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Gravitino and Scalar τ-Lepton Decays in Supersymmetric Models with Broken R-Parity

Diploma Thesis

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Hamburg

January 2010

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Abstract

Mildly broken R-parity is known to provide a solution to the cosmological gravitino problem in supergravity extensions of the Standard Model.

In this work we consider new effects occurring in the R-parity breaking Minimal Supersymmetric Standard Model including right-handed neutrino superfields. We calculate the most general vacuum expectation values of neutral scalar fields including left- and right-handed scalar neutrinos. Additionally, we derive the corresponding mass mixing matrices of the scalar sector. We recalculate the neutrino mass generation mechanisms due to right-handed neutrinos as well as by cause of R-parity breaking. Furthermore, we obtain a, so far, unknown formula for the neutrino masses for the case where both mechanisms are effective.

We then constrain the couplings to bilinear R-parity violating couplings in order to accommodate R-parity breaking to experimental results. In order to constrain the family structure with a $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry we furthermore embed the particle content into an SU(5) Grand Unified Theory. In this model we calculate the signal of decaying gravitino dark matter as well as the dominant decay channel of a likely NLSP, the scalar τ-lepton. Comparing the gravitino signal with results of the Fermi Large Area Telescope enables us to find a lower bound on the decay length of scalar τ-leptons in collider experiments.

Zusammenfassung

Es ist bekannt, dass schwach gebrochene R-Parität eine Lösungsmöglichkeit für das kosmologische Gravitino Problem in Supergravitations-Erweiterungen des Standardmodells darstellt.

Daher berechnen wir in dieser Arbeit die Effekte, die im R-Paritätsbrechenden minimalen supersymmetrischen Standardmodell unter Berücksichtigung von rechtshändigen Neutrinos auftreten. Wir berechnen die allgemeinsten Vakuumerwartungswerte der neutralen Felder inklusive der links- und rechtshändigen skalaren Neutrinos. Zusätzlich berechnen wir die Massenmischungsmatrizen der skalaren Felder. Im fermionischen Sektor berechnen wir sowohl die Neutrinomasse, die durch die rechtshändigen Neutrinos erzeugt wird, als auch diejenige, die durch die R-Paritäts-Brechung erzeugt wird. Dadurch werden wir in die Lage versetzt eine bisher unveröffentlichte Formel für die Neutrinomasse herzuleiten, die gilt, wenn beide Mechanismen wirken.

Um den experimentellen Ergebnissen Rechnung zu tragen führen wir danach bilineare R-Paritäts-Brechung ein. Um die Familienstruktur durch eine $\mathrm{U}(1)_{\widehat{Q}}$-Familiensymmetrie einzuschränken betten wir den Teilcheninhalt in eine SU(5) große vereinheitlichte Theorie ein. In diesem Modell berechnen wir ein mögliches Signal von zerfallender dunkler Materie, sowie den dominanten Zerfallskanal eines möglichen zweitleichtesten supersymmetrischen Teilchens, des skalaren τ-Leptons. Durch Vergleich unserer Rechnung für das Gravitino mit den Ergebnissen des Fermi Large Area Telescope können wir eine untere Schranke für die Zerfallslänge des skalaren τ-Leptons in Beschleunigerexperimenten vorhersagen.

Anna, Anton, Beate, Milena und Miro gewidmet

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Introduction

The *Standard Model* (SM) of particle physics describes the behavior of high energy physics up to the electroweak scale very well [1, 2]. Up to now all particles predicted by the SM, except for the Higgs boson, are found and most parameters it depends on are measured to a high degree of accuracy. Hence it is expected that the Higgs boson will be found in the near future at the *Large Hadron Collider* (LHC). In the SM fermions couple to the *Vacuum Expectation Value* (VEV) of the Higgs boson in order to acquire mass.

However, despite all its successes, it is a well-established fact that the SM must be extended in order to have a full description of nature. One reason is that neutrinos are known to have a tiny mass, as is required to account for neutrino oscillations [3]. This small left-handed neutrino masses can be explained by introducing super-heavy right-handed Majorana neutrino singlets that generate the light neutrino masses via the seesaw mechanism [4, 5]. Furthermore the SM suffers from the hierarchy problem, namely the observation that the Higgs boson is 16 orders of magnitude lighter than the fundamental Planck scale, despite dominant ultraviolet contributions at loop level [6].

The hierarchy problem can be circumvented by imposing *Supersymmetry* (SUSY), which doubles the particle content by introducing new supersymmetric particles whose spins differ by one half from the known particles and which cancel the large loop contributions [6] to the mass of the Higgs boson. In this thesis we consider local SUSY, *i.e.* *Supergravity* (SUGRA). It inevitably predicts the existence of a spin-$3/2$ particle, the gravitino, as the supersymmetric partner of the gauge boson of gravity, the graviton [7, 8].

With imposed SUSY new problems emerge, for instance matter instability and the μ-problem [6]. The latter is a naturalness problem or, in other words, a moderate hierarchy problem, but it is not the topic of this thesis. Matter stability, on the other hand, can be assured by introducing R-parity [9]. However, this symmetry is not governed by a fundamental mechanism and consequently we may take a different approach.

The supersymmetric low-energy mass spectrum is unknown and determined by the chosen SUSY breaking model. Hence, there are numerous different possibilities for the *Lightest Supersymmetric Particle* (LSP), for instance the scalar τ-lepton, the neutralino or the scalar neutrino. Provided the LSP is uncharged, it could amount, on cosmic scales, to the observed *Dark Matter* (DM) density which is crucial to the SM of cosmology [10].

Consistent cosmology needs in addition an inflationary phase, caused by an scalar inflaton field [11]. The decay of the inflaton field can lead to reheating [12]. If one explains the observed baryon asymmetry via leptogenesis one has to deal with a high reheating temperature [13]. This constrains the low-energy mass spectrum. Since the gravitino decays are suppressed by the Planck mass, the late decays of the gravitino into lighter supersymmetric particles spoil *Big Bang Nuclesynthesis* (BBN) predictions [14]. Hence the gravitino must be the LSP, but then the *Next-to-Lightest Supersymmetric Particle* (NLSP) can only decay into the gravitino. This decay, however, is as well suppressed by the Planck mass, hence it spoils BBN predictions likewise [14].

Therefore, one has to open new decay channels. Here we focus on the possibility of R-parity violating couplings and take the experimental constraints as bounds to these couplings. As a side effect these couplings lead to a tiny neutrino mass, without considering right-handed neutrinos [15].

Another approach to extend the SM are *Grand Unified Theories* (GUTs) [16, 17]. In GUTs the SM gauge group is embedded into a larger simple symmetry group, for instance, SU(5) or SO(10) [18, 19]. In supersymmetric GUTs, as opposed to the SM, the gauge couplings approximately unify [1] at the GUT scale. This is a strong indication that both extensions of the SM go along. In GUTs the particle content of the SM is combined in different representations of the larger gauge group.

Supersymmetric GUTs are easily implemented in heterotic string models. These Orbifold GUTs often predict a $\mathrm{U}(1)_{\widehat{Q}}$ Froggatt–Nielsen flavor symmetry. The $\mathrm{U}(1)_{\widehat{Q}}$ charges of representations containing the SM are constrained by, for instance, the CKM-matrix. In models which involve leptogenesis the charges of the right-handed neutrinos can be constrained as well [20].

As explained above, cosmology accommodating leptogenesis and gravitinos requires broken R-parity. Once the $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry is implemented, one is able to build an R-parity violating supersymmetric model, depending on the usual superpotential and SUSY breaking Lagrangian as well as two small parameters, one describing the amount of R-parity breaking and the other one describing the flavor hierarchy of all couplings [21]. Every parameter has to be defined at the GUT scale and run down to the weak scale, where the measurable parameters have to be fitted to observations in order to have a consistent model.

Hence the question arises what the vacuum of R-parity breaking SUSY including right-handed neutrino superfields is. Although the R-parity conserving *Minimal Supersymmetric Standard Model* (MSSM) is very well-known, already for the R-parity breaking case one finds contradicting formulas in the literature. The R-parity breaking theory with right-handed neutrino superfields is even less investigated. For instance, both, the right-handed neutrino and R-parity violation, generate neutrino mass. However, to our knowledge a formula were both mass generation mechanisms are combined is unknown.

Furthermore, we are interested in finding a supersymmetric R-parity breaking model including $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry, which is compatible with the SM of particle physics as well as the SM of cosmology including leptogenesis and BBN. We would like to have a DM candidate as well as an observable signal of the NLSP in detectors. Furthermore, we are looking for a possibility to constrain the predictions for observations in LHC detectors via cosmological observations.

In Chapter 1 we give a rough overview about the theoretical foundations needed in this thesis. We start in Section 1.1 with the SM and its limitations. In Section 1.2 we give a rough overview of SUSY including SUGRA, R-parity breaking, and SUSY breaking. In Section 1.3 we introduce the Froggatt–Nielsen flavor symmetry. We finish the theoretical overview in Section 1.4 with a recollection of the theory of renormalization group equations.

In Chapter 2 we define the superfields and couplings of the MSSM including the right-handed neutrino superfield as well as R-parity violating couplings. In Section 2.1 we introduce the chiral superfields and the superpotentials belonging to these two extensions of the MSSM. In Section 2.2 we describe the rotation freedom of the R-parity breaking Lagrangian and lay the foundations for bilinear R-parity breaking. In Section 2.3 we introduce vector superfields. Finally, we characterize in Section 2.4 the soft SUSY breaking Lagrangian.

In Chapter 3 we evaluate the field mixing in the neutral sector, depending on the couplings and fields chosen. In Section 3.1 we start with the scalar bosons and concentrate in particular on the VEVs, as we expect to clarify the confusion about the formula in the R-parity breaking case. In Section 3.2 we recollect the formulas for the vector bosons in the SM. We come finally in Section 3.3 to the field mixing of the fermions, were we are, in particular, interested in the mass generation mechanisms for neutrinos.

In Chapter 4 we study the constraints on R-parity breaking and introduce bilinear R-parity breaking as a possible solution. In Section 4.1 we examine the bounds on general R-parity breaking due to proton decay. In Section 4.2 we are gathering the constraints on R-parity violation due to cosmology. In Section 4.3 we introduce bilinear R-parity breaking which is in accordance with the constraints mentioned before. Finally, we calculate the gravitino decay width in Section 4.4, as well as the scalar τ-lepton decay width in Section 4.5.

In Chapter 5 we eventually constrain the flavor structure with a $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry. In Section 5.1 we constrain the family structure according to an SU(5) GUT. We derive the flavor dependence of the superpotential parameter in Section 5.2, as well as the dependence of the soft SUSY breaking parameters in Section 5.3. In Section 5.4 we calculate the flavor dependence of the rotated bilinear breaking parameters. In Section 5.5 we finally constrain the family structure of the scalar τ-decay.

Chapter 1

Theoretical Foundation

1.1 The Standard Model and Beyond

The SM of particle physics describes all known phenomena up to the electroweak scale $M_{\text{ew}} = \mathcal{O}(100 \text{ GeV})$ [2]. However, it is expected that new physics occurs at higher scales; definitely at the Planck scale

$$M_P = (8\pi G_{\text{Newton}})^{-1/2} = 2.4 \times 10^{18}\,\text{GeV}\,. \tag{1.1}$$

The large ratio between these two scales give rise to the so called hierarchy problem. In particular the Higgs is rather sensitive to physical effects associated with higher energy via loop corrections. Thus the mass generation mechanism for the SM particles is influenced by this effect. However, one can calculate that the most severe divergences from boson and fermion loops cancel if there is a symmetry which relates these two contributions. Supersymmetry is such a symmetry and since its generator Q connects bosons with fermions [6]

$$Q|\text{boson}\rangle = |\text{fermion}\rangle\,, \qquad Q|\text{fermion}\rangle = |\text{boson}\rangle\,, \tag{1.2}$$

it is fermionic itself. A theory that connects the Poincaré algebra with inner symmetries was formerly ruled out by the Coleman-Mandula theorem [22] which states that for a realistic theory there can be no nontrivial symmetry algebra apart from the Poincaré algebra and the internal symmetry algebra. However, as shown by Haag, Łopuszański and Sohnius [23] this can be circumvented by using a graded Lie algebra. The Poincaré algebra can be extended with the SUSY algebra. The anticommuting generators Q form the (anti)-commutators

$$\left[Q, Q^\dagger\right]_+ = -2\sigma^\mu P_\mu\,, \qquad [Q, P^\mu] = 0\,, \qquad \left[Q^\dagger, P^\mu\right] = 0\,, \tag{1.3}$$

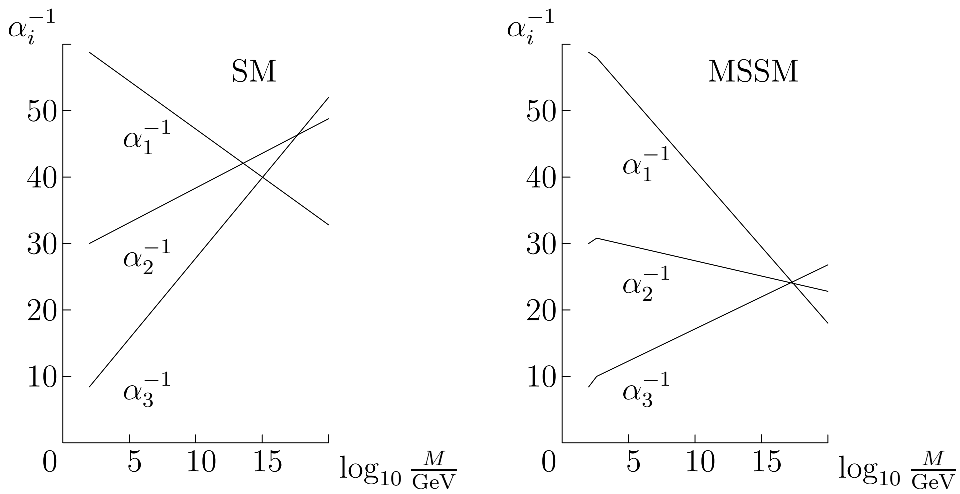

Figure 1.1: Gauge coupling running in the SM (left) and in the MSSM (right) [1]. The gauge coupling $g_{(a)}$ is parametrized as fine structure constant $\alpha_{(a)} = (4\pi)^{-1} g_{(a)}^2$.

involving the generator of translations, the momentum four vector P^μ. The Pauli matrices σ^μ are defined in Appendix B.1.

Additionally, supersymmetric models can be phenomenologically interesting, given that in many cases it turns out that a viable DM candidate can be found in the supersymmetric particle spectrum.

The running gauge coupling constants $g_{(a)}$ of the SM gauge group,

$$G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \ , \tag{1.4}$$

calculated using the *renormalization group equations* (RGE) involving only the SM particle content, do not intersect in a single point. However, with the (MSSM) particle content the coupling constants unify approximately at $M_{\mathrm{GUT}} \simeq 10^{16}$ GeV [1] as can be calculated with the β-function (E.1). This different high energy behavior of the three gauge coupling constants g_a is shown in Figure 1.1, the index (a) runs over the three factors of the SM gauge group, and g_1 is taken in the unification normalization. This unification is a strong evidence that SUSY is consistent with GUTs. In GUTs the SM gauge group is embedded into a simple group with only one gauge coupling constant. The field content of the SM plus the right-handed neutrino superfields can be found in the following representations of the SU(5) [18]

$$\mathbf{10}_i = (Q_i, {U^c}_i, {E^c}_i) \ , \qquad \overline{\mathbf{5}}_i = ({D^c}_i, L_i) \ , \qquad \mathbf{1}_i = {N^c}_i \ , \tag{1.5}$$

where i is the family index. Another possibility are three copies of the $\mathbf{16} = \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$ multiplet of the SO(10) containing the SU(5) multiplets [24].

As one can see, the inclusion of the right-handed neutrino singlet is theoretically well motivated by SO(10) GUT. Experimentally one finds that the left-handed neutrino mass is very small but not negligible, as can be observed from neutrino oscillations [25, 26]. The right-handed neutrino together with the seesaw mechanism provides an elegant way to give such small masses to the left-handed neutrinos [4, 5]. The addition of the right-handed neutrino adds lepton number breaking terms, which can be used to explain the baryon asymmetry in our universe via thermal leptogenesis [13] as briefly explained in Section 4.2.

1.2 Supersymmetry

In SUSY every SM particle obtains a supersymmetric partner, whose spin S differs by $\Delta S = 1/2$ from the SM particle. The supersymmetric particle spectrum must be jointly shifted to higher masses via SUSY breaking.

Chiral fermions are described as two component Weyl spinors. The details of the notation can be found in Appendix B.1 and, for instance, in [27]. A Lagrangian containing Weyl fermions can be written down in such a way that it contains only left-handed fields. The right-handed fields are included with the help of charge conjugation of left-handed fields as can be seen in the representations (1.5).

1.2.1 Global Supersymmetry

The chiral superfield Φ_i is an object that contains a Weyl fermion χ_i, a complex scalar field ϕ_i, and a complex scalar auxiliary field F_i. The index i runs over the whole collection of fields, in particular the family index is included. The component fields are connected via the fermionic super transformation ξ. After the elimination of the auxiliary field via its equation of motion, the SUSY Lagrangian of the chiral multiplet is

$$\begin{aligned} -\mathcal{L}_{\text{chiral}} &= \partial^\mu \phi^{i*} \partial_\mu \phi_i - i{\chi^i}^\dagger \bar{\sigma}^\mu \partial_\mu \chi_i \\ &\quad + \tfrac{1}{2}\left(W^{ij} \chi_i \chi_j + {W_{ij}}^* {\chi^i}^\dagger {\chi^j}^\dagger \right) + W^i {W_i}^* \,, \end{aligned} \tag{1.6}$$

where the superpotential W is defined by

$$W = L^i \Phi_i + \tfrac{1}{2} M^{ij} \Phi_i \Phi_j + \tfrac{1}{6} y^{ijk} \Phi_i \Phi_j \Phi_k \,. \tag{1.7}$$

Here the trilinear term y^{ijk} is a Yukawa coupling, the bilinear term M^{ij} is a mass matrix and the linear term L^i is not needed in our model, but is nonetheless for gauge singlets allowed. The indices on the superpotential in the Lagrangian (1.6) imply a derivative with respect to the scalar fields. The part of the Lagrangian containing the superpotential contribution gives the scalar potential for the chiral multiplet.

The Lagrangian (1.6) is invariant under the phase change

$$\Phi_i \rightarrow \Phi_i \exp\left(iq_i\alpha\right) . \tag{1.8}$$

Making this phase change local $\alpha = \alpha(x)$ requires the introduction of the vector superfield, containing the vector boson A^a_μ, the fermionic gaugino λ^a, and a real bosonic auxiliary field D^a. Here the group index a runs over the adjoint representation of the gauge group. The Lagrangian of the vector superfield components is

$$-\mathcal{L}_{\text{gauge}} = \tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{a\dagger}\bar{\sigma}^\mu D_\mu\lambda^a - \tfrac{1}{2}D^aD^a , \tag{1.9}$$

where $F_{\mu\nu}$ is the usual field strength of A_μ and the auxiliary field has the trivial equation of motion

$$D^a = -g_{(a)}\left(\phi^{i\dagger}(T^a)^j_i\phi_j\right) , \tag{1.10}$$

where T^a are the representation matrices of the gauge groups. In order to couple these two supermultiplets in a gaugeinvariant and supersymmetric way one has to replace the derivative in (1.6) with the covariant derivative used in the Lagrangian (1.9)

$$D_\mu = \partial_\mu - ig_{(a)}A^a_\mu T^a , \tag{1.11}$$

and add the terms

$$-\mathcal{L}_{\text{mix}} = \sqrt{2}g_{(a)}\phi^{i*}(T^a)^j_i\chi_j\lambda^a + \sqrt{2}g_{(a)}\lambda^{a\dagger}\chi^{i\dagger}(T^a)^j_i\phi_j + D^aD^a \tag{1.12}$$

to the SUSY Lagrangian.

The chiral Lagrangian (1.6) forms together with the gauge Lagrangian (1.9) and the mixing Lagrangian (1.12) the complete Lagrangian for global SUSY. Via the Noether procedure one gets the supercurrent of this Lagrangian

$$J^\mu = \sigma^\nu\overline{\sigma}^\mu\chi_i D_\nu\phi^{i*} - i\sigma^\mu\chi^{i\dagger}F_i - \tfrac{1}{2\sqrt{2}}\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^{a\dagger}F^a_{\nu\rho} - \tfrac{i}{\sqrt{2}}D^a\sigma^\mu\lambda^{a\dagger} , \tag{1.13}$$

which is used in local SUSY.

graviton	gravitino	$(\mathrm{SU}(3)_C, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$
$g_{\mu\nu}$	ψ_μ	$(\mathbf{1},\mathbf{1})_0$

Table 1.1: The gravity supermultiplet.

1.2.2 Local Supersymmetry – Supergravity

In order to promote SUSY to a local symmetry $\xi = \xi(x)$ one has to explicitly express the metric tensor in the Lagrangian [8]. After quantization the metric becomes a bosonic spin two field called the graviton,

$$g_{\mu\nu} = \eta_{mn} e_\mu^m e_\nu^n \; . \tag{1.14}$$

Where e_μ^m is the vierbein and $\eta_{mn} = \mathrm{diag}\,(1,-1,-1,-1)$ is the metric tensor of the flat spacetime. In this section $\mu, \nu, \ldots$ denote curved spacetime indices on the manifold and $m, n, \ldots$ denote flat spacetime indices on the tangential space.

The kinetic term for the graviton is the Einstein Hilbert Lagrangian [28]

$$-e^{-1}\mathcal{L}_{\mathrm{EH}} = \tfrac{1}{2} M_P^2 R \; . \tag{1.15}$$

It consists of the contracted Riemann curvature tensor called the Ricci-scalar R and the determinant e of the vierbein e_μ^m.

The off-shell gravitational multiplet contains, except for the spin-2 graviton, the gravitino ψ_μ, a spin $3/2$ particle, as well as two bosonic auxiliary fields M and b^μ. The latter two can be integrated out [29] and the two remaining physical fields are written down in Table 1.1.

The kinetic term for the gravitino is the Rarita–Schwinger Lagrangian [30]

$$e^{-1}\mathcal{L}_{\mathrm{RS}} = \epsilon^{\mu\nu\rho\sigma} {\psi_\mu}^\dagger \overline{\sigma}_\nu \nabla_\rho \psi_\sigma \; . \tag{1.16}$$

Here $\epsilon^{\mu\nu\rho\sigma}$ follows from the four-dimensional Levi–Civita symbol by contracting each flat spacetime index with a vierbein. ∇_ρ is a Lorentz covariant derivative.

The full supergravity Lagrangian is rather lengthy, incorporating, besides general relativity, the global SUSY Lagrangian and new characteristic supergravity terms. As supergravity is non-renormalizable, parts of its Lagrangian depend on the cut-off scale M_P.

The scalar fields of supergravity form a Kähler manifold, which is described by the Kähler potential K. Ultimately the supergravity Lagrangian depends only on two functions. The first is the Kähler function G which is a function of the superpotential W and the Kähler potential K

$$G(\phi, \phi^*) = M_P^{-2} K(\phi, \phi^*) + \ln \left(M_P^{-6} \left| W(\phi) \right|^2 \right) . \tag{1.17}$$

Its second derivative g_{ij^*}, called the Kähler metric, modifies mainly the chiral multiplet part of the SUGRA Lagrangian. The second dependency is the gauge kinetic function f_{ab} which modifies mainly the vector multiplet part of the SUGRA Lagrangian.

Later we are interested in the coupling of the gravitino to the supercurrent of global SUSY (1.13), which describes the gravitino coupling to the particles of the other multiplets. The important interaction terms for this thesis are [31]

$$\begin{aligned} -\mathcal{L}_\psi^J = {} & \tfrac{1}{\sqrt{2}} \kappa g_{ij^*} \mathcal{D}_\nu \phi^{j^*} \chi^i \sigma^\mu \overline{\sigma}^\nu \psi_\mu + \psi_\mu \sigma^\mu {\chi^i}^\dagger {\lambda^a}^\dagger {\lambda^b}^\dagger + \tfrac{i}{4} \kappa \psi_\mu \sigma^{\nu\rho} \sigma^\mu {\lambda_a}^\dagger F_{\mu\nu}^a \\ & + \tfrac{1}{\sqrt{2}} \kappa g_{ij^*} \mathcal{D}_\nu \phi^j {\chi^i}^\dagger \overline{\sigma}^\mu \sigma^\nu {\psi_\mu}^\dagger + \tfrac{i}{4} \kappa {\psi_\mu}^\dagger \overline{\sigma}^{\nu\rho} \overline{\sigma}^\mu \lambda_a F_{\mu\nu}^a + \ldots . \end{aligned} \tag{1.18}$$

Since we are interested in field theories in Minkowski space-time, we require the cosmological constant to vanish. This happens for [32]

$$\langle V \rangle = \left\langle F^i g_{ij^*} F^{*j^*} + \tfrac{1}{2} g^2 D^a D^a - \tfrac{1}{3} M M^* \right\rangle = 0 . \tag{1.19}$$

Therefore we can neglect the interactions of the graviton, ergo we can analyze the limit $R \to 0$ and $e \to 1$. This is the so-called flat limit. The distinction between the curved spacetime indices and the flat spacetime indices vanishes. If D^a or F^i have a nonvanishing VEV, as is required in order to break SUSY (see Section 1.2.5), M must have a nonvanishing VEV as well to fulfill the requirement in Equation (1.19).

