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POST-NEWTONIAN APPROXIMATION OF A SCALAR THEORY OF GRAVITATION AND APPLICATION TO LIGHT RAYS

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Abstract - A scalar, preferred-frame theory of gravitation is summarized. Space-time is endowed with both a flat metric and a curved, "physical" metric. Motion is governed by a natural extension of Newton's second law, which implies geodesic motion only for a static field. The theory predicts Schwarzschild's exterior metric in the spherical static situation. It also predicts gravitation waves with the velocity of light. The equations of motion are recast into the "flat space - uniform time" form, and compared with the geodesic equations of motion. The principles of the post-Newtonian approximation of this theory are given, including the way to account for preferred-frame effects. This approximation is then developed more particularly for photons. It is found that the preferred-frame effects do not occur in this case, nor does the difference between Newton's second law and geodesic assumption. Thus, the post-Newtonian predictions of this theory for photons are indistinguishable from the standard post-Newtonian predictions of general relativity.

1. Introduction

One of the most striking features of Einstein's general relativity is that it predicts three effects of a gravitation field on light rays (or photons), and that these effects are experimentally confirmed. In this paper, it will be shown that a certain *scalar* theory of gravitation, which has a *preferred frame*, predicts post-Newtonian gravitational effects on photons - namely, the red-shift, the deflection, and the echo delay - which are just the same as the standard post-Newtonian predictions of general relativity (GR). Although this does represent an important step for any theory, it is still not enough for claiming that the investigated theory is viable. However, the simplicity of this scalar theory, and the great difference with GR in its physical concept, make this result at least curious, because the effects on light rays often appear as the most "relativistic" predictions of GR. In fact, this theory predicts all the same *exact* effects as the standard effects of GR in the static spherically symmetric situation. This is because, in that situation, the theory predicts the Schwarzschild exterior solution and asserts that free particles follow space-time geodesics - and this is true for mass points [1] as well as for photons [2]. But since the theory has a preferred frame, one must account for the effects of the velocity of the massive attracting body through this "ether": this is done in this paper, for photons and in the first post-Newtonian

approximation. The non-covariance of the theory is a necessary consequence of the fact that it is a scalar theory which accounts for special relativity (SR). Thus, the source of the gravitation field must be an energy and the energy concept of SR does not lead to an invariant scalar, instead it leads to a covariant tensor **T**, the energy-momentum of matter and non-gravitational fields.

One may ask, of course, why it should be interesting to investigate a non-covariant theory, knowing the risks it implies (especially for the explanation of Mercury's advance in perihelion). We mention the following reasons:

(i) This scalar theory is a natural modification of the Newtonian theory to account for "relativistic" effects, when one sees these effects as due to the "absolute" Lorentz contraction of bodies that move through the "ether" - as Lorentz and Poincaré saw these effects. Jánossy [3-4], Prokhovnik [5-7], and others, proved in detail that this "absolute" conception is entirely compatible with the usual formalism and the physical predictions of SR. The Lorentz-Poincaré interpretation of SR was also emphasized by Bell [8]. The theoretical impossibility in measuring the one-way velocity of light independently of a synchronization convention (see Prokhovnik [5] and Zhang [9]) is important in this interpretation.

(ii) This theory does not predict any singularity or infinite density for the case of gravitational collapse in "free fall" with spherical symmetry: instead, the theory predicts a "bounce", i.e. the initial implosion is stopped and is followed by an explosion [2].

(iii) It does lead to a "quadrupole formula" for the gravitational radiation, in the way briefly outlined in ref. [2].

(iv) Some gravitational effects of motion through an "ether" might contribute to explain the strange dynamical behaviours at the scale of galaxies or clusters of galaxies (e.g. the rotation curves). The currently favoured explanation of such behaviours appeals to large amounts of unseen matter [10].

In order to show that the "relativistic" effects on light rays are not masked by the motion of the attracting body through the ether of this theory, we shall proceed thus: in Section 2, we summarize the theory and give the relevant exact equations - in particular, we give the equations of motion of a particle (mass point or photon) in the form of the "flat space - uniform time" acceleration, as is convenient for applications. In Section 3, we present the principles of the asymptotic expansion for the equations of the theory; we apply these principles to expand, first, the metric and the field equation, then the equations of motion for a photon. We find that the effects of absolute motion on photons do not appear in the first post-Newtonian approximation, which is the approximation currently used [11-13] to evaluate these effects in GR and other "metric theories". By the way, we already underline here an important point: although the investigated theory endows the space-time with a curved metric, it is not truly a metric theory in the usual sense [11-13], because "free" particles do not generally follow space-time geodesics (except for a static gravitation field). This means that the parametrized post-Newtonian formalism does not *a priori* apply to this theory.

2. Summary of the Theory and its Main Exact Equations

The starting point of the theory is simple: it is the heuristic interpretation of gravity as the pressure force exerted on matter at the scale of elementary particles by an imagined perfect fluid or "micro-ether". Thus, according to this heuristic view, gravity would be simply "Archimedes' thrust in the ether". It

turns out that Newtonian gravity (NG) was interpreted in this way by Euler in 1746 [14]. However, the point here is not NG, although the present theory does reduce to NG in the limit of weak and slowly varying fields [2]. The construction of the theory was done in refs. [1] and [15] (see ref. [2] for a synthetic account, including new results), except for the general formulation of motion, which has been modified in order to obtain a true conservation equation for the energy [16]. Hereafter, the relevant assumptions and equations are summarized directly in the self-consistent form where they will be used, although this makes them appear less natural.

