry, relating bosons with fermions, requires superpartners of all fields in a theory, that is, fields with different spin, but the same quantum numbers otherwise – up to the charge under the aforementioned R-symmetry. These superpartners are not part of the standard model, so we need to introduce more fields to make the standard model supersymmetric:

- Spin-0 partners for all fermions, with the same quantum numbers: Squarks and sleptons ("scalar fermions"). They are grouped in so-called chiral multiplets, which contain a chiral fermion and a complex scalar.

- Spin-$1/2$ partners for the gauge bosons (the gauginos). Vector fields come in a vector multiplet together with a fermion with non-chiral interactions.

- Supersymmetry requires two Higgs fields, to give masses to the up-type quarks and neutrinos and separately to the down-type quarks and charged leptons. Both Higgs fields are $\mathsf{SU}(2)$ doublets and come with fermionic superpartners, the Higgsinos. After electroweak symmetry breaking, there will be five physical Higgs scalars (two of which are charged), and the Higgsinos mix with the electroweak gauginos to form electrically neutral neutralinos and charged charginos.

So altogether, supersymmetry requires more than doubling the degrees of freedom. For exact supersymmetry, however, there are no new parameters, since masses and couplings of superpartners are fixed. A host of new parameters is introduced by broken supersymmetry.

From the experimental side, however, evidence for supersymmetry is rather slim. In particular, scalar superpartners of the standard model fermions with equal masses are not observed. That means that supersymmetry cannot be exactly realised, but at best in a broken way. Mechanisms for supersymmetry breaking are discussed in the next section.

Supergravity

From the supersymmetry algebra we know that the anticommutator of the supersymmetry generator with itself is the generator of translations, P_μ. From this one can guess that the commutator of local supersymmetry transformations will be a local translation, i.e. a coordinate transformation, and thus that local supersymmetry "implies gravity".

Multiplet	$D=4$		$D=6$		
Gravity	$g_{\mu\nu}$	ψ_μ	G_{MN}	B^+_{MN}	Ψ_M
d.o.f.	2	2	9	3	12
Tensor			B^-_{MN}	Φ	χ
d.o.f.			3	1	4
Matter	ϕ	ψ	$\phi_i\,,\ i=1,2$		Ψ
d.o.f.	2	2	4		4
Vector	A_m	λ	A_M		Λ
d.o.f.	2	2	4		4

There also is a tensor multiplet in four dimensions which we will not consider. The superscripts ± on the two-forms denote (anti-)self-duality of the field strengths.
The matter multiplet is a chiral multiplet in four and a hypermultiplet in six dimensions. The smallest spinor in $D=4$ is a Weyl or Majorana spinor with two degrees of freedom, in $D=6$ the smallest spinor is a Weyl spinor which has four degrees of freedom.
In each multiplet, the number of (on-shell) bosonic and fermionic degrees of freedom is equal.

Table 2.1: *The fields of the relevant multiplets and their degrees of freedom in four and six dimensions.*

Indeed this is the case, and this is why local supersymmetry goes by the name of supergravity. As any local symmetry, supergravity involves a gauge field, but since the transformation parameter is a fermion, the gauge field is a spin-3/2 field, the gravitino, the superpartner of the graviton. In four dimensions, the graviton and the gravitino together form a complete supersymmetry multiplet. In higher dimensions, however, the gravitational multiplet will contain additional bosonic (p-form) fields. In six dimensions, for example, there is the 2-form field B_{MN}. The reason for the additional fields is that a multiplet needs to contain the same number of bosonic and fermionic degrees of freedom. With increasing spacetime dimension, the spinors grow exponentially, and so new bosonic fields have to be added. The same applies to chiral multiplets[1] (vector fields accidentally double their degrees of freedom from four to six dimensions, so the multiplet still contains one vector and one spinor). The field content in four and six dimensions is given in Table 2.1.

The two-form field B^+_{MN} in the six-dimensional gravity multiplet poses a problem: Its field strength $H^+_{MNP}=3\partial_{[M}B^+_{NP]}$ is self-dual, and thus there is no Lorentz-invariant kinetic term to build an action[16][2]. This problem can be solved by adding the tensor multiplet $\left(B^-_{MN},\chi,\Phi\right)$ to the theory: it contains a two-form field with anti-self-dual field strength. The form fields can be combined into a single two-form $B_{MN}=B^+_{MN}+B^-_{MN}$ without duality conditions. In this process, the theory acquires two more new fields, the dilaton Φ and the dilatino χ.

The pure supergravity Lagrangean is fixed by supersymmetry. The coupling of supergravity to matter is also constrained, this time by a geometrical symmetry: The complex scalar fields of the theory (which come in chiral or hypermultiplets) can be regarded as forming a sigma model, and there are strong restrictions on the geometry of the sigma model target space (or in other words, not every matter model can be made

[1] As previously mentioned, six-dimensional simple supersymmetry corresponds to four-dimensional $\mathcal{N}=2$ supersymmetry, which was once introduced as hypersymmetry. The only part of this use that stuck was the name hypermultiplet for the $\mathcal{N}=2$ counterpart of the chiral multiplet

[2] The obvious term $H^+_{MNP}H^{+\,MNP}$ vanishes due to the self-duality relation, as in two or ten dimensions.

locally supersymmetric). These restrictions depend on the dimension and whether one deals with extended or simple supergravity. In the simplest case of four-dimensional simple supergravity, the target space must be a Kähler manifold. A Kähler manifold is a complex manifold with a Riemannian metric and symplectic form, both of which are compatible with the complex structure. For practical purposes, this means that the Kähler metric (i.e. the metric of the sigma model) is given by the derivative of a real scalar function, the Kähler potential (denoting the fields by ϕ^i and the conjugate fields by $\overline{\phi}^{\overline{\imath}}$):

$$\mathscr{L}_{\sigma,\,\mathrm{kin}} = K_{\overline{\imath}j}\,\partial^\mu\overline{\phi}^{\overline{\imath}}\partial_\mu\phi^j\,, \qquad K_{\overline{\imath}j} = \frac{\partial}{\partial\overline{\phi}^{\overline{\imath}}}\frac{\partial}{\partial\phi^j}K\big(\overline{\phi},\phi\big) \equiv \partial_{\overline{\imath}}\partial_j K\big(\overline{\phi},\phi\big)\;. \tag{2.16}$$

The simplest choice $K\big(\overline{\phi},\phi\big) = \overline{\phi}^{\overline{\imath}}\phi^i$ is called the canonical Kähler potential because it leads to canonical kinetic terms, hence to a renormalisable matter sector. If $K\big(\overline{\phi},\phi\big)$ is of higher than second order, the matter sector is nonrenormalisable. Gravity, however, is not renormalisable anyway, so there is no strict justification for a canonical Kähler potential. We can, however, demand that the non canonical terms are suppressed by the Planck scale, such that in the limit $M_{\mathrm{P}} \to \infty$, where gravity decouples, the resulting theory is again renormalisable. We will later see that gravity-induced higher order terms are important for supersymmetry breaking. This discussion holds in the Einstein frame where the gravitational Lagrangean is just given by the Ricci scalar, $\mathscr{L}_{\mathrm{EH}} = \frac{1}{2}\sqrt{-g}\,R$. We will later sometimes use a different function also called Kähler potential, $\Omega = \exp\{-K/3\}$. This is the analogue of K in the supergravity conformal frame where $\mathscr{L}_{\mathrm{EH}} = \frac{1}{2}\sqrt{-g}\,\Omega R$. A canonical Kähler potential corresponds to $\Omega\big(\overline{\phi},\phi\big) = \exp\big\{\overline{\phi}^{\overline{\imath}}\phi^i\big\}$.

There are two more functions that determine a supergravity theory: The gauge kinetic function $f(\phi_i)$ and the superpotential $W(\phi_i)$, both of which are functions of the fields ϕ_i and not of their conjugates. The gauge kinetic function determines the gauge kinetic terms via

$$\mathscr{L}_{\mathrm{gaugekin}} = -\frac{1}{4}\,\mathrm{Re}\,f F_{\mu\nu}F^{\mu\nu} + \frac{\mathrm{i}}{4}\,\mathrm{Im}\,f F_{\mu\nu}\widetilde{F}^{\mu\nu}\,, \tag{2.17}$$

so that the standard gauge kinetic terms are reproduced by $f = 1$. There can be different gauge kinetic functions if the gauge group is composed of several factors. The superpotential contains mass and coupling terms.

We have omitted factors of M_{P} in the formulae above. They can be restored by considering the mass dimensions of the supergravity functions: Ω and f are dimensionless, K has mass dimension 2 and the superpotential is of mass dimension 3.

Supersymmetry Breaking

We now turn to the issue of supersymmetry breaking, which, phenomenologically, certainly is the most important issue in supersymmetric model building since it determines the masses of the superpartners.

As a preliminary remark, let us check signs of supersymmetry breaking: From the supersymmetry algebra, one can infer immediately that for a supersymmetric theory, the vacuum energy cannot be negative, and vanishes if the vacuum state $|0\rangle$ is supersymmetric:

$$E_0 = \langle 0\,|P_0|\,0\rangle = \frac{1}{4}\left(\|Q_1|0\rangle\|^2 + \|Q_2|0\rangle\|^2 + \left\|\overline{Q}_1|0\rangle\right\|^2 + \left\|\overline{Q}_2|0\rangle\right\|^2\right) \geq 0 \qquad (2.18)$$

In supergravity, however, this result is no longer true, and the value of the vacuum energy is not directly related to the question whether supersymmetry is broken or not. The potential of supergravity is not positive semidefinite, and it is given by an expression involving both the Kähler and the superpotential,

$$V = e^K\left(\overline{D_j W} K^{\bar{\jmath}k} D_k W - 3\,|W|^2\right) + \frac{1}{2\,\mathrm{Re}\,f} D^2 \,. \qquad (2.19)$$

Here $D_i W = \partial_i W + K_i W$ is the Kähler covariant derivative[3] and $K^{\bar{\jmath}k}$ is the matrix inverse of the Kähler metric. The D-terms are related to the gauge fields and can be expressed in terms of the scalar fields and the generators of the gauge group, but we will ignore them for now. The sign of supersymmetry breaking in supergravity is a vacuum expectation value of a $D_i W$, or in other words,

$$\left\langle F^i\right\rangle = \left\langle -e^{K/2} K^{i\bar{\jmath}} \overline{D_j W}\right\rangle \neq 0\,. \qquad (2.20)$$

A vacuum expectation value $\langle D\rangle \neq 0$ is also a sign of supersymmetry breaking. Both F and D terms show up in the supersymmetry transformations of fermions, so their vacuum expectation value implies that the vacuum is not invariant. As the potential is not positive semidefinite, one can usually fine-tune the vacuum energy to zero with broken supersymmetry.

A special form of Kähler and superpotential that ensures a vanishing vacuum energy is the so-called no-scale model[17]: For n fields, the Kähler potential is

$$K = -\sum_i \alpha_i \ln\left(\frac{\phi_i + \overline{\phi}_{\bar{\imath}}}{2}\right)\,, \quad \sum_i \alpha_i = 3\,, \qquad (2.21)$$

and the superpotential is constant. For this setup, the scalar potential vanishes identically, since the condition on the α_i enforces the cancellation of the $-3\,|W|^2$ term.

Now for the breaking mechanism. Within a supersymmetric theory, the breaking should be spontaneous rather than explicit, and indeed the O'Raifeartaigh model provides a simple setup for a spontaneous breaking of supersymmetry with only chiral multiplets and renormalisable couplings. However, we run into a phenomenological problem: There is a mass sum rule which says that in such a situation, the supertrace of the mass matrix vanishes. In other words, if the theory has a set of scalars ϕ_i and fermionic partners χ_i,

$$\mathrm{Str}\,M^2 \equiv \sum_{\text{scalars}} (m_{\phi_i})^2 - 2 \sum_{\text{fermions}} (m_{\chi_i})^2 = 0\,. \qquad (2.22)$$

[3] It is covariant under the Kähler transformations $K(\overline{\phi}_{\bar{\imath}}, \phi_i) \to K(\overline{\phi}_{\bar{\imath}}, \phi_i) + F(\phi_i) + \overline{F}(\overline{\phi}_{\bar{\imath}})$ and $W \to \exp\{F(\phi_i)\}\,W$.

This rule holds in the presence of spontaneous supersymmetry breaking, and it holds separately within each quantum number sector, e.g. for the down-type quarks. This clearly is not viable phenomenologically, as it would require the sum masses of the scalar partners to be below $(10\text{GeV})^2$, so they certainly would have been seen in experiment already.

The supertrace theorem, fortunately, tells us how to avoid it: It is valid for renormalisable couplings and at tree level. So the way around employs either nonrenormalisable couplings or radiative corrections, and in both cases, the hidden sector. The hidden sector comprises a set of fields which break supersymmetry, e.g. an O'Raifeartaigh model, but which is hidden, i.e. has no renormalisable couplings to the MSSM at tree level. The breakdown of supersymmetry is instead mediated to the observable fields by some other mechanism. There are several popular such mechanisms:

- Gravity mediation[18]: This is in the first category, nonrenormalisable couplings. Gravity mediation does not refer to graviton exchange, but to all couplings suppressed by the Planck scale. The hidden sector fields are assumed to have Planck-suppressed gravitational couplings to the observable sector, which give rise to superpartner masses. Such couplings might be enforced by the UV completion of the effective theory, and, in particular, are not restricted by the equivalence principle, i.e. need not be flavour-blind. Generically, indeed they are not, and that is one of the major problems of this approach. Here extra dimensions might help.

- Gauge mediation[19]: From the category of radiative corrections, gauge mediation scenarios involve a third sector, the messenger sector, which couples to the hidden sector and is itself coupled to the standard model (or GUT) gauge group, so it induces superpartner masses via loops. These, in contrast to the previous case, are automatically flavour-blind, so in gauge mediated scenarios flavour-changing neutral currents are naturally suppressed.

- Higher-dimensional mechanisms[20]: In higher-dimensional theories, the hidden sector can be hidden by the extra dimension: The observable and hidden sector fields are localised on different branes, with no direct couplings between them. In simple supergravity setups this leads to a special kind of Kähler potential in the low-energy effective theory, the sequestered form (ignoring bulk fields for the moment). Expressed in terms of $\Omega = \exp\{-K/3\}$ the sequestered form is defined by

  $$\Omega\big(\bar{\phi}_{\text{hid}}, \bar{\phi}_{\text{obs}}, \phi_{\text{hid}}, \phi_{\text{obs}}\big) = \Omega_{\text{hid}}\big(\bar{\phi}_{\text{hid}}, \phi_{\text{hid}}\big) + \Omega_{\text{obs}}\big(\bar{\phi}_{\text{obs}}, \phi_{\text{obs}}\big)\ . \tag{2.23}$$

  The Kähler potential at tree level splits into separate potentials for the hidden and observable sector, as might be expected from higher-dimensional locality. Loop corrections do induce couplings between the sectors and hence mediate supersymmetry breaking. In Chapter 4 we will consider a specific scenario: The mediators will be moduli fields, that is, fields in the higher-dimensional metric and gravity multiplet which are related to the size and shape of the internal orbifold.

We will also see, however, that the sequestered form shown above is not a generic outcome of higher dimensional theories. This is actually no surprise: When going to the low-energy effective theory, one explicitly breaks higher-dimensional locality and generically generates a non-local effective Lagrangean. If some fields have non-trivial couplings to the branes, integrating them out will generate direct couplings between the sectors.

If the mediation effects reside in the Kähler potential, it is a variant of gravity mediation. There are other mechanisms in higher dimensions which do not rely on supergravity, but rather employ e.g. gaugino fields living in the bulk to transfer supersymmetry breaking [21].

- Anomaly mediation[22]: The dimensionless couplings of the visible sector are scale invariant on the classical level. In the quantised theory, however, scale invariance is broken by the conformal anomaly. If there is a hidden sector breaking supersymmetry, the anomaly will generate masses in the visible sector which are proportional (but suppressed with respect) to the gravitino mass. These masses depend on the β functions and anomalous dimensions of the fields[4] and are always present. In most models, anomaly mediation is not the dominant mechanism. If there is no other mediation present, however, anomaly mediation effects can be crucial.

As a remark, let us briefly mention the approach usually entertained in the minimal supersymmetric standard model. The key point is that there are certain supersymmetry breaking terms which preserve most of the nice features of supersymmetry, in particular, the absence of quadratic divergences. These are called soft terms and are generated by spontaneously broken supersymmetry. They are mass terms for the gauginos and sfermions as well as bi- and trilinear scalar interaction terms. Since most of these parameters are complex matrices in flavour space, there are 105 new parameters in the MSSM not present in the standard model[5]. This is too much to perform generic parameter space scans, so one usually imposes some restrictions on the parameters, such as universality of sfermion and gaugino masses at the GUT scale. The resulting model goes by the name of CMSSM and has five parameters, and there have been a large number of studies of this or related models.

Grand Unification

The standard model gauge group G_{SM} and representation assignments do not seem as natural as they could be, and there have been attempts to embed them in a simpler framework. The general idea is that G_{SM} actually is the unbroken subgroup of a larger gauge group G_{grand}, which gets broken in some way at a high energy scale. This idea is

[4] The β function dependence of the scalar masses leads to an immediate problem: Sleptons acquire negative squared masses in simplest setups.

[5] This is the final parameter count, since some matrices are Hermitean and some phases can be absorbed in the fields.

known as grand unification, and the resulting theories are called grand unified theories (GUTs). The pattern of standard model representations could follow from the split of representations of G_{grand} into representations of the subgroup G_{SM}.

The idea has been supported by the observation that the three gauge couplings of the standard model evolve with energy scale and (almost) meet at a scale of $\sim 10^{16}$ GeV, which defines the GUT scale M_{GUT}. The running of the couplings of course depends on the particle content, and including only standard model particles, the couplings do not meet exactly. If one, however, assumes supersymmetric partners with masses around a TeV, the couplings meet. This is usually considered an argument in favour of low-energy supersymmetry.

What groups are possible GUT groups? They have to meet some requirements:

- The rank of G_{SM} is four, so G_{grand} must have at least rank four to admit G_{SM} as a subgroup. On the other hand, G_{grand} should not be too large.
- G_{grand} should have complex representations to accommodate the standard model representations without additional mirror fermions. The corresponding representations must combine to anomaly free generations.
- Taking gauge coupling unification serious, G_{grand} should have only one gauge coupling. This is satisfied if G_{grand} is a simple group, or if it is a product of identical groups which are related by discrete symmetries.

Popular GUT groups include $\mathsf{SU}(5)$, $\mathsf{SO}(10)$ and E_6, among others. From the point of view of neutrinos, $\mathsf{SO}(10)$ seems to be the most natural GUT group because it unifies all standard model fermions of one generation, including the right-handed neutrino, in one 16-dimensional spinor representation[6]. Furthermore, it is of rank five, so there is an additional $\mathsf{U}(1)$ which can be combined with hypercharge to a $\mathsf{U}(1)_{(B-L)}$. If this is broken at M_{GUT}, a Majorana mass term of this order for the right-handed neutrino is presumably generated, leading to the see-saw mechanism of Eq. (2.5).

Concrete model of GUTs do have their problems, most notably proton decay. Since quarks and leptons are unified in representations of G_{grand}, there are gauge interactions that turn quarks into leptons and hence, protons can decay, e.g. via $p \to \pi e^+$. Since experimental limits on the proton lifetime are of the order of 10^{33} years (depending on the mode, see [23]), this places strong bounds on M_{GUT} and the details of the model.

There are other problems which are related to the breaking of the GUT symmetry. First of all, G_{grand} needs to be broken down to G_{SM}. If this is done by the Higgs mechanism, the Higgs field usually needs to be inside a very large representation of G_{SM}. Similarly, the standard model Higgs field is paired in a representation of G_{grand} together with coloured partners. In both cases, the symmetry breaking needs to be adjusted not to break e.g. $\mathsf{SU}(3)_C$. Furthermore, the coloured Higgses also mediate proton decay, and hence must be very heavy. This means that inside a single G_{grand} representation we have the usual Higgs fields with electroweak scale masses, and the coloured triplet with GUT scale mass. This is called the problem of doublet-triplet splitting.

