Revised Version

Valence quark distributions in nucleon at low Q^2 in QCD.

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Abstract

Valence u- and d-quarks distributions in proton are calculated in QCD at low Q^2 and intermediate x, basing on the operator product expansion (OPE). The imaginary part of the virtual photon scattering amplitude on quark current with proton quantum numbers is considered. The initial and final virtualities p_1^2 and p_2^2 of the currents are assumed to be large, negative and different, $p_1^2 \neq p_2^2$. The OPE in p_1^2, p_2^2 up to dimension 6 operators was performed. Double dispersion representations in p_1^2, p_2^2 of the amplitudes in terms of physical states contributions are used. Putting them to be equal to those calculated in QCD, the sum rules for quark distributions are found. The double Borel transformations are applied to the sum rules. Leading order perturbative corrections are accounted. Valence quark distributions are found: $u(x)_v$ at $0.15 < x < 65$, $d(x)_v$ at $0.25 < x < 0.55$ with an accuracy $\sim 30\%$ in the middles and $\sim 50\%$ at the ends of these intervals. The quark distributions obtained are in agreement with those found from the analysis of hard processes data.

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1.Introduction

Quark and gluon distributions in hadrons are not fully understood in QCD. QCD predicts the evolution of these distributions with Q^2 in accord with the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [1]-[3] equations, but not the initial values from which this evolution starts. The standard way of determination of quark and gluon distributions in nucleon is the following [4-8] (for the recent review see [9]). At some $Q^2 = Q_0^2$ (usually, at low or intermediate $Q^2 \sim 2 - 5GeV^2$) the form of quark (valence and sea) and gluon distributions is assumed and characterized by the number of free parameters. Then, by using DGLAP equations, quark and gluon distributions are calculated at all Q^2 and x and compared with the whole set of the data on deep inelastic lepton-nucleon scattering (sometimes also with

prompt photon production, jets at high $p_\perp$ etc). The best fit for the parameters is found and, therefore, quark and gluon distributions are determined at all Q^2, including their initial values $q(Q_0^2, x)$, $g(Q_0^2, x)$. Evidently, such an approach is not completely satisfactory from theoretical point of view - it would be desirable to determine the initial distribution directly from QCD. Also, if the form of initial distribution is not divined properly, then the fitting procedure will give the relative minimum, but not the absolute ones and the results, especially at low Q_0^2 could be wrong. (This danger does not, probably, exists for nucleon, but such a situation happens, e.g., for transversally polarized ρ-meson, where unusual form of initial valence quark distribution was found [10]). Finally, extrapolation from high Q^2, to which the main part of the data belongs, to low Q^2 is not a quite stable procedure and may introduce some errors. For all these reasons it is desirable to find quark and gluon distribution in hadrons at low $Q^2 \sim 2 - 5GeV^2$ basing directly on QCD.

In this paper we calculate u and d valence quark distributions in proton. The idea of the method was suggested in [11] and developed in [12-14]. Recently, the method had been improved and valence quark distributions in pion [15] and transversally and longitudinally polarized ρ-meson [10] had been calculated, what was impossible in the initial version of the method. The idea of the approach (in the improved version) is to consider the imaginary part (in s-channel) of a four-point correlator $\Pi(p_1, p_2, q, q')$ corresponding to the non-forward scattering of two quark currents, one of which has the quantum numbers of hadron of interest (in our case – of proton) and the other is electromagnetic (or weak). It is supposed that virtualities of the photon q^2, q'^2 and hadron currents p_1^2, p_2^2 are large and negative $|q^2| = |q'^2| \gg |p_1^2|,\ |p_2^2| \gg R_c^{-2}$, where R_c is the confinement radius. It was shown in [12] that in this case the imaginary part in s-channel $[s = (p_1 + q)^2]$ of $\Pi(p_1, p_2; q_1, q')$ is dominated by a small distance contribution at intermediate x. (The standard notation is used: x is the Bjorken scaling variable, $x = -q^2/2\nu$, $\nu = p_1 q$). The proof of this statement is given in ref.[12]. So, in the mentioned above domain of q^2, q'^2, p_1^2, p_2^2 and intermediate x $Im\Pi(p, p_2; q, q')$ can be calculated using the perturbation theory and the operator product expansion in both sets of variables $q^2 = q'^2$ and p_1^2, p_2^2. The approach is inapplicable at small x and x close to 1. This can be easily understood for physical reasons. In deep inelastic scattering at large $|q^2|$ the main interaction region in space-time is the light-cone domain and longitudinal distances along the light-cone are proportional to $1/x$ and become large at small x [16, 17]. For OPE validity it is necessary for these longitudinal distances along light-cone to be also small, that is not the case at small x. At $1 - x \ll 1$ another condition of applicability of the method is violated. The total energy square $s = Q^2(1/x - 1) + p_1^2$ $Q^2 = -q^2$ is not large at $1 - x \ll 1$. Numerically, the typical values to be used below are $Q^2 \sim 5GeV^2$, $p_1^2 \sim -1GeV^2$. Then, even at $x \approx 0.7$, $s \approx 1GeV^2$, i.e., at such x we are in the resonance, but not in the scaling region. So, one may expect beforehand, that our method could work only up to $x \approx 0.7$. The inapplicability of the method at small and large x manifests itself in the blow-up of higher order terms of OPE. More precise limits on the applicability domain in x will be found from the magnitude of these terms.

The further procedure is common for QCD sum rules. On one hand the four-point correlator $\Pi(p_1, p_2; q, q')$ is calculated by perturbation theory and OPE.On the other hand, the double dispersion representation in p_1^2, p_2^2 in terms of physical states contributions is written for the same correlator and the contribution of the lowest state is extracted using the Borel transformaion. By equalling these two expression the desired quark distribution is found.

In Sec.2 the general outline of the method is presented. It is explained here, why it is

necessary to consider at the beginning the nonequal p_1^2 and p_2^2 and only at the end of the calculation to go to the forward scattering amplitude. Sec.3 presents the results of the bare loop calculation ($d = 0$ term in OPE) and the LO perturbative corrections to it. Sec.4 is devoted to the calculation of power corrections to the QCD side of the sum rules: the contribution of gluonic condensate (d=4), $\alpha_s\langle\bar{\psi}\psi\rangle^2$ term (d=6). The valence u and d-quark distributions are numerically calculated in Sec.5, the errors are estimated and the results are compared with those obtained from the deep inelastic scattering data using the evolution equations. Sec.6 contains our conslusion.