2.1 Basic concepts, space-time metric, field equation

The equations are given in the preferred frame E, which is the rest frame of the reference body M or "macro-ether". [1] The reference body M is assumed to be equipped with an Euclidean space metric $\mathbf{g}^0$. Observers bound to the frame E would measure the spatial distances defined with $\mathbf{g}^0$, and would also measure the "absolute time" t, if there were no gravitation. An "absolute" version of Einstein's equivalence principle implies that physical standards of space (resp. time) are contracted (resp. dilated) in a gravitation field [1-2]. The scalar field of the theory is the contraction factor $b$affecting the time interval $dt_{\mathbf{x}}$ measured by a clock fixed at point $\mathbf{x}$ bound to the frame E (due to the dilation of the clock period in the gravitation field), or the square $f = b^2$. The latter defines the g_{00} component of the physical, curved space-time metric $\mathbf{g}$ in any coordinates (x^a), with $x^0 = ct$ (and c the velocity of light), that are bound to the frame E:

$$dt_{\mathbf{x}} / dt = \sqrt{(g_{00})_{\mathrm{E}}} = \sqrt{f} \equiv b \leq 1. \tag{1}$$

The interpretation of gravity as a pressure force leads to define the acceleration vector $\mathbf{g}$ of gravitation [1-2, 15]. When reexpressed in terms of the Euclidean metric $\mathbf{g}^0$ and the scalar f, the vector $\mathbf{g}$ turns out to be simply [16]

$$\mathbf{g} = -\frac{c^2}{2} \nabla_0 f, \qquad \nabla_0 f \equiv \mathrm{grad}_{\mathbf{g}^0} f \,. \tag{2}$$

Due to the "absolute" version of the equivalence principle, the field $b$defines also the dilation of an infinitesimal distance dl measured in the direction of vector $\mathbf{g}$, as compared with the distance dl^0 evaluated using the Euclidean metric: $dl = dl^0/b$, while the infinitesimal distances in directions perpendicular to $\mathbf{g}$ are unaffected. This assumption may be reexpressed in saying that the physical space metric $\mathbf{g}$in the frame E has the following relation to the Euclidean metric$\mathbf{g}^0$ (in any coordinates (x^a) bound to the frame E): [2]

$$g_{ij} = g^0{}_{ij} + \left(\frac{1}{f} - 1\right) h_{ij}, \quad h_{ij} \equiv \frac{b_{,i}\, b_{,j}}{(g^0)^{kl}\, b_{,k}\, b_{,l}} = \frac{f_{,i}\, f_{,j}}{(g^0)^{kl}\, f_{,k}\, f_{,l}} \tag{3}$$

with $((g^{0)kl})$ the inverse matrix of $(g^0{}_{kl})$. Indeed, Eq. (3) is space-covariant and implies that, in

coordinates orthogonal for the Euclidean metric:

$$(g^0{}_{ij}) = \text{diag}\,(a^0{}_1, a^0{}_2, a^0{}_3),$$

and such that $g_i = 0$ for $i \neq 1$, the physical metric is simply

$$(g_{ij}) = \text{diag}\,(a^0{}_1/f, a^0{}_2, a^0{}_3).$$

The space-time metric writes, in any coordinates (x^a) bound to the frame E, with $x^0 = ct$:

$$ds^2 = g_{lm}dx^l dx^m = f\,(dx^0)^2 - dl^2,\ dl^2 = g_{ij}\,dx^i\,dx^j \quad \Rightarrow \quad g_{00} = f,\ g_{ij} = -g_{ij},\ g_{0i} = 0. \qquad (4)$$

That the g_{0i} components are always zero in the ether frame E, is the natural translation of the assumption that this frame admits a global simultaneity defined with the absolute time t. Hence, a gravitation field that is constant in this frame is necessarily a *static* field. This implies, in turn, that a stationary rotation of a massive body with respect to the ether frame will produce *no Lense-Thirring effect*. One may perhaps infer that this will remain true if, in addition, the mass center **b** of the body has a constant velocity with respect to the ether frame, $d\mathbf{b}/dt = \mathbf{V} =$ Const. However, in that case, the gravitation field *cannot* be static (even in the frame $\mathrm{E}_{\mathbf{V}}$ that follows the translation) since, due to the Lorentz transformation of the non-Galilean metric **g**, the g_{0i} components *cannot* reduce to zero in the frame $\mathrm{E}_{\mathbf{V}}$ [see Eqs. (42)-(43) and (55)-(56) below]. This is independent of the field equation.

In what follows we assume that the "reference ether pressure" $p_e^{\text{¥}}(t)$ [2] does not actually depend on t, which means that we leave aside cosmological problems. [1999 addition: Such problems are considered in ref. 22.] Then the field equation may be written as:

$$\Delta_0 f - \frac{1}{f}\left(\frac{f_{,0}}{f}\right)_{,0} = \frac{8\pi G}{c^2}\,\rho, \qquad (5)$$

with $\mathrm{D}_0 = \mathrm{div}_0\,\mathrm{grad}_0$ the usual Laplace operator, defined with the Euclidean metric $\mathbf{g}^0$, and where $f_{,0} = \partial f/\partial x^0$, G is Newton's gravitation constant, and $\rho = (T^{00})_{\mathrm{E}}$ is the mass-energy density in the ether frame [16]. This equation, which is merely space-covariant, implies that gravitation propagates with the velocity of light (as measured with physical standards affected by gravitation). On the other hand, in any static situation, it associates exactly the Newtonian gravity acceleration $\mathbf{g} = \mathbf{g}_{\mathrm{N}}$ with the given constant mass-energy density $\rho = \rho(\mathbf{x})$. But $\rho = T^{00}$ does not only involve the density e_{pm} of the pure matter energy (e.g., for "dust", it does not only involve the rest-mass and kinetic energy), it is also increased by the gravitation field itself - indeed, one has $\rho = e_{pm}/b = e_{pm}/\sqrt{f}$ [16]. Finally, in the spherical static situation, the space-time metric (4) predicted from the field equation was given in refs. [1-2] (using Eq. (5), it is still easier to get):

$$ds^2 = \left(1 - \frac{2U_N(r)}{c^2}\right) c^2 dt^2 - \frac{1}{1 - \frac{2U_N(r)}{c^2}} dr^2 - r^2 d\Omega^2, \; d\Omega^2 \equiv d\theta^2 + \sin^2\theta \, d\varphi^2, \qquad (6)$$

where $U_N(r)$ is the Newtonian potential associated with the density $r = r(r)$:

$$U_N(r) = \int_r^\infty \frac{GM(u)}{u^2} du, \; M(r) = 4\pi \int_0^r u^2 \, \rho(u) \, du. \qquad (7)$$

Again, as a static solution, this solution assumes that the mass-energy density is time-independent (and spherically symmetric) *in the ether frame*, i.e. the massive body is at rest in the ether or in a stationary rotation with respect to the ether. If $r(r) = 0$ for $r > R$, then $U_N(r) = GM/r$ for $r > R$ (with $M = M(R)$), this is the Schwarzschild exterior solution. As to the Schwarzschild interior solution of GR, it is only known when, in addition, $r(r) =$ Const. for $0 \pounds r < R$, but Eq. (6) does *not* give the Schwarzschild interior solution in that case.