[6]This is the spinor of $\mathsf{SO}(10)$, not a spacetime spinor.

The two latter problems can be solved in higher-dimensional theories, orbifold GUTs[24], where boundary conditions render some of the GUT gauge fields very heavy, thereby eliminating the need for a GUT Higgs boson. Furthermore, boundary conditions can also forbid zero modes for some components of a GUT multiplet, solving the doublet-triplet splitting problem. Actually, two extra dimensions seem to be preferred from the point of view of orbifold breaking of an $\mathsf{SO}(10)$ GUT [25] because the breaking cannot be achieved in five dimensions due to group-theoretical reasons[26].

2.2.2. Top-Down: String Theory Compactifications

Superstring theories[27] live in ten dimensions, and there are five of them: Type I, type IIA, type IIB, heterotic $\mathsf{SO}(32)$ and heterotic $\mathsf{E}_8 \times \mathsf{E}_8$. They are distinguished by the amount of supersymmetry ($\mathcal{N} = 2$ for type II, $\mathcal{N} = 1$ for the rest) and whether they contain open strings (in type I) and gauge fields (type I, type IIA and heterotic). They contain a number of massless bosonic fields: The metric, a scalar called dilaton, and an antisymmetric two-form are generic. There will also be gauge fields (excluding type IIB) and further forms for type II theories (one- and three-form for IIA, zero-, two- and four-form for IIB).

There are various connections between these theories which relate e.g. the strong coupling limit of one theory to the weak coupling limit of another one.

Calabi-Yaus and Orbifolds

The simplest compactification is on a six-dimensional torus. This, however, brings about phenomenological problems, one of them being that the resulting effective theory retains all the supersymmetry of the original string theory. This implies $\mathcal{N} = 4$ supersymmetry in four-dimensional language. This has been a quite popular theory, mainly because it is so symmetric. The theory is finite to all orders and conformal, for example. Unfortunately, the standard model is completely different, so one has to look for more involved compactification schemes. Generally, one is interested in compactifications preserving simple four-dimensional supersymmetry to have some control over perturbative corrections.

A simple alternative to tori are orbifold compactifications[28]: The geometry is almost as simple as a torus, and the orbifold projection breaks some supersymmetries. Furthermore, the orbifold fixed points give rise to localised states (the twisted sectors, strings that “wind around” the fixed points), which can be interpreted as brane-localised matter in higher-dimensional field theories.

In a more general approach, one is led to a certain class of compactification manifolds: If the compactification of the ten-dimensional string theory on a product space $\mathcal{M}_4 \times \mathcal{S}_6$, with $\mathcal{M}_4$ Minkowski space and an internal manifold $\mathcal{S}_6$ is to preserve one quarter of the original supersymmetry in four dimensions, as seems more sensible from the phenomenological point of view, one is led to a certain class of internal spaces called Calabi-Yau spaces[29], which are characterised by two properties: They are Ricci-flat complex manifolds with $\mathsf{SU}(3)$ holonomy. These conditions follow from the transformation law of the gravitino: The ten-dimensional gravitino Ψ_M splits into four four-dimensional gravitini ψ^i_μ, each with a supersymmetry transformation that starts with $\delta\psi^i_\mu = D_\mu \epsilon^i + \cdots$, where the ϵ^i are four spinorial transformation parameters. Unbroken simple supersymmetry means that there should be one ϵ^i that is covariantly constant, $D_\mu \epsilon^i = 0$, which in turn means that the holonomy should not be the whole $\mathsf{SO}(6) = \mathsf{SU}(4)$, but smaller. For type II superstrings, this still yields $\mathcal{N} = 2$ supersymmetry in four dimensions. Calabi-Yau compactifications of the heterotic string, on the other hand, were already widely studied in the 80's.

A given Calabi-Yau manifold usually admits a number of continuous deformations which preserve the Calabi-Yau property. These deformations appear as scalar fields in the theory on this space, and they come in two classes: Kähler moduli and complex structure moduli. Their respective numbers are given by two Hodge numbers $h^{(1,1)}$ and $h^{(1,2)}$ which encode topological information about the manifold. Roughly speaking, the complex structure moduli contain information about the shape of the manifold, while the Kähler moduli are related to its size. After compactification, these moduli fields have no potential, so their value in the ground state is undetermined and needs some stabilisation mechanism.

Orbifolds actually are related to Calabi-Yau spaces: They arise as particular limits, that is, they form special points in the moduli space of a given Calabi-Yau.

Fluxes

One of the problems of string theory compactifications is moduli stabilisation: The moduli demand a potential to fix their values at some minimum. For some of the moduli – the complex structure moduli – a potential is generated by fluxes. Fluxes are vacuum expectation values of p-form field strengths (of which there are several, depending on the type of string theory considered) in the internal directions (so as to preserve 4d Lorentz invariance). As they are field strengths of form fields, they are subject to Bianchi identities as well as equations of motion, and they can be expanded in terms of harmonic functions of the Calabi-Yau, which in turn depend on the complex structure moduli. Thus, integrating over the internal space generates a potential for the complex structure moduli[30]. The Kähler moduli, on the other hand, are not fixed in this way. Furthermore, the fluxes are quantised: the integral of the field strength over a compact submanifold of appropriate dimension must be an integer.

Fluxes also allow warped compactifications with non-Calabi-Yau internal manifolds: The transformation law of the gravitino contains a term involving fluxes, so if they are non-zero, a covariantly constant spinor is no longer required. For an overview of flux compactification, see Ref. [31].

Branes

Important objects in string theory are branes. They can arise in two ways: Open strings have ends, and these ends need boundary conditions, which can be of Dirichlet or von-Neumann type. Imposing Dirichlet boundary conditions in p spatial directions fixes the ends of open strings on $p+1$-dimensional hypersurfaces called Dp-branes. Actually, these branes are more than just boundary conditions: they act as sources for fluxes of the $p+1$-form fields that are present in type I and II. By adding branes to a type II theory, half of the supersymmetry is broken, and open strings are included in the spectrum.

Gauge Groups

A priori, only heterotic and type I strings contain non-Abelian gauge groups (for type I, it is $\mathsf{SO}(32)$), so model building concentrates on these theories, where the heterotic $\mathsf{E}_8 \times \mathsf{E}_8$ theory has been especially popular[32]. In this theory, one E_8 gauge group factor (or its E_6 subgroup) is used as a GUT group in four dimensions, while the other factor is hidden, i.e. visible matter is a singlet with respect to the second E_8. This can lead to models with three families of standard model-like matter[33]. The discovery of D-branes as dynamical objects has recently led to a lot of activity in type II model building, since stacks of N coincident Dp-branes support a $\mathsf{SU(N)}$ gauge theory. For $p=3$, the branes can be extended along the noncompact spacetime, and hence one obtains a non-Abelian gauge theory in four dimensions.

2.2.3. Intermediate Field Theories

The reduction from a full string theory down to the low-energy four-dimensional theory might involve intermediate steps. For a simple example, consider an anisotropic compactification on a six-dimensional torus where d_i radii are of the order R_i, much larger than the remaining $6-d_\mathrm{i}$ which are $\sim R_\mathrm{s}$[34]. In an energy range between R_i^{-1} and R_s^{-1}, the theory will be described by a $(4+d_\mathrm{i})$-dimensional (supergravity) field theory. This also allows orbifold GUTs in any dimensions, where the orbifolding is only applied to the d_i intermediate dimensions. Successful gauge coupling unification, however, prefers one or, in particular, two extra dimensions

In the rest of this thesis, we will – motivated by the preceding discussion – consider in detail six-dimensional supergravity theories. A particularly attractive feature of two extra dimensions is the possibility of conical branes when compactifying to four dimensions, and in the next chapter we will find the general solution of gauged supergravity with any number of conical branes and four-dimensional maximal symmetry. In Chapter 4, we will look to the specific case of a torus orbifold with matter on fixed points and investigate gravity mediated supersymmetry breaking.

3. The General Warped Solution in 6d Supergravity

We now turn to six-dimensional supergravity. The special rôle of six-dimensional models is that the internal space is two-dimensional (since we desire four large dimensions). Two-dimensional spaces admit conical singularities, and branes located in a two-dimensional internal space generate such a singularity, and there is no effective cosmological constant on the brane[35]. This is the idea of self tuning[36], which has been used in various attempts to address the cosmological constant problem. Such a compactification employs a six-dimensional cosmological constant and flux of an Abelian gauge field. First attempts were made in non-supersymmetric theories[37], which find four-dimensional Minkowski vacua if gauge flux and cosmological constant are tuned against each other.

Six-dimensional gauged supergravity[38], on the other hand, includes an Abelian gauge field for the $\mathsf{U}(1)_R$ symmetry, and a positive definite potential. Furthermore, tuning of the gauge flux is automatically enforced by the dilaton equation of motion. Six-dimensional gauged supergravity was compactified (without branes) already in Ref. [39]. It was found in Ref. [40] that four-dimensional Minkowski space is not only possible, but inevitable if one requires maximal symmetry in four dimensions (and the internal space is compact). This analysis was generalised to warped solutions with axial symmetry[41, 42] and to unwarped solutions with many branes[43].

Four-dimensional Minkowski space follows from the compactness of the internal space. For the same reason, however, the gauge flux is quantised, which translates into a fine-tuning condition between the brane tensions of the model[44]. Hence, stable four-dimensional Minkowski vacua require specially chosen brane tensions, which spoils the self-tuning effect[45, 46]. This problem can be avoided by coupling six-dimensional sigma-models to gravity[47]. For anomaly cancellation, further bulk fields are necessary in any case[48].

In this chapter, we will present the general warped solution in six-dimensional supergravity, following Ref. [49]. In the next section, we will introduce the setup and find the solution. In Section 3.2, we will show examples with two and many branes, and we finish the chapter with comments and conclusions.

3.1. The Setup

We consider six-dimensional gauged supergravity with a number of four-dimensional branes. In this chapter the branes only have a brane tension which can arise from a ground state energy of fields living on the branes. Neither do we consider the brane fields themselves, nor do the branes couple other than gravitationally.

The bulk theory contains a combined gravity and tensor multiplet (introduced because of the self-duality condition on the two-form in the gravitational multiplet, see Section 2.2.1) and an Abelian vector multiplet which gauges the $\mathsf{U}(1)_R$ symmetry under which the fermions and the two-form are charged. The component fields of the combined gravity and tensor multiplet are the metric G_{MN}, the antisymmetric tensor B_{MN}, the dilaton Φ, the dilatino χ and the gravitino Ψ_M. The vector multiplet contains a vector A_M and the gaugino Λ. The complete action and transformation laws are given in Appendix B. Here we only give part of the action relevant for our discussion, i.e. omitting fermions and the tensor field strength,

$$S_{\text{bulk}} = \int \mathrm{d}^6X\,\sqrt{-G}\left[\frac{1}{2}R - \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{1}{4}e^{-\Phi}F_{MN}F^{MN} - 2g^2e^{\Phi}\right]. \tag{3.1}$$

We set the six-dimensional Planck scale $M_6 = 1$. The last term in the action is of crucial importance: It is a positive definite potential which acts as a vacuum energy for any finite Φ, and thus as a source for curvature to provide a compact internal manifold. The potential allows us to circumvent the usual no-go theorem for warped compactifications with finite Planck mass. In the limit $g \to 0$, our solution disappears.

The vector field gauges the $\mathsf{U}(1)_R$ symmetry under which all the fermions and the tensor are charged. Its field strength is $F_{MN} = \partial_M A_N - \partial_N A_M$. Its coupling constant is denoted by g.

The branes are characterised by their brane tensions Λ_i, $i = 1, \ldots, n_{\mathrm{b}}$. The branes are located on hypersurfaces $X(x_i^\mu)$, and their action is

$$S_{\text{brane}} = -\sum_i \int \mathrm{d}^4x_i\,\sqrt{-g_i}\,\Lambda_i \tag{3.2}$$

with $g_{i,\mu\nu}$ the metric pulled back to the brane.

The Einstein and field equations following from this action are

$$\begin{aligned} R_{MN} = \partial_M\Phi\partial_N\Phi + g^2e^{\Phi}G_{MN} + e^{-\Phi}\left(F_{MP}F_N{}^P - \frac{1}{8}G_{MN}F_{PQ}F^{PQ}\right) \\ + T^{\text{brane}}_{MN}\,, \end{aligned} \tag{3.3}$$

$$D_M\left(e^{-\Phi}F^{MN}\right) = \frac{1}{\sqrt{-G}}\,\partial_M\left(\sqrt{-G}\,e^{-\Phi}F^{MN}\right) = 0\,, \tag{3.4}$$

$$D_MD^M\Phi = \frac{1}{\sqrt{-G}}\,\partial_M\left(\sqrt{-G}\,\partial^M\Phi\right) = -\frac{1}{4}e^{-\Phi}F_{MN}F^{MN} + 2g^2e^{\Phi}\,. \tag{3.5}$$

T^{brane}_{MN} is the brane contribution to the energy-momentum tensor.

3.1.1. The Ansatz

We now specify the ansatz for the solution: We require a $(4+2)$ split metric with maximal symmetry among the four large dimensions x^μ and a warp factor depending on the extra coordinates y^m. Nothing should depend on the x^μ and the field strength vacuum expectation values should respect four-dimensional Lorentz invariance (this is why we omitted the tensor field strength in the above action). All fermions vanish.

$$\mathrm{d}s^2 = W^2(y)\,\tilde{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \hat{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n\,, \tag{3.6}$$

$$F_{mn} = \sqrt{\hat{g}}\,\epsilon_{mn}F(y)\;, \tag{3.7}$$

$$\Phi = \Phi(y)\;, \tag{3.8}$$

$$G_{MNP} = 0\,, \qquad \text{fermions} = 0\,. \tag{3.9}$$

The requirement of maximal symmetry means that $R_{\mu\nu}(\tilde{g}) = 3\lambda\tilde{g}_{\mu\nu}$, and the slices $y = \text{const.}$ are de Sitter, Minkowski or anti-de Sitter for $\lambda > 0$, $\lambda = 0$ or $\lambda < 0$, respectively.

The Ricci tensor for this metric is

$$R_{\mu\nu}(G) = \left(3\lambda - \frac{1}{4}W^{-2}D_mD^mW^4\right)\tilde{g}_{\mu\nu}\,, \tag{3.10}$$

$$R_{mn}(G) = R_{mn}(\hat{g}) - 4W^{-1}D_mD_nW\,, \tag{3.11}$$

where D_m is the covariant derivative with respect to the extradimensional metric $\hat{g}_{mn}$. The brane energy momentum tensor is

$$T^{\text{brane}}_{\mu\nu} = 0\,, \tag{3.12}$$

$$T^{\text{brane}}_{mn} = \sum_i \frac{1}{\sqrt{\hat{g}}}\Lambda_i\hat{g}_{mn}\delta^2(y-y_i) \tag{3.13}$$

at the brane positions y_i. With this ansatz, the Einstein and field equations (3.3)-(3.5) become

$$3\lambda - \frac{1}{4}W^{-2}D_mD^mW^4 = -\left(\frac{1}{4}F^2e^{-\Phi} - g^2e^{\Phi}\right)W^2\,, \tag{3.14}$$

$$\begin{aligned} R_{mn}(\hat{g}) - 4W^{-1}D_mD_nW &= \partial_m\Phi\partial_n\Phi + \left(\frac{3}{4}F^2e^{-\Phi} + g^2e^{\Phi}\right)\hat{g}_{mn} \\ &\quad + \sum_i \frac{1}{\sqrt{\hat{g}}}\Lambda_i\hat{g}_{mn}\delta^2(y-y_i)\;, \end{aligned} \tag{3.15}$$

$$\partial_m\left(W^4e^{-\Phi}F(y)\right) = 0\,, \tag{3.16}$$

$$W^{-4}D_m\left(W^4D^m\Phi\right) = -\left(\frac{1}{2}F^2e^-\Phi - 2g^2e^{\Phi}\right)\,. \tag{3.17}$$

3.1.2. The Solution

We first solve for the dilaton and gauge flux. Combining Eqs. (3.14) and (3.17), we get

$$6W^2\lambda = D_m\left[W^4D^m\left(\Phi + 2\ln W\right)\right]\,. \tag{3.18}$$

Integrating over the compact manifold, we find $\lambda = 0$, that is, Minkowski space is the unique maximally symmetric solution (for regular warp factor and dilaton). This is completely independent of the brane tensions Λ_i. Inserting $\lambda = 0$ in Eq. (3.18), we find a solution for the dilaton in terms of the warp factor and an integration constant Φ_0:

$$\Phi = \Phi_0 - 2 \ln W \tag{3.19}$$

The gauge flux equation (3.16) is also easily solved:

$$F(y) = f e^{\Phi} W^{-4} = f e^{\Phi_0} W^{-6} \tag{3.20}$$

Here f is another integration constant.

For the Einstein equations, it is convenient to express the metric of the internal space in conformally flat form and introduce a new complex variable $z = y^5 + \mathrm{i} y^6$. We also temporarily write the warp factor as $W = \exp B$, so the metric is

$$\mathrm{d}s^2 = e^{2B(\bar{z},z)} \left(\eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2A(\bar{z},z)} \mathrm{d}z \mathrm{d}\bar{z} \right) . \tag{3.21}$$

With this choice, the Einstein equations (3.14) and (3.15) become (for $R_{\mu\nu}$, $R_{\bar{z}\bar{z}}$ and $R_{z\bar{z}}$):

$$-4e^{-2A} \left(4\partial B \bar{\partial} B + \partial \bar{\partial} B \right) = e^{\Phi_0} \left(g^2 - \frac{1}{4} f^2 e^{-8B} \right) , \tag{3.22}$$

$$-4\bar{\partial}^2 B + 4 \left(\bar{\partial} B \right)^2 + 8 \bar{\partial} B \bar{\partial} A = 4 \left(\bar{\partial} B \right)^2 , \tag{3.23}$$

$$\begin{aligned} -6\partial\bar{\partial} B - 4\partial B \bar{\partial} B - 2\partial\bar{\partial} A = 4\partial B \bar{\partial} B + \frac{1}{2} e^{2A} e^{\Phi_0} \left(g^2 + \frac{3}{4} f^2 e^{-8B} \right) \\ + \sum_i \Lambda_i \delta^2 (z - z_i) . \end{aligned} \tag{3.24}$$

Here $\partial = \frac{\partial}{\partial z}$ and $\bar{\partial} = \frac{\partial}{\partial \bar{z}}$. We can rewrite Eq. (3.23) as

$$e^{2A} \bar{\partial} \left(e^{-2A} \bar{\partial} B \right) = 0 . \tag{3.25}$$

This implies that the combination

$$V(z) = e^{-2A} \bar{\partial} B \tag{3.26}$$

is a holomorphic function[1]. This function vanishes for a non-warped solution, i.e. for constant B which can be rescaled to $B = 0$. In this case, Eq. (3.22) implies $f^2 = 4g^2$, and the internal geometry can be obtained from Eq. (3.24):

$$\partial \bar{\partial} A = -\frac{1}{4} f^2 e^{\Phi_0} e^{2A} - \frac{1}{2} \sum_i \Lambda_i \delta^2 (z - z_i) \tag{3.27}$$

[1] Strictly speaking, it is meromorphic: Isolated singularities are actually required to reproduce the δ-function terms at the brane positions in Eq. (3.24).

This solution reproduces the known unwarped scenarios of Refs. [42, 43].