2. The outline of the method

Consider the 4-current correlator which corresponds to the virtual photon scattering on the quark current with quantum number of proton:

$$T^{\mu\nu}(p_1,p_2,q,q') = -i\int d^4x d^4y d^4z \cdot e^{i(p_1x+qy-p_2z)}$$
$$\cdot\langle 0|T\{\eta(x),\ j_\mu^{u,d}(y), j_\nu^{u,d}(0),\ \bar{\eta}(z)\}|0\rangle, \qquad (1)$$

where $\eta(x)$ is the three-quark current (Ioffe current [18]). Choose the currents in the form $j_\mu^u = \bar{u}\gamma_\mu u,\ \ j_\mu^d = \bar{d}\gamma_\mu d$, i.e. as an electromagnetic current which interacts only with $u(d)$ quark (with unit charges). Such a choice allows us to get sum rules separately for distribuion functions of u and d quarks. The general method of calculation of distribution functions from consideration of the 4-point correlator in the QCD sum rules for light quarks was used in [12]. Unfortunately, this approach being in the form used in [12] did not give completely satisfactory sum rules for quark distribution functions (especially for d-quark in nucleon) and did not allow one to calculate quark distributions in mesons. Recently, in ref. [10,15] was suggested a generlized method of calculation of quark distribution functions, which made it possible to get much more reliable sum rules, what, in particular, made it possible to find valence quark distributions in π and ρ- mesons. The main difference of this method is that the hadronic current momenta are put to be unequal, $p_1^2 \neq p_2^2$ and then independent borelization over p_1^2 and p_2^2 is performed and only at the very end the Borel the parameters M_1^2 and M_2^2 are put to be equal. (Unlike the approach of ref.[12] where p_1 was put to be equal to p_2 and single borelization is performed from the very beginning). The described procedure allows one to kill nondiagonal transitions of the type

$$\langle 0|j^h|h^*\rangle\langle h^*|j_\mu^{el}(y)j_\nu^{el}(0)|h\rangle\langle h|j^h|0\rangle \qquad (2)$$

and thus makes it possible to separate the diagonal transition of interest

$$\langle 0|j^h|h\rangle\langle h|j_\mu^{el}(y)j_\nu^{el}(0)|h\rangle\langle h|j^h|0\rangle, \qquad (3)$$

without using additional tricks like differentiation over Borel parameter M^2, what strongly worsens the accuracy of the sum rules.

Let us briefly remind the main points of derivation of the sum rules in the method under investigaion (for details see [10, 15]. The most general form of the double dispersion relation

(in p_1^2 and p_2^2) for imaginary part of the correlator (1) (we omit for some time all indeces assuming the necessary invariant amplitude to be chosen) has the form

$$ImT(p_1^2,p_2^2,q^2,s)=a(q^2,s)+\int\limits_0^\infty\frac{\psi(q^2,s,u)}{u-p_1^2}du+\int\frac{\psi(q^2,s,u)}{u-p_2^2}du$$

$$+\int\limits_0^\infty du_1\int\limits_0^\infty du_2\,\frac{\rho(q^2,s,u_1,u_2)}{(u_1-p_1^2)(u_2-p_2^2)} \tag{4}$$

(without loss of generality one may put $q^2=q'^2,\quad t=(p_1-p_2)^2=0$). The double Borel transformation in p_1^2 and p_2^2 eliminates three first terms and we have

$$B_{M_1^2}B_{M_2^2}ImT(p_1^2,p_2^2,q^2,s)=\int\limits_0^\infty du_1\int\limits_0^\infty du_2\rho(q^2,s,u_1,u_2)exp\left[-\frac{u_1}{M_1^2}-\frac{u_2}{M_2^2}\right] \tag{5}$$

where M_1^2 and M_2^2 are the squared Borel mass. The integration region with respect to u_1,u_2 may be divided into four areas

I. $u_1<s_0,\ \ u_2<s_0$;

II. $u_1<s_0,\quad u_2>s_0$;

III. $u_1>s_0,\quad u_2<s_0$;

IV. $u_1,\ u_2>s_0$.

Here s_0 is the continuum threshold in the standard QCD sum rule model of the hadronic spectrum with one lowest resonance plus continuum. Area I obviously corresponds to the resonance contribution and spectral density in this area can be written as

$$\rho(u_1,u_2,x,Q^2)=g_h^2\cdot 2\pi F_2(x,Q^2)\delta(u_1-m_h^2)\delta(u_2-m_h^2), \tag{6}$$

where g_h are coupling constants of the corresponding hadronic current. As for the areas II-IV, they are exponentially suppressed, and, using the standard hypothesis of quark-hadron duality, we may estimate them as a bare loop contribution in the same integration region.

Thus, the general form of the sum rule is

$$ImT^0_{QCD}+\text{Power correction}=2\pi F_2(x,Q^2)g_h^2e^{-m_h^2(\frac{1}{M_1^2}+\frac{1}{M_2^2})}$$

$$ImT^0_{QCD}=\int\limits_0^{s_0}\int\limits_0^{s_0}\rho^0(u_1,u_2,x)du_1du_2e^{-(\frac{u_1}{M_1^2}+\frac{u_2}{M_2^2})} \tag{7}$$

where one should go to the limit $M_1^2=M_2^2$.

We choose $M_1^2=M_2^2=2M^2$ following [19] where it was shown that the value of the Borel mass square in the double sum rules is approximately twice as large than in single ones.

Choose now invariant amplitude for the case of the proton current of interest in the correlator (1) (cf.[12]). Its imaginary (in s) part can be written as

$$ImT^{(p)}_{\mu\nu} = \lambda_n^2 \frac{1}{p_1^2 - m^2} \sum_{r,r'} v^r(p_1) \times$$

$$\times Im\{-i \int d^4x e^{-iqy} \cdot \langle p_1, r|T\{j_\mu(y), j_\nu(0)\}|p_2, r'\rangle\} \cdot \bar{v}^{r'}(p_2) \frac{1}{p_2^2 - m^2} \tag{8}$$

where $v^r(p)$ is proton spinor with polarization r and momentum p, λ_N is the coupling constant of proton with the current $\langle 0|\eta|p, r\rangle = \lambda_N v^r(p)$.