1.2.3 Super Higgs Mechanism

The part of the supergravity Lagrangian giving the gravitino mass consists of terms which are quadratic in the fermionic fields [33]

$$\begin{aligned} -\mathcal{L}_F^{(2)} = {} & M_P \left(\psi_\mu \sigma^{\mu\nu} \psi_\nu + {\psi_\mu}^\dagger \overline{\sigma}^{\mu\nu} {\psi_\nu}^\dagger \right) \exp \tfrac{1}{2} G \\ & + \tfrac{1}{2} M_P^{-1} g D^a \left({\psi_\mu}^\dagger \overline{\sigma}^\mu \lambda^a - \psi_\mu \sigma^\mu {\lambda^a}^\dagger \right) \\ & + \tfrac{i}{\sqrt{2}} M_P \left(G_i \chi^i \sigma^\mu {\psi_\mu}^\dagger + G_{i^*} {\chi^i}^\dagger \overline{\sigma}^\mu \psi^\mu \right) \exp \tfrac{1}{2} G \\ & + \tfrac{1}{2} M_P \left(\left(G_{ij} + G_i G_j \right) \chi^i \chi^j + \left(G_{i^* j^*} + G_{i^*} G_{j^*} \right) {\chi^i}^\dagger {\chi^j}^\dagger \right) \exp \tfrac{1}{2} G \\ & + \sqrt{2} i M_P^{-1} g \left(D_i^a \chi^i \lambda^a - D_{i^*}^a {\chi^i}^\dagger {\lambda^a}^\dagger \right) + \ldots . \end{aligned} \tag{1.20}$$

One observes that the gravitino mixes with fermions and gauginos if D^a or F^i have a nonvanishing VEV. However, shifting the gravitino field by

$$\eta^\dagger = \frac{i}{\sqrt{2}} G_{i^*} \chi^{i\dagger} + \tfrac{1}{2} M_P^{-2} g D^a \lambda^{a\dagger} \exp\left(-\tfrac{1}{2}G\right) \tag{1.21}$$

eliminates the mixing, therefore this part of the Lagrangian transforms into

$$\begin{aligned} -\mathcal{L}_F^{(2)} = {} & M_P \left(\psi_\mu + \tfrac{1}{3}\eta^\dagger \overline{\sigma}_\mu\right) \sigma^{\mu\nu} \left(\psi_\nu - \tfrac{1}{3}\sigma_\nu \eta^\dagger\right) \exp \tfrac{1}{2}G \\ & + \tfrac{1}{6} M_P^{-3} g^2 D^a D^b \lambda^{a\dagger} \lambda^{b\dagger} \exp\left(-\tfrac{1}{2}G\right) \\ & + \tfrac{1}{2} M_P \left(G_{i^* j^*} + \tfrac{1}{3} G_{i^*} G_{j^*}\right) \chi^{i\dagger} \chi^{j\dagger} \exp \tfrac{1}{2}G \\ & + \sqrt{2} i M_P^{-1} g \left(\tfrac{1}{3} G_{i^*} D^a - D^a_{i^*}\right) \chi^{i\dagger} \lambda^{a\dagger} + \text{h.c.} + \ldots \,. \end{aligned} \tag{1.22}$$

Now the gravitino mass can be directly read off to be

$$m_{3/2} = \tfrac{1}{3} M_P^{-1} \langle M \rangle = M_P^{-2} \langle W \rangle \exp \tfrac{1}{2} M_P^{-2} \langle K \rangle = M_P \exp \tfrac{1}{2} \langle G \rangle \,. \tag{1.23}$$

The remaining Lagrangian gives masses to the spin $1/2$ fermions.

The field η is a fermionic massless eigenstate of the massmatrix, hence it is a goldstino component. The gravitino field absorbs the goldstino fermion field and acquires a nonvanishing mass. As this mechanism is similar to the Higgs mechanism but includes a spin one half goldstino fermion, it is called super Higgs mechanism. As the graviton stays massless, the gravitino mass is directly related to the SUSY breaking scale.

1.2.4 *R*-Parity

Although it is not governed by an underlying symmetry, the SM does preserve lepton number L and Baryon number B, while the most general supersymmetric Lagrangian does not preserve these quantum numbers. This can introduce some difficulties in model building described in the first two sections of Chapter 4, among them fast proton decay. These problems can be solved by imposing a new symmetry, the most common one is R-parity. For particles with spin S it can be expressed by

$$R_p = (-1)^{3(B-L)+2S} \,. \tag{1.24}$$

It follows that SM particles have even R-parity quantum numbers $R_p = +1$, while the supersymmetric particles have odd R-parity quantum numbers $R_p = -1$. With imposed R-parity the LSP is absolutely stable, thus it is a viable DM candidate [6] provided it is uncharged. If one wants to break R-parity [34], one has to respect experimental bounds. Some are listed in Section 4.2. From these bounds follows that the R-parity violating couplings must be small. A comprehensive introduction into the R-parity violating theory and its constraints is given in [9].

1.2.5 Supersymmetry Breaking

As no supersymmetric particles have been discovered yet, SUSY must be broken. A SUSY breaking mechanism must shift all mass terms of supersymmetric particles to higher energies, preferably in the TeV range. In order to solve the hierarchy problem, these SUSY breaking parameters must be soft. That means that they cannot reintroduce quadratic divergences. As this mechanism cannot occur in the known visible sector of the particle spectrum, it must happen in some hidden sector, containing only particles which couple very weakly to the MSSM [6].

Depending on the breaking mechanism a hidden auxiliary field develops a VEV and causes soft SUSY breaking terms. Consequently this hidden sector VEV causes the gravitino mass (1.23) as well.

As the exact breaking scheme is not known, all allowed terms are taken into account and are treated as free parameters of the theory. They are either of the form of the superpotential terms

$$-\mathcal{L}_{\text{soft}}^{\text{W}} = \tfrac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \tfrac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i + \text{h.c.}\ , \tag{1.25}$$

or are mass terms for the supersymmetric particles

$$-\mathcal{L}_{\text{soft}}^{\text{mass}} = (m^2)^i_j\phi^{j*}\phi_i + \tfrac{1}{2}M_{(a)}\lambda^a\lambda^a + \text{h.c},\ . \tag{1.26}$$

There is another term that is possible in soft SUSY breaking, but it causes quadratic divergences when one includes a chiral multiplet that is a singlet under all gauge symmetries [6]. Hence we are neglecting this term as most authors are doing.

1.3 U(1) Flavor Symmetry

The observed mass hierarchy in the fermion sector, as can be read off of Table A.1, is striking and can be parametrized via a small parameter:

$$\begin{aligned} m_t : m_c : m_u &\approx 1 : \eta^2 : \eta^4 \\ m_b : m_s : m_d &\approx 1 : \eta : \eta^3 \\ m_\tau : m_\mu : m_e &\approx 1 : \eta : \eta^3\ , \end{aligned} \tag{1.27}$$

where η is roughly $\frac{1}{16}$. Connected with the mass hierarchy is the pattern of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, whose definition can be found in Appendix A.1. Its pattern can be roughly parametrized by [35, 36]

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & \eta & \eta^2 \\ \eta & 1 & \eta \\ \eta^2 & \eta & 1 \end{pmatrix}\ . \tag{1.28}$$

Order one parameter have been neglect in this proximate approach, the largest of them is approximately 4. This hierarchy must be explained in the Yukawa matrices but is so far a experimental input for the theory.

One possible explanation is the Froggatt-Nielsen U(1) flavor symmetry [37]. For that we need a single almost conserved Abelian integrally quantized charge $\widehat{Q}$. The theory contains many so far unknown heavy fundamental fermions. The known basic fermions are massless in the limit of exact $\widehat{Q}$ conservation. The heavy particle mass is generated by a VEV $\langle\phi_0\rangle$ of a neutral $\widehat{Q} = 0$ Higgs scalar. The symmetry breaking mechanism responsible for the masses of the light fermions is provided by the VEV $\langle\phi_1\rangle$ of a charged $\widehat{Q} = 1$ Higgs boson. The symmetry breaking parameter is

$$\eta = \frac{g(\mu)}{g(\mu_0)} \frac{\langle\phi_1\rangle}{\langle\phi_0\rangle} \ . \tag{1.29}$$

Here $g(\mu)$ is any running ϕ_1 Yukawa coupling constant at the low energy scale. Electroweak symmetry is broken by $\langle\phi_2\rangle$ with $\widehat{Q} = 0$. The Yukawa couplings are of order unity and random at the fundamental scale μ_0. The charge for the left-handed fields is $\widehat{Q}_{L_j} = c + b_j$ while it is $\widehat{Q}_{R_i} = c - a_i$ for the right-handed fields. For the mass matrix it follows that

$$M_{ij} = g_{ij}\eta^{a_i+b_j} \ , \tag{1.30}$$

where the coefficients g_{ij} are complex numbers of order unity. The mass ratio is to leading order

$$\frac{m_i}{m_j} = \eta^{a_i-a_j+b_i-b_j} \ . \tag{1.31}$$

Finally the CKM matrix is parametrized by

$$V_{ij} \sim \eta^{|b_i-b_j|} \ , \tag{1.32}$$

where order one coefficients are neglected.

1.4 Renormalization Group

The renormalization group is an extremely powerful mechanism to determine the behavior of a theory, known at a specific scale, at a completely different scale. For an introduction see, for instance [2].

In the following we consider a general $N = 1$ supersymmetric Yang-Mills model. The chiral multiplet transforms as a representation R of the gauge group G. The representation matrices $T^a = (T^a)_i^j$ for the gauge group G

form the quadratic Casimir invariant $C(R)$ and the Dynkin index $S(R)$ of a representation R in a way that

$$\begin{aligned}(T^a T^a)^i_j &= C(R)\delta^i_j \\ \mathrm{tr}_R\left(T^a T^b\right) &= S(R)\delta^{ab} \ .\end{aligned} \tag{1.33}$$

As dimensional regularization (DREG) with minimal subtraction $\overline{\text{MS}}$ violates SUSY explicitly, the following is calculated in the dimensional reduction (DRED) with modified minimal subtraction $\overline{\text{DR}}$ [38].

The β function depends in general on all orders of perturbation theory

$$\beta = \frac{1}{(4\pi)^2}\beta^{(1)} + \frac{1}{(4\pi)^4}\beta^{(2)} + \dots \ , \tag{1.34}$$

but in this work just the one loop evaluation is used. The RGE for the gauge couplings is

$$\frac{d}{dt}g_{(a)} = \beta_g \qquad \text{with} \qquad \beta_g^{(1)} = g_{(a)}^3\left(S_{(a)}(R) - 3C(G_{(a)})\right) \ , \tag{1.35}$$

where $t = \log q^2$ and q is the energy. Here the gauge coupling for the $\mathrm{U}(1)_Y$ is normalized for GUTs, that means $g_1 = \sqrt{3/5}\, g_Y$. The RGE for the gaugino mass parameter is

$$\frac{d}{dt}M_{(a)} = \beta_M \qquad \text{with} \qquad \beta_M^{(1)} = g_{(a)}^2\left(2S_{(a)}(R) - 6C(G_{(a)})\right)M_{(a)} \ . \tag{1.36}$$

The β-functions for the superpotential parameters are

$$\begin{aligned}\frac{d}{dt}y^{ijk} &= y^{ijp}\gamma_p^k + (k \leftrightarrow i) + (k \leftrightarrow j) \ , \\ \frac{d}{dt}M^{ij} &= M^{ip}\gamma_p^j + (j \leftrightarrow i) \ , \\ \frac{d}{dt}L^i &= L^p\gamma_p^i \ ,\end{aligned} \tag{1.37}$$

where the anomalous dimension depends on all orders of perturbation theory

$$\gamma = \frac{1}{(4\pi)^2}\gamma^{(1)} + \frac{1}{(4\pi)^4}\gamma^{(2)} + \dots \tag{1.38}$$

and the one loop expression is given by

$$\gamma^{(1)}{}_i^j = \tfrac{1}{2}y_{ipq}{}^* y^{jpq} - 2\delta_i^j \sum_a g_a^2 C_a(i) \ . \tag{1.39}$$

The β functions for the soft SUSY breaking parameters follow a similar scheme and can be found together with the explicit formulas for the RGEs in the R-parity conserving theory in [38] and for the R-parity violating theory in [39]. The explicit formulas for the MSSM with G_{SM} and R-parity violation are noted in Appendix E.

Chapter 2

Supersymmetric Standard Models

So far, the chiral multiplet, the vector multiplet and the gravitational multiplet have been introduced. In Section 2.1 we give the minimal field content needed for the chiral multiplet to incorporate the particle content of the SM, taking also the right-handed neutrino superfield into account. In addition we give the minimal set of supersymmetric interactions, as well as the extensions needed to have R-parity violation and right-handed neutrino superfields. In Section 2.2 we observe the ambiguity of the Higgs Lepton doublets, introduce an adjusted notation and lay the foundations for the bilinear R-parity breaking. In Section 2.3 we give the field content needed for the vector multiplet in order to extended the Standard Model with supersymmetry. Finally we present in Section 2.4 all soft SUSY breaking terms that can occur depending on the interactions chosen in Section 2.1.

2.1 Superpotential and Chiral Superfields

The chiral multiplets of the minimal supersymmetric extension of the SM plus a right-handed neutrino singlet are given in Table 2.1. The standard naming scheme is introduce in Appendix B. These fields can be grouped into SU(5) multiplets as expressed in Equation (1.5).

The most general R-parity violating MSSM superpotential with right-handed neutrino superfields can be written down in the language of a super-

superfields Φ_i	scalars ϕ_i	fermions χ_i	$(\mathrm{SU}(3)_C, \mathrm{SU}(2)_L)_Y$	Q
$Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}$	$\widetilde{Q}_i = \begin{pmatrix} \widetilde{u}_{Li} \\ \widetilde{d}_{Li} \end{pmatrix}$	$\begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$	$(\mathbf{3}, \mathbf{2})_{1/6}$	$\begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$
${U^c}_i$	$\widetilde{u}^c{}_i$	${u^c}_i$	$(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$	$-2/3$
${D^c}_i$	$\widetilde{d}^c{}_i$	${d^c}_i$	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	$1/3$
$L_i = \begin{pmatrix} N_i \\ E_i \end{pmatrix}$	$\widetilde{L}_i = \begin{pmatrix} \widetilde{\nu}_{Li} \\ \widetilde{l}_{Li} \end{pmatrix}$	$\begin{pmatrix} \nu_{Li} \\ l_{Li} \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
${E^c}_i$	$\widetilde{l}^c{}_i$	${l^c}_i$	$(\mathbf{1}, \mathbf{1})_1$	1
${N^c}_i$	$\widetilde{\nu}^c{}_i$	${\nu^c}_i$	$(\mathbf{1}, \mathbf{1})_0$	0
$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	$\widetilde{h}_u = \begin{pmatrix} \widetilde{h}_u^+ \\ \widetilde{h}_u^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{1/2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	$\widetilde{h}_d = \begin{pmatrix} \widetilde{h}_d^0 \\ \widetilde{h}_d^- \end{pmatrix}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Table 2.1: Chiral supermultiplets of the MSSM plus a right-handed neutrino supermultiplet and quantum numbers thereof.

symmetric SU(5) GUT as

$$\begin{aligned} W = {} & \mu H_u(\mathbf{5}) H_d(\overline{\mathbf{5}}) + \mu_i H_u(\mathbf{5})\overline{\mathbf{5}} - \lambda^u_{ij} H_u(\mathbf{5})\mathbf{10}_i\mathbf{10}_j + \lambda^e_{ij} H_d(\overline{\mathbf{5}})\overline{\mathbf{5}}_i\mathbf{10}_j \\ & + \tfrac{1}{2}\lambda_{ijk}\overline{\mathbf{5}}_i\overline{\mathbf{5}}_j\mathbf{10}_k - \lambda^\nu_{ij} H_u(\mathbf{5})\overline{\mathbf{5}}_i\mathbf{1}_j - \lambda^\nu_i H_u(\mathbf{5}) H_d(\overline{\mathbf{5}})\mathbf{1}_i \\ & + \lambda^S_{ij} S(\mathbf{1})\mathbf{1}_i\mathbf{1}_j \ . \end{aligned} \tag{2.1}$$

The VEVs of the Higgs doublets H_u and H_d, which are in the $\mathbf{5}$-plet and the $\overline{\mathbf{5}}$-plet of SU(5), give masses to the SM superfields and the VEV of the singlet Higgs S generates the Majorana mass matrix for right-handed neutrino superfields. The Yukawa coupling λ_{ijk} is antisymmetric under the exchange of $\overline{\mathbf{5}}_i$ and $\overline{\mathbf{5}}_j$, thus it is convenient to introduce a factor of one half.

Translating the superpotential (2.1) into the SM group notation gives for the R-parity conserving superpotential

$$W_{R_p} = \mu H_u H_d + \lambda^e_{ij} H_d L_i {E^c}_j + \lambda^d_{ij} H_d {D^c}_i Q_j - \lambda^u_{ij} H_u Q_i {U^c}_j \ . \tag{2.2}$$

The terms above will give the Lagrangian of the MSSM. This is the minimal set of fields and couplings to make the SM supersymmetric. Due to the embedding into the SU(5) our definition of the coupling λ^d_{ij} is transposed compared to the standard literature, for instance, [6].

The R-parity violating counterpart of the MSSM superpotential is

$$W_{\not{R}_p} = \mu_i H_u L_i + \tfrac{1}{2}\lambda_{ijk} L_i L_j {E^c}_k + \lambda'_{ijk} L_i {D^c}_j Q_k + \tfrac{1}{2}\lambda''_{ijk} {D^c}_i {D^c}_j {U^c}_k \ . \tag{2.3}$$

The couplings λ_{ijk} and λ''_{ijk} are antisymmetric in their first two indices. The first three couplings violate lepton number the last one violates baryon number. Hence R-parity breaking couplings are strongly constrained by observation as shown in Section 4.1 and 4.2.

Compared to the standard literature, for instance [9], we had to transpose the couplings λ'_{ijk} and λ''_{ijk} in the first two indices and the outer indices repectively. As in the R-parity conserving case this is due to the embedding into the SU(5) GUT.

After giving the singlet Higgs S a VEV of order M_{GUT} and contracting it with the Yukawa coupling λ^S_{ij} to a mass term M_{ij} for the right-handed neutrino superfields N^c, the superpotential for Majorana neutrinos is

$$W^{N^c} = \tfrac{1}{2} M_{ij} {N^c}_i {N^c}_j - \lambda^\nu_{ij} H_u L_i {N^c}_j \ . \tag{2.4}$$

The second term couples the right-handed neutrino superfields to the left-handed ones.

Finally one has to add the R-parity violating couplings for the right-handed neutrino superfields

$$W^{N^c}_{\not{R}_p} = -\lambda^\nu_i H_u H_d {N^c}_i \ . \tag{2.5}$$

The combined effects of R-parity breaking and right-handed neutrino superfields are usually neglected, therefore we could find so far unpublished formulas in this sector. But even when both effects are taken into account the superpotential (2.5) is typically ignored [21] as this term gives rather small corrections to the physical fields, as can be seen in the course of Chapter 3 and particularly in Appendix C.

2.2 Choice of the Weak Interaction Basis

As can be seen from the charge assignment in Table 2.1, the distinction between the superfields H_d and L_i vanishes in the absence of R-parity. Thus the weak eigenstate basis can be rotated by a SU(4) transformation [9]

$$\begin{pmatrix} H_d \\ L_i \end{pmatrix} \to \begin{pmatrix} H_d{}' \\ L_i{}' \end{pmatrix} = U_{\alpha\beta} \begin{pmatrix} H_d \\ L_i \end{pmatrix} \ . \tag{2.6}$$

It follows that the superpotential parameters that are connected to H_d and L_i have to be rotated in the same manner. Therefore it is sensible to introduce a notation where the scalars associated to H_d and the three-component tensors connected to L_i are combined to four-component tensors corresponding to L_α with $\alpha = (0, i) = (0, 1, 2, 3)$. After the definition of the tensors

$$\begin{aligned} L_\alpha &= (H_d, L_i)^T , & \mu_\alpha &= (\mu, \mu_i)^T , \\ \lambda^e_{\alpha\beta k} &= \begin{pmatrix} 0_k & \lambda^{e\ \dagger}_{jk} \\ \lambda^e_{ik} & \lambda_{ijk} \end{pmatrix} , & \lambda^d_{\alpha jk} &= \begin{pmatrix} \lambda^d_{jk} \\ \lambda'_{ijk} \end{pmatrix} , & \lambda^\nu_{\alpha j} &= \begin{pmatrix} \lambda^\nu_j \\ \lambda^\nu_{ij} \end{pmatrix} , \end{aligned} \tag{2.7}$$

the superpotentials (2.2), (2.3), (2.4) and (2.5) combine to

$$\begin{aligned} W = {} & \mu_\alpha H_u L_\alpha + \tfrac{1}{2}\lambda^e_{\alpha\beta k} L_\alpha L_\beta {E^c}_k + \lambda^d_{\alpha jk} L_\alpha {D^c}_j Q_k - \lambda^u_{jk} H_u Q_j {U^c}_k \\ & + \tfrac{1}{2}\lambda''_{ijk} {D^c}_i {D^c}_j {U^c}_k + \tfrac{1}{2} M_{ij} {N^c}_i {N^c}_j - \lambda^\nu_{\alpha j} H_u L_\alpha {N^c}_j \, . \end{aligned} \tag{2.8}$$

The rotation (2.6) allows to rotate away one of the bilinear R-parity violating parameters, either μ_i or the sneutrino VEV v_i introduced in Section 3.1.3. In this thesis the rotation fixing is postponed until the introduction of the bilinear R-parity breaking model in Section 4.3.

The notation used here implies that the Lagrangian and the superpotential can only depend on SU(N) scalars. It follows that terms containing SU(N) multiplets products where none of the factors is adjoint, must be contracted with the N dimensional Levi-Civita symbol. Our convention for the SU(2) is $\epsilon_{ab} = \left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$. The same argument holds for SU(3) multiplets and for the SU(5) multiplets in the superpotential (2.1). The contraction gives, for instance, for the superpotential (2.8)

$$\begin{aligned} W = {} & \mu_\alpha \left(H_u^+ E_\alpha - H_u^0 N_\alpha \right) + \tfrac{1}{2}\lambda^e_{\alpha\beta k} \left(N_\alpha E_\beta - E_\alpha N_\beta \right) {E^c}_k \\ & + \lambda^d_{\alpha jk} \left(N_\alpha D_k - E_\alpha U_k \right) {D^c}_j - \lambda^u_{jk} \left(H_u^+ D_j - H_u^0 U_j \right) {U^c}_k \\ & + \tfrac{1}{2}\lambda''_{ijk} \epsilon_{\alpha\beta\gamma} D^{c\alpha}_i D^{c\beta}_j U^{c\gamma}_k \\ & + \tfrac{1}{2} M_{ij} {N^c}_i {N^c}_j - \lambda^\nu_{\alpha j} \left(H_u^+ E_\alpha - H_u^0 N_\alpha \right) {N^c}_j \, . \end{aligned} \tag{2.9}$$

Here it becomes clear why some of the Yukawa couplings are defined with a minus sign. As they will produce mass terms together with fields which acquire a VEV, the signs are chosen in such a way that these mass terms are positive in Equation 2.9.

2.3 Vector Superfields

The gauge group of the Standard Model is

$$G_{\mathrm{SM}} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \tag{2.10}$$

gaugino λ^a	gauge boson A^a_μ	$(\mathrm{SU}(3)_C, \mathrm{SU}(2)_L)_{\mathrm{U}(1)_Y}$
$\widetilde{g}^a$	g^a	$(\mathbf{8},\mathbf{1})_0$
$\widetilde{W}^i$	W^i	$(\mathbf{1},\mathbf{3})_0$
$\widetilde{B}$	B	$(\mathbf{1},\mathbf{1})_0$

Table 2.2: Gauge supermultiplets in the MSSM.

where $\mathrm{SU}(3)_C$ is the color gauge group, $\mathrm{SU}(2)_L$ is the isospin gauge group and $\mathrm{U}(1)_Y$ is the hypercharge group. There is one vector multiplet for every subgroup. These gauge fields and the corresponding gauginos are written down in Table 2.2.

2.4 Soft SUSY Breaking Lagrangian

All soft supersymmetry breaking terms that have to be added to shift the supersymmetry particle masses to higher energies can be found with the help of the Lagrangians given in Section 1.2.5.