We are looking for a warped solution, that is, $V \neq 0$. In this case, we can multiply Eq. (3.22) by V to obtain (reinstating $W = e^B$)

$$\bar{\partial}\left(V\partial W^4\right) = -\frac{1}{4}e^{\Phi_0}g^2\,\bar{\partial}\left(W^4 + \frac{f^2}{4g^2}W^{-4}\right), \tag{3.28}$$

which implies that the two sides differ by at most a holomorphic function $v(z)$,

$$V\partial W^4 = -\frac{1}{4}e^{\Phi_0}g^2\left(W^4 + \frac{f^2}{4g^2}W^{-4} - 2v(z)\right). \tag{3.29}$$

To simplify this equation, we introduce a new holomorphic variable[50]

$$\xi = \int^z \frac{\mathrm{d}z'}{V(z')} \qquad \Rightarrow \qquad \partial \equiv \frac{\partial}{\partial z} = \frac{1}{V}\frac{\partial}{\partial \xi} \tag{3.30}$$

which eliminates the V from the left hand side of Eq. (3.29). We can now exploit the reality of the warp factor by differentiating Eq. (3.29) and its complex conjugate:

$$-\frac{1}{4}e^{\Phi_0}g^2\frac{\partial}{\partial\xi}\left(W^4 + \frac{f^2}{4g^2}W^{-4}\right) = \frac{\partial}{\partial\xi}\frac{\partial}{\partial\bar{\xi}}W^4 = -\frac{1}{4}e^{\Phi_0}g^2\frac{\partial}{\partial\bar{\xi}}\left(W^4 + \frac{f^2}{4g^2}W^{-4}\right) \tag{3.31}$$

which implies

$$\frac{\partial}{\partial\xi}W = \frac{\partial}{\partial\bar{\xi}}W \qquad \text{and hence} \quad v = \bar{v} = \text{const.} \tag{3.32}$$

Since W only depends on the real part of ξ, we define a new variable

$$\zeta = \mathrm{Re}\,\xi = \frac{1}{2}\left(\xi + \bar{\xi}\right), \tag{3.33}$$

in terms of which Eq. (3.29) can be written as an ordinary differential equation,

$$\frac{\mathrm{d}W}{\mathrm{d}\zeta} = \frac{1}{2}\gamma^2\frac{-W^4 - u^2W^{-4} + 2v}{W^3} \equiv \frac{P(W)}{W^3}, \tag{3.34}$$

where

$$\gamma^2 = \frac{1}{4}e^{\Phi_0}g^2, \qquad u^2 = \frac{f^2}{4g^2}. \tag{3.35}$$

The function $P(W)$ on the right hand side of this equation has two real positive roots at

$$W_\pm^4 = v \pm \sqrt{v^2 - u^2}, \tag{3.36}$$

and the warp factor is bounded in this interval $W_-^4 \leq W^4 \leq W_+^4$ for spacelike extra dimensions. This implies the conditions $v > 0$ and $v^2 > u^2$ for the integration constants. The solution to the equation can be given in implicit form as

$$\frac{\left(W^4(\zeta) - W_-^4\right)^{W_-^4}}{\left(W_+^4 - W^4(\zeta)\right)^{W_+^4}} = \exp\left\{2\gamma^2 \left(W_+^4 - W_-^4\right)(\zeta - \zeta_0)\right\} . \tag{3.37}$$

In the limit $g \to 0$, one of the roots of $P(W)$ disappears, the warp factor becomes unbounded and the internal space is noncompact. To keep the warp factor finite, one has to cut off the internal space with a one-dimensional boundary. This corresponds to a four-brane as in Romans supergravity[51], which differs from our model by $g^2 \to -g^2$.

This solution does also satisfy the third equation, Eq. (3.24), up to the δ-function part which will be matched by the deficit angles of conical branes.

To find the internal metric, we look at the definition of V, Eq. (3.26), and the new coordinate ζ and find with Eq. (3.34):

$$e^{2A} = \frac{1}{V}\frac{\bar{\partial} W}{W} = \frac{1}{W}\frac{1}{|V|^2}\underbrace{\frac{\partial W}{\partial \xi}}_{\frac{1}{2}\frac{\mathrm{d}W}{\mathrm{d}\zeta}} = \frac{1}{2\,|V|^2}\frac{P(W)}{W^4} . \tag{3.38}$$

Hence, the internal metric is

$$\mathrm{d}s_2^2 = W^2 e^{2A}\,\mathrm{d}z\mathrm{d}\bar{z} = \frac{1}{2\,|V(z)|^2}\frac{P(W)}{W^2}\,\mathrm{d}z\mathrm{d}\bar{z} . \tag{3.39}$$

Effects of V

We will now analyse the effects of the function $V(z)$. From Eq. (3.37), we can see that the warp factor approaches $W_\pm$ for $\zeta \to \pm\infty$, values which are attained for zeroes of $V(z)$. On the other hand, Eq. (3.39) indicates that for poles of $V(z)$, the extra-dimensional metric vanishes. So zeroes and poles determine singularities in the internal space. More explicitly:

1. Assume $(z) \approx c\,(z - z_0)^\alpha$ around z_0. Since $V(z)$ is holomorphic, α must be an integer, and it is restricted to be ± 1: For $\alpha > 1$, the new variable ζ and hence the warp factor are discontinuous around z_0, while for $\alpha < -1$, there are curvature singularities worse than delta functions. Thus, $V(z)$ should have only simple zeroes or poles.

2. Now let us look at the two cases. Consider first a simple zero,

$$V(z) \approx c\,(z - z_0) \qquad \curvearrowright \qquad \zeta \approx \frac{1}{2c}\ln|z - z_0|^2 . \tag{3.40}$$

   c must be real. On approaching z_0, $\zeta \to \pm\infty$ and the warp factor $W \to W_\pm$ for $c \lesseqgtr 0$.

The internal geometry exhibits a conical singularity at z_0. The metric around z_0 becomes

$$\mathrm{d}s_2^2 = |z-z_0|^{2(\beta_\pm - 1)}\,\mathrm{d}z\mathrm{d}\bar{z}\,, \qquad \text{with} \quad \beta_\pm = \gamma^2\,|c|\,\frac{W_+^4 - W_-^4}{W_\pm^4}\,. \tag{3.41}$$

On a change of coordinates to $\rho = |z-z_0|^{\beta_\pm}/\beta_\pm$ and $\theta = \mathrm{Arg}(z-z_0)$, the metric changes to $\mathrm{d}s_2^2 = \mathrm{d}\rho^2 + \beta_\pm^2\rho^2\mathrm{d}\theta^2$, which is precisely the form of a conical singularity with deficit angle $2\pi\,(1-\beta_\pm)$ (see Eq. (2.13)).

This is where the singular terms in the Einstein equations come into play. From Eq. (3.15) or (3.24), we can match the deficit angle to the brane tension. In two dimensions, we have $\partial\bar{\partial}\ln|z|^2 = 2\pi\delta^2(z)$ and $R_{mn}(\hat{g}) = (1-\beta_\pm)\,\partial\bar{\partial}\ln|z-z_0|^2$. Thus, we find that the brane tension is equal to the deficit angle:

$$\Lambda = 2\pi\left(1 - \gamma^2\,|c|\,\frac{W_+^4 - W_-^4}{W_\pm^4}\right)\,. \tag{3.42}$$

3. At a simple pole $V(z) \approx c\,(z-z_0)^{-1}$, nothing special happens to ζ and the warp factor. The presence of $|V|^{-2}$ in the internal metric (3.39), however, implies that the metric around z_0 behaves like

$$\mathrm{d}s_s^s \approx \underbrace{\frac{P(W(z_0))}{2\,|c|^2\,W^2(z_0)}}_{=:k^2}|z-z_0|^2\,\mathrm{d}z\mathrm{d}\bar{z} = k^2r^2\left(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2\right)\,, \quad z - z_0 = re^{\mathrm{i}\theta}\,. \tag{3.43}$$

The overall factor k^2 could be scaled away. Now, changing the radial coordinate to $\rho = r^2/2$, the metric takes the form $\mathrm{d}s_2^2 \propto \mathrm{d}\rho^2 + 4\rho^2\mathrm{d}\theta^2$, i.e. $\beta = 2$ and the deficit angle is fixed to -2π.

4. Finally, we have to consider the behaviour at infinity. If, for large z, $V(z) \propto z^N$, with $N > 2$, there will be another brane at $z = \infty$ with fixed deficit angle

$$\Lambda_\infty = 2\pi\,(2-N)\,. \tag{3.44}$$

For $N = 1$, $\zeta \to \pm\infty$ logarithmically as $z \to \infty$, and there will be a conical brane with deficit angle $2\pi\,(1-\beta_\pm)$, depending on the sign of ζ.

This extra brane with negative tension can be understood as a consequence of the Gauss–Bonnet theorem which states that the sum of brane tensions cannot exceed 4π, but is limited by the integral over the Ricci curvature over the internal space (for a genus-0 surface),

$$\sum_i \Lambda^i + \Lambda^\infty = 4\pi - \frac{1}{2}\int\sqrt{\hat{g}}R\,\mathrm{d}^2y\,. \tag{3.45}$$

For N conical branes, the sum over the deficit angles (3.42) contains a term $2\pi N$, and hence there must be some brane tension contribution $-2\pi N$ to cancel this. Explicitly, we have

$$4\pi - 2\pi\gamma^2 \sum_{\text{branes}} \frac{|c|}{W_\pm^4} = 4\pi - \frac{1}{2}\int \sqrt{\hat{g}}\, R\, \mathrm{d}^2 y\,. \tag{3.46}$$

In the sum it is W_+^4 for $c < 0$ W_-^4 for $c > 0$.

3.2. Examples

Now we will discuss specific examples of the solution, that is, we choose special forms for the function $V(z)$. First we recover known warped solutions with two branes and discuss flux quantisation and the unwarped limit. In the next section, we find new solutions with more than two branes.

3.2.1. Two-Brane Solutions, Axial Symmetry

The Solution

For the simplest solution, we choose $V(z)$ to be

$$V(z) = -\frac{z}{c}\,. \tag{3.47}$$

This will lead to conical branes at $z = 0$ and at $z = \infty$, with warp factors W_+ and W_-, respectively[2].

We can perform a global change of coordinates to ζ and θ, provided c is real. In the following, we assume $c > 0$. The new coordinates are

$$\zeta = -\frac{c}{2}\ln|z|^2\,, \qquad \theta = -\frac{c}{2\mathrm{i}}\ln\frac{z}{\bar{z}}\,, \tag{3.48}$$

and the metric takes the form

$$\mathrm{d}s_2^2 = \frac{P(W)}{2W^2}\left(\mathrm{d}\zeta^2 + \mathrm{d}\theta^2\right)\,. \tag{3.49}$$

To find the warp factor, we have to solve Eq. (3.34). To find the explicit solution, it is actually convenient to rescale the coordinates ζ and θ to

$$\mathrm{d}\eta = \frac{1}{c}W^{-4}\mathrm{d}\zeta\,, \qquad \mathrm{d}\psi = -\frac{1}{c}\mathrm{d}\theta\,. \tag{3.50}$$

[2]The point $z = \infty$ is at a finite proper distance from $z = 0$ and corresponds to the other pole of the compact space.

As W did not depend on θ, neither does it depend on ψ, and the equation becomes

$$\mathrm{d}\eta = -\frac{\mathrm{d}\left(W^4\right)}{2\gamma^2\left(W^8 - 2vW^4 + u^2\right)}\,, \tag{3.51}$$

which can easily be integrated over $\mathrm{d}\left(W^4\right)$ to yield

$$\eta - \eta_0 = \frac{1}{2\gamma^2}\frac{1}{\sqrt{v^2-u^2}}\,\mathrm{Artanh}\left(\frac{W^4 - v}{\sqrt{v^2-u^2}}\right)\,. \tag{3.52}$$

Inverting this and expressing v and u^2 in terms of $W_\pm^4$, we can get the warp factor as a function of η. This can be inserted into the overall factor in Eq. (3.49), where the tanh-behaviour of W^4 leads to a $\cosh^{-2}$-dependence. The constant η_0 can be set to zero by demanding that $W^4(\eta = \pm\infty) = W_\pm^4$.

So finally, the full metric is

$$\mathrm{d}s^2 = W^2\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + a^2\left(W^8\mathrm{d}\eta^2 + \mathrm{d}\psi^2\right)\,, \tag{3.53}$$

where

$$W^4 = \frac{1}{2}\left(W_+^4 + W_-^4\right) + \frac{1}{2}\left(W_+^4 - W_-^4\right)\tanh\!\left[c\gamma^2\left(W_+^4 - W_-^4\right)\eta\right]\,, \tag{3.54}$$

$$a^2 = \frac{c^2}{2}\frac{P(W)}{W^2} = \frac{c^2\gamma^2}{16}\frac{\left(W_+^4 - W_-^4\right)^2}{\cosh^2\!\left[c\gamma^2\left(W_+^4 - W_-^4\right)\eta\right]}\,. \tag{3.55}$$

Nothing depends on ψ, the axial symmetry is manifest.

Brane Asymptotics

We can explicitly check Eq. (3.41) in our case to see that the deficit angle does not depend on the coordinates. Consider the limit $\eta \to \infty$ (the case $\eta \to -\infty$ can be obtained by switching $W_+ \leftrightarrow W_-$ in the following discussion). The warp factor and a^2 go to

$$W^4 \longrightarrow W_+^4 - \left(W_+^4 - W_-^4\right)\exp\left\{-\gamma^2 c\left(W_+^4 - W_-^4\right)\eta\right\}\,, \tag{3.56}$$

$$a^2 \longrightarrow \frac{1}{16}\gamma^2 c^2\frac{\left(W_+^4 - W_-^4\right)^2}{W_+^6}\exp\left\{-\gamma^2 c\left(W_+^4 - W_-^4\right)\eta\right\}\,. \tag{3.57}$$

To show the conicality, we change the radial coordinate to

$$\mathrm{d}\rho_+ = -aW^4\mathrm{d}\eta = -\frac{1}{2}\gamma c W_+\left(W_+^4 - W_-^4\right)\exp\left\{-\gamma^2 c\left(W_+^4 - W_-^4\right)\eta\right\}\mathrm{d}\eta \tag{3.58}$$

and the metric becomes

$$\mathrm{d}s_2^2 \longrightarrow \mathrm{d}\rho_+^2 + \underbrace{\left[\gamma^2 c\frac{W_+^4 - W_-^4}{W_+^4}\right]^2}_{\beta_+^2}\rho_+^2\mathrm{d}\psi^2\,. \tag{3.59}$$

Hence, the brane tensions at $\eta = \pm\infty$ are indeed

$$\Lambda_\pm = 2\pi \left(1 - \gamma^2 c \frac{W_+^4 - W_-^4}{W_\pm^4}\right) . \tag{3.60}$$

The brane conditions can be exploited to fix some parameters of the solution in terms of the brane tensions $\Lambda_\pm$ and the gauge coupling g, which are input parameters in the Lagrangean. There will be two conditions, which can be obtained by solving for W_+^4/W_-^4 and eliminating it:

$$e^{\Phi_0} c = \frac{8\pi}{g^2 (\Lambda_+ - \Lambda_-)} \left(1 - \frac{\Lambda_+}{2\pi}\right)\left(1 - \frac{\Lambda_-}{2\pi}\right) , \tag{3.61}$$

$$\frac{f}{v} = \pm 4g \frac{\sqrt{\kappa}}{(1+\kappa)} , \qquad \text{with} \quad \kappa = \frac{W_+^4}{W_-^4} = \frac{2\pi - \Lambda_-}{2\pi - \Lambda_+} . \tag{3.62}$$

Flux Quantisation and Planck Mass

Unfortunately, we will see that flux quantisation does not give further conditions on the free parameters. The general flux quantisation condition for a compact two-dimensional space is

$$\int_{\mathcal{M}_2} F_2 = \frac{2\pi n}{g} , \tag{3.63}$$

with an integer n. For this discussion, it is convenient to use the warp factor itself as the radial coordinate, by inserting

$$W^4 \mathrm{d}\eta = \frac{1}{c} \frac{W^3}{P(W)} \mathrm{d}W \tag{3.64}$$

into Eq. (3.53) (via ζ and Eq. (3.34)) to obtain the internal metric

$$\mathrm{d}s_2^2 = \frac{W^4}{2P(W)} \mathrm{d}W^2 + \frac{P(W)}{2W^2} \mathrm{d}\psi^2 . \tag{3.65}$$

Then, the flux is

$$F_{W\psi} = \sqrt{\hat{g}}\, \epsilon_{W\psi} f e^{\Phi_0} W^{-6} = \frac{1}{2} \epsilon_{W\psi} f e^{\Phi_0} W^{-5} , \tag{3.66}$$

which can easily be integrated to give the quantisation condition

$$\frac{1}{8} f e^{\Phi_0} \left(\frac{W_+^4 - W_-^4}{W_+^4 W_-^4}\right) \underbrace{\int \mathrm{d}\theta}_{2\pi c} = \frac{2\pi n}{g} . \tag{3.67}$$

This, however, does not give a new relation between the parameters, but only a fine-tuning condition of the brane tensions,

$$\left(1-\frac{\Lambda_+}{2\pi}\right)\left(1-\frac{\Lambda_-}{2\pi}\right)=n^2\,. \tag{3.68}$$

Finally we can consider the Planck mass in this solution by dimensionally reducing the Einstein-Hilbert term in the Lagrangean. Integrating over the internal space in (W,ψ) coordinates, we obtain

$$M_{\mathrm{P}}=\frac{1}{2}M_6^4\int\limits_{W_-}^{W_+}W^3\mathrm{d}W\cdot\int\mathrm{d}\psi=\frac{\pi}{4}M_6^4c\left(W_+^4-W_-^4\right)\,. \tag{3.69}$$

So in the end we have only fixed two out of the four parameters c, f, v and Φ_0 by Eqs. (3.61) and (3.62). The parameter c, however, can be absorbed in a rescaling of η and ψ, so only one unfixed modulus remains.

The Unwarped Limit

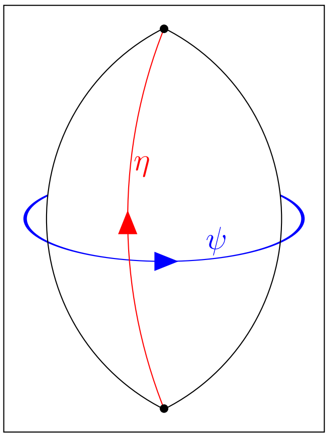


Figure 3.1: *The unwarped rugby ball. η runs from $-\infty$ at the lower end to $+\infty$ at the top.*

Although we could scale c away, it is sensible to keep it explicit when performing the unwarped limit. The reason is that, in the limit of constant warp factor, the function V should vanish, and c provides a handle for that, by taking $c\to\infty$.

We need to be careful, however, when taking this limit: Several quantities explicitly depend on the parameter c. On the other hand, obviously $W_+\to W_-$ for the unwarped case, or in other words,

$$\left(W_+^4-W_-^4\right)\longrightarrow 0 \quad\Leftrightarrow\quad \frac{f^2}{v^2}\longrightarrow 4g^2\,. \tag{3.70}$$

Hence, a consistent limit is obtained by taking $c\to\infty$ and $W_+\to W_-$ at the same time, while keeping

$$k=c\left(W_+^4-W_-^4\right) \tag{3.71}$$

finite. It this way, $V\to 0$, and quantities like the Planck mass and metric coefficients stay finite. It is also consistent with the parameter fixing conditions (3.61) and (3.62), since $\Lambda_+-\Lambda_-\propto 1\left(W_+^4-W_-^4\right)k$ and $\kappa\to 1$. The metric (3.53) becomes

$$\mathrm{d}s^2\longrightarrow W_+^2\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu+\frac{\gamma^2}{16}k^2W_+^2\frac{\mathrm{d}\eta^2+W_+^{-8}\mathrm{d}\psi^2}{\cosh^2(\gamma^2k\eta)}\,. \tag{3.72}$$

This is a metric with two conical singularities of equal deficit angle, as can be seen by a change of coordinate to $\mathrm{d}\rho=k\gamma^2\cosh^{-1}(k\gamma^2\eta)\,\mathrm{d}\eta$, which converts the metric to

$$\mathrm{d}s^2\longrightarrow W_+^2\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu+\frac{1}{16\gamma^2}W_+^2\left(\mathrm{d}\rho^2+\underbrace{\left(\frac{k\gamma^2}{W_+^4}\right)^2}_{\beta^2}\sin\rho\;\mathrm{d}\psi^2\right)\,. \tag{3.73}$$

This describes a rugby ball shaped internal space (see Fig. (3.1) with two branes of equal tension Λ, where

$$\Lambda = 2\pi\left(1-\beta\right) . \tag{3.74}$$

Here the internal curvature is constant, $R = 32\gamma^2 W_+^{-2}$. This is consistent with the Gauss–Bonnet theorem (3.46), where the volume of the internal space is $\pi k / \left(4W_+^2\right)$.