In order to choose the most suitable invariant amplitude, rewrite eq.(8) in the form

$$ImT_{\mu\nu} = \lambda_N^2 \frac{1}{(p_1^2 - m^2)(p_2^2 - m^2)} \sum_{r,r'} v^r(p_1) \cdot$$

$$\cdot \left(\bar{v}^r(p_1) Im\ \tilde{T}^{(p)}_{\mu\nu}\ v^{r'}(p_2) \right) \bar{v}^{r'}(p_2) \tag{9}$$

where $Im\ \tilde{T}^{(p)}_{\mu\nu}$ is the amplitude before averaging in proton spin, m is the proton mass.

The general form of this amplitude is

$$Im\ \tilde{T}^{(p)}_{\mu\nu}(p_1, p_2) = C_1 P_\mu P_\nu + C_2(P_\mu \gamma_\nu + P_\mu \gamma_\nu) + C_3 \hat{P} P_\mu P_\nu + ...$$
$$+ (\text{terms with } r) \tag{10}$$

where $P = \frac{p_1 + p_2}{2}$; $\quad r = p_1 - p_2$.

Let us now take into account that we are interested in such a combination of invariant amplitudes which in the spin-averaged matrix element

$$\sum_r \bar{v}^r(p_1) Im\ \tilde{T}^{(p)}_{\mu\nu} v^r(p_2) \tag{11}$$

appears at the kinematical structure $P^\mu P^\nu$ (since this structure in the limit $p_1 \to p_2 \equiv p$ transforms into $p^\mu p^\nu$, the coefficient at which is just $F_2(x)$).

Using the equation of motion

$$\hat{p}_{1,2} v^r(p_{1,2}) = \frac{1}{2}(\hat{p}_{1,2} + m) v^r(p_{1,2}) \tag{12}$$

and

$$\sum_r v^r_\alpha(p_{1,2}) \bar{v}^r_\beta(p_{1,2}) = (\hat{p}_{1,2} + m)_{\alpha\beta} \tag{13}$$

it can be seen that the combination of invariant amplitudes in eq.(10), which appears in eq.(9) at the kinematical structure $\hat{P} P^\mu P^\nu$ coincides (up to numerical factor) with the combination of invariant functions at the structure $P_\mu P_\nu$ in (11).

Thus, we come to a conclusion that the sum rules should be written for invariant amplitude at the kinematical structure $\hat{P} P^\mu P^\nu$ (in what follows we will denote it ImT/ν).

So, the sum rules for nucleon have the form

$$\frac{2\pi}{4M^4} \frac{\bar{\lambda}_N^2}{32\pi^4} xq^{u,d}(x) e^{-m^2/M^2} = ImT^0_{u,d} + \text{Power corrections} \tag{14}$$

where $\bar{\lambda}_N^2 = 32\pi^4 \lambda_N^2$; $\quad q^{u,d}(x)$ are distribution functions of $u(d)$ quark in nucleon, $Im\tilde{T}^0$ is perturbative contribution, i.e. of a bare loop with perturbative corrections. (The continuum

contribution, i.e., of II, III, IV regions should be subtracted from $Im\tilde{T}$. Note, that really, the contribution of regions II and III to the bare loop is zero, since $\rho^0 \sim \delta(u_1 - u_2)$).

3. Bare loop contribution and leading order perturbative corrections.

Bare loop contribution to the sum rules is represented in Fig.1. In the calculation we use the technique described in ref.15, Appendix. The following formulae are exploited:

$$\int \frac{d^4k}{(p_1-k)^2(p_2-k)^2}\delta[(p_1+q-k)^2]\theta(k^2) = \frac{\pi}{4\nu x}(1-x)\int du \frac{u}{(p_1^2-u)(p_2^2-u)}$$

$$\int \frac{d^4k\cdot k^2}{(p_1-k)^2(p_2-k)^2}\delta[(p_1+q-k)^2]\theta(k^2) = \frac{\pi}{8\nu x}(1-x)^2\int du \frac{u^2}{(p_1^2-u)(p_2^2-u)}$$

$$\int \frac{d^4k(p_1-k)p_2}{(p_1-k)^2(p_2-k)^2}\delta[(p_1+q-k)^2]\theta(k^2) = \frac{\pi}{16\nu x}(1-x^2)\int du \frac{u^2}{(p_1^2-u)(p_2^2-u)} \qquad (15)$$

(The terms, vanishing at the double Borel transformation are omitted). The results after the double Borel transformation are the same as in the case for equal $p_1 = p_2$ [12]:

$$ImT^0_{u(d)} = \varphi_0^{u(d)}(x)\frac{M^2}{32\pi^3}\ E_2(s_0/M^2) \qquad (16)$$

where

$$\varphi_0^u(x) = x(1-x)^2(1+8x), \quad \varphi_0^d(x) = x(1-x)^2(1-2x), \qquad (17)$$

s_0 is the continuum threshold

$$E_2(z) = 1-(1+z+z^2/2)e^{-z} \qquad (18)$$

The substitution of eq.(16) into the sum rules (14) results in

$$xq(x)_0^{u(d)} = \frac{2M^6e^{m^2/M^2}}{\bar{\lambda}_N^2}\ \varphi_0^{u(d)}(x)\cdot E_2(\frac{s_0}{M^2}) \qquad (19)$$

In the bare loop approximation the moments of the quark structure function are equal to

$$\int_0^1 q_0^d(x)dx = \frac{M^6e^{m^2/M^2}}{\bar{\lambda}_N^2}E_2$$

$$\int_0^1 q_0^u(x)dx = 2\frac{M^6e^{m^2/M^2}}{\bar{\lambda}_N^2}E_2 \qquad (20)$$

Making use of relation $\bar{\lambda}_N^2 e^{-m^2/M^2} = M^6E_2$ which follows from the sum rule for the nucleon mass (see [18]) in the same approximation, we get

$$\int\limits_0^1 q_0^d(x)dx = 1$$

$$\int\limits_0^1 q_0^u(x)dx = 2 \tag{21}$$

In the bare loop approximation there also appears the sum rule for the second moment:

$$\int\limits_0^1 x(q_0^u(x) + q_0^d(x))dx = 1 \tag{22}$$

Analogously to [12] one can show that relations (21),(22) hold also when taking into account power corrections proportional to the quark condensate square in the sum rules for the 4-point correlator (Fig.2) and in the sum rules for the nucleon mass. Relations (21) reflect the fact that proton has two u-quarks and one d-quark. Relation (22) expresses the momentum conservation law – in the bare loop approximation all momentum is carried by valence quarks. Therefore, the sum rules (21),(22) demonstrate that the zero order approximation is reasonable. In the real physical theory the regions $x \ll 1$ and $1 - x \ll 1$ are off the frames of our consideration. However, in the noninteracting quark model which corresponds to the bare loop approximation, the whole region $0 \leq x \leq 1$ should be considered and relations (21),(22) should take place.

