Quantum mechanics forbids an initial or final singularity in a closed FRW universe

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The canonical Hamiltonian quantization of standard cosmology by Elbaz et al [1] and Novello et al [2] avoids the complexities of the old approach to quantum cosmology involving superspace (or mini-superspace) and the Wheeler-De Witt equation. Novello et al [2] show the quantum dynamics of a closed homogeneous and isotropic radiation-dominated universe is equivalent to the evolution of a particle moving in one dimensions in a potential

$$V(q) = -\frac{b^2}{4q^2},$$

where q is proportional to the radius of curvature of the universe and b is a constant. This should not be surprising. The Friedmann equation [3] for the radius of curvature R of a closed homogeneous, isotropic radiation-dominated universe is

$$\left(\frac{dR}{dt}\right)^2 - \frac{A}{R^2} = -c^2 \qquad (1)$$

where A is a constant and c = 3 x 10^{10} cm/sec. If the Friedmann equation is multiplied by ½ , it is a condition on the energy of a classical particle of unit mass bound in a $-\frac{A}{2R^2}$ potential well, moving in one dimension with speed $\frac{dR}{dt}$. The Schrodinger equation for the motion of this fictitious unit mass particle has the same form as the Schrodinger equation identified by Elbaz et al and Novello et al.

The Schrodinger equation corresponding to equation (1) (and, likewise, the Schrodinger equation of Elbaz et al and Novello et al) for a closed radiation-dominated FRW universe can be written

$$\frac{d^2}{dR^2}\psi + \left[-\kappa^2 + \frac{\gamma^2}{R^2}\right]\psi = 0 \qquad (2)$$

Morse and Feshbach [4] show that the solution to equation (2) is

$$\psi = KRh_{ip-\frac{1}{2}}(i\kappa R)$$

where

$$p = \sqrt{\gamma^2 - \frac{1}{4}},$$

h is the spherical Hankel function and *K* is a normalization constant. This stationary state wave function is zero at $R = 0$. A time-dependent wave function describing the evolution of a closed radiation-dominated FRW universe must be a wave packet centered around the "effective energy" $\kappa = -\frac{c^2}{2}$, and that superposition of stationary state solutions to equation (2) must also be zero at $R = 0$.

If general relativity is assumed valid at distances less than the Planck length, the Schrodinger equation of Elbaz et al should be valid at these distances and quantum mechanics seems to forbid singularities in a closed FRW universe. Since the eigensolutions of the Schrodinger equations for flat and open FRW universes are also zero at $R = 0$, it appears this conclusion can be extended to flat and open FRW universes.

A quantum mechanical model that allows the universe to begin with a non-zero radius, by a quantum fluctuation from the vacuum, is outlined elsewhere [5].

References

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