3.2.2. Multi-brane Solution

We can generalise the solution of the preceding section to more than two branes. To this end we require a function V with several zeroes. However, we encounter some restrictions on the positions of the branes and a fixed-tension brane at $z=\infty$.

The Solution

We take an obvious generalisation of the simple ansatz (3.47),

$$V(z) = \frac{1}{c}\prod_{i=1}^{N}(z-z_i) \tag{3.75}$$

with N complex numbers (brane positions) z_i.

The new coordinate ζ becomes

$$\zeta = \int^{z} \frac{\mathrm{d}z'}{V(z')} + \text{c.c.} = c\int^{z} \mathrm{d}z' \prod_{i=1}^{N}\frac{1}{(z-z_i)} + \text{c.c.} \tag{3.76}$$

This can be integrated using

$$\prod_{i=1}^{N}\frac{1}{(z-z_i)} = \sum_{i=1}^{N}\left(\prod_{j\neq i}^{N}\frac{1}{z_i-z_j}\right)\frac{1}{z-z_i}\,, \tag{3.77}$$

so

$$\zeta = \frac{1}{2}\sum_{i=1}^{N}\left\{\underbrace{c\left(\prod_{j\neq i}^{N}\frac{1}{z_i-z_j}\right)}_{a_i}\ln(z-z_i) + \underbrace{\bar{c}\left(\prod_{j\neq i}^{N}\frac{1}{\bar{z}_i-\bar{z}_j}\right)}_{\bar{a}_i}\ln(\bar{z}-\bar{z}_i)\right\}. \tag{3.78}$$

If this is to be single-valued, the coefficients a_i have to be real, $a_i = \bar{a}_i$ for all i. This implies that the brane positions have to be aligned along a straight line, $z_i = |z_i|\, e^{\mathrm{i}\phi}$, where ϕ is the same for all i. To compensate this, $c = |c|\, e^{-\mathrm{i}(N-1)\phi}$. By a change of coordinates, we can choose the phase $\phi = 0$, that is, the branes are aligned along the real axis, and c is real. Thus ζ is

$$\zeta = \frac{1}{2}\sum_{i=1}^{N} a_i \ln|z-z_i|^2 \,. \tag{3.79}$$

Note that $\sum_{i=1}^{N} a_i = 0$.

For the explicit warp factor, consider Eq. (3.37) (with $\zeta_0 = 0$) and define a new "coordinate" χ by

$$e^{W_+^4 \chi} = \left(W^4 - W_-^4\right) e^{2\gamma^2 \zeta}\,, \tag{3.80a}$$

$$e^{W_-^4 \chi} = \left(W_+^4 - W^4\right) e^{2\gamma^2 \zeta}\,. \tag{3.80b}$$

χ is subject to the constraint

$$e^{W_+^4 \chi} + e^{W_-^4 \chi} = \left(W_+^4 - W_-^4\right) e^{2\gamma^2 \zeta}\,. \tag{3.80c}$$

The warp factor can be written in terms of χ in a form reminiscent of (3.54),

$$W^4 = \frac{1}{2}\left(W_+^4 + W_-^4\right) + \frac{1}{2}\left(W_+^4 - W_-^4\right) \tanh\left[\frac{1}{2}\left(W_+^4 - W_-^4\right)\chi\right]\,. \tag{3.81}$$

Brane Asymptotics

The brane tensions can be found from the general discussion at the end of Section 3.1.2, or by expanding the warp factor around each brane. Here we follow the general approach.

From Eq. (3.79), we can see that $\zeta \to \pm\infty$ at z_i. The sign of ζ and hence the warp factor at the brane, that is, W_+ or W_-, is determined by the sign of a_i: For $a_i < 0$, $\zeta \to +\infty$ and $W \to W_+$, and opposite for positive a_i. Each brane tension is given by Eq. (3.42), so writing $\Lambda_\pm$ for branes with warp factor $W_\pm$, we have

$$\Lambda_\pm^i = 2\pi\left(1 - \gamma^2 \left|a_i\right| \frac{W_+^4 - W_-^4}{W_\pm^4}\right)\,. \tag{3.82}$$

This is completely analogous to the two-brane case.

In contrast to the two-brane case, however, for N branes there also is a brane at $z = \infty$. This is mapped to $\zeta = 0$ because $\sum_i a_i = 0$, and the brane tension is fixed by the number of branes (see Eq. (3.44)):

$$\Lambda_\infty = 2\pi\left(2 - N\right)\,. \tag{3.83}$$

We will now use Eq. (3.82) to fix some of the parameters of the model. Assume the first k a_i's are negative and the rest is positive, so that there are k branes with tensions Λ_+^i and $N-k$ with tensions Λ_-^j. We can get a set of relations similar to (3.61) and (3.62):

$$e^{\Phi_0}\left|a_i\right| = \frac{4}{g^2} \frac{\left(1 - \frac{1}{2\pi}\Lambda_+^i\right)\left(N - k - \frac{1}{2\pi}\sum_{j=k+1}^{N}\Lambda_-^j\right)}{N - 2k + \frac{1}{2\pi}\left(\sum_{j=1}^{k}\Lambda_+^j - \sum_{l=k+1}^{N}\Lambda_-^l\right)} \qquad i = 1, \ldots, k\,, \tag{3.84}$$

$$e^{\Phi_0}\left|a_i\right| = \frac{4}{g^2} \frac{\left(1 - \frac{1}{2\pi}\sum_{j=1}^{k}\Lambda_+^j\right)\left(k - \frac{1}{2\pi}\Lambda_-^i\right)}{N - 2k + \frac{1}{2\pi}\left(\sum_{j=1}^{k}\Lambda_+^j - \sum_{l=k+1}^{N}\Lambda_-^l\right)} \qquad i = k+1, \ldots, N\,, \tag{3.85}$$

$$\frac{f}{v} = \pm 4g\frac{\sqrt{\tilde\kappa}}{1 + \tilde\kappa}\,, \qquad \text{where} \quad \tilde\kappa = \frac{2\pi\left(N - k\right) - \sum_{i=k+1}^{N}\Lambda_-^i}{2\pi k - \sum_{i=1}^{k}\Lambda_+^i}\,. \tag{3.86}$$

We find $N+1$ relations. Furthermore, two brane positions can be chosen at $z_0 = 0$ and $z_1 = 1$. The overall factor c cannot be absorbed into V, so one parameter remains unfixed, as in the unwarped case.

Flux Quantisation and Planck Mass

We again consider flux quantisation. This time, we will employ a more general procedure than in the two-brane case, since we cannot use the warp factor as a coordinate. We will stick to the original complex coordinate z and metric (3.21). The flux is

$$F_{z\bar{z}} = \epsilon_{z\bar{z}}\sqrt{\hat{g}} f e^{\Phi_0} W^{-6} = \frac{1}{2}\epsilon_{z\bar{z}} f e^{\Phi_0} W^{-4} e^{2A} = -\frac{1}{8}\epsilon_{z\bar{z}} f e^{\Phi_0} \frac{\overline{\partial} W^{-4}}{V}\,, \tag{3.87}$$

and we can insert this in the flux quantisation condition (3.63) to obtain the condition

$$-\frac{1}{8} f e^{\Phi_0} \int \mathrm{d}z\mathrm{d}\bar{z}\,\frac{\overline{\partial} W^{-4}}{V} = \frac{2\pi n}{V}\,. \tag{3.88}$$

To evaluate this integral, we use the divergence theorem for a complex function J and a domain of integration $R \subset \mathbb{C}$,

$$\int_R \mathrm{d}z\mathrm{d}\bar{z}\left(\partial \overline{J} + \overline{\partial} J\right) = \mathrm{i}\oint_{\partial R}\left(\overline{J}\mathrm{d}\bar{z} - J\mathrm{d}z\right)\,. \tag{3.89}$$

To apply this to the flux quantisation integral, we have to split the internal space into several patches and find that the only contributions come from the singularities where $W = W_\pm$, and there is an additional factor of $2\pi\mathrm{i}$ from the residue. Hence, the condition (3.88) gives

$$f e_0^{\Phi}\left(\sum_{i=1}^{k}|a_i|\right)\left(W_+^4 - W_-^4\right) = \frac{8n}{g} \tag{3.90}$$

since $\sum_{i=1}^{k}|a_i| = \sum_{i=k+1}^{N}|a_i|$. The a_i's are given in Eq. (3.84), and we find a weakened fine-tuning condition between the brane tensions:

$$\left(1 - \frac{1}{2\pi}\sum_{j=1}^{k}\Lambda_+^j\right)\left(N - k - \frac{1}{2\pi}\sum_{j=k+1}^{N}\Lambda_-^j\right) = n^2\,. \tag{3.91}$$

A similar integral has to be performed for the Planck mass: In the same coordinates as before, the Planck mass is given by

$$M_\mathrm{P} = \frac{1}{8}M_6^4\int \mathrm{d}z\mathrm{d}\bar{z}\,\frac{\overline{\partial} W^4}{V} = \frac{\pi}{4}M_6^4\left(\sum_{i=1}^{k}|a_i|\right)\left(W_+^4 - W_-^4\right)\,. \tag{3.92}$$

The Unwarped Limit

For the unwarped limit, we again have to balance two limits: we take $\left(W_+^4 - W_-^4\right) \to 0$ and $c \to \infty$, i.e. $|a_i| \to \infty$, while keeping

$$\beta_i = \gamma^2 a_i \frac{W_+^4 - W_-^4}{W_+^4} \tag{3.93}$$

finite (no distinction between $i = 1, \ldots, k$ and $i = k+1, \ldots, N$ is necessary in this limit). This keeps the Planck mass and brane tensions (3.82) finite,

$$M_\mathrm{P} \longrightarrow \frac{\pi}{4\gamma^2} M_6^4 \left(\sum_{i=1}^{k} |\beta_i| \right) , \tag{3.94}$$

$$\Lambda^i \longrightarrow 2\pi \left(1 - |\beta_i|\right) . \tag{3.95}$$

The brane tension at $z = \infty$ is not affected. As $c \to \infty$, indeed $V \to 0$ as required.

For the metric, we can use the warp factor of Eq. (3.81) to rewrite the function $P(W)$,

$$P(W) = \frac{1}{2}\gamma^2 W^{-4} \left(W_+^4 - W^4\right)\left(W^4 - W_-^4\right) \tag{3.96}$$

$$= \frac{1}{8}\gamma^2 \frac{\left(W_+^4 - W_-^4\right)^2}{W^4} \frac{1}{\cosh^2\left[\frac{1}{2}\left(W_+^4 - W_-^4\right)\chi\right]} . \tag{3.97}$$

Hence, in the unwarped limit, the metric goes to

$$\mathrm{d}s^2 \longrightarrow W_+^2 \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \frac{\gamma^2}{16} \frac{\left(W_+^4 - W_-^4\right)^2}{|V(z)|^2} \cosh^{-2}\left[\gamma^2 \frac{W_+^4 - W_-^4}{W_+^4} \eta\right] \mathrm{d}z \mathrm{d}\bar{z} . \tag{3.98}$$

In each patch of coordinates, we can change this into a more convenient form using a holomorphic function

$$\omega = \exp\left\{\gamma^2 \frac{W_+^4 - W_-^4}{W_+^4} \xi\right\} , \tag{3.99}$$

so that the internal metric becomes

$$\mathrm{d}s_2^2 = \frac{1}{4\gamma^2} W_+^2 \frac{|\partial\omega|^2}{\left(1 + |\omega|^2\right)} \mathrm{d}z \mathrm{d}\bar{z} . \tag{3.100}$$

For our case, ξ can be inferred from ζ, Eq. (3.79), and ω is

$$\omega = \prod_{i=1}^{N} (z - z_i)^{\beta_i} , \quad \text{with} \quad \beta_i = \gamma^2 a_i \frac{W_+^4 - W_-^4}{W_+^4} , \tag{3.101}$$

so the brane tensions are given by the exponents,

$$\Lambda^i \longrightarrow 2\pi \left(1 - |\beta_i|\right) , \Lambda^\infty \longrightarrow 2\pi \left(1 - N\right) . \tag{3.102}$$

Note that this is again consistent with the expression from the Gauss–Bonnet theorem, Eq. (3.46), since the scalar curvature of the internal space goes to $R = 32\gamma^2 W_+^{-2}$.

3.3. Possible Generalisations

We have found the general warped solution in six-dimensional gauged supergravity with four-dimensional maximal symmetry. The four-dimensional geometry is fixed to be flat. Essential properties of the internal geometry, such as the number of conical branes, are determined by a holomorphic function.

We can reproduce the known warped solution[40, 41] with two branes. Unfortunately, brane tension fine tuning from flux quantisation is not removed. For a more general function, we can also incorporate many branes in a warped geometry. In this case, there is still one fine-tuning condition involving all brane tensions. There also appears a negative tension brane, as required by the Gauss-Bonnet theorem. For both situations, we can safely take the unwarped limit so that the internal volume and Planck mass stay finite.

Interesting generalisations would involve more general functions, in particular, periodic functions to generate other topologies like toric orbifolds. Another route would be to consider couplings of bulk fields to the brane tensions, as suggested by the next chapter's results.

4. Gravity Mediated Supersymmetry Breaking in 6d

In this chapter we will analyse gravity mediated supersymmetry breaking in a six-dimensional supergravity theory: Ungauged supergravity on a $T^2/\mathbb{Z}_2$ orbifold, with matter located at fixed points.

One motivation for considering brane world supersymmetry breaking is the problem of flavour-changing neutral currents (FCNCs) which generically are present and strong in gravity mediated models. The reason is that the Kähler potential $\Omega = \exp\{-K/3\}$, which encodes the soft masses, introduces mixings via the coefficient of $Q_i^\dagger Q_j$, where Q_i are visible sector scalars:

$$\Omega_{Q^\dagger Q} = -\frac{1}{3M_\mathrm{P}^2} Q_i^\dagger Q_j \left(\delta_{ij} + C_{ij}\right) , \tag{4.1}$$

where the contact terms C_{ij} depend on the hidden sector fields which break supersymmetry. Generally, this will lead to unacceptably large FCNCs, which can be avoided only if we assume a special structure of the contact terms, diagonal or aligned with the fermion Yukawa couplings. This is a strong assumption and needs to be justified.

A possible justification is the so-called sequestering[20], where the Kähler potential is simply the sum of hidden and observable sector contributions, so $C_{ij} = 0$. Sequestered Kähler potentials can arise in higher-dimensional models where the hidden and observable sectors are located on different branes. The separation in the extra dimension can guarantee the absence of contact terms. However, sequestering is not a generic result of higher-dimensional theories, for two reasons. First, the bulk contains the gravity multiplet, which couples to the branes. Integrating out the higher Kaluza-Klein modes can generate contact terms, which are benign in the sense that they are calculable and flavour-blind since gravitational fields couple universally. Second, the bulk might contain other fields with nontrivial brane couplings, and these fields may spoil sequestering and introduce dangerous contact terms. Such contact terms seem non-local, but of course integrating out the higher Kaluza-Klein states generates a non-local effective theory, and the branes are not "far apart" in any sense[52, 53]. The contact terms so generated are suppressed by the volume of the internal space, but so is the gravitino mass which is the scale of supersymmetry breaking. Hence, generically one might expect non-universal contact terms of the order of the gravitino mass.

Specific models of brane-world supersymmetry breaking have been widely studied in five dimensions[54], where the tree-level result indeed is sequestered,

$$\Omega_{5d} = \frac{1}{2}\left(T + \overline{T}\right) - \frac{1}{3}\Omega_\mathrm{obs} - \frac{1}{3}\Omega_\mathrm{hid} \, , \tag{4.2}$$

with the radion modulus T which does not couple to the branes. Hence, at tree-level there is no mediation of supersymmetry breaking, and one-loop corrections[55] are important. Unfortunately, they turn out to produce negative $(\text{masses})^2$ unless large brane-localised gravity kinetic terms are introduced[56].

Here we will generalise these studies to six dimensional supergravity on the orbifold $T^2/\mathbb{Z}_2$, presenting the results of Ref. [57]. For the bulk-brane coupling we start from globally supersymmetric brane theories and iteratively make them locally supersymmetric by the Noether method[58]. This will involve coupling to bulk fields. We will follow this procedure to two-fermion terms. For the combined bulk-brane action we will find the low-energy effective action at tree level and include one-loop corrections. In this setup, we will analyse the effects of gravity mediated supersymmetry breaking.

4.1. Bulk-Brane Coupling

4.1.1. Bulk Action

The action of ungauged supergravity follows from the general action in Appendix B by setting $g \to 0$ and omitting the gauge fields A_M and Λ. The remaining field content is the metric G_{MN} which we trade for the sechsbein[1] ${e_M}^A$, the gravitino Ψ_M, the two-form B_{MN}, with field strength $H_{MNP} = 3\partial_{[M}B_{NP]}$, the dilaton Φ and its fermionic partner, the dilatino χ. The fermions are chiral in the six-dimensional sense. The Lagrangean is

$$\begin{aligned}\mathscr{L}_{\text{bulk}} = e_6 \bigg\{ &\frac{1}{2}R_6 - \mathrm{i}\overline{\Psi}_M\Gamma^{MNP}D_N\Psi_P + \frac{e^{-\Phi}}{12}H_{MNP}H^{MNP} + \mathrm{i}\overline{\chi}\Gamma^M D_M\chi \\ &+ \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{\mathrm{i}e^{-\Phi}}{12\sqrt{2}}\overline{\Psi}_M\Gamma^{MNPQR}\psi_N H_{PQR} \\ &+ \frac{\mathrm{i}e^{-\Phi}}{2\sqrt{2}}\overline{\Psi}^M\Gamma^N\overline{\Psi}^P H_{MNP} + \frac{e^{-\Phi}}{12\sqrt{2}}\overline{\Psi}_M\Gamma^{MNPQ}\chi H_{NPQ} + \text{H.c.} \\ &- \frac{e^{-\Phi}}{4\sqrt{2}}\overline{\Psi}^M\Gamma^{NP}\chi H_{MNP} + \text{H.c.} + \frac{\mathrm{i}e^{-\Phi}}{12\sqrt{2}}\overline{\chi}\Gamma^{MNP}\chi H_{MNP} \\ &- \frac{1}{2}\overline{\chi}\Gamma^M\Gamma^N\Psi_M\partial_N\Phi + \text{H.c.} + (\text{four-fermion terms})\bigg\} \,. \end{aligned} \tag{4.3}$$

It is invariant under local supersymmetry transformations with chiral parameter ε (up to three-fermion terms):

$$\delta {e_M}^A = \frac{\mathrm{i}}{2}\overline{\Psi}_M\Gamma^A\varepsilon + \text{H.c.}\,, \tag{4.4a}$$

$$\delta\Psi_M = D_M\varepsilon + \frac{e^{-\Phi}}{24\sqrt{2}}\left(\Gamma_{MNPQ} - 3G_{MN}\Gamma_{PQ}\right)\varepsilon H^{NPQ}\,, \tag{4.4b}$$

[1] $M = (\mu, m)$ etc. are Einstein indices while $A = (\alpha, a)$ etc. are Lorentz indices. Both range over $0, \ldots, 3, 5, 6$, and numerical Einstein indices are indicated by $\dot{5}$ etc. Details can be found in the appendix.