Let us calculate the perturbative corrections to bare loop and restrict ourselves by the leading order (LO) corrections proportional to lnQ_0^2/μ^2, where Q_0^2 is the point, where the quark distributions $q(x, Q_0^2)$ is calculated and μ^2 is the normalization point. In our case it is reasonable to choose μ^2 to be equal to the Borel parameter $\mu^2 = M^2$. The results take the form:

$$d^{LO}(x) = d_0(x)\left\{1 + \frac{4}{3}ln(Q_0^2/M^2) \cdot \frac{\alpha_s(Q_0^2)}{2\pi} \cdot \right.$$

$$\left.\left[1/2 + x + ln((1-x)^2/x) + \frac{-5 - 17x + 16x^2 + 12x^3}{6(1-x)(1+2x)} - \frac{(3-2x)x^2 ln(1/x)}{(1-x)^2(1+2x)}\right]\right\} \tag{23}$$

$$u^{LO}(x) = u_0(x) \cdot \left\{1 + \frac{4}{3}\,\frac{\alpha_s(Q_0^2)}{2\pi} ln(Q_0^2/M^2)\left[1/2 + x + ln(1-x)^2/x + \right.\right.$$

$$\left.\left.\frac{7 - 59x + 46x^2 + 48x^3}{6(1-x)(1+8x)} - \frac{(15-8x)x^2 ln(1/x)}{(1-x)^2(1+8x)}\right]\right\} \tag{24}$$

where $u_0(x)$ and $d_0(x)$ are bare loop contributions, given by (19).

4.Power corrections

In this Section the power corrections to the sum rules are calculated.

The lowest dimension (d=4) power corrections are the corrections due to gluon condensate $\langle 0|(\alpha_s/\pi)G^n_{\mu\nu}\ G^n_{\mu\nu}|0\rangle$. Some examples of the corresponding diagrams are given in Fig.3. The calculations are performed in the fixed point (Fock-Schwinger) gauge. The corresponding expression for quark propagator is given in ref.12, eq.(24). (The error in the coefficient in front of the last term was corrected: it should be 1/288 instead of 1/96). In order to be sure in the final results the fixed point was chosen in two ways: at the upper and lower left-hand vertices of Fig.1 diagrams. The results for the sum of diagrams coincide in these two cases, as it should be. However, the contributions of separate diagrams are different. Particularly, if the fixed point is chosen at the upper left-hand vertex, then all diagrams where the soft gluon is emitted from the upper horizontal line (i.e., by active quark) in Fig.1, are zero in the lowest twist approximation. However, if the fixed point is at the lower left-hand vertex, the diagrams with soft gluon, emitted by active quark are nonzero. [1] Therefore, it is generally untrue the folklore statement, that in the lowest twist approximation one may neglect in the forward scattering amplitude the active quark interaction between absorption and emission of virtual photon – this statement is gauge dependent.

All other technique is the same as in ref.15. The contributions of gluon condensate to u and d-quarks distribution were found to be (the ratios to bare loop contributions are presented):

$$\frac{u(x)_{\langle G^2\rangle}}{u_0(x)} = \frac{\langle(\alpha_s/\pi)G^2\rangle}{M^4}\cdot\frac{\pi^2}{12}\ \frac{(11+4x-31x^2)}{x(1-x)^2(1+8x)}\cdot(1-e^{-s_0/M^2})/E_2(\frac{s_0}{M^2}) \tag{25}$$

$$\frac{d(x)_{\langle G^2\rangle}}{d_0(x)} = -\frac{\langle(\alpha_s/\pi)G^2\rangle}{M^4}\frac{\pi^2}{6}\ \frac{(1-2x^2)}{x^2(1-x)^2(1+2x)}(1-e^{-s_0/M^2})/E_2(\frac{s_0}{M^2}) \tag{26}$$

(The factor $(1-e^{-s_0/M^2})$ appears since gluonic condensate also contributes to continuum and this contribution should be subtracted).

Consider now the contributions of d=6 operators.

We start from studying the diagrams of Fig.4 and Fig.5, the contribution of which is proportional to the vacuum mean values of the type $\langle\bar\psi_\alpha\psi_\beta D_\rho G^n_{\mu\nu}\rangle$, $\langle\bar\psi_\alpha\nabla_D\psi_\beta G^n_{\mu\nu}\rangle$, $\langle\bar\psi\nabla\nabla\nabla\psi\rangle$ ets. All of them are expressed through $\langle\bar\psi\psi\rangle^2$ (see [12]). Here it should be necessary to make the following remark: since the approach is inapplicable at $x\approx 1$, then the diagrams of the type of Fig.2, the imaginary part of which is proportional to $\delta(1-x)$, should not to be taken into account. But the diagrams of Fig.4 arising by evolution of the diagram Fig.2, should not be taken into account too. The diagrams which contribute to intermediate region x (and not being evolution of the corresponding non-loop ones) are given in Fig.5 (cf. Fig.5b in [12]). These diagrams are one-loop ones and for this reason it may be expected that their contribution will be dominant among contributions of d=6 operators. The contribution of Fig.5 diagram into sum rules for quark distribution in proton appeared to be equal to:

[1] May be, this observation is related to the Brodsky et al. [20] statement, that structure functions are influent by active quark interactions with spectators.

$$\frac{u(x)_{\alpha_s\langle\bar\psi\psi\rangle^2}}{u_0(x)}=\frac{\alpha_s a^2(215-867x+172x^2+288(1-x)ln2)}{M^6\cdot 81\pi\cdot 8x(1-x)^3(1+8x)}\frac{1}{E_2(s_0/M^2)} \tag{27}$$

$$\frac{d(x)_{\alpha_s\langle\bar\psi\psi\rangle^2}}{d_0(x)}=-\frac{\alpha_s a^2(19-43x+36x^2)}{M^6 81\pi x(1-x)^3(1+2x)}\ \frac{1}{E_2(s_0/M^2)}, \tag{28}$$