The R-parity conserving soft SUSY breaking Lagrangian is

$$\begin{aligned}
-\mathcal{L}^{\mathrm{soft}}_{R_p} = {}& (m^2_{\widetilde{Q}})_{ij}\widetilde{Q}_i^{\dagger}\widetilde{Q}_j + (m^2_{\widetilde{u}^c})_{ij}\widetilde{u}^c{}_i^{\dagger}\widetilde{u}^c{}_j + (m^2_{\widetilde{d}^c})_{ij}\widetilde{d}^c{}_i^{\dagger}\widetilde{d}^c{}_j \\
&+ (m^2_{\widetilde{L}})_{ij}\widetilde{L}_i^{\dagger}\widetilde{L}_j + (m^2_{\widetilde{l}^c})_{ij}\widetilde{l}^c{}_i^{\dagger}\widetilde{l}^c{}_j + \widetilde{m}^2_d h_d{}^{\dagger} h_d + \widetilde{m}^2_u h_u{}^{\dagger} h_u \\
&+ (B h_u h_d + \mathrm{h.c.}) \\
&+ \left(A^e_{ij} h_d \widetilde{L}_i \widetilde{l}^c{}_j + A^d_{ij} h_d \widetilde{d}^c{}_i \widetilde{Q}_j + A^u_{ij} h_u \widetilde{Q}_i \widetilde{u}^c{}_j + \mathrm{h.c.}\right) \\
&+ \tfrac{1}{2}\left(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}^i\widetilde{W}^i + M_3 \widetilde{g}^a\widetilde{g}^a + \mathrm{h.c.}\right) .
\end{aligned} \tag{2.11}$$

The R-parity violating soft SUSY breaking Lagrangian is according to the superpotential (2.3) and the process of Section 1.2.5

$$\begin{aligned}
-\mathcal{L}^{\mathrm{soft}}_{\not{R}_p} = {}& \left(\widetilde{m}^2_{di} h_d{}^{\dagger}\widetilde{L}_i + B_i h_u \widetilde{L}_i + \mathrm{h.c.}\right) \\
&+ \left(\tfrac{1}{2}A_{ijk}\widetilde{L}_i\widetilde{L}_j\widetilde{l}^c{}_k + A'_{ijk}\widetilde{L}_i\widetilde{d}^c{}_j\widetilde{Q}_k + \tfrac{1}{2}A''_{ijk}\widetilde{d}^c{}_i\widetilde{d}^c{}_j\widetilde{u}^c{}_k + \mathrm{h.c.}\right) .
\end{aligned} \tag{2.12}$$

If one adds right-handed neutrinos to the field content, the scalar neutrinos get contributions to their Lagrangian according to

$$-\mathcal{L}^{\mathrm{soft}}_{N^c} = (m^2_{\widetilde{\nu}^c})_{ij}\widetilde{\nu}^c{}_i^{\dagger}\widetilde{\nu}^c + \left(B^N_{ij}\widetilde{\nu}^c{}_i\widetilde{\nu}^c{}_j + A^\nu_{ij} h_u \widetilde{L}_i \widetilde{\nu}^c{}_j + \mathrm{h.c.}\right) . \tag{2.13}$$

The only R-parity breaking counterpart is the trilinear contribution

$$-\mathcal{L}^{\text{soft}}_{\not{R}_p N^c} = A^\nu_j h_u h_d \widetilde{\nu}^c{}_j + \text{h.c.} \,. \tag{2.14}$$

Using the notation based on the rotation (2.6) one can combine some Lagrangian terms into four-component objects:

$$\begin{aligned}
\widetilde{m}^2_{\alpha\beta} &= \begin{pmatrix} \widetilde{m}^2_d & \widetilde{m}^{2\,\dagger}_{dj} \\ \widetilde{m}^2_{di} & (m^2_{\widetilde{L}})_{ij} \end{pmatrix} \,, \qquad B_\alpha = \begin{pmatrix} B \\ B_i \end{pmatrix} \,, \qquad \widetilde{L}_\alpha = \begin{pmatrix} h_d \\ \widetilde{L}_i \end{pmatrix} \,, \\
A^e_{\alpha\beta k} &= \begin{pmatrix} 0_k & A^{e\,\dagger}_{jk} \\ A^e_{ik} & A_{ijk} \end{pmatrix} \,, \qquad A^d_{\alpha jk} = \begin{pmatrix} A^d_{jk} \\ A'_{ijk} \end{pmatrix} \,, \qquad A^\nu_{\alpha j} = \begin{pmatrix} A^\nu_j \\ A^\nu_{ij} \end{pmatrix} \,.
\end{aligned} \tag{2.15}$$

This simplifies the soft SUSY breaking Lagrangians (2.11), (2.12), (2.13) and (2.14) to

$$\begin{aligned}
\mathcal{L}^{\text{soft}} = {} & \widetilde{m}^2_{\alpha\beta} \widetilde{L}_\alpha{}^\dagger \widetilde{L}_\beta + \widetilde{m}^2_u h_u{}^\dagger h_u + (m^2_{\widetilde{Q}})_{ij} \widetilde{Q}_i{}^\dagger \widetilde{Q}_j + (m^2_{\widetilde{u}^c})_{ij} \widetilde{u}^c{}_i{}^\dagger \widetilde{u}^c{}_j \\
& + (m^2_{\widetilde{d}^c})_{ij} \widetilde{d}^c{}_i{}^\dagger \widetilde{d}^c{}_j + (m^2_{\widetilde{l}^c})_{ij} \widetilde{l}^c{}_i{}^\dagger \widetilde{l}^c{}_j + (m^2_{\widetilde{\nu}^c})_{ij} \widetilde{\nu}^c{}_i{}^\dagger \widetilde{\nu}^c \\
& + \left(B_\alpha h_u \widetilde{L}_\alpha + B^N_{ij} \widetilde{\nu}^c{}_i \widetilde{\nu}^c{}_j + \text{h.c.} \right) \\
& + \left(\tfrac{1}{2} A^e_{\alpha\beta k} \widetilde{L}_\alpha \widetilde{L}_\beta \widetilde{l}^c{}_k + A^d_{\alpha jk} \widetilde{L}_\alpha \widetilde{d}^c{}_j \widetilde{Q}_k + A^u_{ij} h_u \widetilde{Q}_i \widetilde{u}^c{}_j \right. \\
& \left. + \tfrac{1}{2} A''_{ijk} \widetilde{d}^c{}_i \widetilde{d}^c{}_j \widetilde{u}^c{}_k + A^\nu_{\alpha j} h_u \widetilde{L}_\alpha \widetilde{\nu}^c{}_j + \text{h.c.} \right) \\
& + \tfrac{1}{2} \left(M_1 \widetilde{B} \widetilde{B} + M_2 \widetilde{W}^i \widetilde{W}^i + M_3 \widetilde{g}^a \widetilde{g}^a + \text{h.c.} \right) \,.
\end{aligned} \tag{2.16}$$

In the following we will make use of the four-component notation whenever possible.

Chapter 3

Symmetry Breaking: Higgs and Sneutrinos

In the SM electroweak symmetry is spontaneously broken down to electromagnetism:

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \rightarrow \mathrm{U}(1)_{\mathrm{em}} \,. \tag{3.1}$$

In this chapter we give the consequences of this transition in the supersymmetric extensions considered in this thesis depending on the superpotentials and field content examined. Therefore we give an insight how the fields given in Table 2.1 mix to the mass eigenstates and which of them acquire a VEV.

An overview of the well known calculations required for the MSSM can be found, for instance, in [6, 40]. We recalculate the most important formulas as well, but merely as references to compare what changes when new terms are taken into account.

The new mixing terms in the R-parity violating theory are summarized in [9, 41]. In this sector the emphasis for us lies on the sneutrino VEVs, as one can find contradictory formulas in the literature. Another focus lies on the neutrino masses induced by R-parity breaking.

The implications of the Majorana neutrino superpotential for the fermion sector are well known from the SM [3] and the implications for the supersymmetric theory are shown, for instance, in [42, 43]. Nevertheless, we recalculate the result to have a common ground to extend on.

The case where both extensions are in effect is not studied very well, therefore our attention is focused on this area. In the scalar sector we calculate the most general mass mixing matrix, but the more important part is that we calculate a so far unknown mass formula for neutrinos where both mass generation effects are combined.

We also calculate the R-parity breaking effects on the charged fields, but except for larger mixing matrices due to the lepton number breaking couplings no new effects occur. Nonetheless we have listed the mixing matrices in Appendix D.

3.1 Scalar Bosons

The neutral scalar bosons are responsible for the mass generation of all fermions in the SM. In the following we discuss the modifications of the Higgs mechanism due to SUSY, right-handed neutrinos and broken R-parity.

This is modified by SUSY but the Higgs mechanism is just as well important in this case.

3.1.1 The Minimal Supersymmetric Standard Model

In the MSSM the neutral Higgs fields h_u^0 and h_d^0 acquire VEVs

$$\left\langle h_u^0 \right\rangle = \frac{1}{\sqrt{2}} v_u \; , \qquad \left\langle h_d^0 \right\rangle = \frac{1}{\sqrt{2}} v_d \; , \tag{3.2}$$

which breaks electroweak symmetry. Their values are connected to the SM Higgs VEV v via $v^2 = v_u^2 + v_d^2$. The ratio between these two VEVs is commonly referred to as $\tan\beta = \frac{v_u}{v_d}$.

The minimum of the R-parity conserving scalar potential satisfies

$$\begin{aligned} 0 &= \widetilde{m}_u^2 + |\mu|^2 - B\cot\beta - \tfrac{1}{2} m_Z^2 \cos 2\beta \; , \\ 0 &= \widetilde{m}_d^2 + |\mu|^2 - B\tan\beta + \tfrac{1}{2} m_Z^2 \cos 2\beta \; . \end{aligned} \tag{3.3}$$

After diagonalizing the mass matrices, the gauge eigenstate fields can be expressed in terms of the VEVs, the CP-even mass eigenstate fields $(h^0, H^0)^T$ and the CP-odd mass eigenstate fields $(G^0, A^0)^T$ as shown in [6, 40],

$$\begin{pmatrix} h_u^0 \\ h_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} v_u \\ v_d \end{pmatrix} + R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + i R_\beta \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \right) \; . \tag{3.4}$$

Here R_α and R_β are rotation matrices. The would-be Nambu Goldstone boson G^0 remains massless and becomes the longitudinal mode of the Z gauge boson. The CP-odd neutral scalar obtains the mass

$$m_{A^0}^2 = \frac{2B}{\sin 2\beta} \; . \tag{3.5}$$

The masses of the CP-even neutral scalars are slightly more complicated and fulfill the relation $m_{h^0} < m_{H^0}$.

The sneutrino sector is separated from the neutral Higgs sector and the sneutrinos satisfy the mass relation

$$m_{ij}^{\widetilde{\nu}} = \left(m_{\widetilde{L}}^2\right)_{ij} + \tfrac{1}{2} m_Z^2 \cos 2\beta \ . \tag{3.6}$$

The first term comes from the soft SUSY breaking Lagrangian, the second term is the D-term explained before Equation (3.14).

3.1.2 Adding Majorana Sneutrinos

When one adds the right-handed neutrino superfield to the MSSM, one has to calculate the sneutrino mass for the right and left-handed sneutrinos once for CP-even and once for CP-odd fields

$$\left(\Re\widetilde{\nu}^c{}_i, \Re\widetilde{\nu}_i, \Im\widetilde{\nu}^c{}_i, \Im\widetilde{\nu}_i\right) \ . \tag{3.7}$$

Assuming CP-conservation, the resulting mass matrix separates into a CP-even and a CP-odd block

$$M^2 = \begin{pmatrix} M_+^2 & 0 \\ 0 & M_-^2 \end{pmatrix} , \tag{3.8}$$

where the submatrices are given by [44, 45]

$$M_\pm^2 = \begin{pmatrix} \left(m_{\widetilde{L}}^2\right)_{ij} + \frac{1}{2} m_Z^2 \cos 2\beta \delta_{ij} + \lambda_{ik}^{\nu}{}^\dagger h_u^0 \lambda_{kj}^\nu h_u^0 & \pm A_{ij}^\nu h_u^0 - M_{ik}{}^\dagger \lambda_{kj}^\nu h_u^0 \mp \mu^\dagger \lambda_{ij}^\nu h_u^0 \cot\beta \\ \pm A_{ij}^\nu{}^\dagger h_u^0 - \lambda_{ik}^\nu{}^\dagger h_u^0 M_{kj} \mp \mu \lambda_{ij}^\nu{}^\dagger h_u^0 \cot\beta & \pm 2 B_{ij}^N + M_{ik}{}^\dagger M_{kj} + \lambda_{kj}^\nu{}^\dagger h_u^0 \lambda_{ik}^\nu h_u^0 + \left(m_{\widetilde{\nu}^c}^2\right)_{ij} \end{pmatrix} . \tag{3.9}$$

Here and in the following the notation $\lambda_{ij}^\nu h_u^0 = \sqrt{2} m_{ij}^\nu$ means that a Yukawa coupling and a field that acquires a VEV form a mass matrix.

As the submatrices are hierarchic they can be diagonalized via the seesaw mechanism sketched in Appendix G.1. The resulting mass matrix for the left-handed sneutrinos is [46]

$$\begin{aligned} m_{ij} \simeq \ & \left(m_{\widetilde{L}}^2\right)_{ij} + \tfrac{1}{2} m_Z^2 \cos 2\beta \delta_{ij} + \lambda_{ik}^\nu{}^\dagger h_u^0 \lambda_{kj}^\nu h_u^0 \\ & - \frac{\left|\pm A_{ij}^\nu h_u^0 - M_{ik}{}^\dagger \lambda_{kj}^\nu h_u^0 \mp \mu^\dagger \lambda_{ij}^\nu h_u^0 \cot\beta\right|^2}{\pm 2 B_{ij}^N + M_{ik}{}^\dagger M_{kj} + \lambda_{kj}^\nu{}^\dagger h_u^0 \lambda_{ik}^\nu h_u^0 + \left(m_{\widetilde{\nu}^c}^2\right)_{ij}} \ . \end{aligned} \tag{3.10}$$

The second line is the first order correction due to the right-handed sneutrinos. The mass of the right-handed sneutrinos are slightly modified as well.

3.1.3 MSSM Including *R*-Parity Violating Couplings

Now we turn to R-parity violation without taking into account the right-handed sneutrinos. Due to the lepton number violating couplings the left-handed sneutrinos acquire a small VEV

$$\langle \nu_i \rangle = \frac{1}{\sqrt{2}} v_i \ . \tag{3.11}$$

Using the notation from Section 2.2 one can combine the VEV of the down-type Higgs (3.2) and the sneutrino VEVs into a four-component vector

$$v_\alpha = (v_d, v_i)^T \ . \tag{3.12}$$

We use this opportunity to give a small step-by-step instruction how to obtain the scalar potential in a supersymmetric theory: The general approach is the same for scalar fields and fermionic fields. The details in this section, however, differ slightly from the fermionic case explained in Section 3.3.3. First, one needs the F-terms which are obtained by inserting the superpotential (2.9) into the last term of the Lagrangian (1.6). In our case the F-terms are

$$V_F = |\mu_\alpha|^2 \left|h_u^0\right|^2 + {\mu_\beta}^\dagger \mu_\alpha {\widetilde{\nu}_\beta}^\dagger \widetilde{\nu}_\alpha \ . \tag{3.13}$$

Secondly, one needs the D-terms, which are calculated by inserting the equation of motion (1.10) into the combined chiral Lagrangian (1.6) and gauge Lagrangian (1.9) taking the mixing terms (1.12) into account,

$$V_D = \tfrac{1}{8}\left(g_1^2 + g_2^2\right)\left(\left|h_u^0\right|^2 - |\widetilde{\nu}_\alpha|^2\right)^2 \ . \tag{3.14}$$

In the end one has to add the soft SUSY breaking terms (2.16). These three contributions give the scalar potential for the neutral scalar fields

$$\begin{aligned} V_{\text{scalar}} = {} & \left(|\mu_\alpha|^2 + \widetilde{m}_u^2\right)\left|h_u^0\right|^2 + \left({\mu_\beta}^\dagger \mu_\alpha + \widetilde{m}_{\alpha\beta}^2\right) {\widetilde{\nu}_\beta}^\dagger \widetilde{\nu}_\alpha \\ & - B_\alpha h_u^0 \widetilde{\nu}_\alpha - {B_\beta}^\dagger {\widetilde{\nu}_\beta}^\dagger {h_u^0}^\dagger + \tfrac{1}{8}\left(g_1^2 + g_2^2\right)\left(\left|h_u^0\right|^2 - |\widetilde{\nu}_\alpha|^2\right)^2 \ . \end{aligned} \tag{3.15}$$

The only terms that depend on the phases of the fields are the B_α, therefore a redefinition of the phases of L_α can make B_α real and positive. The minimum of the potential satisfies as in [47–49]

$$\begin{aligned} v_u &= \frac{B_\alpha v_\alpha}{|\mu_\alpha|^2 + \widetilde{m}_u^2 + \frac{1}{2} m_Z^2 \left(\frac{v_u^2}{v^2} - \frac{|v_\alpha|^2}{v^2}\right)} \\ v_\alpha &= \frac{B_\alpha v_u}{|\mu_\alpha|^2 + \widetilde{m}_{\alpha\alpha}^2 - \frac{1}{2} m_Z^2 \left(\frac{v_u^2}{v^2} - \frac{|v_\alpha|^2}{v^2}\right)} \ . \end{aligned} \tag{3.16}$$

In order to extend this to the common MSSM notation one has to mark that for the case of small R-parity violation,

$$\widehat{\epsilon}_i = \frac{v_i}{v_d} \ll 1 \ , \tag{3.17}$$

the R-parity violating VEVs are connected to the R-parity conserving VEV ratio $\tan\beta$ as in

$$\left(\frac{v_u^2}{v^2} - \frac{|v_\alpha|^2}{v^2} \right) = (1 - \rho + \rho \cos 2\beta) \simeq \cos 2\beta \ , \tag{3.18}$$

with ρ given by

$$\rho = 2 \left(2 + |\widehat{\epsilon}_i|^2 + |\widehat{\epsilon}_i|^2 \cos 2\beta \right)^{-1} \simeq 1 \ . \tag{3.19}$$

Finally one gets three equations for the minimum of the scalar potential [15, 50]:

$$\begin{aligned} 0 &\simeq \widetilde{m}_u^2 + \left(|\mu|^2 + |\mu_i|^2 \right) - (B + \widehat{\epsilon}_i B_i) \cot\beta - \tfrac{1}{2} m_Z^2 \cos 2\beta \ , \\ 0 &\simeq \left(\widetilde{m}_d^2 + \widehat{\epsilon}_i \widetilde{m}_{di}^2 \right) + \left(|\mu|^2 + \widehat{\epsilon}_i \mu_i \mu \right) - B \tan\beta + \tfrac{1}{2} m_Z^2 \cos 2\beta \ , \\ 0 &\simeq \left(\widetilde{m}_{id}^{2\,*} + \widehat{\epsilon}_i \widetilde{m}_{ii}^2 \right) + \left(\mu {\mu_i}^* + \widehat{\epsilon}_i \, |\mu_i|^2 \right) - B_i \tan\beta + \widehat{\epsilon}_i \tfrac{1}{2} m_Z^2 \cos 2\beta \ . \end{aligned} \tag{3.20}$$

The first two equations are just the slightly modified R-parity conserving equations (3.3) for the minimum of the potential. The last equation is new and gives the most general value of the sneutrino VEV,

$$\widehat{\epsilon}_i = \frac{v_i}{v_d} \simeq \frac{B_i \tan\beta - \widetilde{m}_{id}^{2\,*} - \mu {\mu_i}^*}{|\mu_i|^2 + \widetilde{m}_{ii}^2 + \frac{1}{2} m_Z^2 \cos 2\beta} \ , \tag{3.21}$$

in terms of $\tan\beta$ and Lagrangian parameters. One still has to constrain the parameters to fulfill the bounds on the R-parity violating theory [9] (for an example see Section 4.1), as well as to rotate away one of the bilinear parameters as done in Section 4.3. Other authors did not take into account all soft SUSY breaking parameters [51–53] or found other signs for some of the parameters in the sneutrino VEV [54–57].

The potential has no CP-violation at tree level [48], hence the CP-even and CP-odd mass eigenstates can be calculated. The CP-odd neutral scalar fields

$$\left(\Im h_u^0, \Im \widetilde{\nu}_\alpha \right)^T = \left(\Im h_u^0, \Im h_d^0, \Im \widetilde{\nu}_i \right)^T \tag{3.22}$$

have the mass mixing matrix *c.f.* (3.18)

$$
\begin{aligned}
M^2_{\Im\phi^0} &= \begin{pmatrix} \widetilde{m}_u^2+|\mu_\alpha|^2+\frac{1}{2}m_Z^2\left(\frac{v_u^2}{v^2}-\frac{|v_\alpha|^2}{v^2}\right) & {B_\beta}^T \\ B_\alpha & \widetilde{m}^2_{\alpha\beta}+{\mu_\beta}^\dagger\mu_\alpha-\frac{1}{2}m_Z^2\left(\frac{v_u^2}{v^2}-\frac{|v_\alpha|^2}{v^2}\right)\delta_{\alpha\beta} \end{pmatrix} \\
&\simeq \begin{pmatrix} \widetilde{m}_u^2+\mu^2+|\mu_i|^2-\frac{1}{2}m_Z^2c_{2\beta} & -B & -{B_j}^T \\ -B & \widetilde{m}_d^2+\mu^2+\frac{1}{2}m_Z^2c_{2\beta} & {\widetilde{m}^2_{dj}}{}^\dagger+\mu{\mu_j}^\dagger \\ -B_i & \widetilde{m}^2_{di}+\mu\mu_i & \widetilde{m}^2_{ij}+{\mu_j}^\dagger\mu_i+\frac{1}{2}m_Z^2c_{2\beta}\delta_{ij} \end{pmatrix} .
\end{aligned}
\tag{3.23}
$$

Here and in the following $c_{2\beta}$ and $s_{2\beta}$ stand for $\cos 2\beta$ and $\sin 2\beta$, respectively. The CP-even fields

$$
\left(\Re h_u^0, \Re\widetilde{\nu}_\alpha\right)^T = \left(\Re h_u^0, \Re h_d^0, \Re\widetilde{\nu}_i\right)^T \tag{3.24}
$$

have the mass mixing matrix

$$
\begin{aligned}
M^2_{\Re\phi^0} &= \begin{pmatrix} \widetilde{m}_u^2+|\mu_\alpha|^2+\frac{1}{2}m_Z^2\left(3\frac{v_u^2}{v^2}-\frac{|v_\alpha|^2}{v^2}\right) & -{B_\beta}^T-m_Z^2\frac{v_u{v_\beta}^T}{v^2} \\ -B_\alpha-m_Z^2\frac{v_uv_\alpha}{v^2} & \widetilde{m}^2_{\alpha\beta}+{\mu_\beta}^\dagger\mu_\alpha-\frac{1}{2}m_Z^2\left(\frac{v_u^2}{v^2}-3\frac{|v_\alpha|^2}{v^2}\right) \end{pmatrix} \\
&\simeq \begin{pmatrix} \widetilde{m}_u^2+\mu^2+|\mu_i|^2+\frac{1}{2}m_Z^2\left(1-c_{2\beta}\right) & -B-m_Z^2s_{2\beta} & -{B_j}^T-\widehat{\epsilon}_j^Tm_Z^2s_{2\beta} \\ -B-m_Z^2s_{2\beta} & \widetilde{m}_d^2+\mu^2+\frac{1}{2}m_Z^2\left(1+2c_{2\beta}\right) & {\widetilde{m}^2_{dj}}{}^\dagger+\mu{\mu_j}^\dagger+\frac{1}{2}\widehat{\epsilon}_j^Tm_Z^2\left(1+c_{2\beta}\right) \\ -B_i-\widehat{\epsilon}_im_Z^2s_{2\beta} & \widetilde{m}^2_{di}+\mu\mu_i+\frac{1}{2}\widehat{\epsilon}_im_Z^2\left(1+c_{2\beta}\right) & \widetilde{m}^2_{ij}+{\mu_j}^\dagger\mu_i+\frac{1}{2}m_Z^2c_{2\beta}\delta_{ij} \end{pmatrix} .
\end{aligned}
\tag{3.25}
$$

As in the R-parity conserving case these matrices can be diagonalized in order to express the gauge eigenstates via VEVs and mass eigenstates. However, this cannot be done analytically without assumptions on the sneutrino Lagrangian terms.

3.1.4 Majorana Sneutrinos and *R*-Parity Violation

If one takes into account right-handed sneutrinos in addition to R-parity violation and assumes CP-conservation, the VEVs (3.16) are once more modified to

$$
\begin{aligned}
\left\langle h_u^0\right\rangle &= \left\langle\nu_\alpha\right\rangle \frac{B_\alpha+M_{ij}\lambda_{\alpha i}\langle {\nu^c}_j\rangle-A_{\alpha i}\langle {\nu^c}_i\rangle}{\widetilde{m}_u^2+m_Z^2\left(\frac{\langle h_u^0\rangle^2}{v^2}-\frac{|\nu_\alpha|^2}{v^2}\right)+|\lambda_{\alpha i}\langle\nu_\alpha\rangle|^2+|\mu_\alpha+\lambda_{\alpha i}\langle {\nu^c}_i\rangle|^2} , \\
\left\langle\nu_\alpha\right\rangle &= \left\langle h_u^0\right\rangle \frac{B_\alpha+M_{ji}\lambda_{\alpha i}\langle {\nu^c}_j\rangle-A_{\alpha i}\langle {\nu^c}_i\rangle}{\widetilde{m}^2_{\alpha\alpha}-m_Z^2\left(\frac{\langle h_u^0\rangle^2}{v^2}-\frac{|\nu_\alpha|^2}{v^2}\right)+|\lambda_{\alpha i}\langle h_u^0\rangle|^2+|\mu_\alpha+\lambda_{\alpha i}\langle {\nu^c}_i\rangle|^2} .
\end{aligned}
\tag{3.26}
$$

In this case the right-handed sneutrinos acquire a VEV as well:

$$
\langle {\widetilde{\nu}^c}_j\rangle = \left\langle h_u^0\right\rangle \frac{M_{jk}\lambda^\nu_{\alpha k}\langle\widetilde{\nu}_\alpha\rangle-A^\nu_{\alpha j}\langle\widetilde{\nu}_\alpha\rangle-\mu_\alpha\lambda^\nu_{\alpha j}\left\langle h_u^0\right\rangle-\lambda^\nu_{\beta j}\mu_\alpha\frac{\langle\widetilde{\nu}_\alpha\rangle\left\langle\widetilde{\nu}_\beta\right\rangle}{\langle h_u^0\rangle}}{B^N_{jj}+{B^N_{jj}}^\dagger+\left|\lambda^\nu_{\alpha j}\langle h_u^0\rangle\right|^2+|M_{ij}|^2+\left|\lambda^\nu_{\alpha j}\langle\widetilde{\nu}_\alpha\rangle\right|^2+(m^2_{\widetilde{\nu}^c})_{ij}\delta_{ij}} . \tag{3.27}
$$

Due to the large mass M_{ij} in the denominator this VEV is so small that the phenomenological consequences can be neglected. This can be seen, for instance, in the small modifications in the VEVs (3.26) due to the right-handed sneutrino VEVs.

The mass matrices which include the right-handed sneutrinos and R-parity violation are given in Appendix C.1.

3.2 Gauge Bosons

The derivation of the physical gauge bosons in the SM can be found, for instance, in [2].

The $\mathrm{U}(1)_Y$ gauge force is mediated by the gauge boson B_μ. The representation matrix in this simple case is $(T^a)_i^j = Y\delta_i^j\delta_1^a$. We name the coupling constant $g_{(a)} = g_1$.

The $\mathrm{SU}(2)_L$ gauge force is mediated by the W_μ^a gauge bosons, where the gauge index runs over $a = 1, 2, 3$. The representation matrices are $(T^a)_i^j = \frac{1}{2}(\sigma^a)_i^j$ and the coupling constant is called $g_{(a)} = g_2$.