2.2 Equations of motion

Motion is governed by a natural extension of Newton's second law. In the case of a constant gravitation field, it is relatively straightforward to extend the special-relativistic form of Newton's second law ($\mathbf{F} = d\mathbf{P}/dt$ with $\mathbf{P}$ the momentum) to the situation with gravitation, because the time-derivative of a vector (like $\mathbf{P}$) must undoubtedly be defined as the "absolute derivative", the latter being induced from the covariant derivative with respect to the constant space metric. The absolute derivative of a vector $\mathbf{w}$ depending on the parameter c along a trajectory in a Riemannian space equipped with a fixed metric $\mathbf{g}$ is given by:

$$\left(\frac{D\mathbf{w}}{D\chi}\right)^i \equiv \frac{dw^i}{d\chi} + \Gamma^i_{jk} w^j \frac{dx^k}{d\chi}, \qquad (8)$$

with Γ^i_{jk} the Christoffel symbols of metric $\mathbf{g}$. This derivative was used by Landau & Lifchitz [18] to rewrite the spatial part of Einstein's geodesic assumption as Newton's second law, for a constant gravitation field. In the case of a variable gravitation field, we still have, in the frame E (as actually in any other possible reference frame), a well-defined space metric $\mathbf{g}$, but now it depends on time, and we have a local time $t_{\mathbf{x}}$. It turns out that a unique definition may be given for the time-derivative $D\mathbf{w}/Dc$ of a space vector $\mathbf{w}$ depending on the parameter c along a trajectory in the given frame, under the following essential requirements: (i) it must be a space vector depending linearly on $\mathbf{w}$, (ii) it must reduce to the absolute derivative if $\mathbf{g}$ does not depend on time, (iii) it must be expressed in terms of the space metric $\mathbf{g}$ and its derivatives, and (iv) it must obey Leibniz' derivation rule for a scalar product, i.e.

$$\frac{d}{d\chi}\big(\mathbf{g}(\mathbf{w},\mathbf{z})\big) = \mathbf{g}(\mathbf{w}, \frac{D\mathbf{z}}{D\chi}) + \mathbf{g}(\frac{D\mathbf{w}}{D\chi}, \mathbf{z}).$$

In coordinates, this unique definition is as follows [16] :

$$\left(\frac{D\mathbf{w}}{D\chi}\right)^i \equiv \frac{dw^i}{d\chi} + \Gamma^i_{jk}\, w^j \frac{dx^k}{d\chi} + \frac{1}{2} g^{ij} g_{jk,0} \frac{dx^0}{d\chi} w^k, \tag{9}$$

with G^i_{jk} the Christoffel symbols of the space metric **g**. To define Newton's second law, the trajectory of the test particle is parametrized with the local time $c = t_{\mathbf{x}}$, synchronized along the trajectory as defined by Landau & Lifchitz [18]. Actually, definition (9) is also valid (and compelling under the above requirements) for any reference frame in any theory with a curved space-time metric [19]. In the present case, the synchronized local time is simply obtained by integration of Eq. (1) along the trajectory, because the absolute time t is globally synchronized in the ether frame E (i.e., because the g_{0i} components are assumed zero in E).

There is also no ambiguity for the definition of the momentum. This is

$$\mathbf{P} \circ m(v)\, \mathbf{v},\ m(v) \circ m(v=0).g_v \circ m(0).(1 - v^2/c^{2)-1/2}, \tag{10}$$

the velocity **v** of the test particle (relative to the frame E) being measured with the local time $t_{\mathbf{x}}$ and its modulus v being defined with the space metric **g**:

$$v^i \circ dx^i/dt_{\mathbf{x}}\ ,\ v \circ [\mathbf{g}(\mathbf{v}, \mathbf{v})]^{1/2} = (g_{ij} v^i v^j)^{1/2}. \tag{11}$$

The gravitation force is assumed to be $\mathbf{F}_g = m(v)\mathbf{g}$ with **g** given by Eq. (2), and there may be also a non-gravitational force $\mathbf{F}_0$. Thus, the extension of Newton's second law is obtained as

$$\mathbf{F}_0 + m(v)\mathbf{g} = D\mathbf{P}/Dt_{\mathbf{x}}\ . \tag{12}$$

For a photon, the velocity **v** is still defined by (11) (and now its modulus is *constrained* to $v = c$), but the mass content of the energy $E = hn$ is substituted for the inertial mass, the frequency n (number n of oscillations per unit time) being measured with the local time:

$$n \circ dn/dt_{\mathbf{x}},\quad E \circ hn, \tag{13}$$

$$\mathbf{F}_0 + (E/c^2)\mathbf{g} = D\mathbf{P}/Dt_{\mathbf{x}}\ ,\ \mathbf{P} \circ (E/c^2)\ \mathbf{v}. \tag{14}$$

Defining the "pure energy" of a mass point (i.e., not accounting for the potential energy in the gravitation field) as $E = m(v)\, c^2$, we see that Eq. (14) applies to both mass points and photons. Together with the assumed vector **g** (Eq. (2)) and the assumed metric (Eqs. (1), (3) and (4)), Eq. (14) implies that

free particles ($\mathbf{F}_0 = 0$) follow space-time geodesics in a *static* gravitation field ($f_{,0} = 0$): see ref. [1] or ref. [19] for mass points and see ref. [2] for photons.

We now compute the "coordinate acceleration" for a *free particle* (mass point or photon) in a general gravitation field:

$$a^i \circ (du^i/dt)_{i=1,2,3},\ u^i \circ dx^i/dt\ . \tag{15}$$

Note that $\mathbf{u} = (u^i)$ is a space vector, the "absolute velocity", but (a^i) is not a space vector. The "absolute acceleration" $\mathbf{a}_0$ may be defined in arbitrary space coordinates as the absolute derivative of $\mathbf{u}$ with respect to (the time t and) the Euclidean metric $\mathbf{g}^0$, and its components are indeed a^i in locally *Cartesian* coordinates x^i, i.e. such that $g^0{}_{ij} = d_{ij}$ and $g^0{}_{ij,k} = 0$ at the point considered. Using definitions (1), (2) and (9) in Eq. (14) and multiplying by $f = b^2$ obtains

$$-\frac{c^2}{2} f\,(\nabla_0 f)^i = \frac{\beta}{E}\frac{d}{dt}\left(\frac{E}{\beta}u^i\right) + \Gamma^i_{jk}\,u^j\,u^k + \frac{1}{2}g^{ij}g_{jk,0}\,c\,u^k\,. \tag{16}$$