where

$$a=-(2\pi)^2\langle 0|\bar\psi\psi|0\rangle \tag{29}$$

Besides the considered above contribution of the quark condensate square $\alpha_s\langle 0|\bar\psi\psi|0\rangle^2$, there exist contributions of d=6 operators described by two-loop diagrams. Examples of these diagrams are shown in Fig.6. The contributions of these diagrams are expressed via $\alpha_s^2\langle 0|\bar\psi\psi|0\rangle^2$ and $\langle 0|g^3f^{abc}G^a_{\mu\nu}G^b_{\nu\lambda}G^c_{\lambda\mu}|0\rangle$. The diagrams of the first type are suppressed in comparison with (27), (28) by α_s/π and by numerical factor $\sim 1/2\pi$, because they are two loop diagrams. The diagrams of the second type, proportional to and $\langle 0|g^3f^{abc}G^a_{\mu\nu}G^b_{\nu\lambda}G^c_{\lambda\mu}|0\rangle$ are also suppressed by two-loop factor $1/2\pi$, but vacuum expectation value of G^3 operator is poorly known: there is only instanton estimate [21]

$$\langle g^3f^{abc}G^uG^bG^c\rangle=-\frac{48\pi^2}{5}\ (1/\rho_c^2)\langle 0|(\alpha_s/\pi)G^2|0\rangle \tag{30}$$

where ρ_c is the effective radius of instanton. The magnitude of ρ_c is not well known: various estimations result in essenially different values, varying from 1/3 [22] to 1 fm [21], Also, one may have doubts [23] if instantons quantitatively describe physical processes at the scale $\sim 1GeV^2$, where we are working. Moreover, some of these diagrams are infrared divergent (for examples see Fig.6b).

For these reasons we restrict ourselves to the estimate of $\langle G^3\rangle$ contribution only. Such estimate shows, that it may be essential in the domain of small $x\lesssim 0.2$ and is positive in both cases – for u- and d-quarks.

The final result for valence quark distribution in proton are of the form

$$xu(x)=\frac{M^6e^{m^2/M^2}}{\bar\lambda_N^2}2x(1-x)^2(1+8x)E_2(\frac{s_0}{M^2})\left\{\left[1+\frac{u^{LO}(x,Q_0^2)}{u_0(x)}\right]+\right.$$
$$\left.+\frac{1}{u_0(x)}\left[u(x)_{\langle G^2\rangle}+u(x)_{\alpha_s\langle\bar\psi\psi\rangle^2}\right]\right\} \tag{31}$$

$$xd(x)=\frac{M^6e^{m^2/M^2}}{\bar\lambda_N^2}2x(1-x)^2(1+2x)E_2(\frac{s_0}{M^2})\left\{\left[1+\frac{d^{LO}(x,Q_0^2)}{d_0(x)}\right]+\right.$$
$$\left.+\frac{1}{d_0(x)}\left[d(x)_{\langle G^2\rangle}+d(x)_{\alpha_s\langle\bar\psi\psi\rangle^2}\right]\right\} \tag{32}$$

5. Valence u- and d-quark distributions at intermediate x.

We are now in a position to calculate numerically valence u- and d-quark distributions using eqs.(31),(32). The following values of the parameters were taken: $\bar{\lambda}_N^2 = 2.1 GeV^6$, $s_0 = 2.3 GeV^2$ [24], $\lambda_{QCD} = 250 MeV$. The latter is the effective one-loop value, which gives the same $\alpha_s(Q^2)$ as many-loop calculations at low $Q^2 = 1-5 GeV^2$ and particularly, $\alpha_s(m_\tau^2) = 0.355$ found in [23]. The parameter $\alpha_s\langle 0|\bar{\psi}\psi|o\rangle^2$ was varied from $8.10^{-5} GeV^6$ to $2.2.10^{-4} GeV^6$, i.e, $\alpha_s a^2$ varied from $0.13 GeV^6$ to $0.34 GeV^6$. (The lower limit is the old value [25], the upper limit corresponds to the recent determination of this parameter from τ-decay data [26],[27]). The important parameter is the value of gluon condensate $b = \langle 0|(\alpha_s/\pi)G_{\mu\nu}^2|0\rangle$. We allow it to vary from 0 up to $0.012 GeV^4$. (The recent determination of $\langle 0|(\alpha_s/\pi)G_{\mu\nu}^2|0\rangle$ from the analysis of the τ-decay data results in: $\langle 0|(\alpha_s/\pi)G_{\mu\nu}^2|0\rangle = 0.009 \pm 0.007 GeV^4$ [23], the value $0.012 GeV^4$ is the old result [28]). All results are presented at $Q_0^2 = 5 GeV^2$.

Let us start with studying of the Borel mass dependence of quark distributions at various x. (Fig.7; the chosen values of the parameters are: $a^2 = 0.34 GeV^6$, $b = 0.006 GeV^4$ – our favourite values – see below.) As is seen from Fig.7, for u-quark the stability in M^2 is good or satisfactory at $0.1 \lesssim x \lesssim 0.5$, at $x = 0.6$ the M^2 dependence is rather strong. For d-quark distribution the stability interval is narrower; $0.25 \lesssim x \lesssim 0.55$.

Check now how much are the contributions of power corrections in comparison with $d = 0$ OPE term (bare loop). For u-quark the applicability domain at low x is limited by gluon condensate contribution – it comprises about 30% at $x = 0.15$ (and $M^2 = 1.1 GeV^2$). At the same $x = 0.15$ the contribution of $\langle G^3\rangle$ term becomes large, although uncertain. At large x the limits for u-quark come from perturbative corrections, they comprise about 40% at $x = 0.65$. For d-quark the corresponding limits are $0.25 \lesssim x \lesssim 0.60$. (All the mentioned above values are given for $\alpha_s a^2 = 0.34 GeV^6$, $b = 0.006 GeV^4$, $M^2 = 1.1 GeV^2$).

Fig.8a,b shows the dependence of $u(x)$, $d(x)$ on the magnitude of gluon condensate. One can see from Fig.8, that the data following from the analysis of deep inelastic scattering are best described at $b = 0.006 GeV^4$. The zero value of gluon condensate cannot be excluded with certainty, but higher values, $b \gtrsim 0.012 GeV^4$ give much worse description of the data. The curves in Fig.8a,b were calculated at $\alpha_s a^2 = 0.34 GeV^6$ (and $M^2 = 1.1. GeV^2$).The dependence of quark distributions on the magnitude of quark condensate square times α_s – $\alpha_s a^2 = (2\pi)^4 \alpha_s \langle|\bar{\psi}\psi|0\rangle^2$ is plotted in Fig.9a,b (at $b = 0.006 GeV^4$). The best fit is obtained at $\alpha_s a^2 = 0.34 GeV^6$. Therefore, as follows from the analysis, our favourite values of gluon condensate and α_s times quark condensate square are

$$\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle = 0.006 GeV^4$$
$$\alpha_s\langle 0|\bar{\psi}\psi|0\rangle^2 = 2.2. \cdot 10^{-4} GeV^6 \qquad (33)$$

which agree with the recent result from the τ-decay analysis [23], [26], [27].