In the SM the neutral fields are broken to the massless photon

$$\gamma_\mu = \frac{1}{\sqrt{g_2^2 + g_1^2}} \left(g_1 W_\mu^3 + g_2 B_\mu \right) \tag{3.28}$$

and the Z_μ boson

$$Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} \left(g_2 W_\mu^3 - g_1 B_\mu \right) \tag{3.29}$$

which has the mass

$$m_Z^2 = \tfrac{1}{4} \left(g_2^2 + g_1^2 \right) v^2 \ . \tag{3.30}$$

The charged fields are the $W_\mu^\pm$ bosons

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right) \tag{3.31}$$

with mass

$$m_W^2 = m_Z^2 \cos^2\theta_w = \tfrac{1}{4} g_2^2 v^2 \ . \tag{3.32}$$

The mixing between W_μ^3 and B_μ with the constraint to form the massless photon defines the weak mixing angle θ_w. The rotation matrix T_w has the form

$$\begin{pmatrix} \gamma \\ Z \end{pmatrix} = T_w \begin{pmatrix} B \\ W^3 \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} B \\ W^3 \end{pmatrix} = \begin{pmatrix} Bc_w + W^3 s_w \\ W^3 c_w - B s_w \end{pmatrix} \ . \tag{3.33}$$

Here and in the following c_w and s_w symbolizes $\cos\theta_w$ and $\sin\theta_w$, respectively. The angle θ_w can be expressed by the gauge couplings with the help of

$$\cos\theta_w = \frac{g_2}{\sqrt{g_1^2+g_2^2}} \,, \qquad \sin\theta_w = \frac{g_1}{\sqrt{g_1^2+g_2^2}} \,. \tag{3.34}$$

The electric charge is then

$$e = \frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}} = g_1\cos\theta_w = g_2\sin\theta_w \,. \tag{3.35}$$

The electric charge quantum number is calculated using the Gell-Mann-Nishijima formula

$$Q = T^3 + Y \,. \tag{3.36}$$

As the MSSM does not add any new propagating vector fields this sector of the model does not change.

3.3 Neutralinos and Neutrinos

In this section we give an overview about the neutral fermionic fields which are the fermionic components of either chiral superfields or vector superfields.

3.3.1 Neutralinos in the MSSM

In the R-parity conserving case the wino and the bino are mixing with the Higgsino due to the electroweak symmetry breaking to the four neutralinos $\chi^0_{R_p}$. The Lagrangian in the gauge eigenstate basis

$$\chi^0_{R_p} = \left(\widetilde{B}, \widetilde{W}^3, \widetilde{h}^0_u, \widetilde{h}^0_d\right)^T \tag{3.37}$$

is given by

$$-\mathcal{L}_{\text{neutralino}} = \tfrac{1}{2}{\chi^0_{R_p}}^T M_{\chi^0_{R_p}} \chi^0_{R_p} \,, \tag{3.38}$$

where the symmetric neutralino mass mixing matrix is

$$M_{\chi^0_{R_p}} = \begin{pmatrix} M_1 & 0 & \frac{1}{2}g_1 v_u & -\frac{1}{2}g_1 v_d \\ 0 & M_2 & -\frac{1}{2}g_2 v_u & \frac{1}{2}g v_d \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & 0 & -\mu \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & -\mu & 0 \end{pmatrix} \,. \tag{3.39}$$

Expressing the coupling constant with the help of (3.30) via m_Z and rotating the gauginos by T_w and the Higgsinos by an analog matrix composed of $\cos\beta$

and $\sin\beta$, called T_β, into the photino-zino and the (anti)symmetric Higgs basis

$$\begin{aligned}
{\chi^0_{R_p}}' = \begin{pmatrix} \widetilde{\gamma} \\ \widetilde{Z} \\ \widetilde{h}^0_S \\ \widetilde{h}^0_A \end{pmatrix} &= \begin{pmatrix} T_w & 0 \\ 0 & T_\beta \end{pmatrix} \begin{pmatrix} \widetilde{B} \\ \widetilde{W}^3 \\ \widetilde{h}^0_u \\ \widetilde{h}^0_d \end{pmatrix} \\
&= \begin{pmatrix} c_w & s_w & 0 & 0 \\ -s_w & c_w & 0 & 0 \\ 0 & 0 & c_\beta & s_\beta \\ 0 & 0 & -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \widetilde{B} \\ \widetilde{W}^3 \\ \widetilde{h}^0_u \\ \widetilde{h}^0_d \end{pmatrix} = \begin{pmatrix} \widetilde{B} c_w + \widetilde{W}^3 s_w \\ \widetilde{W}^3 c_w - \widetilde{B} s_w \\ h^0_u c_\beta + h^0_d s_\beta \\ h^0_d c_\beta - h^0_u s_\beta \end{pmatrix}
\end{aligned}, \tag{3.40}$$

the neutralino mixing matrix simplifies to

$$\begin{aligned}
{M_{\chi^0_{R_p}}}' &= T_w T_\beta M_{\chi^0_{R_p}} {T_\beta}^T {T_w}^T \\
&= \begin{pmatrix} M_1 c_w^2 + M_2 s_w^2 & \frac{1}{2}(M_2 - M_1) s_{2w} & 0 & 0 \\ \frac{1}{2}(M_2 - M_1) s_{2w} & M_2 c_w^2 + M_1 s_w^2 & 0 & m_Z \\ 0 & 0 & -\mu s_{2\beta} & -\mu c_{2\beta} \\ 0 & m_Z & -\mu c_{2\beta} & \mu s_{2\beta} \end{pmatrix} .
\end{aligned} \tag{3.41}$$

This is a 4×4 matrix, hence an analytic diagonalization with a unitary matrix S is in general possible [58–61]. In order to get positive mass eigenvalues one has to multiply a diagonal unitary matrix P. Therefore the diagonalization is done by multiplying the matrices

$$P^* S^* M_{\chi^0_{R_p}} S^\dagger P^\dagger = \operatorname{diag}\left(m_{\chi^0_{1R_p}}, m_{\chi^0_{2R_p}}, m_{\chi^0_{3R_p}}, m_{\chi^0_{4R_p}} \right) . \tag{3.42}$$

The mass eigenstate is then given by

$$\chi^{0,\mathrm{mass}}_{R_p} = P S {\chi^0_{R_p}}' . \tag{3.43}$$

The complete formulas as well as the proceeding how to derive them are written down in Appendix C.2. However, the formulas are in general very complex and not very helpfull. Therefore an approximating diagonalization for small mixing gives more insight [62]. In this approach, where $m_Z \ll \mu$, one gets to first order

$$\begin{aligned}
m_{\chi^0_{1R_p}} &= M_1 - \frac{m_Z^2 \sin^2\theta_w \,(M_1 + \mu \sin 2\beta)}{\mu^2 - M_1^2} + \dots\,, \\
m_{\chi^0_{2R_p}} &= M_2 - \frac{m_Z^2 \cos^2\theta_w \,(M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \dots\,, \\
\left.\begin{matrix} m_{\chi^0_{3R_p}} \\ m_{\chi^0_{4R_p}} \end{matrix}\right\} &= \begin{cases} |\mu| + \dfrac{m_Z^2\,(\mathrm{sgn}\,(\mu) - \sin 2\beta)\,(\mu + M_{\tilde\gamma})}{2\,(\mu + M_1)\,(\mu + M_2)} + \dots\,, \\ |\mu| + \dfrac{m_Z^2\,(\mathrm{sgn}\,(\mu) + \sin 2\beta)\,(\mu - M_{\tilde\gamma})}{2\,(\mu - M_1)\,(\mu - M_2)} + \dots\,. \end{cases}
\end{aligned} \tag{3.44}$$

Here $M_{\tilde\gamma} = M_1 \cos^2\theta_w + M_2 \sin^2\theta_w$ is the photino mass, a gauge eigenstate mass parameter that proves helpful to parametrize the weak mixing angle θ_w. For the case $M_1 < M_2 < |\mu|$ the neutralino masses $m_{\chi^0_{1R_p}} < m_{\chi^0_{2R_p}} < m_{\chi^0_{3R_p}} < m_{\chi^0_{4R_p}}$ are sorted by their size.

3.3.2 Neutrino Mass Generation via Majorana Masses

In the R-parity conserving theory, left-handed neutrino masses must be generated with the help of right-handed neutrinos via the seesaw mechanism sketched in Appendix G.1.

The scalar potential for left and right-handed neutrinos is

$$\begin{aligned}
V_{\text{scalar}} &= \left(\tfrac{1}{2} M_{ij} {\nu^c}_i {\nu^c}_j + \lambda^\nu_{ij} h^0_u \nu_i {\nu^c}_j + \text{h.c.}\right) \\
&= \tfrac{1}{2}\left(\nu_i {\nu^c}_i\right) M_\nu \begin{pmatrix} \nu_j \\ {\nu^c}_j \end{pmatrix} + \text{h.c.} \\
&= \tfrac{1}{2}\left(\nu_i {\nu^c}_i\right) \begin{pmatrix} 0 & {\lambda^\nu_{ij}}^T h^0_u \\ \lambda^\nu_{ij} h^0_u & M_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ {\nu^c}_j \end{pmatrix} + \text{h.c.}\,.
\end{aligned} \tag{3.45}$$

If M_{ij} is much larger then the weak scale one can calculate the neutrino masses directly from this scalar potential. Due to $\lambda^\nu_{ij} h^0_u \ll M_{ij}$ the seesaw mechanism can be used. This gives the neutrino mass matrix

$$M^\nu_{ij} = -\lambda^\nu_{ik} h^0_u M^{-1}_{kl} {\lambda^\nu_{lj}}^T h^0_u\,. \tag{3.46}$$

This result is well known from SM physics [4, 5]. For an overview in the SM see [3, 63], for the supersymmetric case see [46].

3.3.3 Neutrino Mass Generation via *R*-Parity Violation

In the R-parity violating case at least one neutrino acquires a mass at tree level without the help of right-handed neutrinos. The other two neutrinos become massive at one or two loop level, respectively. This effect is based on the mixing of the neutrinos with other neutralinos via the bilinear R-parity violating parameter μ_i and the sneutrino VEV v_i.

As an example how to calculate a supersymmetric fermionic Lagrangian we perform the calculation for the R-parity breaking neutralino Lagrangian explicitly. The gauge eigenstate basis gets extended to

$$\chi^0_{\not{R}_p} = \left(\widetilde{B}, \widetilde{W}^3, \widetilde{h}^0_u, \nu_\alpha\right)^T = \left(\widetilde{B}, \widetilde{W}^3, \widetilde{h}^0_u, \widetilde{h}^0_d, \nu_i\right)^T . \tag{3.47}$$

The F-terms for the fermions come from inserting the superpotential (2.9) into the second to last term in the Lagrangian (1.6).

$$V_F = \tfrac{1}{2}\left(\mu_\alpha \widetilde{h}^0_u \nu_\alpha + \text{h.c.}\right) \tag{3.48}$$

The D-terms come from the first two terms of the Lagrangian (1.12) which is needed to combine the chiral supermultiplet with the vector multiplet in a supersymmetric way. The D-terms are

$$\begin{aligned} V_D = {} & \tfrac{1}{2} g_1 v_u \widetilde{h}^0_u \widetilde{B} + \tfrac{1}{2} g_1 v_u \widetilde{B}^* \widetilde{h}^0_u - \tfrac{1}{2} g_1 {v_\alpha}^T \nu_\alpha \widetilde{B} - \tfrac{1}{2} g_1 v_\alpha \widetilde{B}^* {\nu_\alpha}^\dagger \\ & - \tfrac{1}{2} g_2 v_u \widetilde{h}^0_u \widetilde{W}^3 - \tfrac{1}{2} g v_u \widetilde{W}^{3\dagger} \widetilde{h}^0_u + \tfrac{1}{2} g_2 {v_\alpha}^T \nu \widetilde{W}^3 + \tfrac{1}{2} g_2 v_\alpha \widetilde{W}^{3\dagger} {\nu_\alpha}^\dagger . \end{aligned} \tag{3.49}$$

Finally the soft susy breaking terms are

$$-\mathcal{L}^{\text{soft}} = \left(\tfrac{1}{2} M_1 \widetilde{B}\widetilde{B} + \tfrac{1}{2} M_2 \widetilde{W}^3 \widetilde{W}^3 + \text{h.c.}\right) . \tag{3.50}$$

Hence the R-parity violating neutralino Lagrangian is as in the R-parity conserving case (3.38)

$$-\mathcal{L}_{\text{neutralino}} = \tfrac{1}{2} {\chi^0_{\not{R}_p}}^T M_{\chi^0_{\not{R}_p}} \chi^0_{\not{R}_p} , \tag{3.51}$$

where the mass matrix is given by

$$M_{\chi^0_{\not{R}_p}} = \begin{pmatrix} M_1 & 0 & \frac{1}{2} g_1 v_u & -\frac{1}{2} g_1 {v_\beta}^T \\ 0 & M_2 & -\frac{1}{2} g_2 v_u & \frac{1}{2} g_2 {v_\beta}^T \\ \frac{1}{2} g_1 v_u & -\frac{1}{2} g_2 v_u & 0 & -{\mu_\beta}^T \\ -\frac{1}{2} g_1 v_\alpha & \frac{1}{2} g_2 v_\alpha & -\mu_\alpha & 0_{\alpha\beta} \end{pmatrix} . \tag{3.52}$$

Here $0_{\alpha\beta}$ means a 4 times 4 matrix containing only zeros. This matrix has a 3 times 3 submatrix 0_{ij} which reveals that the neutrinos do not have a direct mass term. In the case where the diagonal terms containing μ_i and v_i

are much smaller than the R-parity conserving submatrix (3.39) the seesaw mechanism can be used and gives

$$M^{\nu}_{ij} = -m^T M^{-1}_{\chi^0_{R_p}} m = \frac{M_{\widetilde{\gamma}} m_Z^2}{\det M_{\chi^0_{R_p}}} \frac{\cos^2\beta}{\tan^2\beta} a_i a_j \; . \tag{3.53}$$

The previous expression introduces the misalignment vector

$$a_i = \left(\mu \frac{v_i}{v_d} - \mu_i\right) \tan\beta \; . \tag{3.54}$$

The determinant of the R-parity conserving neutralino mass matrix

$$\det M_{\chi^0_{R_p}} = \mu \left(M_{\widetilde{\gamma}} m_Z^2 \sin 2\beta - M_1 M_2 \mu\right) \tag{3.55}$$

emerges due to the relation $A^{-1} = A^{\dagger} \det^{-1} A$.

The matrix (3.53) has two massless neutrino eigenvalues. The third eigenvalue of this matrix built from the misalignment vector (3.54) equals

$$|a_i|^2 = \mu^2 \left|\widehat{\epsilon}_i - \epsilon_i\right|^2 \tan^2\beta \; , \tag{3.56}$$

where the parameter $\epsilon_i = \frac{\mu_i}{\mu}$ is introduced. The product of the nonzero eigenvalues of the R-parity violating neutral fermion mixing matrix defines the reduced determinant $\det' M_{\chi^0_{\not R_p}}$. Therefore the neutrino mass is given by

$$m^{\not R_p}_{\nu_3} = \frac{\det' M_{\chi^0_{\not R_p}}}{\det M_{\chi^0_{R_p}}} = \frac{M_{\widetilde{\gamma}} m_Z^2 \mu^2 \left|\widehat{\epsilon}_i - \epsilon_i\right|^2}{\left|\mu \left(M_{\widetilde{\gamma}} m_Z^2 \sin 2\beta - M_1 M_2 \mu\right)\right|} \cos^2\beta \; . \tag{3.57}$$

This is the most general formula for the neutrino mass induced by R-parity violation as, for instance, in [15, 41, 51, 64, 65]. Other authors use a one neutrino model [50] or stay in the four component-notation [45, 47, 49, 66–68]. In the latter case, however, one has to introduce new notations like a misalignment angle in order to describe the three-component neutrino mass in the four-component framework.

Finally one has to rotate away either the sneutrino VEV v_i [69, 70] or the bilinear coupling μ_i as we will do in Section 4.3 (see there for references).

3.3.4 Neutrino Mass Generation via Majorana Neutrinos and *R*-Parity Violation

The neutralino Lagrangian including right-handed neutrinos

$$\chi^0_{\not R_p N^c} = \left(\widetilde{B}, \widetilde{W}^0, \widetilde{h}^0_u, {\nu^c}_i, \nu_\alpha\right)^T = \left(\widetilde{B}, \widetilde{W}^0, \widetilde{h}^0_u, {\nu^c}_i, h^0_d, \nu_i\right)^T \tag{3.58}$$

can again be expressed by a formula analog to (3.38)

$$-\mathcal{L}_{\chi^0}^{R\!\!\!/_p N^c} = \tfrac{1}{2} {\chi^0_{R\!\!\!/_p N^c}}^T M_{\chi^0_{R\!\!\!/_p N^c}} \chi^0_{R\!\!\!/_p N^c} \;, \tag{3.59}$$

but the mixing matrix is extended. Going directly into the photino-zino basis and staying in the four-component notation for the Higgsino-sneutrino fields, the mixing matrix reads

$$M_{\chi^0_{R\!\!\!/_p N^c}}{}' = \begin{pmatrix} M_1 c_w^2 + M_2 s_w^2 & \frac{1}{2}(M_2 - M_1) s_{2w} & 0 & 0_j & 0_\beta \\ \frac{1}{2}(M_2 - M_1) s_{2w} & M_2 c_w^2 + M_1 s_w^2 & -m_Z \frac{v_u}{v} & 0_j & m_Z \frac{v_\alpha}{v}^T \\ 0 & -m_Z \frac{v_u}{v} & 0 & {\lambda^\nu_{\gamma j}}^\dagger {\widetilde{\nu}_\gamma}^T & {\lambda^\nu_{\beta k}}^\dagger {\widetilde{\nu}^c{}_k}^T - {\mu_\beta}^\dagger \\ 0_i & 0_i & \lambda^\nu_{\gamma i} \widetilde{\nu}_\gamma & M_{ij} & {\lambda^\nu_{\beta i}}^\dagger h_u^0 \\ 0_\alpha & m_Z \frac{v_\alpha}{v} & \lambda^\nu_{\alpha k} \widetilde{\nu}^c{}_k - \mu_\alpha & \lambda^\nu_{\alpha j} h_u^0 & 0_{\alpha\beta} \end{pmatrix} . \tag{3.60}$$

Because of the hierarchy

$$\lambda^\nu_{ij} \widetilde{\nu}^c{}_i \ll \lambda^\nu_{ij} \widetilde{\nu}_i \ll \lambda^\nu_{ij} h_u^0 \tag{3.61}$$

in the matrix and the fact that we have set $\lambda_i^\nu h_d^0 = 0$, it is justified to give first mass to the left-handed neutrinos via the SM seesaw mechanism and then calculate the mass contribution to the neutrino mass from the R-parity violating couplings. This procedure results in

$$M_{ij}^{R\!\!\!/_p + N^c} = \frac{m^\nu_{R\!\!\!/_p} a_i a_j + M^\nu_{ij} \left|a\right|^2 + 2 m^\nu_{R\!\!\!/_p} M^\nu_{ij} \widehat{\epsilon}_k a_k}{\left|a\right|^2 - m^\nu_{R\!\!\!/_p} \widehat{\epsilon}_i M^\nu_{ij} \widehat{\epsilon}_j} \;, \tag{3.62}$$

where we neglected the small contribution of the right-handed sneutrino VEVs and the small coupling λ_i^ν. In the R-parity conserving limit this expression becomes the standard seesaw formula (3.46). In the $M_{ij} \to \infty$ limit it becomes the R-parity violating formula (3.57). Therefore Equation (3.62) generalizes the partial results in the literature.

Chapter 4

Constraining SUSY with Bilinear *R*-Parity Breaking

In order to fulfill the experimental bounds it is necessary to constrain the R-parity violating couplings. In this thesis we work in the bilinear R-parity violating model.

In the first two sections of this chapter we give the reasons for this choice and the constraints it is subject to. In the third section we will derive the relevant terms of the bilinear R-parity breaking and calculate two decays in the bilinear R-parity violating model in section four and five.

4.1 Proton Decay

The R-parity violating couplings are strongly constrained by experiment [9]. That is why most work is done in the R-parity conserving theory. The strongest constraint comes from the longevity of the proton.

The coupling $\lambda'_{ijm}{\lambda''_{1j1}}^{\dagger}$ with $m = 1, 2$ would lead to proton decay via tree-level down squark exchange as shown in Figure 4.1. The decay rate would

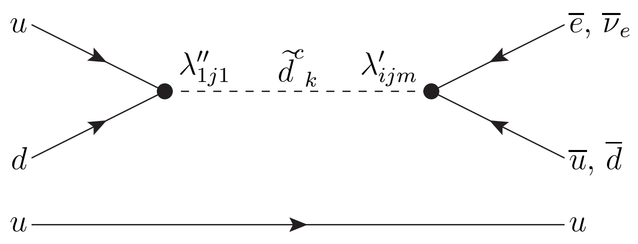

Figure 4.1: R-parity violating proton decay channel.

be much larger [9] than the limits given by observation [1] unless $\left|\lambda'_{ijm}\lambda''^{\dagger}_{1j1}\right|$ is smaller than $\mathcal{O}\left(10^{-26}\right)$ for squark masses around 300 GeV. This strong constraint can be dealt with by finding a model where $\lambda''_{ijk} = 0$. In such a case only lepton number violating couplings are present in the superpotential.

4.2 Gravitino Dark Matter

In this section we recollect the constraints on R-parity violation due to gravitino dark matter [21, 71] and cosmology. For a rough overview about cosmology see Appendix F. For an introduction see, for instance [11].

Thermal leptogenesis [13] is an attractive theory to explain the observed asymmetry between baryons and antibaryons [11]:

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.1 \pm 0.3) \times 10^{-10} \, . \tag{4.1}$$

In thermal leptogenesis heavy Majorana neutrinos generate a $B - L$ asymmetry, which can be converted into a baryon asymmetry via sphaleron processes [72]. Thermal leptogenesis requires a high reheating temperature $T_R \simeq 10^{10}$ GeV after inflation.

So far we have only dealt with the particles that are added by SUSY. Supergravity adds the gravitino to the mass spectrum, which can cause some problems, as explained in the following.

In the early universe the gravitino is in equilibrium with the thermal bath. Calculation of the relic density shows that the gravitino density would be larger than the critical density, hence the density parameter would be $\Omega_{3/2} > 1$, and the universe would overclose. Postulating inflation dilutes the gravitinos, but they are reproduced in the reheating phase after inflation. This generates the gravitino density [73]

$$\Omega_{3/2} h^2 \simeq 0.3 \left(\frac{T_R}{10^{10}\,\mathrm{GeV}}\right) \left(\frac{m_{3/2}}{100\,\mathrm{GeV}}\right)^{-1} \left(\frac{m_{\widetilde{g}}}{1\,\mathrm{TeV}}\right) \, . \tag{4.2}$$

Gravitino DM is compatible with thermal leptogenesis, since for sensible values of the gluino mass $m_{\widetilde{g}} \simeq \mathcal{O}\left(1\,\mathrm{TeV}\right)$, a reheating temperature of order 10^{10} GeV is needed in order to explain DM solely with thermal gravitinos. The gravitino mass has to be of order 100 GeV to cause the observed DM amount.

Without R-parity breaking the gravitino has a lifetime of [74]

$$\tau_{3/2} \sim 3 \text{ years} \left(\frac{m_{3/2}}{100\,\mathrm{GeV}}\right)^{-3} \, , \tag{4.3}$$

due to the Planck mass suppression if it is not the LSP. The late decay of the gravitino can spoil the successful predictions of the big bang nucleosynthesis [14]. In order to avoid this, the reheating temperature must be below 10^5 GeV. However, this requirement is ruled out in a thermal leptogenesis model.

If the gravitino is the LSP, the NLSP can only decay into the gravitino in R-parity conserving theories. This decay, being also suppressed by the Planck mass, leads to an NLSP lifetime of [6]

$$\tau_{\mathrm{NLSP}} \sim 9 \text{ days} \left(\frac{m_{3/2}}{10\,\mathrm{GeV}}\right)^2 \left(\frac{m_{\mathrm{NLSP}}}{150\,\mathrm{GeV}}\right)^{-5} . \tag{4.4}$$

This can be circumvented with R-parity breaking because then the NLSP can decay via the R-parity violating channels. In order to avoid the dilution of an existing baryon asymmetry in the early universe before the electroweak phase transition the R-parity violating couplings must fulfill [75–77]

$$\lambda, \lambda' < 10^{-7} . \tag{4.5}$$

This is a sufficient condition, which can be relaxed for some flavor structures. On the other hand the slow decay of the NLSP can affect BBN. In order to make sure that the decay happens fast enough, the couplings must fulfill [21]

$$10^{-14} < \lambda, \lambda' . \tag{4.6}$$

Summing up, thermal leptogenesis demands that R-parity violation must be small, but cannot be absent.