To evaluate $(d/dt)(E/b)$, we use the energy equation of the theory [16], which is deduced from the extended Newton law (14) for a free particle (mass point or photon):

$$\frac{de_m}{dt} = \frac{e_m}{\beta}\frac{\partial\beta}{\partial t}\ , \qquad \left(e_m \equiv E\,\beta \Rightarrow \frac{E}{\beta} = \frac{1}{f}e_m\ \right). \tag{17}$$

Inserting Eq. (17) into Eq. (16) gives after rearranging

$$\frac{du^i}{dt} = \frac{1}{\beta}\left(\frac{\partial\beta}{\partial t} + 2\,\beta_{,j}\,u^j\right)u^i - \Gamma^i_{jk}\,u^j\,u^k - \frac{1}{2}g^{ij}\,\frac{\partial g_{jk}}{\partial t}\,u^k - \frac{c^2}{2}f\,(\nabla_0 f)^i\,. \tag{18}$$

This is the form of the equation of motion that we shall use for the pN approximation. However, it will be useful also to have this exact equation in a form that allows a comparison with the following equation of motion, deduced (for arbitrary space-time coordinates) by Weinberg [11] from Einstein's assumption that particles follow space-time geodesics:

$$\frac{du^i}{dt} = -\Gamma'^{\,i}_{00} - 2\Gamma'^{\,i}_{0j}\,u^j - \Gamma'^{\,i}_{jk}\,u^j\,u^k + \left(\Gamma'^{\,0}_{00} + 2\Gamma'^{\,0}_{0j}\,u^j + \Gamma'^{\,0}_{jk}\,u^j u^k\right)u^i\,. \quad (t \circ x^0) \tag{19}$$

The Christoffel symbols G ' of the space-time metric (4), with $x^0 = ct$, are [19] :

$$\Gamma'^{0}_{00}=\frac{\phi_{,0}}{\phi},\quad \Gamma'^{0}_{0j}=\frac{\phi_{,j}}{\phi},\quad \Gamma'^{i}_{00}=-\frac{1}{c^2}g^{i}f=\frac{1}{2}f\,(\nabla_0 f)^{i},\Gamma'^{i}_{0k}=\frac{1}{2}g^{ij}g_{jk,0}. \qquad (20)$$

Therefore, when changing the time coordinate to $x^0 = t$ (which is convenient for the pN expansion), the existing c or c^2 factors disappear and we may rewrite Eq. (18) as

$$\frac{du^i}{dt}=-\Gamma'^{i}_{00}-\Gamma'^{i}_{0j}u^j-\Gamma^{i}_{jk}u^j u^k+\left(\Gamma'^{0}_{00}+2\Gamma'^{0}_{0j}u^j\right)u^i \qquad (x^0=t)\,. \qquad (21)$$

This equation is only valid in coordinates (x^a) bound to E, with $x^0 = t$, thus $g_{0i} \neq 0$, whence $\Gamma'^{i}_{jk} = \Gamma^{i}_{jk}$. Thus, the difference between (19) and (21) is that: (i) the

$$\Gamma'^{i}_{0j}u^j$$

term is not multiplied by 2 in Eq. (21) as it is in Eq. (19), and (ii) the last term of Eq. (19) is missing in Eq. (21).

3. Post-Newtonian Approximation

3.1 Principle of the approximation

The principle of the post-Newtonian (pN) expansion is classical (see Fock [20], Chandrasekhar [21], Weinberg [11], Misner *et al.* [12], Will [13]) : it is assumed (i) that the gravitation field is weak so that the Newtonian approximation gives the main terms and moreover the Newtonian potential U is small as compared with the square c^2, thus $U_{\max}/c^2 \sim e^2$ with e a small parameter. (ii) That the velocities of the massive bodies or mass test particles are small as compared with c; this assumption is not independent from (i) since, in a frame moving with the gravitationally bound system, the square of the orbital velocity will be of the order U. Hence, in such a frame, the velocities will be at most of the order $e\,c$. So any relevant quantity will be expanded in powers of the small parameter e. The derivative of any relevant quantity A, with respect to any space coordinate, will be of the same order as A with respect to e, of course. It is still assumed that (iii) the derivative of any relevant quantity A with respect to the time coordinate $x^0 = ct$ is of the order e´ (order of A). This amounts to assuming that $|\partial A/\partial x^0| \leq e\,O(|\tilde{N}A|)$. It is *not* valid in the "wave zone", i.e. for distances from the center of the system that are *large* as compared with the wave length of the gravitation waves emitted by the system, $l \sim 2\pi\, lc/v \sim 2\pi l e^{-1}$ with l the typical radius of the system and v the typical orbital velocity (see Landau & Lifchitz [18] and Misner *et al*. [12]; note that the typical dimension of the system is thus *small* as compared with l). All these assumptions are taken into account automatically if *we simply expand all equations with* $e' \sim 1/c$ *as the small parameter, and with* $x^0 = t$ *as the time coordinate*, as proposed by Fock [20]. This is what we shall do in the actual calculations. We note that, choosing the units such that the maximum value of the Newtonian potential is $U_{\max} = 1$, we get indeed $e = e'$.

There is something more to be said in a preferred-frame theory like the present one: the absolute velocity V of the mass-center of the system with respect to the ether should not exceed the order $e\,c$, as is the case for the typical orbital velocity v in the mass-center frame. For the solar system, we have $e^2 \sim$

$U_{max}/c^2 \approx 10^{-5}$ if we take the absolute maximum, at the center of the Sun [12], whereas the maximum of the "useful" values of U is rather the value at the surface of the Sun, which would give $U_{max}/c^2 \approx 2.10^{-6}$. On the other hand, we may take as a working assumption that the ether frame is bound to the mean motion of matter, which is consistent with the present theory [16]. To take this assumption into account, we may further assess V by assuming that the ether frame is at rest with respect to the cosmic microwave background. Then V is approximately 300 km/s [13], so that $V/c \approx 10^{-3}$ is indeed smaller than e. Strictly speaking, however, the exact value of V should be determined internally to the present theory, from the pN effects of V on celestial mechanics that this theory will predict: as it will be shown here, the effect on photons does not appear in the pN approximation. But, for celestial mechanics, we have also to account for what is known about the motion of the solar system itself with respect to the Galaxy, etc. That is, we could not simply come to the conclusion that $V \approx 0$, for example, unless it were found to be consistent with that kinematics in the framework of the present theory (although this seems *a priori* unlikely). The determination, internal to this theory, of the motion (including, perhaps, some straining) of an astronomical reference frame with respect to the ether might thus demand a careful analysis and somewhat complex calculations. However, it is very reasonable to assume that the velocity V of the mass center of the solar system is constant, i.e. does not vary significantly over the relevant time scales (at most a century, say), and is small enough so that the pN approximation is valid.