Besides, the uncertainties arising from the spread of possible values of b and a^2, there are additional sourses of errors – the uncertainty in $\bar{\lambda}_N^2$ and those coming from continuum contribution. The value of $\bar{\lambda}_N^2 = 2.1 GeV^6$ was found in [24] by the best fit of the proton QCD sum rules. However, in [24] the old value of $\alpha_s\langle 0|\psi\psi|0\rangle^2 = 8.10^{-5} GeV^6$ was used and α_s corrections were not accounted. This may result in $10-20\%$ deviation of $\bar{\lambda}_N^2$ from the above accepted value. Continuum contribution comprises about 30% of the total in u-quark

case and about 60% in d-quark case. So, one may estimate possible errors from uncertainty of continuum model as 10% and 20%, correspondingly, for u- and d- quarks. Therefore, our estimates of total errors are: for u-quarks distributions, in the middle of x-interval – $0.25 < x < 0.45$, about 25%, at the ends of intervals – $x = 0.15$ and 0.65 – about 50%; for d quarks distributions, in the middle of x-interval – $0.3 < x < 0.45$ about 30%, at the ends of intervals – $x = 0.25$ and 0.55 by a factor of 2. Taking into account these errors, the agreement with u- and d-quark distributions, found in [4] from experimental data (solid curve in Fig.8) is satisfactory. It should be mentioned, that the account of $\langle G^3 \rangle$ terms would improve the agreement in the domain of low x.

A few additional remarks are in order. In Figs.8,9 we performed the comparison with valence quark distributions found from hard processes, we used only one group results [4]. We limited our comparison to these data not because we consider these results as more confident or reliable, than those of other groups [5-9]. Quite opposite, we believe, that the precision of new analysis (see especially [29],[30]) is better than the older ones. The reason is that theoretical errors in determination of u- and d- quark distribution exceed the differences in various treatment of the data – all of them are in the limit of our theoretical uncertainties. We chosed for comparison the LO results of [4], because our perturbative calculations are done also in LO. Our results can be compared directly with experimental data by considering the difference

$$F_2^p(x) - F_2^n(x) = \frac{1}{3}[u(x) - d(x)]_{val} + \frac{1}{3}[\bar{u}(x) - \bar{d}(x)] \tag{34}$$

$F_2^p(x) - F_2^n(x)$ was measured by NMC [31]. At $x > 0.30$ one may expect, that sea quark contribution – the last term in (36) – is negligible and the data can be compared with valence quark distributions. The result is that at $0.3 < x < 0.55$ the theoretical curve (at $b = 0.006$ GeV^6) is about 30-40% higher than the data points. Such disagreement is in the limit of estimated errors. (At $b = 0$ the agreement is better.)

6. Conclusion

The distributions of valence u- and d-quarks at low Q^2 and intermediate x were theoretically calculated basing on the first principles of QCD: perturbation theory and operator product expansion (OPE). No experimental data and no fitting parameters were used. New technique, suggested in ref.15, was exploited: the double Borel transformation in virtualities of two hadronic currents in 4-point function eq.(1), which allows one to kill the background non-diagonal transition amplitudes. In OPE the operators of dimension 4 (gluon condensate) and dimension 6 (quark condensate square times α_s) were accounted. The theoretical analysis of the obtained valence quark distributions at $Q^2 = 5GeV^2$ showed, that u-quark distribution is reliable at $0.15 < x < 0.65$, its accuracy is about 25% in the middle of this interval and decreases to 50% at the ends of interval; d-quark distribution is reliable at $0.25 < x < 0.55$ with an accuracy of about 30% in the middle and is given by a factor of 2 at the ends of interval. In the limit of these accuracies the theoretically calculated valence quark distributions are in agreement with those found from deep inelastic scattering and other hard processes data. The account of G^3 contribution, probably, improves the agreement, especially at small x.

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References

[1] V.N.Gribov and L.N.Lipatov, Sov.J.Nucl.Phys. **15** (1972) 438.

[2] Yu.L.Dokshitzer, Sov.Phys.JETP, **46** (1977) 641.

[3] G.Altarelli and G.Parisi, Nucl.Phys. **B126** (1977) 298.

[4] M.Glück E.Reya and A.Vogt, Z.Phys. **C53** (1992) 127.

[5] H.L.Lai et al. (CTEQ Collab.) Eur.Phys. J. **C12** (2000) 375.

[6] A.D.Martin, R.G.Roberts, W.J.Stirling and R.S.Thorne, Eur.Phys. J.C.**4** (1998) 463.

[7] M.Glück, E.Reya and A.Vogt, Eur.Phys. J.**C5** (1998) 461.

[8] A.M.Cooper-Sarkar, R.C.E.Davenish and A.De Roeck, Int.J.Mod.Phys. **A13** (1998) 3385.

[9] Wu-Ki Tung , At the Frontier of Particle Physics, in: Handbook of QCD, Boris Ioffe Festschrift, ed. by M.Shifman, World Sci. 2001, v.2, p.887.

[10] B.L.Ioffe and A.G.Oganesian, Phys.Rev. **D63** (2001, 096006.

[11] B.L.Ioffe, Pis'ma Zh.Eksp.Fiz. **42** (1985) 266, [JETP Lett **42** (1985) 327].

[12] V.M.Belyaev and B.L.Ioffe, Nucl.Phys. B310 (1988) 310.

[13] A.S.Gorsky, B.L.Ioffe, A.Yu.Khodjamirian and A.G.Oganesian, Z.Phys. **C44** (1989) 523.

[14] B.L.Ioffe and A.Yu.Khodjamirian, Phys.Rev. **D51** (1995) 3373.

[15] B.L.Ioffe and A.Oganesian, Eur.Phys.J. **C13** (2000) 485.

[16] V.N.Gribov, B.L.Ioffe and I.Ya.Pomeranchuk, Yad.Fiz. **2** (1965) 768. [Sov.J.Nucl.Phys. **2** (1966) 549].

[17] B.L.Ioffe, Phys.Lett. **30 B** (1969) 123.

[18] B.L.Ioffe, Nucl.Phys. **188** (1981) 317.

[19] B.L.Ioffe and A.V.Smilga, Nucl.Phys. **B216** (1983) 373.