Field	even		odd	
$e_M{}^A$	$e_\mu{}^\alpha$	$e_m{}^a$	$e_m{}^\alpha$	$e_\mu{}^a$
Ψ_M	ψ_μ^+	ψ_m^+	ψ_μ^-	ψ_m^-
B_{MN}	B_{mn}	$B_{\mu\nu}$	$B_{\mu m}$	
χ	χ^+		χ^-	
ε	ε^+		ε^-	
Φ	Φ			

Table 4.1: *Decomposition of bulk fields into even and odd components. Ψ_M and χ have opposite six-dimensional chirality, ψ_μ^+, ψ_m^- and χ^- are four-dimensionally right-handed. Details about Γ-matrices and the spinor decomposition are given in Appendix A.*

$$\delta B_{MN} = \frac{\mathrm{i}e^\Phi}{\sqrt{2}}\overline{\Psi}_{[M}\Gamma_{N]}\varepsilon - \frac{e^\Phi}{2\sqrt{2}}\overline{\chi}\Gamma_{MN}\varepsilon + \text{H.c.}\,, \tag{4.4c}$$

$$\delta\chi = -\frac{i}{2}\Gamma^M\varepsilon\partial_M\Phi + \frac{\mathrm{i}e^{-\Phi}}{12\sqrt{2}}\Gamma_{MNP}\,\varepsilon H^{MNP}\,, \tag{4.4d}$$

$$\delta\Phi = \frac{1}{2}\overline{\chi}\varepsilon + \text{H.c.} \tag{4.4e}$$

We compactify the theory on the torus orbifold $T^2/\mathbb{Z}_2$. We take a symmetric orbifold $(y^5, y^6) \in (-\pi R, \pi R] \times (-\pi R, \pi R]$, and the $\mathbb{Z}_2$ acts by $(y^5, y^6) \mapsto -(y^5, y^6)$. The fields – and the transformation parameter – are decomposed into even and odd fields as given in Table 4.1.

The extradimensional metric is parametrised as follows,

$$g_{mn} = \frac{A}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}\,, \tag{4.5}$$

so that the volume of the orbifold is $V_{T^2/\mathbb{Z}_2} = 2\,(\pi R)^2\,A$. The other two moduli can be combined into $\tau = \tau_2 + \mathrm{i}\tau_1$ which describes the shape of the torus (the ratio of lengths via $|\tau|^2$, the angle by τ_1/τ_2). This parametrisation is actually redundant, i.e. under a $\mathsf{SL}(2,\mathbb{Z})$ transformation $\tau \to (a\tau - \mathrm{i}b)\,/\,(\mathrm{i}c\tau + d)$ the torus goes over to itself.

4.1.2. Brane Action and the Noether Method

In this section we will couple the branes to the bulk via the Noether method. This method has e.g. been used to couple chiral multiplets to supergravity in four dimensions and branes to a supergravity bulk in five and eleven dimensions[58, 55], but the six-dimensional bulk-brane coupling has not been derived before. Off-shell methods might be more powerful when applicable, but the Noether method is a simple and efficient procedure to derive the coupling even when there is no off-shell formulation of supergravity at hand, like in six dimensions. Furthermore, we will find that it is sufficient to compute zero- and two-fermion terms to uniquely fix the low-energy effective theory.

The procedure involves several steps, and we will explicitly find the coupling of a chiral multiplet on the brane to the bulk supergravity. This section will be rather technical, and the result is Eq. (4.29). For the case of a charged chiral multiplet plus vector multiplet we just give the result.

Brane Chiral Multiplet

We put a chiral multiplet with complex scalar Q and chiral fermion η on the fixed point $(y^5, y^6) = (0, 0)$. We take the starting action to be globally supersymmetric:

$$\mathscr{L}_0 = e_4 \delta_{56} f \left(\partial_\mu Q^\dagger \partial^\mu Q + \left(\frac{\mathrm{i}}{2} \overline{\eta} \gamma^\mu \partial_\mu \eta + \text{H.c.} \right) \right) . \tag{4.6}$$

Here we abbreviated $\delta_{56} = \delta(y^5)\,\delta(y^6)$ and included a factor f, which is a function of the bulk bosonic fields to be specified later. This is invariant under these global transformations:

$$\delta Q^\dagger = \frac{1}{\sqrt{2}} \overline{\eta} \tilde{\varepsilon} \tag{4.7a}$$

$$\delta \eta = -\frac{\mathrm{i}}{\sqrt{2}} \gamma^\mu \left(\partial_\mu Q \right) \tilde{\varepsilon} \tag{4.7b}$$

The parameter ε is a four-dimensional chiral spinor which we identify with the positive parity component of the six-dimensional supersymmetry variation parameter, $\tilde{\varepsilon} = \varepsilon^+$.

Now for the Noether procedure. The idea is to vary the action (4.6) with a local variation parameter. The action will not be invariant, and we will add further terms involving couplings to brane fields and augment some bulk quantities with brane-localised terms until the action is invariant. We will pursue this method to two-fermion terms, i.e. there will be uncancelled four-fermion variations left. The rationale for that is that the zero- and two-fermion terms are all we need to infer the Kähler potential and gauge kinetic function of the low-energy effective theory. Since we know the effective theory is $\mathcal{N} = 1$ supersymmetric, knowledge of these two functions completely determines the theory. Hence, it should be possible to find the complete action, including four-fermion term, so that it is locally supersymmetric.

But now to the brane! The variation of the action (4.6) under a local transformation, where $\tilde{\varepsilon} = \varepsilon^+ = \varepsilon^+(x)$ depends on the four-dimensional coordinates, splits into several distinct pieces:

$$\begin{aligned} \delta \mathscr{L}_0 = {} & \underbrace{\frac{e_4 \delta_{56}}{\sqrt{2}} \left(f \overbrace{\overline{\eta} \gamma^\mu \gamma^\nu \partial_\nu Q}^{\propto J^\mu_{\text{super}}} \partial_\mu \varepsilon^+ + \frac{1}{2} \partial_\mu f \overline{\eta} \left(\gamma^{\mu\nu} - g^{\mu\nu} \right) \partial_\nu Q \, \varepsilon \right)}_{\delta_{\eta,Q} \mathscr{L}_0} + \underbrace{e_4 \delta_{56} \partial_\mu Q^\dagger \partial^\mu Q \delta f}_{\delta_f \mathscr{L}_0} \\ & + \underbrace{\frac{\mathrm{i} e_4 \delta_{56} f}{2} \partial_\mu Q^\dagger \partial_\nu Q \, \overline{\psi}^+_\rho \left(\gamma^\rho g^{\mu\nu} - \gamma^\nu g^{\rho\mu} - \gamma^\mu g^{\nu\rho} \right) \varepsilon^+}_{\delta_e \mathscr{L}_0} + \text{H.c.} \end{aligned} \tag{4.8}$$

The last part stems from the variation of the vierbein determinant, and we only keep the scalar kinetic term since we only consider two-fermion variations. The second part contains the yet unknown variation of f, while the first contribution comes from the variation of Q and η by Eq. (4.7a). The term $\propto \partial_\mu f$ comes from an integration by parts, while the first term is the supercurrent times the derivative of the transformation parameter. Such a term always comes about when one varies a globally symmetric Lagrangean with a local parameter, and is cancelled by a coupling of the associated gauge field which is the gravitino for supergravity. For the parameter $\varepsilon^+(x)$, the correct component actually is the right-handed positive parity component ψ_μ^+, and the coupling is the Noether term

$$\mathscr{L}_{\mathrm{N}} = -\frac{e_4\delta_{56}f}{\sqrt{2}}\overline{\eta}\gamma^\mu\gamma^\nu\partial_\nu Q\,\psi_\mu^+ + \mathrm{H.c.} \tag{4.9}$$

This term, however, again, has further variations (again, we only consider the variations of the fermions in $\mathscr{L}_{\mathrm{N}}$, since the variations of e_4 and f are four-fermion terms):

$$\begin{aligned}\delta\mathscr{L}_{\mathrm{N}} = &-\frac{e_4\delta_{56}}{\sqrt{2}}\overline{\eta}\gamma^\mu\gamma^\nu\partial_\nu Q\partial_\mu\varepsilon^+ \\ &-\frac{e_4\delta_{56}fe^{-\Phi}}{48}\partial_\lambda\overline{\eta}\Big(H^{\nu\rho\sigma}\left(\mathrm{i}\gamma^\mu\gamma^\lambda\epsilon_{\mu\nu\rho\sigma} - 3\gamma_\nu\gamma^\lambda\gamma_{\rho\sigma}\right) \\ &\qquad\qquad + 6\mathrm{i}e_2 H^{\nu\dot{5}\dot{6}}\left(\gamma^\mu\gamma^\lambda\gamma_{\mu\nu} - \gamma_\nu\gamma^\lambda\right)\Big)\varepsilon^+ \\ &+\frac{\mathrm{i}e_4\delta_{56}f}{2}\partial_\mu Q^\dagger\partial_\nu Q\,\overline{\psi}_\rho^+\left(-\gamma^{\mu\nu\rho} - \gamma^\rho g^{\mu\nu} + \gamma^\nu g^{\rho\mu} + \gamma^\mu g^{\nu\rho}\right)\varepsilon^+ + \mathrm{H.c.}\end{aligned} \tag{4.10}$$

Here the last line is $\delta_\eta\mathscr{L}_{\mathrm{N}}$, the variation of the Noether term from the variation of η. The second and third line follow from the H_{MNP} variation of the gravitino. The determinant of the zweibein follows from the contractions of the form $H^{MNP}\Gamma_{MNP}$, e.g.

$$\begin{aligned}H^{MNP}\Gamma_{MNP} &= H^{\mu\nu\rho}\Gamma_{\mu\nu\rho} + 3H^{\mu\nu r}\Gamma_{\mu\nu r} + 3H^{\mu nr}\Gamma_{\mu nr} \\ &= H^{\mu\nu\rho}\Gamma_{\mu\nu\rho} + 3H^{\mu\nu r}\Gamma_{\mu\nu r} + H^{\mu\dot{5}\dot{6}}\epsilon^{nr}e_n{}^a e_r{}^b\Gamma_\mu\epsilon_{ab}\Gamma_{56}\,,\end{aligned} \tag{4.11}$$

where the Einstein and Lorentz indices are connected via zweibeins. The $\Gamma_{\mu\nu r}$-term is off-diagonal and hence vanishes on ε^+.

The last line of (4.10) is of the same form as $\delta_e\mathscr{L}_0$ in Eq. (4.8). They combine to

$$\delta_e\mathscr{L}_0 + \delta_\eta\mathscr{L}_{\mathrm{N}} = -\frac{\mathrm{i}e_4\delta_{56}f}{2}\partial_\mu Q^\dagger\partial_\nu Q\,\overline{\psi}_\rho^+\gamma^{\mu\nu\rho}\varepsilon^+ + \mathrm{H.c.} \tag{4.12}$$

We will not cancel this by further terms, but modify the bulk fields. To be precise, the variation of the gravitino kinetic term in the bulk action (4.3) contains a piece that vanishes due to the Bianchi identity,

$$\begin{aligned}\delta^1\mathscr{L}_{\mathrm{bulk}} &= \frac{\mathrm{i}e_6e^{-\Phi}}{12\sqrt{2}}\overline{\Psi}_M\Gamma^{MNPQR}\varepsilon\partial_N H_{PQR} + \mathrm{H.c.} \\ &= -\frac{e_4e^{-\Phi}}{2\sqrt{2}}\overline{\psi}_\mu^+\gamma^{\mu\nu\rho}\varepsilon^+\partial_\nu H_{\rho\dot{5}\dot{6}} + \frac{\mathrm{i}e_2e^{-\Phi}}{12\sqrt{2}}\epsilon^{\mu\nu\rho\sigma}\partial_\mu H_{\nu\rho\sigma}\overline{\psi}^+\varepsilon^+ + \mathrm{H.c.}\end{aligned} \tag{4.13}$$

where $\psi^+ = -\left(\psi_5^+ + \mathrm{i}\psi_6^+\right)$ is a linear combination of the higher-dimensional gravitino components, and there are additional terms in the second line that vanish on the brane. We exploit the above variation by replacing $H_{MNP} \to \widehat{H}_{MNP}$, where $\widehat{H}_{MNP}$ does not satisfy the Bianchi identity, but rather

$$\partial_{[\mu}\widehat{H}_{\nu]\dot{5}\dot{6}} = -\sqrt{2}\, i\delta_{56} f e^{\Phi} \partial_{[\mu} Q^\dagger \partial_{\nu]} Q + (\text{2-fermion-terms})\,, \tag{4.14}$$

while $\partial_{[\mu}\widehat{H}_{\mu\nu\rho]} = 0 + (\text{2-fermion-terms})$. To find a $\widehat{H}$ which will do that, we first set $f = e^{-\Phi}$. This implies

$$\delta_{Q,\eta}\mathscr{L}_0\Big|_{\partial f} = -\frac{e_4\delta_{56}e^{-\Phi}}{2\sqrt{2}}\partial_\mu\Phi\partial_\nu Q\overline{\eta}\left(\gamma^{\mu\nu} - g^{\mu\nu}\right)\varepsilon^+ + \text{H.c.} \tag{4.15}$$

$$\delta_f\mathscr{L}_0 = -\frac{e_4\delta_{56}e^{-\Phi}}{2}\partial_\mu Q^\dagger\partial^\mu Q\overline{\chi}^+\varepsilon^+ + \text{H.c.} \tag{4.16}$$

The condition (4.14) for the modified Bianchi identity is satisfied by

$$\widehat{H}_{\mu\dot{5}\dot{6}} = H_{\mu\dot{5}\dot{6}} - \frac{\mathrm{i}\delta_{56}}{\sqrt{2}}\left(Q^\dagger\partial_\mu Q - \partial_\mu Q^\dagger Q\right) + (\text{2-fermion-terms})\,. \tag{4.17}$$

The modified Bianchi identity cancels the variations $\delta_e\mathscr{L}_0 + \delta_\eta\mathscr{L}_{\mathrm{N}}$. However, there is a price to pay: $\delta\mathscr{L}_{\text{bulk}}$ contains another term which relies on the Bianchi identity to vanish,

$$\begin{aligned}\delta^2\mathscr{L}_{\text{bulk}} &= -\frac{e_6e^{-\Phi}}{12\sqrt{2}}\overline{\chi}\Gamma^{MNPQ}\varepsilon\partial_M H_{NPQ} + \text{H.c.}\\ &= -\frac{\mathrm{i}e_4e^{-\Phi}}{2\sqrt{2}}\overline{\chi}_\mu^+\gamma^{\mu\nu}\varepsilon^+\partial_\mu H_{\nu\dot{5}\dot{6}} + \text{H.c.}\\ &= -\frac{1}{2}e_4\delta_{56}e^{-\Phi}\partial_\mu Q^\dagger\partial_\nu Q\overline{\chi}^+\gamma^{\mu\nu}\varepsilon^+ + \text{H.c.}\end{aligned} \tag{4.18}$$

To cancel this, we have to include another coupling, which is to the dilatino. This comes as no surprise, since the dilaton couples to the brane, and so should its superpartner. The term is

$$\mathscr{L}_{\text{dino}} = \frac{\mathrm{i}e_4\delta_{56}e^{-\Phi}}{\sqrt{2}}\overline{\chi}\gamma^\mu\eta\,\partial_\mu Q^\dagger + \text{H.c.} \tag{4.19}$$

It has variations from $\delta\eta \sim \partial Q$ and $\delta\chi \sim \partial\Phi$ which cancel $\delta^2\mathscr{L}_{\text{bulk}}$, $\delta_f\mathscr{L}_0$ and the $\partial_\mu f$-part of $\delta_{Q,\eta}\mathscr{L}_0$. On the other hand, there is another variation from $\delta\chi \sim H_{MNP}$ which is not cancelled.

So by now, the brane Lagrangean is

$$\mathscr{L}_{\text{brane}} = \mathscr{L}_0 + \mathscr{L}_{\mathrm{N}} + \mathscr{L}_{\text{dino}}\,, \tag{4.20}$$

which, together with the modified Bianchi identity, has a remaining variation

$$\delta\mathscr{L}_{\text{brane}} = \frac{e_4\delta_{56}e^{-2\Phi}}{4}\left(\frac{\mathrm{i}}{e_2}\partial_\mu Q H_{\nu\dot{5}\dot{6}}\overline{\eta}\left(\gamma^{\mu\nu}+3g^{\mu\nu}\right)\varepsilon^+ \right. \tag{4.21}$$

$$\left. +\frac{1}{6}H_{\mu\nu\rho}\partial_\sigma Q\,\overline{\eta}\gamma^{\mu\nu\rho}\gamma^\sigma\varepsilon^+\right) + \text{H.c.} \tag{4.22}$$

This can be cancelled by a further modification of the three-form field strength which changes its supersymmetry transformation law. The variation of the bulk field strength kinetic term is

$$\delta_{H^2}\mathscr{L}_{\text{bulk}} = \frac{e_6e^{-2\Phi}}{6}\left(H_{\mu\nu\rho}\delta H^{\mu\nu\rho} + 3H_{\mu\nu m}\delta H^{\mu\nu m} + 6H_{\mu\dot{5}\dot{6}}\delta H^{\mu\dot{5}\dot{6}}\right). \tag{4.23}$$

So we can cancel $\delta\mathscr{L}_{\text{brane}}$ if we find a further two-fermion modification of $\widehat{H}$ which has the additional variations

$$\delta_{\text{extra}}\widehat{H}^{\mu}{}_{\dot{5}\dot{6}} = -\frac{\mathrm{i}\delta_{56}}{4}\partial_\nu Q\overline{\eta}\left(\gamma^{\nu\mu}+3g^{\nu\mu}\right)\varepsilon^+ + \text{H.c.} \tag{4.24}$$

$$\delta_{\text{extra}}\widehat{H}^{\mu\nu\rho} = \frac{\delta_{56}}{4e_2}\partial_\lambda Q\,\overline{\eta}\gamma^{\mu\nu\rho}\gamma^\lambda\varepsilon^+ + \text{H.c.} \tag{4.25}$$

Such a modification is provided by the following terms:

$$\widehat{H}_{\mu\dot{5}\dot{6}}\Big|_{2-\text{fermion}} = -\frac{\delta_{56}}{2\sqrt{2}}\overline{\eta}\gamma_\mu\eta \tag{4.26}$$

$$\widehat{H}_{\mu\nu\rho}\Big|_{2-\text{fermion}} = -\frac{\mathrm{i}\delta_{56}}{2\sqrt{2}}\overline{\eta}\gamma_{\mu\nu\rho}\eta \tag{4.27}$$

together with a modified transformation law of the two-form field,

$$\delta_{\text{extra}}B_{\dot{5}\dot{6}} = -\frac{\mathrm{i}\delta_{56}}{2}\overline{\eta}\varepsilon^+Q + \text{H.c.} \tag{4.28}$$

This was the final step. To summarise, the brane Lagrangean up to four-fermion terms is

$$\begin{aligned}\mathscr{L}_{\text{brane}} &= \mathscr{L}_0 + \mathscr{L}_{\text{N}} + \mathscr{L}_{\text{dino}}\\ &= e_4\delta_{56}e^{-\Phi}\left[\partial_\mu Q^\dagger\partial^\mu Q + \left(\frac{\mathrm{i}}{2}\overline{\eta}\gamma^\mu\partial_\mu\eta\right.\right.\\ &\qquad\left.\left. -\frac{1}{\sqrt{2}}\overline{\eta}\gamma^\mu\gamma^\nu\partial_\nu Q\psi_\mu^+ + \frac{\mathrm{i}}{\sqrt{2}}\overline{\chi}\gamma^\mu\eta\,\partial_\mu Q^\dagger + \text{H.c.}\right)\right].\end{aligned} \tag{4.29}$$

In the bulk Lagrangean, we have replaced $H \to \widehat{H}$, where

$$\widehat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - \frac{\mathrm{i}\delta_{56}}{2\sqrt{2}}\overline{\eta}\gamma_{\mu\nu\rho}\eta\,, \tag{4.30}$$

$$\widehat{H}_{\mu\dot{5}\dot{6}} = H_{\mu\dot{5}\dot{6}} - \frac{\mathrm{i}\delta_{56}}{\sqrt{2}}\left(Q^\dagger\partial_\mu Q - \partial Q^\dagger\,Q - \frac{\mathrm{i}}{2}\overline{\eta}\gamma_\mu\eta\right), \tag{4.31}$$

and modified the transformation law to (4.28).

Brane Vector Multiplet and Charged Matter

A similar, but more complicated procedure can be carried out for the case of a brane Lagrangean involving a $\mathsf{U}(1)$ vector multiplet and a charged chiral multiplet. Here we only quote the result.