As to the allowable powers of the small parameter e that must enter the actual expansions, it is known that, from the behaviour of terms under time reversal, some quantities are expected to take only even powers of e, e.g. g_{00} and g_{ij}, and other ones only odd powers, e.g. g_{0i} [12]. In the present theory, there is an additional argument: in it, the non-Newtonian effects all come from the "ether compressibility", $K \propto 1/c^2$ [1-2, 15]. So K itself (or $U_{max}K$, if the units are not arranged so that $U_{max} = 1$) could be considered as the small parameter, whence the appearance of only one from two successive terms in any expansion with respect to $1/c = \sqrt{K}$ - the leading (Newtonian) term giving the parity.

3.2 Expansion of the metric and the field equation in the preferred frame

The leading expansion is that of the scalar field, b or $f = b^2$:

$$b = 1 - U/c^2 + S/c^4 + \dots , \quad f = 1 - 2U/c^2 + (U^2 + 2S)/c^4 + \dots , \tag{22}$$

whence the space metric by Eq. (3) (and with $g^0{}_{ij} = g^{0\,ij} = d_{ij}$ in Cartesian coordinates):

$$h_{ij} = h^{(1)}{}_{ij} + O(1/c^2), \qquad h^{(1)}{}_{ij} \propto (U_{,i}U_{,j})/(g^{0\,kl}U_{,k}U_{,l}), \tag{23}$$

$$g_{ij} = g^0{}_{ij} + (2U/c^2)h^{(1)}{}_{ij} + O(1/c^4), \tag{24a}$$

$$g^{ij} = g^{0ij} - (2U/c^2)m^{ij} + O(1/c^4), m^{ij} \propto g^{0\,ik}h^{(1)}{}_{kl}\,g^{0\,lj} \tag{24b}$$

and the space-time metric by Eqs. (4a) and (4c) (remembering that now $x^0 = t$):

$$g_{00} = c^2 f \ = c^2[1 - 2U/c^2 + (U^2 + 2S)/c^4 + ...], \quad g_{0i} = 0, \quad g_{ij} = -g_{ij}, \tag{25}$$

$$g^{00} = 1/g_{00} = (1/c^2)[1 + 2U/c^2 + (3\,U^2 - 2S)/\,c^4 + ...], \; g^{0i} = 0\,, \; g^{ij} = -\,g^{ij}. \tag{26}$$

To expand the field equation, we must also expand the mass-energy density $r = (T^{00})_E$. For the present purpose, it will be enough to recognize that this may be written in the form

$$r = r_N + w_1/c^2 + ..., \tag{27}$$

with r_N the Newtonian mass density, which only involves rest mass and obeys the usual continuity equation involving the velocity $\mathbf{v}_N$ found at the Newtonian approximation:

$$\frac{\partial \rho_N}{\partial t} + \mathrm{div}_0(\rho_N \mathbf{v}_N) = 0, \; \mathrm{div}_0 \circ \mathrm{div}_{g^0}\,, \tag{28}$$

and where w_1 is a correction (involving the Newtonian energy). This is *a priori* obvious, and may be readily verified from the local conservation equation for the energy which applies, in this theory, to a continuous medium [16].

The expansion of the field equation (5) follows easily from Eqs. (22) and (27): we have

$$D_0 f = (1/c^2)\,[-2\,D_0 U + (1/c^2)\,D_0(U^2 + 2S) + O(1/c^4)]\,, \; (1/f)(f_{,0}/f)_{,0} = -\,2\,U_{,0,0}/c^2 + O(1/c^4),$$

whence (remembering that now $x^0 = t$):

$$D_0 U = -\,4\pi G r_N\,, \tag{29a}$$

$$D_0 S = 4\pi G\, w_1 - D_0 U^2/2 - \partial^2 U/\partial\, t^2. \tag{29b}$$

Thus, what we have noted U in the expansion (22) of the scalar field is indeed the Newtonian potential associated with the Newtonian rest-mass density. The second-order coefficient S is obtained by solving a Poisson equation, too, which means that the propagation of gravity is still instantaneous at the (first) pN approximation, in this theory as also in GR.

3.3 Post-Newtonian equations of motion for a photon in the preferred frame

For a photon, the velocity u^i is O(c), of course. Using Eqs. (22) and (24), one gets easily the main part of the coefficients in Eq. (18):

$$\frac{1}{\rho}\frac{\partial\rho}{\partial t} = -\frac{1}{c^2}\frac{\partial U}{\partial t} + O\left(\frac{1}{c^4}\right), \qquad \frac{\rho_{,j}}{\rho} = -\frac{1}{c^2}U_{,j} + O\left(\frac{1}{c^4}\right), \tag{30}$$

$$g^{ij}\frac{\partial g_{jk}}{\partial t} = O\left(\frac{1}{c^2}\right), \qquad c^2 f\left(\nabla_0 f\right)^i = -2g^{0ij}U_{,j} + O\left(\frac{1}{c^2}\right), \tag{31}$$

although one of them has a somewhat complex expression in general space coordinates x^i:

$$\Gamma^i_{jk} = \left(\Gamma^0\right)^i_{jk} + \frac{2}{c^2}\Delta^i_{jk} - \frac{2U}{c^2}m^{il}\left(\Gamma^0\right)_{ljk} + O\left(\frac{1}{c^4}\right), \tag{32}$$

with $\left(\Gamma^0\right)_{ijk}$ and $\left(\Gamma^0\right)^i_{jk}$ the (first and second kind) Christoffel symbols of the Euclidean metric $\mathbf{g}^0$ in the space coordinates x^i, and where

$$\Delta^i_{jk} \equiv \frac{1}{2}g^{0il}\left((Uh^{(1)}_{lj})_{,k} + (Uh^{(1)}_{lk})_{,j} - (Uh^{(1)}_{jk})_{,l}\right). \tag{33}$$

In Cartesian coordinates, this becomes:

$$\Gamma^i_{jk} = \frac{1}{c^2}\left((Uh^{(1)}_{ij})_{,k} + (Uh^{(1)}_{ik})_{,j} - (Uh^{(1)}_{jk})_{,i}\right) + O\left(\frac{1}{c^4}\right). \tag{34}$$