4.3 Bilinear *R*-Parity Breaking

In the bilinear R-parity breaking SUSY only bilinear R-parity violating terms are nonzero at the beginning, hence the superpotential is composed of the MSSM Yukawa couplings, the right handed neutrino terms as well as the bilinear R-parity violating term.

$$\begin{aligned} W_{\mathrm{bil}} = \mu H_u H_d &+ \lambda^e_{ij} H_d L_i {E^c}_j + \lambda^d_{ij} H_d {D^c}_i Q_j - \lambda^u_{ij} H_u Q_i {U^c}_j \\ &+ \tfrac{1}{2} M_{ij} {N^c}_i {N^c}_j - \lambda^\nu_{ij} H_u L_i {N^c}_j + \mu_i H_u L_i \, . \end{aligned} \tag{4.7}$$

Using the freedom (2.6) to rotate H_d and L_i with $\epsilon_i = \frac{\mu_i}{\mu}$ via

$$H_d = {H_d}' - \epsilon_i {L_i}' \, , \qquad\qquad L_i = {L_i}' + \epsilon_i {H_d}' \, , \tag{4.8}$$

gives the rotated superpotential

$$\begin{aligned} W'_{\text{bil}} = &\lambda^e_{ij} H_d' L_i' E^c{}_j + \lambda^d_{ij} H_d' D^c{}_i Q_j - \lambda^u_{ij} H_u Q_i U^c{}_j - \lambda^\nu_{ij} H_u L_i N^c{}_j \\ &- \epsilon_i \lambda^e_{jk} L_i' L_j' E^c{}_k - \epsilon_i \lambda^d_{jk} L_i' D^c{}_j Q_k - \epsilon_i \lambda^\nu_{ij} H_u H_d' N^c{}_j \\ &+ \mu H_u H_d' + \tfrac{1}{2} M_{ij} N^c{}_i N^c{}_j + \mathcal{O}\left(\epsilon^2\right) \ . \end{aligned} \tag{4.9}$$

The term containing $H_d' H_d'$ vanishes because of the SU(2) symmetry. This superpotential contains the lepton number breaking trilinear couplings as well as the R-parity breaking right-handed neutrino coupling

$$\lambda_{ijk} = \epsilon_i \lambda^e_{jk} \ , \qquad \lambda'_{ijk} = \epsilon_i \lambda^d_{jk} \ , \qquad \lambda^\nu_j = \epsilon_i \lambda^\nu_{ij} \ . \tag{4.10}$$

Combining the constraint (4.5) with the requirement, that the Yukawa-couplings together with the Higgs VEVs have to amount to the masses of the third generation, gives an upper limit on the bilinear R-parity breaking parameter ϵ_i [21],

$$\left(\frac{\epsilon_i}{10^{-6}}\right)\left(\frac{\tan\beta}{10}\right) \lesssim 1 \ . \tag{4.11}$$

In the superpotential (4.9) the bilinear lepton number breaking term μ_i is absent by construction. As the superpotential (4.7) does not contain any baryon number breaking term, λ''_{ijk} could not emerge. Furthermore they are not reintroduced via RGE running because the RGE for λ''_{ijk} (E.12) is self-contained and the RGE for μ_i (E.8) does only depend on terms that are proportional to μ_i in the bilinear R-parity breaking as shown in Appendix E.3. Hence the sneutrino vacuum expectation value is the only bilinear R-parity violating term and the proton is stable.

We neglect the right-handed sneutrino VEV, because of its double suppression first in the mass term M_{ij} and second in ϵ_i.

The relevant bilinear soft SUSY breaking Lagrangian is

$$\begin{aligned} -\mathcal{L}^{\text{bil}}_{\text{soft}} = &\widetilde{m}^2_u {h_u}^\dagger h_u + \widetilde{m}^2_d {h_d}^\dagger h_d + (m^2_{\widetilde{L}})_{ij} {\widetilde{L}_i}^\dagger \widetilde{L}_j \\ &+ \Big(A^e_{ij} h_d \widetilde{L}_i \widetilde{l}^c{}_j + A^d_{ij} h_d \widetilde{d}^c{}_i \widetilde{Q}_j + A^u_{ij} h_u \widetilde{Q}_i \widetilde{u}^c{}_j + A^\nu_{ij} h_u \widetilde{L}_i \widetilde{\nu}^c{}_j \\ &+ B h_u h_d + B_i h_u \widetilde{L}_i + \widetilde{m}^2_{di} {h_d}^\dagger \widetilde{L}_i + \text{h.c.} \Big) + \dots \ . \end{aligned} \tag{4.12}$$

After rotating according to (4.8) the Lagrangian becomes

$$\begin{aligned} -\mathcal{L}'^{\text{bil}}_{\text{soft}} = &\widetilde{m}^2_u {h_u}^\dagger h_u + \widetilde{m}^2_{d'} {h'_d}^\dagger h'_d + (m^2_{\widetilde{L}'})_{ij} {\widetilde{L}_i}'^\dagger {\widetilde{L}_j}' \\ &+ \Big(A^e_{ij} h_d' \widetilde{L}_i' \widetilde{l}'^c{}_j + A^d_{ij} h_d' \widetilde{d}^c{}_i \widetilde{Q}_j + A^u_{ij} h_u \widetilde{Q}_i \widetilde{u}^c{}_j + A^\nu_{ij} h_u \widetilde{L}_i \widetilde{\nu}^c{}_j \\ &- \epsilon_i A^e_{jk} {\widetilde{L}_i}' {\widetilde{L}_j}' \widetilde{l}'^c{}_k + \epsilon_i A^d_{jk} {\widetilde{L}_i}' \widetilde{d}^c{}_j \widetilde{Q}_k + \epsilon_i A^\nu_{ij} h_u h_d' \widetilde{\nu}^c{}_j \\ &+ B' h_u h'_d + B'_i h_u {\widetilde{L}_i}' + \widetilde{m}^2_{d'i} {h_d}'^\dagger {\widetilde{L}_i}' + \mathcal{O}\left(\epsilon^2\right) + \text{h.c.} \Big) + \dots \ . \end{aligned} \tag{4.13}$$

The rotated bilinear soft SUSY breaking terms are defined by

$$\widetilde{m}^2_{d'} = \widetilde{m}^2_d + \epsilon_i \widetilde{m}^2_{di} + \mathcal{O}\left(\epsilon^2\right) , \tag{4.14}$$

$$(m^2_{\widetilde{L}'})_{ij} = (m^2_{\widetilde{L}})_{ij} - \epsilon_j \widetilde{m}^2_{di} + \mathcal{O}\left(\epsilon^2\right) , \tag{4.15}$$

$$B' = B + \epsilon_i B_i , \tag{4.16}$$

$$B'_i = B_i - \epsilon_i B , \tag{4.17}$$

$$\widetilde{m}^2_{d'i} = \widetilde{m}^2_{di} + \epsilon_j (m^2_{\widetilde{L}})_{ji} - \epsilon_i \widetilde{m}^2_d + \mathcal{O}\left(\epsilon^2\right) . \tag{4.18}$$

Again the trilinear lepton number breaking terms are reintroduced:

$$A_{ijk} = \epsilon_i A^e_{jk} , \qquad A'_{ijk} = \epsilon_i A^d_{jk} , \qquad A^\nu_j = \epsilon_i A^\nu_{ij} . \tag{4.19}$$

In the bilinear R-parity breaking theory the sneutrino VEV (3.21) simplifies to [78–80]

$$\widehat{\epsilon}_i = \frac{v_i}{v_d} \simeq \frac{B'_i \tan\beta - \widetilde{m}^{2\,*}_{d'i}}{\widetilde{m}^2_{i'i'} + \frac{1}{2} m^2_Z \cos 2\beta} . \tag{4.20}$$

The neutrino mass induced by R-parity violation is [78–81]

$$m^{\not{R}_p}_{\nu_3} \simeq \frac{\mu M_{\widetilde{\gamma}} m^2_Z \cos^2\beta}{\left|\mu M_1 M_2 - M_{\widetilde{\gamma}} m^2_Z \sin 2\beta\right|} \left|\widehat{\epsilon}_i\right|^2 . \tag{4.21}$$

The general neutrino mass (3.62) in bilinear R-parity breaking is

$$M^{\not{R}_p + N^c}_{ij} \simeq \frac{m^\nu_{\not{R}_p} \mu^2 \widehat{\epsilon}_i \widehat{\epsilon}_j + M^\nu_{ij} \mu^2 \left|\widehat{\epsilon}_k\right|^2 + 2\mu m^\nu_{\not{R}_p} M^\nu_{ij} \left|\widehat{\epsilon}_k\right|^2 \cot\beta}{\mu^2 \tan^2\beta \left|\widehat{\epsilon}_i\right|^2 - m^\nu_{\not{R}_p} \widehat{\epsilon}_i M^\nu_{ij} \widehat{\epsilon}_j} \tan^2\beta . \tag{4.22}$$

These formulas are bilinear results derived from the most general equations given in Chapter 3. They are valid as long as the bilinear breaking parameter is constrained by (4.11). In order to get an overview about the behavior of these values we will simplify the parameter further in Chapter 5.

4.4 Gravitino Decay into Photon and Neutrino

As shown in Section 4.2 the gravitino must be the LSP and the R-parity violating couplings must be small in order to be compatible with standard cosmology. Hence the R-parity violating effects can be calculated as a perturbation of the R-parity conserving theory. Therefore we can calculate the gravitino ψ decay via R-parity conserving couplings into a photon γ and a

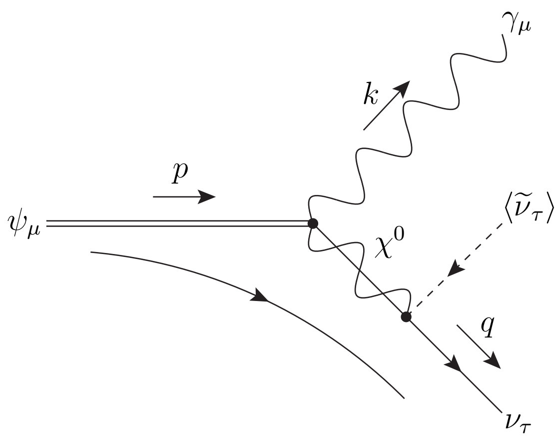

Figure 4.2: The gravitino decays into a photon and a neutrino via an intermediate neutralino coupling to the sneutrino VEV.

virtual photino $\widetilde{\gamma}$. The latter mixes immediately to a zino $\widetilde{Z}$ which in turn mixes via the R-parity violating sneutrino VEV υ_τ into a τ-neutrino ν_τ. That the coupling to the third family is the largest will be shown in Chapter 5. The Feynman diagram of this process can be found in Figure 4.2. The calculation has been done before in, for instance,[54, 71, 82–84]. Hence we give just a rough overview about the calculation. The amplitude of this process reads in four spinor-notation

$$\begin{aligned} i\mathcal{M} = &-\overline{u}^r(q)i\sqrt{2}\,\langle\widetilde{\nu}_\tau\rangle\left(g_2\tfrac{1}{2}\sigma_{3,11}\cos\theta_w - g_1 Y_{\nu L}\sin\theta_w\right)P_R \\ &\times\left(\sum_{i,j,\alpha=1}^{4} S_{\widetilde{Z}i}{}^{*}P_{i\alpha}{}^{*}\frac{i\left(\not{q}+m_{\chi^0_\alpha}\right)}{q^2-m^2_{\chi^0_\alpha}}P_{\alpha j}S_{j\widetilde{\gamma}}\right)\frac{i}{4}\frac{1}{M_P}\gamma^\mu\left[\not{k},\gamma^\rho\right]\psi^{+s}_\mu(p)\epsilon^{\lambda *}_\rho(k) \\ \simeq &-\frac{i}{8\sqrt{2}}\frac{1}{M_P}g_Z\,\langle\widetilde{\nu}_\tau\rangle\left(\sum_{\alpha=1}^{4}\frac{S_{\widetilde{Z}\alpha}{}^{*}S_{\alpha\widetilde{\gamma}}}{m_{\chi^0_\alpha}}\right) \\ &\times\overline{u}^r(q)\left(1+\gamma^5\right)\gamma^\mu\left[\not{k},\gamma^\rho\right]\psi^{+s}_\mu(p)\epsilon^{\lambda *}_\rho(k)\,. \end{aligned} \tag{4.23}$$

For details of the four-spinor notation see Appendix B.2 and [2]. For details of the continuous fermion flow used in Figure 4.2 see [85]. By virtue of the properties of the Dirac γ-matrices the square of the amplitude reduces to

$$|\mathcal{M}|^2 \simeq \frac{1}{12}\widehat{\epsilon}^2_\tau\left|U_{\widetilde{\gamma}\widetilde{Z}}\right|^2\frac{\left(m^2_{3/2}-m^2_\nu\right)^2\left(3m^2_{3/2}+m^2_\nu\right)}{m^2_{3/2}M^2_P}\cos^2\beta\,. \tag{4.24}$$

Finally the decay width for the decay into photon and (anti) neutrino is

$$\Gamma(\psi \to \gamma\nu_\tau) \simeq \frac{1}{32\pi}\widehat{\epsilon}_\tau^2 \left|U_{\widetilde{\gamma}\widetilde{Z}}\right|^2 \frac{m_{3/2}^3}{M_P^2}\cos^2\beta \; . \tag{4.25}$$

In this calculation we have introduced the photino-zino mixing parameter

$$\left|U_{\widetilde{\gamma}\widetilde{Z}}\right| = m_Z \sum_{\alpha=1}^{4} \frac{S_{\alpha\widetilde{Z}}{}^{\dagger} S_{\alpha\widetilde{\gamma}}}{m_{\chi_\alpha^0}} \; , \tag{4.26}$$

which contains the neutralino diagonalization matrices (3.42) for the zino and the photino in the R-parity conserving case:

$$\widetilde{\gamma} = \sum_{i,\alpha=1}^{4} S_{\widetilde{\gamma}i}{}^{*} P_{i\alpha}{}^{*} \chi_\alpha^0 \; , \qquad \widetilde{Z} = \sum_{i,\alpha=1}^{4} S_{\widetilde{Z}i}{}^{*} P_{i\alpha}{}^{*} \chi_\alpha^0 \; . \tag{4.27}$$

The mixing parameter becomes in the weak coupling limit $m_Z \to 0$

$$\left|U_{\widetilde{\gamma}\widetilde{Z}}\right| \simeq m_Z \frac{|M_1 - M_2|}{2M_1M_2} \sin 2\theta_w \; . \tag{4.28}$$

The Higgsino mass modifies this only in the third order. The previous result can be easily improved by using numerical diagonalization, but the analytical formula has the advantage, that one can keep track of the dominant dependencies. Equation (4.28) is a more elegant solution than the often used more heuristic formula, depending on the neutralino mass, as in, for instance, [71] or the solution which comes from a more simple approach [82].

Taking the GUT relation $M_1 = M_2$ one is led to the low energy relation

$$M_2 = \tfrac{3}{5}\cot^2\theta_w M_1 \simeq 2.2 \; M_1 \tag{4.29}$$

for the soft SUSY breaking neutralino masses. In this case the mixing parameter simplifies to

$$\begin{aligned} \left|U_{\widetilde{\gamma}\widetilde{Z}}\right| &\simeq \frac{1}{3}\frac{m_Z}{M_1}\,|1 - 4\cos 2\theta|\tan\theta \\ &\simeq 0.128 \left(\frac{M_1}{150\,\mathrm{GeV}}\right)^{-1} \; . \end{aligned} \tag{4.30}$$

The gravitino lifetime becomes

$$\tau_{3/2}^{\text{2-body}} = 2.08 \times 10^{27}\,\mathrm{s} \left(\frac{\widehat{\epsilon}_\tau}{10^{-7}}\right)^{-2} \left(\frac{M_1}{150\,\mathrm{GeV}}\right)^{2} \left(\frac{m_{3/2}}{10\,\mathrm{GeV}}\right)^{-3} \; . \tag{4.31}$$

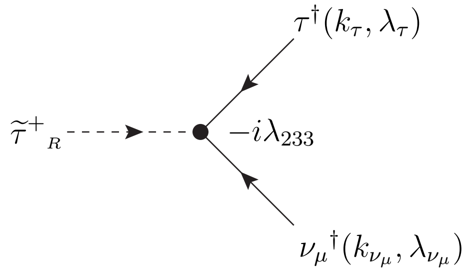

Figure 4.3: The right-handed $\widetilde{\tau}$-slepton decays into a right-handed τ-lepton and an anti μ-neutrino.

The decay width (4.25) depends only on the soft SUSY breaking neutralino masses and the gravitino mass. It is suppressed by the square of the Planck mass as well as the square of the τ-sneutrino VEV. This double suppression makes the gravitino, once produced, very long-lived as can be observed in the lifetime formula (4.31). As it might be generated in a hot phase after inflation, it can still be present in our universe, and can be responsible for the observed amount of DM.

Recent results of Fermi LAT give a preliminary lower bound of about

$$\tau \gtrsim 10^{29}\,\mathrm{s} \tag{4.32}$$

on the lifetime of a DM candidate which decays into photons [86]. Hence we are able to further constrain one of the parameters in the gravitino lifetime (4.31). If we do not change, for instance, $m_{3/2}$ and M_1, we can constrain the sneutrino VEV parameter to be

$$\widehat{\epsilon}_\tau \lesssim 10^{-8}\ . \tag{4.33}$$

For the case where $\widehat{\epsilon} \approx \epsilon$ this is one order of magnitude better than the constraint coming from non-erasure of primordial baryogenesis.

4.5 Scalar τ-Lepton Decay into τ-Lepton and Neutrino

For large parameter ranges the $\widetilde{\tau}$-slepton is the NLSP [87]. It can decay very slowly via R-parity conserving channels into the gravitino and SM particles, however due to the huge suppression by the Planck mass this decay conflicts with BBN (see Section 4.2).

A possible R-parity breaking channel is the decay into a τ-lepton and an anti μ-neutrino, as shown in Figure 4.3 and calculated, for instance, in [27]. Why exactly this decay channel is dominant is shown in Chapter 5.

Inserting the R-parity violating superpotential (2.3) into the chiral Lagrangian (1.6) gives for the trilinear lepton coupling λ_{ijk} the Lagrangian which couples one scalar field to two fermions

$$-\mathcal{L}_{LLE^c} = \lambda_{ijk}\left(\nu_i l_j \widetilde{l}^c{}_k + \nu_i \widetilde{l}_j l^c{}_k + \widetilde{\nu}_i l_j l^c{}_k\right) + \text{h.c.} \tag{4.34}$$

The Feynman diagram in Figure 4.3 shows an incoming scalar field that couples with $-i\lambda_{233}$ to two outgoing fermions, y_τ and y_{ν_μ} where the external commuting two-component spinor wave functions are $y = y(\vec{k}, \lambda)$. For details of the notation see Appendix B.1, in particular Equation (B.7), and [27]. Why we take this flavor structure to be the dominant decay channel will be explained in the next chapter. The amplitude is given by

$$i\mathcal{M} = -i\lambda_{233} y_\tau y_{\nu_\mu} \tag{4.35}$$

and squared amplitude is

$$|\mathcal{M}|^2 = |\lambda_{233}|^2 \, y_\tau{}^\dagger y_{\nu_\mu}{}^\dagger y_\tau y_{\nu_\mu} \, . \tag{4.36}$$

The sum over the spins gives

$$\sum_{\lambda_\tau, \lambda_{\nu_\mu}} |\mathcal{M}|^2 = |\lambda_{233}|^2 \left(m^2_{\widetilde{\tau}^c} - m^2_\tau - m^2_{\nu_\mu}\right) \, . \tag{4.37}$$

It follows that the decay rate is

$$\begin{aligned} \Gamma\left(\widetilde{\tau}^c \to \tau\nu_\mu\right) &= \frac{1}{16\pi}\frac{1}{m_{\widetilde{\tau}^c}}\left(\sum_{\lambda_\tau,\lambda_{\nu_\mu}} |\mathcal{M}|^2\right) \\ &= \tfrac{1}{16\pi} |\lambda_{233}|^2 \, m^{-1}_{\widetilde{\tau}^c}\left(m^2_{\widetilde{\tau}^c} - m^2_\tau - m^2_{\nu_\mu}\right) \\ &\simeq \tfrac{1}{16\pi} |\lambda_{233}|^2 \, m_{\widetilde{\tau}^c} \, . \end{aligned} \tag{4.38}$$

The leptonic decay length of the right-handed $\widetilde{\tau}$-slepton is then approximated by [21]

$$c\tau^{\text{lep}}_{\widetilde{\tau}} = 25\,\text{cm}\left(\frac{m_{\widetilde{\tau}}}{200\,\text{GeV}}\right)^{-1}\left(\frac{\epsilon_2}{10^{-7}}\right)^{-2}\left(\frac{\tan\beta}{10}\right)^{-2} \, . \tag{4.39}$$

For this approximation we have taken into account the decays $\widetilde{\tau}_R \to \tau\nu_\mu, \mu\nu_\tau$. One can see that this decay if it happend in a detector would be observable as a displaced vertex in ATLAS or CMS.

Chapter 5

SU(5) Grand Unified Theory with $\mathrm{U}(1)_{\widehat{Q}}$ Flavor Symmetry

In order to make simple calculations for the purpose of getting an overview about the behavior of the particles we restrict the family structure in particular in the Yukawa matrices via the $\mathrm{U}(1)_{\widehat{Q}}$ flavor Froggatt-Nielsen symmetry described in Section 1.3. With this symmetry one can predict the unknown hierarchy of the unmeasured Yukawa matrices depending only on the measured hierarchies and on the model chosen.

5.1 $\mathrm{U}(1)_{\widehat{Q}}$ Flavor Charge Assignment

One possible $\mathrm{U}(1)_{\widehat{Q}}$ flavor charge distribution in a SU(5) GUT is developed in [36]. For concreteness we shall adopt this model here, reducing the complexity of flavor structure to a single parameter. The charges are denoted in Table 5.1. The value of the b-quark mass allows for $a = 0$ or $a = 1$. As shown in [21] there are two at low energies indistinguishable consistent models

$$\begin{array}{llll} 1) & a = b = 0\ , & c = 1\ , & d = 2 \\ 2) & b = c = 0\ , & a = d = 1\ , & \end{array}$$

Φ_i	$(\mathbf{10}_1, \mathbf{10}_2, \mathbf{10}_3)$	$(\overline{\mathbf{5}}_1, \overline{\mathbf{5}}_2, \overline{\mathbf{5}}_3)$	$(\mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3)$
$\widehat{Q}_i$	$(2, 1, 0)$	$(a+1, a, a)$	(d, c, b)

Table 5.1: $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry charge distribution for the chiral multiplets in SU(5) notation. The Higgs fields have charge zero.

The model also predicts the observed baryon asymmetry via leptogenesis for the case where $a + d = 2$. As it turns out the first model is inconsistent with the constraint from neutrino masses and baryogenesis washout.

In the following $\sim$ means that a matrix of order one has to be multiplied. In principle most of the factors in different formulas depend on each other, but for the sake of clarity we will not write this out.

5.2 Superpotential Parameters

In order to define the flavor dependency of the superpotential (2.1) at the GUT scale, one has to assign the charges given in Table 5.1 to Equation (1.30).

5.2.1 *R*-Parity Conserving Parameters

The SM Yukawa matrices turn out to be

$$\lambda^u_{ij} \sim \eta^{\widehat{Q}(\mathbf{10})_i + \widehat{Q}(\mathbf{10})_j} \sim \begin{pmatrix} \eta^4 & \eta^3 & \eta^2 \\ \eta^3 & \eta^2 & \eta \\ \eta^2 & \eta & 1 \end{pmatrix} , \tag{5.1}$$

and

$$\lambda^e_{ij} \sim \lambda^d_{ij} \sim \eta^{\widehat{Q}(\overline{\mathbf{5}})_i + \widehat{Q}(\mathbf{10})_j} \sim \eta^a \begin{pmatrix} \eta^3 & \eta^2 & \eta \\ \eta^2 & \eta & 1 \\ \eta^2 & \eta & 1 \end{pmatrix} . \tag{5.2}$$

Hence the CKM-Matrix (1.32) is [36]

$$V_{\mathrm{CKM}} \sim \eta^{\left|\widehat{Q}(\mathbf{10})_i - \widehat{Q}(\mathbf{10})_j\right|} \sim \begin{pmatrix} 1 & \eta & \eta^2 \\ \eta & 1 & \eta \\ \eta^2 & \eta & 1 \end{pmatrix} . \tag{5.3}$$

If one fits this matrix to the measured CKM-matrix (A.6), one gets the best result for

$$\eta = \tfrac{1}{16} \tag{5.4}$$

at the weak scale. The value of the largest $\mathcal{O}(1)$ parameter is approximately 4.

The neutrino Yukawa coupling is [20]

$$\lambda^\nu_{ij} \sim \eta^{\widehat{Q}(\overline{\mathbf{5}})_i + \widehat{Q}(\mathbf{1})_j} \sim \eta^a \begin{pmatrix} \eta^{d+1} & \eta^{c+1} & \eta^{b+1} \\ \eta^d & \eta^c & \eta^b \\ \eta^d & \eta^c & \eta^b \end{pmatrix} . \tag{5.5}$$

The Majorana mass matrix $M_{ij} = \lambda^S_{ij}\langle S\rangle$ for the right-handed neutrinos depends on

$$\lambda^S_{ij} \sim \eta^{\widehat{Q}(\mathbf{1})_i+\widehat{Q}(\mathbf{1})_j} \sim \begin{pmatrix} \eta^{2d} & 0 & 0 \\ 0 & \eta^{2c} & 0 \\ 0 & 0 & \eta^{2b} \end{pmatrix} . \tag{5.6}$$

Without loss of generality we have chosen a basis where M_{ij} is diagonal and real. The diagonalization does not change the hierarchy structure of the neutrino Yukawa couplings. Hence in the the R-parity conserving case the Majorana mass matrix of the light neutrinos is given by

$$m^\nu_{ij} \sim \lambda^\nu_{ik}(\lambda^S)^{-1}_{kl}\lambda^\nu_{lj} \sim \eta^{2a}\begin{pmatrix} \eta^2 & \eta & \eta \\ \eta & 1 & 1 \\ \eta & 1 & 1 \end{pmatrix} . \tag{5.7}$$

The dependence on the charges of the heavy neutrinos drops out. Diagonalization yields a large mixing between the second and third family which is needed in order to explain the atmospheric neutrino deficit by $\nu_\mu - \nu_\tau$ oscillations.