The initial Lagrangean contains kinetic terms for vector field A_μ, gaugino λ, scalar Q and chiral fermion η, couplings of matter to the vector via covariant derivatives, a Yukawa-like term $\sim Q\overline{\eta}\lambda$ and a quartic scalar term. There are two pieces to be added, a Noether term which now involves Λ and the field strength $F_{\mu\nu}$ and a dilatino coupling to η and λ. We again change the three-form $H \to \widehat{H}$ which does not satisfy the Bianchi identity and has a modified transformation law. The complete Lagrangean is

$$\mathscr{L}_{\text{brane}} = e_4\delta_{56}\left\{\mathscr{L}_0' + \mathscr{L}_{\text{N}}' + \mathscr{L}_{\text{dino}}'\right\} , \tag{4.32}$$

where

$$\begin{aligned}\mathscr{L}_0' = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(\frac{\mathrm{i}}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda + \text{H.c.}\right)\\ &+ e^{-\Phi}\left[D_\mu Q^\dagger D^\mu Q + \left(\frac{\mathrm{i}}{2}e^{-\Phi}\overline{\eta}\gamma^\mu D_\mu\eta + \text{H.c.}\right)\right]\\ &+ e^{-\Phi}\left(\sqrt{2}\,\mathrm{i}ge^{-\Phi}Q^\dagger\bar{\lambda}\eta + \text{H.c.}\right) - \frac{1}{2}g^2e^{-2\Phi}\left|Q\right|^4 ,\end{aligned} \tag{4.33a}$$

$$\begin{aligned}\mathscr{L}_{\text{N}}' = &-\frac{1}{2}\overline{\Lambda}\gamma^\mu\left(\frac{\mathrm{i}}{2}F_{\rho\sigma}\gamma^\rho\sigma - ge^{-\Phi}\left|Q\right|^2\gamma^\mu\right)\psi_\mu^+\\ &+ \frac{1}{\sqrt{2}}e^{-\Phi}\overline{\eta}\gamma^\mu\gamma^\nu D_\nu Q\,\psi_\mu^+ + \text{H.c.}\,,\end{aligned} \tag{4.33b}$$

$$\mathscr{L}_{\text{dino}}' = -\mathrm{i}ge^{-\Phi}\left|Q\right|^2\bar{\lambda}\chi - \frac{\mathrm{i}}{\sqrt{2}}e^{-\Phi}D_\mu Q\overline{\eta}\gamma^\mu\chi + \text{H.c.} \tag{4.33c}$$

The modified field strength is

$$\widehat{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - \frac{\mathrm{i}\delta_{56}}{2\sqrt{2}\,e_2}\left(e^\Phi\bar{\lambda}\gamma_{\mu\nu\rho}\lambda + \overline{\eta}\gamma_{\mu\nu\rho}\eta\right) , \tag{4.34a}$$

$$\widehat{H}_{\mu\dot{5}\dot{6}} = H_{\mu\dot{5}\dot{6}} + \frac{\delta_{56}}{2\sqrt{2}}\left[-2\mathrm{i}\left(Q^\dagger D_\mu Q - D_\mu Q^\dagger Q\right) - \overline{\eta}\gamma_\mu\eta + e^\Phi\bar{\lambda}\gamma_\mu\lambda\right] , \tag{4.34b}$$

$$\delta_{\text{extra}}B_{\dot{5}\dot{6}} = -\frac{\mathrm{i}\delta_{56}}{2}Q\overline{\eta}\varepsilon^+ + \text{H.c.} \tag{4.34c}$$

Brane Superpotential

We can also include a brane superpotential. Here we give the simplest case, a constant superpotential, i.e. a gravitino mass term W_0. The starting point is the brane action

$$\mathscr{L}_{W,0} = -\frac{1}{2}e_4\delta_{56}fW_0\overline{\psi}_\mu\gamma^{\mu\nu}C\overline{\psi}_\nu^T + \text{H.c.} \tag{4.35}$$

The procedure is similar to the previous one, but this time it involves couplings of the extra-dimensional components of the gravitino to the brane. We also have to take the spin connection into account, which results in a coupling of the shape modulus. The final brane action is

$$\mathscr{L}_W = \frac{1}{2} e_4 \delta_{56} \sqrt{\frac{A}{\tau_2}} W_0 \left(-\overline{\psi}_\mu \gamma^{\mu\nu} C \overline{\psi}_\nu^T + \mathrm{i} \overline{\psi}_{5-6} \gamma^\mu C \overline{\psi}_\mu^T + \text{H.c.} \right) . \tag{4.36}$$

Here $\psi_{5\pm6} = -\left(\psi_5^+ \pm \mathrm{i}\psi_6^+\right)$. The transformation laws of these gravitino components also get an additional brane-localised term,

$$\delta_{\text{extra}} \psi_{5\pm6} = \pm \delta_{56} \frac{1}{\sqrt{A\tau_2}} W_0 C \overline{\varepsilon}^T . \tag{4.37}$$

From this result we can infer how a field-dependent bulk superpotential couples to the bulk moduli. In particular, if the brane theory develops a vacuum expectation value of F- or D-terms, the coupling is

$$\mathscr{L}_{F,D} = -\frac{1}{2} e_4 \delta_{56} \left(e^{-2\Phi} g^2 D^2 + \frac{2Ae^\Phi}{\tau_2} |F|^2 \right) . \tag{4.38}$$

4.2. The 4d Effective Theory

We now find the low-energy effective action for the theory. The light degrees of freedom are the zero modes of the even fields: The bosonic ones are the four-dimensional metric, the volume and shape moduli, the dilaton, the two-form components $B_{\mu\nu}$ and $B_{5\dot{6}}$, while the fermionic zero modes comprise the four-dimensional gravitino, the even extra-dimensional gravitino components and the dilatino.

4.2.1. Tree Level

Our aim is to find the Kähler potential, superpotential and gauge kinetic function of the low-energy theory. To this end it is sufficient to check the kinetic terms of the boson zero modes. We take the background to be

$$\mathrm{d}s^2 = \frac{1}{A} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu - g_{mn} \mathrm{d}y^m \mathrm{d}y^n , \tag{4.39}$$

and reinsert the Planck scale. The six-dimensional fundamental scale M_6 is the cut-off of the effective six-dimensional theory. The compactification scale, that is, the mass of the lowest Kaluza-Klein modes, depends on the volume of the internal space, $M_{\text{KK}} \sim V_{T^2/\mathbb{Z}_2}^{-1/2} \sim A^{-1/2} R^{-1}$. For the six-dimensional theory to be sensible, we require $M_6/M_{\text{KK}} \gg 1$. The coupling of graviton zero modes is gouverned by M_6^2/M_{KK}, rather than by M_6 itself.

The above scale considerations are valid in the Einstein frame. When compactifying to four dimensions, the Ricci tensor is multiplied by A coming from the determinant of the metric in the integration measure. To get rid of this, one has to perform a Weyl rescaling to again go to the Einstein frame. This endows all masses with a factor of $A^{-1/2}$, so that the compactification scale becomes $M_{\mathrm{KK}} \sim (AR)^{-1}$, while the four-dimensional Planck scale is determined from the graviton zero mode coupling and is $M_{\mathrm{P}} = 2\pi R M_6^2$.

Now for the action. The higher modes of the fields can be integrated out trivially, with the exception of the odd components of the two-form. They need special treatment because of the replacement (4.34), which results in the appearance of

$$\begin{aligned} \frac{e_6 e^{-2\Phi}}{12} \widehat{H}_{MNP} \widehat{H}^{MNP} &= \frac{e_4 e^{-2\Phi}}{2A^2} \widehat{H}_{\mu\dot{5}\dot{6}} \widehat{H}^{\mu}{}_{\dot{5}\dot{6}} + \cdots \\ &= \frac{e_4 e^{-2\Phi}}{2A^2} \Bigg(\partial_\mu B_{\dot{5}\dot{6}} + \partial_{\dot{6}} B_{\mu\dot{5}} + \partial_{\dot{5}} B_{\mu\dot{6}} - \delta_{56} \underbrace{\frac{\mathrm{i}}{\sqrt{2}} \left(Q^\dagger D_\mu Q - D_\mu Q^\dagger Q \right)}_{j_\mu} \Bigg)^2 + \cdots . \end{aligned} \tag{4.40}$$

The ellipses represent the other components of $\widehat{H}$ and fermionic terms in the brane-localised additions. Hence, there are couplings $\sim \delta_{56} \partial_{\dot{5}} B_{\mu\dot{6}} j^\mu$ of the odd components of the two-form to the brane. Additionally, there appear potentially dangerous δ_{56}^2-terms, but they will cancel while integrating out the Kaluza-Klein modes of $B_{\mu m}$. The derivative translates into a mass of the Kaluza-Klein modes, so the coupling is proportional to the mass, which compensates the mass suppression of the propagator, and the heavy states do not decouple. In other words, the appearance of the brane coupling modifies the boundary conditions and thus the Kaluza-Klein expansion. To find a new expansion, one first has to solve the equation of motion of $B_{\mu m}\left(y^{\dot{5}}, y^{\dot{6}}\right)$, and then reinsert the solution into the action before integrating out the extra dimensions. In this procedure, one has to treat the even fields j_μ and $\partial_\mu B_{\dot{5}\dot{6}}$ as constant sources. The relevant equations for the field strength are

$$\partial^{\dot{5}} \widehat{H}_{\mu\dot{5}\dot{6}} = 0 \,, \qquad\qquad \partial^{\dot{6}} \widehat{H}_{\mu\dot{5}\dot{6}} = 0 \,. \tag{4.41}$$

So the field strength is constant, $\widehat{H}_{\mu\dot{5}\dot{6}} = C_\mu = \text{const}$. We can solve this equation if we assume a function W_μ with $B_{\mu\dot{5}} = -\partial_{\dot{6}} W_\mu$ and $B_{\mu\dot{6}} = \partial_{\dot{5}} W_\mu$, so that $\widehat{H}_{\mu\dot{5}\dot{6}} = C_\mu$ becomes

$$\left(\partial_{\dot{5}}^2 + \partial_{\dot{6}}^2\right) W_\mu = \partial_\mu B_{\dot{5}\dot{6}} - C_\mu + \delta_{56} j_\mu \,. \tag{4.42}$$

The left hand side vanishes when integrated over the internal space. Hence we find the integration constant (remember $B_{\dot{5}\dot{6}}$ is a zero mode)

$$C_\mu = \partial_\mu B_{\dot{5}\dot{6}} + \frac{1}{(2\pi R)^2} j_\mu \,. \tag{4.43}$$

Hence, we see that $\widehat{H}_{\mu\dot{5}\dot{6}}$ does not contain a δ_{56} contribution. The brane-localised part was cancelled by a contribution of the odd $B_{\mu m}$. We could solve the equation of motion explicitly, but actually, C_μ is all we need.

Inserting C_μ in the action and the zero modes for all other fields, we find the low-energy effective action for the bosonic fields:

$$\begin{aligned}\mathscr{L}_{\text{eff,bos}} = e_4 M_\text{P}^2 \Bigg\{ &\frac{1}{2}R + \frac{1}{2A^2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{4\tau_2^2}\partial_\mu\tau_2\partial^\mu\tau_2 + \frac{1}{4\tau_2^2}\partial_\mu\tau_1\partial^\mu\tau_1 \\ &+ \frac{e^{-\Phi}}{M_\text{P}^2 A} D_\mu Q^\dagger D^\mu Q - \frac{1}{4M_\text{P}^2}F_{\mu\nu}F^{\mu\nu} + \frac{e^{-2\Phi}}{12A^2}H_{\mu\nu\rho}H^{\mu\nu\rho} \\ &+ \frac{e^{-2\Phi}}{2A^2}\left(\partial_\mu B_{\dot{5}\dot{6}} + \frac{\mathrm{i}}{\sqrt{2}\,M_\text{P}^2}\left(Q^\dagger D_\mu Q - D_\mu Q^\dagger Q\right)\right)^2 \Bigg\}\end{aligned} \tag{4.44}$$

It is actually convenient to redefine some fields: The three-form is dual to a pseudoscalar σ via $e^{-2\Phi}H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda}\partial^\lambda\sigma$, and the metric determinant and dilaton are combined to new fields t and s by

$$t = Ae^{\Phi} \qquad\qquad s = Ae^{-\Phi} \tag{4.45}$$

to disentangle the coupling to $B_{\dot{5}\dot{6}}$ and σ. The new Lagrangean reads

$$\begin{aligned}\mathscr{L}_{\text{eff,bos}} = e_4 M_\text{P}^2 \Bigg\{ &\frac{1}{2}R + \frac{1}{4t^2}\partial_\mu t\,\partial^\mu t + \frac{1}{4s^2}\partial_\mu s\,\partial^\mu s + \frac{1}{4\tau_2^2}\partial_\mu\tau_2\,\partial^\mu\tau_2 + \frac{1}{4\tau_2^2}\partial_\mu\tau_1\,\partial^\mu\tau_1 \\ &+ \frac{1}{M_\text{P}^2 t}\partial_\mu Q^\dagger \partial^\mu Q - \frac{1}{4M_\text{P}^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2s^2}\partial_\mu\sigma\,\partial^\mu\sigma \\ &+ \frac{1}{2t^2}\left(\partial_\mu B_{\dot{5}\dot{6}} + \frac{\mathrm{i}}{\sqrt{2}\,M_\text{P}^2}\left(Q^\dagger D_\mu Q - D_\mu Q^\dagger Q\right)\right)^2 \Bigg\}\,.\end{aligned} \tag{4.46}$$

Since we know that the four-dimensional effective theory has to be $\mathcal{N} = 1$ supergravity, the kinetic terms must be derivable from a Kähler potential, which is a function of complex fields and their conjugates, and a gauge kinetic function which exclusively depends on the complex fields. To obtain complex fields, we can group the scalar and pseudoscalar fields[2] together to form the moduli multiplets

$$T = t + \mathrm{i}\sqrt{2}\,B_{\dot{5}\dot{6}} + \frac{|Q|^2}{M_\text{P}^2}\,, \qquad S = s + \mathrm{i}\sigma\,, \qquad \tau = \tau_2 + \mathrm{i}\tau_1\,. \tag{4.47}$$

These combinations are actually dictated from the kinetic terms in Eq. (4.46), where e.g. t appears in the coefficient of the kinetic term of $B_{\dot{5}\dot{6}}$.

In terms of the Kähler potential $K\big(T, \overline{T}, S, \overline{S}, \tau, \overline{\tau}\big)$ and the gauge kinetic function $f(T, S, \tau)$ the kinetic terms can be written as

$$\mathscr{L}_{\text{bos,kin}} = e_4 M_\text{P} \left\{ \frac{1}{2}R + \sum_{\phi=T,S,\tau} \frac{\partial^2 K}{\partial\overline{\phi}\,\partial\phi}\,D_\mu\phi D^\mu\overline{\phi} - \frac{1}{4}\,\mathrm{Re}\,f\,F_{\mu\nu}F^{\mu\nu} \right\}\,. \tag{4.48}$$

[2] $B_{\dot{5}\dot{6}}$ can be seen to be a pseudoscalar because CP in the extra dimensions amounts to an orientation reversal, under which the antisymmetric two-form changes sign.

Comparison with the Lagrangean (4.46) gives

$$K = -\ln\left(\frac{1}{2}\left(T+\overline{T}\right) - \frac{|Q|^2}{M_{\mathrm{P}}^2}\right) - \ln\left(\frac{1}{2}\left(S+\overline{S}\right)\right) - \ln\left(\frac{1}{2}\left(\tau+\overline{\tau}\right)\right) , \tag{4.49}$$

$$f = 1 . \tag{4.50}$$

Compactifying the gravitino mass term from the superpotential Lagrangean (4.36), we get

$$\mathscr{L}_W = -\frac{e_4}{2A\sqrt{\tau_2}} W_0 \overline{\psi}_\mu \gamma^{\mu\nu} \overline{\psi}_\nu^T + \text{H.c.} \tag{4.51}$$

This precisely corresponds to a constant superpotential $W = W_0$ in four-dimensional supergravity.

Later we will study the mediation of supersymmetry breaking from the hidden sector on one brane to other branes. To do so, we need to find the supergravity functions for the case where there is matter on more than one brane. Fortunately, the other branes' matter couples just as the one we considered, so the previous result can be generalised straightforwardly.

Assume that on brane a (where $a = 1, \ldots, 4$ labels the branes), there is matter consisting of scalars Q_a and gauge fields A_a^μ. The brane kinetic terms are fixed by $\Omega_a\big(Q_a, Q_a^\dagger\big)$ for the scalars[3] and $f_a(Q_a)$ for the gauge fields, and the brane superpotential is denoted by $W_a(Q_q)$. The supergravity functions for the low-energy effective theory are

$$\begin{aligned} K = &-\ln\left(\frac{1}{2}\left(T+\overline{T}\right) - \sum_a \Omega_a\big(Q_a, Q_a^\dagger\big)\right) \\ &- \ln\left(\frac{1}{2}\left(S+\overline{S}\right)\right) - \ln\left(\frac{1}{2}\left(\tau+\overline{\tau}\right)\right) , \end{aligned} \tag{4.52a}$$

$$f = \sum_a f_a(Q_a) , \tag{4.52b}$$

$$W = \sum_a W_a(Q_a) . \tag{4.52c}$$

In Section 4.3, we will use this result to analyse gravity mediated supersymmetry breaking. Some remarks about general features of this solution are in order.

1. The first observation is that the Kähler potential is not sequestered as in Eq. (2.23). This means that in principle, there can be tree-level mediation of supersymmetry breaking. However, we will see in Section 4.3 that the issue is more subtle. The non-sequestering is not so severe that the effective theory would contain direct contact terms $\sim |Q_a|^2 |Q_b|^2$ between different branes. There are, however, vector-vector terms $\sim (\Omega_a' \partial_\mu Q_a)(\Omega_b' \partial^\mu Q_b)$ which arise from integrating out the two-form field in $\widehat{H}_{\mu\dot{5}\dot{6}}$ along the lines of the discussion before Eq. (4.43). Such operators do not contribute to supersymmetry breaking mediation.

[3] Ω is the Kähler potential on the brane. For canonical kinetic terms, $\Omega_a\big(Q_a, Q_a^\dagger\big) = Q^\dagger Q$.

2. The moduli couple flavour-blindly, that is, universally to all brane fields. This is enforced by six-dimensional supergravity without any freedom.

3. The superpotential only depends on the brane fields and not on the bulk moduli.

4. The Kähler potential is “partially no-scale”: It is no-scale with respect to the bulk moduli but not to the brane fields. In the scalar potential, hence, the $-3\,|W|^2$ part is cancelled, and the only moduli dependence of the scalar potential comes from the $\exp\{K\}$ factor:

$$V = \frac{1}{st\tau_2} K^{i\bar{j}} \partial_i W \partial_{\bar{j}} \overline{W} \tag{4.53}$$

   Here i, j only range over the brane fields. This result holds as long as W is independent of the moduli. The full scalar potential is given in Eq. (4.69).

4.2.2. Loop Corrections

As the tree-level system (4.52) is not no-scale, the loop corrections to the Kähler potential will generically not play a decisive rôle in the mediation of supersymmetry breaking. In specific situations, on the other hand, the system is effectively sequestered in the sense that there is no tree-level mediation of supersymmetry breaking, and loop corrections might be relevant, so we list them for completeness.

The one-loop corrections are most easily expressed in terms of $\Omega = \exp\{-K/3\}$. At one loop, $\Omega = \Omega_{\text{tree}} + \Delta\Omega$, and $\Delta\Omega$ can be conveniently calculated from[59]

$$\Delta\Omega = \frac{N_j}{3} \frac{\Gamma(1-d/2)}{M_{\text{P}}^2\,(4\pi)^{d/2}} \sum m_n^{d-2}\,. \tag{4.54}$$

Here m_n is the Kaluza-Klein mass spectrum depending on $n = (n_5, n_6)$ in the so-called supergravity conformal frame where the Einstein-Hilbert term is

$$\mathscr{L}_{\text{conf}} = \frac{1}{2} M_{\text{P}}^2 e_4 \Omega R = \frac{1}{2} M_{\text{P}}^2 e_4 A^{2/3} \tau_2^{1/3} R\,. \tag{4.55}$$

The factor N_j depends on the matter present. Each Dirac fermion contributes $N_{1/2} = 1$, while a gauge boson has a negative contribution, $N_1 = -2$. The formula is to be understood in dimensional regularisation, so $d = 4 + \epsilon$.