Inserting Eqs. (30)-(32) into Eq. (18), we get (remembering that $u^i = O(c)$):

$$\frac{du^i}{dt} + \left(\Gamma^0\right)^i_{jk}u^j u^k \equiv a_0{}^i = g^{0ij}U_{,j} - 2\left(U_{,j}u^j\right)\frac{u^i}{c^2} - \frac{2}{c^2}\left(\Delta^i_{jk} - Um^{il}\left(\Gamma^0\right)_{ljk}\right)u^j u^k + O\left(\frac{1}{c}\right). \tag{35}$$

(Note that the left-hand side is indeed the absolute acceleration.) In Cartesian coordinates:

$$\frac{du^i}{dt} = a_0{}^i = U_{,i} - 2\left(U_{,j}u^j\right)\frac{u^i}{c^2} - \left((Uh^{(1)}_{ij})_{,k} + (Uh^{(1)}_{ik})_{,j} - (Uh^{(1)}_{jk})_{,i}\right)\frac{u^j u^k}{c^2} + O\left(\frac{1}{c}\right). \tag{36a}$$

This equation of motion has to be augmented with the constraint $ds^2 = 0$, or $v^2 \circ g_{ij}\, v^i\, v^j = c^2$, of course. To the required order, this may be written in arbitrary space coordinates as

$$g_{ij}\, u^i\, u^j = c^2 + O(1). \tag{36b}$$

It is interesting to notice that *this same pN equation of motion for a photon would be obtained in the present theory if, in the place of Newton's second law (14), we would postulate that photons follow the (null) space-time geodesics.* Indeed, only two terms make the difference between the "Newtonian" acceleration (21) and the "Einsteinian" acceleration (19), and both terms are of order O(1/c): this is true for the $G'^{i}_{0j}\,u^{j}$ term, because (see Eq. (31))

$$G'^{i}_{0j} = (g^{ik}\partial g_{jk}/\partial t)/2 = O(1/c^2);$$

and this is true for the $G'^{0}_{jk}\,u^{j}\,u^{k}\,u^{i}$ term, because (see Eqs. (24a) and (26))

$$G'^{0}_{jk} = (g^{00}\partial g_{jk}/\partial t)/2 = O(1/c^4).$$

3.4 Transition to a moving frame: principle and exact equations

In the investigated theory, there is a global flat space-time metric $\mathbf{g}^0$, with line element

$$(ds^0)^2 = g^0_{lm}\,dy^l\,dy^m \qquad (37)$$

in arbitrary space-time coordinates (y^l). By definition, this metric is such that

$$(ds^0)^2 = h_{lm}\,dx^l\,dx^m = c^2\,dt^2 - (dl^0)^2, \qquad (38a)$$

i.e.

$$g^0_{lm} = h_{lm} \text{ with } (h_{lm}) = \text{diag}\,(1, -1, -1, -1), \qquad (38b)$$

when expressed in space-time coordinates (x^a) that are bound to the preferred frame E, with $x^0 = ct$ and with (x^i) being global Cartesian space coordinates on the reference body M (i.e. $g^0_{ij} = d_{ij}$). Hence, there are also global Lorentz transformations of the space-time $\mathbf{R}'$M, which are those transformations of the space-time coordinates (x^a) that leave the flat metric (g^0_{lm}) in the standard form, $g^0_{lm} = h_{lm}$. These transformations give the transition between space-time coordinates (x^a) that are bound to the preferred frame E, with $x^0 = ct$ and with the x^i's being Cartesian space coordinates on the reference body M, and space-time coordinates (x'^a) that are bound to a uniformly moving frame $E_{\mathbf{V}}$, with $x'^0 = ct'$ and with the x'^i's being Cartesian space coordinates on the reference body $M_{\mathbf{V}}$ moving with uniform velocity $\mathbf{V}$ with respect to E (the x'^i's are such that $g'^0_{ij} = d_{ij}$ with $\mathbf{g}'^0$ the Euclidean space metric on $M_{\mathbf{V}}$, associated with $\mathbf{g}^0$). Of course, one may also pass from a uniformly moving frame $E_{\mathbf{V}}$ to another one, E , by a such transformation.

$E_{\mathbf{V}}$,, by a such transformation.

Thus, in order to evaluate the preferred-frame effects that this theory predicts, we pass from E to that frame $E_{\mathbf{V}}$ which is assumed to follow the motion of the mass-center of the gravitating system (see § 3.1), by using a special Lorentz transformation of the flat metric $\mathbf{g}^0$. That is, **V** being the absolute velocity of the mass center, we take Cartesian coordinates on M such that $V^1 = V$, $V^2 = V^3 = 0$, and we define a new coordinate system thus (using low indices for Cartesian coordinates from now on):

$$x'^1 = g_V\ (x_1 - Vt),\ t' = g_V(t - Vx_1/c^2),\ x'_2 = x_2\ ,\ x'_3 = x_3\ . \tag{39}$$

The Euclidean space metric $\mathbf{g'}^0$ attached with the moving reference body $M_{\mathbf{V}}$ is such that $g'^0_{ij} = d_{ij}$, hence

$$(dl'^0)^2 = dx'_i\, dx'_i. \tag{40}$$

We transform the physical, curved space-time metric **g** [Eqs. (3) and (4c)] to the x'_a coordinates (with $x'_0 = ct'$):

$$g'_{00} \circ g_V^2\,[g_{00} + 2\,(V/c)\,g_{01} + (V^2/c^2)g_{11}] = g_V^2\,\{f - (V^2/c^2)[1 + \left(\frac{1}{f} - 1\right)h_{11}]\}, \tag{41}$$

$$g'_{01} \circ g_V^2\,[(V/c)\,g_{00} + (1 + V^2/c^2)\,g_{01} + (V/c)\,g_{11}] = g_V^2\,(V/c)\,[\,f - 1 - \left(\frac{1}{f} - 1\right)h_{11}], \tag{42}$$

$$g'_{0i} \circ g_V\,[\,g_{0i} + (V/c)\,g_{1i}] = -g_V\,(V/c)\left(\frac{1}{f} - 1\right)h_{1i} \qquad (i = 2,3), \tag{43}$$

$$g'_{11} \circ g_V^2\,[(V^2/c^2)g_{00} + 2\,(V/c)\,g_{01} + g_{11}] = g_V^2\,[(V^2/c^2)f - 1 - \left(\frac{1}{f} - 1\right)h_{11}], \tag{44}$$

$$g'_{1i} \circ g_V[(V/c)g_{0i} + g_{1i}] = -\,g_V\left(\frac{1}{f} - 1\right)h_{1i} \qquad (i = 2,3), \tag{45}$$

$$g'_{ij} = g_{ij} = -\,1 - \left(\frac{1}{f} - 1\right)h_{ij} \qquad (i, j = 2, 3). \tag{46}$$