[20] S.J.Brodsky et al., Phys.Rev. **D65** (2002) 114025.

[21] V.A.Novikov, M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Phys.Lett. **B86** (1979) 347.

[22] T.Schäfer and E.V.Shuryak, Rev.Mod.Phys. **70** (1998) 323.

[23] B.V.Geshkenbein, B.L.Ioffe and K.N.Zyablyuk, Phys.Rev. **D64** (2001) 093009.

[24] B.L.Ioffe and A.V.Smilga, Nucl.Phys. **B232** (1984) 109.

[25] B.L.Ioffe, Lecture at XXIII Cracow School of Physics, Acta Physica Polonica **B16** (1985) 543.

[26] B.L.Ioffe and K.N.Zyablyuk, Nucl.Phys. **A687** (2001) 437.

[27] B.L.Ioffe, hep-ph/0207 191, Phys.At.Nucl., in press.

[28] M.A.Shifman, A.I.Vainstein, V.I.Zakharov, Nucl.Phys. **147** (1979) 448.

[29] A.Martin et al. ep-ph/0201127.

[30] J.Pumplin et al. hep-ph/0201195.

[31] P.Amandruz et al.Phys.Rev.Lett.**66** (1991) 2712.

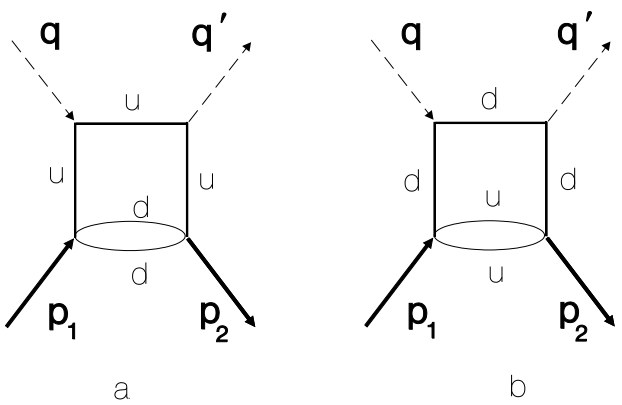
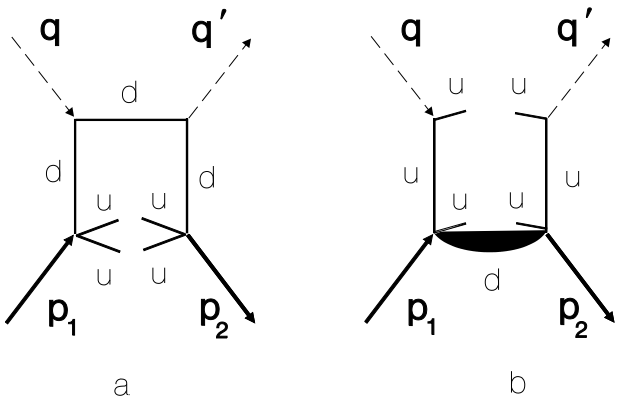


Figure 1: Bare loop diagrams, corresponding to unit operator contribution for u- and d-quarks (respectively, a) and b)).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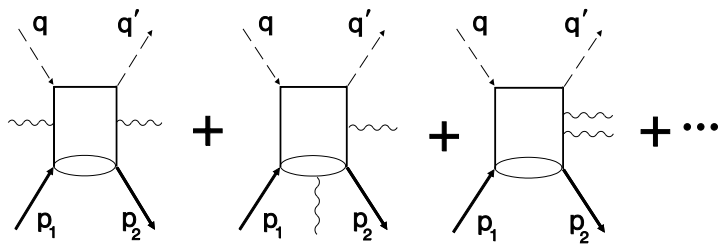

Figure 3: Examples of diagrams for $d = 4$ operator contribution, wavy lines correspond to gluon.

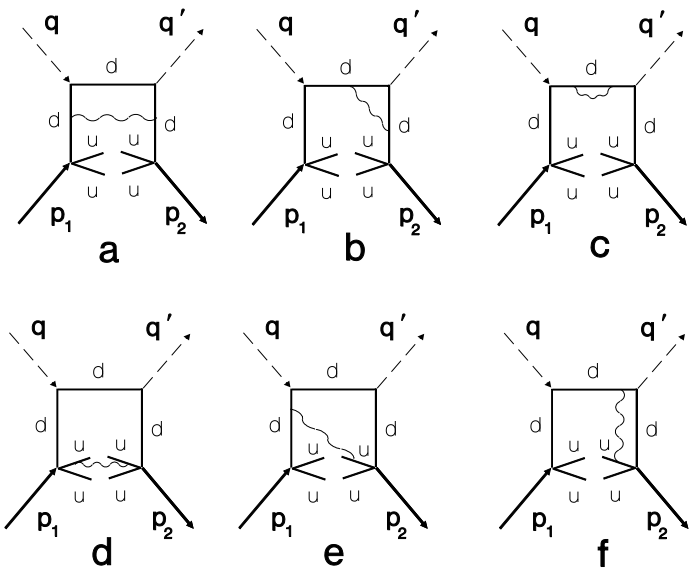

Figure 4: Diagrams for $d = 6$ contribution, which are treated as perturbative correction to non-loop ones and should be omitted. Other notations as in Fig.3. (For diagrams b), d), e), f) symmetrical diagrams are not shown).

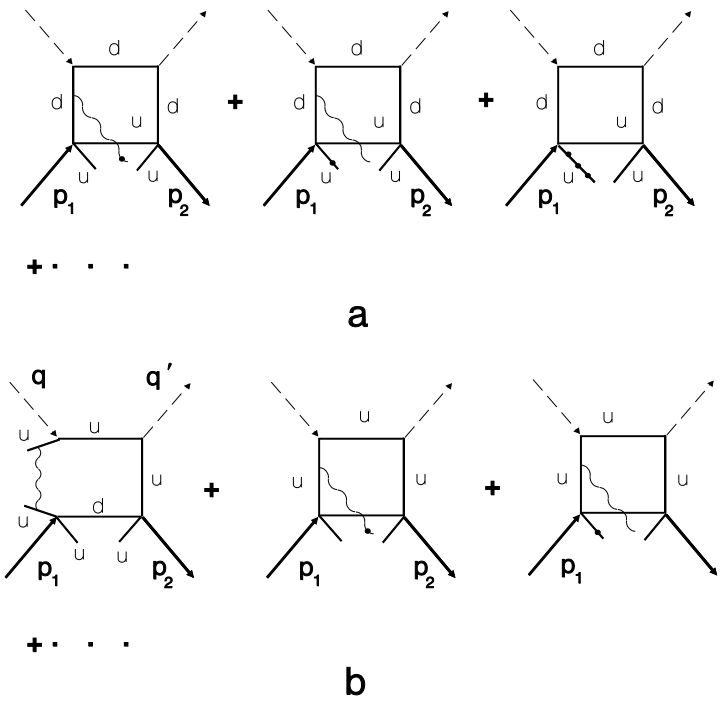

Figure 5: Examples of $d = 6$ diagrams, which are taken into consideration, non-labelled quark propagator means that it can be both of u, d-quarks, other notations – as in Fig.3.