5.2.2 *R*-Parity Breaking Parameters

The $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry enables us to write the bilinear R-parity breaking superpotential parameter μ_i as a product of a vector containing the family structure and a small parameter ϵ of order of the R-parity breaking suppression

$$\frac{\mu_i}{\mu} = \epsilon_i \sim \epsilon\eta^{\widehat{Q}(\overline{\mathbf{5}})_i} \sim \epsilon\eta^a(\eta,1,1)^T . \tag{5.8}$$

In the case where $a = 1$ the constraint (4.11) leads with this definition to

$$\left(\frac{\epsilon}{10^{-5}}\right)\left(\frac{\eta}{1/16}\right)^{2a} \lesssim 2.56 . \tag{5.9}$$

Equation (5.8) yields with (4.10) the trilinear R-parity violating couplings

$$\begin{aligned} \lambda_{ijk} = \lambda'_{ijk} = \epsilon_i\lambda^{d/e}_{jk} &\sim \epsilon\eta^{\widehat{Q}(\overline{\mathbf{5}})_i+\widehat{Q}(\overline{\mathbf{5}})_j+\widehat{Q}(\mathbf{10})_k} \\ &\sim \epsilon\eta^{2a}\big(\eta^1,1,1\big)^T\begin{pmatrix} \eta^3 & \eta^2 & \eta \\ \eta^2 & \eta & 1 \\ \eta^2 & \eta & 1 \end{pmatrix} \end{aligned} \tag{5.10}$$

and the R-parity breaking right-handed neutrino coupling

$$\lambda^\nu_i \sim \epsilon\eta^{\widehat{Q}(\mathbf{1})_i} \sim \epsilon\big(\eta^d,\eta^c,\eta^b\big)^T . \tag{5.11}$$

Since the λ_{ijk}-matrix must be antisymmetric in its first two indices and the flavor parameter matrix is symmetric in these indices, the coefficient matrix must be antisymmetric.

5.3 Soft Supersymmetry Breaking Masses

In order to reduce the parameter space further, we have to impose the flavor hierarchy from Table 5.1 in the soft SUSY breaking sector and constrain the soft SUSY breaking masses at the GUT scale.

5.3.1 *R*-Parity Conserving Masses

At the GUT scale we assume universal scalar masses that are diagonal and proportional to the gravitino mass $m_{3/2}$. This mass is usually called m_0:

$$(m^2_{\widetilde{u}^c})_{ij} = (m^2_{\widetilde{d}^c})_{ij} = (m^2_{\widetilde{l}^c})_{ij} = (m^2_{\widetilde{L}})_{ij} = (m^2_{\widetilde{Q}})_{ij} = m_0^2 1_{ij} \; . \tag{5.12}$$

The Higgs mass parameters are equal to the gravitino mass,

$$\widetilde{m}_d^2 = \widetilde{m}_u^2 = m_0^2 \; , \tag{5.13}$$

and the Higgs B term is equal to

$$B = m_0 \mu \; . \tag{5.14}$$

The dimension of the cubic scalar couplings is given by the scalar A and the pattern is proportional to the corresponding Yukawa couplings

$$A^e_{ij} = A\lambda^e_{ij} \; , \qquad A^\nu_{ij} = A\lambda^\nu_{ij} \; , \qquad A^d_{ij} = A\lambda^d_{ij} \; , \qquad A^u_{ij} = A\lambda^u_{ij} \; . \tag{5.15}$$

Hence the nontrivial flavor structure for the R-parity conserving soft SUSY Lagrangian is determined by Equations (5.1), (5.2) and (5.5)

5.3.2 *R*-Parity Breaking Masses

The charge distribution for bilinear R-parity breaking parameters is

$$\begin{aligned} \widetilde{m}^2_{di} &\sim \epsilon \eta^{\widehat{Q}(\overline{\mathbf{5}})_i} \widetilde{m}^2_d \sim \epsilon \eta^a (\eta, 1, 1)^T m_0^2 \\ B_i &\sim \epsilon \eta^{\widehat{Q}(\overline{\mathbf{5}})_i} B \sim \epsilon \eta^a (\eta, 1, 1)^T m_0 \mu \; . \end{aligned} \tag{5.16}$$

We take the trilinear couplings to be

$$A_{ijk} \sim A\lambda_{ijk} \; , \qquad A'_{ijk} \sim A\lambda'_{ijk} \; , \qquad A''_{ijk} \sim A\lambda''_{ijk} \; . \tag{5.17}$$

This is the analog definition to (5.15).

5.4 SU(5) GUT with U(1) Flavor Symmetry and Bilinear *R*-Parity Breaking

After introducing the $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry into our supersymmetric model we can express the bilinear terms found in Section 4.3 at the GUT scale with the help of this symmetry. The rotated bilinear soft SUSY breaking parameter B_i (4.17) is

$$B_i{}' \sim \epsilon\eta^a(\eta,1,1)^T \mu m_0 \ . \tag{5.18}$$

The rotated Higgs slepton mixing mass parameter (4.18) is

$$\begin{aligned}\widetilde{m}^2_{d'i} \sim &\epsilon\eta^a(\eta,1,1)^T m_0^2 + \epsilon\eta^a\,(\eta,1,1)\,1_{ij} m_0^2 - \epsilon\eta^a(\eta,1,1)^T m_0^2 \\ \sim &\epsilon\eta^a(\eta,1,1)^T m_0^2 \ .\end{aligned} \tag{5.19}$$

and the rotated slepton mass parameter (4.15) is

$$(m^2_{\widetilde{L}'})_{ij} \sim m_0^2 1_{ij} - \epsilon^2\eta^{2a}\begin{pmatrix}\eta^2 & \eta & \eta\\ \eta & 1 & 1\\ \eta & 1 & 1\end{pmatrix} m_0^2 \ . \tag{5.20}$$

The sneutrino VEVs (4.20) follows

$$\widehat{\epsilon}_i \sim \frac{\mu m_0 \tan\beta - m_0^2}{3m_0^2 + \frac{1}{2}m_Z^2\cos 2\beta}\epsilon\eta^a(\eta,1,1)^T \ . \tag{5.21}$$

Finally we get the square of the sneutrino VEV parameter just in terms of scalars to leading order in the flavor parameter η

$$|\widehat{\epsilon}|^2 \sim \frac{\left(\mu m_0\tan\beta - m_0^2\right)^2}{\left(3m_0^2 + \frac{1}{2}m_Z^2\cos 2\beta\right)^2}\epsilon^2\eta^{2a}(\eta^2+2) \ . \tag{5.22}$$

It depends only on the known parameter m_Z, the model-dependend parameters μ, $m_{3/2}$ and $\tan\beta$ as well as the small scalars ϵ and η. With this formula and the constraint (4.33) one is able to derive an upper bound on the R-parity breaking scalar ϵ. However, depending on the parameters chosen, the bound on ϵ may not be more rigorous then the constraint coming from non-erasure of primordial baryogenesis. Nonetheless, ϵ can be inserted into the $\widetilde{\tau}$ decay length in order to bound it from below.

In order to get accurate results RGE running must be taken into account (for the formulas see Appendix E). Therefore, the result (5.22) gives only a rough overview over the dependence on the small parameters ϵ and η.

η^{2a}	η^{2a+1}	η^{2a+2}	η^{2a+3}
$\widetilde{\tau}^-{}_R \to \tau_L \nu_\mu$	$\widetilde{\tau}^-{}_R \to \mu_L \nu_e$	$\widetilde{\tau}^-{}_L \to \mu_R \overline{\nu}_e$	$\widetilde{\tau}^-{}_L \to e_R \overline{\nu}_e$
$\widetilde{\tau}^-{}_R \to \mu_L \nu_\tau$	$\widetilde{\tau}^-{}_R \to \tau_L \nu_e$	$\widetilde{\tau}^-{}_L \to e_R \overline{\nu}_\mu$	$\widetilde{\tau}^-{}_L \to \overline{u}_L d_R$
$\widetilde{\tau}^-{}_L \to \tau_R \overline{\nu}_\mu$	$\widetilde{\tau}^-{}_R \to e_L \nu_\mu$	$\widetilde{\tau}^-{}_L \to \overline{c}_L d_R$	
$\widetilde{\tau}^-{}_L \to \overline{t}_L b_R$	$\widetilde{\tau}^-{}_R \to e_L \nu_\tau$	$\widetilde{\tau}^-{}_L \to \overline{u}_L b_R$	
$\widetilde{\tau}^-{}_L \to \overline{t}_L s_R$	$\widetilde{\tau}^-{}_L \to \mu_R \overline{\nu}_\mu$	$\widetilde{\tau}^-{}_L \to \overline{u}_L s_R$	
	$\widetilde{\tau}^-{}_L \to \tau_R \overline{\nu}_e$		
	$\widetilde{\tau}^-{}_L \to \overline{t}_L d_R$		
	$\widetilde{\tau}^-{}_L \to \overline{c}_L b_R$		
	$\widetilde{\tau}^-{}_L \to \overline{c}_L s_R$		

Table 5.2: Flavor suppression of two-body $\widetilde{\tau}^-$-slepton decays.

We achieve the precise calculation with a version of SOFTSUSY [88] extend with R-parity breaking couplings [89], that we have modified to correctly run the small parameters according to the special pattern of the R-parity breaking couplings predicted by our model.

5.5 Flavor Structure of $\widetilde{\tau}$-Slepton Decays

The flavor structure developed in the present chapter allows the investigation of the main decay channels of the $\widetilde{\tau}$-slepton.

The decay width (4.38) depends on the parameters of the Lagrangian unified at the GUT scale and their RGE running. We performed our calculation with the modified version of SOFTSUSY mentioned above. If one calculates the $\widetilde{\tau}$-decay length in this model the coupling is to lowest order proportional to $\lambda_{ijk} \propto \eta^{2a}$ due to the flavor structure in (5.10). Hence the stau decay length is given by

$$c\tau_{\widetilde{\tau}}^{\text{lep}} = 25\,\text{cm} \left(\frac{m_{\widetilde{\tau}}}{200\,\text{GeV}}\right)^{-1} \left(\frac{\epsilon}{2.56 \times 10^{-6}}\right)^{-2} \left(\frac{\eta^{2a}}{1/16}\right)^{-2} \tag{5.23}$$

In order to allow comparison between our model and the more phenomenological prediction in (4.39) and [21] we take the R-parity breaking parameter to be $\epsilon = 2.56 \times 10^{-6}$, which corresponds exactly to the R-parity breaking coupling in the phenomenological prediction.

In Table 5.2 we list all allowed decay channels sorted by their power in η. In Equation (5.23) only the dominant decay channels are taken into account in contrast to Figure 5.1 which contains all contributions to the two-body decays. The dominance of the channels $\widetilde{\tau}_R \to \tau_L \nu_\mu, \mu_L \nu_\tau$ can be seen in the

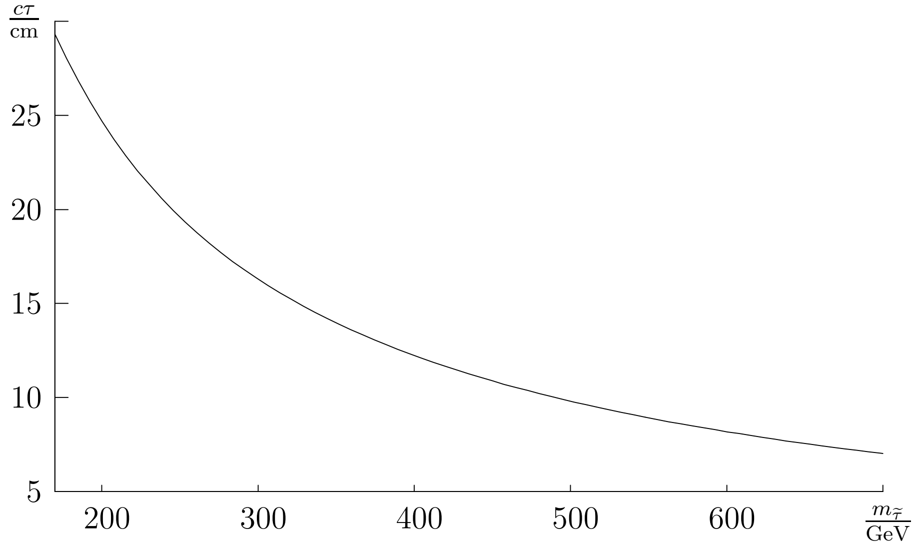

Figure 5.1: $\widetilde{\tau}$-slepton decay length in cm as function of the $\widetilde{\tau}$-slepton mass in GeV. All orders in η are included the R-parity breaking Parameter is 2.56×10^{-6}.

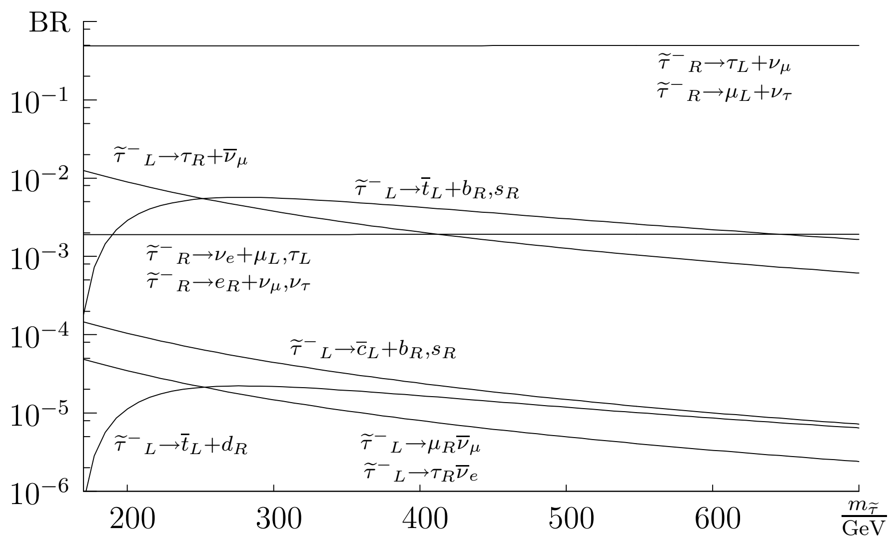

Figure 5.2: $\widetilde{\tau}$-slepton branching ratio in order η^{2a} and η^{2a+1} as function of the $\widetilde{\tau}$-slepton mass in GeV. The Parameters are chosen to be $\epsilon = 2.56 \times 10^{-6}$ and $a = 1$.

Branching Ratio (BR), which is plotted up to the second order in η in Figure 5.2.

The present model yields a unique signal as it predicts a heavy charged scalar that decays approximately 25 cm away from the interaction point. The signals featuring these displaced vertices have a negligible background, and can therefore lead to a rapid discovery, as long as the $\widetilde{\tau}$ mass stays small enough. The simple flavor structure of the model predicts additionally, that the heavy scalar decays equally into τ- and μ-leptons. Hence one should observe a corresponding ratio between τ-jets and μ-leptons. The combination of a heavy ionizing charged track and these characteristic branching ratios allows a discrimination from SUSY models with conserved R-Parity.

The hadronic channel is more unlikely to be discovered as its width is suppressed by a factor larger than 100 leading to a hadronic decay length of order 100 larger then the leptonic decay length.

Conclusion and Outlook

We calculated the effects occurring in supersymmetric extensions of the SM. We have focused on two extensions to the MSSM: R-parity breaking as well as right-handed neutrino superfields.

We rederived formulas for the R-parity breaking sneutrino VEVs (3.21), coming to a result which differs from formulas found in parts of the literature. It is a useful result that we obtained the sneutrino VEVs without restricting the interaction terms of the Lagrangian prior to that. Therefore, the model-dependent restrictions can be applied later without recalculating the VEVs. As we have also taken into account the right-handed sneutrinos we were able to derive the VEVs as well as the mass mixing matrices for this extension of the MSSM. To the best of our knowledge, we are the first to derive all mixing effects when both extensions are taken into account.

As both expansions generate neutrino masses separately most authors settle with one of them. Hence we were able to derive a new formula where both mass generating mechanisms are in effect (3.62). In the limiting cases this formula simplifies, respectively, to the known formulas (3.46) and (3.57). Here we want to emphasize that the pure R-parity breaking formula seems not to be known well enough, as many authors use a more heuristic approach, depending on the unknown neutralino masses.

Afterwards we calculated an R-parity breaking supersymmetric model. In order to obey experimental constraints and to have one single R-parity breaking parameter we constrained bilinear R-parity breaking with a $\mathrm{U}(1)_{\widehat{Q}}$ flavor symmetry in an SU(5) GUT. This enabled us to derive the lifetime of the gravitino LSP. By comparison with recent Fermi LAT results we could restrict the order of R-parity breaking (4.33) from above. We also derived the decay length of one possible NLSP, the $\widetilde{\tau}$-slepton (4.39). This decay is as well governed by R-parity breaking, hence the satellite results bound it from below. We came to the conclusion that it would have a spectacular signal at LHC.

Although we were able to derive numerical results, concerning the connection between gravitino DM and $\widetilde{\tau}$ NLSP decays, we were neither able to analyze the gravitino DM restrictions on R-parity breaking, nor the resulting constraints for scalar τ-leptons as deeply as we would have liked to. Hence we see a good opportunity to extend this study in order to derive more solid predictions for the scalar τ-lepton in detectors based on gravitino DM searches.

Appendices

Appendix A

Units and Physical Constants in the Standard Model

In table A.1 we summarize the values of the physical constants used in this thesis. The values are taken from [1].

We are using a system of units where the reduced Planck constant $\hbar = 6.58211899(16) \times 10^{-25}$ GeV s , the speed of light $c = 299792458\,{}^{\mathrm{m}}\!/\!{}_{\mathrm{s}}$, and the Boltzmann constant $k_B = 8.617343(15) \times 10^{-11}\,{}^{\mathrm{GeV}}\!/\!{}_{\mathrm{K}}$ equal one: $\hbar = c = k_B = 1$. In this way every physical quantity can be measured in powers of the mass m. In high energy physics the mass is measured in electron volts (eV). The only remaining dimensionful coupling constant is the gravitational constant G_N, which defines the reduced Planck mass $M_P = (8\pi G_N)^{-1/2}$.

A.1 Cabibbo-Kobayashi-Maskawa Matrix

The Cabibbo-Kobayashi-Maskawa (CKM) matrix parametrizes the flavor changing charged current in the quark sector of the standard model. It arises from the diagonalization of the Yukawa matrices.

The square of the matrix λ_u can be diagonalized with two matrices U and W:

$$\lambda_u {\lambda_u}^\dagger = U_u D_u^2 {U_u}^\dagger \,, \qquad {\lambda_u}^\dagger \lambda_u = W_u D_u^2 {W_u}^\dagger \,. \tag{A.1}$$

Hence the matrix λ_u can be diagonalized by

$$\lambda_u = U_u D_u {W_u}^\dagger \,. \tag{A.2}$$

The same applies to λ_d

$$\lambda_d {\lambda_d}^\dagger = U_d D_d^2 {U_d}^\dagger \,, \qquad {\lambda_d}^\dagger \lambda_d = W_d D_d^2 {W_d}^\dagger \,. \tag{A.3}$$

Quantity	Symbol	Value
gravitational constant	G_N	$6.70887(67) \times 10^{-39}\,\mathrm{GeV}^{-2}$
reduced Planck mass	M_P	$\simeq 2.4 \times 10^{18}\,\mathrm{GeV}$
electron mass	m_e	$0.510998910(13)\,\mathrm{MeV}$
muon mass	m_μ	$105.658367(4)\,\mathrm{MeV}$
τ lepton mass	m_τ	$1.77684(17)\,\mathrm{GeV}$
u quark mass	m_u	$1.5 - 3.3\,\mathrm{MeV}$
d quark mass	m_d	$3.5 - 6.0\,\mathrm{MeV}$
s quark mass	m_s	$104(34)\,\mathrm{MeV}$
c quark mass	m_c	$1.27(11)\,\mathrm{GeV}$
b quark mass	m_b	$4.20(17)\,\mathrm{GeV}$
t quark mass	m_t	$171.2(2.1)\,\mathrm{GeV}$
$W^\pm$ mass	m_W	$80.398(25)\,\mathrm{GeV}$
Z mass	m_Z	$91.1876(21)\,\mathrm{GeV}$
neutrino mass	m_ν	$< 2\,\mathrm{eV}$
SM Higgs VEV	v	$\simeq 174\,\mathrm{GeV}$
proton lifetime	τ_p	$> 2.1 \times 10^{29}$ years
weak mixing angle	$\sin\theta_w$	$0.23119(14)$

Table A.1: Physical constants.

Therefore, it can be diagonalized by

$$\lambda_d = U_d D_d {W_d}^\dagger \; . \tag{A.4}$$

Finally the CKM-Matrix can be defined by

$$V_{\mathrm{CKM}} = {U_u}^\dagger U_d \; . \tag{A.5}$$

The observed values are

$$V_{\mathrm{CKM}} \simeq \begin{pmatrix} 0.97418(27) & 0.2255(19) & 0.00393(36) \\ 0.230(11) & 1.04(6) & 0.0412(11) \\ 0.0081(6) & 0.0387(23) & 0.77(24) \end{pmatrix} \; . \tag{A.6}$$

One possible parametrization of the obvious hierarchy can be found in Section 1.3.

Appendix B

Notation

In this thesis we are using letters from the middle of the Greek alphabet $\mu, \nu, \ldots = 0, 1, 2, 3$ for the indices of space-time four vectors. We are using the space-time metric $\eta_{\mu\nu} = \mathrm{diag}\,(+1, -1, -1, -1)$.

B.1 Two-Component Spinor Notation

As supersymmetric spinors are mostly noted as two-component spinors we will give a rough overview about important formulas in this area. We are closely following [27], see there for a more complete introduction.

A left-handed $(1/2, 0)$ two-component Weyl spinor denoted as χ_α transforms as $\chi_\alpha \to M_\alpha{}^\beta \chi^\beta$ under the representation matrix $M_\alpha{}^\beta$ of the Lorentz group. A right-handed $(0, 1/2)$ spinor denoted as $\chi_{\dot\alpha}{}^\dagger$ transforms as $\chi_{\dot\alpha}{}^\dagger \to M^*{}_{\dot\alpha}{}^{\dot\beta} \chi_{\dot\beta}{}^\dagger$ under the Lorentz group representation. The spinor indices are denoted by letters of the beginning of the Greek alphabet $\alpha, \beta, \ldots = 1, 2$. Undotted indices represent left-handed spinors and dotted indices right-handed spinors. Majorana spinors can be composed of either representation. The free-field Lagrangian for a free neutral massive anticommuting spin-$1/2$ Majorana field $\xi_\alpha(x)$ is

$$-\mathcal{L} = -i\xi^\dagger \overline{\sigma}^\mu \partial_\mu \xi + \tfrac{1}{2} m \left(\xi\xi + \xi^\dagger \xi^\dagger\right) \ . \tag{B.1}$$

On-shell, ξ_α satisfies the free-field Dirac equation

$$i\overline{\sigma}^{\mu\dot\alpha\beta} \partial_\mu \xi_\beta = m\xi^{\dot\alpha\dagger} \ . \tag{B.2}$$

After quantization, ξ_α can be expanded in a Fourier series

$$\xi_\alpha(x) = \sum_s \int \frac{d^3\vec{p}}{(2\pi)^{3/2}(2E_p)^{1/2}} \left(x_\alpha(\vec{p}, s) a(\vec{p}, s) e^{-ipx} + y_\alpha(\vec{p}, s) a^\dagger(\vec{p}, s) e^{ipx}\right) \ , \tag{B.3}$$

where $E_p = \sqrt{|\vec{p}|^2 + m^2}$, and the creation and annihilation operators a and $a^\dagger$ satisfy the canonical anticommutation relation.

Dirac spinors combine two Weyl spinors of equal mass into a reducible representation of the form $(1/2, 0) \oplus (0, 1/2)$. Consider the case of two mass-degenerate massive fermion fields. Then the Lagrangian (B.1) possesses a global internal O(2) flavor symmetry. The conserved hermitian Noether current

$$J^\mu = i\left(\xi^{1\dagger}\overline{\sigma}^\mu\xi_2 - \xi^{2\dagger}\overline{\sigma}^\mu\xi_1\right) , \tag{B.4}$$

corresponds this symmetry. In the basis

$$\chi = \tfrac{1}{\sqrt{2}}\left(\xi_1 + i\xi_2\right) , \qquad \eta = \tfrac{1}{\sqrt{2}}\left(\xi_1 - i\xi_2\right) , \tag{B.5}$$

the Noether current is diagonal. The Lagrangian (B.1) in this basis becomes

$$-\mathcal{L} = -i\chi^\dagger\overline{\sigma}^\mu\partial_\mu\chi - i\eta^\dagger\overline{\sigma}^\mu\partial_\mu\eta + m\left(\chi\eta + \chi^\dagger\eta^\dagger\right) . \tag{B.6}$$

On-shell, χ and η satisfy the free-field Dirac equation (B.2). Together, χ and $\eta^\dagger$ constitute a single Dirac fermion. We can then write

$$\begin{aligned}\chi_\alpha(x) &= \sum_s \int \frac{d^3\vec{p}}{(2\pi)^{3/2}(2E_p)^{1/2}}\left(x_\alpha(\vec{p},s)a(\vec{p},s)e^{-ipx} + y_\alpha(\vec{p},s)b^\dagger(\vec{p},s)e^{ipx}\right) ,\\ \eta_\alpha(x) &= \sum_s \int \frac{d^3\vec{p}}{(2\pi)^{3/2}(2E_p)^{1/2}}\left(x_\alpha(\vec{p},s)b(\vec{p},s)e^{-ipx} + y_\alpha(\vec{p},s)a^\dagger(\vec{p},s)e^{ipx}\right) .\end{aligned} \tag{B.7}$$

All four creation and annihilation operators satisfy canonical anticommutation relations.