At large distance from the massive bodies, we have $f \to 1$ and $h_{11} = O(1)$ [see Eqs. (22) and (23)], hence $g'_{00} \to 1$. Thus, the time measured by a clock bound to $E_{\mathbf{V}}$ and outside the gravitation field (far enough) is indeed t'. The time measured by a clock bound to $E_{\mathbf{V}}$ and in the gravitation field is affected in the ratio

$$dt'_{\mathbf{x}}/dt' = (g'_{00})^{1/2}. \tag{47}$$

It is easy to verify in the same way that dl'^0 is also the distance which is locally measured by observers that are bound to the moving frame $E_{\mathbf{V}}$ and remote from the massive bodies. The relative velocity $u'_i \equiv dx'_i/dt'$ and the relative acceleration $a'_i \equiv du'_i/dt'$, if needed, may be deduced from the absolute velocity $u_i \equiv dx_i/dt$ and the absolute acceleration $a_{0i} = a_i \equiv du_i/dt$ by using the usual transformations of SR, which are obtained simply by differentiating (39). These formulae are valid independently of the velocity measured with "unaffected" standards, $u \equiv (u_i u_i)^{1/2}$, hence they are valid for photons (V must be smaller than c, but the unique condition on $\mathbf{u}$ is that $Vu_1 \ll c^2$). To transform the acceleration completely, one would also have to reexpress the space and time derivatives as relative to the coordinates x'_a in the moving frame. If, in the preferred frame, the derivatives of some function $f(x_a)$ play a rôle, we first define a new function, say $f'(x'_a)$:

$$f'(x'_0, ..., x'_3) \equiv f(x_0(x'_0, ..., x'_3), ..., x_3(x'_0, ..., x'_3)) \tag{48}$$

and we obtain from Eq. (39):

$$\frac{\partial \phi}{\partial t} = \nu V \left(\frac{\partial \phi'}{\partial t'} - V \frac{\partial \phi'}{\partial x'_1} \right), \tag{49a}$$

$$\frac{\partial \phi}{\partial x_1} = \nu V \left(\frac{\partial \phi'}{\partial x'_1} - \frac{V}{c^2} \frac{\partial \phi'}{\partial t'} \right), \tag{49b}$$

$$\frac{\partial \phi}{\partial x_i} = \frac{\partial \phi'}{\partial x'_i} \quad (i = 2,3). \tag{49c}$$

3.5 "Post-Newtonian Lorentz-invariance" of the pN approximation for photons

We begin with the gravitational red-shift. As is known, this effect is calculated, in GR, for a stationary metric ($g_{lm,0} = 0$), using the fact that $ds^2 = 0$ for a photon, and the assumption that the frequency measured at the place of emission (proper frequency) is independent of the gravitation field. It is first shown that the frequency n_2, measured at point 1, of light emitted at point 2, differs from the proper frequency $n_0 = n_1$(emitted and measured at point 1) in the ratio

$$\frac{\nu_2}{\nu_1} = \left(\frac{\gamma_{00}(\mathbf{x}_2)}{\gamma_{00}(\mathbf{x}_1)} \right)^{1/2} \tag{50}$$

(see Weinberg [11]). This first reasoning holds exactly the same in the present theory since the "equivalence principle" is true in the theory, in the sense that the laws of non-gravitational physics (that rule the behaviour of light emitters and detectors) are expressed in terms of the curved physical space-time metric **g** [19]. However, the metric cannot *a priori* be stated to be stationary (to an approximation equivalent to that in GR) in the preferred frame, but instead in some uniformly moving frame $E_{\mathbf{V}}$.The second step, in evaluating the red-shift in GR, is to show that the g_{00} term has the following expansion:

$$g_{00} = 1 - 2U/c^2 + O(1/c^4), \tag{51}$$

with U the Newtonian potential. In the present theory, we enter the expansion of f [Eq. (22)] into the expression of g'_{00} in the moving frame $E_{\mathbf{V}}$ [Eq. (41)] and obtain also

$$\gamma'_{00} = \frac{1}{1 - V^2/c^2} \left[1 - \frac{2U}{c^2} + O\left(\frac{1}{c^4}\right) - \frac{V^2}{c^2}\left(1 + O\left(\frac{1}{c^2}\right)\right) \right] = 1 - \frac{2U}{c^2} + O\left(\frac{1}{c^4}\right). \tag{52}$$

To be complete, we mention that Eq. (49) gives for $V/c < e^{\circ} (U_{\max}/c^2)^{1/2}$ (see § 3.1):

$$\frac{\partial \phi'}{\partial x_1} = \frac{\partial \phi''}{\partial x'_1} + O\left(\frac{1}{c^2}\right), \quad \frac{\partial \phi'}{\partial x_i} = \frac{\partial \phi''}{\partial x'_i} \quad (i = 2,3) \tag{53}$$

so that the Poisson equation (29a), obtained in the preferred frame, is also Lorentz-invariant, if one neglects $O(1/c^2)$ terms (since the same is true for the Newtonian mass density, of course). Hence, at this approximation, U is indeed the Newtonian potential as defined in $E_{\mathbf{V}}$. We conclude that, *to just the same level of approximation as in the pN approximation of GR*, more precisely: neglecting $O(1/c^4)$ terms in the basic equations (50) and (52), *the present theory predicts exactly the same gravitational red-shift as does general relativity, and this is true accounting for the preferred-frame effects (which do not appear at this approximation).*

The other two "classical" effects of gravitation on photons are the deflection of electromagnetic radiation by a gravitation field, and the gravitational increase in the time of flight (the delay of the radar echo on a forth-and-back travel, as compared with the echo "in the absence of the gravitation field"; in practice the comparison is with a flight in a weaker gravitation field). Both effects are calculated, in GR, using the pN equation of motion for a photon, which depends only on the Newtonian potential, and using the further approximation of the Newtonian potential of the main massive body by the spherical (and constant) one, i.e. $U = GM/r$ (see Weinberg [11], Misner *et al.* [12], and Will [13]). Actually,

Weinberg [11, chapter 8] calculates both effects with the Schwarzschild metric (retaining the pN terms in the geodesic equations of motion), whereas Misner *et al.* [12] and Will [13] present the calculation in the more general context of the "parametrized pN formalism", with the additional setting $U = GM/r$.