The relation (4.54) can be derived by considering the contribution of a field of mass m and spin j to the renormalisation of the Einstein-Hilbert term which is

$$\Delta\mathscr{L}_{\text{EH}} = \frac{N_j}{3} \frac{\Gamma(1-d/2)\, m^{d-2}}{M_{\text{P}}^2\,(4\pi)^{d/2}} \sqrt{-g} R\,. \tag{4.56}$$

This has to be summed over all Kaluza-Klein modes. Now one can see why the supergravity conformal frame is convenient: The $\Delta\mathscr{L}_{\text{EH}}$ terms can be understood as contributions to Ω, and so (4.54) follows. For a more explicit derivation, see [59].

The mass spectrum in this frame can be found from the kinetic term of a scalar for simplicity. Its equation of motion is

$$\partial_M \left(e_6 G^{MN} \partial_N \phi\right) = 0 \tag{4.57}$$

Inserting the Kaluza-Klein expansion for ϕ results in the replacements $\Box_4 \phi \to -m_n^2 \phi$ and $\partial_m \phi \to \mathrm{i} n_m \phi$. After Weyl rescaling to the supergravity conformal frame, the mass spectrum is

$$m_{n_5,n_6}^2 = \frac{1}{A^{4/3} \tau_2^{2/3} R^2} \left|n_6 + \mathrm{i} n_5 \tau\right|^2 . \tag{4.58}$$

To find $\Delta\Omega$, we need to evaluate the sum

$$\Sigma(s) = \frac{1}{2} \sum_{n_5,n_6 \in \mathbb{Z}}{}' m_{n_5,n_6}^{2s} \tag{4.59}$$

at $s = d/2 - 1$. The sum is computed in Appendix C. The result for $\Delta\Omega$ is

$$\begin{aligned} \Delta\Omega = &-\frac{32}{3\,(4\pi)^2} \frac{1}{(2\pi R M_{\mathrm{P}})^2} \\ &\times \frac{\zeta(3) + \frac{\pi^3}{360} (\tau + \bar{\tau})^3 + \sum\limits_{m_5,m_6 \in \mathbb{N}} \left[\frac{1 + \pi m_5 m_6 (\tau + \bar{\tau})}{m_6^3} \left(e^{-2\pi m_5 m_6 \tau} + \mathrm{H.c.}\right)\right]}{\left(T + \bar{T} - 2\Omega_{\mathrm{branes}}/M_{\mathrm{P}}^2\right)^{2/3} \left(S + \bar{S}\right)^{2/3} \left(\tau + \bar{\tau}\right)^{2/3}} . \end{aligned} \tag{4.60}$$

This contribution is suppressed by a loop factor and the volume, $(4\pi)^2 (2\pi R A M_{\mathrm{P}})^2$. There is also a divergent part of the one-loop correction which simply leads to a renormalisation of the tree-level parameters which depends on the UV completion of the theory.

4.3. Gravity Mediated Supersymmetry Breaking

Now we can turn to the issue of gravity mediation. The setup is given by the effective theory defined by the supergravity functions K, f and W as given in E. (4.52). We restrict to matter on two branes, denoted by Q^V and Q^H for the observable and the hidden sector. The tree-level Kähler function is

$$\Omega = \frac{1}{2} \left(T + \bar{T} - 2\Omega_{\mathrm{obs}}\left(\bar{Q}^{\bar{V}}, Q^V\right) - 2\Omega_{\mathrm{hid}}\left(\bar{Q}^{\bar{H}}, Q^H\right)\right)^{1/3} \left(S + \bar{S}\right)^{1/3} \left(\tau + \bar{\tau}\right)^{1/3} . \tag{4.61}$$

As stressed before, it is not sequestered, and the matter fields are not separated from the T modulus either. The "partially no-scale" structure, visible from the fact that the exponents add up to one, means that in the scalar potential the $-3\left|W\right|^2$ part will be cancelled.

The scalar potential generally is given by Eq. (2.19),

$$V = \frac{1}{M_{\mathrm{P}}^2} e^K \left(\overline{D_j W} K^{\bar{j}k} D_k W - 3\,|W|^2 \right) + \frac{1}{2\,\mathrm{Re}\, f} D^2$$
$$= \frac{1}{M_{\mathrm{P}}^2} \overline{F}^{\bar{\imath}} K_{\bar{\imath}j} F^j - 3 M_{\mathrm{P}}^2 m_{3/2}^2 + \frac{1}{2\,\mathrm{Re}\, f} D^2 \,.$$

Here we have inserted the expression (2.20) for the F-terms and the gravitino mass

$$m_{3/2}^2 = \frac{1}{M_{\mathrm{P}}^4} e^K \, |W|^2 \,, \tag{4.62}$$

since these are the order parameters for supersymmetry breaking. We assume there are no relevant D-terms. The F-terms and the Kähler metric are listed in Appendix D.

We assume supersymmetry is broken by a nonvanishing vacuum expectation value for the F-component of the hidden sector fields or any modulus field. Further, we assume that the breaking leads to a vanishing vacuum energy, $\langle V \rangle = 0$. This allows us to express the gravitino mass in terms of the F-terms,

$$m_{3/2}^2 = \frac{1}{3} \left[\frac{\left|F^S\right|^2}{\left(S+\overline{S}\right)^2} + \frac{\left|F^\tau\right|^2}{(\tau+\overline{\tau})^2} + \frac{\left|F^T\right|^2}{\left(T+\overline{T}\right)^2} + 2 \frac{\left|F^H\right|^2}{M_{\mathrm{P}}^2 \left(T+\overline{T}\right)} \right] . \tag{4.63}$$

We now turn to the soft supersymmetry breaking masses. For the brane theories, we assume that the Kähler potentials start off canonical,

$$\Omega_{\mathrm{obs}} = \overline{Q}^{\overline{V}} Q^V + \mathcal{O}\left(\left(\overline{Q}^{\overline{V}} Q^V \right)^2 \right) , \qquad \Omega_{\mathrm{hid}} = \overline{Q}^{\overline{H}} Q^H + \mathcal{O}\left(\left(\overline{Q}^{\overline{H}} Q^H \right)^2 \right) . \tag{4.64}$$

To calculate the soft masses, we expand the Kähler function in powers of $\overline{Q}^{\overline{V}} Q^V$,

$$\Omega = \Omega_0\left(T, S, \tau, Q^H\right) - \frac{1}{3} Y_V\left(T, S, \tau, Q^H\right) \overline{Q}^{\overline{V}} Q^V + \mathcal{O}\left(\left(\overline{Q}^{\overline{V}} Q^V \right)^2 \right) , \tag{4.65}$$

and find the visible sector masses by[60]

$$m_{\mathrm{obs}}^2 = -F^i \overline{F}^{\bar{\jmath}} \partial_i \partial_{\bar{\jmath}} \ln(Y_V) \,. \tag{4.66}$$

Here the index i ranges over the moduli and the hidden matter, $i \in \{T, S, \tau, H\}$. In our case,

$$Y = \frac{\left(S+\overline{S}\right)^{1/3} \left(\tau+\overline{\tau}\right)^{1/3}}{\left(T+\overline{T}-2\Omega_{\mathrm{hid}}\right)^{2/3}} \,, \tag{4.67}$$

and hence the soft mass is

$$m_{\mathrm{obs}}^2 = \frac{1}{3} \left(\frac{|F_S|^2}{\left(S+\overline{S}\right)^2} + \frac{|F_\tau|^2}{\left(\tau+\overline{\tau}\right)^2} - 2 \frac{|F_T|^2}{\left(T+\overline{T}\right)^2} - 4 \frac{|F_H|^2}{M_{\mathrm{P}}^2 \left(T+\overline{T}\right)} \right) \tag{4.68}$$

Here we assume that the hidden sector fields have vanishing vacuum expectation values.

First of all, we see that the masses can turn out negative, so the radion and hidden sector vacuum expectation values cannot be the only contribution but must be compensated by the other moduli. The other important observation concerns the last term: It corresponds to tree-level mediation of supersymmetry breaking, where the symmetry breakdown, signalled by $|F_H|^2 \neq 0$, is immediately transferred to the visible scalars. This is the effect of the non-sequestered form of the Kähler function. However, one needs to be careful: The F-terms are not necessarily independent. Indeed, we can explicitly calculate the scalar potential and obtain

$$\begin{aligned} V = & \frac{1}{M_{\mathrm{P}}^2}\frac{1}{t}\left[(\Omega_{\mathrm{obs}})_{V\overline{V}}\left|F_V\right|^2 + (\Omega_{\mathrm{hid}})_{H\overline{H}}\left|F_H\right|^2\right] \\ & + \frac{1}{M_{\mathrm{P}}^2}\frac{2}{s\,t\tau_2}\left[t\left(2t\left|\partial_T W\right|^2 - \left(\overline{W}\partial_T W + W\partial_{\overline{T}}\overline{W}\right)\right)\right. \\ & \qquad + s\left(2s\left|\partial_S W\right|^2 - \left(\overline{W}\partial_S W + W\partial_{\overline{S}}\overline{W}\right)\right) \\ & \qquad \left. + \tau_2\left(2\tau_2\left|\partial_\tau W\right|^2 - \left(\overline{W}\partial_\tau W + W\partial_{\overline{\tau}}\overline{W}\right)\right)\right] . \end{aligned} \tag{4.69}$$

The mass of the visible scalar must show up in the potential. Yet, if the superpotential is moduli-independent — which it is on the perturbative level —, only the first line remains and the hidden sector does not interact with the observable one. Hence, $m_{\mathrm{obs}} \sim \partial_v \partial_{\overline{V}} V$ is independent of $\left|F^H\right|^2$, and there is no tree-level mediation.

Actually, this persists even in the presence of non-perturbative effects as long as the superpotential is T-independent: The Kähler potential depends on $2t = T + \overline{T} - 2\Omega_{\mathrm{obs}} - 2\Omega_{\mathrm{hid}}$, but not on T and Q^V separately. The superpotential will in general depend on Q_V, but the masses inside the superpotential are supersymmetric masses. Hence, to consider soft breaking masses, we can assume $\partial_Q W = 0$, and hence T and Q^V only appear in the combination $\frac{1}{2}\left(T + \overline{T}\right) - \overline{Q}^{\overline{V}} Q^V$. The soft mass is proportional to

$$\partial_V \partial_{\overline{V}} V \sim \partial_T V . \tag{4.70}$$

Assuming that the potential stabilises T, that is, there is a minimum of V with respect to T, we have $\partial_T V = 0$, and hence the soft masses vanish. Hence, there is a relation between moduli stabilisation and mediation of supersymmetry breaking (as was to be expected, since soft masses really can only be computed in a stable vacuum). It should be noted that the soft masses are flavour-blind even if we introduce more than one visible field, since neither the moduli nor the hidden sector fields have any freedom to couple to the observable sector.

In the remainder of this section we briefly comment on possibilities of moduli stabilisation: We can stabilise S and τ by gaugino condensation, if there are two non-Abelian gauge groups in the bulk. Generically, for bulk gauge groups the low-energy effective gauge kinetic function is $f = S$, as can be seen from the kinetic term

$$\mathscr{L}_6 = -\frac{1}{4} e_6 e^{-\Phi} F_{MN} F^{MN} \xrightarrow{\text{compactification}} -\frac{1}{4} e_4 \underbrace{A e^{-\Phi}}_{S} F_{\mu\nu} F^{\mu\nu} . \tag{4.71}$$

There are loop corrections to this result[61], which give $\Delta f = \frac{b}{4\pi^2} \ln \eta(\mathrm{i}\tau)$. Here, b is the beta function coefficient and η is the Dedekind η function. Gaugino condensation results in an effective S-dependent superpotential, which for two condensing gauge groups is of the racetrack form,

$$W_{\text{eff}} = \frac{1}{\eta(\mathrm{i}\tau)^2} \left(\Lambda_1 \exp\left\{ -\frac{8\pi^2}{b_1} S \right\} - \Lambda_2 \exp\left\{ -\frac{8\pi^2}{b_2} S \right\} \right) . \tag{4.72}$$

This fixes both S and τ. To stabilise T, we can invoke the Green-Schwarz anomaly cancellation mechanism. In six dimensions, this requires additional terms in the Lagrangean[62], coupling the two-form B_{MN} to the gauge field strength. This leads to a T-dependent gauge kinetic function in the low-energy effective theory, since $B_{\dot{5}\dot{6}}$ is the imaginary part of T. By gaugino condensation, this can generate a T-dependent superpotential which leads to a stabilisation of the radion. Such models, however, are rather contrived, since they involve several gaugino condensates and additional hidden sector D-terms to cancel the effective cosmological constant.

4.4. Conclusion

Here we have presented gravity mediated supersymmetry breaking in six dimensions. We first found the locally supersymmetric coupling of brane matter to the bulk by the Noether method and derived the low-energy effective theory, which is uniquely fixed by supersymmetry. Hence, the contact terms are fixed and calculable in terms of the moduli.

The Kähler potential is neither sequestered nor no-scale. It is no-scale for the moduli in the sense that they appear $\sim \left(S + \overline{S}\right)$ with exponents adding up to one, but the radion modulus T acquires an admixture of the brane matter which spoils the no-scale structure. Since the superpotential is modulus-independent perturbatively, the scalar potential remembers the higher-dimensional separation and exhibits no tree-level mediation of supersymmetry breaking. Nonperturbatively, however, the superpotential can pick up moduli dependence, as is necessary to stabilise the moduli. Gaugino condensates can stabilise the S and τ moduli, but to generate a T-dependent superpotential, we need to add higher order operators in the six-dimensional action which couple the two-form to the gauge fields. In this direction it is possible to stabilise the moduli and generate acceptable soft masses (where anomaly mediation is present as well to generate gaugino masses), albeit in a rather complicated way.

5. Conclusion and Discussion

We have analysed two aspects of six-dimensional supergravity: We have found the general warped solution with four-dimensional maximal symmetry, and we have investigated gravity mediated supersymmetry breaking.

Such six-dimensional models are interesting, for example, in the light of anisotropic string theory compactifications, where successful gauge coupling unification can be realised if the internal space has two large radii, i.e. of the order of the inverse GUT scale, while the remaining ones are small, $\sim M_{\mathrm{P}}^{-1}$. This might help to bridge the gap between the string and GUT scale. Accordingly, in a recent compactification of the heterotic string which yields the standard model at low energies[63] (see also [64]), there can be intermediate supersymmetric orbifold GUTs in any dimension from five to ten. However, the resulting low-energy theory in such models relies on a discrete choice of orbifold. A dynamical mechanism to find such an orbifold configuration as a ground state of a theory might be a desirable generalisation.

The warped solution we find is characterised by a free holomorphic function: Its zeroes and poles fix the singularities of the internal space, and its functional form determines the warp factor. We have presented two-brane solutions with axial symmetry and multi-brane solutions which are not axially symmetric. In both cases, we found a finite Planck mass. Flux quantisation gives a fine-tuning-condition on the brane tensions. In the unwarped limit, the multi-brane case reproduces known solutions. The stability of such solutions has been analysed recently[65]. A promising direction for future work would be a systematic study of possible functions to choose. In particular, elliptic (i.e. doubly periodic) functions would allow to incorporate solutions similar to torus orbifolds in this formalism. Furthermore, we found in the analysis of the Noether coupling that bulk moduli, in particular the dilaton, might couple to the brane. This would provide an important generalisation of our approach.

On the more phenomenological side, the issue of supersymmetry breaking is far from resolved, in particular, the mediation mechanism is unknown. Gravity mediation is a theoretically favoured way because it is very generic: gravity is out there. However, for viable models it is important to understand how soft masses are generated and why they do not induce strong flavour-changing neutral currents. In this respect, extra dimensions offer the obvious possibility to separate the supersymmetry breaking hidden sector from the observable fields, and hence to suppress flavour changing contact terms. In five dimensions, it turned out that a supergravity setup along these lines does not work: the masses come out negative, unless there are large brane-localised gravity mass terms. In six dimensions, the situation is different.

To study gravity mediation, we first found the coupling of brane-localised matter to the bulk supergravity by the Noether procedure. This involved adding a gravitino and dilatino coupling term to the Lagrangean, as well as a modification of the bulk two-form field and its field strength, which acquired a brane-localised piece. This in turn leads to a non-sequestered low energy effective theory, since the odd components of the two-form get a mass-dependent coupling which compensates the mass suppression of the propagator and spoils sequestering, in spite of the spatial separation between the hidden and observable sector. The supergravity functions — Kähler potential, superpotential and gauge kinetic function — are uniquely fixed. In particular, the moduli couple flavour-blind to the branes, and the superpotential is simply the sum of the brane superpotentials; it does not depend on the moduli (in perturbation theory).

Sequestering forbids contact terms and hence tree-level mediation of supersymmetry breaking. However, even though our model is non-sequestered, tree-level gravity mediation does not occur as long as the superpotential is independent of the radion modulus T and the radius is stabilised. Non-perturbative effects such as gaugino condensation, can induce a T dependence in the superpotential, which is required in any case to stabilise the moduli. Hence we need to add more matter or gauge fields in the bulk. Models of moduli stabilisation based purely on the ungauged supergravity of Chapter 4 require quite baroque mechanisms (multiple gaugino condensates and higher dimensional operators). However, we are confident that embedding this scenario in the context of flux compactifications along the lines of Chapter 3 will lead to more attractive models.

A. Notation and Conventions

A.1. Indices and Metric

Dimension	6	4	2
Einstein	$M, N, \ldots$	$\mu, \nu, \ldots$	$m, n, \ldots$
Lorentz	$A, B, \ldots$	$\alpha, \beta, \ldots$	$a, b, \ldots$

Table A.1: *Index conventions*

We use the "mostly minus" signature $\eta_{AB} = \mathrm{diag}(+,-,-,-,-,-)$. The indices are chosen as given in Table A.1. When in doubt, we denote explicit Einstein indices by a dot, e.g. $B_{\dot{5}\dot{6}}$.

The vielbein (sechsbein, vierbein, zweibein) has a lower Einstein index, e_M^A and satisfies $G_{MN} = e_M^A e_N^B \eta_{AB}$. The inverse vielbein e_A^M is distinguished by index placement and fulfills $e_M^A e_A^N = \delta_M^N$ and $e_A^M e_M^B = \delta_A^B$.

The extra-dimensional metric in Chapter 4 is parametrised as

$$g_{mn} = \frac{A}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix} , \qquad \Rightarrow \qquad g^{mn} = \frac{1}{A\tau_2} \begin{pmatrix} \tau_1^2 + \tau_2^2 & -\tau_1 \\ -\tau_1 & 1 \end{pmatrix} . \tag{A.1}$$

A.2. Γ-Matrices and Fermion Decomposition

The six-dimensional Γ^A matrices are eight-dimensional, while we use four-dimensional four-dimensional γ^a-matrices. The algebra is $\left\{\Gamma^A, \Gamma^B\right\} = 2\eta^{AB}$ and $\left\{\gamma^a, \gamma^b\right\} = 2\eta^{ab}$. The chirality projectors are

- in six dimensions: $P_\pm = \frac{1}{2}\left(1 \pm \Gamma^7\right)$ with $\Gamma^7 = \mathrm{diag}(-1,-1,1,1,1,1,-1,-1)$,
- in four dimensions $P_{R/L} = \frac{1}{2}\left(1 \pm \gamma^5\right)$ with $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \mathrm{diag}(-1,-1,1,1)$.

The Γ^A are given in terms of the γ^a as follows:

$$\Gamma^a = \begin{pmatrix} \gamma^a & 0 \\ 0 & \gamma^a \end{pmatrix} \qquad \Gamma^5 = \begin{pmatrix} 0 & \mathrm{i}\gamma^5 \\ \mathrm{i}\gamma^5 & 0 \end{pmatrix} \qquad \Gamma^6 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix} \tag{A.2}$$

Now we will discuss the decomposition of six-dimensional fermions into four-dimensional ones. The gravitino Ψ_A and the gaugino Λ have negative six-dimensional chirality, $P_-\Psi_M = \Psi_M$ and $P_-\Lambda = \Lambda$, while the dilatino has opposite chirality, $P_+\chi = \chi$.