Two spinors are combined to a Lorentz vector with the help of the Pauli matrices $\sigma^\mu_{\alpha\dot\beta}$ and $\overline{\sigma}^{\mu\alpha\dot\beta}$ which are defined by

$$\sigma^0 = \overline{\sigma}^0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} , \qquad \sigma^1 = -\overline{\sigma}^1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} , \tag{B.8}$$

$$\sigma^2 = -\overline{\sigma}^2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} , \qquad \sigma^3 = -\overline{\sigma}^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} . \tag{B.9}$$

Contraction of the Pauli matrices gives

$$\begin{aligned}\sigma^\mu_{\alpha\dot\alpha}\overline{\sigma}^{\beta\dot\beta}_\mu &= 2\delta_\alpha{}^\beta\delta^{\dot\beta}{}_{\dot\alpha} ,\\ \sigma^\mu_{\alpha\dot\alpha}\sigma_{\mu\beta\dot\beta} &= 2\epsilon_{\alpha\beta}\epsilon_{\dot\alpha\dot\beta} ,\\ \overline{\sigma}^{\mu\dot\alpha\alpha}\overline{\sigma}^{\dot\beta\beta}_\mu &= 2\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta} .\end{aligned} \tag{B.10}$$

particle symbol	fermion name	two-component field
l^-	lepton	$l, l^{c\dagger}$
l^+	antilepton	$l^c, l^\dagger$
ν	neutrino	$\nu, \nu^{c\dagger}$
$\overline{\nu}$	antineutrino	$\nu^c, \nu^\dagger$
q	quark	$q, q^{c\dagger}$
$\overline{q}$	antiquark	$q^c, q^\dagger$
$\widetilde{N}$	neutralino	$\chi^0, \chi^{0\dagger}$
$\widetilde{C}^+$	chargino	$\chi^+, \chi^{-\dagger}$
$\widetilde{C}^-$	antichargino	$\chi^-, \chi^{+\dagger}$
$\widetilde{g}$	gluino	$\widetilde{g}, \widetilde{g}^\dagger$

Table B.1: (Anti-) fermion names and two-component fields.

Pauli matrices with two vector indices are defined by

$$\begin{aligned} \sigma^{\mu\nu}{}_\alpha{}^\beta &= \tfrac{1}{4}\left(\sigma^\mu_{\alpha\dot{\gamma}}\overline{\sigma}^{\nu\dot{\gamma}\beta} - \sigma^\nu_{\alpha\dot{\gamma}}\overline{\sigma}^{\mu\dot{\gamma}\beta}\right) \, , \\ \overline{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}} &= \tfrac{1}{4}\left(\overline{\sigma}^{\mu\dot{\alpha}\gamma}\sigma^\nu_{\gamma\dot{\beta}} - \overline{\sigma}^{\nu\dot{\alpha}\gamma}\sigma^\mu_{\gamma\dot{\beta}}\right) \, . \end{aligned} \tag{B.11}$$

For the sake of simplicity we have suppressed the spinor indices in this thesis. The common fermion particle names and how they are connected to the two-component spinor notation are written down in Table B.1.

B.2 Four-Component Spinor Notation

For an introduction in the four-component spinor notation see, for instance, [2]. A four-component Dirac spinor Ψ consists of two mass-degenerate two-component spinors χ_α and η_α of opposite U(1) charge, which is based on the rotation freedom leading to the Noether current (B.4),

$$\Psi = \left(\chi_\alpha, \eta^{\dot{\alpha}\dagger}\right)^T \, . \tag{B.12}$$

To project onto the left-handed,

$$\Psi_L = P_L\Psi = (\chi_\alpha, 0)^T \, , \tag{B.13}$$

and right-handed,

$$\Psi_R = P_R\Psi = \left(0, \eta^{\dot{\alpha}\dagger}\right)^T \, , \tag{B.14}$$

components, one has to define the projection operators

$$P_L = \tfrac{1}{2}\,(1-\gamma_5) = \begin{pmatrix} \delta_\alpha{}^\beta & 0 \\ 0 & 0 \end{pmatrix}\ , \qquad P_R = \tfrac{1}{2}\,(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}\ . \tag{B.15}$$

Here the gamma matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \overline{\sigma}^{\mu\dot\alpha\beta} & 0 \end{pmatrix}\ , \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\delta_\alpha{}^\beta & 0 \\ 0 & \delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix} \tag{B.16}$$

are used.

Appendix C

Addendum to Neutral Field Mixing

We have given a thorough insight of the mass mixing phenomena in the neutral sector in Chapter 3. However we have neither given the most general scalar mass mixing matrix for R-parity breaking and right-handed sneutrinos nor the analytic diagonalization of the R-parity conserving neutralino mass mixing matrix.

C.1 Most General Mass Mixing Matrices of the Scalar Neutral Fields

The most general scalar potential for neutral scalar fields in the R-parity violating Minimal Supersymmetric Standard Model with right-handed neutrino superfields is

$$\begin{aligned} V_{\text{scalar}} = &\left(|\mu_\alpha|^2 + \widetilde{m}_u^2\right) \left|h_u^0\right|^2 + \left({\mu_\alpha}^\dagger \mu_\beta + \widetilde{m}_{\alpha\beta}^2\right) \widetilde{\nu}_\alpha \widetilde{\nu}_\beta \\ &- B_\alpha h_u^0 \widetilde{\nu}_\alpha - {B_\alpha}^\dagger {\widetilde{\nu}_\alpha}^\dagger {h_u^0}^\dagger + B_{ij}^N {\widetilde{\nu}^c}_i {\widetilde{\nu}^c}_j + {B_{ij}^N}^\dagger {{\widetilde{\nu}^c}_i}^\dagger {{\widetilde{\nu}^c}_j}^\dagger \\ &+ \tfrac{1}{8}\left(g_1^2 + g_2^2\right)\left(\left|h_u^0\right|^2 - |\widetilde{\nu}_\alpha|^2\right)^2 \\ &+ \left|M_{ii} {\widetilde{\nu}^c}_i + \lambda_{\alpha j}^\nu h_u^0 \widetilde{\nu}_i\right|^2 + \left|\lambda_{\alpha j}^\nu \widetilde{\nu}_\alpha {\widetilde{\nu}^c}_j\right|^2 + (m_{\widetilde{\nu}^c}^2)_{ij} {{\widetilde{\nu}^c}_i}^\dagger {\widetilde{\nu}^c}_j \\ &+ \left|\lambda_{\alpha j}^\nu h_u^0 {\widetilde{\nu}^c}_j\right|^2 - A_{\alpha j}^\nu h_u^0 \widetilde{\nu}_\alpha {\widetilde{\nu}^c}_j - {A_{\alpha j}^\nu}^\dagger {{\widetilde{\nu}^c}_j}^\dagger {\widetilde{\nu}_\alpha}^\dagger {h_u^0}^\dagger \,. \end{aligned} \tag{C.1}$$

The first derivatives with respect to the fields give modified VEVs, see (3.26) and (3.27).

If one assumes CP-conservation, the second derivative gives for

$$(h_u, {\widetilde{\nu}^c}_i, \widetilde{\nu}_\alpha) = (h_u, {\widetilde{\nu}^c}_i, h_d, \widetilde{\nu}_i) \tag{C.2}$$

the CP-odd mass matrix

$$M = \begin{pmatrix} M_u & -M_{kj}\lambda_{\beta k}{}^\dagger \widetilde{\nu}_\beta{}^\dagger - A_{\beta j}{}^\dagger \widetilde{\nu}_\beta{}^\dagger & +B_\beta{}^\dagger - M_{kl}\lambda_{\beta k}{}^\dagger \widetilde{\nu}^c{}_l{}^\dagger \\ -M_{ik}{}^\dagger \lambda_{\beta k}\widetilde{\nu}_\beta - A_{\beta i}\widetilde{\nu}_\beta & M_{ij} & -A_{\alpha j}{}^\dagger h_u^0 - M_{kj}\lambda_{\alpha k}{}^\dagger \left\langle h_u^0 \right\rangle \\ +B_\alpha - M_{kl}{}^\dagger \lambda_{\alpha k}\widetilde{\nu}^c{}_l & -A_{\alpha j}h_u^0 - M_{kj}{}^\dagger \lambda_{\alpha k}\left\langle h_u^0 \right\rangle & M_{\alpha\beta} \end{pmatrix} . \tag{C.3}$$

The diagonal terms are

$$\begin{aligned} M_u =& \widetilde{m}_u^2 + \left|\mu_\gamma\right|^2 + m_Z^2 \left(\frac{\left|h_u^0\right|^2}{v^2} + \frac{\left|\widetilde{\nu}_\gamma\right|^2}{v^2} \right) \\ & + \lambda_{\gamma j}{}^\dagger \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\gamma i}\widetilde{\nu}^c{}_i + \lambda_{\beta k}{}^\dagger \widetilde{\nu}_\beta{}^\dagger \lambda_{\alpha k}\widetilde{\nu}_\alpha + \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\gamma j}{}^\dagger \mu_\gamma + \mu_\gamma{}^\dagger \widetilde{\nu}^c{}_i \lambda_{\gamma i} \; , \\ M_{ij} =& - B_{ij}{}^\dagger - B_{ij} + M_{ij}^2 + (m_{\widetilde{\nu}^c})_{ij}^2 + \lambda_{\beta j}{}^\dagger \widetilde{\nu}_\beta{}^\dagger \lambda_{\alpha i}\widetilde{\nu}_\alpha + \lambda_{\gamma j}{}^\dagger h_u^{0\dagger} \lambda_{\gamma i} h_u^0 \; , \\ M_{\alpha\beta} =& \widetilde{m}_{\alpha\beta}^2 + \mu_\beta{}^\dagger \mu_\alpha + m_Z^2 \left(\frac{\widetilde{\nu}_\beta{}^\dagger \widetilde{\nu}_\alpha}{v^2} - \frac{\left|h_u^0\right|^2}{v^2}\delta_{\alpha\beta} \right) \\ & + \lambda_{\beta k}{}^\dagger \lambda_{\alpha k} \left|h_u^0\right|^2 + \lambda_{\beta j}{}^\dagger \lambda_{\alpha i}\widetilde{\nu}^c{}_j{}^\dagger \widetilde{\nu}^c{}_i + \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\beta j}{}^\dagger \mu_\alpha + \mu_\beta{}^\dagger \widetilde{\nu}^c{}_i \lambda_{\alpha i} \; . \end{aligned} \tag{C.4}$$

For the CP-even mass matrix we find

$$M = \begin{pmatrix} M_u & M_j & M_\beta \\ M_i & M_{ij} & M_{i\beta} \\ M_\alpha & M_{\alpha j} & M_{\alpha\beta} \end{pmatrix} . \tag{C.5}$$

In this case the diagonal terms are

$$\begin{aligned} M_u =& \widetilde{m}_u^2 + \left|\mu_\gamma\right|^2 + m_Z^2 \left(3\frac{\left|h_u^0\right|^2}{v^2} - \frac{\left|\widetilde{\nu}_\gamma\right|^2}{v^2} \right) \\ & + \lambda_{\gamma j}{}^\dagger \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\gamma i}\widetilde{\nu}^c{}_i + \lambda_{\beta k}{}^\dagger \widetilde{\nu}_\beta{}^\dagger \lambda_{\alpha k}\widetilde{\nu}_\alpha + \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\gamma j}{}^\dagger \mu_\gamma + \mu_\gamma{}^\dagger \widetilde{\nu}^c{}_i \lambda_{\gamma i} \; , \\ M_{ij} =& B_{ij}{}^\dagger + B_{ij} + M_{ij}^2 + (m_{\widetilde{\nu}^c})_{ij}^2 + \lambda_{\beta j}{}^\dagger \widetilde{\nu}_\beta{}^\dagger \lambda_{\alpha i}\widetilde{\nu}_\alpha + \lambda_{\gamma j}{}^\dagger h_u^{0\dagger} \lambda_{\gamma i} h_u^0 \; , \\ M_{\alpha\beta} =& \widetilde{m}_{\alpha\beta}^2 + \mu_\beta{}^\dagger \mu_\alpha + m_Z^2 \left(3\frac{\widetilde{\nu}_\beta{}^\dagger \widetilde{\nu}_\alpha}{v^2} - \frac{\left|h_u^0\right|^2}{v^2}\delta_{\alpha\beta} \right) \\ & + \lambda_{\beta k}{}^\dagger \lambda_{\alpha k} \left|h_u^0\right|^2 + \lambda_{\beta j}{}^\dagger \lambda_{\alpha i}\widetilde{\nu}^c{}_j{}^\dagger \widetilde{\nu}^c{}_i + \widetilde{\nu}^c{}_j{}^\dagger \lambda_{\beta j}{}^\dagger \mu_\alpha + \mu_\beta{}^\dagger \widetilde{\nu}^c{}_i \lambda_{\alpha i} \; . \end{aligned} \tag{C.6}$$

The off-diagonal terms are

$$\begin{aligned} M_j &= -M_{kj}\lambda_{\beta k}{}^\dagger \widetilde{\nu}_\beta{}^\dagger - A_{\beta j}{}^\dagger \widetilde{\nu}_\beta{}^\dagger + 2\lambda_{\beta i} h_u^0 \mu_\beta + 2\lambda_{\beta i}\lambda_{\beta j} h_u^0 \widetilde{\nu}^c{}_j \; , \\ M_\beta &= -B_\beta{}^\dagger + M_{kl}\lambda_{\beta k}{}^\dagger \widetilde{\nu}^c{}_l{}^\dagger + A_{\alpha j}{}^\dagger \widetilde{\nu}^c{}_i{}^\dagger + 2\lambda_{\alpha k}{}^\dagger h_u^0 \lambda_{\gamma k}\nu_\gamma + 2m_Z^2 \frac{h_u^0 \nu_\beta{}^\dagger}{v^2} \; , \\ M_{i\beta} &= -A_{\alpha j}{}^\dagger h_u^0 - M_{kj}\lambda_{\alpha k}{}^\dagger h_u^0 + 2\lambda_{\alpha j}\widetilde{\nu}_\alpha \lambda_{\beta i}{}^\dagger \widetilde{\nu}^c{}_i{}^\dagger + 2\lambda_{\gamma i}\mu_\beta \nu_\gamma{}^\dagger \; , \end{aligned} \tag{C.7}$$

with the corresponding adjoint values for the other off-diagonal terms.

C.2 Neutralino Mass Mixing Matrix

As stated in Section 3.3.1 the R-parity conserving neutralino mass mixing matrix (3.39) is analytically diagonalizable. In this section we give an idea how to perform the analytic diagonalization. In Section 3.3 we have indicated by indices as $\chi^0_{R_p}$ in which theory we are working. In the following we are working in the R-parity conserving MSSM but are supressing the indication for the sake of clarity.

The square of the diagonalization equation (3.42) gives the eigenvalue equation,

$$\left(M^\dagger M - m_i^2\right)(PS)_i = 0\ , \tag{C.8}$$

which one can transform into the characteristic equation

$$m_i^8 - a m_i^6 + b m_i^4 - c m_i^2 + d = 0\ . \tag{C.9}$$

This is a quartic equation in m_i^2 that has four invariants. The first is given by

$$a = \mathrm{tr}\,(X) = m_1^2 + m_2^2 + m_3^2 + m_4^2 = M_1^2 + M_2^2 + 2\mu^2 + 2m_Z^2\ , \tag{C.10}$$

where we have used the abriviation $MM^\dagger = X$. The second invariant is given by

$$\begin{aligned} b =& \tfrac{1}{2}\left(\mathrm{tr}\,(X)^2 - \mathrm{tr}\left(X^2\right)\right) \\ =& m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2 + m_1^2 m_4^2 + m_2^2 m_4^2 + m_3^2 m_4^2 \\ =& \left(M_1^2 + m_Z^2\right)\left(M_2^2 + m_Z^2\right) + 2\left(M_1^2 + M_2^2 + m_Z^2\right)\mu^2 + \mu^4 \\ & + m_Z^2\left((M_1 - M_2)\cos 2\theta_w\left(M_1 + M_2 + \mu\sin 2\beta\right) - (M_1 + M_2)\,\mu\sin 2\beta\right)\ , \end{aligned} \tag{C.11}$$

and the third invariant is given by

$$\begin{aligned} c =& \tfrac{1}{6}\left(\mathrm{tr}\,(X)^3 - 3\,\mathrm{tr}\,(X)\,\mathrm{tr}\left(X^2\right) + 2\,\mathrm{tr}\left(X^3\right)\right) \\ =& m_1^2 m_2^2 m_3^2 + m_1^2 m_2^2 m_4^2 + m_1^2 m_3^2 m_4^2 + m_2^2 m_3^2 m_4^2 \\ =& \tfrac{1}{8}\left(3M_1^2 + 2M_1 M_2 + 3M_2^2\right) m_Z^4 + \tfrac{1}{2}\left(2M_1^2 + m_Z^2\right)\left(2M_2^2 + m_Z^2\right)\mu^2 \\ & + \left(M_1^2 + M_2^2\right)\mu^4 - \tfrac{1}{2} m_Z^4 \mu^2 \cos 4\beta \\ & + m_Z^2\left(\tfrac{1}{8}\left(M_1 - M_2\right)^2 m_Z^2 \cos 4\theta_w - (M_1 + M_2)\,\mu\left(M_1 M_2 + \mu^2\right)\sin 2\beta\right. \\ & + \tfrac{1}{2}\left(M_1 - M_2\right)\cos 2\theta_w \\ & \left.\times\left((M_1 + M_2)\left(m_Z^2 + 2\mu^2\right) + 2\mu\left(\mu^2 - M_1 M_2\right)\sin 2\beta\right)\right)\ . \end{aligned} \tag{C.12}$$

By using the photino mass

$$M_{\tilde{\gamma}} = M_1 \cos^2\theta_w + M_2 \sin^2\theta_w \tag{C.13}$$

the fourth invariant is given by

$$d = \det X = m_1^2 m_2^2 m_3^2 m_4^2 = \mu^2 \left(m_Z^2 M_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu\right)^2 \ . \tag{C.14}$$

As sketched in the next section the solutions of the characteristic equation (C.9) are as for every other quartic equation given by

$$m_i^2 = \tfrac{1}{4}a \pm_1 A \pm_2 B \ . \tag{C.15}$$

As stated in Section 3.3.1 these formulas are too lengthy to gain intuition on the neutralino diagonalization.

C.2.1 Solving Quartic Equations

As we could not find a canonical notation for the solution of a quartic equation, we will give a rough overview how the solution is composed.

The solution for the quadratic equation

$$m^2 + p_2 m + q_2 = 0 \tag{C.16}$$

is given by

$$m_i = -\tfrac{1}{2}p_2 \pm_2 \sqrt{\tfrac{1}{4}p_2^2 - q_2} \ . \tag{C.17}$$

In order to solve the cubic equation

$$a_3 x^3 + b_3 x^2 + c_3 x + d_3 = 0 \tag{C.18}$$

it has to be converted into the depressed cubic equation

$$y^3 + 3p_3 y + 2q_3 = 0 \ . \tag{C.19}$$

Here we have defined

$$\begin{aligned} y &= x + \frac{1}{3}\frac{b_3}{a_3} \ , \\ p_3 &= \frac{3a_3c_3 - b_3^2}{9a_3^2} \ , \\ q_3 &= \frac{1}{27}\frac{b^3}{a^3} - \frac{1}{6}\frac{b_3c_3}{a_3^2} + \frac{1}{2}\frac{d_3}{a_3} \ . \end{aligned} \tag{C.20}$$

The real solution of the depressed cubic equation is given by

$$y = \sqrt[3]{-q_3 + \sqrt{q_3^2 + p_3^3}} + \sqrt[3]{-q_3 - \sqrt{q_3^2 + p_3^3}} \,. \tag{C.21}$$

The roots of the quartic equation

$$m^4 - am^3 + bm^2 - cm + d = 0 \tag{C.22}$$

are identical [90] to the roots of the quadratic equation (C.17) with

$$p_2 = \pm_1 2A - \tfrac{1}{2}a \,, \qquad\qquad q_2 = x \pm_1 \frac{c - ax}{4A} \,, \tag{C.23}$$

where

$$x = y - \frac{1}{3}\frac{b_3}{a_3} = y + \frac{1}{6}b \tag{C.24}$$

is the real root of the cubic Equation (C.18) with

$$a_3 = 1 \,, \quad b_3 = -\tfrac{1}{2}b \,, \quad c_3 = \tfrac{1}{4}ac - d \,, \quad d_3 = d\left(\tfrac{1}{2}b - \tfrac{1}{8}a^2\right) - \tfrac{1}{8}c^2 \,. \tag{C.25}$$

In terms of the depressed cubic equation (C.19) it is

$$\begin{aligned} p_3 &= \tfrac{1}{12}ac - \tfrac{1}{3}d - \tfrac{1}{12}b^2 \,, \\ q_3 &= -\tfrac{1}{216}b^3 + \tfrac{1}{12}b\left(\tfrac{1}{4}ac - d\right) + d\left(\tfrac{1}{4}b - \tfrac{1}{16}a^2\right) - \tfrac{1}{16}c^2 \,. \end{aligned} \tag{C.26}$$

With the definition of

$$A = \sqrt{\tfrac{1}{2}x + \tfrac{1}{16}a^2 - \tfrac{1}{4}b} \,, \quad B = \sqrt{\frac{1}{16}\left(a \pm_1 4A\right)^2 - x \pm_1 \frac{1}{4}\frac{c - ax}{A}} \,, \tag{C.27}$$

the four solutions for the quartic equation (C.22) read

$$m_i = \tfrac{1}{4}a \pm_1 A \pm_2 B \tag{C.28}$$

These solutions are used in (C.15).

Appendix D

Field Mixing in the Charged Sector

We have also calculated the R-parity violation induced mixing of charged fields in the field content of the MSSM.

D.1 Scalar Fields

The charged scalar fields do not acquire a VEV as that would lead to a charged vacuum. The mass mixing matrices for the R-parity conserving and R-parity breaking theory are calculated in this section.

D.1.1 R-Parity Conserving Case

In the R-parity conserving theory just the neutral Higgs acquire a VEV. The charged Higgs $\left(h_u^+, h_d^-\right)$ form the mass matrix

$$M_{h^\pm}^2 = \begin{pmatrix} B\cot\beta + m_W^2\cos^2\beta & B + m_W^2\cos\beta\sin\beta \\ B + m_W^2\cos\beta\sin\beta & B\tan\beta + m_W^2\sin^2\beta \end{pmatrix} . \tag{D.1}$$

This can be diagonalized in two massless would-be Nambu Goldstone bosons $G^\pm$ and two massive charged scalars $H^\pm$ with $m_{H^\pm} = m_W^2 + m_{A^0}^2$. These two masses are given in Table A.1 and Equation (3.5)

The sleptons $\left(\widetilde{l}_i, \widetilde{l}^c{}_j\right)^T$ have the mass matrix

$$M_{\widetilde{l}}^2 = \begin{pmatrix} \lambda^e_{ik}{}^\dagger \lambda^e_{jk}\left|h_d^0\right|^2 + (m^2_{\widetilde{L}})_{ij} - \frac{1}{2}m_Z^2 c_{2\beta}c_{2\theta} & -\mu\lambda^e_{ij}{}^\dagger h_u^0 + A^e_{ij}{}^\dagger h_d^{0\dagger} \\ -\mu^\dagger\lambda^e_{ij} h_u^{0\,\dagger} + A^e_{ij} h_d^0 & \lambda^e_{ik}{}^\dagger\lambda^e_{jk}\left|h_d^0\right|^2 + (m^2_{\widetilde{l}^c})_{ij} - m_Z^2 c_{2\beta} s_\theta^2 \end{pmatrix} . \tag{D.2}$$

The up squarks $(\widetilde{u}_i, \widetilde{u}^c{}_j)^T$ have the mass matrix

$$M^2_{\widetilde{u}} = \begin{pmatrix} \lambda^u_{ik}{}^\dagger \lambda^u_{jk} \left|h^0_u\right|^2 + (m^2_{\widetilde{Q}})_{ij} + \frac{1}{6} m^2_Z c_{2\beta}(1+2c_{2\theta}) & -\mu \lambda^u_{ij}{}^\dagger h^0_d - A^u_{ij}{}^\dagger h^0_u{}^\dagger \\ -\mu^\dagger \lambda^u_{ij} h^0_d{}^\dagger - A^u_{ij} h^0_u & \lambda^u_{ik}{}^\dagger \lambda^u_{jk} \left|h^0_u\right|^2 + (m^2_{\widetilde{u}^c})_{ij} + \frac{2}{3} m^2_Z c_{2\beta} s^2_\theta \end{pmatrix} \tag{D.3}$$

and the down squarks $\left(\widetilde{d}_i, \widetilde{d}^c{}_j\right)^T$ have the mass matrix

$$M^2_{\widetilde{d}} = \begin{pmatrix} \lambda^d_{ki}{}^\dagger \lambda^d_{kj} \left|h^0_d\right|^2 + (m^2_{\widetilde{Q}})_{ij} - \frac{1}{6} m^2_Z c_{2\beta}(2+c_{2\theta}) & -\mu \lambda^d_{ij}{}^\dagger h^0_u + A^d_{ij}{}^\dagger h^0_d{}^\dagger \\ -\mu^\dagger \lambda^d_{ij} h^0_u{}^\dagger + A^d_{ij} h^0_d & \lambda^d_{ki}{}^\dagger \lambda^d_{kj} \left|h^0_d\right|^2 + (m^2_{\widetilde{d}^c})_{ij} - \frac{1}{3} m^2_Z c_{2\beta} s^2_\theta \end{pmatrix} . \tag{D.4}$$

As in the neutral sector these matrices are well-known.

D.1.2 *R*-Parity Violating Case

The charged Higgs and sleptons mix in the R-parity violating theory.