First, we recall that, to pN approximation, photons obey the geodesic equation in the present theory (see after Eq. 36), and this is generally-covariant. Therefore, preferred-frame effects could exist in the pN approximation only in the case that they would affect the space-time metric in the moving frame E_V. As Eqs. (41)-(46) show, this is certainly the case for the exact metric (i.e., the velocity V enters the exact metric). However, expanding these equations with the help of Eqs. (22) and (24a), one finds immediately that (with $x'_0 = ct'$)

$$g'_{00} = 1 - 2U/c^2 + O(1/c^4), \tag{52}$$

$$g'_{ij} = -d_{ij} - (2U/c^2)h^{(1)}_{ij} + O(1/c^4) \qquad (i, j = 1, 2, 3), \tag{54}$$

$$g'_{01} = -2(UV/c^3)(1 + h^{(1)}_{11}) + O(1/c^5) = O(1/c^3), \tag{55}$$

$$g'_{0i} = -2(UV/c^3)h^{(1)}_{1i} + O(1/c^5) = O(1/c^3) \qquad (i = 2,3). \tag{56}$$

Putting $U = GM/r$ in these equations (as is done in GR), we verify that this is the expansion of the Schwarzschild metric, when one neglects $O(1/c^3)$ terms. Indeed, Eq. (54), written in Cartesian coordinates for the metric $\mathbf{g'}^0$, i.e. such that $g'^{\,0}_{ij} = d_{ij}$, is immediately rewritten in arbitrary space coordinates ($y'^{\,i}$) as

$$g'_{ij} = -g'^{\,0}_{ij} - (2U/c^2)h^{(1)}_{ij} + O(1/c^4) \qquad (i, j = 1, 2, 3). \tag{57}$$

In spherical coordinates, $y'^{\,1} = r$, $y'^{\,2} = q$, $y'^{\,3} = f$, we have

$$(g'^{\,0}_{ij}) = \mathrm{diag}\,(1, r^2, r^2 \sin^2 q), \; (g'^{\,0\,ij}) = \mathrm{diag}\,(1, 1/r^2, 1/(r^2 \sin^2 q)). \tag{58}$$

With $U = U(r)$, Eq. (23) gives thus [3]

$$(h^{(1)}_{ij}) = \mathrm{diag}\,(1, 0, 0) + O(1/c^2). \tag{59}$$

So, with $U = GM/r$, the space-time metric writes in spherical coordinates:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r} + O\left(\frac{1}{c^4}\right)\right)c^2 dt'^2 - \left(1 + \frac{2GM}{c^2 r} + O\left(\frac{1}{c^4}\right)\right)dr^2 - r^2 d\Omega^2 + O(1/c^3), \tag{60}$$

which is the Schwarzschild metric (6), at the corresponding approximation. The order of the neglected terms is actually $O(1/c^4)$, except for the neglected g'_{0i} components, which are indeed $O(1/c^3)$. Such g'_{0i} components of order $O(1/c^3)$ also exist in the pN approximation of GR (they are obtained from the well-known vector potential), and they do not play any rôle in the pN approximation for photons [11-13]. *We conclude that, to just the same level of approximation as in the pN approximation of GR,* in particular neglecting the g_{0i} components of the metric, which are $O(1/c^3)$, *the present theory predicts exactly the same light deflection and the same echo delay as does standard general relativity, and this is true accounting for the preferred-frame effects (which do not appear at this approximation).* Here, "standard" GR means that the present theory recovers the predictions made with the Schwarzschild metric, as given, e.g., by Weinberg [11, chapter 8].

4. Conclusion

It has been shown that the present theory is simply indistinguishable from standard GR for the post-Newtonian approximation of gravitational effects on photons, which is the approximation used to compare GR with the observation of these effects. This is not enough to qualify this theory as viable, although the effects of gravitation on photons are among the most striking and the best established predictions of GR. It is interesting to note that this set of effects, which may be regarded as a brilliant confirmation of Einstein's theory, can also be accounted for by a scalar theory with a preferred frame, which derives motion from an extension of Newton's second law instead of assuming that free particles follow space-time geodesics.

Endnotes

1. A frame can be physically thought as a spatial network of observers equipped with measuring rods *and clocks*, allowing to define space-time coordinates (z^a) $(a = 0,...,3)$ such that each observer has constant space coordinates z^i $(i = 1, 2, 3)$. Such coordinates are called "adapted to the frame", see Cattaneo [17]. The 3-D reference body associated with a frame may be defined as the set of the world-lines of the observers, and may be described by the space coordinates (z^i).

2. Equation (3) does not apply when $\tilde{N}_0 f = 0$. The point is that the assumed contraction occurs in the direction of the gravity acceleration **g**, and thus is *a priori* undefined at those places where $\mathbf{g} = 0$ [unless the place is one where the scalar potential f has the same value ($f = 1$) it has "at infinity", in which case there is no contraction at all]. In the generic, physically realistic case, the vector **g** vanishes merely at isolated points, e.g. at one point between two massive bodies, and also at one point near the centre of each massive body. At such isolated points where $\mathbf{g} = 0$, the direction of contraction cannot be defined uniquely. But Eq. (3a) applies and shows that the metric **g** is discontinuous at this isolated point, but remains bounded in its neighborhood. Hence, geometrical elements such as the length of a line are defined as with a continuous metric. Moreover, the examination of the case with spherical symmetry shows that no difficulty occurs in the dynamical equations either. A more difficult case occurs in the highly idealized situation of "collapse in free fall" : **g** vanishes in a bounded volume domain [2]. This case was easily solved in ref. [2] for the relevant subcase of spherical symmetry, and the way of solution can be extended to the case without any symmetry.

3. We may use Eq. (23) if we neglect $O(1/c^2)$ terms in **h**, for the following reason. In the Cartesian coordinates (x^i) in the preferred frame, we have

$$h^{(1)}_{ij} = U_{,i}\, U_{,j}/(U_{,k}U_{,k}).$$

Then Eq. (53) gives us the same expression, plus $O(1/c^2)$ terms, in the Lorentz-transformed Cartesian coordinates $(x'^{\,i})$ in the moving frame, and this may be rewritten as

$$h^{(1)}_{ij} = (U_{,i}\, U_{,j})/(g'^{\,0}_{kl}\, U_{,k}U_{,l}) + O(1/c^2)$$

in arbitrary space coordinates of the moving frame.

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