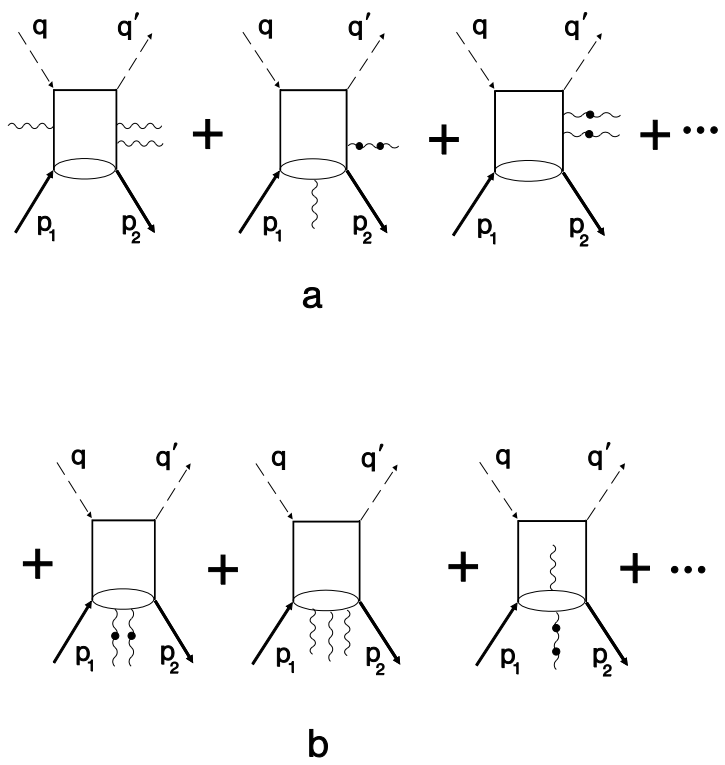

Figure 6: Examples of two-loop diagrams of $(d = 6)$,(a,b). Diagrams b) are examples of infrared divergent diagrams.

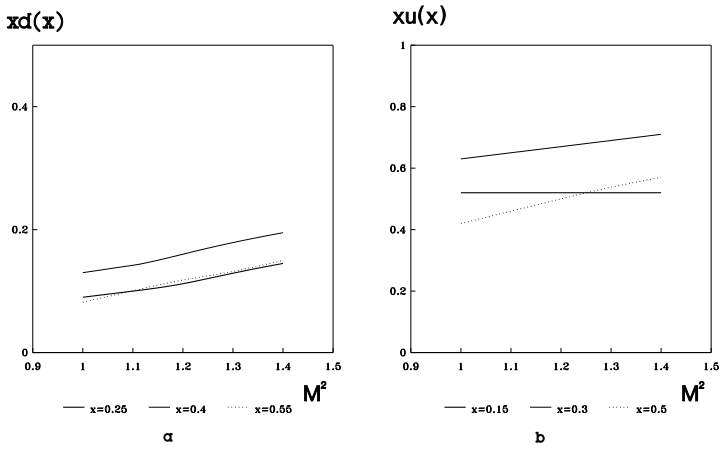

Figure 7: Borel mass dependence of sum rules for u- and d-quark distributions at various x.

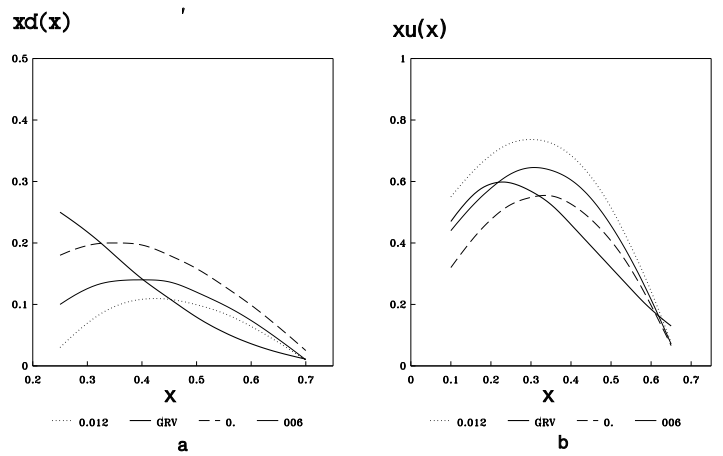

Figure 8: d- and u-quark distribution at various values of gluon condensate ($b = 0.012, 0.06$ and 0 GeV^4, respectively dotted, solid and dashed lines). Thick solid line corresponds to the results of [4].

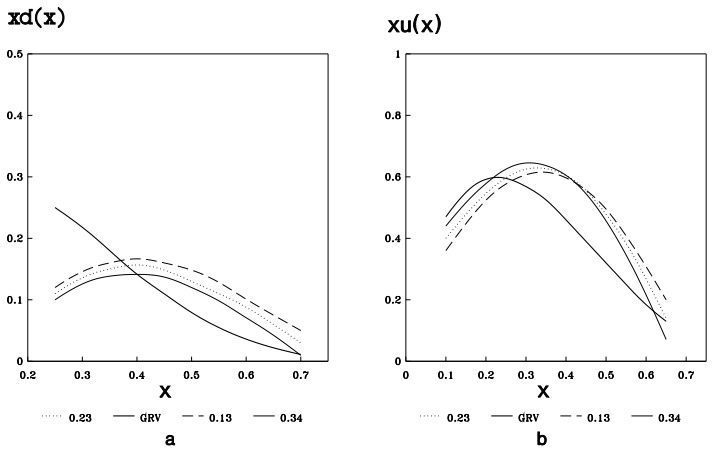

Figure 9: d- and u-quark distribution at various values of quark condensate $\alpha_s a^2 = 0.13,\ 0.23,\ 0.34$ GeV^6 in comparison with the [4] result (the curve denoted as GRV).