Chiral six-dimensional spinors correspond to two chiral ones in four dimensions. In this case, they can be decomposed as

$$\Psi_\mu = \begin{pmatrix} \psi_\mu^- \\ \psi_\mu^+ \end{pmatrix} , \qquad \Lambda = \begin{pmatrix} \Lambda^- \\ \Lambda^+ \end{pmatrix} , \qquad \Psi_m = \begin{pmatrix} \psi_m^+ \\ \psi_m^- \end{pmatrix} , \qquad \chi = \begin{pmatrix} \chi^- \\ \chi^+ \end{pmatrix} . \tag{A.3}$$

The lower component of negative chirality spinors, i.e. ψ_μ^+, ψ_m^- and Λ^+, are four-dimensionally right-handed ($P_R\psi = \psi$), as well as the upper component of the positive-chirality spinor, χ^-. Conversely, ψ_μ^-, ψ_m^+, Λ^- and χ^+ are left-handed ($P_L\psi = \psi$). The $\pm$ superscript refers to $\mathbb{Z}_2$ parity.

B. 6d Supergravity: Lagrangean and Transformations

B.1. Gauged Six-Dimensional Supergravity

The theory contains the following fields: The metric G_{MN}, the gravitino Ψ_M, the two-form B_{MN}, the dilaton Φ, the dilatino χ, the vector field A_M and the gaugino Λ. The field strength of the gauge field is

$$F_{MN} = \partial_M A_N - \partial_N A_M \,, \tag{B.1}$$

while the two-form has a three-form field strength with a Chern-Simons-like coupling,

$$G_{MNP} = \partial_M B_{NP} + \frac{1}{\sqrt{2}} F_{MN} A_P + \text{cyclic} = \underbrace{3\partial_{[M} B_{NP]}}_{H_{MNP}} + \underbrace{\frac{3}{\sqrt{2}} F_{[MN} A_{P]}}_{J_{MNP}} \,. \tag{B.2}$$

The fermions are chiral, with Ψ_M and χ being of opposite chirality. The two-form and the fermions are charged under the $\mathsf{U}(1)$, the covariant derivatives of the fermions are

$$D_M \Psi_N = \left(\partial_M + \frac{1}{4} \omega_M{}^{AB} \Gamma_{AB} + \mathrm{i} g A_M \right) \Psi_N \tag{B.3}$$

and the same for χ and Λ. The variation of the two-form is

$$\delta B_{MN} = -\alpha F_{MN} \quad \text{for} \quad \delta A_M = \partial_M \alpha \,. \tag{B.4}$$

Hence, G_{MNP} is gauge invariant due to the Bianchi identity for F_{MN}.

The supersymmetry transformation laws are (up to three-fermion terms in $\delta\Psi_M$):

$$\delta e_M{}^A = \tfrac{\mathrm{i}}{2} \overline{\psi}_M \Gamma^A \varepsilon + \text{H.c.} \tag{B.5a}$$

$$\delta \psi_M = D_M \varepsilon + \frac{e^{-\Phi}}{24\sqrt{2}} G^{NPQ} \left(\Gamma_{MNPQ} - 3 g_{MN} \Gamma_{PQ} \right) \varepsilon \tag{B.5b}$$

$$\delta B_{MN} = \sqrt{2}\, A_{[M} \delta A_{N]} + \frac{\mathrm{i} e^{\Phi}}{\sqrt{2}} \overline{\psi}_{[M} \Gamma_{N]} \varepsilon - \frac{e^{\Phi}}{2\sqrt{2}} \overline{\chi} \Gamma_{MN} \varepsilon + \text{H.c.} \tag{B.5c}$$

$$\delta \chi = -\tfrac{\mathrm{i}}{2} \Gamma^M \varepsilon \partial_M \Phi + \frac{\mathrm{i} e^{-\Phi}}{12\sqrt{2}} \Gamma^{MNP} \varepsilon G_{MNP} \tag{B.5d}$$

$$\delta \Phi = \tfrac{1}{2} \overline{\varepsilon} \chi + \text{H.c.} \tag{B.5e}$$

$$\delta A_M = \frac{\mathrm{i}}{2} \left(\overline{\varepsilon} \Gamma_M \Lambda - \overline{\Lambda} \Gamma_M \varepsilon \right) \tag{B.5f}$$

$$\delta \Lambda = \frac{1}{4} F_{MN} \Gamma^{MN} \varepsilon - \mathrm{i} g e^{\Phi} \varepsilon \,. \tag{B.5g}$$

The bulk Lagrangean invariant under these transformations up to four-fermion terms is

$$\begin{aligned}\mathscr{L}_{\text{bulk}} = M_6^4 e_6 \Bigg\{ & \frac{1}{2}R_6 - \mathrm{i}\overline{\psi}_M \Gamma^{MNP} D_N \psi_P + \frac{e^{-2\Phi}}{12} G_{MNP} G^{MNP} + \mathrm{i}\overline{\chi}\Gamma^M D_M \chi \\ & + \frac{1}{2}\partial_M \Phi \partial^M \Phi - \frac{e^{-\Phi}}{4} F_{MN} F^{MN} - 2g^2 e^{\Phi} + \mathrm{i}e^{-\Phi}\overline{\Lambda}\Gamma^M D_M \Lambda \\ & - \frac{\mathrm{i}e^{-2\Phi}}{12\sqrt{2}} G_{MNP}\overline{\Lambda}\Gamma^{MNP}\Lambda - \frac{\mathrm{i}e^{-\Phi}}{2}\overline{\Lambda}\Gamma^M \Lambda \partial_M \Phi \\ & - \frac{\mathrm{i}e^{-\Phi}}{12\sqrt{2}} \overline{\psi}_M \Gamma^{MNPQR} \psi_N G_{PQR} + \frac{\mathrm{i}e^{-\Phi}}{2\sqrt{2}} \overline{\psi}^M \Gamma^N \psi^P G_{MNP} \\ & + \frac{e^{-\Phi}}{12\sqrt{2}} \overline{\psi}_M \Gamma^{MNPQ} \chi G_{NPQ} - \frac{e^{-\Phi}}{4\sqrt{2}} \overline{\psi}^M \Gamma^{NP} \chi G_{MNP} + \text{H.c.} \\ & + \frac{\mathrm{i}e^{-\Phi}}{12\sqrt{2}} \overline{\chi}\Gamma^{MNP}\chi G_{MNP} - \frac{1}{2}\overline{\chi}\Gamma^M \Gamma^N \psi_M \partial_N \Phi + \text{H.c.} \\ & - \frac{e^{-\Phi}}{4} F_{MN}\left(\mathrm{i}\overline{\psi}_P \Gamma^{MN}\Gamma^P \Lambda + \mathrm{i}\overline{\Lambda}\Gamma^P \Gamma^{MN}\psi_P - \overline{\chi}\Gamma^{MN}\Lambda + \overline{\Lambda}\Gamma^{MN}\chi\right) \\ & + \mathrm{i}g\left(\mathrm{i}\overline{\psi}_M \Gamma^M \Lambda + \mathrm{i}\overline{\Lambda}\Gamma^M \psi_M + \overline{\chi}\Lambda - \overline{\Lambda}\chi\right) \Bigg\} \,. \end{aligned} \tag{B.6}$$

B.2. Brane Supersymmetry Lagrangean

The starting point for the coupling of the bulk supergravity to the gauged chiral multiplet on the brane is the theory involving a brane chiral multiplet (Q, η) and a vector multiplet (A_μ, λ) gauging a $\mathsf{U}(1)$ symmetry (no R-symmetry) with gauge coupling g. The covariant derivative acts as

$$D_\mu Q = (\partial_\mu + \mathrm{i}gA_\mu) \,, \qquad D_\mu \eta = \left(\partial_\mu + \mathrm{i}gA_\mu + \frac{1}{4}\omega_{\mu mn}\gamma^{mn}\right)\eta \,, \tag{B.7}$$

and the Lagrangean reads

$$\begin{aligned}\mathscr{L}_0 = e_4 \delta_{56} \Bigg\{ & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(\frac{\mathrm{i}}{2}\bar{\lambda}\gamma^\mu D_\mu \lambda + \text{H.c.}\right) \\ & + e^{-\Phi}\left[D_\mu Q^\dagger D^\mu Q + \left(\frac{\mathrm{i}}{2}e^{-\Phi}\overline{\eta}\gamma^\mu D_\mu \eta + \text{H.c.}\right)\right] \\ & + e^{-\Phi}\left(\sqrt{2}\,\mathrm{i}g e^{-\Phi} Q^\dagger \bar{\lambda}\eta + \text{H.c.}\right) - \frac{1}{2}g^2 e^{-2\Phi}\left|Q\right|^4 \Bigg\} \,. \end{aligned} \tag{B.8}$$

The supersymmetry transformations are (with constant parameter ε)

$$\delta Q = \tfrac{1}{\sqrt{2}}\overline{\varepsilon}\eta \qquad \delta Q^\dagger = \tfrac{1}{\sqrt{2}}\overline{\eta}\varepsilon \tag{B.9a}$$

$$\delta\eta = -\tfrac{\mathrm{i}}{\sqrt{2}}\gamma^\mu D_\mu Q\varepsilon \qquad \delta\overline{\eta} = \tfrac{\mathrm{i}}{\sqrt{2}}\overline{\varepsilon}\gamma^\mu D_\mu Q^\dagger \tag{B.9b}$$

$$\delta A_\mu = -\tfrac{\mathrm{i}}{2}\overline{\Lambda}\gamma_\mu\varepsilon + \text{H.c.} \qquad \delta F_{\mu\nu} = -\mathrm{i}\partial_{[\mu}\left(\overline{\Lambda}\gamma_{\nu]}\varepsilon\right)\text{H.c.} \tag{B.9c}$$

$$\delta\Lambda = \tfrac{1}{4}\gamma^{\mu\nu}\varepsilon F_{\mu\nu} + \tfrac{\mathrm{i}}{2}gf|Q|^2\varepsilon \qquad \delta\overline{\Lambda} = -\tfrac{1}{4}\overline{\varepsilon}\gamma^{\mu\nu}F_{\mu\nu} - \tfrac{\mathrm{i}}{2}gf|Q|^2\overline{\varepsilon}\,. \tag{B.9d}$$

C. Derivation of Loop Corrections

We give a brief derivation of the one-loop corrections to the Kähler potential, Eq. (4.60), using methods analogous to ζ function regularisation[66].

The general formula (4.54) gives the one-loop corrections to the Kähler potential,

$$\Delta\Omega = \frac{N_j}{3} \frac{\Gamma(1-d/2)}{M_\mathrm{P}^2 \, (4\pi)^{d/2}} \sum m_n^{d-2} \, .$$

The mass spectrum is given by Eq. (4.58),

$$\Rightarrow \quad m^2_{n_5,n_6} = \frac{1}{A^{4/3}\tau_2^{2/3} R^2} \left| n_6 + \mathrm{i} n_5 \tau \right|^2 \, .$$

Here we will compute the sum over the Kaluza-Klein masses:

$$\begin{aligned}\Sigma(s) &= \frac{1}{2} \sum_{n_5,n_6\in\mathbb{Z}}{}' m^2_{n_5,n_6}{}^s = \frac{1}{2} \sum_{n_5,n_6}{}' \frac{1}{\Gamma(-s)} \int \mathrm{d}t' \, t'^{-s-1} e^{-t' m^2_{n_5,n_6}} \\ &= \frac{1}{2} \frac{1}{\left(A^{4/3} t_2^{2/3} R^2\right)^s} \frac{1}{\Gamma(-s)} \int \mathrm{d}t \, t^{-s-1} \underbrace{\sum_{n_5,n_6}{}' e^{-t|n_6+\mathrm{i}\tau n_5|^2}}_{I(\tau)}\end{aligned} \tag{C.1}$$

In the last step we have rescaled $t' = t/\left(A^{4/3} t_2^{2/3} R^2\right)$. We will now split $I(\tau)$: The sum extends over all $(n_5, n_6) \in \mathbb{Z}$ except for $(0,0)$. The sum can be split into one over $(0, n_6 \neq 0)$ and one over $(n_5 \neq 0, n_6)$. In the second sum, the exponent can be split into a term $\sim n_5^2$ and a piece involving both n_5 and n_6, in which we perform a Poisson resummation:

$$\begin{aligned}\sum_{\substack{n_5\neq 0\\ n_6\in\mathbb{Z}}} e^{-t|n_6+\mathrm{i}\tau n_5|^2} &= \sum_{n_5\neq 0} e^{-t\tau_2^2 n_5^2} \sum_{n_6\in\mathbb{Z}} e^{-t(n_6-n_5\tau_1)^2} \\ &= \sum_{n_5\neq 0} e^{-t\tau_2^2 n_5^2} \sqrt{\frac{\pi}{t}} \sum_{m_6\in\mathbb{Z}} e^{-\pi^2 m_6^2/t - 2\mathrm{i}\pi\tau_1 n_5 m_6}\end{aligned} \tag{C.2}$$

Taking all this together, we have

$$\begin{aligned}
\sum_{n_5,n_6}{}' e^{-t|n_6+\mathrm{i}\tau n_5|^2} &= \sum_{n_6\neq 0} e^{-tn_6^2} + \sqrt{\frac{\pi}{t}}\sum_{n_5\neq 0} e^{-t\tau_2^2 n_5^2} \\
&\quad + \sqrt{\frac{\pi}{t}} \sum_{\substack{n_5\neq 0\\ m_6\neq 0}} \exp\left\{-t\tau_2^2 n_5^2 - \frac{\pi^2 m_6^2}{t} - 2\mathrm{i}\pi n_5 m_6 \tau_1\right\} \\
&= I_1(\tau) + I_2(\tau) + I_3(\tau)
\end{aligned} \tag{C.3}$$

Reinserting the expression back into Eq. (C.1), we get three components of $\Sigma(s)$,

$$\Sigma(s) = \frac{1}{A^{4/3} t_2^{2/3} R^2} \left(\Sigma_1(s) + \Sigma_2(s) + \Sigma_3(s)\right) \tag{C.4a}$$

with

$$\Sigma_1(s) = \zeta(-2s)\ , \tag{C.4b}$$

$$\Sigma_2(s) = \sqrt{\pi}\, \tau_2^{2s+1} \frac{\Gamma\left(-s-\frac{1}{2}\right)\zeta(-2s-1)}{\Gamma(-s)}\,, \tag{C.4c}$$

$$\Sigma_3(s) = \frac{2\tau_2^{s+1/2}}{\pi^2\Gamma(-s)} \sum_{n_5,p_6\in\mathbb{N}} \left[\left(\frac{n_5}{p_6}\right)^{s+1/2} K_{s+1/2}(2\pi\tau_2 n_5 p_6)\, e^{-2\pi\mathrm{i}\tau_1 n_6 p_6} + \mathrm{H.c.}\right]\,, \tag{C.4d}$$

where $K_{s+1/2}$ is the modified Bessel function of second kind. This Bessel function arises from the integral

$$\int \mathrm{d}t\, t^n e^{-a^2 t - b^2/t} = 2\left(\frac{a}{b}\right)^{-1-n} K_{(-1-n)}(2ab) \tag{C.5}$$

from integrating $I_3(\tau)$.

D. Kähler Metric and F-Terms

In this appendix we list some explicit expressions for the Kähler metric and its inverse, and the F-terms of the model of Chapter 4.

The Kähler metric in both forms is

$$\Omega = \left[\left(\frac{T+\overline{T}}{2} - \Omega_{\text{obs}} - \Omega_{\text{hid}}\right)\left(\frac{S+\overline{S}}{2}\right)\left(\frac{\tau+\overline{\tau}}{2}\right)\right]^{1/3}, \tag{D.1a}$$

$$K = -3\ln\Omega = -\ln\left(\frac{T+\overline{T}}{2} - \Omega_{\text{obs}} - \Omega_{\text{hid}}\right) - \ln\left(\frac{S+\overline{S}}{2}\right) - \ln\left(\frac{\tau+\overline{\tau}}{2}\right). \tag{D.1b}$$

For brevity we use the following expressions for the real parts of the moduli multiplets:

$$t = \frac{T+\overline{T}}{2} - 2\Omega_{\text{obs}} - 2\Omega_{\text{hid}} \qquad s = \frac{S+\overline{S}}{2} \qquad \tau_2 = \frac{\tau+\overline{\tau}}{2} \tag{D.2a}$$

The Kähler metric $K_{i\bar{\jmath}}$ is given by the second derivatives of K with respect to the fields ($i = V, H, T, S, \tau$):

$$K_{i\bar{\jmath}} = \begin{pmatrix} \frac{(\Omega_{\text{obs}})_{V\overline{V}}}{t} + \frac{\left|(\Omega_{\text{obs}})_V\right|^2}{t^2} & \frac{(\Omega_{\text{obs}})_V(\Omega_{\text{hid}})_{\overline{H}}}{t^2} & -\frac{(\Omega_{\text{obs}})_V}{2t^2} & 0 & 0 \\ \frac{(\Omega_{\text{hid}})_H(\Omega_{\text{obs}})_{\overline{V}}}{t^2} & \frac{(\Omega_{\text{hid}})_{H\overline{H}}}{t} + \frac{\left|(\Omega_{\text{hid}})_H\right|^2}{t^2} & -\frac{(\Omega_{\text{hid}})_H}{2t^2} & 0 & 0 \\ -\frac{(\Omega_{\text{obs}})_{\overline{V}}}{2t^2} & -\frac{(\Omega_{\text{hid}})_{\overline{H}}}{2t^2} & \frac{1}{4t^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4s^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4\tau_2^2} \end{pmatrix}. \tag{D.3}$$

The inverse metric is

$$K^{\bar{\imath}j} = \begin{pmatrix} \frac{t}{(\Omega_{\text{obs}})_{V\overline{V}}} & 0 & \frac{2(\Omega_{\text{obs}})_V}{(\Omega_{\text{obs}})_{V\overline{V}}}t & 0 & 0 \\ 0 & \frac{t}{(\Omega_{\text{hid}})_{H\overline{H}}} & \frac{2(\Omega_{\text{hid}})_H}{(\Omega_{\text{hid}})_{H\overline{H}}}t & 0 & 0 \\ \frac{2(\Omega_{\text{obs}})_{\overline{V}}}{(\Omega_{\text{obs}})_{V\overline{V}}}t & \frac{2(\Omega_{\text{hid}})_{\overline{H}}}{(\Omega_{\text{hid}})_{H\overline{H}}}t & 4t\left(t + \frac{\left|(\Omega_{\text{obs}})_V\right|^2}{(\Omega_{\text{obs}})_{V\overline{V}}} + \frac{\left|(\Omega_{\text{hid}})_H\right|^2}{(\Omega_{\text{hid}})_{H\overline{H}}}\right) & 0 & 0 \\ 0 & 0 & 0 & 4s^2 & 0 \\ 0 & 0 & 0 & 0 & 4\tau_2^2 \end{pmatrix}. \tag{D.4}$$

Finally, the F-terms:

$$F^V = -\sqrt{\frac{t}{s\tau_2}}\,\frac{1}{(\Omega_{\rm obs})_{V\bar{V}}}\left(D_{\bar{V}}\overline{W} + 2\,(\Omega_{\rm obs})_{\bar{V}}\,D_{\bar{T}}\overline{W}\right) \tag{D.5}$$

$$F^H = -\sqrt{\frac{t}{s\tau_2}}\,\frac{1}{(\Omega_{\rm hid})_{H\bar{H}}}\left(D_{\bar{H}}\overline{W} + 2\,(\Omega_{\rm hid})_{\bar{H}}\,D_{\bar{T}}\overline{W}\right) \tag{D.6}$$

$$F^T = -4\sqrt{\frac{t^3}{s\tau_2}}D_{\bar{T}}\overline{W} + 2\left((\Omega_{\rm obs})_V\,F^V + (\Omega_{\rm hid})_H\,F^H\right) \tag{D.7}$$

$$F^S = -4\sqrt{\frac{s^3}{t\tau_2}}D_{\bar{S}}\overline{W} \tag{D.8}$$

$$F^\tau = -4\sqrt{\frac{\tau_2^3}{ts}}D_{\bar{\tau}}\overline{W} \tag{D.9}$$

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