The eight-dimensional mass matrix for

$$\left(h^+_u, \widetilde{l}^c{}_i, \widetilde{l}_\alpha\right)^T = \left(h^+_u, \widetilde{l}^c{}_i, h^-_d, \widetilde{l}_i\right)^T \tag{D.5}$$

is given by

$$M^2_{h^\pm \widetilde{l}} =$$
$$\begin{pmatrix} |\mu_\gamma|^2 + \widetilde{m}^2_u + \frac{1}{2} m^2_Z c_{2\beta} c_{2\theta} & -\mu_\gamma{}^\dagger \lambda^e_{\gamma\delta j} \widetilde{\nu}_\delta & B_\beta{}^T \\ -\mu_\gamma \lambda^e_{\gamma\delta i}{}^\dagger \widetilde{\nu}_\delta{}^\dagger & (m^2_{\widetilde{l}^c})_{ij} + \left|\lambda^e_{\gamma\delta} \widetilde{\nu}_\delta\right|^2_{ij} - m^2_Z c_{2\beta} s^2_\theta & -\mu_\gamma{}^\dagger \lambda^e_{\gamma\beta i} h^0_u + A^e_{\gamma\beta i}{}^\dagger \widetilde{\nu}_\gamma \\ B_\alpha & -\mu_\gamma \lambda^e_{\gamma\alpha j}{}^\dagger h^0_u + A^e_{\gamma\alpha j} \widetilde{\nu}_\gamma & |\mu|^2_{\alpha\beta} + \widetilde{m}^2_{\alpha\beta} + \left|\lambda^e_{\gamma k} \widetilde{\nu}_\gamma\right|^2_{\alpha\beta} - \frac{1}{2} m^2_Z c_{2\beta} c_{2\theta} \end{pmatrix} . \tag{D.6}$$

The up squarks $(\widetilde{u}^c{}_i, \widetilde{u}_j)^T$ do not mix with other fields in the R-parity violating case, but there arise new terms in the mass matrix [91]:

$$M^2_{\widetilde{u}} = \begin{pmatrix} (m^2_{\widetilde{u}^c})_{ij} + \lambda^u_{ki}{}^\dagger \lambda^u_{kj} h^0_u{}^\dagger h^0_u + \frac{2}{3} m^2_Z c_{2\beta} s^2_\theta & -A^u_{ij} h^0_u - \mu_\gamma{}^\dagger \lambda^u_{ij} \widetilde{\nu}_\gamma{}^\dagger \\ -A^u_{ij}{}^\dagger h^0_u{}^\dagger - \mu_\gamma \lambda^u_{ij}{}^\dagger \widetilde{\nu}_\gamma & (m^2_{\widetilde{Q}})_{ij} + \lambda^u_{ik}{}^\dagger \lambda^u_{jk} h^0_u{}^\dagger h^0_u + \frac{1}{6} m^2_Z c_{2\beta}(1+2c_{2\theta}) \end{pmatrix} . \tag{D.7}$$

The same holds for the down squarks $\left(\widetilde{d}^c{}_i, \widetilde{d}_i\right)^T$. Their mass matrix is

$$M^2_{\widetilde{d}} = \begin{pmatrix} \lambda^d_{\gamma ki}{}^\dagger \lambda^d_{\delta kj} \widetilde{\nu}_\gamma{}^\dagger \widetilde{\nu}_\gamma + (m^2_{\widetilde{d}^c})_{ij} - \frac{1}{3} m^2_Z c_{2\beta} s^2_\theta & A^d_{\alpha jk} \widetilde{\nu}_\alpha - \mu_\gamma{}^\dagger \lambda^d_{\gamma ij} h^0_u{}^\dagger \\ A^d_{\alpha jk}{}^\dagger \widetilde{\nu}_\alpha{}^\dagger - \mu_\gamma \lambda^d_{\gamma ij}{}^\dagger h^0_u & \lambda^d_{\gamma ki}{}^\dagger \lambda^d_{\delta jk} \widetilde{\nu}^\dagger{}_\gamma \widetilde{\nu}_\gamma + (m^2_{\widetilde{Q}})_{ij} - \frac{1}{6} m^2_Z c_{2\beta}(2+c_{2\theta}) \end{pmatrix} . \tag{D.8}$$

Comparing the R-parity conserving formulas with the R-parity violating mass matrices one finds basically that one has to replace scalar expressions with corresponding four-component tensor expressions in order to get the R-parity violating expressions.

D.2 Fermionic Fields

The charged fermionic fields consist mainly of the well-known fermions of the SM. Their masses should not be altered too much in extensions to the SM.

D.2.1 *R*-Parity Conserving Case

The charged Higgsinos and the charged winos mix in the R-parity conserving theory and form the charginos

$$\chi^+_{R_p} = \left(\widetilde{W}^+, \widetilde{h}^+_u\right)^T , \qquad \chi^-_{R_p} = \left(\widetilde{W}^-, \widetilde{h}^-_d\right)^T . \tag{D.9}$$

Their mass term in the Lagrangian is

$$-\mathcal{L}_{\chi^\pm} = {\chi^-_{R_p}}^T M_{\chi^\pm_{R_p}} \chi^+_{R_p} + \text{h.c.} , \tag{D.10}$$

where the mixing matrix is

$$M_{\chi^\pm_{R_p}} = \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_2 v_u \\ \frac{1}{\sqrt{2}} g_2 v_d & \mu \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2} m_W s_\beta \\ \sqrt{2} m_W c_\beta & \mu \end{pmatrix} . \tag{D.11}$$

Diagonalization yields the two chargino masses

$$\begin{aligned} 2m^2_{\chi^\pm_{R_p}} &= |M_2|^2 + |\mu|^2 + 2m_W^2 \\ &\mp \sqrt{\left(|M_2|^2 + |\mu|^2 + 2m_W^2\right)^2 - 4\left|\mu M_2 - m_W^2 \sin 2\beta\right|^2} . \end{aligned} \tag{D.12}$$

The leptons get the same mass as in the Standard Model

$$V = \lambda^e_{ij} h^0_d l_i {l^c}_j + \text{h.c.} = m^e_{ij} l_i {l^c}_j + \text{h.c.} . \tag{D.13}$$

The same holds for the down quarks,

$$V = \lambda^d_{ij} h^0_d d_i {d^c}_j + \text{h.c.} = m^d_{ij} d_i {d^c}_j + \text{h.c.} , \tag{D.14}$$

and the up quarks,

$$V = \lambda^u_{ij} h^0_u u_i {u^c}_j + \text{h.c.} = m^u_{ij} u_i {u^c}_j + \text{h.c.} . \tag{D.15}$$

As there is no way to form another color-neutral combination, the gluino does not mix with the other particles:

$$V = \tfrac{1}{2} \left(M_3 \widetilde{g}^a \widetilde{g}^a + \text{h.c.} \right) . \tag{D.16}$$

The above formulas are partially known from the SM.

D.2.2 *R*-Parity Violating Case

In the R-parity violating case all fermionic fields with the same charge $Q = \pm 1$ mix. The mass matrix part of the Lagrangian for the charged fermions

$$\chi^-_{\not R_p} = \left(\widetilde{W}^-, \widetilde{h}^-_u, l_i\right)^T = \left(\widetilde{W}^-, l_\alpha\right)^T, \qquad \chi^+_{\not R_p} = \left(\widetilde{W}^+, \widetilde{h}^+_u\right)^T \tag{D.17}$$

is

$$-\mathcal{L}_{\chi^\pm_{\not R_p}} = \chi^{-\,T}_{\not R_p} M_{\chi^\pm_{\not R_p}} \chi^+_{\not R_p} + \text{h.c.}\,. \tag{D.18}$$

The mass matrix has the form

$$\begin{aligned} M_{\chi^\pm_{\not R_p}} &= \begin{pmatrix} M_2 & \frac{1}{\sqrt{2}} g_2 v_u & 0_j \\ \frac{1}{\sqrt{2}} g_2 v_\alpha & \mu_\alpha & \frac{1}{\sqrt{2}} \lambda^e_{\gamma\alpha j} v_\gamma \end{pmatrix} \\ &= \begin{pmatrix} M_2 & \sqrt{2} m_W \frac{v_u}{v} & 0_j \\ \sqrt{2} m_W \frac{v_\alpha}{v} & \mu_\alpha & \sqrt{2} \lambda^e_{\gamma\alpha j} \frac{v_\gamma}{v} \end{pmatrix} \\ &\simeq \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta & 0_j \\ \sqrt{2} m_W \cos\beta & \mu & \sqrt{2}\lambda^e_{ij}\cos\beta + \sqrt{2}\lambda_{kij}\epsilon^v_k \cos\beta \\ \sqrt{2} m_W \epsilon^v_i \cos\beta & \mu_i & \sqrt{2}\lambda^e_{ij}\cos\beta + \sqrt{2}\lambda_{kij}\epsilon^v_k \cos\beta \end{pmatrix}\,. \end{aligned} \tag{D.19}$$

The masses in the quark sector do not change as there are no new mass generating couplings in the R-parity breaking theory. As in the R-parity conserving case, the gluinos do not mix with the other particles as such a combination would not be color neutral.

Appendix E

Renormalization Group Equations

E.1 RGEs in the Minimal Supersymmetric Standard Model

The RGE running for the three gauge couplings in the MSSM is governed by the β function

$$\frac{d}{dt} g_{(a)} = \beta_g \ , \tag{E.1}$$

which is to first order given by

$$\beta_g^{(1)} = g_{(a)}^3 B_{(a)} \ , \tag{E.2}$$

where the parameter

$$B_{(a)} = (33/5, 1, -3) \tag{E.3}$$

is used. Here (a) runs over $\mathrm{U}(1)_Y$ in a GUT normalization, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(3)_C$. Opposed to the SM this leads to an unification of the gauge couplings shown in Figure 1.1.

The β function for the three gaugino mass parameters

$$\frac{d}{dt} M_a = \beta_M \tag{E.4}$$

is to first order proportional to the same parameter:

$$\beta_M^{(1)} = 2 g_{(a)}^2 B_{(a)} M_a \tag{E.5}$$

The running of the other parameters is modified by R-parity breaking so that we are not listing them in this section. For an overview see [38].

E.2 RGEs in the R-Parity Breaking MSSM

The complete RGEs for the R-parity breaking theory can be found in [39]. As in the rest of this thesis we are using the four-component vector notation. This has the advantage that the dependencies of the coupling running on different couplings becomes clearer. For the bilinear couplings the RGE is

$$\frac{d}{dt}\mu_\alpha = \mu_\alpha \gamma_{uu} + \mu_\beta \gamma_{\alpha\beta} \ . \tag{E.6}$$

In components this decouples in a slightly modified R-parity conserving formula and an R-parity violating counterpart

$$\frac{d}{dt}\mu = \mu\gamma_{uu} + \mu\gamma_{dd} + \mu_j \gamma_{dj} \ , \tag{E.7}$$

$$\frac{d}{dt}\mu_i = \mu_i \gamma_{uu} + \mu\gamma_{id} + \mu_j \gamma_{ij} \ . \tag{E.8}$$

The renormalization group equation for the combined lepton Yukawa coupling and the R-parity breaking counterpart is

$$\frac{d}{dt}\lambda^e_{\alpha\beta k} = \lambda^e_{\alpha\beta l}\gamma_{E_l E_k} + \lambda^e_{\alpha\delta k}\gamma_{\delta\beta} + \lambda^e_{\gamma\beta k}\gamma_{\gamma\alpha} \ . \tag{E.9}$$

The RGE for the combined down quark Yukawa and the R-parity breaking counterpart is

$$\frac{d}{dt}\lambda^d_{\alpha jk} = \lambda^d_{\alpha jl}\gamma_{D_l D_k} + \lambda^d_{\alpha lk}\gamma_{Q_l Q_j} + \lambda^d_{\gamma jk}\gamma_{\gamma\alpha} \ , \tag{E.10}$$

the RGE for the up quark Yukawa coupling is

$$\frac{d}{dt}\lambda^u_{ij} = \lambda^u_{ik}\gamma_{U_j U_k} + \lambda^u_{ij}\gamma_{uu} + \lambda^u_{kj}\gamma_{Q_i Q_k} \ , \tag{E.11}$$

and the RGE for the baryon number breaking trilinear coupling is

$$\frac{d}{dt}\lambda''_{ijk} = \lambda''_{ilk}\gamma_{D_j D_l} + \lambda''_{ljk}\gamma_{U_i U_l} + \lambda''_{ijk}\gamma_{D_k D_l} \ . \tag{E.12}$$

The one-loop anomalous dimensions for the quarks are given by

$$\gamma^{(1)}_{Q_kQ_l} = \lambda^e_{\alpha jk}{\lambda^e_{\alpha jl}}^\dagger + \lambda^u_{ik}\lambda^{u\dagger}_{il} - \delta_{ij}\left(\tfrac{1}{30}g_1^2 + \tfrac{3}{2}g_2^2 + \tfrac{8}{3}g_3^2\right) \tag{E.13}$$

$$= \left(\lambda^d\lambda^{d\dagger}\right)_{lk} + \left(\lambda^u\lambda^{u\dagger}\right)_{lk} + \left(\lambda'^\dagger\lambda'\right)_{kl} - \delta_{ij}\left(\tfrac{1}{30}g_1^2 + \tfrac{3}{2}g_2^2 + \tfrac{8}{3}g_3^2\right) , \tag{E.14}$$

$$\gamma^{(1)}_{D_kD_l} = 2{\lambda^d_{\alpha jk}}^\dagger\lambda^d_{\alpha jl} + 2{\lambda''_{ijk}}^\dagger\lambda''_{ijl} - \delta_{ij}\left(\tfrac{2}{15}g_1^2 + \tfrac{8}{3}g_3^2\right) \tag{E.15}$$

$$= 2\left(\lambda^{d\dagger}\lambda^d\right)_{ij} + 2\,\mathrm{tr}\left(\lambda_i'^\dagger\lambda_j'\right) + 2\left(\lambda''\lambda''^\dagger\right)_{ji} - \delta_{ij}\left(\tfrac{2}{15}g_1^2 + \tfrac{8}{3}g_3^2\right) , \tag{E.16}$$

$$\gamma^{(1)}_{U_iU_j} = 2{\lambda^u_{ik}}^\dagger\lambda^u_{jk} + {\lambda''_{ikl}}^\dagger\lambda''_{jkl} + \delta_{ij}\left(\tfrac{8}{15}g_1^2 + \tfrac{8}{3}g_3^2\right) . \tag{E.17}$$

The anomalous dimension for the right- handed leptons is

$$\begin{aligned}\gamma^{(1)}_{E_kE_l} &= \lambda_{\alpha\beta k}{\lambda_{\alpha\beta l}}^\dagger - \delta_{kl}\tfrac{6}{5}g_1^2 \\ &= 2\left(\lambda^e\lambda^{e\dagger}\right)_{lk} + \mathrm{tr}\left(\lambda_{ijk}{\lambda_{ijl}}^\dagger\right) - \delta_{kl}\tfrac{6}{5}g_1^2\end{aligned} \tag{E.18}$$

and the one for the up-type Higgs is

$$\gamma^{(1)}_{uu} = 3\lambda^u_{ij}\lambda^u_{ij} - \left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right) . \tag{E.19}$$

The four-component lepton anomalous dimension is given by

$$\gamma^{(1)}_{\alpha\beta} = {\lambda^e_{\alpha\gamma l}}^\dagger\lambda^e_{\beta\gamma l} + 3{\lambda^d_{\alpha kl}}^\dagger\lambda^d_{\beta kl} - \delta_{\alpha\beta}\left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right) \tag{E.20}$$

which is a symbolic expression for the four components

$$\gamma^{(1)}_{dd} = {\lambda^e_{kl}}^\dagger\lambda^e_{kl} + 3{\lambda^d_{kl}}^\dagger\lambda^d_{kl} - \left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right) , \tag{E.21}$$

$$\gamma^{(1)}_{ij} = {\lambda^e_{il}}^\dagger\lambda^e_{jl} + {\lambda_{ikl}}^\dagger\lambda_{jkl} + 3{\lambda'_{ikl}}^\dagger\lambda'_{jkl} - \delta_{ij}\left(\tfrac{3}{10}g_1^2 + \tfrac{3}{2}g_2^2\right) , \tag{E.22}$$

$$\gamma^{(1)}_{dj} = -{\lambda^e_{kl}}^\dagger\lambda_{jkl} - 3{\lambda^d_{kl}}^\dagger\lambda'_{jkl} , \tag{E.23}$$

$$\gamma^{(1)}_{id} = -{\lambda_{ikl}}^\dagger\lambda^e_{kl} - 3{\lambda'_{ikl}}^\dagger\lambda^d_{kl} . \tag{E.24}$$

In the general R-parity breaking theory the parameters that can be combined in a four-component parameter, are created mutually in the RGE running if not collectively set to zero.

E.3 Bilinear R-Parity Breaking RGEs

In the bilinear R-parity breaking the mixing between the Higgsino mass parameter μ and the lepton number breaking mass term in Equation (E.6) vanish to first order. The Higgsino mass term gets only corrections of order μ_i^2 and the dependency of μ_i on μ drops out completely.

E.4 RGEs in the SU(5) Flavor Model

Integrating the renormalization group equation from the GUT scale and taking into account the decoupling of heavy fermions at their respective masses M_k for scales $\mu \ll M_k$ one gets the leading logarithmic order for the scalar masses [36]:

$$\begin{aligned}
(\delta m^2_{\widetilde{L}})_{ij} &\simeq -\frac{1}{2(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\lambda^{\nu\,\dagger}_{ik}\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\lambda^{\nu}_{kj} \\
&\sim -\frac{1}{2(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\begin{pmatrix}\eta^2 & \eta & \eta\\ \eta & 1 & 1\\ \eta & 1 & 1\end{pmatrix}\,, \\
(\delta m^2_{\widetilde{Q}})_{ij} &\simeq -\frac{1}{2(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\lambda^{u\,*}_{ik}\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\lambda^{u\,T}_{kj} \\
&\sim -\frac{1}{2(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\ln\frac{\Lambda_{\mathrm{GUT}}}{v}\begin{pmatrix}\eta^4 & \eta^3 & \eta^2\\ \eta^3 & \eta^2 & \eta\\ \eta^2 & \eta & 1\end{pmatrix}\,, \\
(\delta m^2_{\widetilde{u}^c})_{ij} &\simeq -\frac{1}{(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\lambda^{u\,T}_{ik}\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\lambda^{u\,*}_{kj} \\
&\sim -\frac{1}{(2\pi)^2}\left(3m^2_{3/2}+A^2\right)\ln\frac{\Lambda_{\mathrm{GUT}}}{v}\begin{pmatrix}\eta^4 & \eta^3 & \eta^2\\ \eta^3 & \eta^2 & \eta\\ \eta^2 & \eta & 1\end{pmatrix}\,.
\end{aligned}\tag{E.25}$$

The leading logarithmic order for the cubic scalar couplings is

$$\begin{aligned}
\delta A^d_{ij}v_d &\simeq -\frac{3}{4(2\pi)^2}Av_d\left(h^u h^{u\dagger}\right)_{ik}\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\lambda^d_{kj} \\
&\sim -\frac{1}{4(2\pi)^2}Am_b\ln\frac{\Lambda_{\mathrm{GUT}}}{v}\begin{pmatrix}\eta^7 & \eta^4 & \eta^2\\ \eta^6 & \eta^3 & \eta\\ \eta^5 & \eta^2 & 1\end{pmatrix}\,, \\
\delta A^e_{ij}v_d &\simeq -\frac{1}{2(2\pi)^2}Av_d\left(h^e h^{\nu\dagger}\right)_{ik}\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\lambda^{\nu}_{kj} \\
&\sim -\frac{1}{2(2\pi)^2}Am_\tau\ln\frac{\Lambda_{\mathrm{GUT}}}{M_k}\begin{pmatrix}\eta^5 & \eta^4 & \eta^4\\ \eta^2 & \eta & \eta\\ \eta & 1 & 1\end{pmatrix}\,.
\end{aligned}\tag{E.26}$$

Appendix F

Cosmology

The basics of cosmology can be found in most books about general relativity, for instance in [92]. The Einstein-Hilbert Lagrangian (1.15) gives with a matter Lagrangian the Einstein equations

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = M_P^{-2} T_{\mu\nu} \ , \tag{F.1}$$

where $T_{\mu\nu}$ is the energy-momentum tensor. In order to solve this set of equations for the whole universe we assume that we live in an isotropic and homogeneous universe. This assumption is justified by the measurement of the cosmic microwave background (CMB) and large scale galaxy surveys. The most general space-time metric compatible with this assumptions is the Robertson–Walker metric. To simplify the problem further we assume that the matter and energy densities of the universe are compatible with a perfect fluid on large scales

$$T_\mu^\nu = \mathrm{diag}\,(\rho, -p, -p, -p) \ . \tag{F.2}$$

Solving the Einstein equations with these assumptions gives the Friedmann equations. Depending on the Hubble constant H, which is the proportionality constant for the linear expansion of our universe, the energy content of the universe can be characterized by the equation of state:

$$p_i = w_i \rho_i \ . \tag{F.3}$$

Relativistic particles have $w_r = 1/3$, nonrelativistic particles have $w_m = 0$ and the cosmological constant can be parametrized with $w_\Lambda = -1$. In order to have a flat universe the density must be critical. The present-day critical density is given by

$$\rho_c = 3H_0^2 M_P^2 \simeq 1.05 \times 10^{-5} h^2 \,\mathrm{GeV\ cm}^{-3} \ , \tag{F.4}$$

where H_0 is the present-day Hubble constant which usually is parametrized by

$$H_0 = 100\, h\, \mathrm{km\ s^{-1} Mpc^{-1}} \tag{F.5}$$

with $h \simeq 0.7$. The unit pc stands for parsec, the common astrophysical unit of length. The Friedmann equation can be expressed in terms of the critical density by introducing the density parameter

$$\Omega_i = \frac{\rho_i}{\rho_c} \; . \tag{F.6}$$

As observation shows that we live in a spatially flat universe, the Friedmann equation gives the sum rule

$$1 \simeq \Omega_{\mathrm{tot}} \simeq \Omega_m + \Omega_\Lambda + \Omega_r \; . \tag{F.7}$$

In the present-day universe the radiation contribution can be neglected and the matter density makes up for $\Omega_m = 0.24$ of the total density. However, observable baryonic matter can only explain $\Omega_b = 0.0425$, therefore, the rest of the matter must consists of dark matter. One possible explanation for particle dark matter is introduced in Section 4.2.

Standard cosmology needs very specific initial conditions to give the observed results. In particular it suffers under the flatness problem and the homogeneity problem. We will not specify this further, but to solve this problems one introduces inflation, where the scale factor of the universe grows by a factor of about e^{60}. Afterwards the density of the particles is diluted. However, after inflation the inflaton field decays into, among others, SM particles. This process is called reheating and the reheating temperature constraints which particles can be produced in the thermal bath [12].

As opposed to the observable universe, matter and antimatter are still in equilibrium at this stage. The observed baryon-photon ratio (4.1) can be explained by baryogenesis via thermal leptogenesis [13]. In this theory CP-violating out-of-equilibrium decays of heavy right-handed neutrinos creates nonvanishing lepton number, which can be converted into nonvanishing baryon number via sphaleron processes.

When the universe is cold enough to allow particles to form atoms, BBN takes place [93]. It predicts the abundance of the light elements. These predictions agree with the observed abundances. Hence extensions to the SM must not spoil these predictions.

Appendix G

Matrix Diagonalization

Any matrix M can be diagonalized by two unitary matrices U, V via

$$V^\dagger M U = M' = \mathrm{diag}\,(m_1, m_2, ..., m_n) = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \ , \tag{G.1}$$

where m_i^2 are the eigenvalues of $MM^\dagger$ and M_1, M_2 are two diagonal submatrices. Hence, one can always rewrite M in the form

$$M = VM'U^\dagger = VA^\dagger AM'B^\dagger BU^\dagger \ , \tag{G.2}$$

where A and B are unitary matrices of the form

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \ , \qquad\qquad B = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix} \ . \tag{G.3}$$

the submatrices $A_{1,2}$ and $B_{1,2}$ which are also unitary. Thus we can write

$$M = QM_BP^\dagger \ , \tag{G.4}$$

where

$$M_B = AM'B^\dagger = \begin{pmatrix} A_1 M_1 {B_1}^\dagger & 0 \\ 0 & A_2 M_2 {B_2}^\dagger \end{pmatrix} \tag{G.5}$$

is block-diagonal and $Q = VA^\dagger$ and $P = UB^\dagger$. Of course, M_B is not unique.

G.1 Hierarchical Matrices

For the case of hierarchical matrices one can find an analytical approximate formula for P and Q.

For example the matrix

$$M = \begin{pmatrix} m_A & m_B \\ m_B{}^\dagger & m_C \end{pmatrix} \ , \tag{G.6}$$

where

$$\begin{aligned} m_A &= m_A{}^\dagger \ , \\ m_C &= m_C{}^\dagger \ , \\ |m_A|^2 &\gg |m_B|^2 \, , |m_C|^2 \ , \end{aligned} \tag{G.7}$$

is transformed by

$$U = \begin{pmatrix} 1 & -m_A^{-1} m_B \\ m_B{}^\dagger m_A^{-1} & 1 \end{pmatrix} \tag{G.8}$$

into approximately block-diagonal form [43]:

$$\begin{aligned} U^\dagger M U = &\begin{pmatrix} m_A + m_A^{-1} m_B m_B{}^\dagger + m_B m_B{}^\dagger m_A^{-1} & m_A^{-1} m_B m_C \\ m_C{}^\dagger m_B{}^\dagger m_A^{-1} & m_C - m_B{}^\dagger m_A^{-1} m_B \end{pmatrix} \\ &+ \mathcal{O}\left(\frac{|m_{B,C}|^3}{|m_A|^2} \right) \\ \simeq &\begin{pmatrix} m_A & 0 \\ 0 & m_C - m_B{}^\dagger m_A^{-1} m_B \end{pmatrix} + \mathcal{O}\left(\frac{|m_{B,C}|^2}{|m_A|} \right) \ . \end{aligned} \tag{G.9}$$

As one can see, to lowest order only the small diagonal block gets modifications.

Acknowledgments

First of all let me thank my supervisor Wilfried Buchmüller for giving me the opportunity to work on this exciting topic. I am grateful for good advice, continuous and valuable support and his excellent mentoring throughout the entire period I worked on this thesis.

Furthermore, I would like to thank Jan Louis for acting as second examiner for this thesis, who thereby gave me the opportunity to carry out this thesis at the DESY Theory Group.

I am also very thankful to Sergei Bobrovskyi and Jonas Schmidt for the very pleasant collaboration. In addition I want to thank my office colleague Christoph Weniger for many fruitful and pleasant discussions. Many thanks also to Michael Grefe and Jasper Hasenkamp for many profound discussions and a enjoyable time at DESY. In particular I would like to thank Jonas, Michael, Sergei and Daniel Baker for proofreading and commenting on this thesis.

Finally, I want to thank all the people at DESY for the inspiring environment, with doors that are always open for questions.

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